**Galois' Theory**
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**Lecture No 56**
**Embeddings**

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Ok now we prove the proposition which I have stated last time which characterizes

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perfect fields in terms of the irreducible polynomial is being separable. So let us recall what we are proving.

We are proving this proposition.

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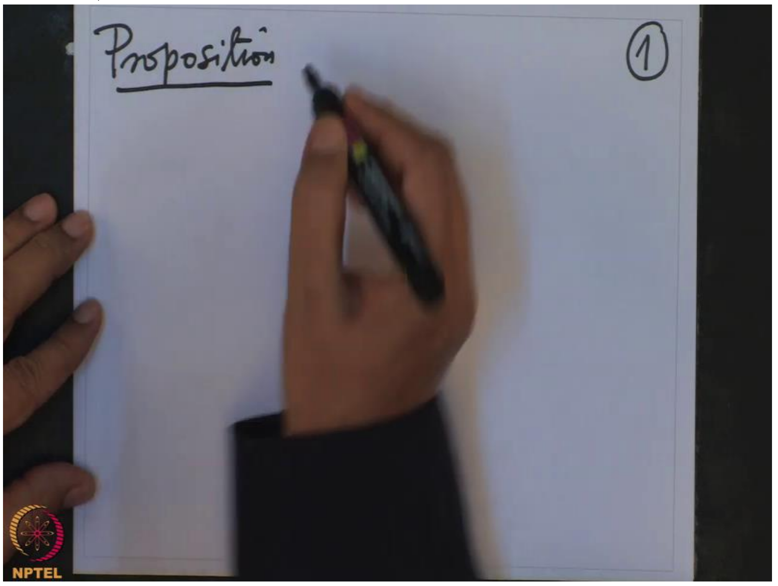


Let K be a field.

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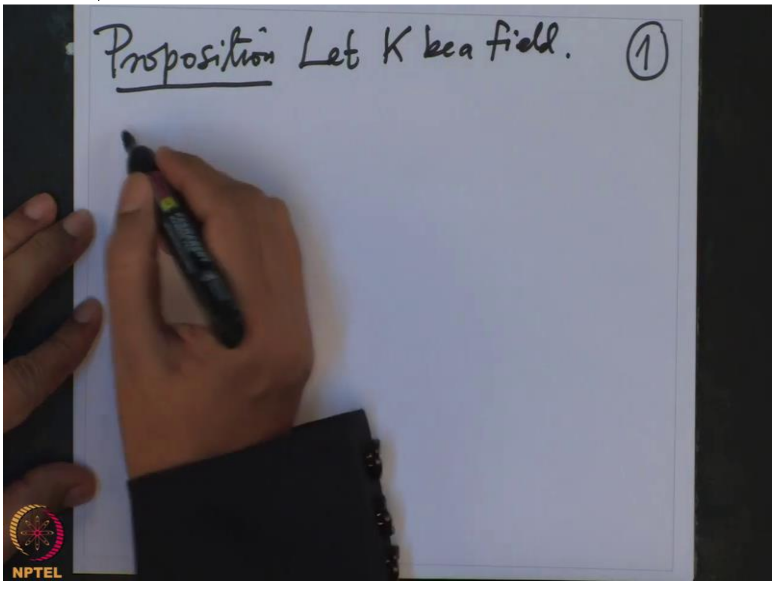


Then K is perfect if and only if every irreducible polynomial in K X is separable.

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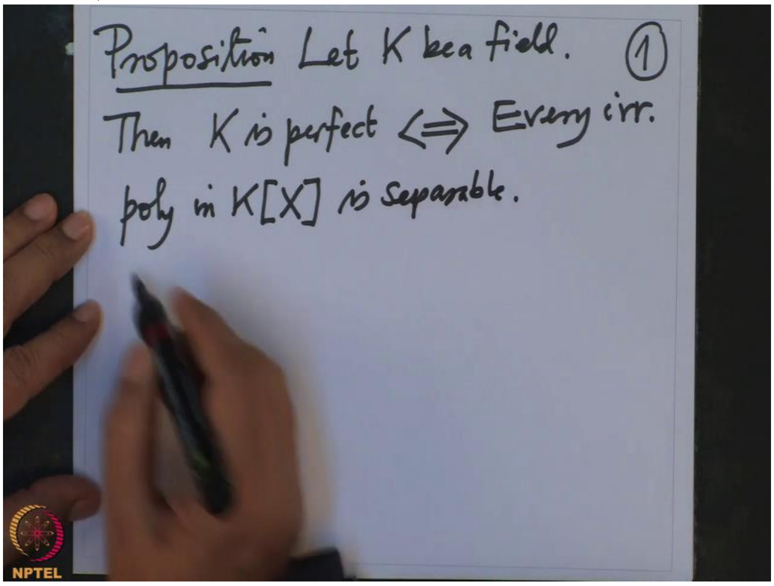


So proof

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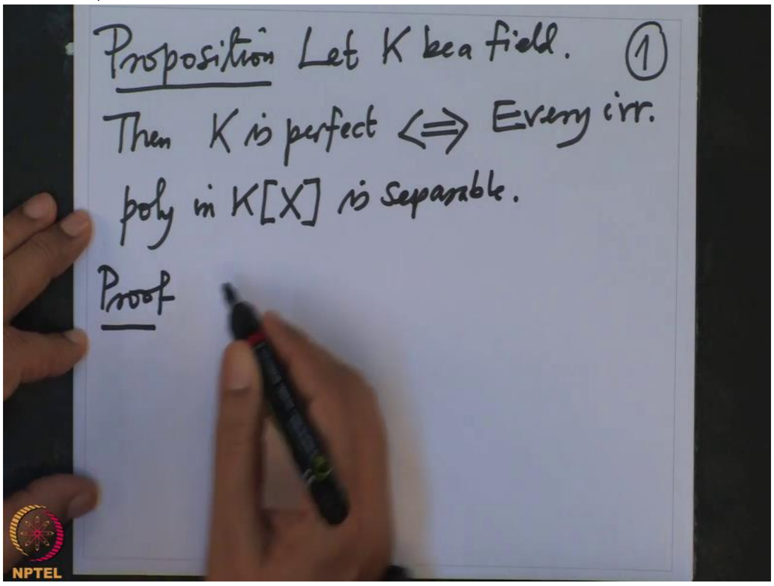


We may assume characteristic of the field is p positive. Because if the

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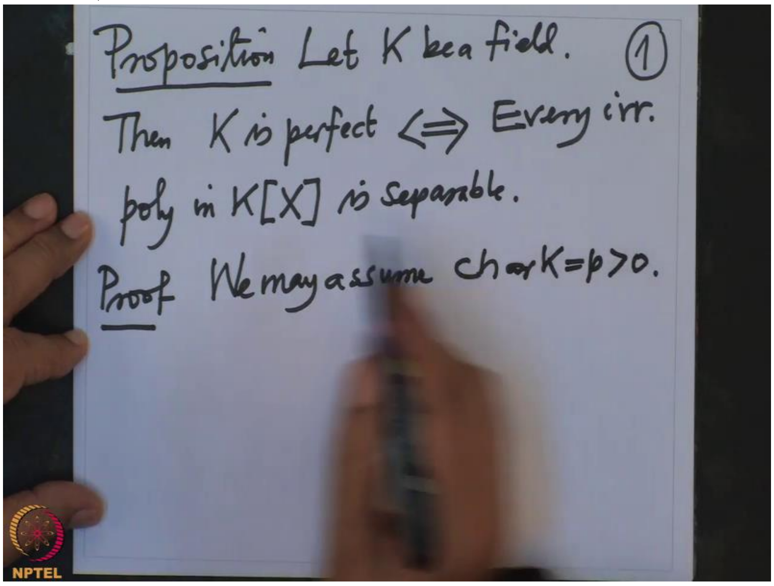


field is characteristic 0 then we know every polynomial, every irreducible polynomial is separable over characteristic 0 field because the derivative is non-zero and then the degree will be strictly smaller than f.

Therefore from there it is clear that irreducible polynomials are separable over characteristic 0 fields. Now therefore we assume characteristic p is positive. Now we will first prove this implication.

That we are assuming K is perfect and we want to show that every irreducible polynomial is separable. So we are assuming K perfect that means we are assuming, assume the Frobenius map which is from K to K is bijective.

That is the definition we made, perfect definition.

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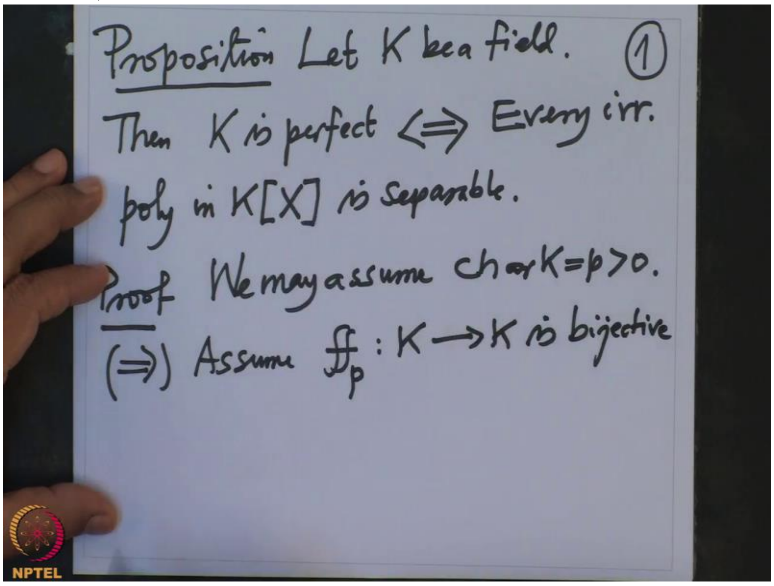


Field is perfect means this Frobenius map is bijective. Now suppose we want to prove what? Every irreducible polynomial is separable. So let f in K X be irreducible.

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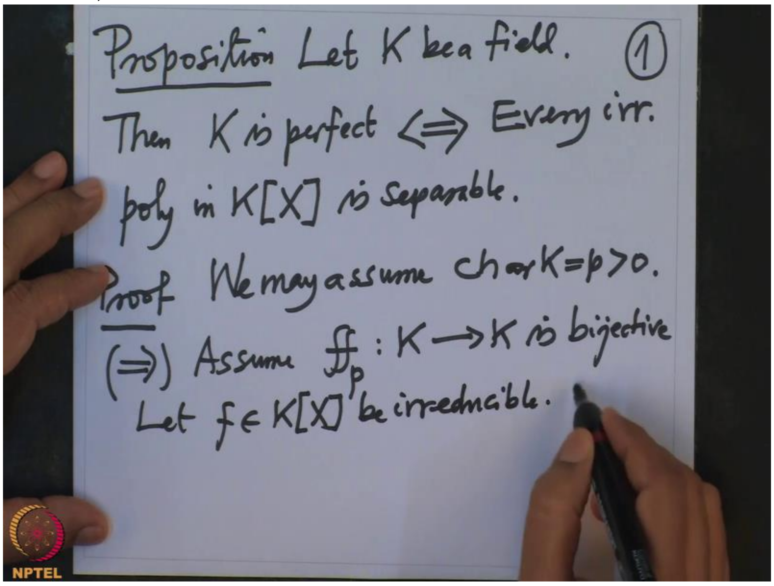


Suppose on the contrary we want to prove that, we want to prove that this f is separable. So suppose on the contrary that f is not separable, then we are looking for a contradiction.

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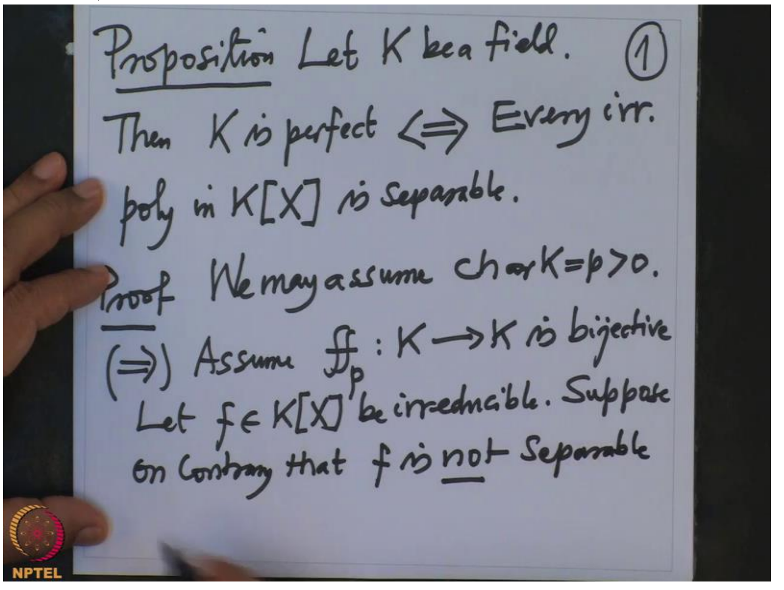


We are looking for, for a contradiction.

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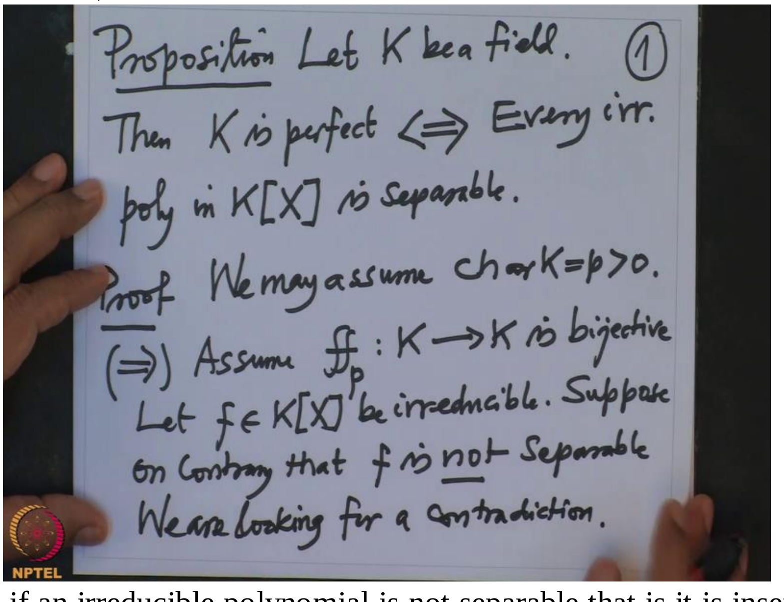


So we have seen if an irreducible polynomial is not separable that is it is inseparable then we know f prime has to be 0. This is by earlier observation.

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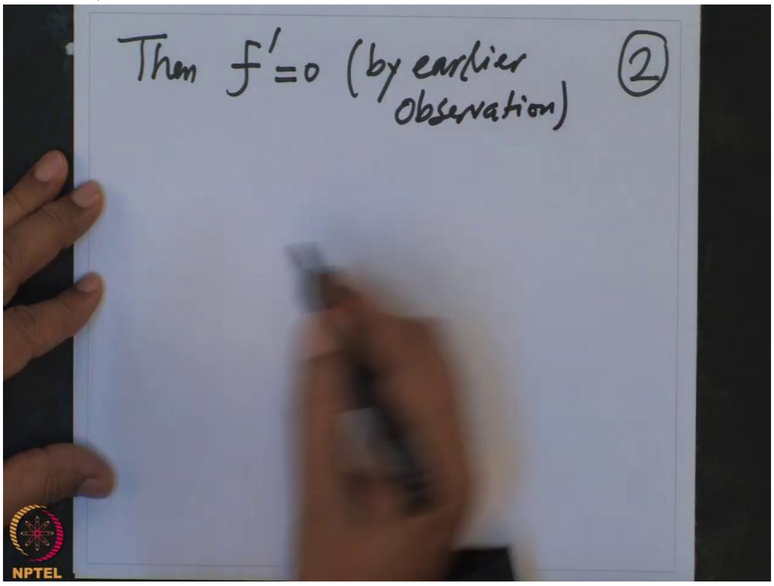


The derivative is 0 and now we want to get a contradiction, contradiction to what we will get, contradiction to the fact that f is irreducible.

So f prime is 0 that will mean that f will be a polynomial so that will imply, f is a polynomial in X power p with g is a polynomial a i X power i, i from 0 to d. This is in K X.

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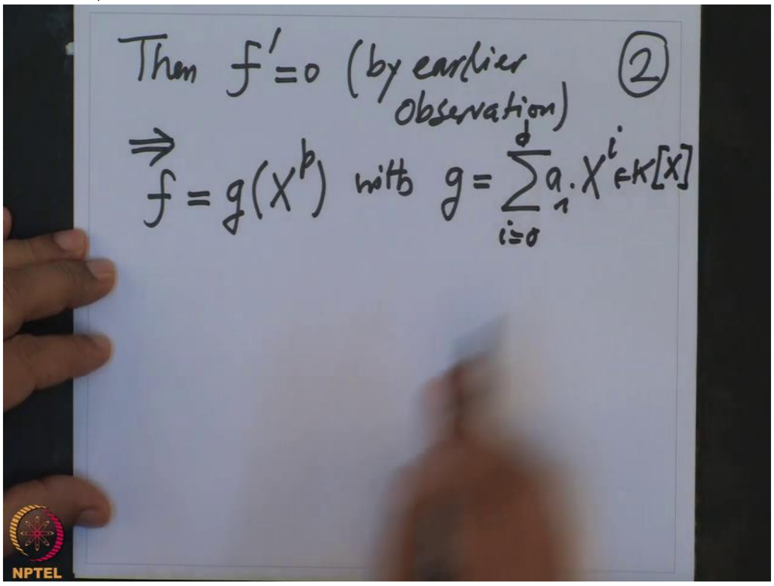


That means other

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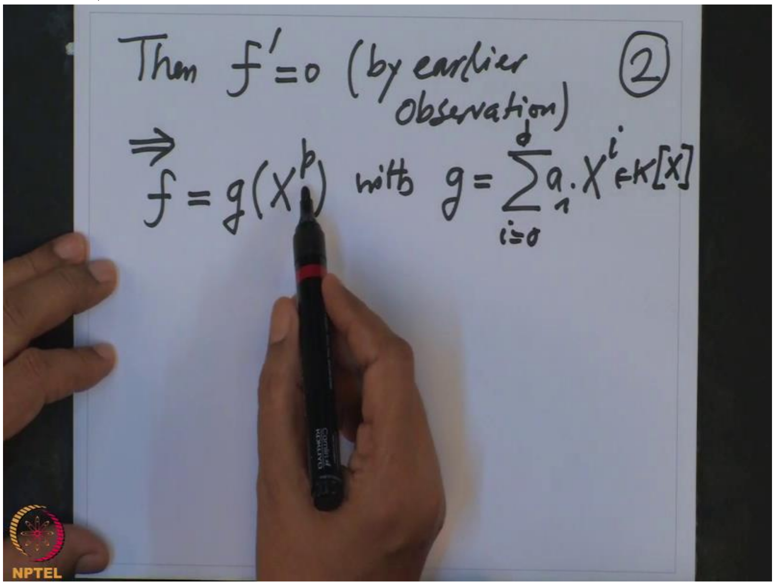


powers which are the powers of X which are not in, not multiples of p;

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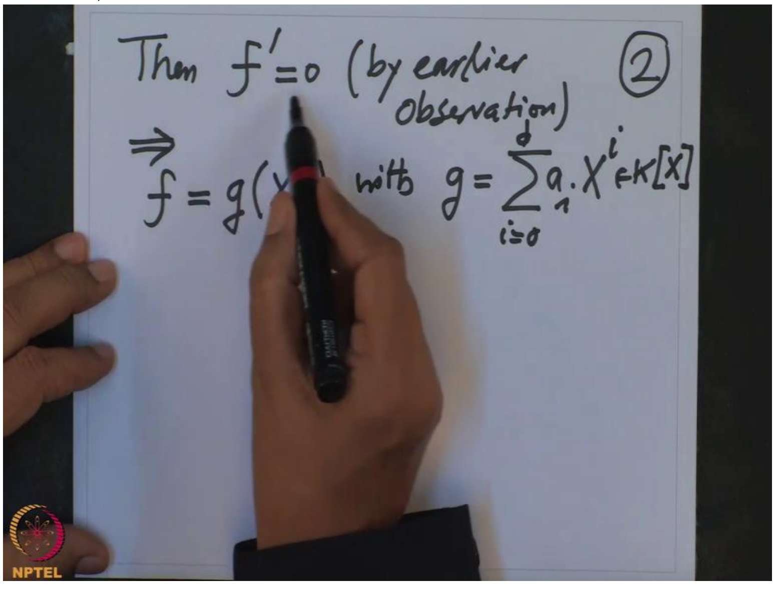


they will be 0 because f prime is 0.

Now

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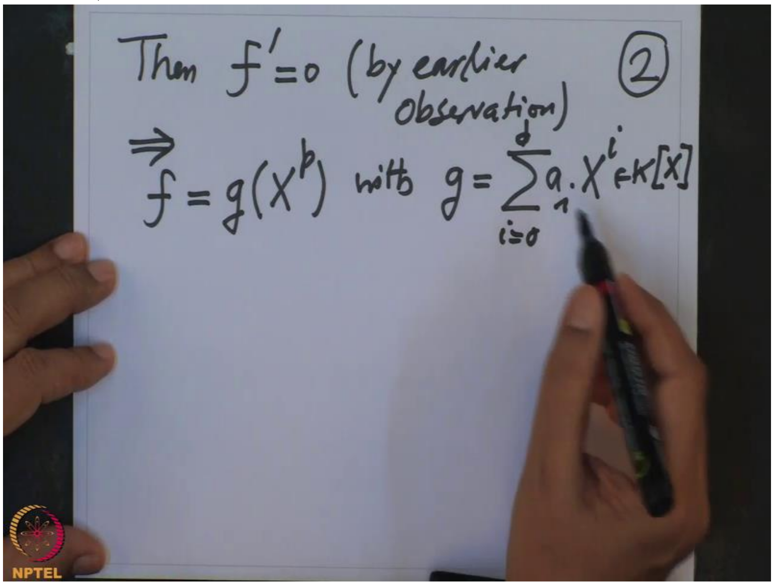


these a is are coefficients in a, so a is belong to K but K is the image of the Frobenius

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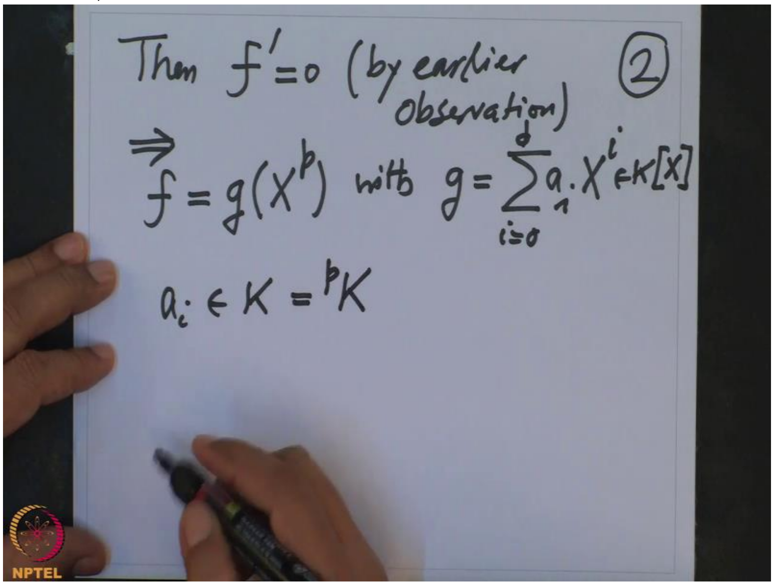


because Frobenius is surjective. Therefore each a i, I will write it as some pth power of some element b i. So i is from 0 to d.

And

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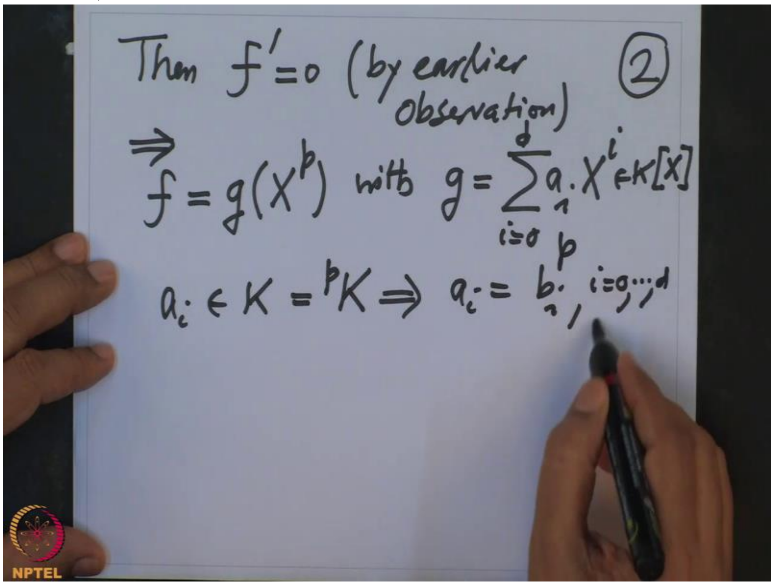


b is are element in K.

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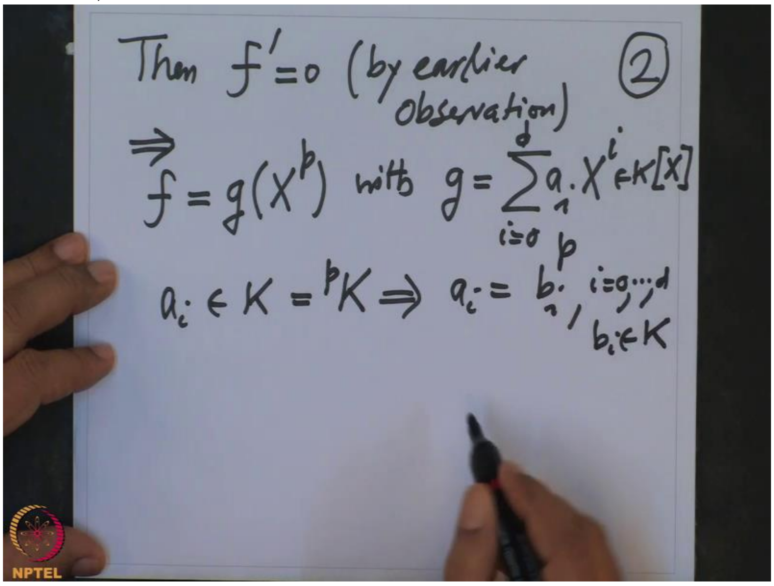


So this is because the image is

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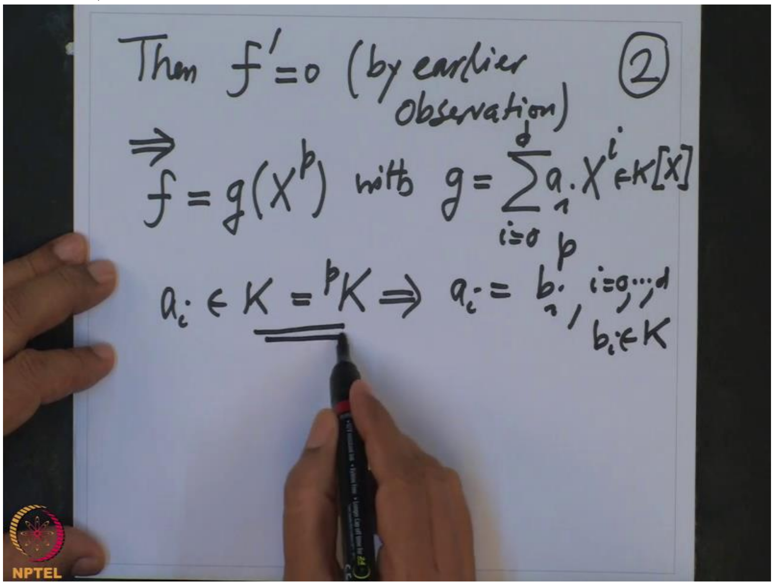


everybody, so p is surjective therefore this, so therefore now look at these b is and look at the polynomial h I am defining.

This is the polynomial b i X power i.

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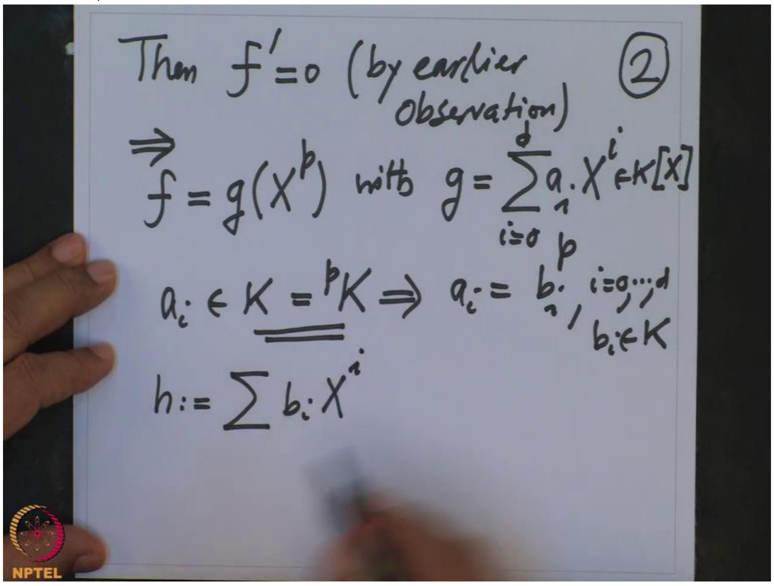


Look at this polynomial. This is a polynomial in K X, Ok

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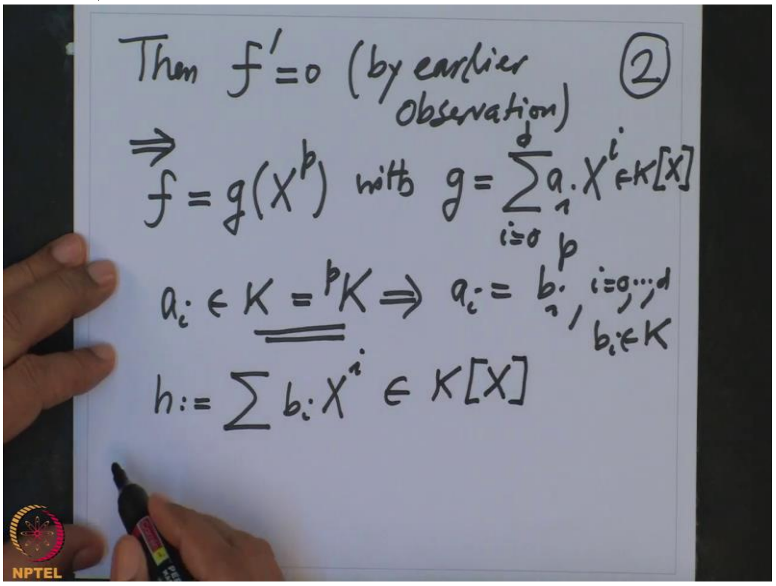


and then what is f? Look at f. We want to get, we want to check that f is not irreducible now. That will be the contradiction.

f equal to g of X power p,

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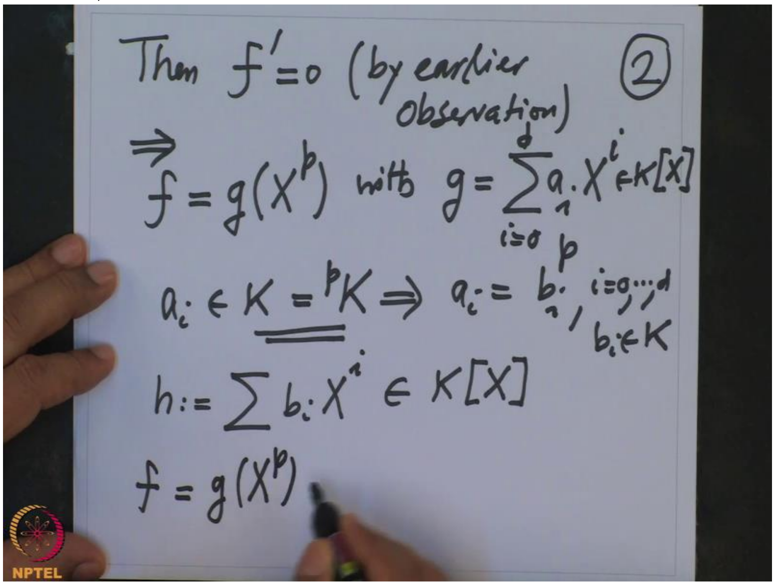


that means in g I should

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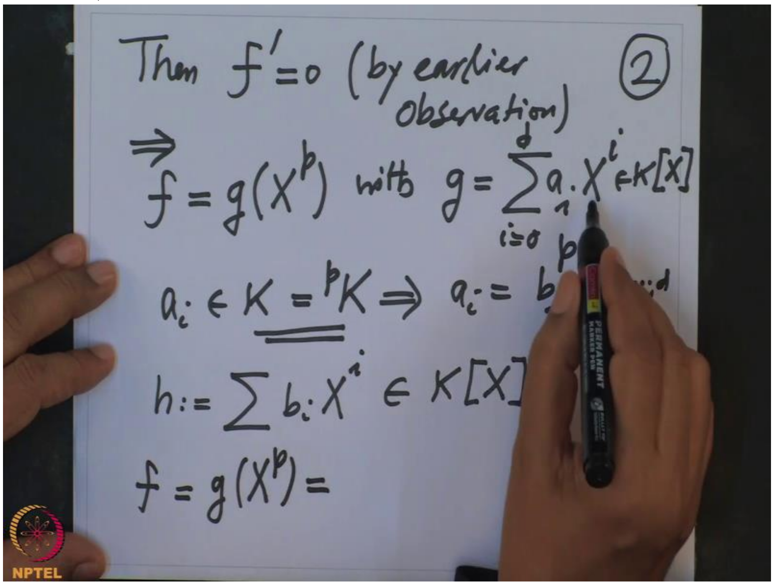


put instead of X, X power p. So this is summation i equal to 0 to d and these a is are b i power, so this is b i power p and X i, X I will replace by X power p, so this is X p i.

But now it is characteristic p, therefore

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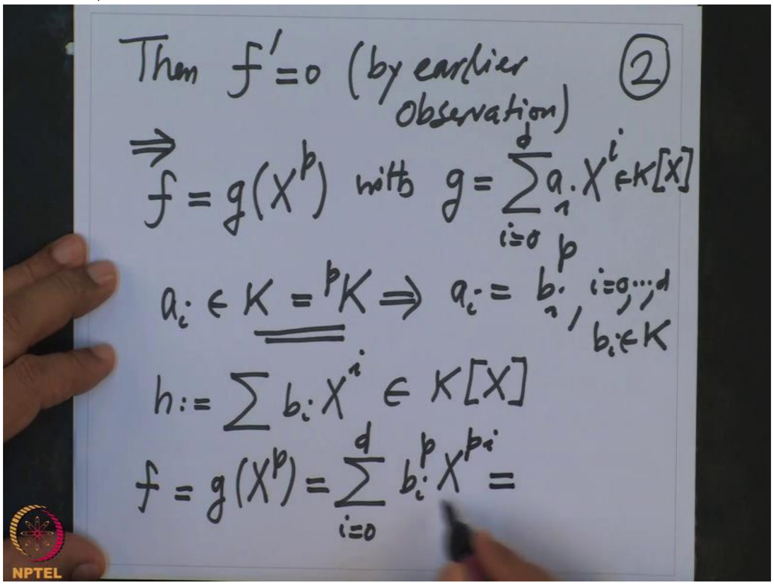


this sum is same thing as i equal to 0 to d, b i X power i and then I have taken that p out of the sum.

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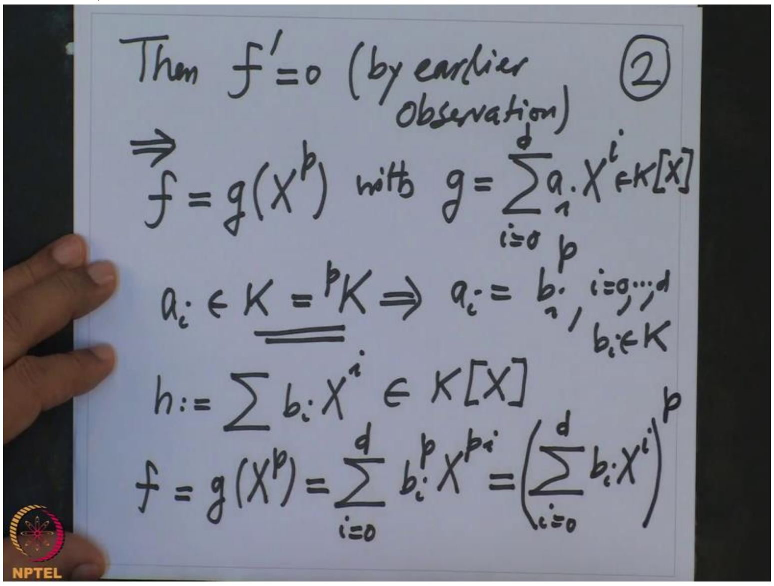


That is because p is a characteristic of the field. This equality follows from the fact that characteristic p is positive.

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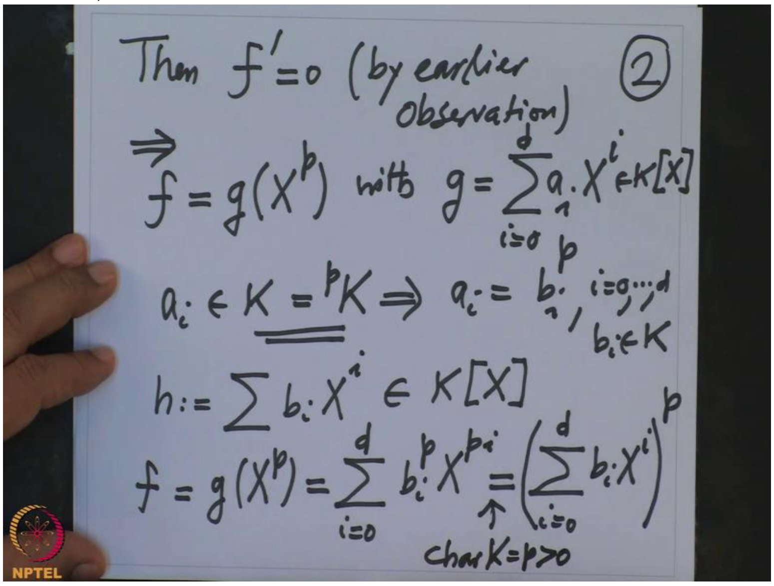


This is when you expand by binomial, middle terms which are

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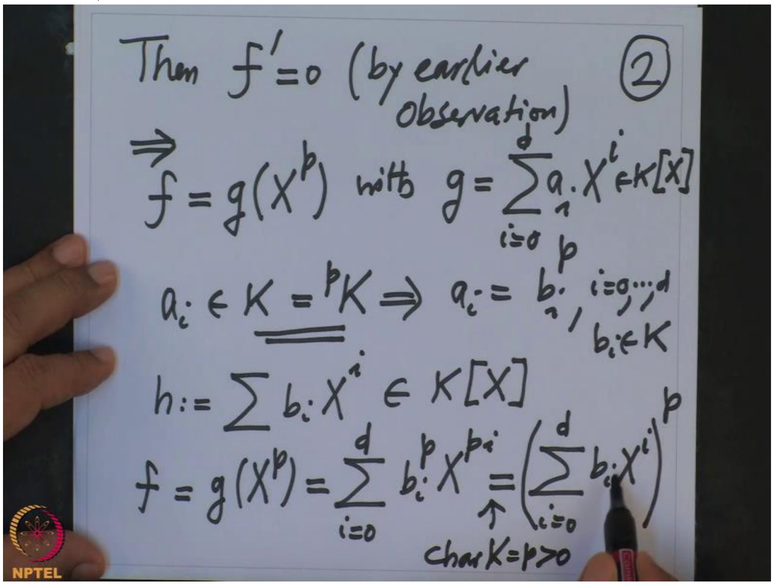

binomial coefficients, middle binomial coefficients they are divisible by p therefore they are 0 in K. That is why, this equality.

But what is this? This is h power p.

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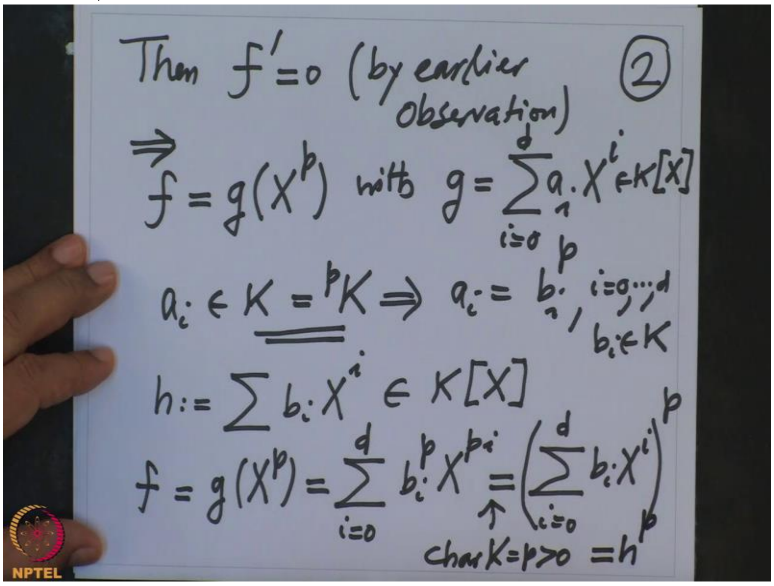

This is precisely the definition of h. This is h power p which contradicts the irreducibility of f. p is at least 2.

So this f is not irreducible in K X. Therefore we, I have finished the

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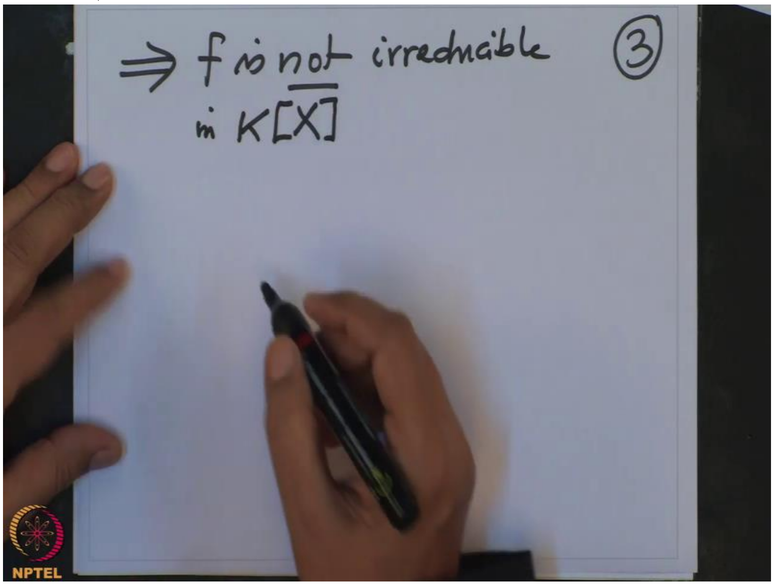


proof of one implication. Conversely we need to prove that if f is; if every polynomial is irreducible then we want to prove that K is perfect.

Conversely assume that every irreducible polynomial in K X is separable.

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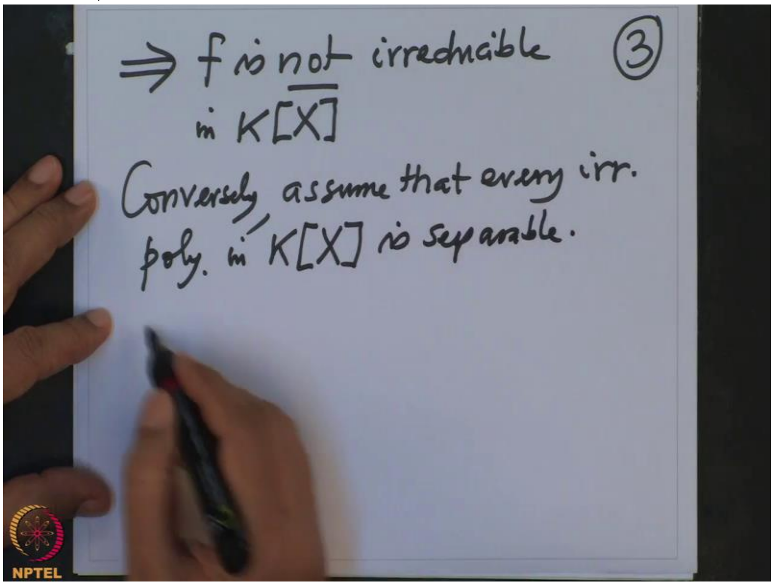


And now to prove that f p Frobenius map is surjective. This is what we want to prove,

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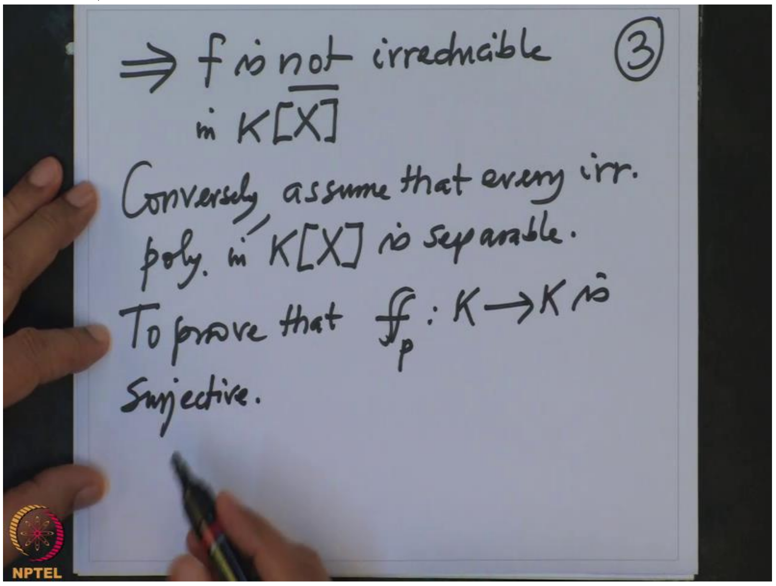


alright.

So suppose it is not surjective. So suppose, then we should get a contradiction, suppose so the image of the Frobenius is K p. So suppose I have an element a in K which is not in the image of Frobenius.

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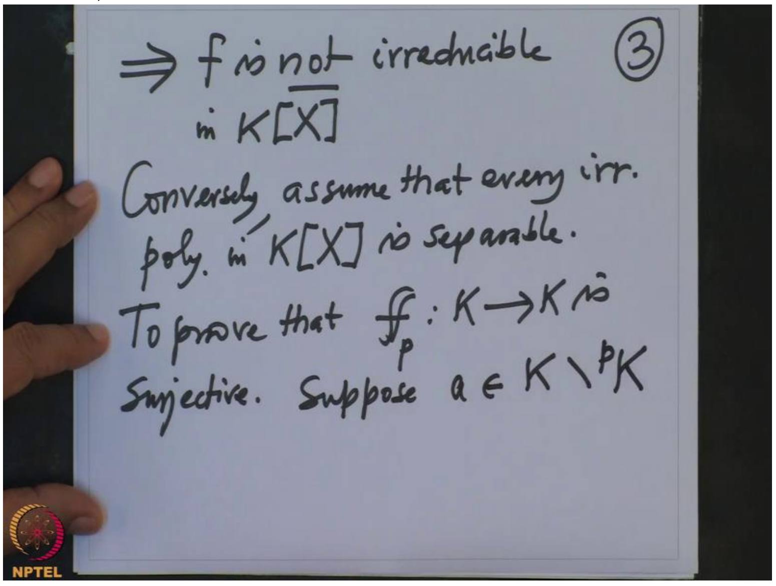


Then I am looking for a contradiction.

So, now we have an element in the field which is not the pth power. So I look at the polynomial, I enlarge the field. So let L over K be finite field extension

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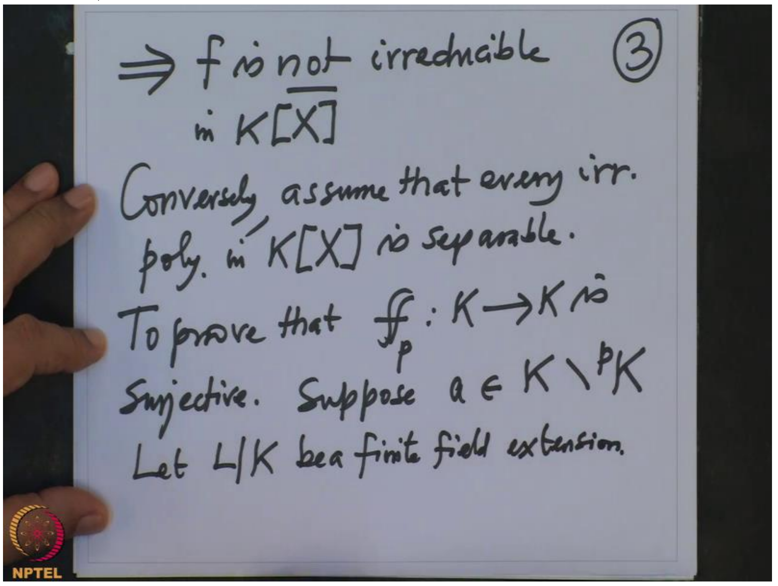


which is obtained by adjoining pth root of a

So adjoining x, x power p equal to L. So that is,

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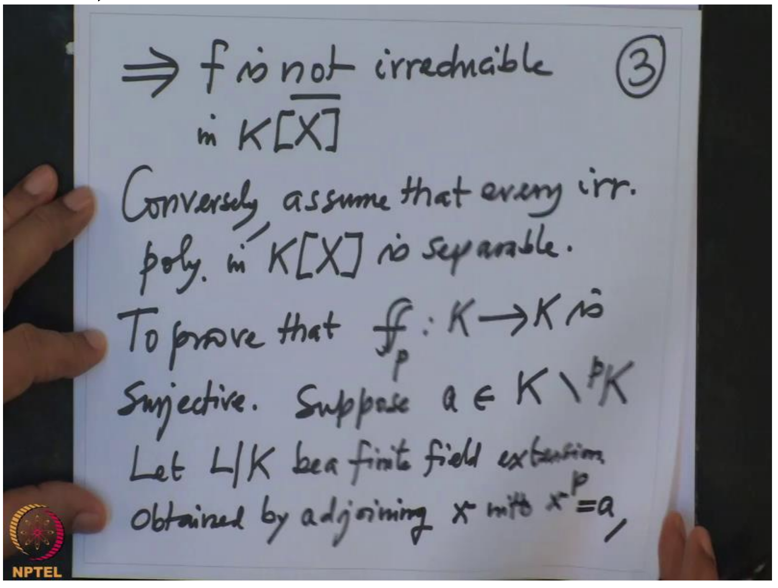


what I am saying is, look at the field extension L. L is the simple extension generated over K by x and x power p equal to a.

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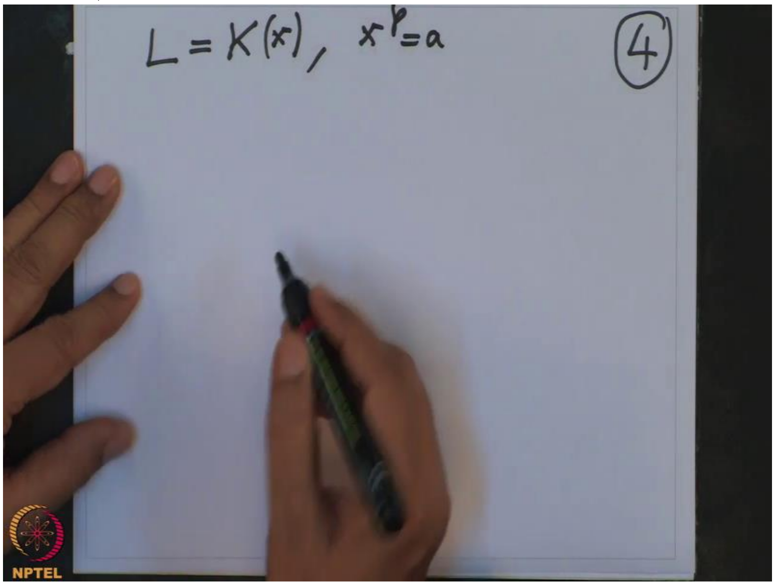


So this is a field extension and this,

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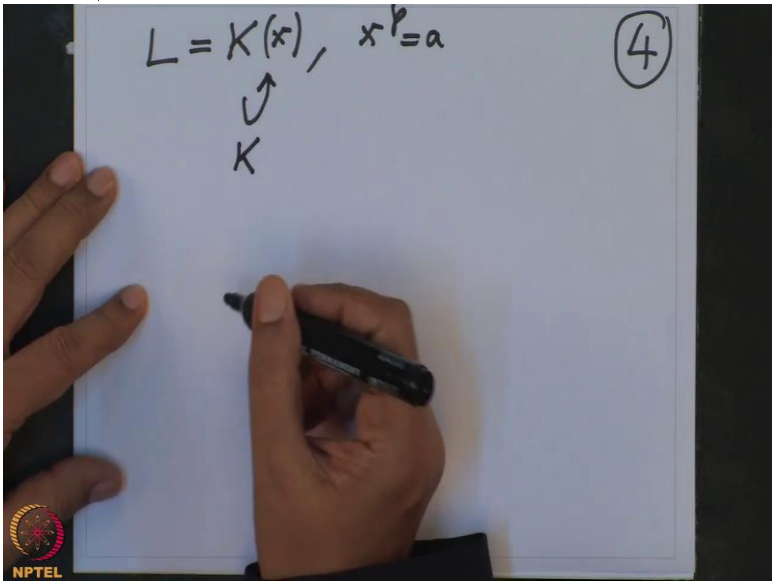


this is the finite field extension.

So what is irreducible, so x belongs to the 0 set of the polynomial X power p minus a. It is one 0

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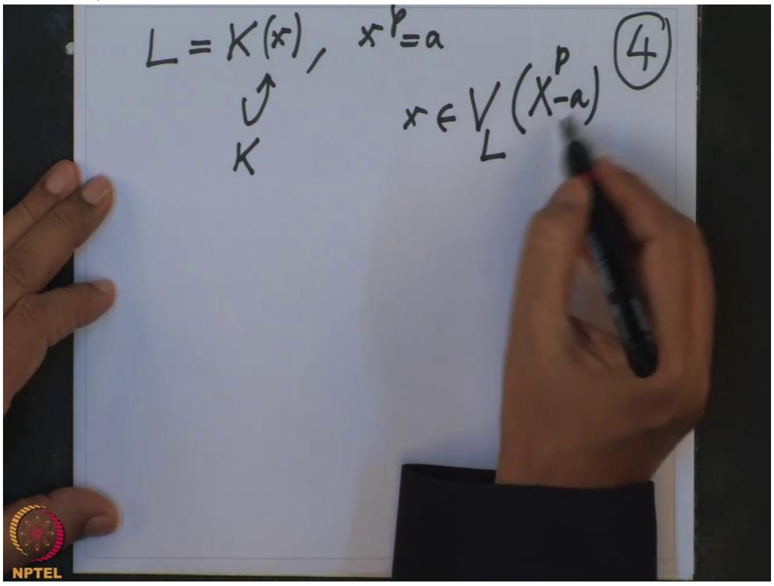


of the polynomial. This is in L that is x.

So look at the polynomial, this polynomial you want to study X power p minus a. This polynomial, when I write like this, X power p minus small x power p,

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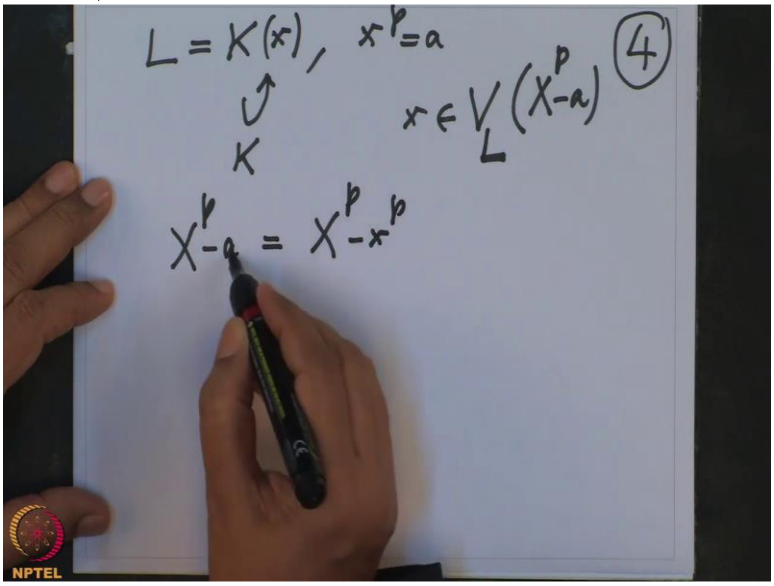


I will just a equal to x power p, this is X minus x power p.

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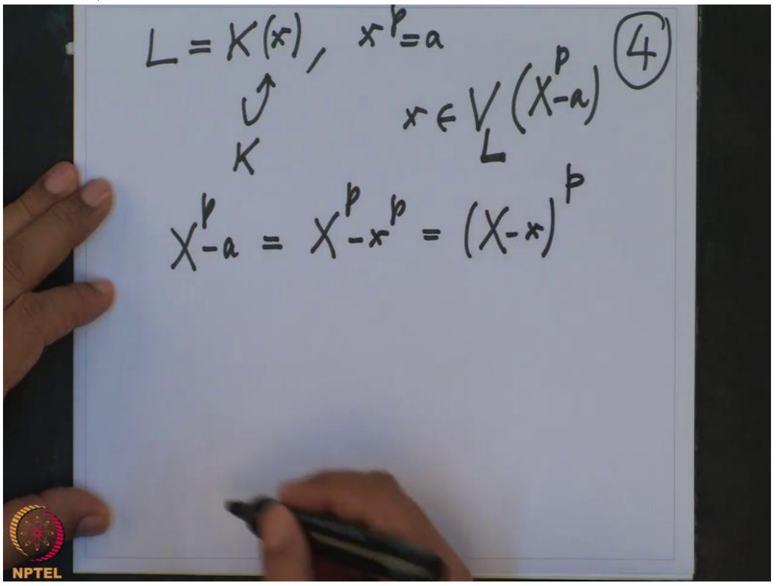


Therefore see, therefore what we know from this equation, every factor of X power p minus a is of the form X minus x power r

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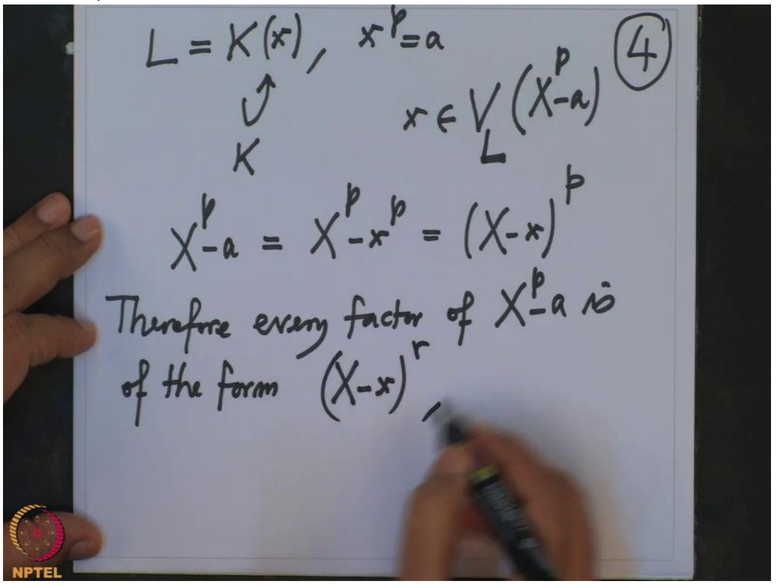


where r is from 0 to,

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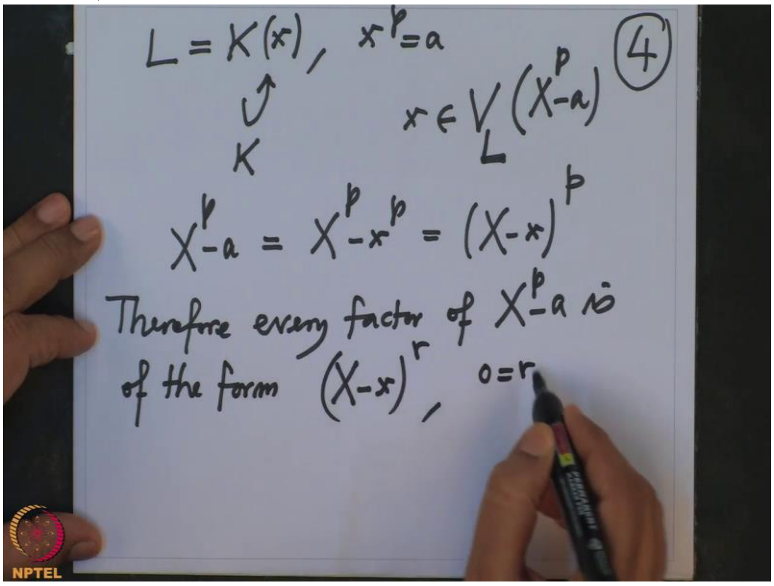


r is from 0 to p.

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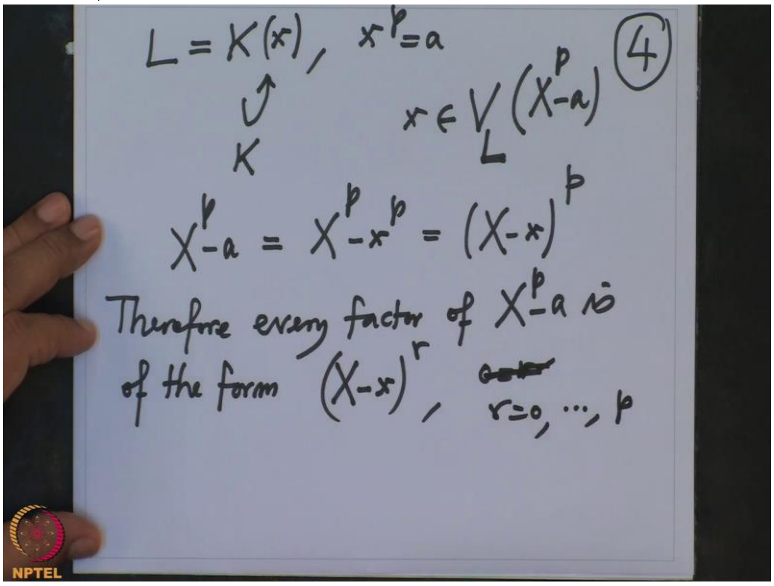


Well, when r is equal to 0, this is just 1 which is a constant factor. So we are not interested in that. So this is

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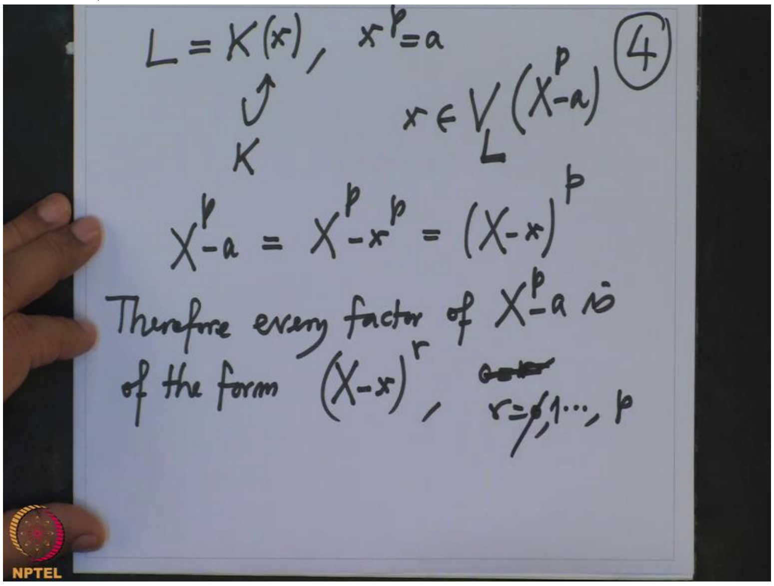


not interesting for us, so 1 to p.

And therefore what we know is every irreducible factor, every irreducible factor of this polynomial X power p minus a, this is a polynomial in K X

And I am considering irreducible factor also in K X.

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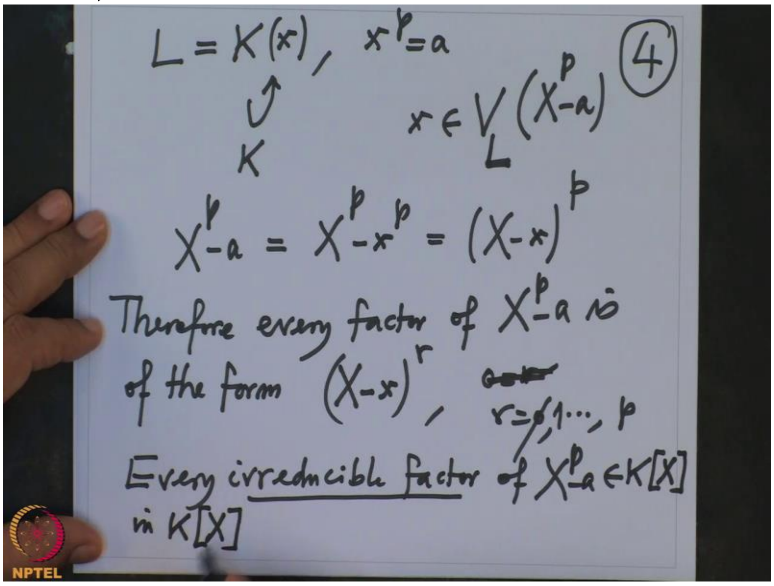


And it will be not

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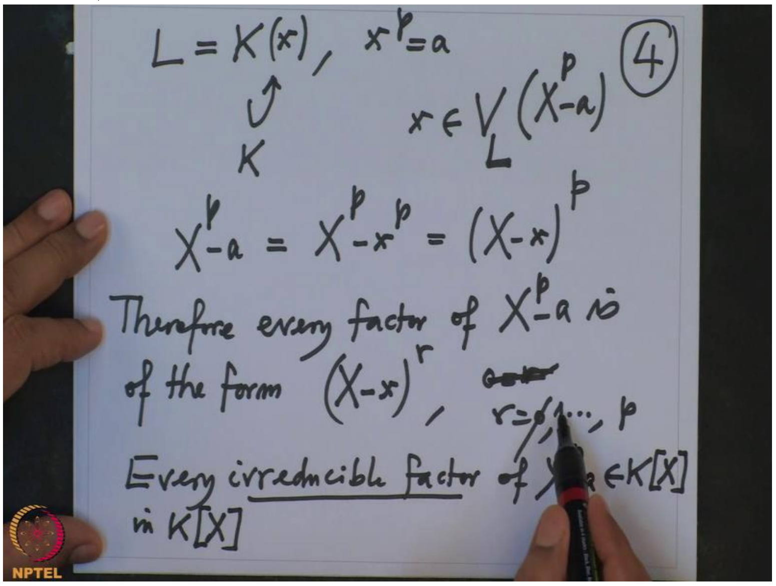


r equal to 1 so every irreducible factor has a multiple zero in L.

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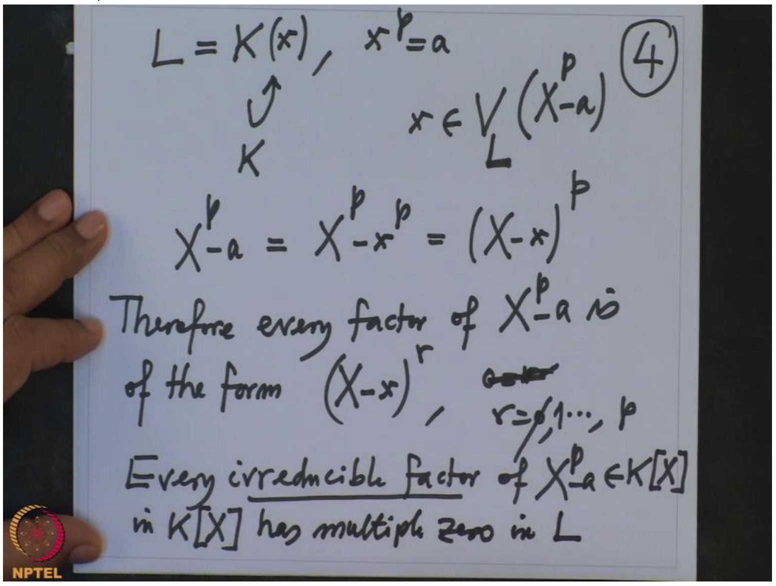


So the same X will be multiple zero because if you take irreducible factor of this polynomial,

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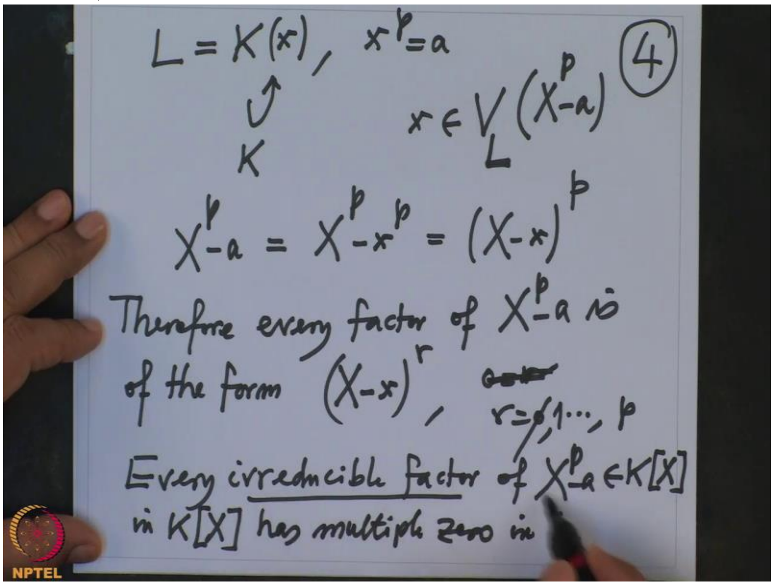


that, first of all

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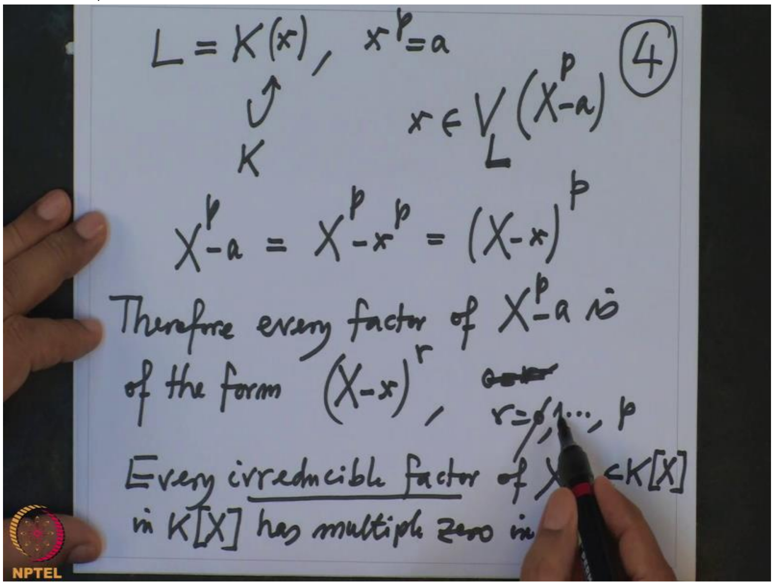


that is not X minus x because that X minus x is not a polynomial in K X.

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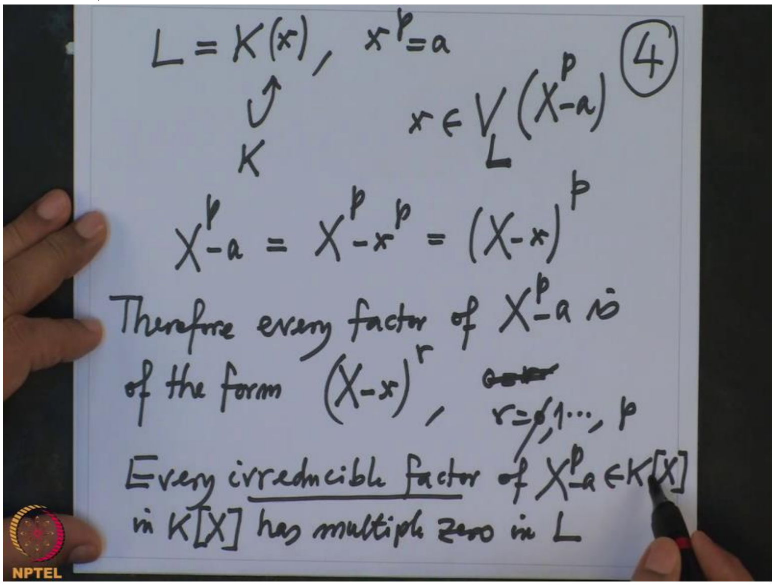


So since, since this polynomial X minus x, this is not in K X, so all irreducible factors in K X,

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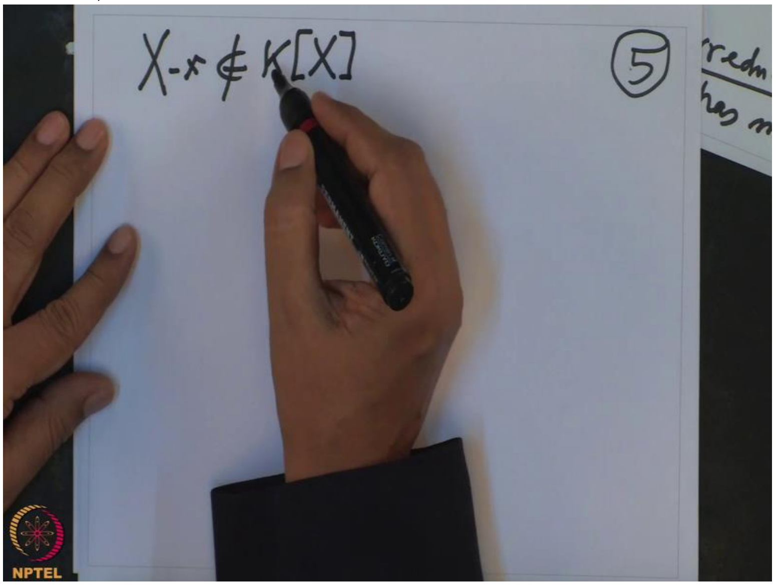


they will be higher powers of this. Therefore they have multiple zeroes.

So therefore they are not separable, so what we proved is in particular irreducible factors of X power p minus a in K X are not separable.

This is a contradiction

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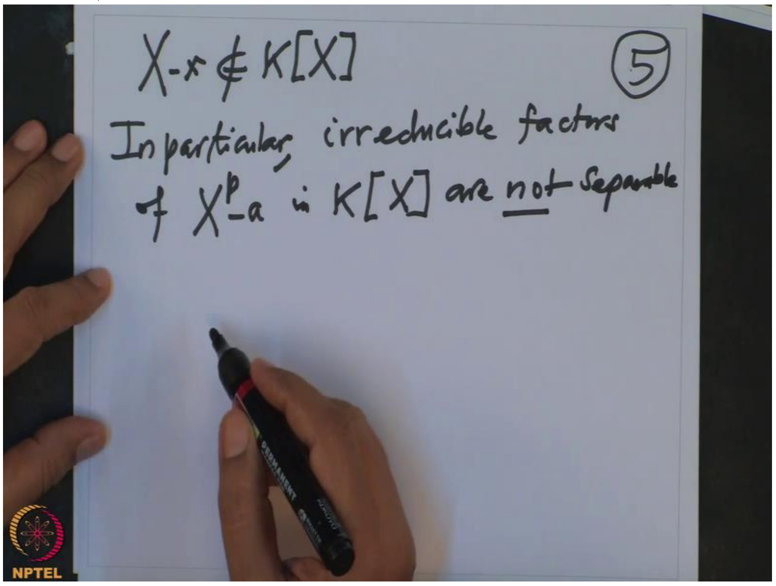


to our assumption. We are assuming that every irreducible polynomial in K X are separable. But here we produce irreducible polynomials in K X which are not separable.

This is a contradiction,

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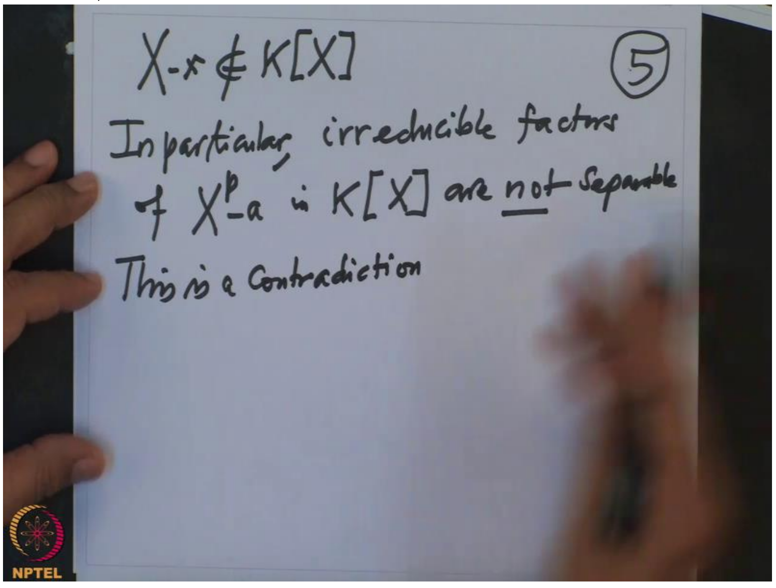


so remind you we are assuming that

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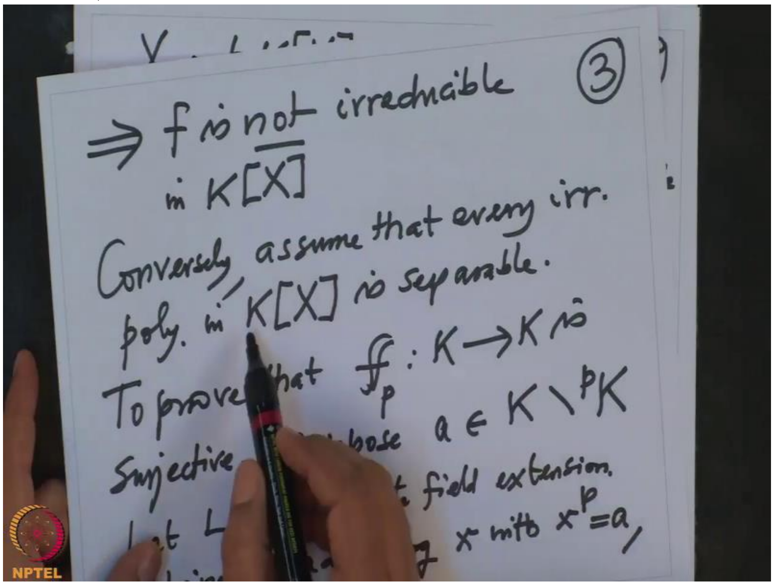


every irreducible polynomial in K X is separable but here we produce irreducible

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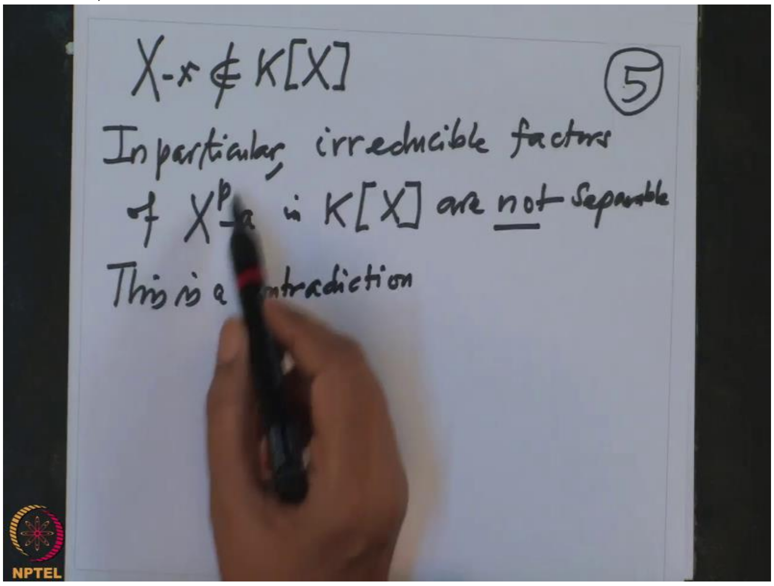


polynomial which is a factor of this could be this itself.

This is a contradiction to assumption. So this completes,

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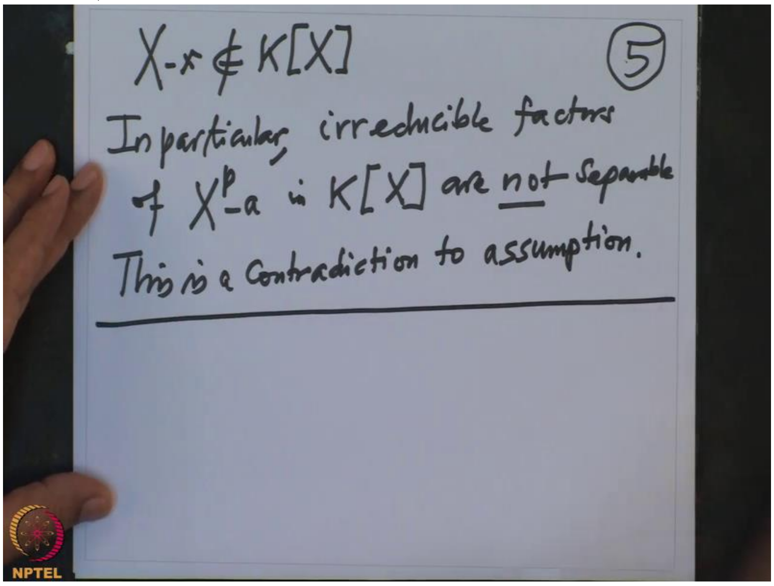


this completes the characterization of perfect field. Now I want to characterize separable field extensions.

So next is we want to characterize finite separable field extensions. This is what we want to do.

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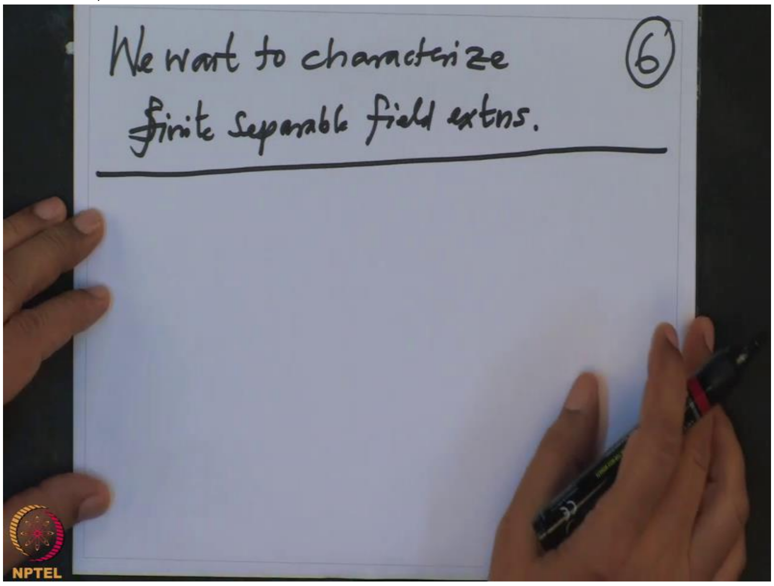


So recall that when we say finite field extension is called separable if every polynomial, every element is separable. That means minimal polynomials are separable polynomials. But this is too much checking.

So I want to find economical way to check that a given finite field extension is separable, possibly in terms of the degree of the field extensions. So this is what I want to do it. So first of all note that let us do checking for simple extension.

So let L equal to K x over K be a simple extension.

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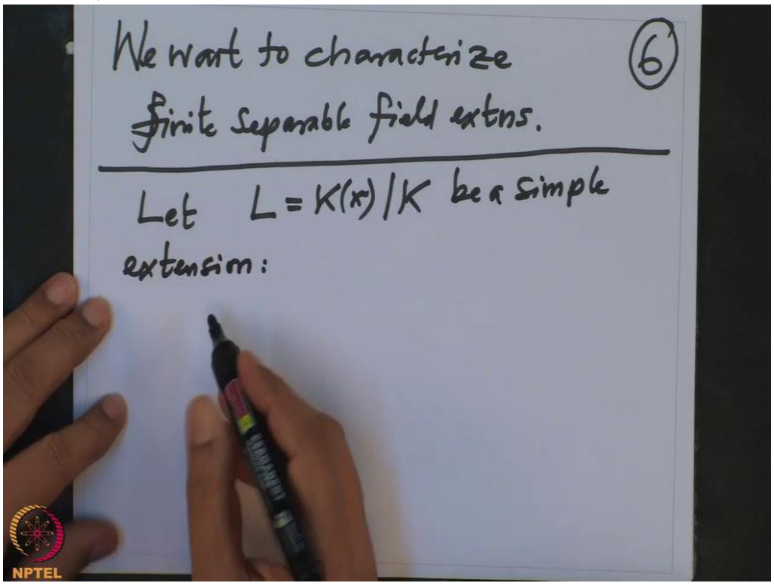


Then we want to, so x is separable over K, that is if and only if, this is a definition of separability of an element mu x over K is a separable polynomial.

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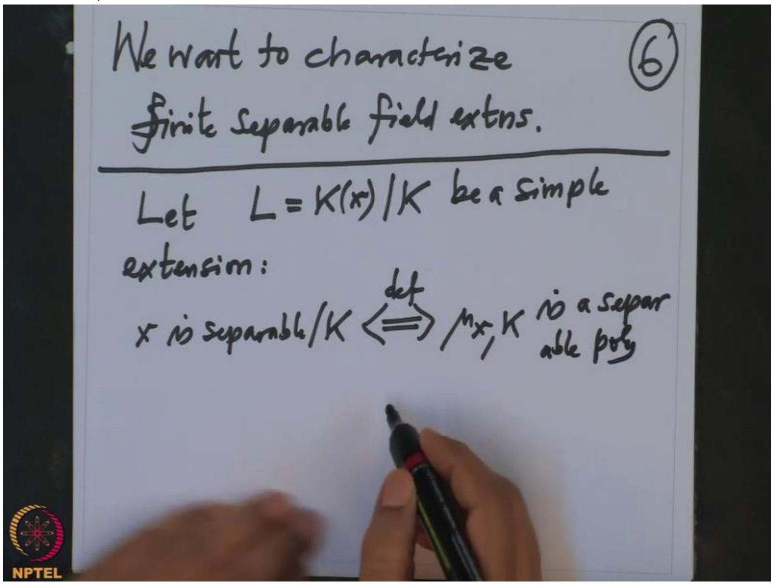


That is if and only if the 0 set of mu x in K bar, this cardinality equal to the degree of the mu x,

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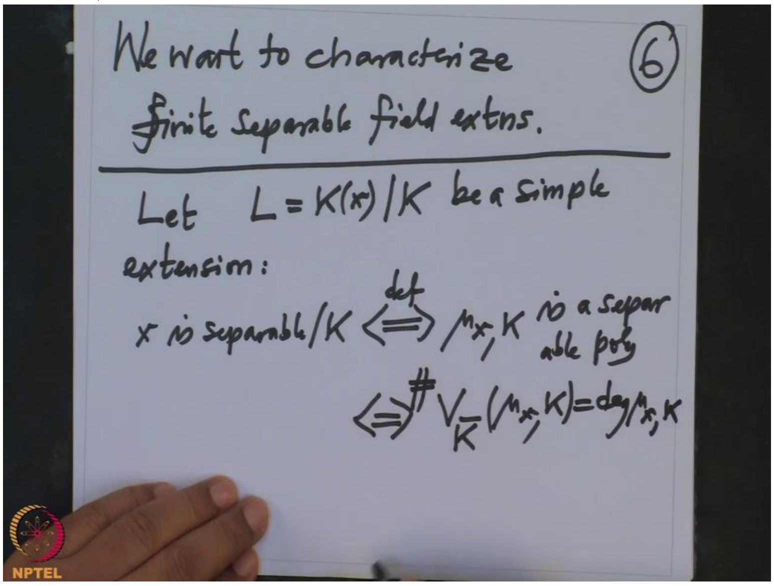


as many as zeroes. But this degree of mu x is equal to the degree of K x over K; that is L over K.

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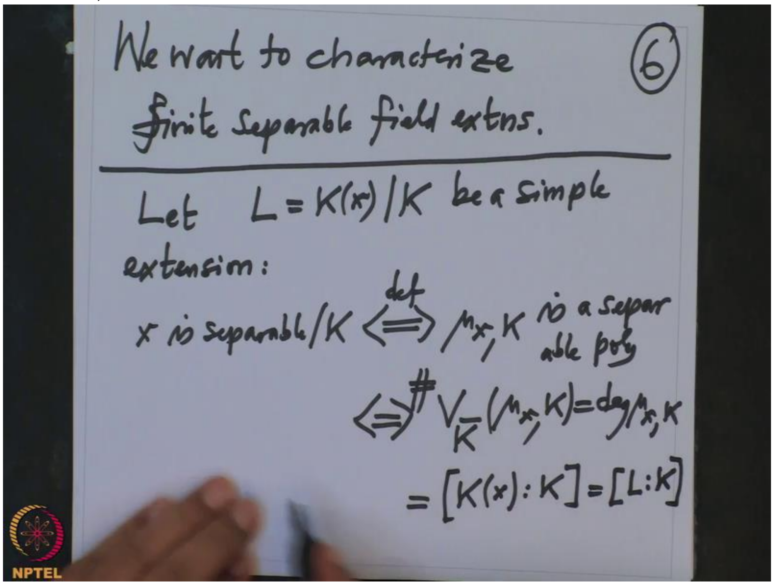


So an element, a generating

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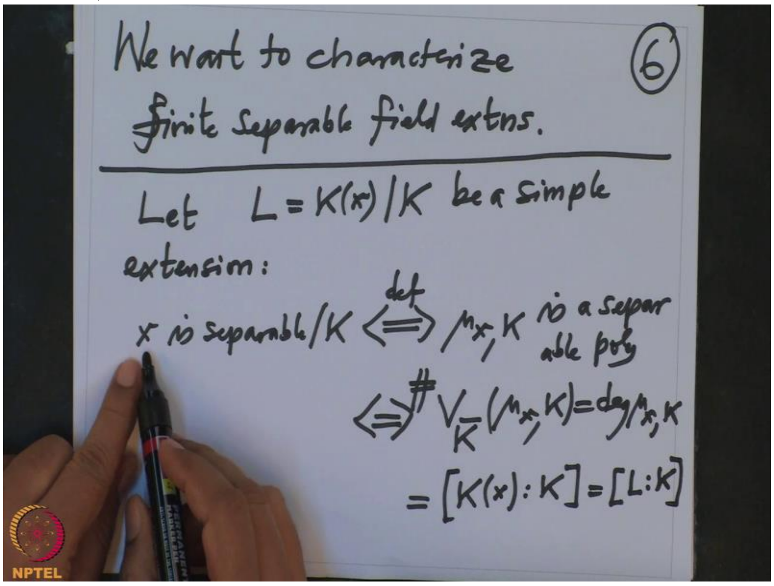


element is a separable over K that is, if and only if the degree of the field extension is exactly equal to the,

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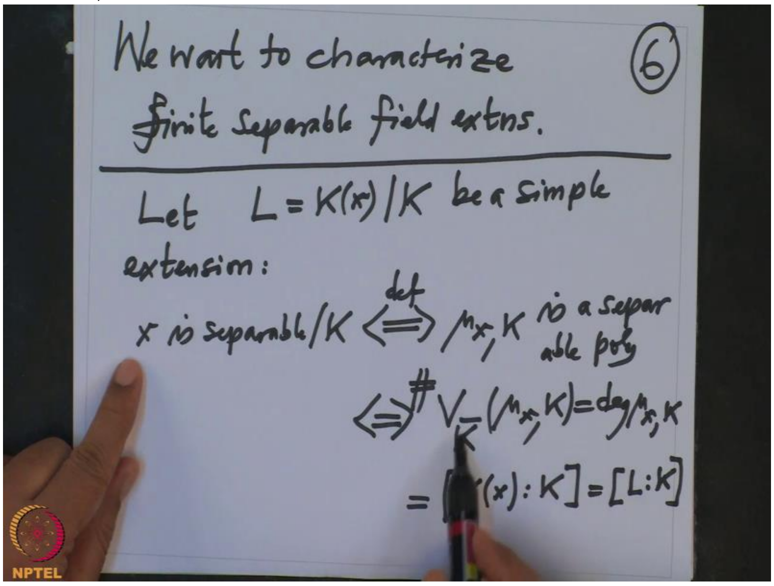


the number of zeroes of the irreducible polynomial of x inside the algebraic closure of K. So here K bar is an algebraic closure of K. So this

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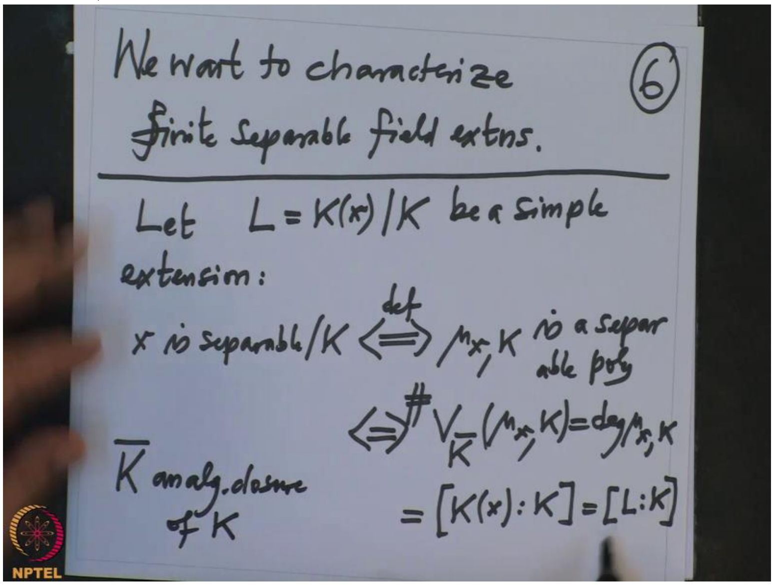


Is one way to check the number of zeroes equal to the degree of the polynomial. This is one possibility.

So I am going to enlarge on this. But before I do that I want to note one observation. So note that, this is remark.

This separability of a field extension, of a field extension is a relative concept. What does that mean?

That means it depends

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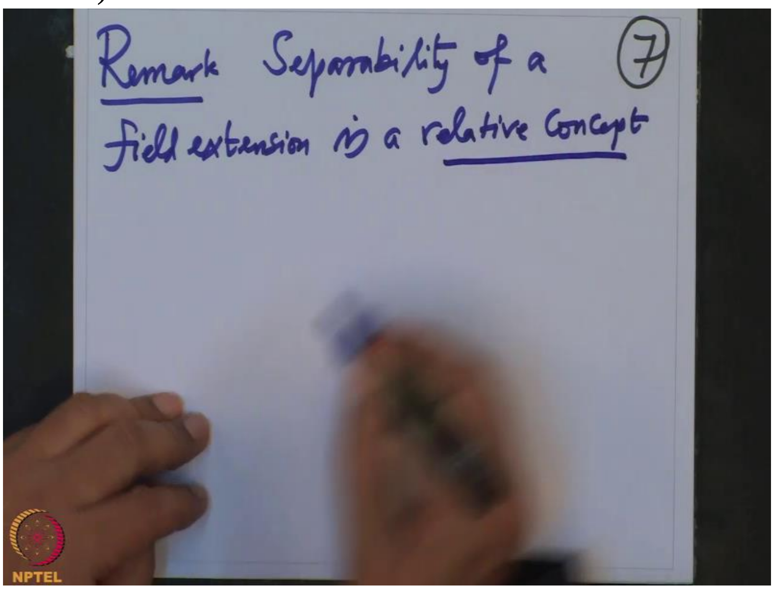

on the base field. So, so for example suppose K is field of characteristic p, positive and

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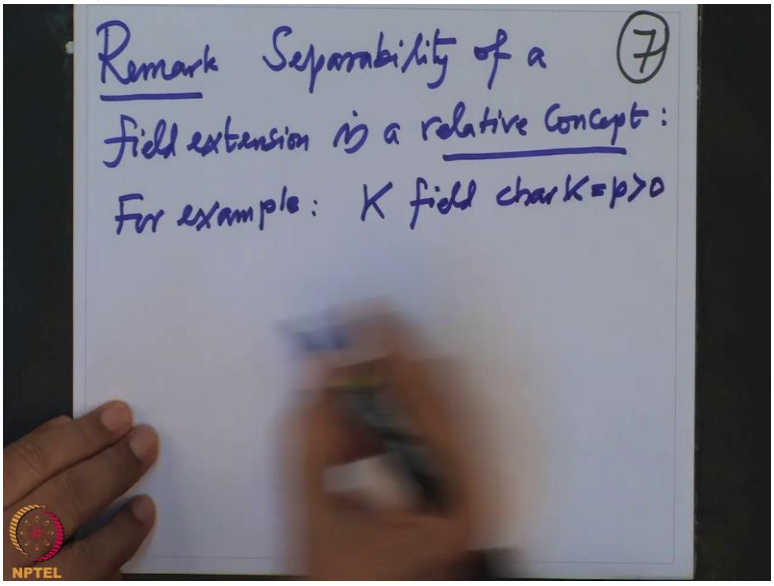


we have taken an element a in K minus pth powers of K.

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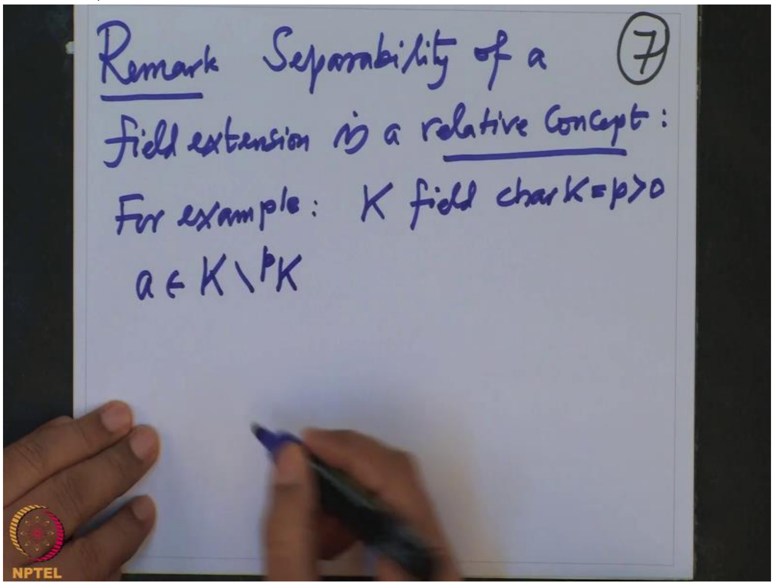


This is a image of the Frobenius map.

Suppose I have taken K. Like we have taken in one of the proof and suppose we have taken x is a 0 of X power p minus a. This 0 is in algebraic closure.

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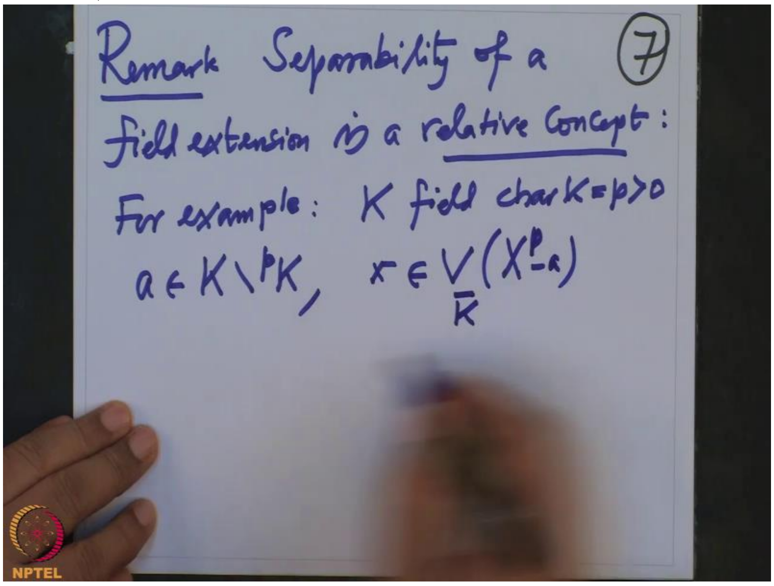


So K bar is algebraic closure of K.

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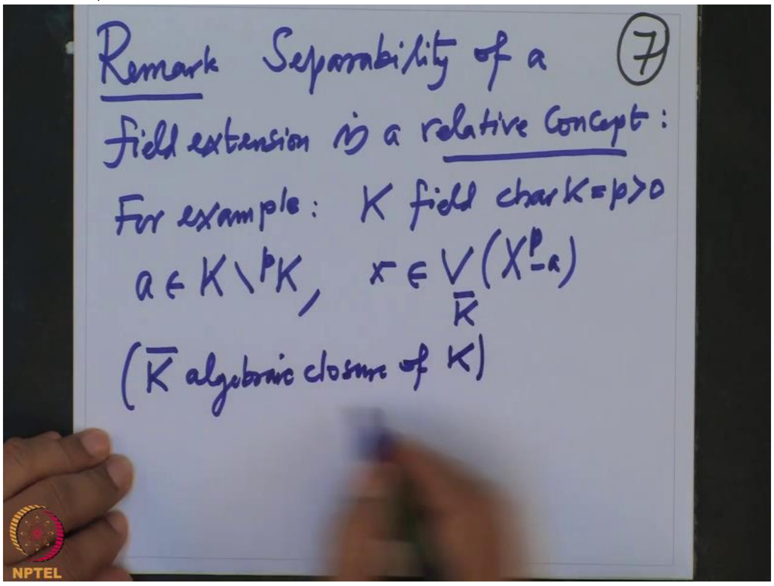


We choose an algebraic closure. We know it exists. And so

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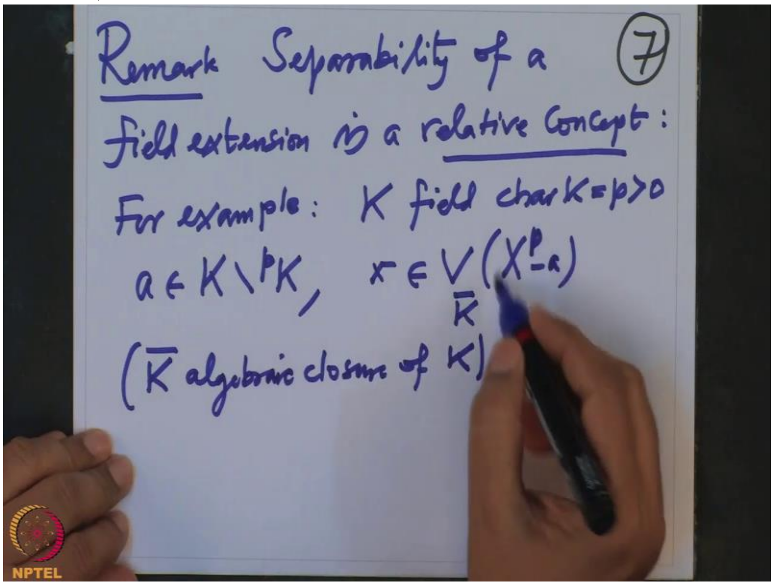


this polynomial is a polynomial in K bar X and therefore it has 0, so I choose that 0. And now look at the simple extension L equal to K x. This K x, this is a simple extension.

Then we know L over K is not separable. Because we have seen that

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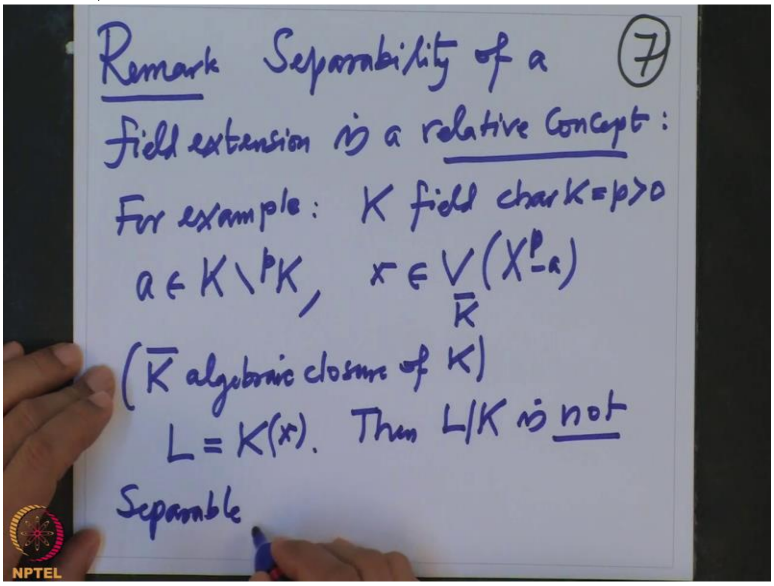


the minimal polynomial mu x over K has multiple zeroes.

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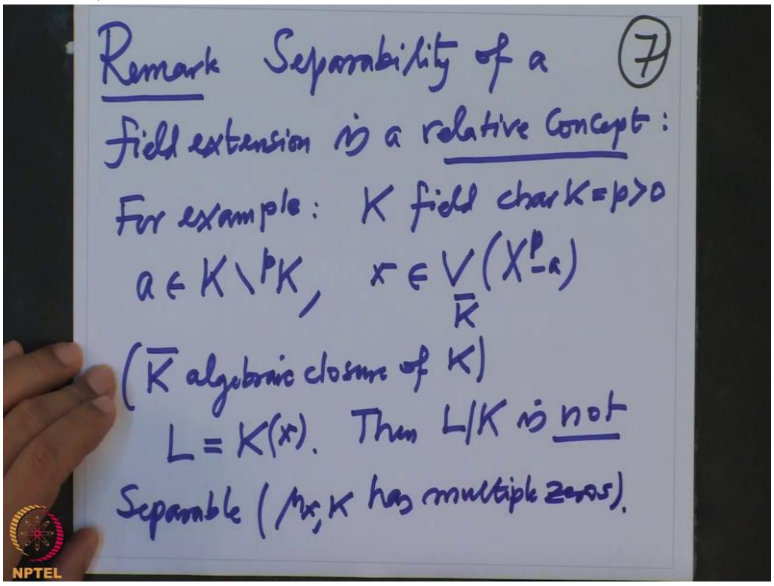


This x is not in K,

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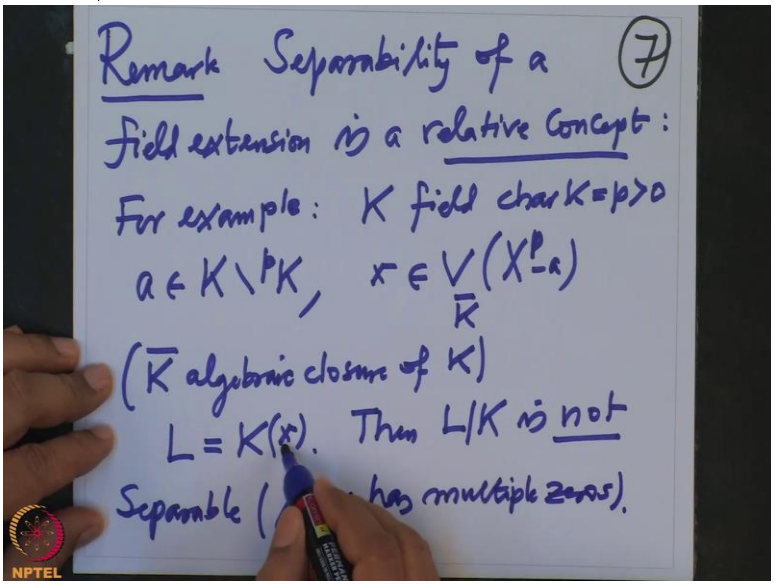


this x; this x is not in K because

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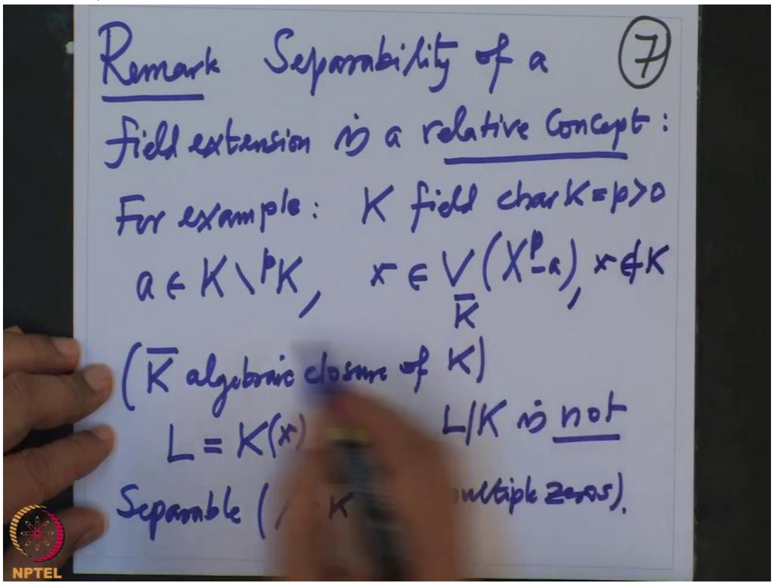


it is not a pth root. a is not a pth root.

But now, so therefore this L over K is not a separable extension. Or an element x, so element x in L is not separable over K.

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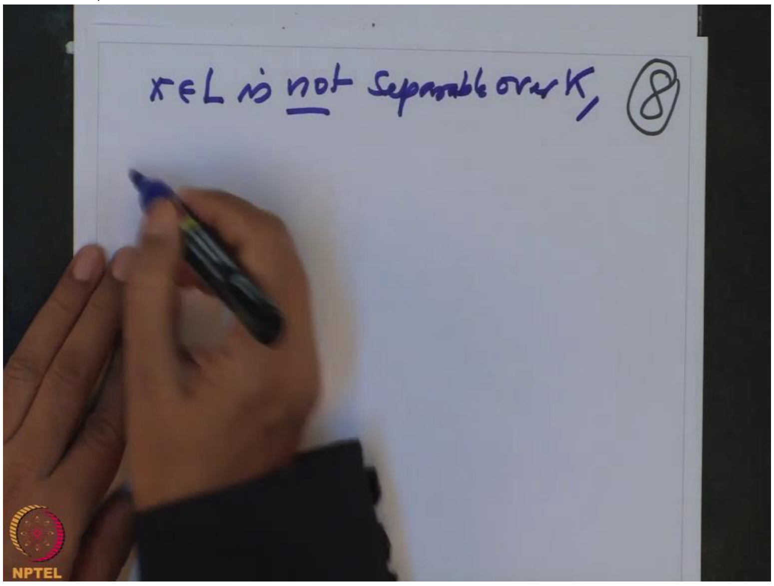


But the element x in L is separable over L

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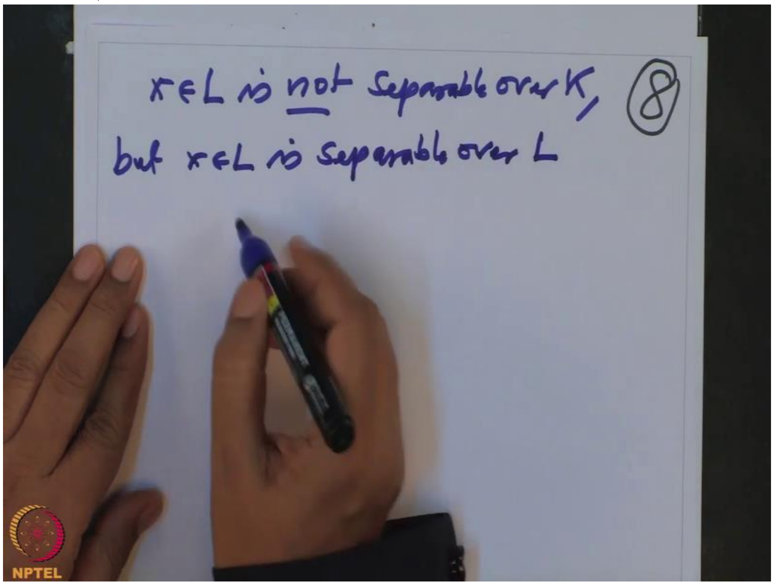


because what is the minimal polynomial of x over L? This is the simple X minus x

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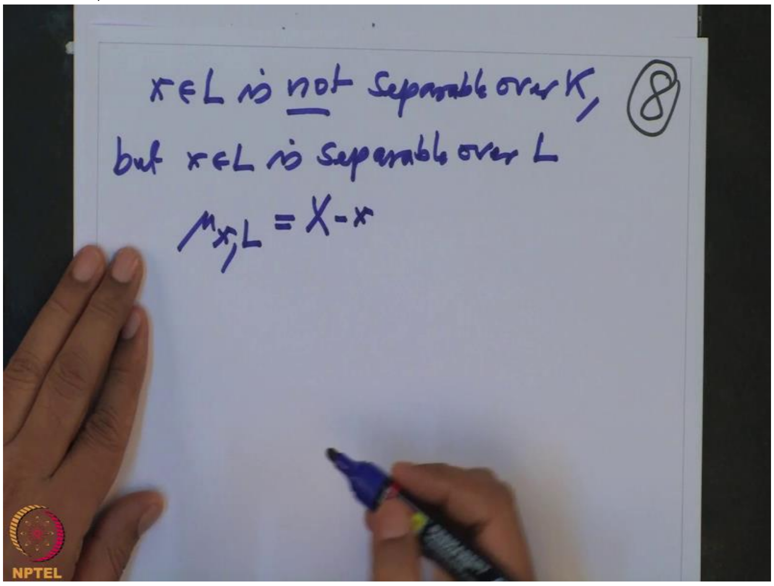


polynomial only because X is, this is a polynomial in L X and this is a

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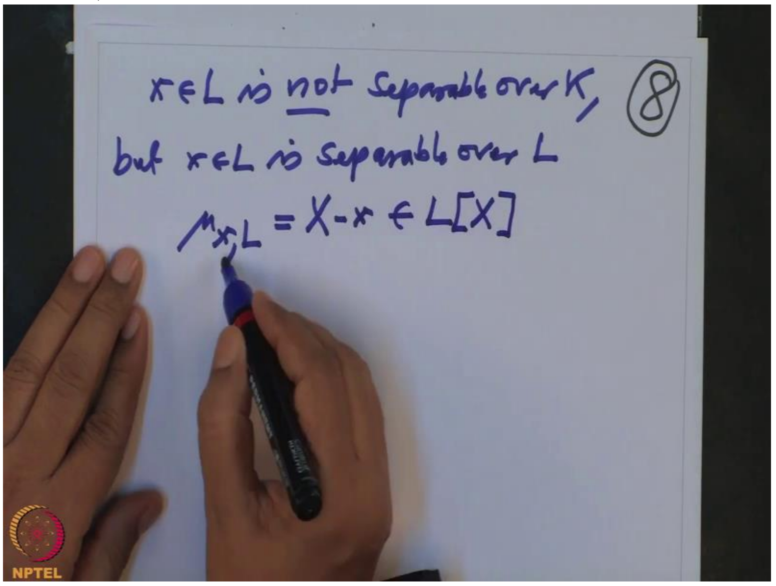


minimal polynomial of x over L which has only the simple zeroes.

So therefore element x is separable over L but the same element x is not separable over K. So this concept, separability of an element, it depends on the base field. It depends on the relative, so it is a relative notion.

So this was the comment I wanted to make, alright. Now to check the numerical criterion, I want; I am looking for easy checking condition which checks that the field extension is separable.

So for that I want to recall, recall that, so in earlier lecture we have considered

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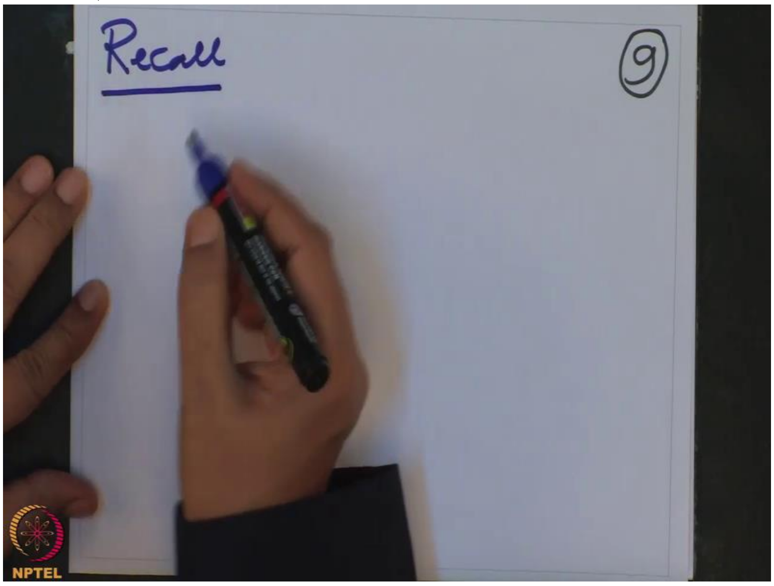


embeddings, K embeddings.

When we have a field extension L over K finite field extension, finite field extension, and

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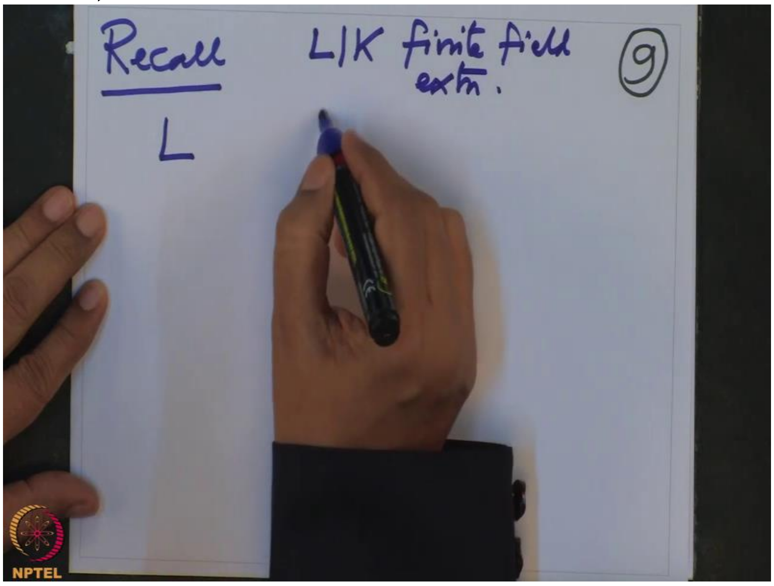


if we have any other extension E over K and E is algebraically closed, algebraically closed field extension.

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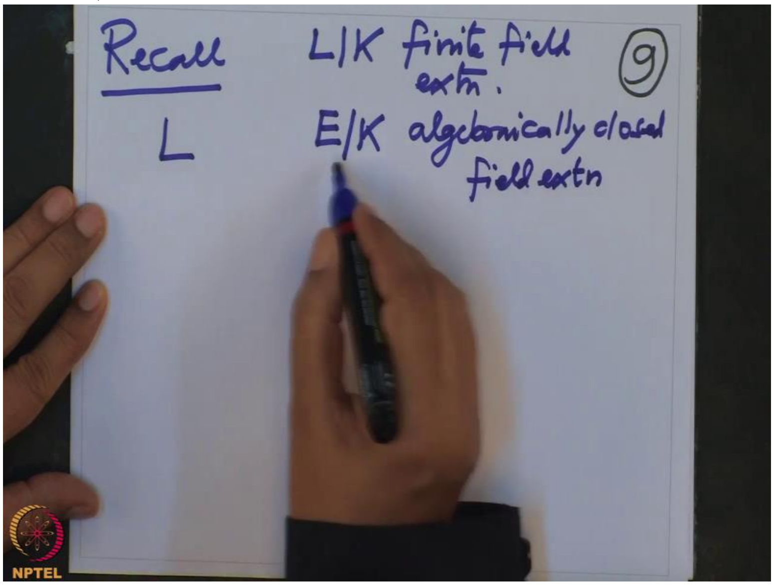


When I write like this that means E is algebraically closed and it is

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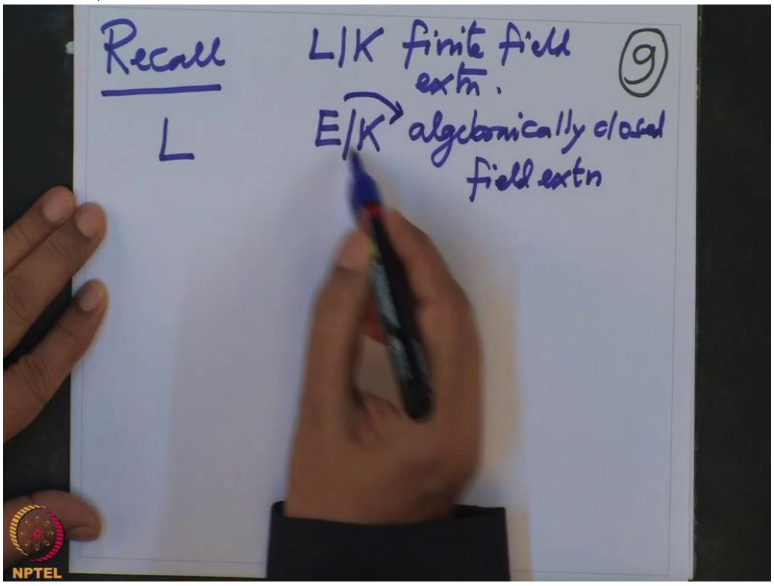


an extension of K. I am not saying E over K is algebraic, Ok so I will keep writing an algebraically closed field extension of K

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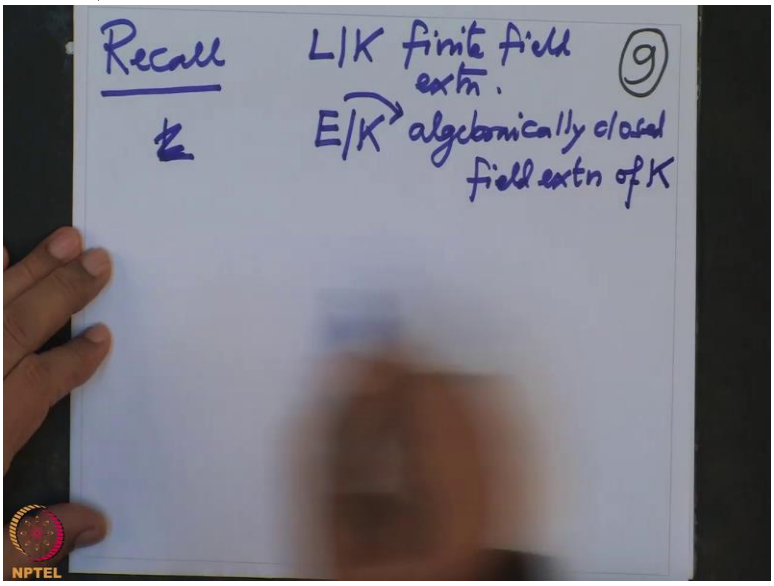


that may not be algebraical closure.

But definitely if I look at K is here,

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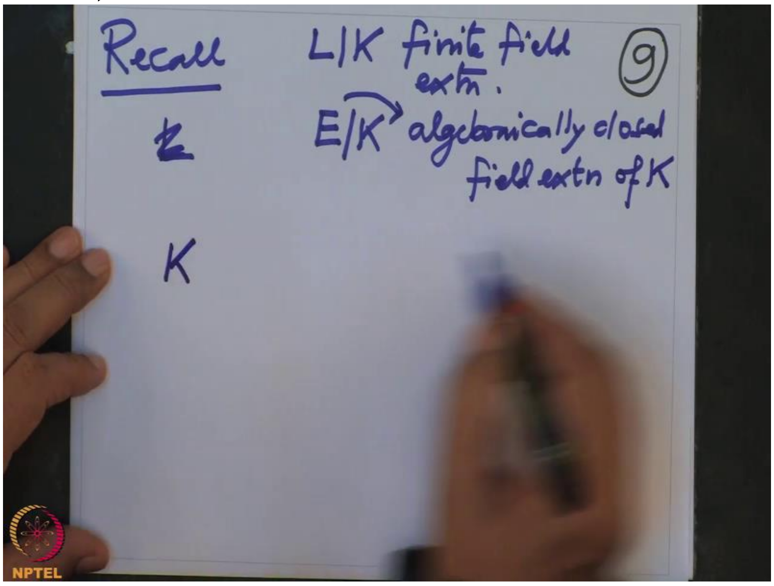


E is here

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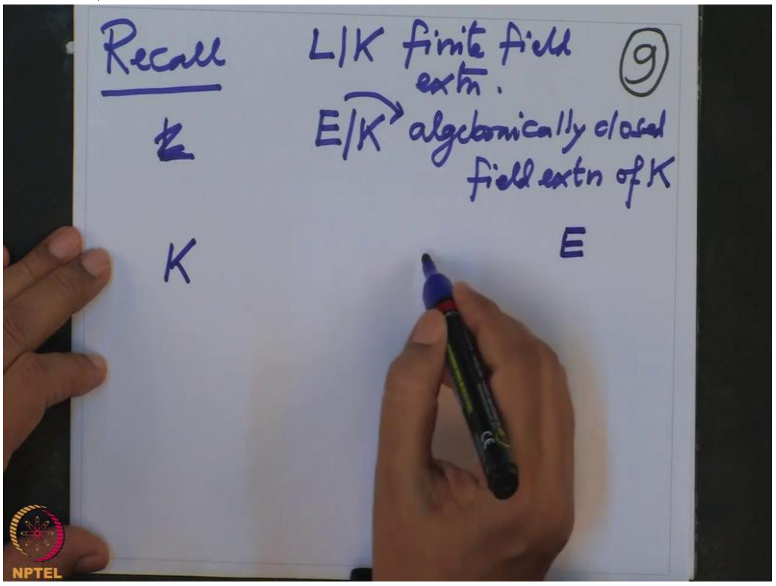


and if I look at the relative algebraic closure a l g, this is the set of

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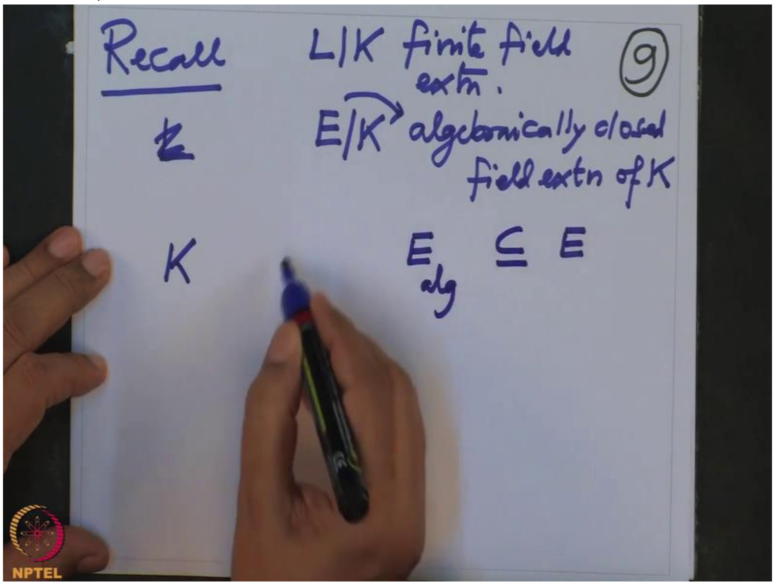


all algebraic

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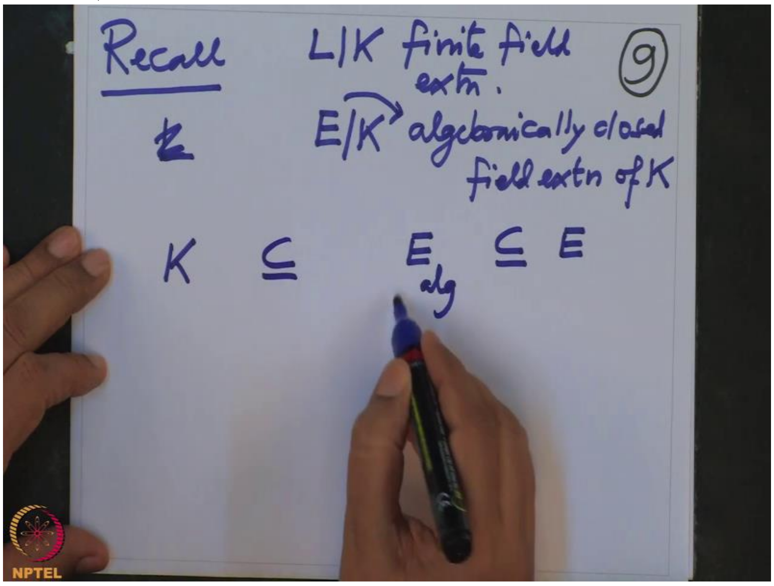


elements, all those x in E such that x is algebraic over K then

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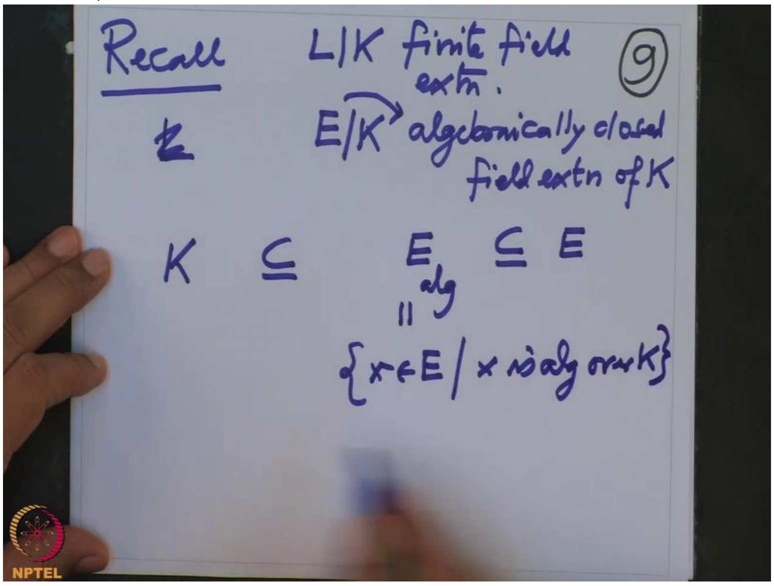


this E a l g is an algebraic closure of K inside in E.

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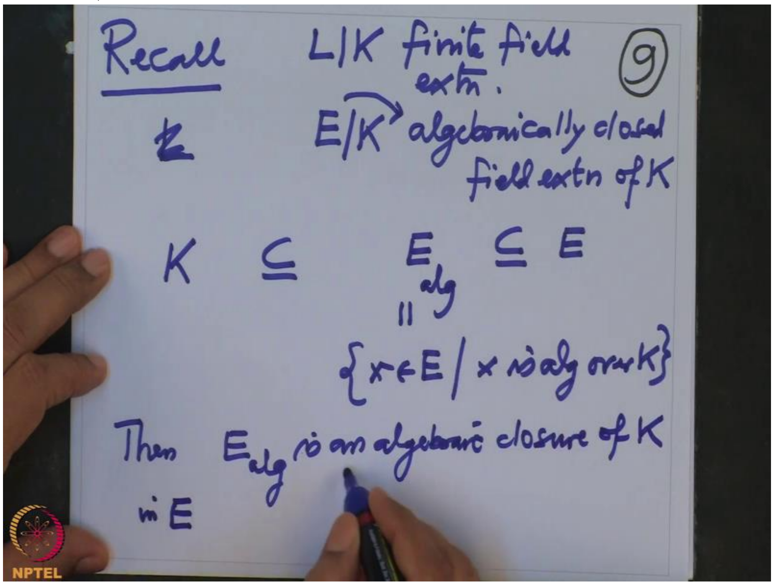


So this is easy to check, so easy to check. So what do we need to check?

Let me spell out what do we need to check.

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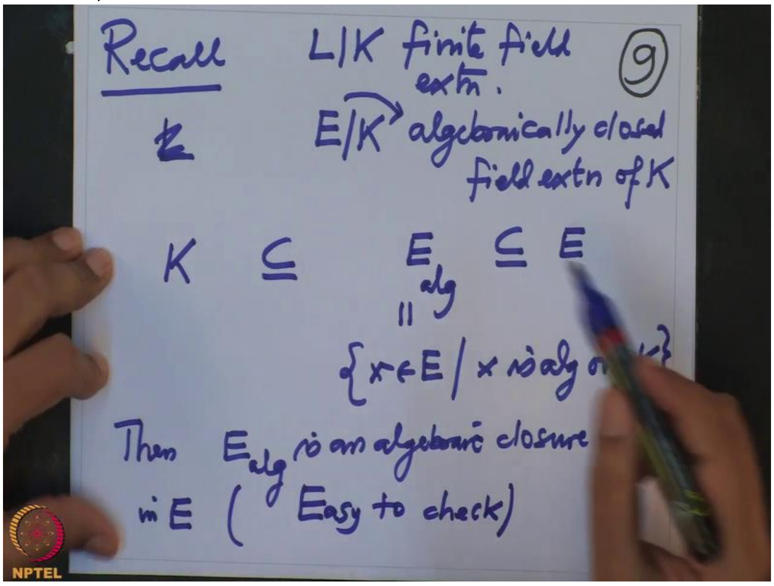


First of all, because this extension is now algebraic, because I have taken only those elements which are

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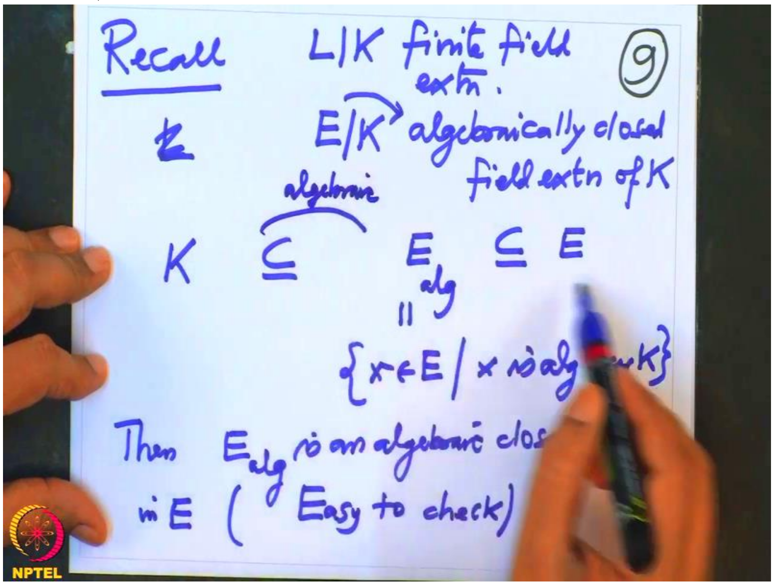


algebraic over K. So this extension is algebraic.

All that we

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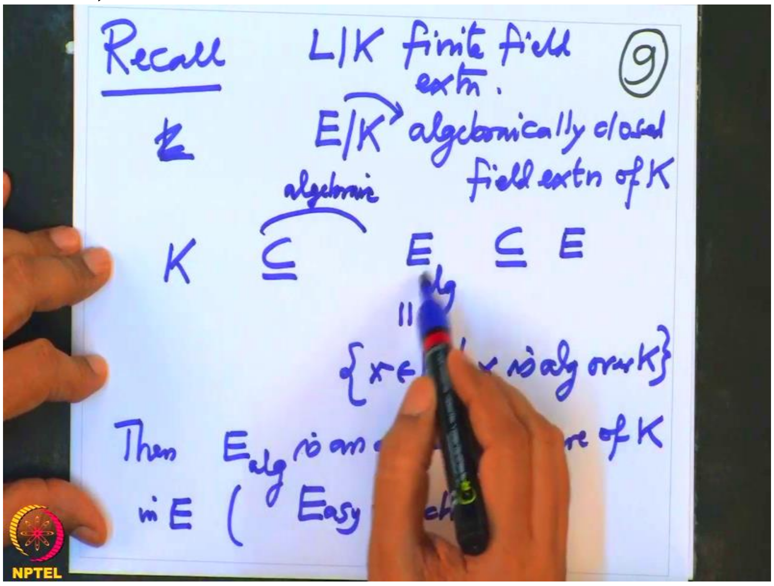


need to check is that this field, this is a field also, we know. So this field is algebraically closed. That is what we need to check.

And how does one check the field is algebraically closed?

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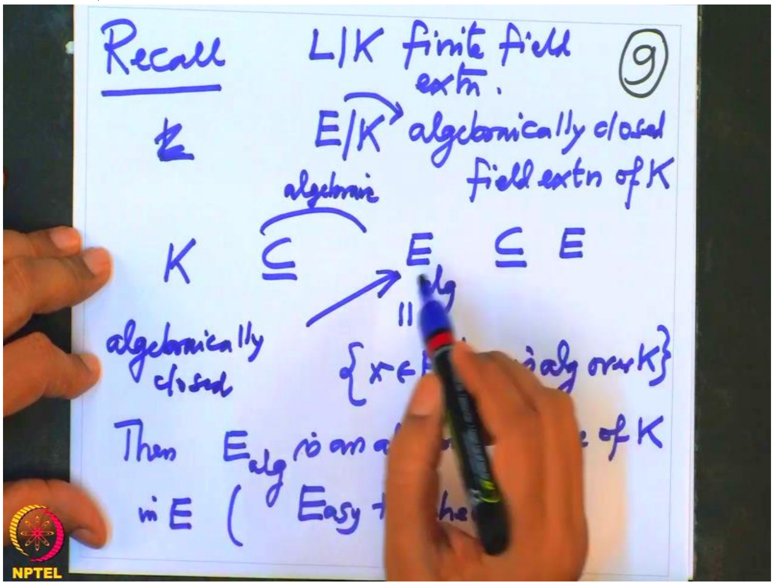


We take a polynomial with coefficients in that field. And we want to check that it has a 0 inside this field. That is enough to check.

Or in other words we want to check that any irreducible polynomial here is linear. So if you take any, any element here we need to check that, we need to check that this field is algebraically closed.

That means you take a polynomial f and consider that

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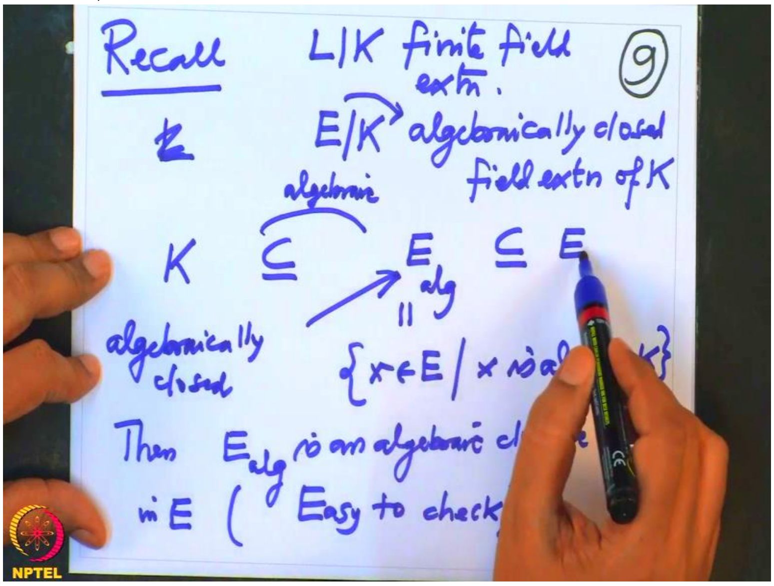


as an polynomial in with coefficients in E, so and E is algebraically closed. Therefore definitely that polynomial has a 0 here. We want to prove that, that 0 is

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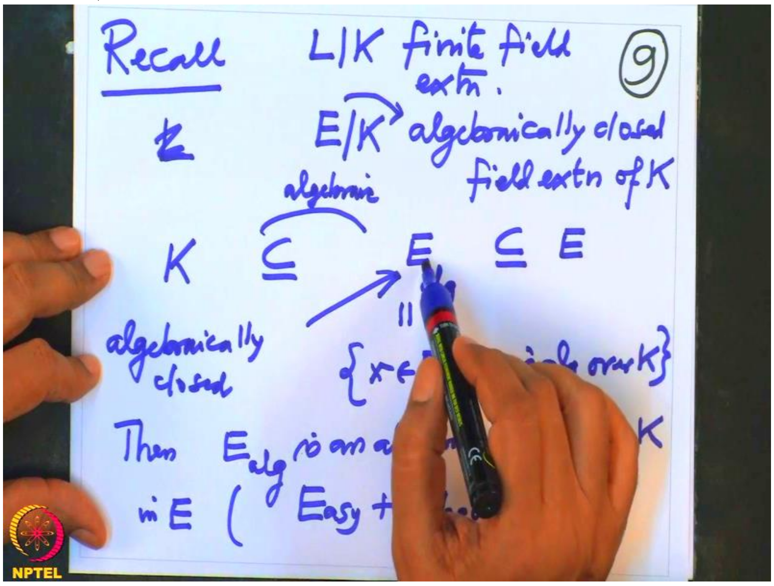


automatically here.

But that will follow from the transitivity of algebraic elements, so which I will not write down the proof here. This is easy to check that, this is one way to restrict it to algebraic closure of K, alright.

So now we want to do that? So we have considered the embeddings, embeddings of, K embeddings of L inside E. Remember E is algebraically closed.

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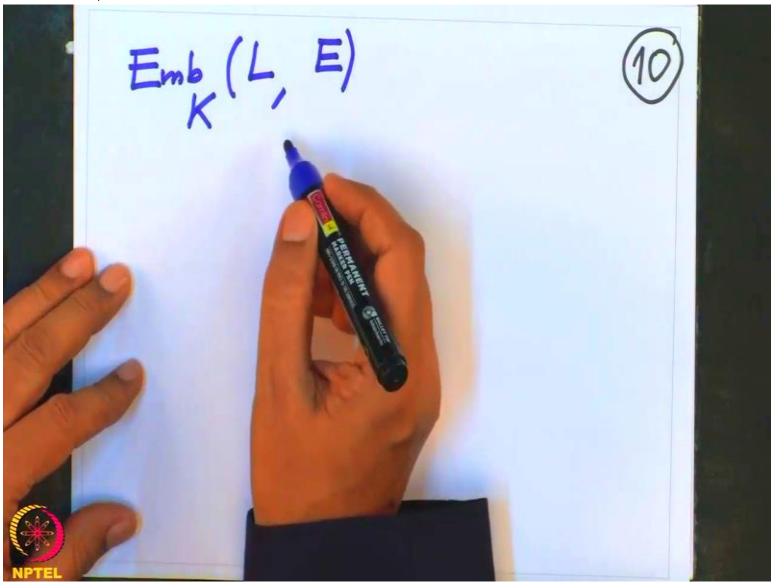


So K embed, this set I want to, I want to prove the theorem, this theorem I want to prove.

This theorem has

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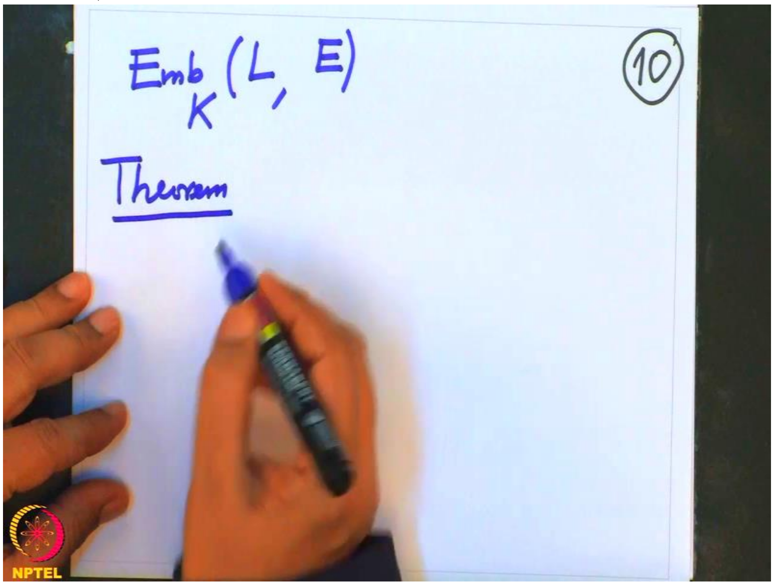


two parts. Part 1, the number of embeddings of L in E, this number smaller equal to the degree of L over K

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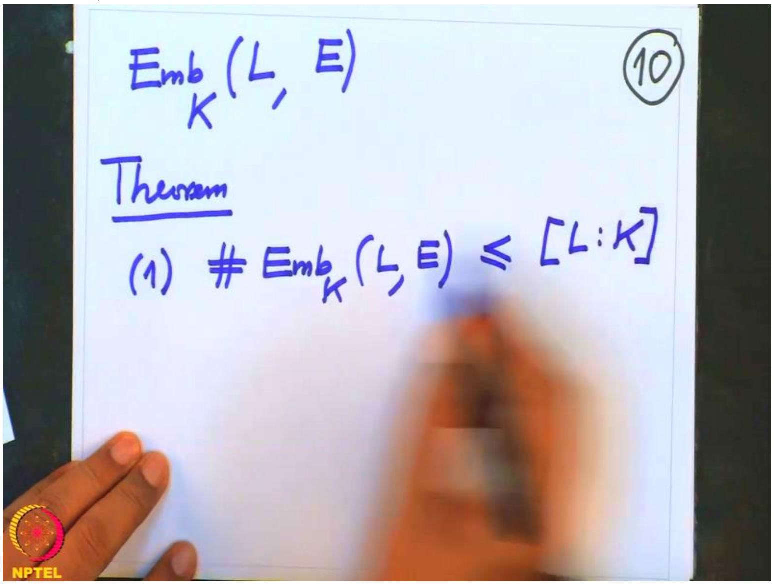


and this is L over K finite extension and E over K algebraically closed field extension. This is part 1

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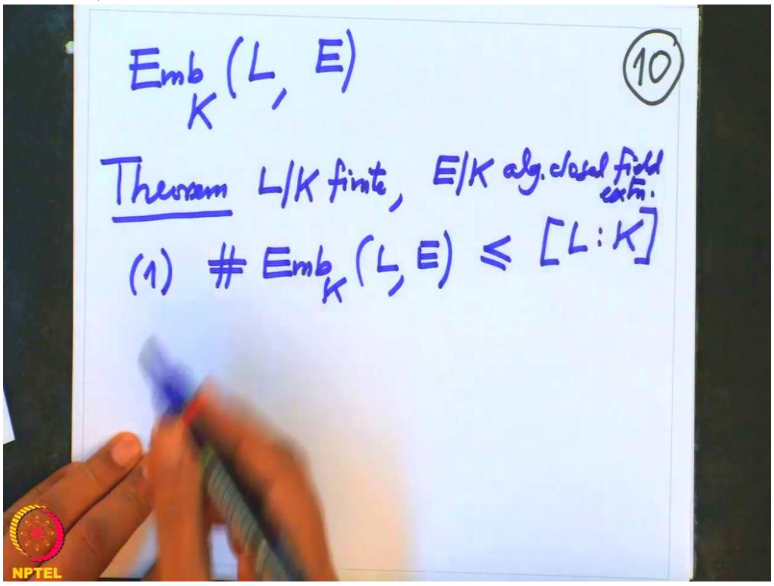


And part 2, I want to show that this number, number of embeddings is independent of E over K. That means that is, if E prime over K prime is another algebraically closed field extension, then these two numbers are equal, number of K embeddings of L in E is same thing as number of K embeddings of L in E prime.

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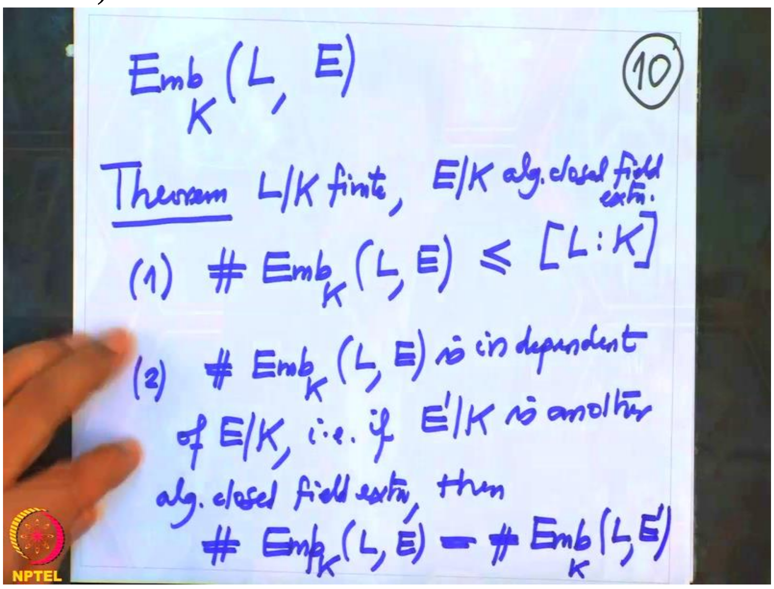


And then that number is called the degree of separability of L over K if I prove this.

So let us prove at least first part. So it is very easy and proof, what I will use is

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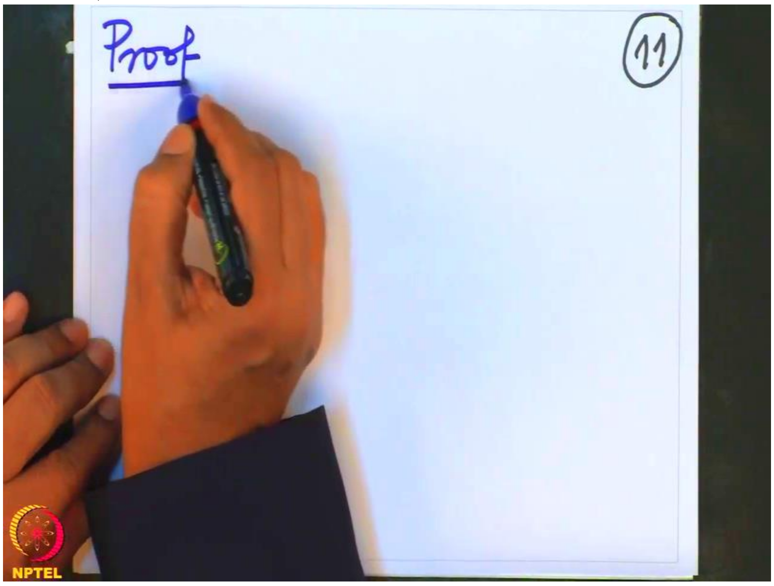


old long time back we have proved Dedekind and Artin lemma and I will use that again here. So note that what are the embeddings.

The embeddings of L in E, they are precisely the K-algebra homomorphisms from L to E

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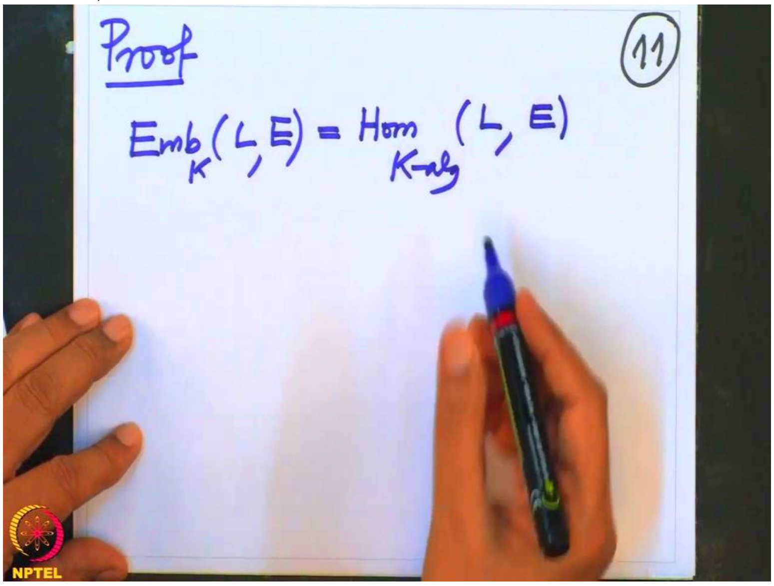


and this is a subset of K linear maps from L to E, because K-algebra means it is a ring homomorphism and K linear and this is just K linear map, they need not respect the multiplication.

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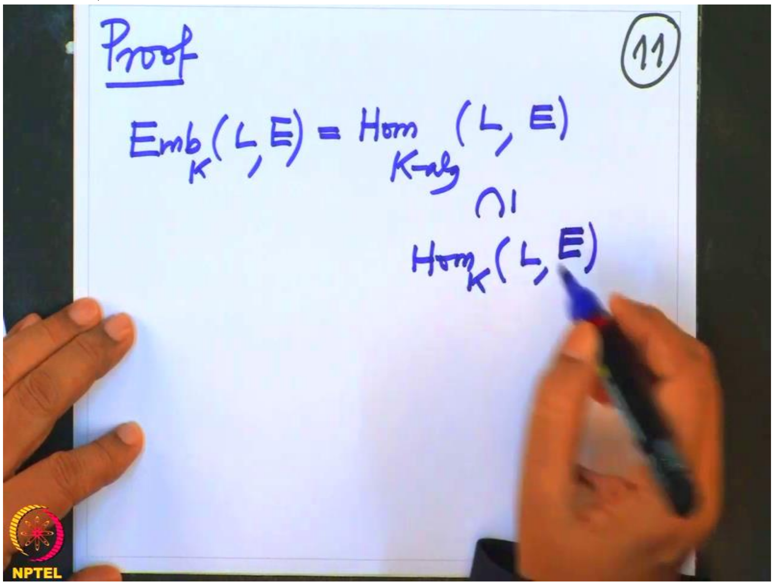


So this is clearly subset here and what Dedekind and Artin say? Dedekind and Artin say, Dedekind and Artin lemma, that says that

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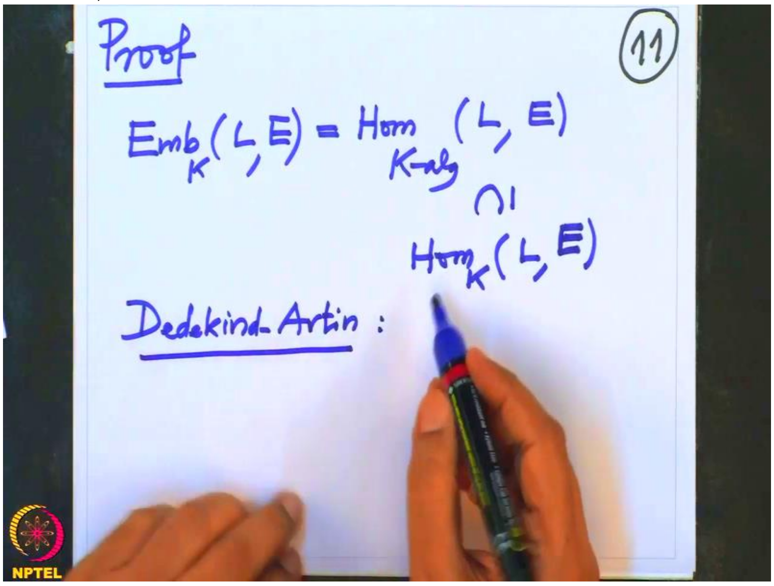


this set Hom K-algebra of L in E, this set is linearly independent subset, linear independent over K, over E not over K, over E subset of the E vector space Hom K L E,

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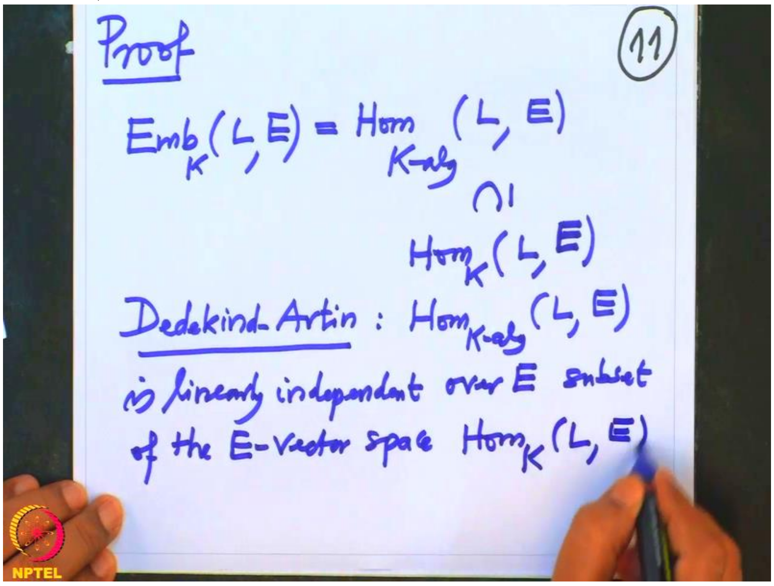


this.

This is E vector, not only K vector space, this is E vector space and what was the vector space multiplication?

That is if I have z in E and a K linear map f L to E, K linear then how did we

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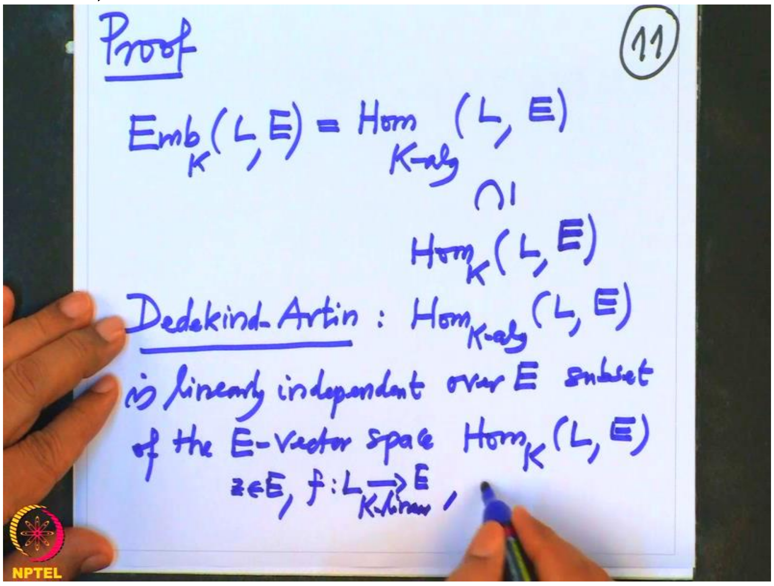


define z times f, this on only x in L? It is defined z times f x

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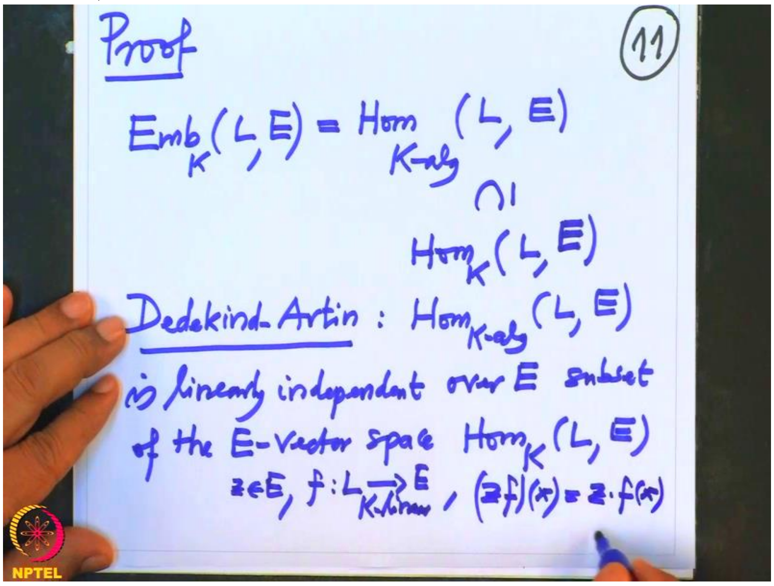


for every x in L. This is z is in E, x is in E therefore

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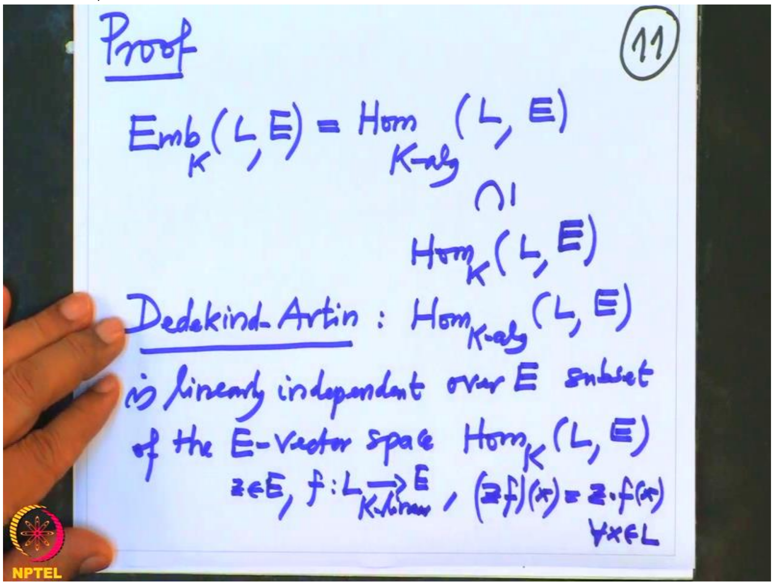


this is defined with a multiplication in E.

So these are linearly independent, particular the cardinality

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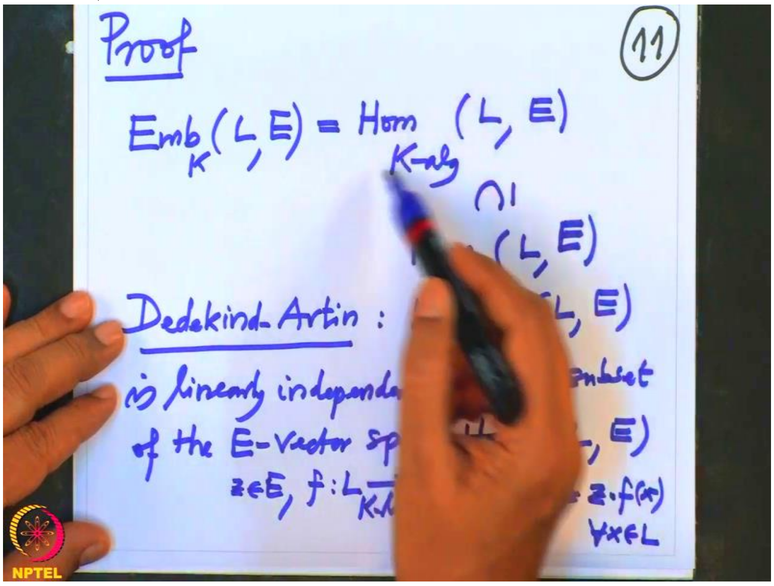


of this set will be less equal to

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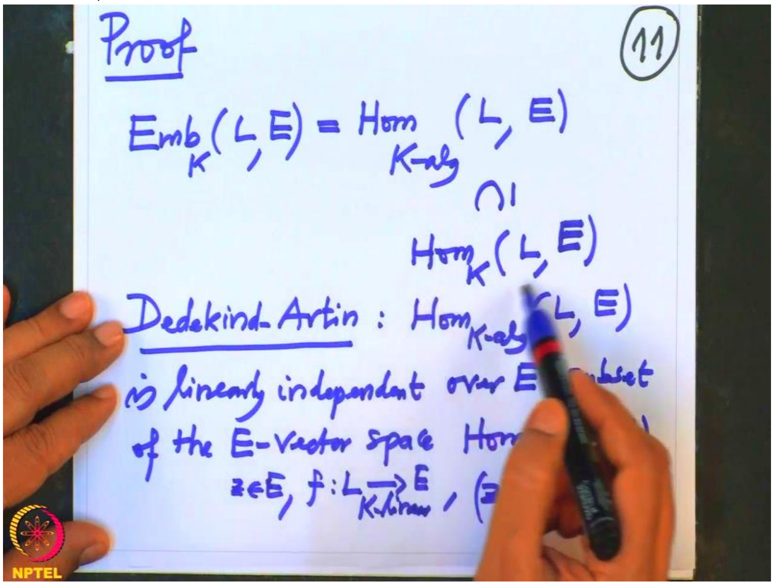


cardinality, dimension of this vector space.

So in particular cardinality of Hom K-algebra L E, this cardinality

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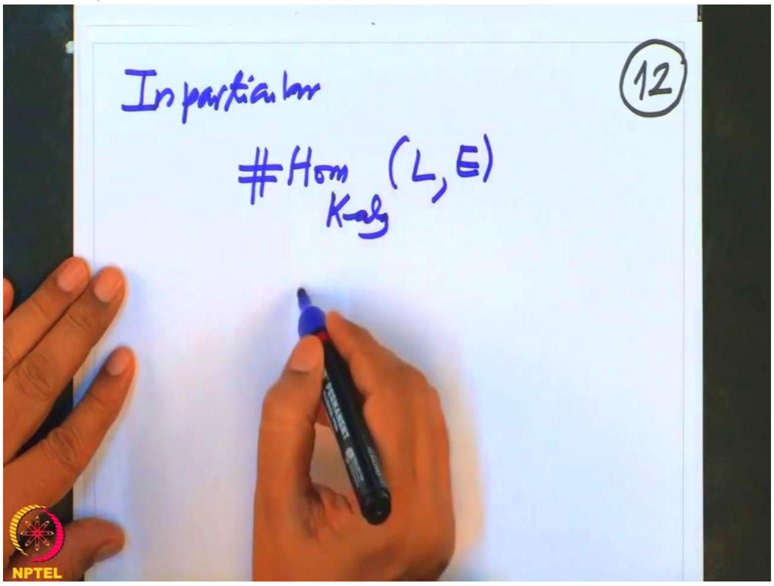


will be less equal to dimension of, E dimension of the vector space Hom K L E

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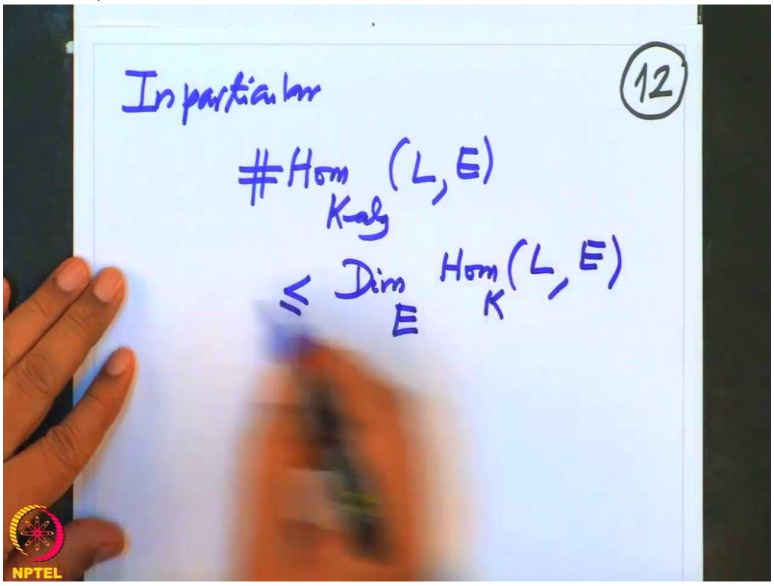


and this is precisely the number of embeddings.

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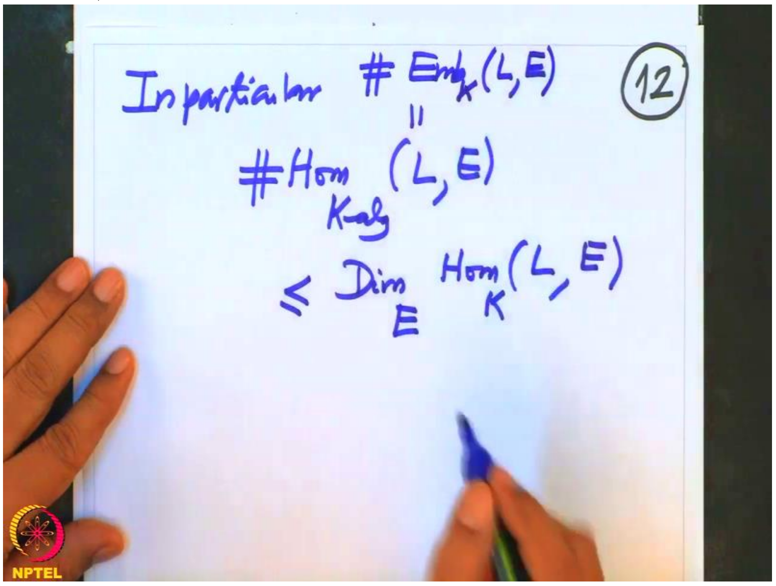


And this, what is this dimension? I want to compute. So let us compute this dimension, this is equal, I claim that this dimension is precisely equal to the degree L over K.

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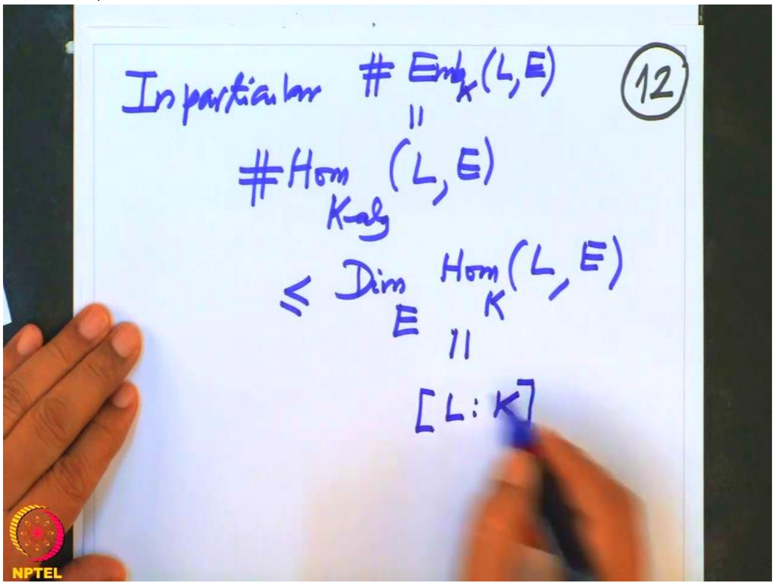


This equality follows from the following. So look at this vector space, Hom there are K linear maps on K to E.

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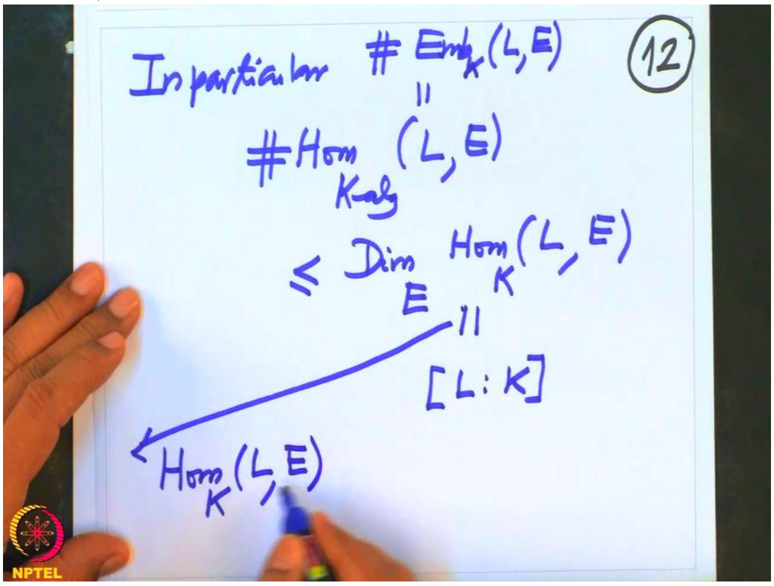


But E, L is a finite extension therefore L as a vector space is isomorphic to K power L, degree of L, degree of L over K because

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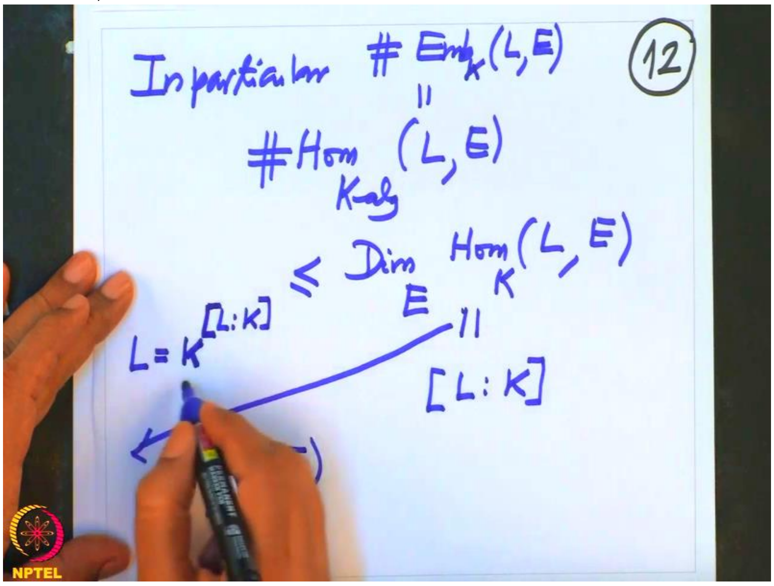


this L is a vector space of this dimension.

So therefore this is same thing as Hom K K power L over K E but

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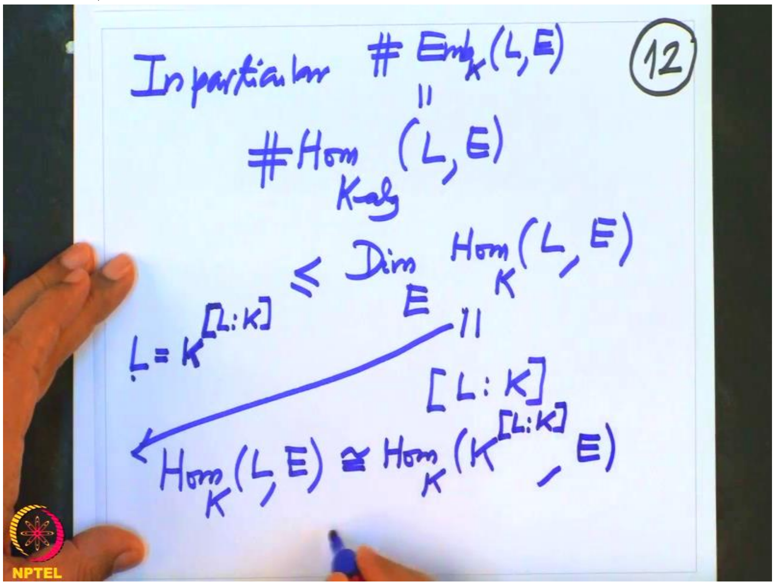


this is isomorphic to, clearly this degree will come out. This is Hom K K E power L over K degree

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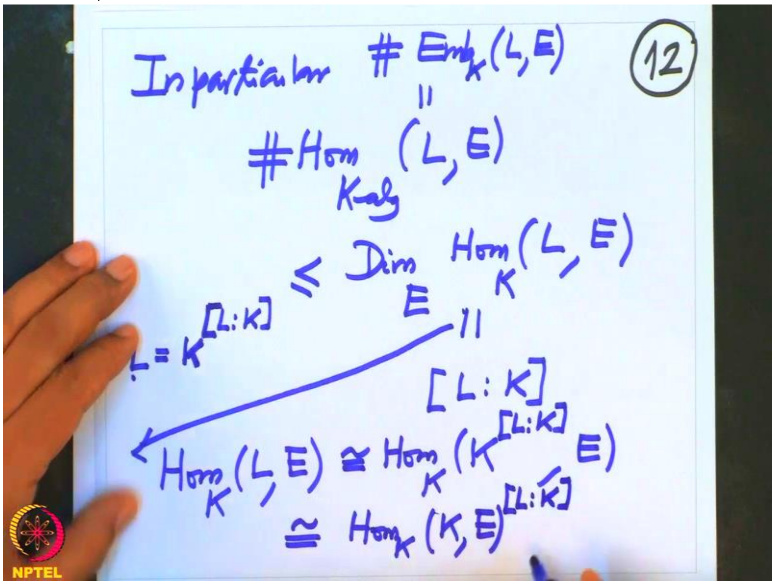


but this is, this is isomorphic to E. So this is E power L over K

So it is

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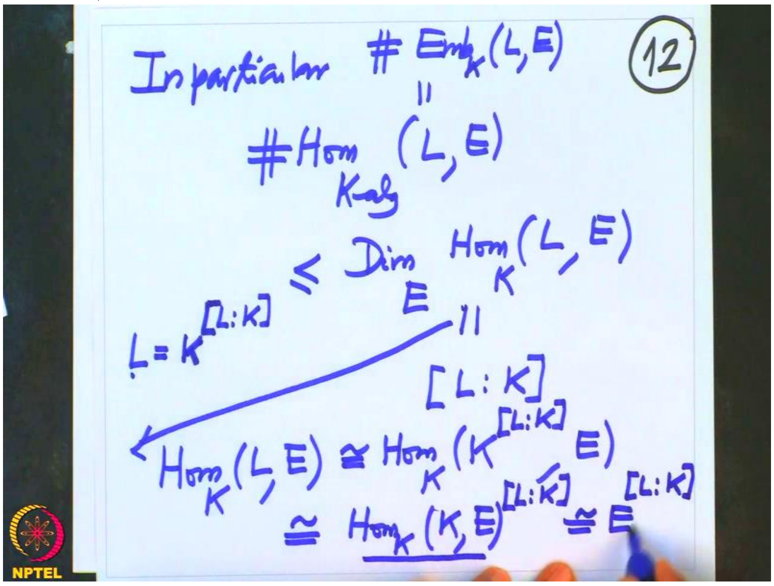


vector space E power, so all these isomorphisms are, isomorphism as, this is isomorphism as K vector spaces,

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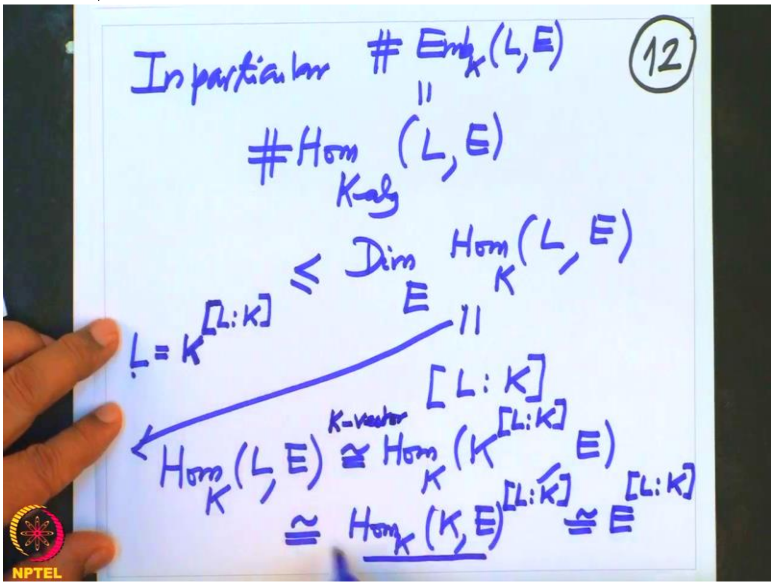


and this is isomorphism as K vector space and the last one is E isomorphism. Therefore

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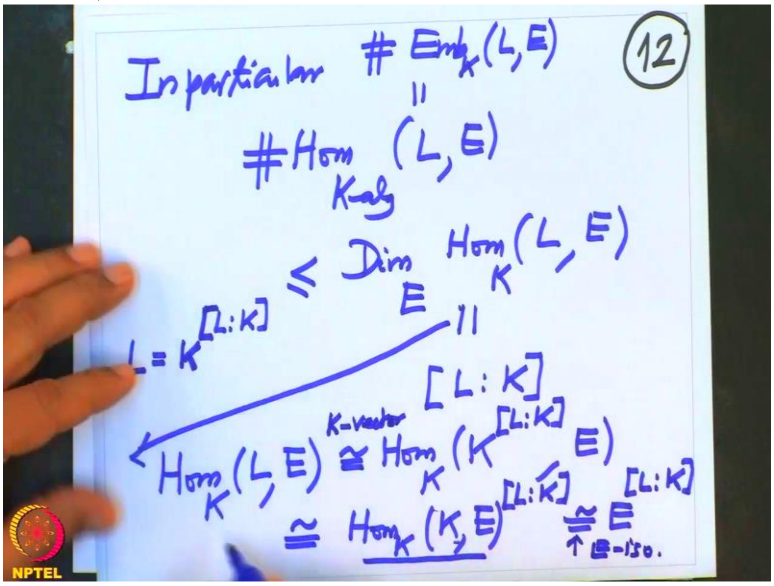


altogether this will be all E isomorphism.

These are all E isomorphisms, sorry E.

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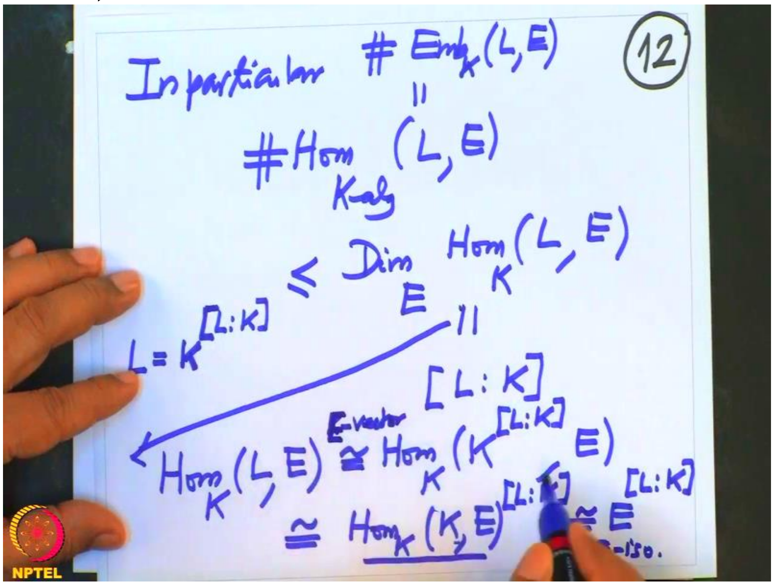


This is also E isomorphism,

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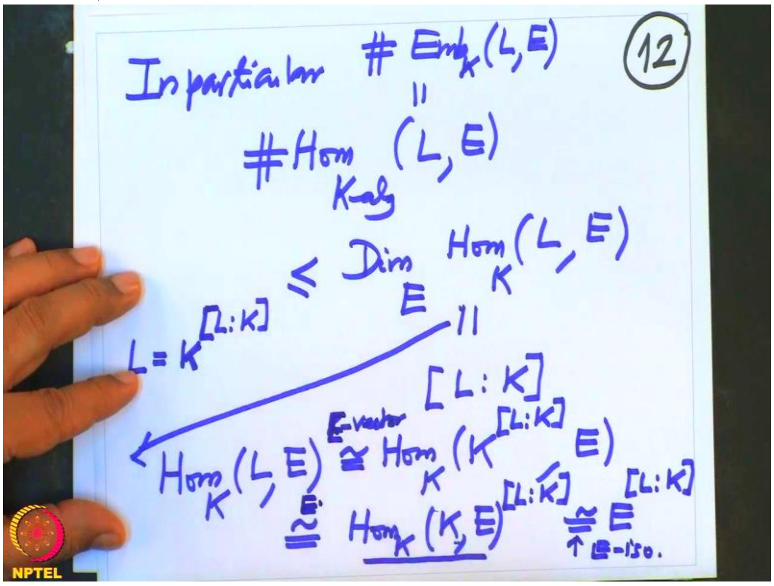


this is also E isomorphism therefore the, this dimension equal to dimension of this but which is clear that L over K,

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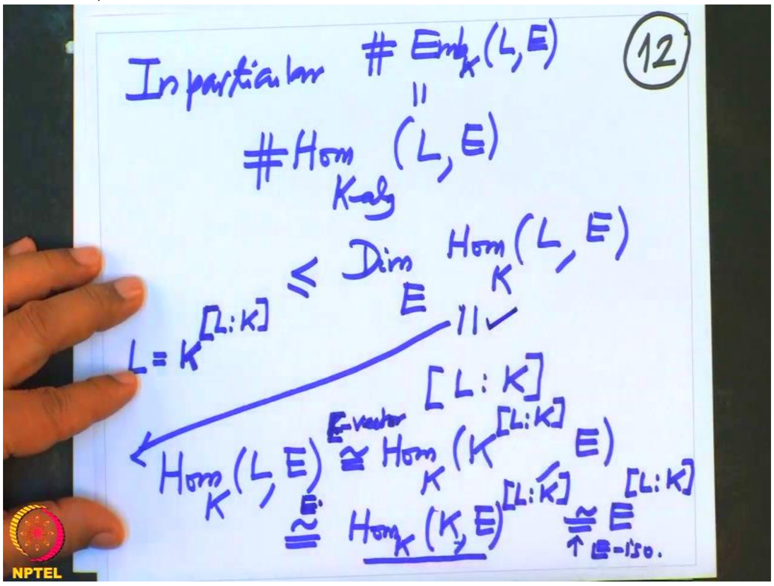


therefore we know this.

So therefore we proved the equality in, inequality in 1 and I will indicate the, inequality now 2. Remember we now want to prove that this is independent of E algebraically closed extension of K. So I want to prove that, so this is the proof of 2.

The cardinality of embeddings of L in E or

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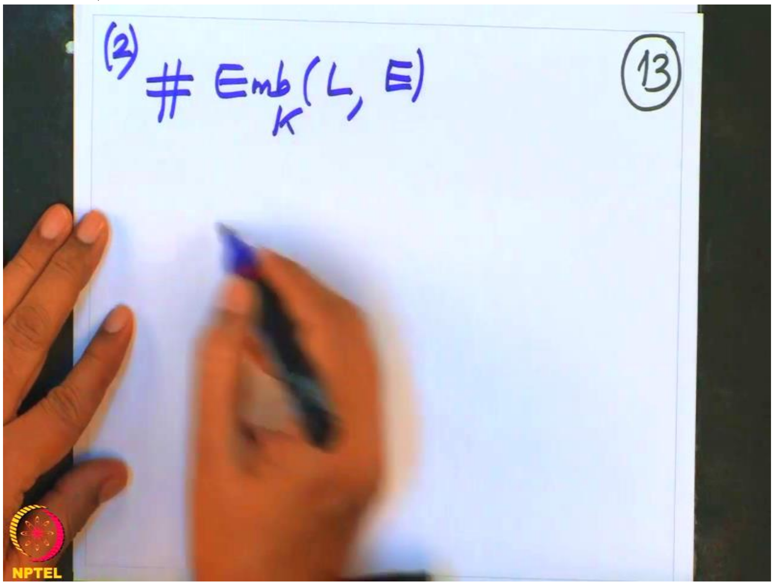


cardinality of the embeddings of L in some other algebraically closed extension; I want

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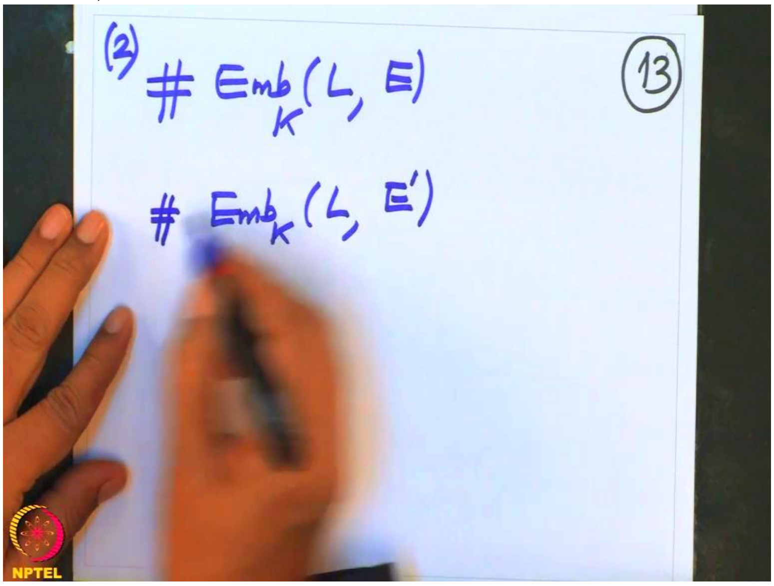


to prove these are equal,

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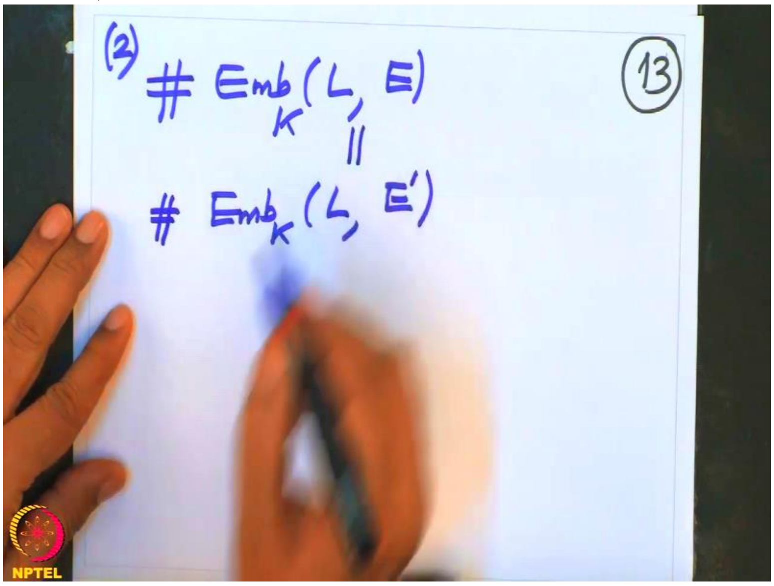


Ok.

But that is also very simple because now look at the embedding sigma L in E. So

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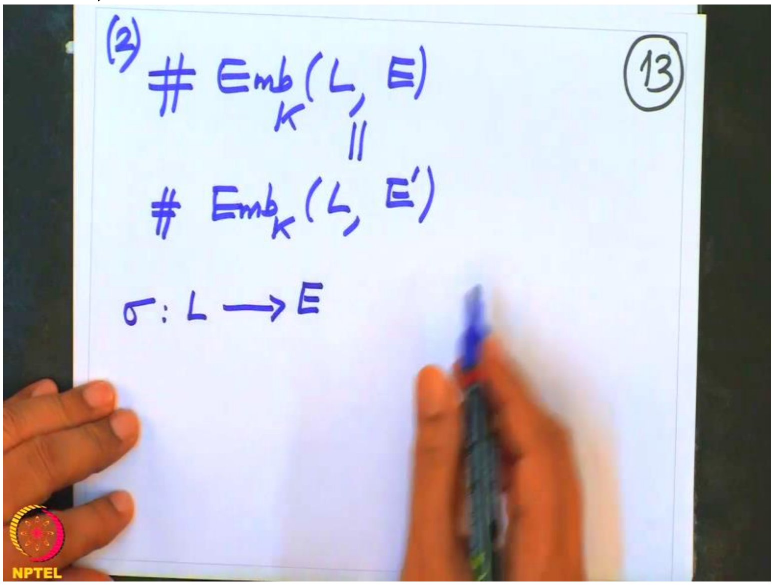


look at the image in sigma that is sigma L. So L is isomorphic to sigma L and this is contained in E.

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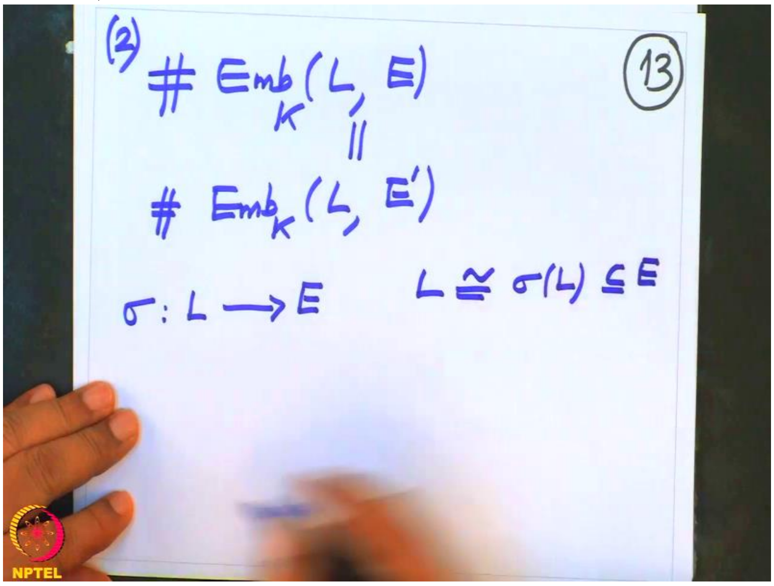


So I am going to replace and this one we have E a l g here, what is E a l g?

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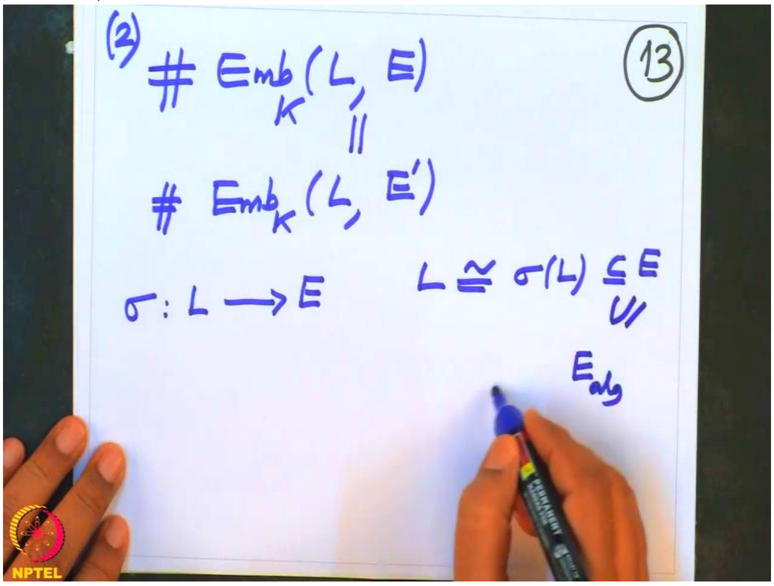


That is an algebraic closure of

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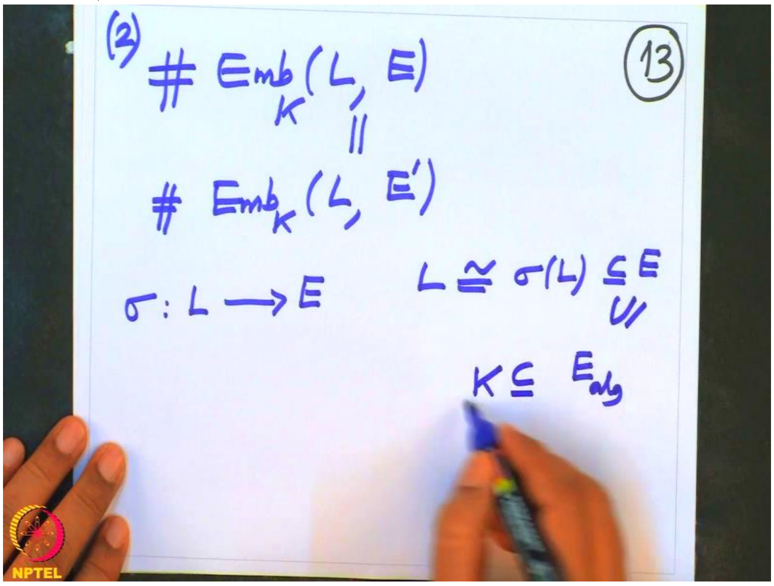


K in E. This is algebraic closure of K in E. We have considered it earlier also. Now because L is finite,

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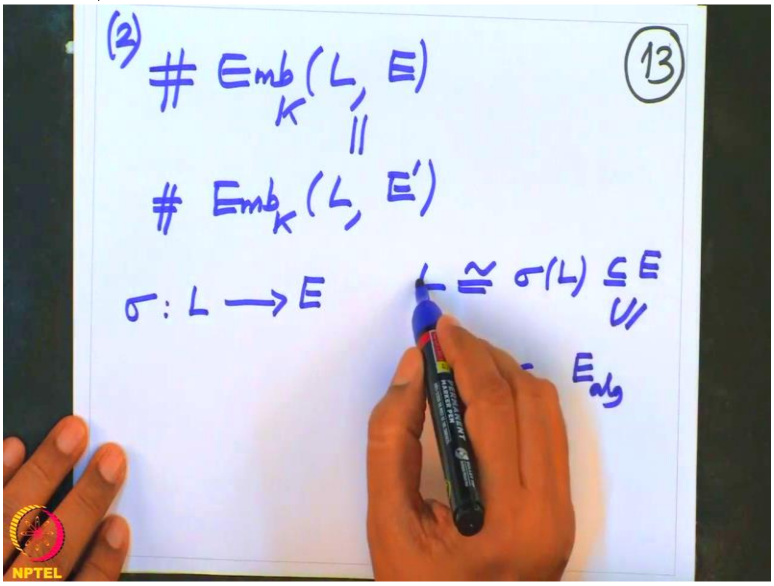


this is algebraic over K.

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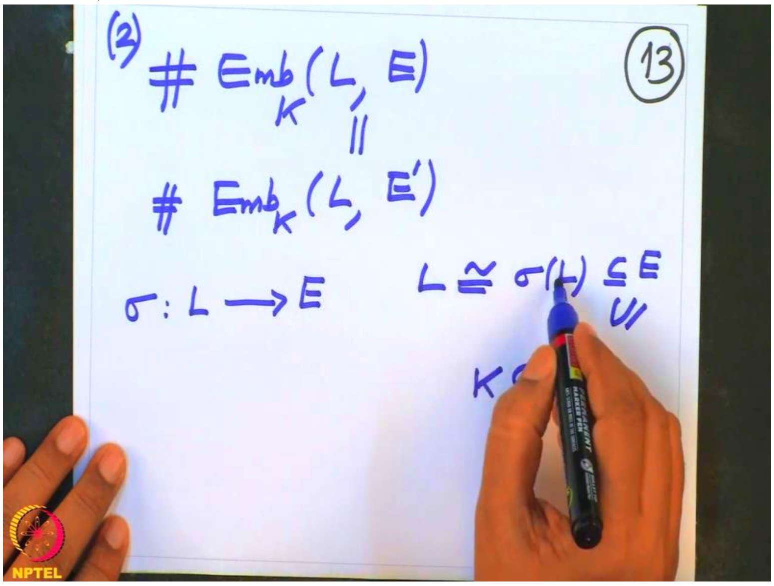


Sigma L is an element,

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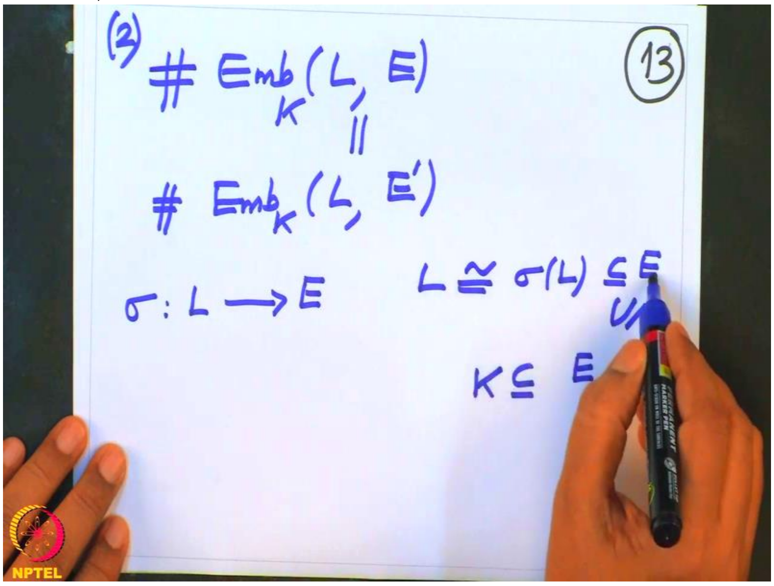


elements of E which are algebraic over K because the elements of L are algebraic over K, this extension is algebraic.

Therefore this element, this sigma L is actually contained here.

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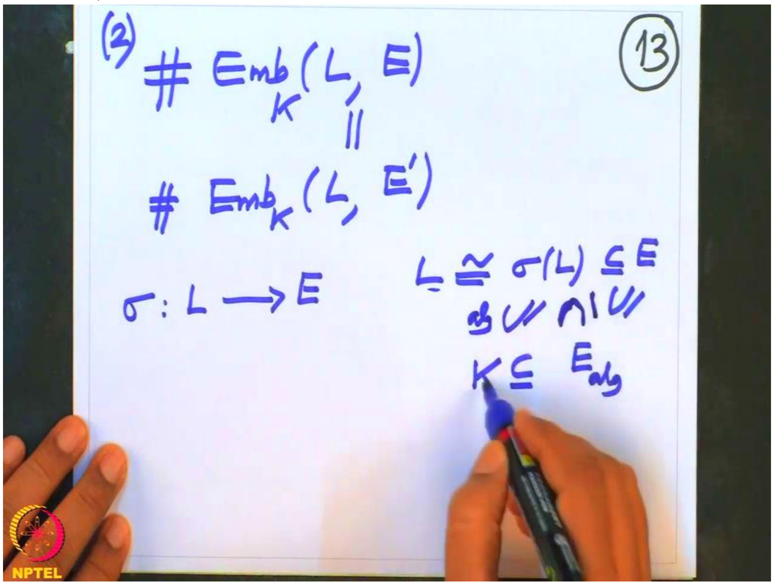


Therefore I am going to replace, this is true for every sigma. So replace E by E a l g

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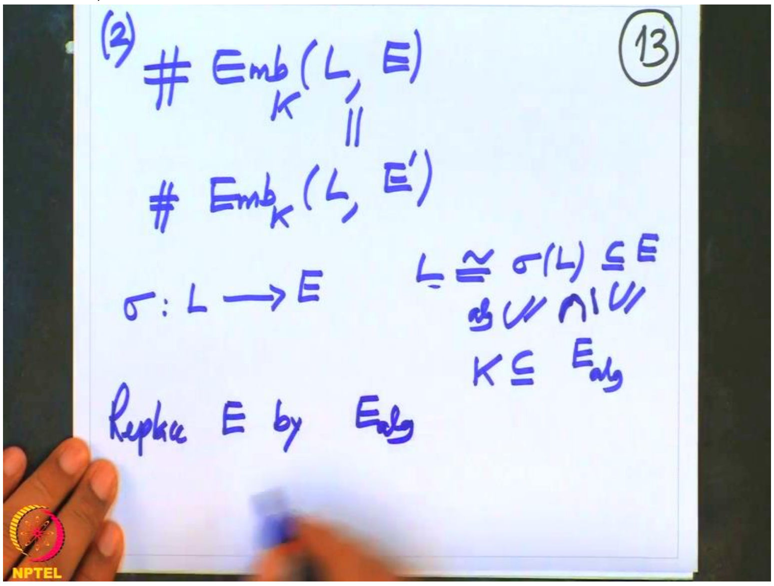


and E prime by E prime a l g.

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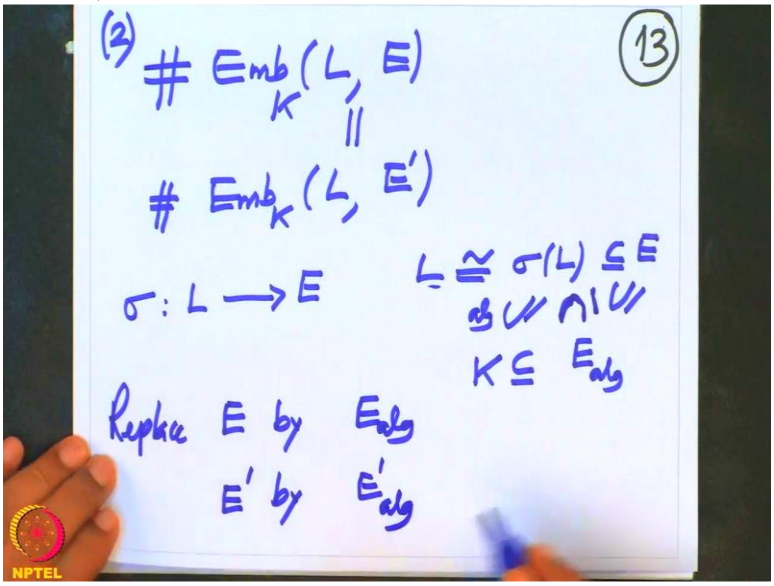


The advantage is now these are algebraic closures of K.

These are algebraic closures of K and

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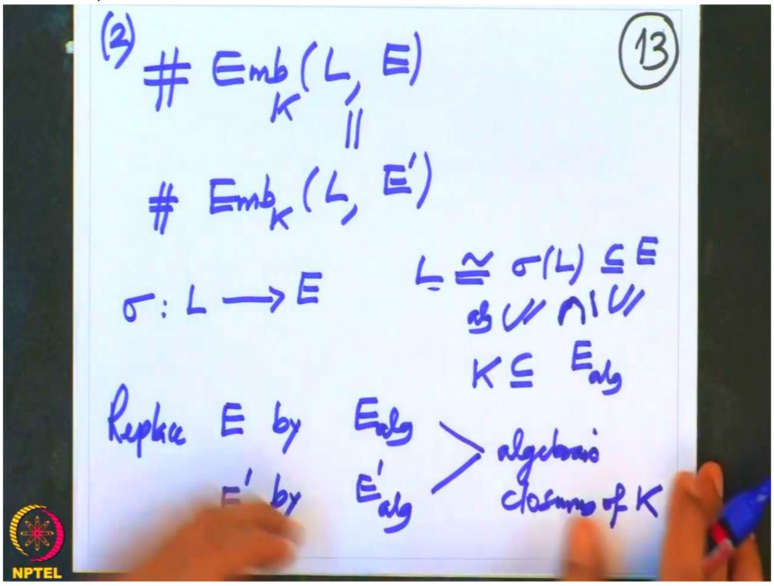


also the embeddings, the number of embeddings I have not changed. And also note that E embeddings, K embeddings of L in E

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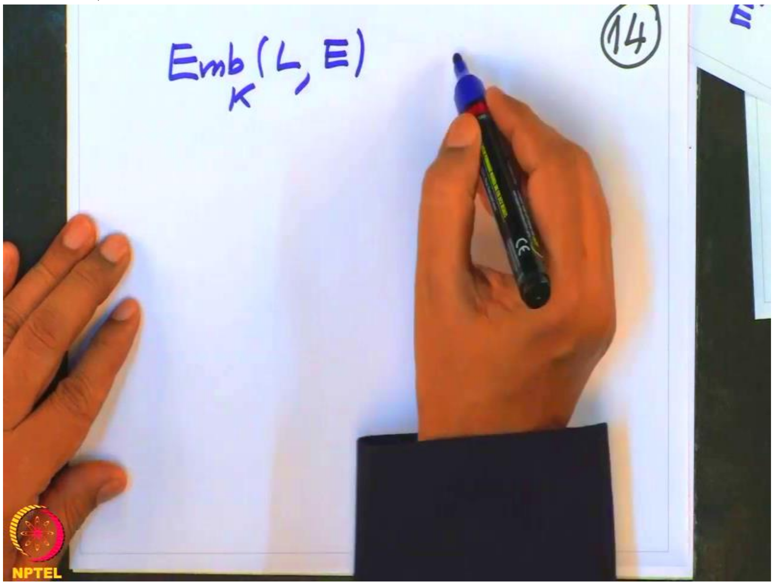


and K embeddings of L in E a l g,

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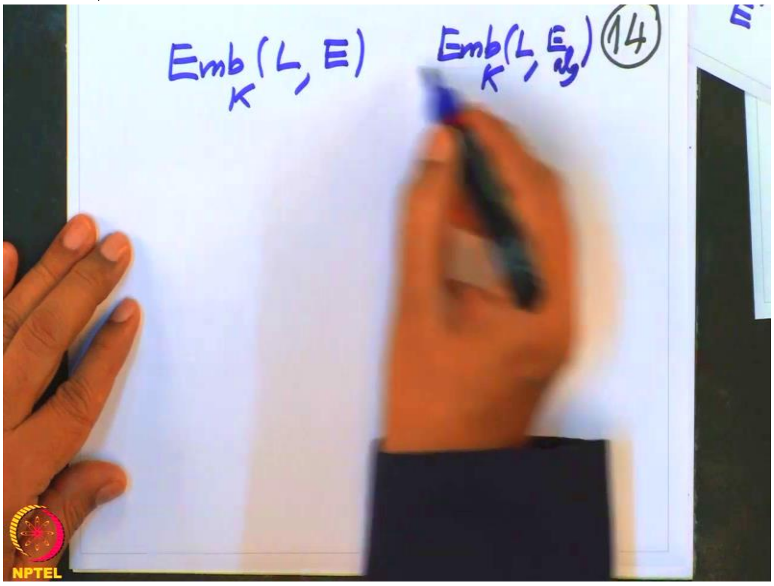


this is just a

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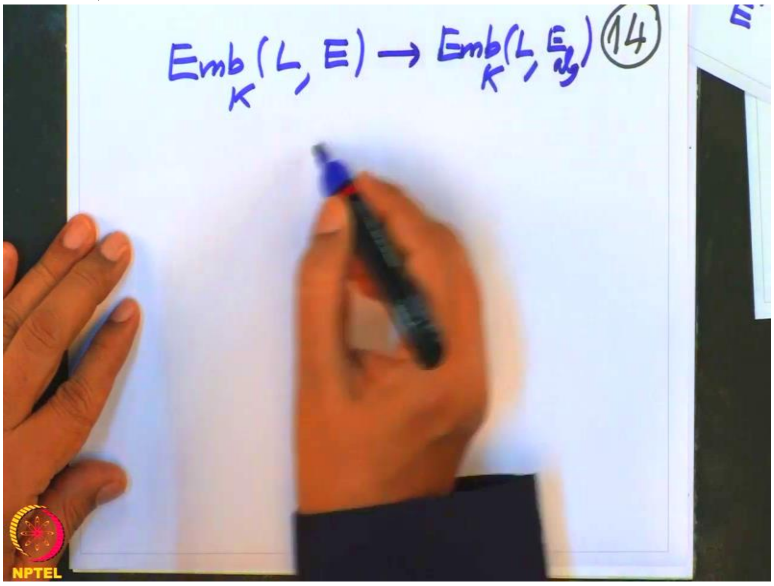


restriction, this sigma going to the restriction.

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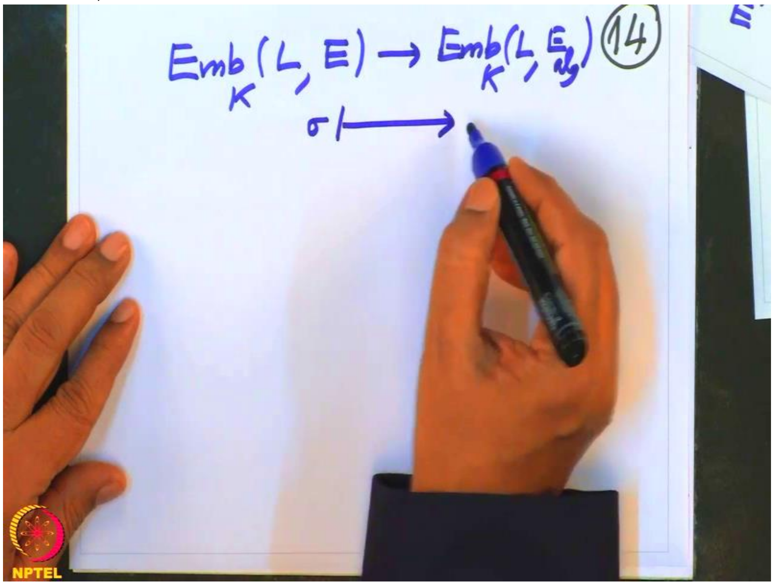


Now this is the other way. So this is, these are bijectives.

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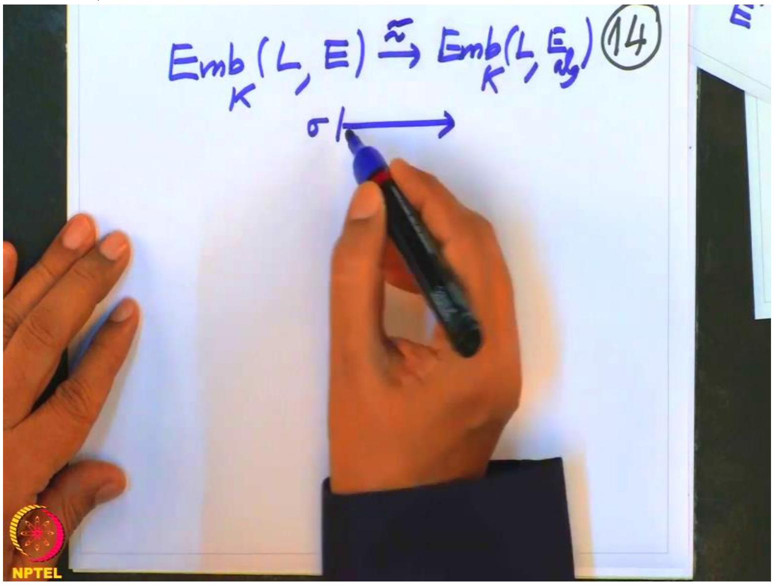


This has not changed. Similarly the other. So I can assume E, so we may assume, what is the advantage? We may assume therefore that E and E prime are two algebraic closures of K.

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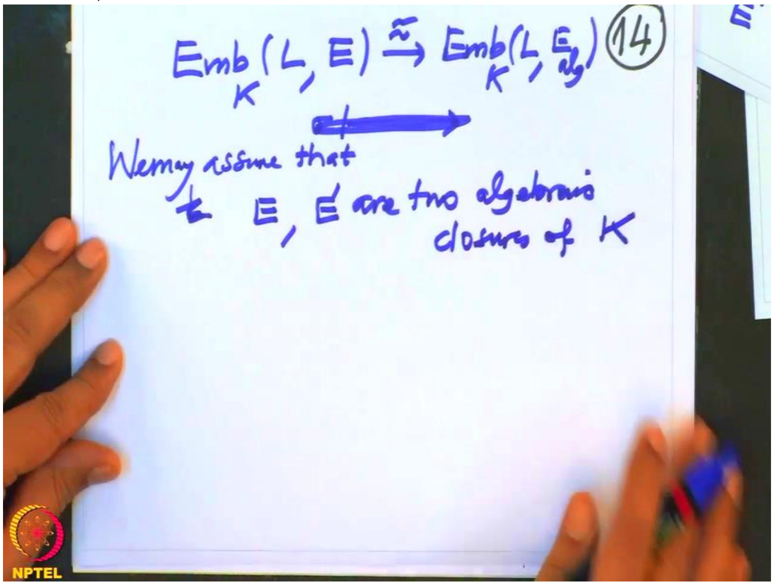


But then we know by Steinitz Theorem that there is an isomorphism. So therefore by Steinitz Theorem, E and E prime, they are isomorphic over K; that is there exists a K isomorphism rho

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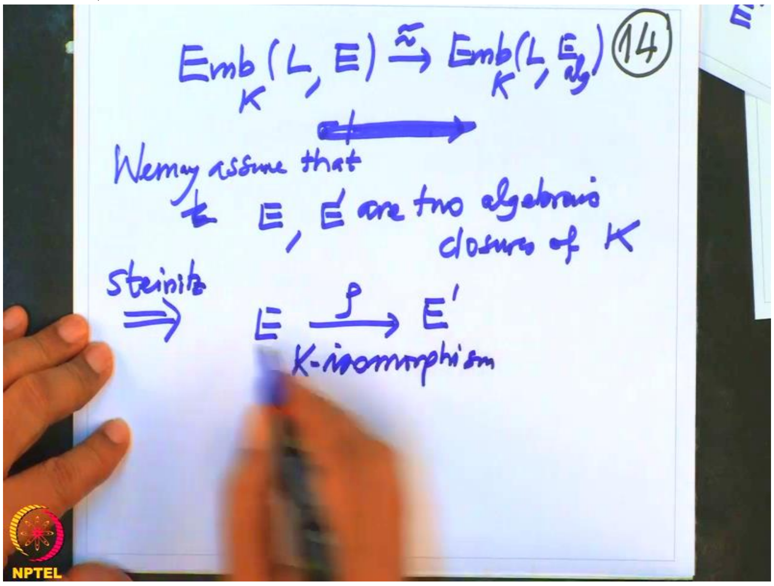


from E to E prime.

This we know rho. But then we can give a bijective map from embeddings of L in E to embeddings of L in E prime.

This is just a map,

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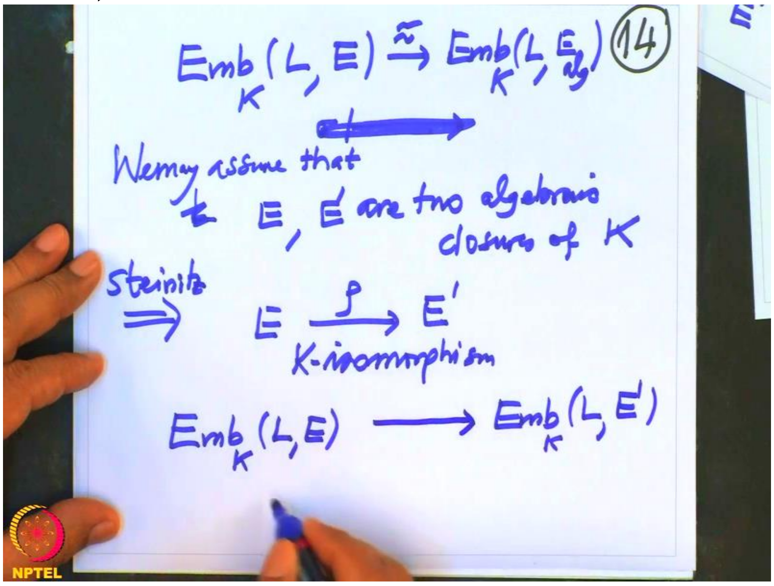


any sigma here going to rho compose sigma.

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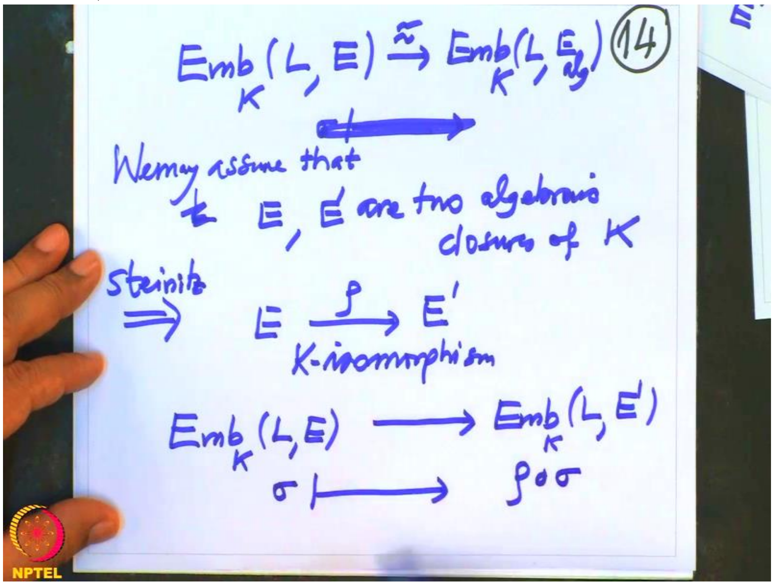


And obviously it is bijective because other way map is if we have tau here; that will be tau inverse, tau of, rho inverse of tau.

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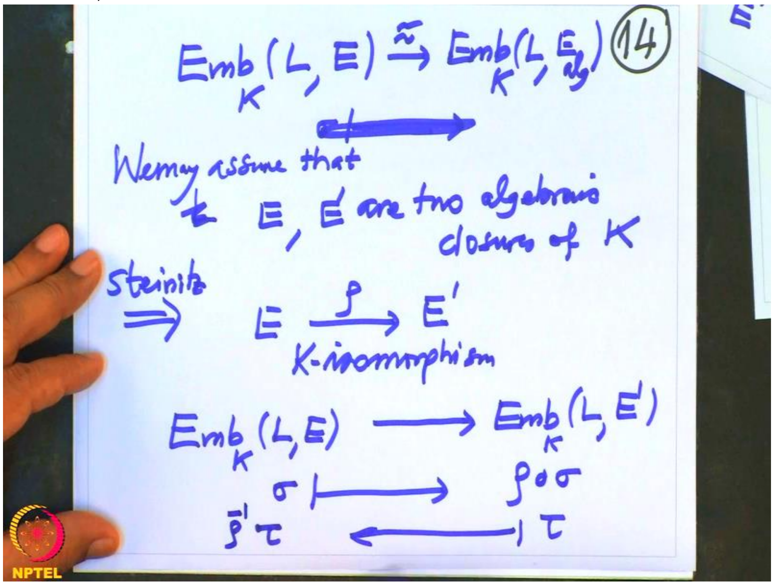


This is therefore bijective

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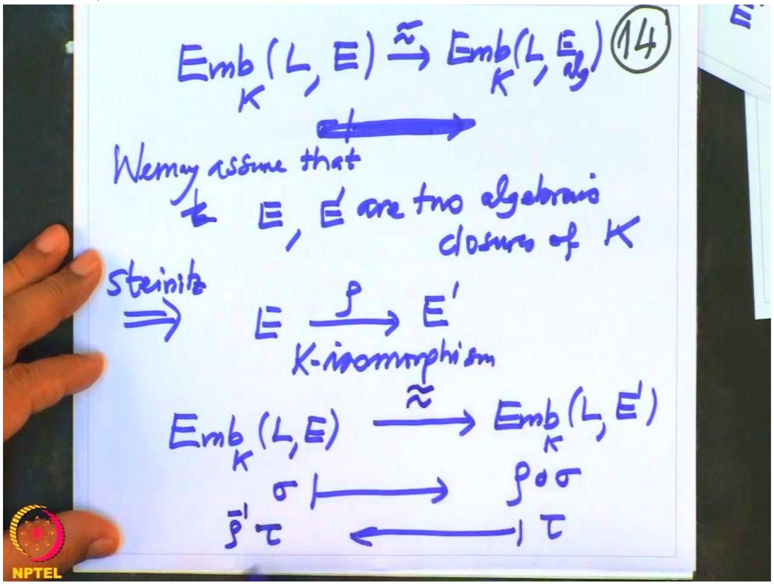


map. Therefore in particular their cardinalities are equal. That is what we wanted to prove.

And note that this is isomorphism

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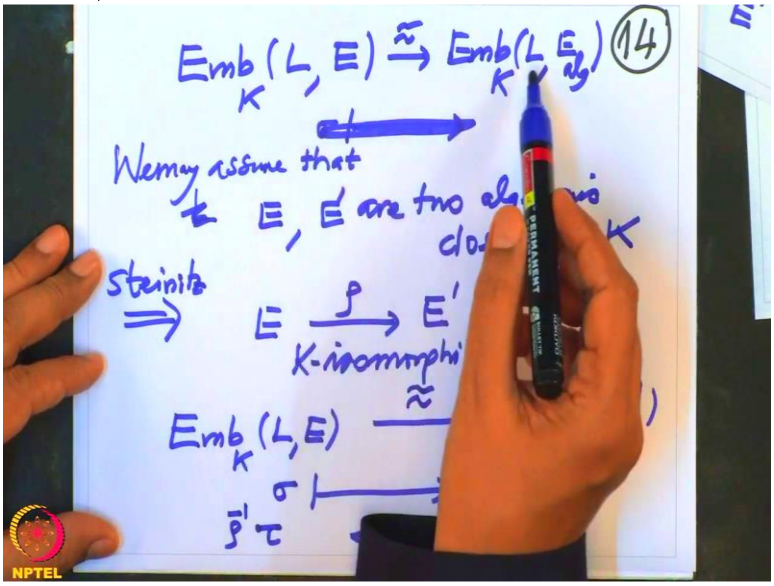


because, you know, because this, this E is here, and E a l g is here

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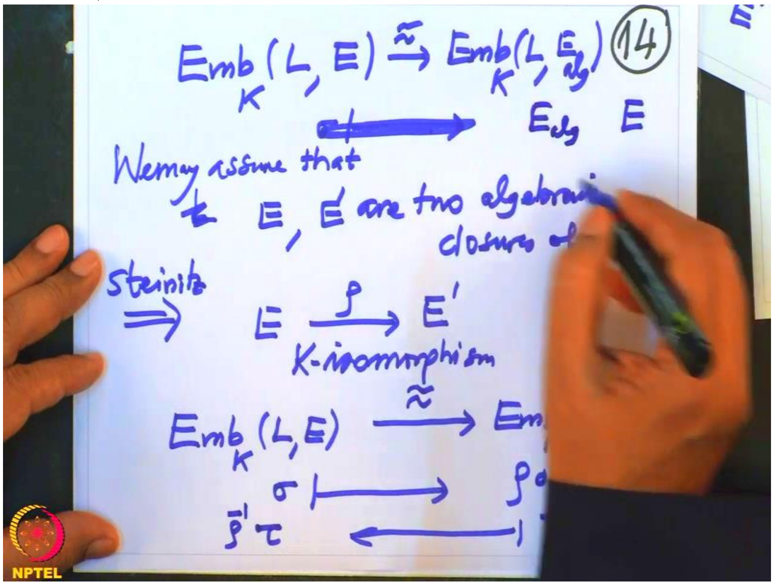


L is here and K is here, so whether you take

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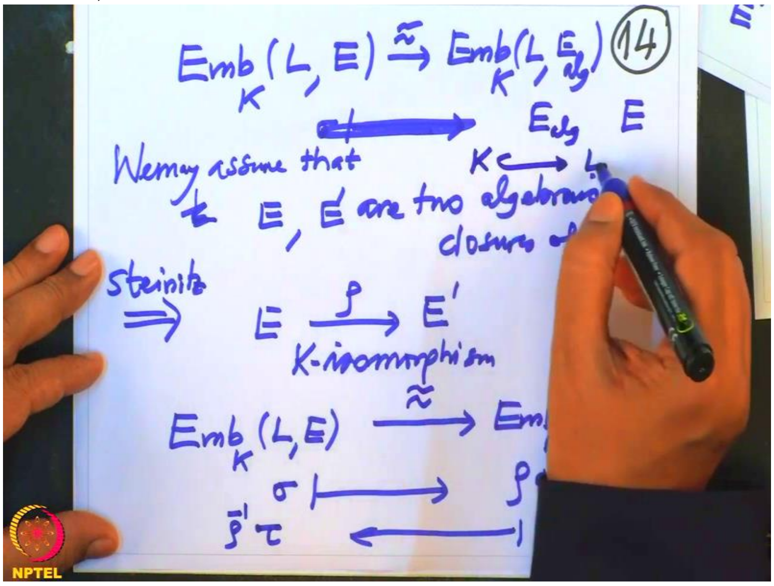


embedding, all embeddings of L in

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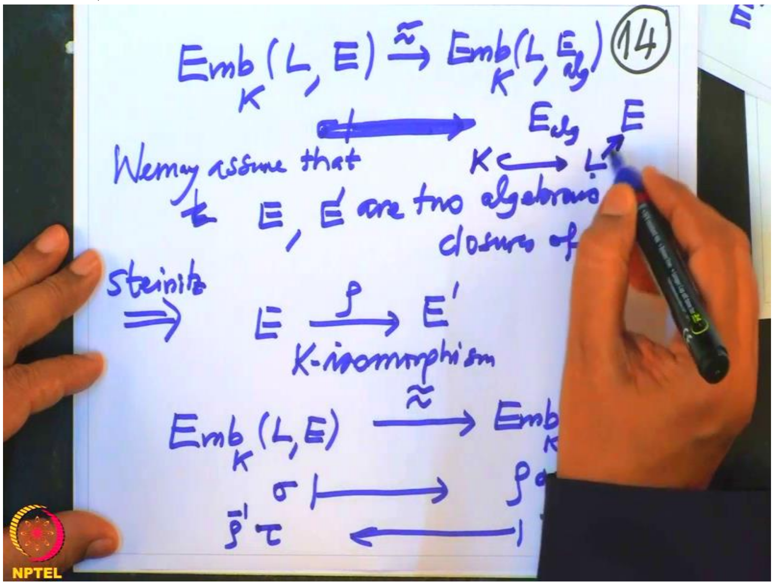


E they will already factor through this

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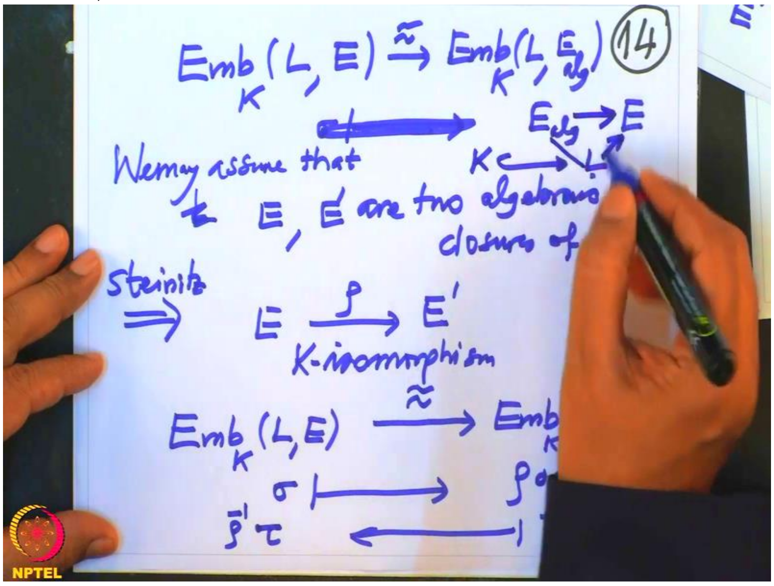


because elements of L are algebraic, therefore elements of sigma L also algebraic.

Therefore the images of sigma,

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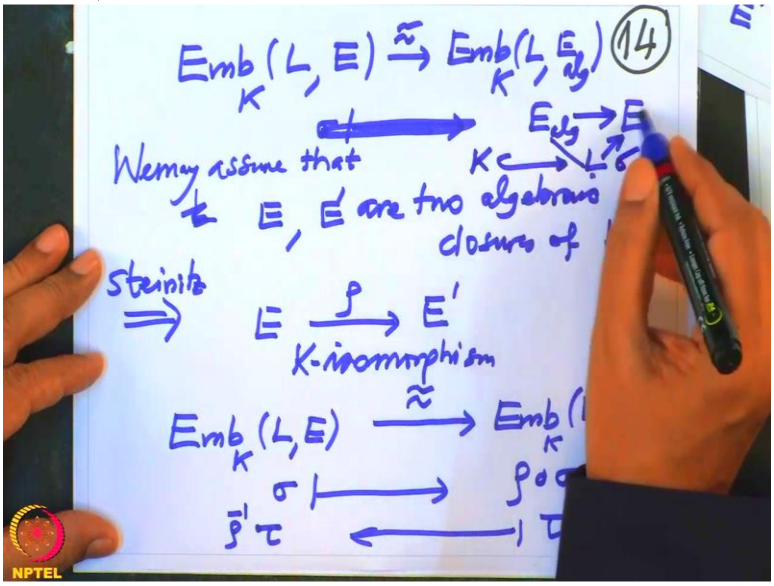


they will factor through this. Therefore you can identify elements of this as elements of this, so therefore this is a bijective map.

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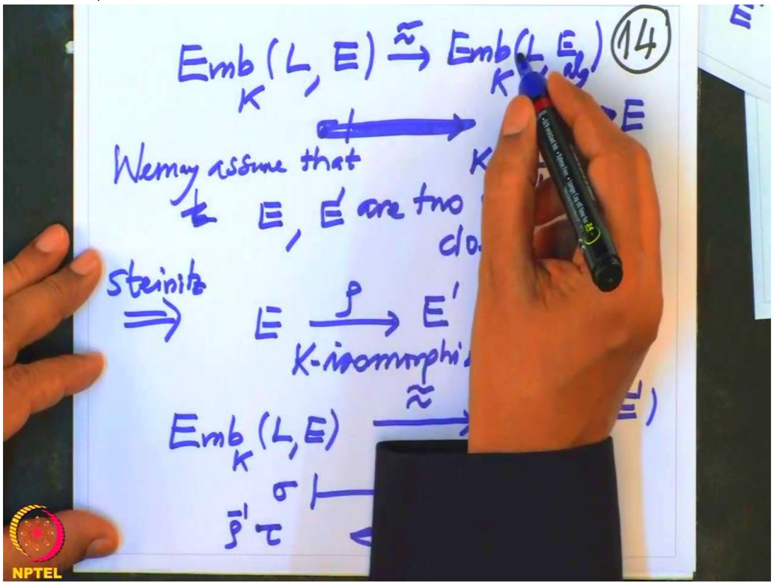


Therefore we can replace E and E prime by the algebraic closures of K in them and therefore by Steinitz Theorem we have replaced, we know that E and E prime are isomorphic and that isomorphism will give us a bijection

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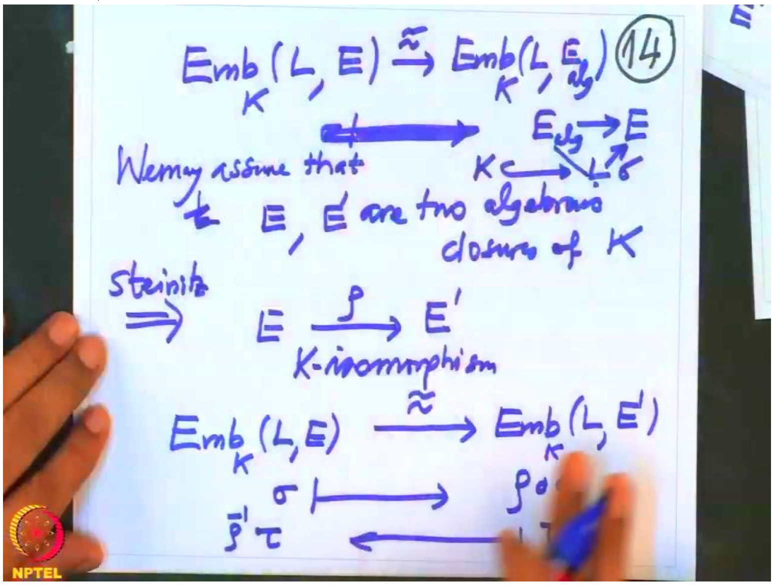


So we have therefore proved completely that the number of embeddings is a good invariant

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of the finite field extension L over E and now we will use this fact to prove that separable extensions have primitive elements so that we will do it in the next lecture. Thank you very much.