

Introduction to Algebraic Topology (Part-II)
Prof. Anant R. Shastri
Department of Mathematics
Indian Institute of Technology-Bombay

Lecture-41
Some Applications of Homology

(Refer Slide Time: 00:12)



So, today we will start a new chapter namely applications of homology. Here, we will do a few popular applications of homology. Some of these results one can prove in different methods also. And some of them you have already seen in part I, without using the homology theory. Ok? So, having said that, the idea of proving these things is to emphasize how homology theory actually helps in many problem solving in topology especially in algebraic topology.

So, we begin with the famous Brouwer's fixed point theorem, then go on to prove the Lefschetz's fixed point theorem which is an improvement on Brouwer's theorem. And then we prove some popular results such as hairy ball theorem, Jordan-Brouwer separation theorem, Jordan-Brouwer invariance of domain etc. So, these are all very, very popular results. When these things were proved, algebraic topology itself became extremely popular because of such results. Ok?

(Refer Slide Time: 01:44)

All Questions
 Comments and Feedback
 Download Slides
 Other Homology groups
 Abstract
 Examples
 Summary
 Michael St. John, Department of Mathematics
 University of Toronto, and I have recently received
 Michael St. John, Department of Mathematics
 University of Toronto

Lemma 4.4
 Let A be a retract of X . Then $H_*(A)$ is a retract of $H_*(X)$, i.e.,
 $H_*(A)$ is a direct summand of $H_*(X)$.

Proof: Let $r : X \rightarrow A$ be a retraction. Then $r_* : H_*(X) \rightarrow H_*(A)$
 is a retraction, by functoriality, viz., $r_* \circ i_* = (r \circ i)_* = (Id_A)_* = Id_{H_*(A)}$.
 Here $i : A \rightarrow X$ denotes the inclusion map.

Home > Search > Related Courses > Faculty Department of Mathematics > Michael St. John, Department of Mathematics > Algebraic Topology, Part II > HFTS, Lecture

So, we begin with a lemma which has reminiscent of certain things that we have done in part I using the fundamental group. Instead, we now use the homology groups, ok? Let A be a retract of X , then $H_*(A)$ will be a retract of $H_*(X)$, that is, $H_*(A)$ is a direct summand of $H_*(X)$, which is somewhat stronger, Ok? For topological spaces A and X , A is a subspace of X , A is a retract of X does not yield such decomposition. All that you have is a continuous function from X to A which is identity on A , ok? r is the continuous function from X to A , i is inclusion map of A into X , start with i and then follow by r that is the identity of A . This is the meaning of saying that A is a retract of X .

When you pass to the homology, it will tell you that $H_*(A)$ is a retract of $H_*(X)$. That means there is a homomorphism from $H_*(X)$ to $H_*(A)$, etc., where to look for it? It is nothing but r_* . r itself induces a homomorphism and that will have this property $r_* \circ i_* = Id_{H_*(A)}$. Ok? So, r from X to A is a retraction, then r_* from $H_*(X)$ to $H_*(A)$ is a retraction. $r_* \circ i_* = Id_{H_*(A)}$. So, that is the meaning of saying that r_* is the retraction. And whenever you have retraction of abelian groups, the subgroup $H_*(A)$ becomes a direct summand, ok? This is a general theory for abelian groups. Ok?

(Refer Slide Time: 03:50)

As a corollary we have: For any $n \geq 0$, S^n is not a retract of \mathbb{D}^{n+1} .

Proof: We have seen that, $H_n(S^n)$ is 'non trivial' and hence cannot be a subgroup of the 'trivial' group $H_n(\mathbb{D}^{n+1})$.

As a corollary we have: For any $n \geq 0$, S^n is not a retract of \mathbb{D}^{n+1} . So, this sounds like a negative result but this is a very positive result. ok? Remember, in part I, this was a stepping stone in the proof of Brouwer-fixed point theorem using simplicial approximation and Sperner lemma. So, here the same thing comes very easily. No need for a deep result like Sperner lemma and so on. But of course we have to use whatever homology theory that we have done. Namely, the simplest thing that we have done was the computation of $H_n(S^n)$ and $H_n(\mathbb{D}^{n+1})$. We know that $H_n(\mathbb{D}^{n+1}) = 0$ because the disc is contractible, and $H_n(S^n)$ is infinite cyclic. Ok? If S^n were a retract of \mathbb{D}^{n+1} , then $H_n(S^n)$ would have been a subgroup of $H_n(\mathbb{D}^{n+1})$, which is 0. But the subgroup is not 0. That contradiction proves the corollary.

So, observe that the functoriality of the homology is strongly used here in this theorem. See from the hypothesis that $r \circ i$ is identity, in topology we have got the corresponding statement that $r_* \circ i_*$ is identity in the group theory, in the homology, Ok?

(Refer Slide Time: 05:38)

Remark 4.12

Recall that, in Part-I, we have proved the above result in a different way using simplicial approximation and the Sperner lemma. We have also seen that the above result is actually equivalent to Brouwer's fixed point theorem. The present proof is obviously shorter though it uses the big machine of homology.

I have already told you, but I will repeat it. In part I we have proved the above result in a different way using simplicial approximation and Sperner lemma. The present proof is obviously shorter though it uses big machine of homology. Ok?

(Refer Slide Time: 06:05)

Lefschetz-Fixed-Point Theorem

Our next application is a generalization of this result on any compact polyhedron. Of course, now we shall have some hypothesis on the map itself. Recall that, given an endomorphism of a finite type graded module $\tau : C \rightarrow C$, we define the Lefschetz number $L(\tau)$ to be the alternate sum of the traces (see Exercise 3.1).

Also recall that, if τ_* denotes the homomorphism induced on the homology groups, then $L(\tau) = L(\tau_*)$. For any continuous map $f : X \rightarrow X$ on a space X that has finitely generated homology groups, we now define $L(f) = L(f_*)$. The well-known Lefschetz fixed point theorem reads as follows:

We have also seen that the above corollary is actually equivalent to Brouwer's fixed point theorem. So, we shall not bother about proving Brouwer's fixed point theorem again, once you have got this corollary, ok? You have got then of Brouwer's fixed point theorem. Namely, any continuous map from \mathbb{D}^{n+1} to \mathbb{D}^{n+1} has a fixed point. Ok? So, I am not going to discuss this one anymore, because it was done very thoroughly in part I.

So, our next application is a generalization of this BFT is for any any compact polyhedron. Since \mathbb{D}^n is a compact polyhedron, it will be applicable to \mathbb{D}^n also. Now we shall have some hypothesis on the map itself. Ok? On the polyhedron, only compactness is the assumption. On the map we have some additional assumption.

So, I am just recalling a result. Recall that if τ_* denote the homomorphism induced on the homology groups of the chain complex C , then $L(\tau) = L(\tau_*)$, ok? So, this Lefschetz number will be used now in the statement of Lefschetz fixed point theorem. For any continuous map f from X to X on a space X that has finitely generate homology groups, we now define $L(f) = L(f_*)$. The well known Lefschetz fixed point theorem will be the following. Ok?

FIXED POINT THEOREMS READS AS: TOWARDS:

Let X be a compact polyhedron, which means what? There is a finite simplicial complex K , $|K|$ is homeomorphic to X . ok? (So, presently, do not worry about the simplicial structure here, just take the underlying topological space X . Let f from X to X be a continuous function. If $L(f)$ is not 0, then f must have a fixed point.

In Brouwer's fixed point theorem, there was no assumption on f except that f is continuous, whereas X itself was very special viz., just the closed the unit disc. Any thing homeomorphic to a disc will also do ok? But here this X is any the underlying $|K|$ where K is arbitrary finite simplicial complex. The extra assumption was on the continuous function. So, how do you derive Brouwer's fixed point theorem from Lefschetz's?

So, if X is \mathbb{D}^n , ok, any map f from X to X is homotopic to the identity map, this $L(f)$ is a homotopy invariant, because it depends only on f_* on homology groups. Therefore, $L(f)$ will be equal to $L(Id_X)$. Now L of the identity map is easy to compute. In fact, on a path connected space X , we have proved that for any continuous map f from X to X , f_* from $H_0(X)$ to $H_0(X)$ is the identity map. Since $H_0(\mathbb{D}^n)$ is infinite cyclic, it follows that trace of $f_* = 1 = L(f)$. So, the hypothesis for LFT viz., that $L(f)$ not zero, is satisfied. That gives an easy proof of Brouwer's fixed point theorem from this theorem. Ok?

(Refer Slide Time: 10:51)

Remark 4.13

Notice that, since X is a compact polyhedron, its homology is finitely generated and so $L(f) = L(f_*)$ makes sense. Secondly, if $X = \mathbb{D}^n$, then f is homotopic to the identity map of \mathbb{D}^n . Since $L(f)$ is a homotopy invariant, it follows that $L(f) = L(Id)$. But $L(Id)$ is nothing but the Euler characteristic of the space \mathbb{D}^n . Therefore, $L(Id) = 1$ since \mathbb{D}^n is contractible. Thus the requirement of the theorem is satisfied. So, in conclusion, we can say that f has a fixed point. This shows that Lefschetz fixed point theorem is a generalization of Brouwer's fixed point theorem. Indeed, we have just derived a stronger version of BFT, viz., any self-map of a compact contractible polyhedron has a fixed point.

So, let us go to the proof of this itself now. So, this is what I have said here, I will repeat it. If X is \mathbb{D}^n then f is homotopic to identity map of \mathbb{D}^n , because \mathbb{D}^n is contractible. Any two functions into \mathbb{D}^n are homotopic. Since $L(f)$ is a homotopy invariant, it follows that $L(f)$ is $L(Id)$. But L of identity map is nothing but Euler characteristic \mathbb{D}^n which is equal to 1, because \mathbb{D}^n is contractible. Ok? Thus the requirement of the theorem is satisfied. So, in conclusion, we can say that f has a fixed point. This shows that Lefschetz fixed point theorem is a generalization of Brouwer's fixed point theorem.

Indeed, we have just derived a stronger version of Brouwer's fixed point theorem, namely any self map of a compact contractible polyhedron has a fixed point. We never used that X is actually homeomorphic to \mathbb{D}^n , only contractibility was used. Ok So, this is another intermediary result, you may note down which follows from Lefschetz fixed point theorem. So, let us see some more results also. But before that let us try to complete a proof of this.

(Refer Slide Time: 12:21)

Other Homology groups
Homotopy and Homology
Thomson's theorem

Module 21: Some Applications of Homology
Homotopy and Homology

Proof: Given a compact polyhedron X and a continuous self-map $f: X \rightarrow X$ with no fixed points, we shall show that $L(f) = 0$.
Fixing some linear metric on X , we can find $\epsilon > 0$ such that,
 $d(x, f(x)) > \epsilon, \forall x \in X$. (Why?)
Let K be a simplicial complex that triangulates X and such that

$$\text{mesh } K = \max \{ \text{diam } F : F \in K \} < \epsilon/3.$$

[For instance, we can start with any simplicial complex that triangulates X and then take K to be the (sufficiently often) iterated barycentric subdivision of it to make its mesh as small as we please. (This is immediate from the one of the lemmas proved in Part-I.)]

Asian & Trans-Pacific Economic Forum Department of Mathematics, University of Hyderabad, Hyderabad, India

Left navigation
Contents and Resources
Homology Groups
Other Homology groups
Homotopy and Homology

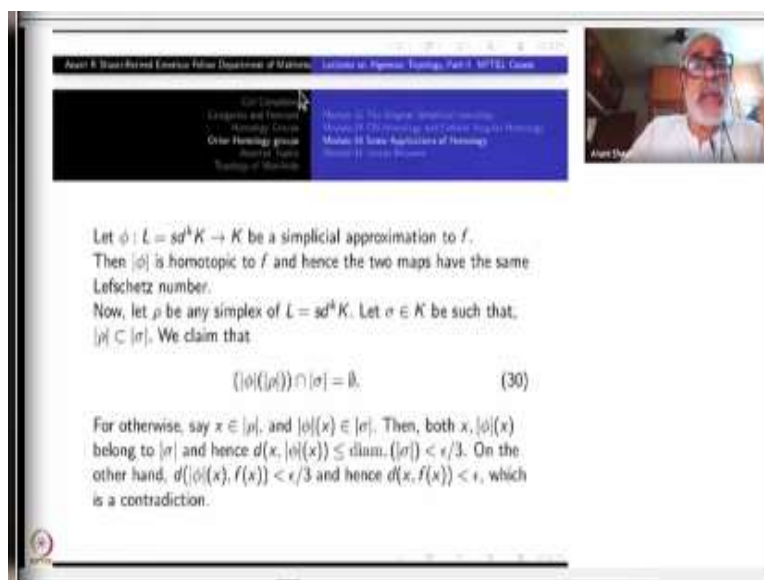
Module 21: Some Applications of Homology
Homotopy and Homology

Given a compact polyhedron and a continuous self map, suppose it has no fixed points. Then we want to show that $L(f)$ is 0. Now this being a compact polyhedron, viz., we have a finite simplicial complex K such that $|K| = X$. We have seen that there is always a linear metric, ok, you can choose any linear metric and find an ϵ such that distance between x and $f(x)$ is bigger than this ϵ for every x belonging to X . Alright, why this is true?

You have to do some topology here. On a compact metric space, the distance between x and $f(x)$ is a continuous function, ok? A real valued continuous function, positive. So, it attains its minimum and that means that minimum will be strictly positive. Take that positive number to be epsilon. Then everything else will be bigger than that. ok?

Now, I will fix a finite simplicial structure on X , I do not care what it actually is but that it is a finite triangulation on X . Now, X is nothing but $|K|$, Ok? Next I can choose a subdivision K' of K , (by repeating barycentric subdivision as many times as required, for instance) so that the mesh of K' which is the by definition, maximum of the diameters of all the simplexes, is less than epsilon by 3. Ok. We have done this in part I. All that you have to do is to keep taking barycentric subdivisions. Each time the diameter becomes r times original one where $r < 1$. So, if you repeat it several times then it will be less than given number. Ok

(Refer Slide Time: 15:21)



Let $\phi : L = sd^k K \rightarrow K$ be a simplicial approximation to f .
Then $|\phi|$ is homotopic to f and hence the two maps have the same Lefschetz number.
Now, let σ be any simplex of $L = sd^k K$. Let $\tau \in K$ be such that $|\sigma| \subset |\tau|$. We claim that

$$L(|\phi|(|\sigma|)) \cap |\tau| = \emptyset. \quad (30)$$

For otherwise, say $x \in |\sigma|$, and $|\phi|(x) \in |\tau|$. Then, both $x, |\phi|(x)$ belong to $|\sigma|$ and hence $d(x, |\phi|(x)) \leq \text{diam}(|\sigma|) < \epsilon/3$. On the other hand, $d(|\phi|(x), f(x)) < \epsilon/3$ and hence $d(x, f(x)) < \epsilon$, which is a contradiction.

Replacing K' by K , we can as well assume that mesh of K itself is less than $\epsilon/3$. Now by simplicial approximation theorem applied to the continuous function f from $|K|$ to $|K|$, we get a integer k and a simplicial map ϕ from $sd^k(K)$ to K to the function f . That means, in particular that $|\phi|$ is homotopic to f . Therefore, $L(|\phi|)$ is the same thing as $L(f)$. You want to show that $L(f)$ is 0. We can do that by showing that $L(|\phi|)$ is 0. Ok? So, that is what we are trying to do now. How do we use this information all this information? Why all this was done?

So, the first thing we claim is the following. Take ρ to be any simplex of $L := sd^k(K)$. Then there is a σ belonging to K such that $|\rho|$ is contained inside $|\sigma|$. That is a property of subdivisions. The claim is that $|\phi|(|\rho|) \cap |\sigma|$ is empty. For this we have to use the facts that ϕ is a simplicial approximation to f , f has no fixed point, ϵ is chosen in a particular way and then the simplicial complex is chosen in a particular way viz, mesh of K is smaller than $\epsilon/3$, etc. Since ρ was inside σ , where as points of ρ will be carried away from σ . This is a strong claim that we want to prove now. ok?

Suppose this is not true, that could mean that there is a point x in the intersection. That means x in $|\rho|$ and such that $|\phi|(x)$ belongs to $|\sigma|$. Ok? Then both x and $|\phi|(x)$ will be inside $|\sigma|$. Now the diameter of $|\sigma|$ is less than $\epsilon/3$. Therefore distance between x and $|\phi|(x)$ will be less than $\epsilon/3$. On the other hand ϕ is a simplicial approximation to f . So, $f(x)$ is $|\phi|(x)$ will be the same same simplex of K . Since the mesh of K is less than $\epsilon/3$, it follows that distance between $f(x)$ and $|\phi|(x)$ is less than $\epsilon/3$. By triangle inequality it follows that the distance between x and $f(x)$ will be less than $2\epsilon/3$ which is a contradiction to the choice of ϵ , being the minimum of such distances. In particular, it follows that the simplicial map ϕ has the property that ρ and $\phi(\rho)$ are disjoint for every simplex ρ in $sd^k(K)$.

(Refer Slide Time: 20:06)



Header: Ramanathan Sundararaman, Department of Mathematics, University of Hyderabad, Hyderabad, India. Course: Algebraic Topology, Part II, SPT20, Online.

Left Column:

- 1.10. Lecture
- 1.11. Lecture
- 1.12. Lecture
- 1.13. Lecture
- 1.14. Lecture
- 1.15. Lecture
- 1.16. Lecture
- 1.17. Lecture
- 1.18. Lecture
- 1.19. Lecture
- 1.20. Lecture
- 1.21. Lecture
- 1.22. Lecture
- 1.23. Lecture
- 1.24. Lecture
- 1.25. Lecture
- 1.26. Lecture
- 1.27. Lecture
- 1.28. Lecture
- 1.29. Lecture
- 1.30. Lecture
- 1.31. Lecture
- 1.32. Lecture
- 1.33. Lecture
- 1.34. Lecture
- 1.35. Lecture
- 1.36. Lecture
- 1.37. Lecture
- 1.38. Lecture
- 1.39. Lecture
- 1.40. Lecture
- 1.41. Lecture
- 1.42. Lecture
- 1.43. Lecture
- 1.44. Lecture
- 1.45. Lecture
- 1.46. Lecture
- 1.47. Lecture
- 1.48. Lecture
- 1.49. Lecture
- 1.50. Lecture
- 1.51. Lecture
- 1.52. Lecture
- 1.53. Lecture
- 1.54. Lecture
- 1.55. Lecture
- 1.56. Lecture
- 1.57. Lecture
- 1.58. Lecture
- 1.59. Lecture
- 1.60. Lecture
- 1.61. Lecture
- 1.62. Lecture
- 1.63. Lecture
- 1.64. Lecture
- 1.65. Lecture
- 1.66. Lecture
- 1.67. Lecture
- 1.68. Lecture
- 1.69. Lecture
- 1.70. Lecture
- 1.71. Lecture
- 1.72. Lecture
- 1.73. Lecture
- 1.74. Lecture
- 1.75. Lecture
- 1.76. Lecture
- 1.77. Lecture
- 1.78. Lecture
- 1.79. Lecture
- 1.80. Lecture
- 1.81. Lecture
- 1.82. Lecture
- 1.83. Lecture
- 1.84. Lecture
- 1.85. Lecture
- 1.86. Lecture
- 1.87. Lecture
- 1.88. Lecture
- 1.89. Lecture
- 1.90. Lecture
- 1.91. Lecture
- 1.92. Lecture
- 1.93. Lecture
- 1.94. Lecture
- 1.95. Lecture
- 1.96. Lecture
- 1.97. Lecture
- 1.98. Lecture
- 1.99. Lecture
- 1.100. Lecture

Right Column:

- 1.101. Lecture
- 1.102. Lecture
- 1.103. Lecture
- 1.104. Lecture
- 1.105. Lecture
- 1.106. Lecture
- 1.107. Lecture
- 1.108. Lecture
- 1.109. Lecture
- 1.110. Lecture
- 1.111. Lecture
- 1.112. Lecture
- 1.113. Lecture
- 1.114. Lecture
- 1.115. Lecture
- 1.116. Lecture
- 1.117. Lecture
- 1.118. Lecture
- 1.119. Lecture
- 1.120. Lecture
- 1.121. Lecture
- 1.122. Lecture
- 1.123. Lecture
- 1.124. Lecture
- 1.125. Lecture
- 1.126. Lecture
- 1.127. Lecture
- 1.128. Lecture
- 1.129. Lecture
- 1.130. Lecture
- 1.131. Lecture
- 1.132. Lecture
- 1.133. Lecture
- 1.134. Lecture
- 1.135. Lecture
- 1.136. Lecture
- 1.137. Lecture
- 1.138. Lecture
- 1.139. Lecture
- 1.140. Lecture
- 1.141. Lecture
- 1.142. Lecture
- 1.143. Lecture
- 1.144. Lecture
- 1.145. Lecture
- 1.146. Lecture
- 1.147. Lecture
- 1.148. Lecture
- 1.149. Lecture
- 1.150. Lecture
- 1.151. Lecture
- 1.152. Lecture
- 1.153. Lecture
- 1.154. Lecture
- 1.155. Lecture
- 1.156. Lecture
- 1.157. Lecture
- 1.158. Lecture
- 1.159. Lecture
- 1.160. Lecture
- 1.161. Lecture
- 1.162. Lecture
- 1.163. Lecture
- 1.164. Lecture
- 1.165. Lecture
- 1.166. Lecture
- 1.167. Lecture
- 1.168. Lecture
- 1.169. Lecture
- 1.170. Lecture
- 1.171. Lecture
- 1.172. Lecture
- 1.173. Lecture
- 1.174. Lecture
- 1.175. Lecture
- 1.176. Lecture
- 1.177. Lecture
- 1.178. Lecture
- 1.179. Lecture
- 1.180. Lecture
- 1.181. Lecture
- 1.182. Lecture
- 1.183. Lecture
- 1.184. Lecture
- 1.185. Lecture
- 1.186. Lecture
- 1.187. Lecture
- 1.188. Lecture
- 1.189. Lecture
- 1.190. Lecture
- 1.191. Lecture
- 1.192. Lecture
- 1.193. Lecture
- 1.194. Lecture
- 1.195. Lecture
- 1.196. Lecture
- 1.197. Lecture
- 1.198. Lecture
- 1.199. Lecture
- 1.200. Lecture

Our task has been complicated by the fact that, while obtaining the simplicial approximation ϕ , we had to subdivide only the domain, and so ϕ almost never has the same domain and range. This difficulty is overcome at least in two different ways:

Now the only problem is that when you take the simplicial approximation, you have taken a subdivision of K and then the domain complex and codomain are different. Thus the chain

complexes of the domain and the codomain are different. So, there is no concept of trace of a morphism. Trace is defined only for an endomorphism of the module to itself and not for an arbitrary homomorphism. So, that is the difficulty here and it has to be overcome somehow and there are different methods of doing that. So, I will give you two methods here which I like, some other methods I do not like and I even doubt them.

(Refer Slide Time: 21:10)

Method I:

We can use the CW-chain complex associated with K . Since ϕ is a simplicial map $L \rightarrow K$, it follows that $|\phi|$ is a cellular map of the CW-complex K . Let $\alpha_n : H_n(K^{(n)}, K^{(n-1)}) \rightarrow H_n(K^{(n)}, K^{(n-1)})$ be the homomorphism induced by $|\phi|$. If $\tau \in K$ is an oriented n -simplex, it follows from (30), that $|\phi|(|\tau|) \cap |\tau| = \emptyset$. Therefore, the coefficient of τ in $\alpha_n(\tau)$ will be zero. It follows that the trace of the matrix representing α_n is zero. Therefore $L(|\phi|) = 0$.

So, in the first method I use the CW-structure on X associated with the simplicial complex K on both domain and codomain. Then it follows that $|\phi|$ from X to X is a cellular map. Any k -cell will be mapped into the union of k -cells by $|\phi|$ since it is simplicial from $sd^k(K)$ to K . Therefore we can now pass of the morphism induced by $|\phi|$ on $C_n^{CW}(X) = H_n(K^{(n)}, K^{(n-1)})$.

Let this map be denoted by α_n from $H_n(K^{(n)}, K^{(n-1)})$ to itself. So, I have the domain and codomain are same here. If an n -simplex τ is a generator, (this is the meaning of oriented n -simplex, ok), it follows from the above property of $|\phi|$ that $\tau \cap |\phi|(\tau)$ is empty, right? That $\alpha_n(\tau)$ is a finite sum of certain oriented n -simplexes the support of none of which will intersect $|\tau|$. But this will mean that coefficient of τ in the $\sum \alpha_n(\tau) = 0$.

It follows that if you write down the matrix for α_n using the set of n -simplexes in K as the basis, the diagonal entries of this matrix will be all 0. That means the trace of α_n is 0. Therefore trace of

α_n is 0. This is true for all n right? Therefore, the alternating sums will be also 0. That means $L(|\phi|) = 0$. That is what we wanted to prove.

(Refer Slide Time: 25:53)

Method II

Consider the subdivision chain map $Sd : C(K) \rightarrow C(sd K)$, as defined on page no. 512. We have seen that Sd induces identity isomorphism on homology. By repeated application of this, we know that $Sd^k = Sd \circ \dots \circ Sd$ (k -copies) induces identity isomorphism on homology. Therefore,

$L(f) = L(|\phi|) = L(Sd^k \circ |\phi|)$. We can compute the Lefschetz number of $|Sd^k \circ \phi| : |sd^k K| \rightarrow |sd^k K|$ at the chain level using the chain complex $C(sd^k K)$ with the basis elements from the simplexes of $sd^k K$.

The second method is slightly more elaborate, but it teaches you something about the subdivision chain map also. So, how to use subdivision map? Ok. The property of subdivision chain map, whatever, we are going to use here, will be useful elsewhere also. So, here take the subdivision chain map Sd , which you have defined from $C(K)$ to $C(sd(K))$. Little sd is the barycentric subdivision, whereas Sd is the subdivision chain map. Ok.

We have seen that this Sd induces identity homomorphism in the homology. That is the beauty. You can forget about this C . and go to $H_*(|K|)$ to $H_*(|K|)$, it is identity morphism there. By repeated application of this, take compositions of Sd with itself k times, it follows that Sd^k from $C(K)$ to $C(sd^k(K))$ induces the identity map on the homology.

So, I can compose it with $|\phi|_*$ from $H_*(|sd^k(K)|)$ to $H_*(K)$ to get an endomorphism of $C(Sd^k(K))$ to $C(Sd^k(K))$, viz., $Sd^k \circ \phi$. When you pass on to homology, it follows that the induced morphism is the same as $|\phi|_*$. Therefore, $L(|\phi|) = L(Sd^k \circ \phi)$ which can be computed at the chain complex level.

(Refer Slide Time: 29:29)

the chain complex $C_n(Sd^k K) \rightarrow C_n(Sd^k K)$ with the basis elements from the simplexes of $Sd^k K$.

Apurva B. Shrivastava
Department of Mathematics
Lectures on Algebraic Topology, Part I
MOTET Course

<ul style="list-style-type: none"> Cell Complexes Categories and Functors Homology Groups Other Homology groups Algebraic Topology Topology of Manifolds 	<ul style="list-style-type: none"> Review of The Algebraic Topology Review of Cell Homology and Cellular Topology Module 35 Some Applications of Homology Module 36 Some Applications of Homology Module 37 Some Applications of Homology
--	--

Let $\lambda : C_n(Sd^k K) \rightarrow C_n(Sd^k K)$ be the chain map induced by $Sd^k \circ \phi$. From (30), it follows that for any n -simplex ρ of $Sd^k K$, in the expression for $\lambda(\rho)$, the term ρ does not occur at all. Hence, the matrix of λ on $C_n(Sd^k K)$ has all its diagonal entries 0. Since this is true for all n , we conclude that, $L(f) = L(Sd^k \circ \phi) = 0$.

Taking the n -simplexes of $Sd^k(K)$ as a basis, consider the matrix associated to $\lambda = (Sd^k \circ \phi)_n$ at the n -chain group. Property (35) here will tell you that if you write the expression for $\lambda(\rho)$ for any simplex ρ in $Sd^k(K)$, it will not consist of ρ at all. That argument is the same as in the earlier case, ok? Hence the matrix of λ will have all diagonal entries 0, so that its trace is 0. Alternate sum of these traces will be also 0.

(Refer Slide Time: 30:25)

Let $\lambda : C_n(Sd^k K) \rightarrow C_n(Sd^k K)$ be the chain map induced by $Sd^k \circ \phi$. From (30), it follows that for any n -simplex ρ of $Sd^k K$, in the expression for $\lambda(\rho)$, the term ρ does not occur at all. Hence, the matrix of λ on $C_n(Sd^k K)$ has all its diagonal entries 0. Since this is true for all n , we conclude that, $L(f) = L(Sd^k \circ \phi) = 0$.

Apurva B. Shrivastava
Department of Mathematics
Lectures on Algebraic Topology, Part I
MOTET Course

<ul style="list-style-type: none"> Cell Complexes Categories and Functors Homology Groups Other Homology groups Algebraic Topology Topology of Manifolds 	<ul style="list-style-type: none"> Review of The Algebraic Topology Review of Cell Homology and Cellular Topology Module 35 Some Applications of Homology Module 36 Some Applications of Homology Module 37 Some Applications of Homology
--	--

Module-40 Applications of LFT

So I think I will have stop here. We shall see some interesting applications of the Lefschetz fixed point theorem itself next time. Thank you.