Partial Differential Equations
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Lecture – 1.1
Basic Concepts and Nomenclature

Welcome to this course on partial differential equations. In this lecture, we are going to discuss some basic concepts and the nomenclature that is involved in the study of partial differential equations.

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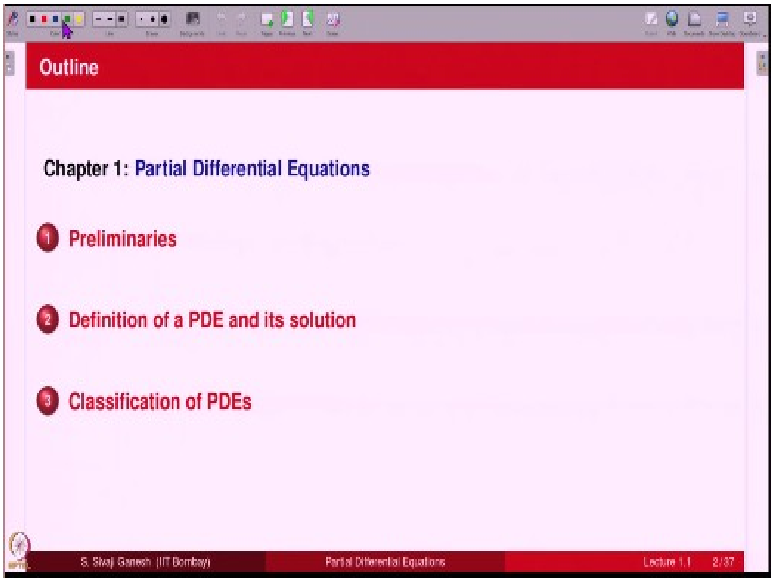

So, the outline of today's lecture is first we discuss certain preliminaries, then we go on to define what is a PDE and what do we mean by solution of the PDE and then we discuss classification of partial differential equations.

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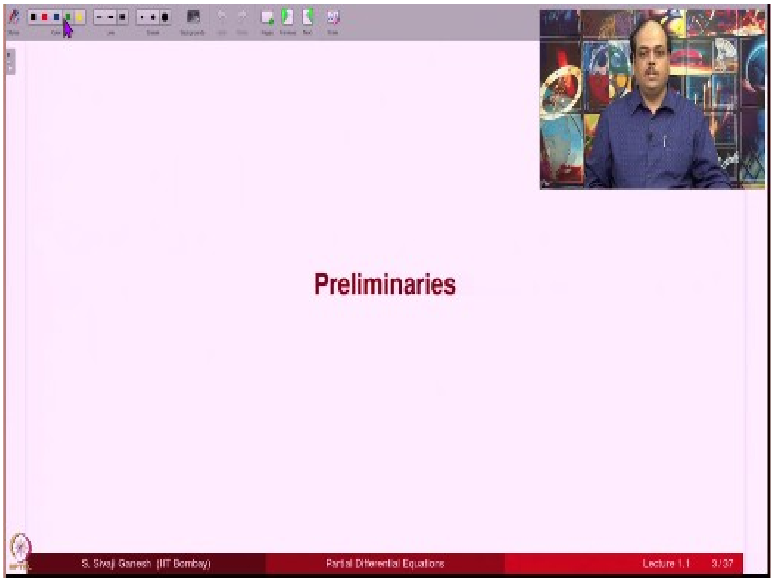

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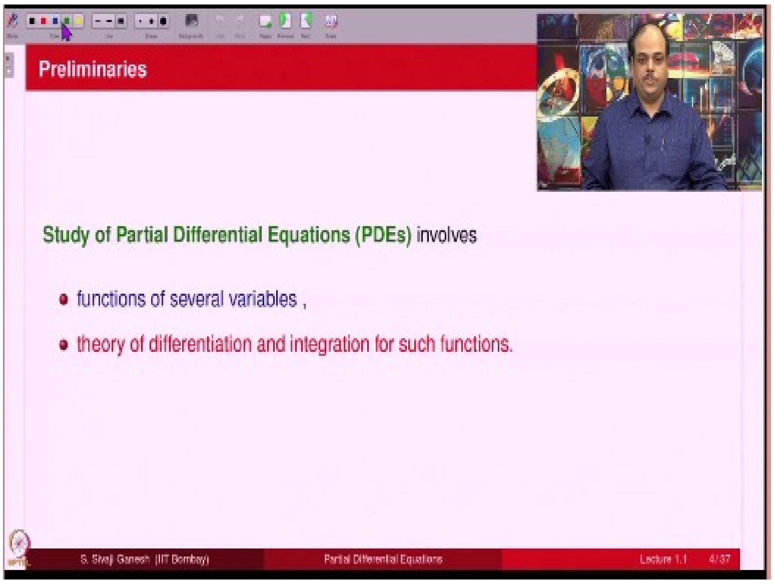

So, a study of partial differential equations involves functions of several variables, theory of differentiation and integration for such functions.

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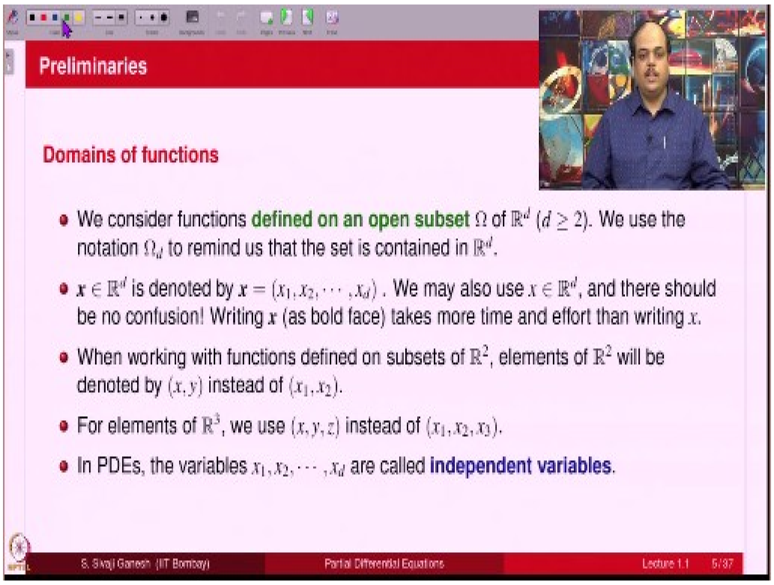

Therefore, let us discuss what are the domains of those functions are going to be. So, we consider functions defined on an open subset omega of $\mathbb{R}^d$. Since, we are studying partial differential equations functions are of several variables, therefore, d is greater than or equal to 2. We use a notation omega d to remind us that the set is contained in $\mathbb{R}^d$ Because later on we are going to see that in a certain problem, we will see many sets one is lying in a higher $\mathbb{R}^d$ higher dimension and one is lying in lower dimension.

Therefore, this will keep track or it will remind us that we are in certain dimension. So, an element of $\mathbb{R}^d$, x is written in boldface. It is not easy for us to write on paper, but in print it is possible. So, the boldface x actually stands for d tuple of real numbers, $x_1, x_2, \cdots, x_d$, we may also use normal x, x in $\mathbb{R}^d$. And there should be no confusion. Because the moment you know we are dealing with functions in $\mathbb{R}^d$.

And you have a function let us say u of x, it means x is a d tuple. So, writing x takes more time and effort than writing a very simple x. So, when working with functions defined on subset of $\mathbb{R}^2$ elements of $\mathbb{R}^2$ will be denoted by x y. That is the standard practice and we will not use x 1, x 2. We stick to the standard notation x y. Similarly, when we are working with the functions defined $\mathbb{R}^3$, we use the notation x y z.

The 3 tuple is denoted as x y z instead of x 1, x 2, x 3. And in the context of PDEs, the variables x 1, x 2, x d, these are called independent variables.

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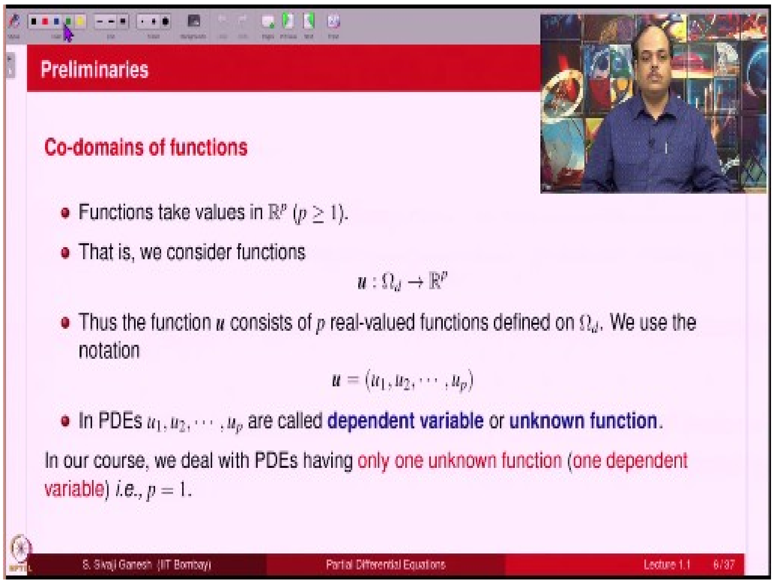

Now, we have discussed the domains of functions, let us discuss the co-domains of the function that we are going to deal with. So, co-domains of functions means the function where the values it takes. So, we assume that the functions take values in $\mathbb{R}^p$, p greater than or equal to 1. That is we consider functions u defined on omega d, I need not tell you that omega d is a subset of rd again because the notation tells us and takes values in $\mathbb{R}^p$.

Thus, the function u consists of p real-valued functions defined on omega d and we use this notation u = u 1, u 2, u p. So, the function u is given in terms of the sometimes what is called coordinate functions u 1, u 2, etc up to p. In partial differential equations u 1, u 2, u p are called dependent variable or unknown functions. In our course, we are going to deal with PDEs which have only one unknown function.

That is only one dependent variable, in other words p = 1. So, are the PDFs that we are going to study in this course are going to be a single equations. There is only one unknown function, not systems. Some more notations.

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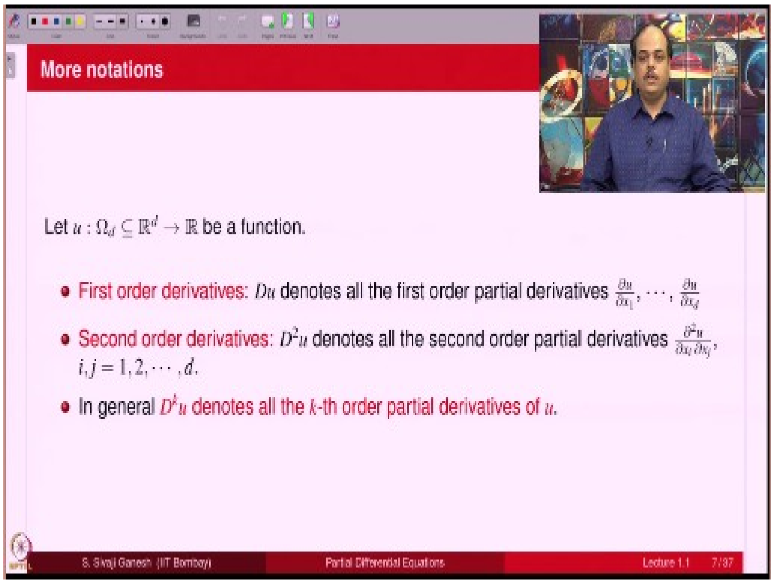

Let u be a function defined on omega d taking real values. Then first order derivatives the notation we use here is D u. It denotes all the first order partial derivatives which are described here, dou u by dou x 1, do u by dou x 2 upto dou u by dou x t. Then the second order derivatives are denoted with D 2 u. This notation D 2 u stands for all second order partial derivatives of the unknown function u which is dou 2 u by dou x i dou x j, i and j vary from 1 to d.

Now, in general D k u for k greater than or equal to 1 denotes all k-th order partial derivatives of u.

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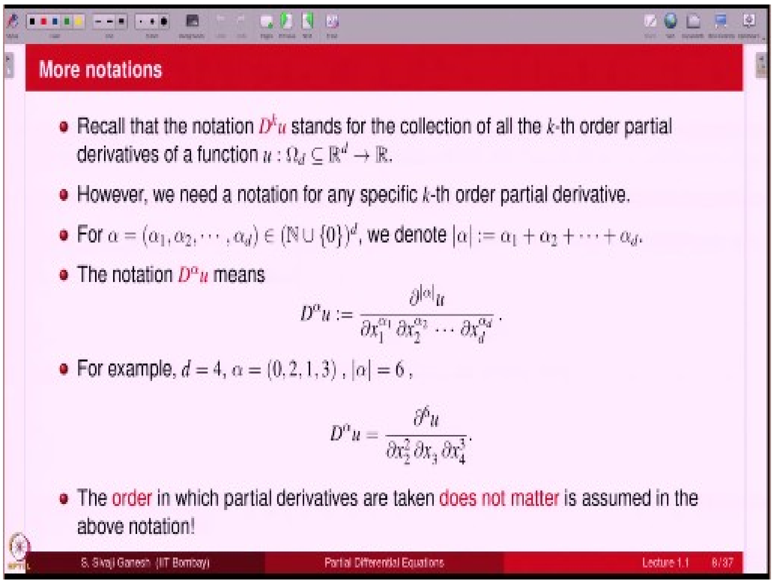

So, recall that the notation D k u stands for the collection of all k-th order partial derivatives. However, we need a notation for any specific k-th order partial derivative. Therefore, we are going to introduce that notation. To do that what we do is let us fix a d tuple of numbers which are either natural numbers or 0. That is alpha is a vector which is equal to alpha 1 up to alpha d.

It is a d tuple. Where does it belong? N union zero power d. That means, each of them is a whole number either a natural number or 0. For such an alpha of d tuple of numbers, whole numbers, we denote mod alpha equal to the sum of alpha 'i's. So, mod alpha equal to alpha 1 + alpha 2 + alpha 3 up to alpha d. So, the notation d alpha u means, I am going to differentiate u with respect to x 1, alpha 1 times, with respect to x 2, alpha 2 times and so on up to with respect to x d, alpha d times.

So, that is the notation here. d alpha u where alpha is not a natural number or a whole number. Alpha is a D tuple of whole numbers. d alpha u is equal to dou mod alpha u divided by dou x 1 alpha 1 dou x 2 alpha 2 up to dou x d alpha d. Let us see an example, assume that d = 4. That means, we are dealing with functions of 4 variables. So, the independent variables are denoted by x 1, x 2, x 3, x 4.

Let us take alpha equal to 0,2,1,3. And we want to differentiate u. This alpha, d alpha u. What is that? It tells us that do not differentiate with respect to x 1, with respect to x 2 differentiate 2 times, with respect to x 3 differentiate 1 time and with respect to x 4 differentiate 3 times. So, sum of the entries of alpha is 6.0 + 2 + 1 + 3 that is 6. That means, what it defines is a particular sixth order partial derivative of u.

So, d alpha u is dou 6 u by dou x 2 square, because the second entry here is 2. That is why second variable differentiate 2 times third, where third entry here is 1. Therefore, differentiate once with respect to the third variable. And similarly, with respect to x 4 variable differentiate 3 times because there is a 3 here. So, there is a room for confusion, because we are using D k u at the top of the slide to stand for collection of all k-th order partial derivatives.

And here also D alpha u, to denote this particular partial derivative, but there should be no reason to be confused. Because this D k u stands make sense only when k is a natural number. This D alpha u makes sense only if alpha is a d tuple of whole numbers. So, there is no room for confusion here. So, the order in which partial derivatives are taken does not matter.

For example, if I want to differentiate with respect to x 3first in the above example, and with respect to x 2 later there is no way to take care of that in this notation. In other words, this notation does not care the order in which you take the differentiation which is not okay. We know that partial derivatives do depend the order in which they are taken, but for reasonable functions they coincide.

So, we are going to consider such functions only where the order in which you differentiate does not matter,. For example, if you are considering a C to function, then it does not matter in which order you differentiate. That is the theorem in multivariable calculus. Maybe that is reason why this notation does not respect the order in which you will differentiate.

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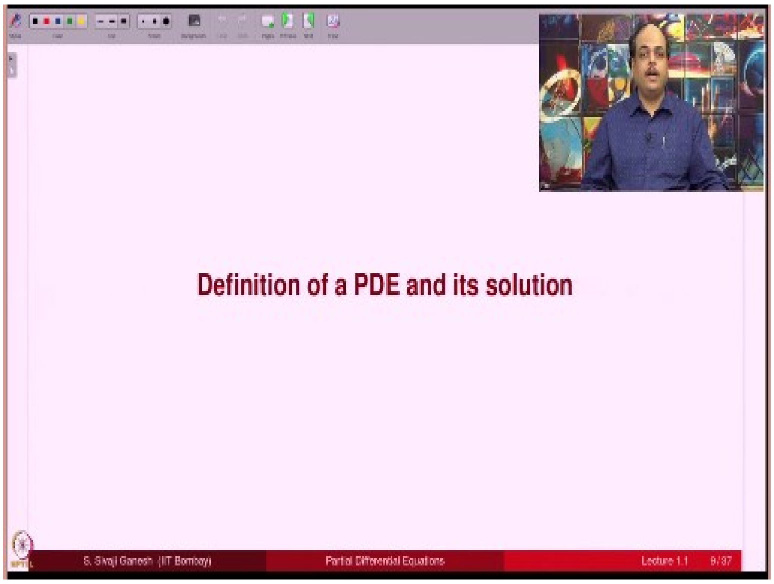

Now, let us go to the definition of a partial differential equation and what do we mean by its solution?

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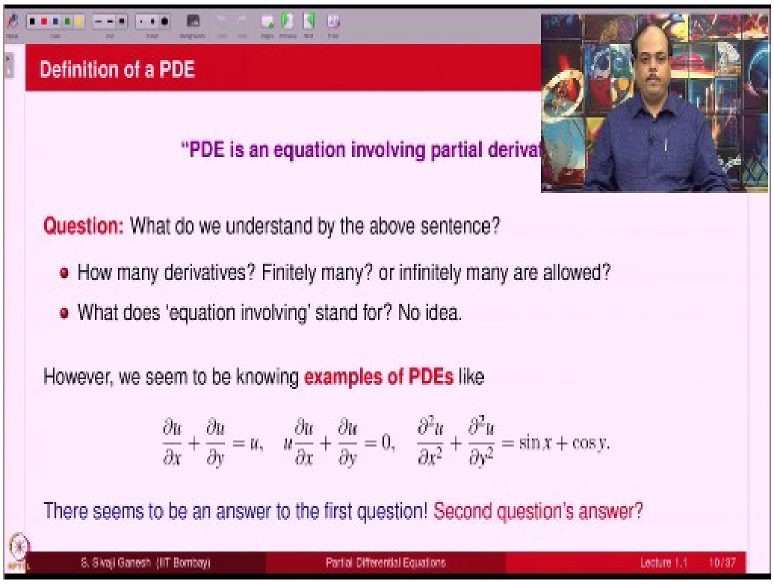

One normally says PDE is an equation involving partial derivatives. This can be accepted as a definition provided we understand the statement. Not only that, everybody understand the statement the same way. Question: What do we understand by the above sentence? We are saying involving partial derivatives. So, question is how many derivatives? Finitely many or infinitely many is also allowed. There is one question there.

Second question is, what does ‘equation involving’ stand for? What does that mean? No idea. However, we seem to be knowing examples of PDEs. Therefore, we do not question this kind of a definition. We found we do not question because we know what are PDEs. We know. So, how do we know? We have examples. And this English statement seems to be okay.

But the problem is this English statement might allow more types of equations, which you do not think is a PDE. We will see one of them as an example. Therefore, these are the examples that we seem to be knowing dou u by dou x + dou u by dou y = u. u dou u by dou x + dou u by dou y = 0. And another equation dou 2 u by dou x square + dou 2 u by dou y square = sin x + cos y and so on.

Many such examples we know and the above sentence PDE is an equation involving partial derivatives seems to be acceptable. Therefore, there seems to be an answer to the first question that means, how many derivatives should be there in the equation? Because, whenever we are writing a example of PDE we are not thinking of writing the equation which involves infinitely many derivatives.

Therefore, we know that a PDE should have only finitely many derivatives appearing in the equation. What about second question's answer? Still no idea. So, we need to capture what we are writing here in terms of some definition, which is very rigid.

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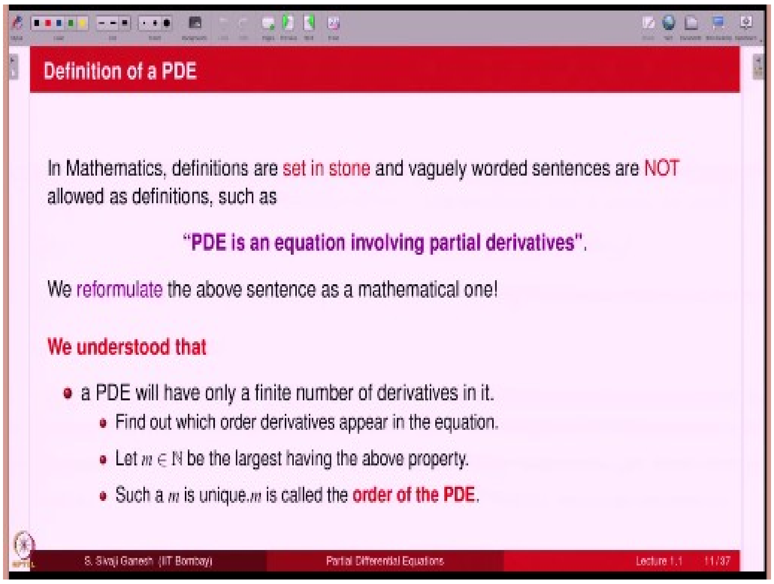

So, in mathematics as we know definitions are set in stones and vaguely worded sentences are not allowed as definitions, such as this. PDE is an equation involving partial derivatives. So, we reformulate the above sentence or rather we give a mathematical meaning of the sentence or a possible mathematical meaning of the above sentence. And once we write down as a definition we stick to that. That is a definition.

So, we understood that PDE will have only finite limited derivatives in it. Therefore, find out the order of the derivatives which appear in the equation. Look at the equation, ask which th derivative appears and let m be the largest having the above property. Being the largest it will be unique and this largest derivative which appears largest order derivative, which appears in this. That particular order is unique and that is called the order of the partial differential equation.

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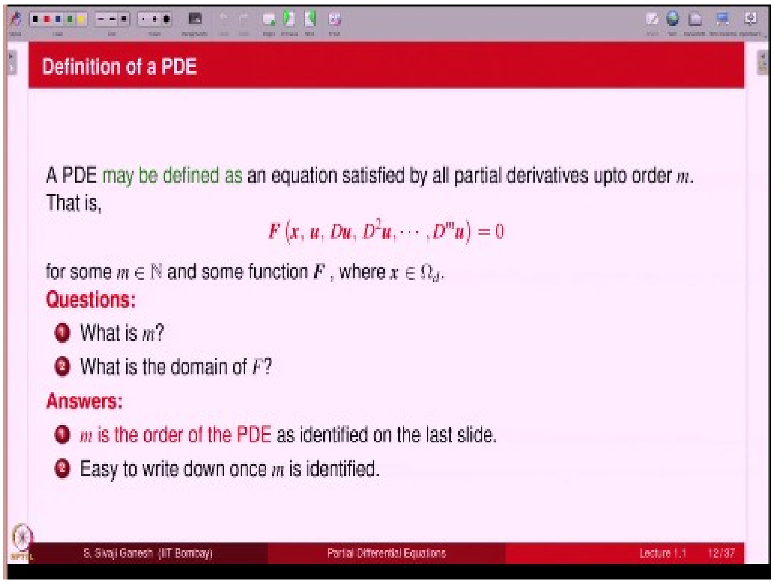

So, now, we need to define. A PDE may be defined as an equation satisfied by all partial derivatives upto order m including the function as well. So, a relation of this type capital F of x, u, D u, $\mathbb{D}^2$u , $\mathbb{D}^m$ u = 0. Recall what is D u collection of all first order derivatives, $\mathbb{D}^2$u collection of all second order derivatives and so on. This is collection of all m-th order derivatives.

So, for some m and some function F, we should be able to cast our equation in this form. Now, where x is in omega d. Questions : What is m? What is the domain of F? So, m is the order of the PDE. So, this is how we plan to define. We can if you have a PDE, which involves only first order, you can also think I can write this equation with m = 2. But such a thing should not be allowed.

Our definition will not allow because this m, we would require that m should be the order of the PDE as identified on the last slide. m is the order of the PDE means some m-th order partial data appears and no higher order partial derivatives appear in the equation. That is why that m is unique. Now, once you know this, it must be easy to write down the domain of F. Once you see the equation it is easy to write.

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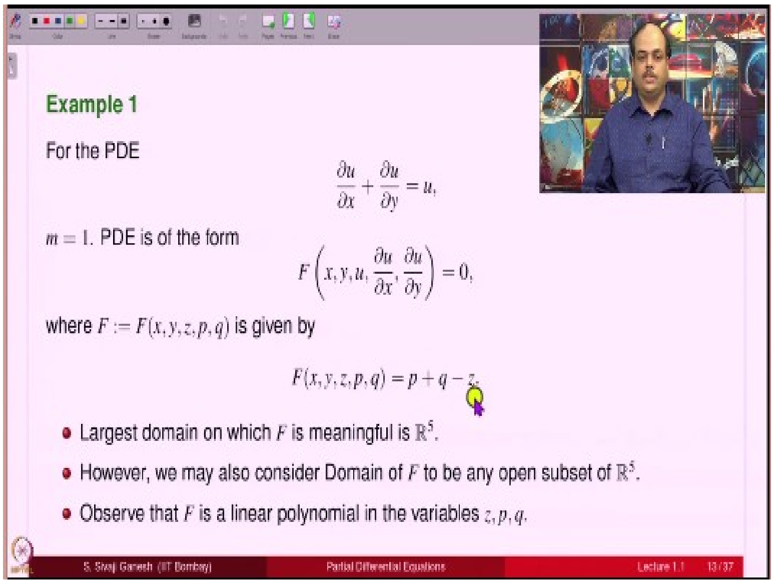

Let us look at some examples. Examples. It is always easy. So, for this PDE dou u by dou x + dou u by dou y equal to u m is 1 that means first order PDE. So, first order PDE means it has to be expressed as x, y, u, d u = 0. d u means all first order partial derivatives which are dou u by dou x and dou u by dou y. So, it should be written in this form. What is F? F should be some function of how many variables? 1,2,3,4,5.

F should be a function of 5 variables. So, this is always a good practice to write any such function like this, F is F of x, y, z, p, q. x, y is independent variables. It means we are working with a PDE in 2 independent variables here. z is the location where you are going to substitute u, p and q are the places where we are going to substitute for dou u by dou x and dou u by dou y respectively.

It is important to stick to this kind of notation, understand this notation. So, what is F of x, y, z, p, q in this example? It is p + q – z. Therefore, if you take F equal to this and consider this L.H.S that will become dou u by dou x + dou u by dou y – u. And hence you get the PDE. Now, largest domain on which this f is meaningful is $\mathbb{R}^5$, because it is defined for every 5 tuple x, y, z, p, q, this function is defined.

However, we may also consider the domain to be any open subset of $\mathbb{R}^5$,. That is also okay. Observe something nice about this F. It is a linear polynomial in this variable p, q and z.

Of course, x y does not appear therefore, it is still, you can say it is a linear polynomial in all the variables.

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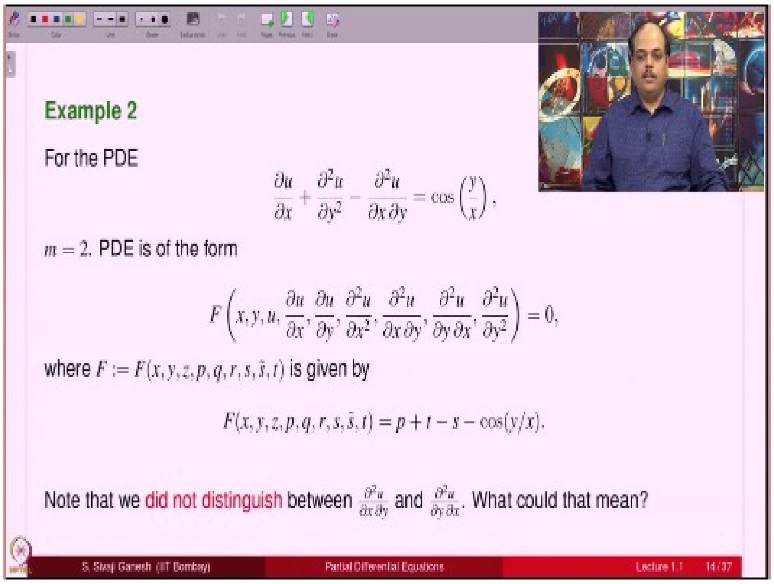

Let us look at a second example. It is dou u by dou x + dou 2 u by dou y square – dou 2 u by dou x dou y = cos of y by x. What is the m here? It is 2, because the second order derivative appears and higher order derivatives other than 2 are not appearing, only 2. 3, 4, 5 etc nothing appears. So, m = 2. Once you are decided m = 2, what should be the F? F should look like this.

x, y, u. This is corresponding to d u that is dou u by dou x and dou u by dou y. And rest corresponds to all second order partial derivatives of u. Therefore, F must be written like this. I told you x, y, z is a place we are going to put u. p, q for the first order derivatives r s, t for the second order derivatives. I have included sand s tilde, just a placeholder for dou 2 u by dou x dou y and dou 2 u by dou y dou x. A

As I mentioned earlier, we are not going to really distinguish between these 2. So, after some time we will not write s tilde, we will simply write F of x, y, z, p, q, r, s, t. s tilde will be forgotten. We will not write that. Because we will, maybe we will consider functions for which both are same. For reasonable class of functions this mixed partial derivatives coincide. And what is the formula for F? One can easily write down.

It is now we will ask what is the domain of F, then there is a problem here, because x = 0 should not be allowed, everything else is allowed. And what is the domain of F? It is how many tuples? A tuple of what size? 2 here, 3, + 2 first order derivatives and 4 second order derivatives. So, 2 + 1 that is 3, + 2, 5, + 4, 9. So, domain of F will be in $\mathbb{R}^9$ but it should not contain x = 0.

Here I have written we did not, what I am saying is we will not distinguish. I told you what that means already.

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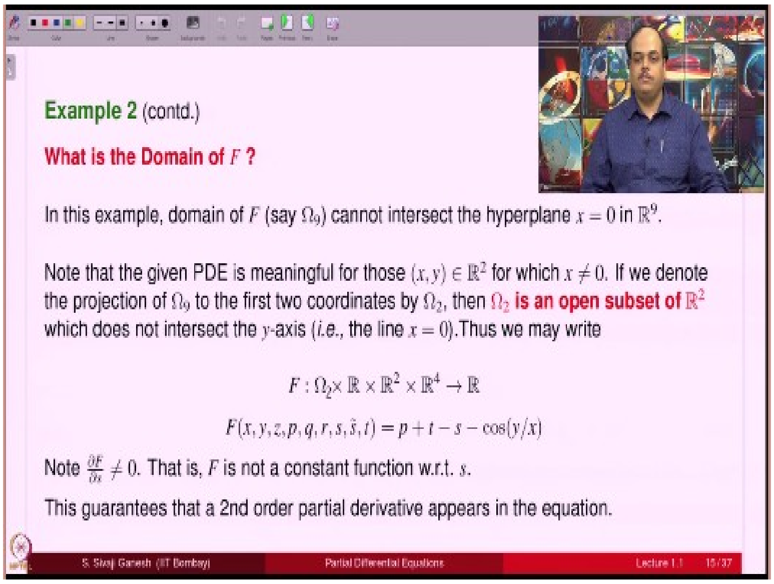

What is the domain of F? So domain of F, let us call omega 9 because it is sitting in $\mathbb{R}^9$ as we have written. Because we have included F is a function of x, y, z, p, q, r s, s tilde, t. So nine variables and x = 0 is not allowed. So, that is what we say not that the given PDE is meaningful for those x, y, z 0 two for which x is not 0. So, if we denote the projection of omega 9 to the first 2 coordinates by omega 2, then omega 2 is an open subset of $\mathbb{R}^2$ which does not intersect the y axis.

Thus, we may write the domain omega 9 to be like this. F is from omega 2 cross R cross $\mathbb{R}^2$ cross $\mathbb{R}^4$. For this omega 2 transfer the independent variables x, y vary here. This R stands for the unknown function u which we do not place any restriction on what values it takes. So, therefore, it can take any real number. $\mathbb{R}^2$ stands for the derivatives. The p, q variables where dou u by dou x and dou u by dou y are going to come, $\mathbb{R}^4$ is all 4, second order partial derivatives.

So, we can further write omega 9 in this form. Now, dou F by dou s is not 0. Infact dou F by dou t is also non 0. It means F is not a constant function in s. It means s variable is used by F. It is not constant. It depends on s. It means it is a second order PDE. This guarantees that some second order partial derivative appears in the equation.

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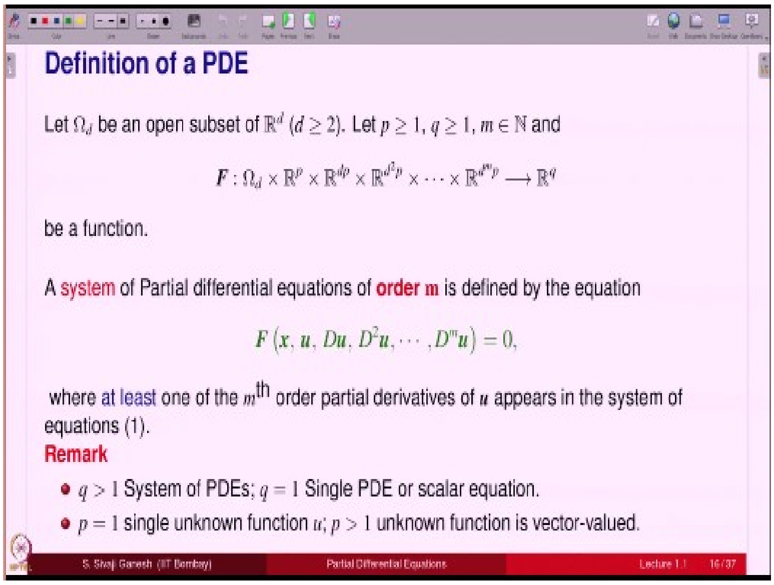

So now we are going to write the definition. Let omega d be an open subset of $\mathbb{R}^d$ and let F be like this omega d cross $\mathbb{R}^P$ and cross $\mathbb{R}^d$ p, d square p and d m p, taking values in R q be a function. So it means F is taking values in $\mathbb{R}^q$ that means there are q functions in this. This omega d is where x is going to sit that means I am considering x in $\mathbb{R}^d$, so d independent variables and this is $\mathbb{R}^P$.

That means the unknown function has p components, u 1, u 2, u p. It is a vector valued unknown function and this corresponds to these derivatives because u is a mapping from omega t omega d to $\mathbb{R}^P$. Therefore, the derivative will be of size d p, d p many R first order derivatives will be there. This will be corresponding to second order derivatives. And this corresponds to m th order partial derivatives of the unknown function.

So, a system of partial differential equations why system? Because we are considering q greater than or equal to 1. So there can be more than one equation. That is why in general we call it a system or partial differential equation, order m, because we have d power m here, is defined by this equation. So D m u are all m-th order partial derivatives, which are so many. I am not discounting the symmetries like earlier I have done I am writing full.

So where at least one of the m-th order partial derivatives of u appears in the system of equations only then we are going to say it is m-th order PDE. So, as I said, if q is bigger than 1, it is actually system of PDEs, q can be equal to 1, then it is a single PDE or scalar equation. And if p equal to 1 is a single unknown function, if p is bigger than 1 unknown function is vector valued.

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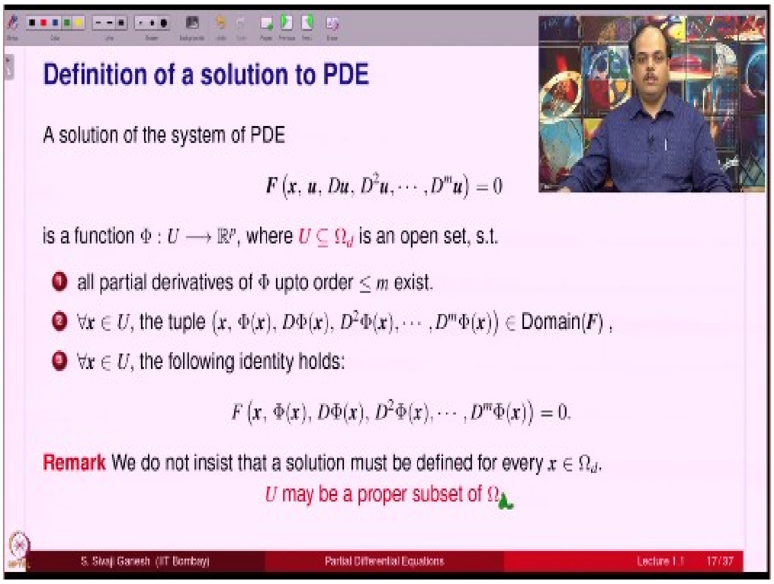

So now we have defined what is the solution of the PDE. A solution of the system of PDE, which is defined here is a function. Solution is a function, when we say a function with some domain, so it is a function phi with domain U. Of course, here we have written system for vector variable, dependent variable is vector valued. Therefore, this phi is also vector valued $\mathbb{R}^p$. With what property? U is a subset of omega d.

What is omega d? omega d is that set where this equation is meaningful for x belongs to omega d, the domain of F was omega d cross something’s. But a solution can be a function which is defined even on a proper subset of omega d. It is a function with some properties,? What should be the properties? All partial derivatives of phi upto order m, it should exist, including m.

Because I am thinking of m-th order equation that for all partial differential exists up to order less than or equal to m. Second, once it exists, look at this tuple. I am going to substitute inside this x, phi x, D phi x, D 2 phi x, D m phi x. Therefore, I would require that this tuple belongs to the domain of F and I have the to apply F to that, when I apply I get 0. That means this equation is satisfied in this way by this function phi.

Note phi, we are demanding that it should be defined on a subset of omega d and we are not demanding that it should be defined on omega d. That is very important to be noted. So, we do not insist that a solution must be defined for every x in omega d. U may be proper subset of omega d, it should be omega d.

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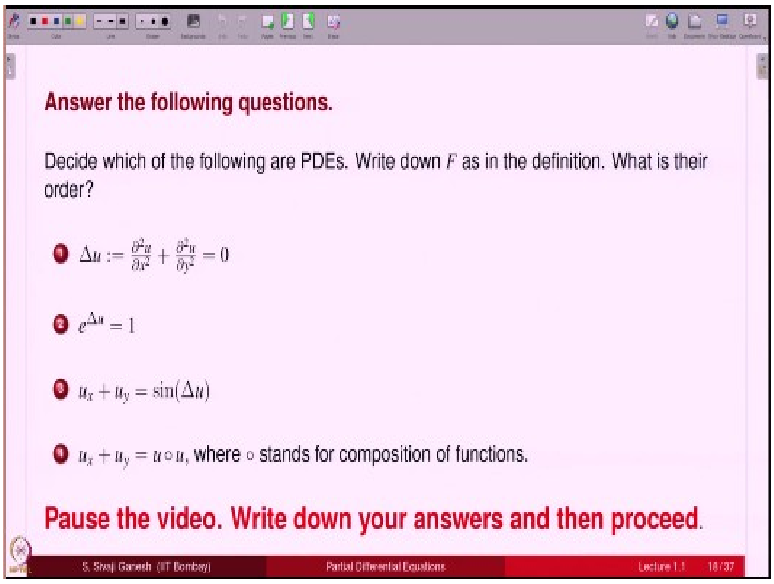

Now, answer the following questions. Decide which of the following are PDES, write down capital F as in the definition. How do you say somebody is a PDE? You have to go to the definition of the PDE that involves a capital F. It comes with this domain. You should be able to give such a function F and then this becomes a PDE.

So Laplacian u the first one is dou 2 u by dou x square + dou 2 u by dou y square = 0. Next is e power Laplacian u = 1. Laplacian u is defined above and this is u x plus u y = sine of Laplacian u. This is u x plus u i value composed with u. So, this circle stands for the composition of functions. Now at this point you must stop the video, write down, you think about the answers.

Write down answers, if you have and then proceed. Because answer is going to come on the next slide.

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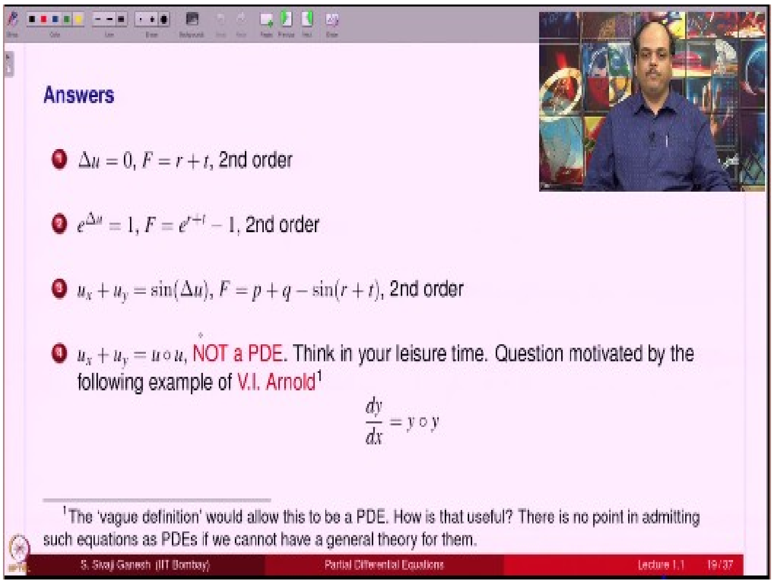

What are the answers? The F is r + t is a second order PDE. e power Laplacian u = 1. F = e power r + t minus one, second order. u x + u y sine of Laplacian u, F= p + q minus sin of r + t, second order and this one is not a PDE. Think in your leisure time. If you do not get this answer, do not worry, keep thinking about this. Question motivated by the following example of Arnauld ODEs, d y by d x = y circle y. He says it is not an ODE.

So, the vague definition would allow this as a PDE because it just says it should involve unknown function and its derivatives, which this equation does involve, u x + u y = u circle u. So, there is no point in admitting such equations, if you cannot have a general theory for some special class of such equations. So, maybe that is the reason why Arnauld called this as not as a ODE. So, if you understand the answer to this, the answer of this will be immediate.

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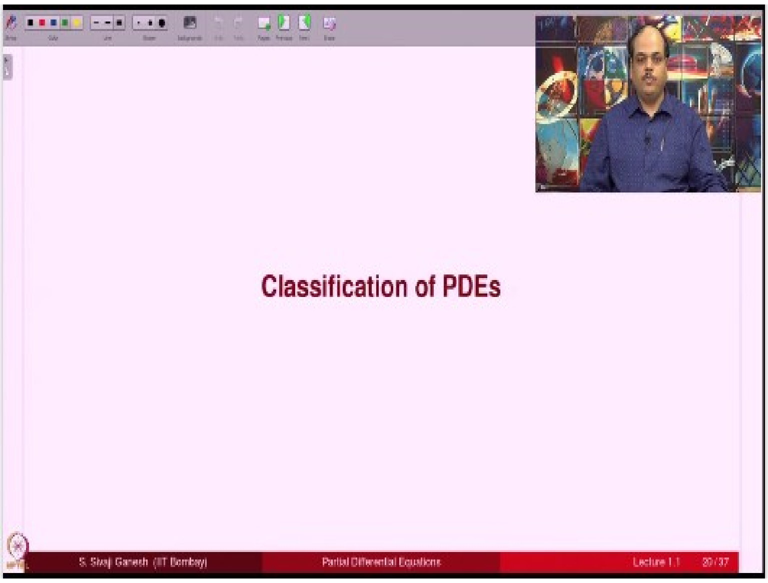

Now, we move on to classification of PDEs.

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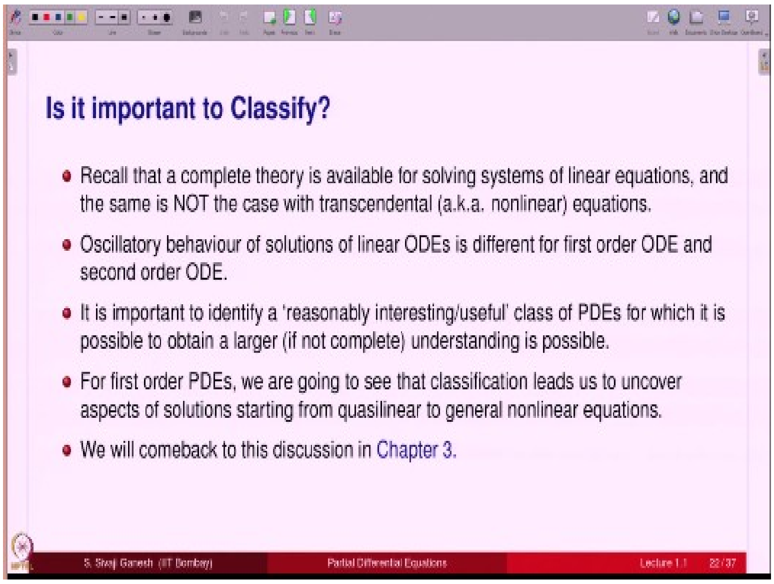

So, what is classification? Classification of PDEs means creating Bags with Labels and associate each PDE to exactly one of the bags. Examples of a few methods of classification, we are going to see each one of them, based on the number of equations, based on the order of PDEs, based on algebra, I will mention this details when we come there. So, question is, is it important to classify?

Recall that a complete theory is available for solving systems of linear equations ax = b and the same is not the case with transcendental equations, which are also known as nonlinear equations. You have to hold each and every equation and analyse for solutions. Now, coming to linear ODEs there is a oscillatory behaviour of solutions. That is different for first order ODE and second order ODE.

Look at simple equation like y dash = y and y double dash + y = 0. I write y double dash equal to y. Think of the solutions, some of them are oscillatory. So, it changes. So, it is important to identify a reasonably interesting or useful Class of PDEs for which it is possible to obtain a larger understanding. Possible is possible, may not be complete, but at least a larger understanding, good understanding.

So, for first order PDEs we are going to see that classification leads us to uncover certain aspects or solutions starting from the quasilinear to the general nonlinear equations. We will see that in the next chapter. So, we will come back to this discussion on classification later on. Also in chapter 3, where we are going to classify second order PDEs, we asked the same question. And we have some better answers there, we will see that.

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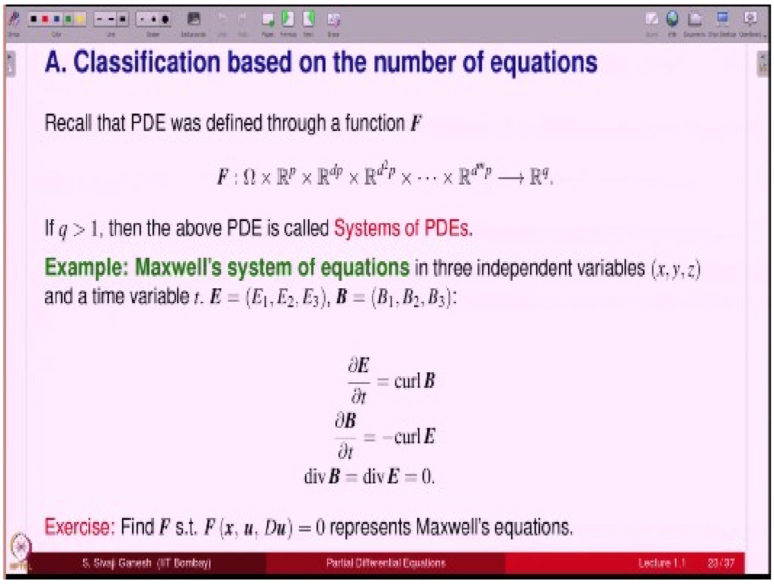

So, now a classification based on the number of equations, recall that the PDE is defined to be a function of this type? If q is bigger than one, it is called system. If q is equal to one, it is called a scalar. So, this is an example is a Maxwell system of equations in 3 independent variables x, y, z and a time variable. That means there are 4 independent variables here. And E is a 3 vector E 1, E 2, E 3, 3 functions.

B is 3 functions B 1, B 2, B 3 and dou E by dou t = curl B, dou B by dou t = minus curl E. Divergence of B and divergence of E are 0. So, this is indeed a PDE. You must write in this setup. And that is left as an exercise to you.

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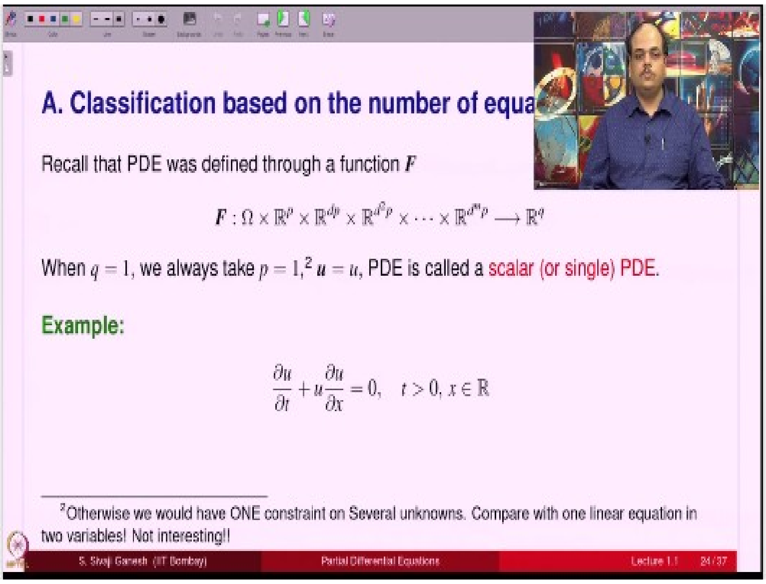

I had told you when q = 1 that called a scalar PDE, but when q = 1, we also take p = 1. That means the unknown function also is a single unknown function. For this otherwise, what happens is that you have one equation because you set q =1. If you have more than one unknown, it means that information is I mean conditions are very less. For example, just recall what happens with 1 linear equation 2 variables. Not so much interesting?

I mean, x – y = 1 infinitely many solutions that is one of the aspects. So, this is an example of a single equation. It is very efficient. An example we will be studying in depth later on.

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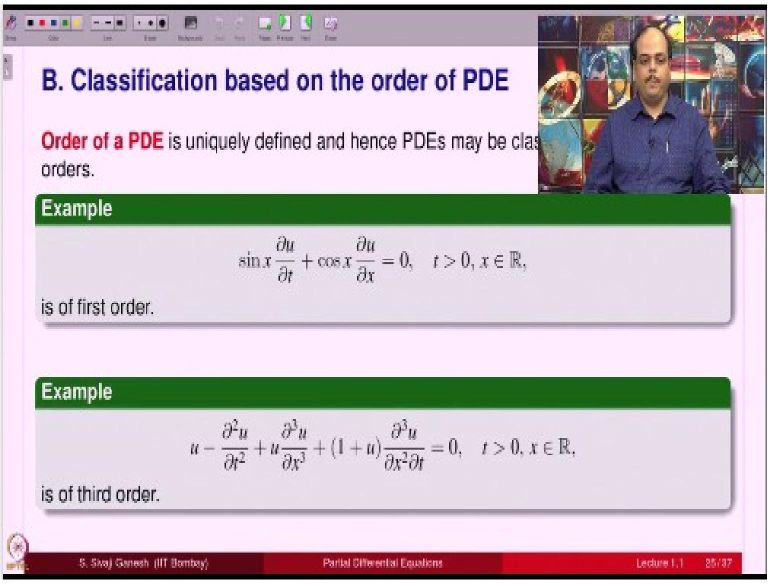

Now, second thing is the classification based on the order of the PDE. So order of PDE as we know is uniquely defined and hence they may be classified based on their orders. This is first order PDE. This is a third order PDE.

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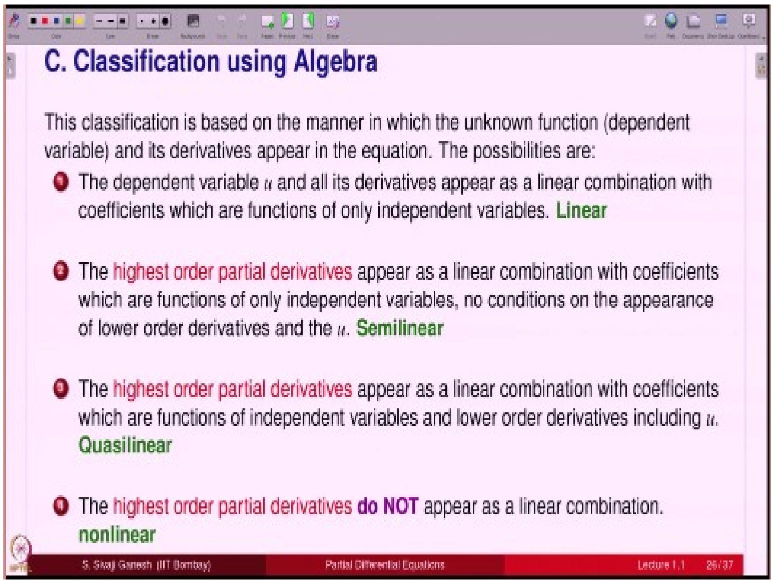

Now, this is the important classification which uses algebra. So, this classification is based on the manner in which the unknown function and its derivatives appear in the equation. What are the possibilities? First possibility: dependent variable and all its derivatives whichever appear in the equation they all appear as a linear combination with coefficients which are functions of only independent variables.

That equation which has this feature will be called linear equations. Second option is highest order partial derivatives appear as a linear combination with coefficients which are functions of only independent variables. And no conditions on the appearance of lower order derivatives and the function u. This may be called semi linear equations. Now highest order partial derivatives appear as a linear combination.

But now, the coefficients depend on independent variables and also on a lower order derivatives including u. So, that will be called quasilinear equations. The highest order partial derivatives do not appear as a linear combination at all. So, such equations will be called nonlinear equations.

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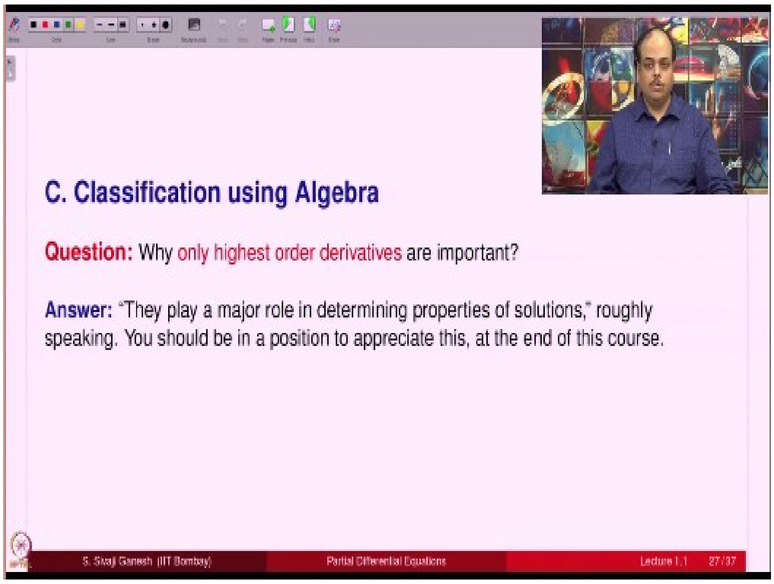

Now, you may ask this question why only highest order derivatives are important for the last 3 classifications. They play a major role in determining properties or functions is a vague answer., you should be in a position to appreciate this at the end of this course.

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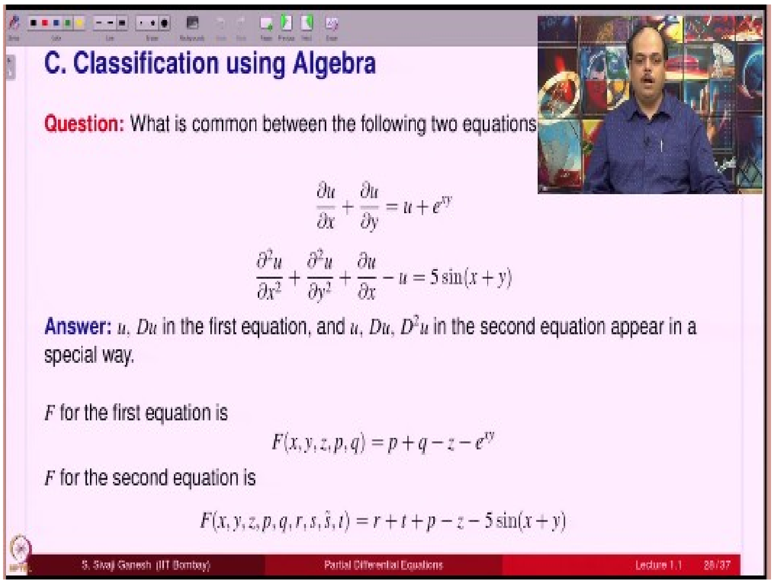

Now, let us look at these equations, these 2 equations, one is the first order PDE and this is the second order PDE. What is common between them is that u and D u in the first equation u D u and D 2 u in the second equation, they appear linearly,. It is a linear combination of those derivatives and the function. x and y we do not ask any questions. The classification is based on z, p, q variables. In this for the second order PDE it will be z, p, q, r, s, s tilde, t.

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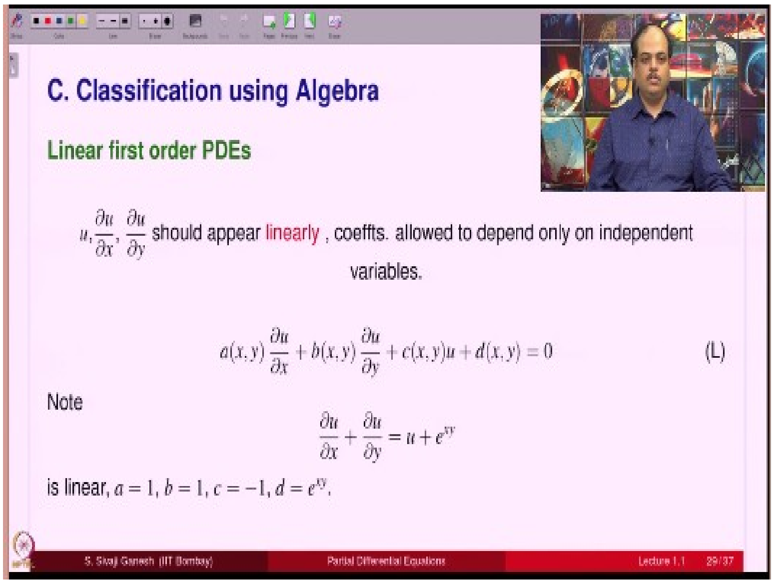

So, we will see very specific examples for first order PDEs, the classifications and we will see most general possible equations that we can write of that type. So, linear first order PDE means the unknown function and the first order derivative should appear linearly. That is, coefficients are allowed to depend only on independent variables. So, they appear like this a x y dou u by dou x + b x y dou u by dou y + c x y u + d x y = 0. This is an example.

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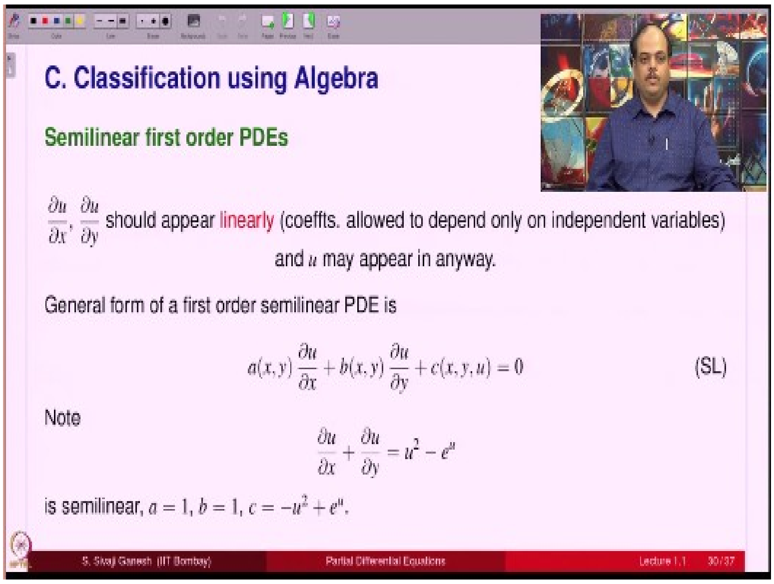

Semi linear. Here what we ask is dou u by dou x dou u by dou y should appear linearly. Coefficients are allowed to depend only on the independent variables. And u may appear in any way, but not in the coefficients of dou u by dou x and dou u by dou y. So, therefore, the most general possible semi linear equation of first order is like this. This is an example.

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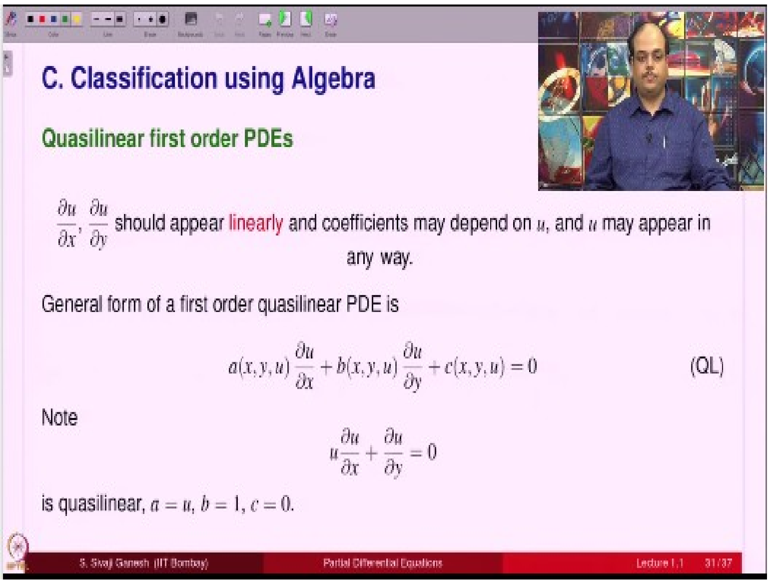

Now, comes the quasilinear first order PDEs. Here the first order derivatives dou u by dou x and dou u by dou y should appear linearly. Fine. Coefficients are now allowed to depend on the independent variables as well as the lowest lower order derivatives. If this is the first order derivative, what is the lower derivative? That is function itself. They are allowed. So, therefore, this is the general possible quasilinear first order PDE. This is an example.

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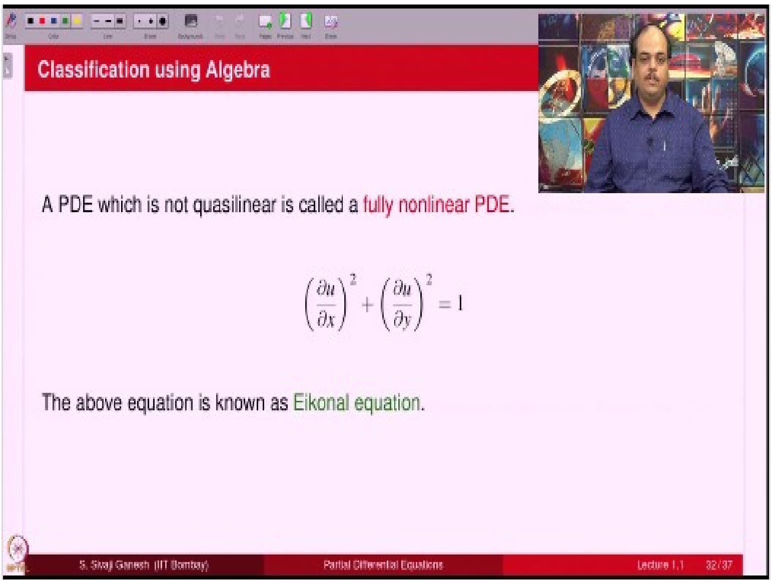

Now, a PDE which is not quasilinear is called a fully nonlinear PDE. So which is basically the complement of quasilinear equations is an example. Very important example. It is called Eikonal equation.

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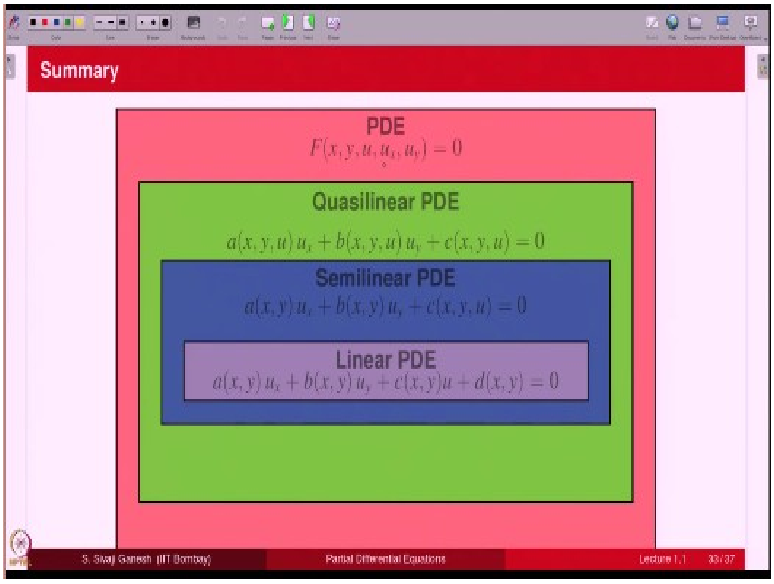

Now, this is the picture, a Venn diagram. First is linear. This is a formula how the general linear PDE looks like. Linear is also semi linear. There is a bigger class. Bigger class is quasilinear out of quasilinear will be full PDEs. This colour red colour, but not in green will be fully nonlinear. Otherwise, these are all general. First order PDE will look like this. That was a definition.

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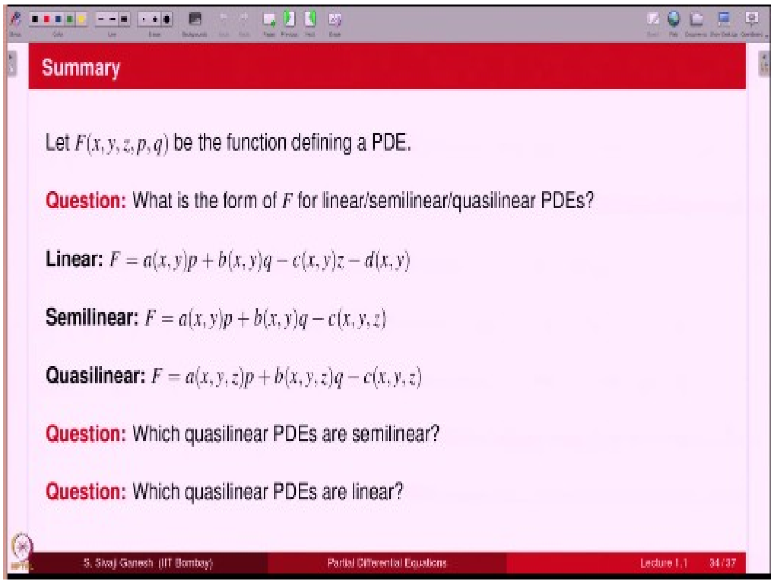

Now, for a linear PDE first order PDE, F will be of this type. F of x, y, z, p, q will be like this. This is for semi linear and this is for quasilinear. Now, we can ask the questions like which quasilinear PDEs are semilinear. When this a does not depend on z, b does not depend on z causing linear equation becomes semilinear. Similarly, there is another question. Please answer yourself.

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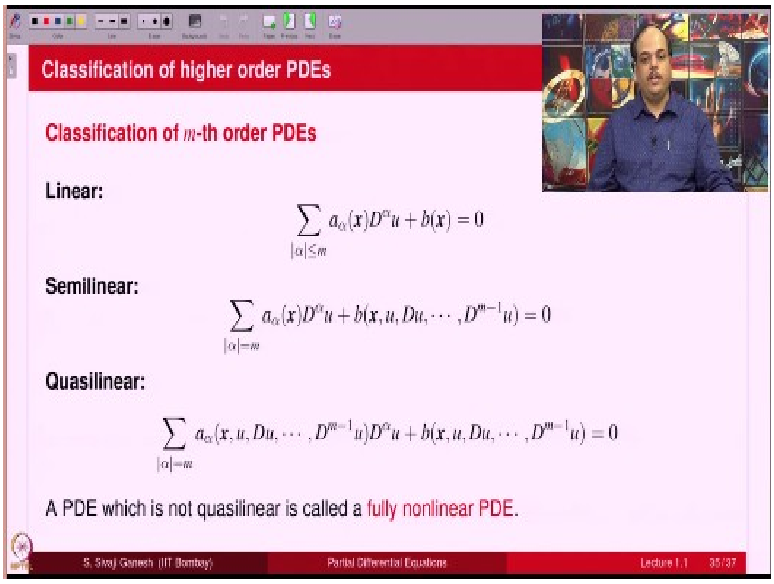

Now, we can classify m-th order PDEs. We have explicitly seen first order PDEs classification. But same thing can be done for m-th order PDEs. What are the guiding principles? The 1, 2, 3, 4 that we have written down. How unknown functional derivatives appears? If all of them appear as a linear combination, it is called linear PDE. If that is not the case, we straightaway go for highest order derivative and ask how they appear.

Then we had 2, 3, 4. So, we will write down here once again. Linear, see here, mod alpha less than equal to m. So, when our alpha equal to 0 0 0 that is m = 0, the tuple alpha is 0 0 0 that means, no derivatives which means, it is a function. So, here it says function as well as the derivatives have to appear as a linear combination, coefficient depend only on the independent variables.

Semilinear. Once again highest order partial derivative that is m-th order appear linear combination, coefficients are functions of independent variables. Rest however, it appears no restriction. Quasilinear once again is on the highest order derivatives coefficients are allowed to depend on apart from independent variable x, u, D u upto D m – 1. That is m – 1 order derivatives, then this is called quasilinear.

Outside that it can appear in any way they want, but as coefficients are the highest order partial derivatives, they can only be like this. Now, PDE which is not quasi linear is called fully nonlinear PDE.

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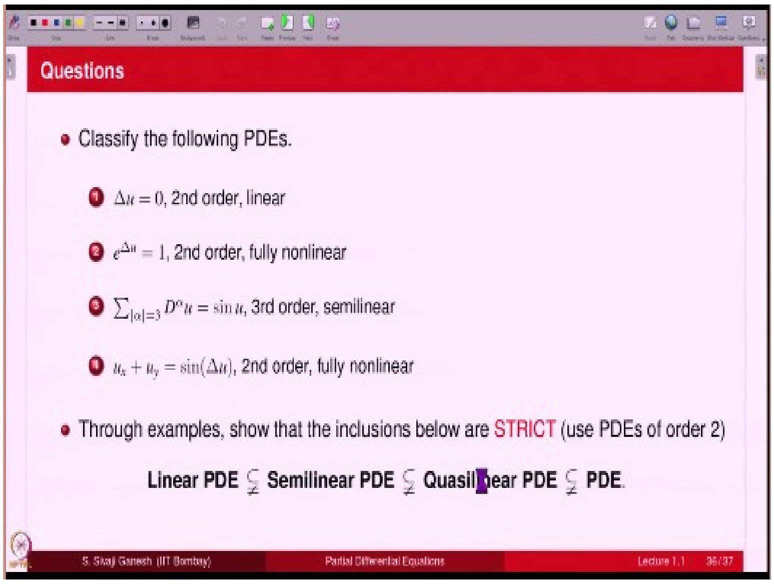

So, Laplacian u. It is second order and linear. e power Laplacian u = 1, it is second order fully nonlinear. And this equation third order, semilinear. Of course, you may also say quasilinear, but when I say semilinear it has more information, because semilinear, a much much smaller set. This equation second order fully nonlinear. So, this is an exercise for you 2 examples. Show that the following inclusions are very strict, strict inclusions.

That means find a semilinear PDE which is not linear, find a quasilinear which is not semilinear. I am sure that there are PDEs which are not quasilinear. That means give an example which is not causing it. These are the questions for you and thank you.