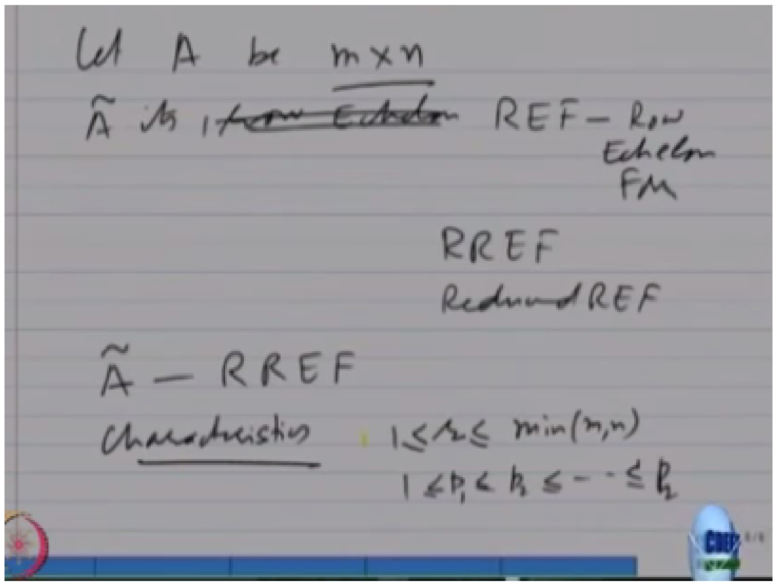
Basic Linear Algebra
Prof. Inder K. Rana
Department of Mathematics
Indian Institute of Technology- Bombay

Lecture – 09
Reduced Row Echelon Form and Rank of a Matrix - III

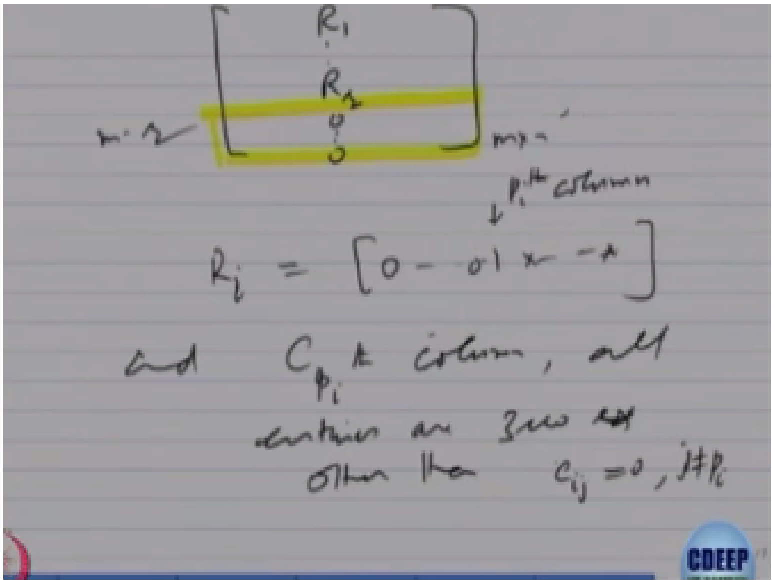
(Refer Slide Time: 00:30)



So, let us look at; so let us a let A be m cross n, A tilde its row echelon, let us write; so we will write now onward REF that is for row echelon form and RREF will be reduced REF, now REF is already defined, so it is a new command, okay, RREF. So, let us look at A tilde to be the reduced row echelon form, so it is characterise; what are the characteristics; what are the characteristics of this?

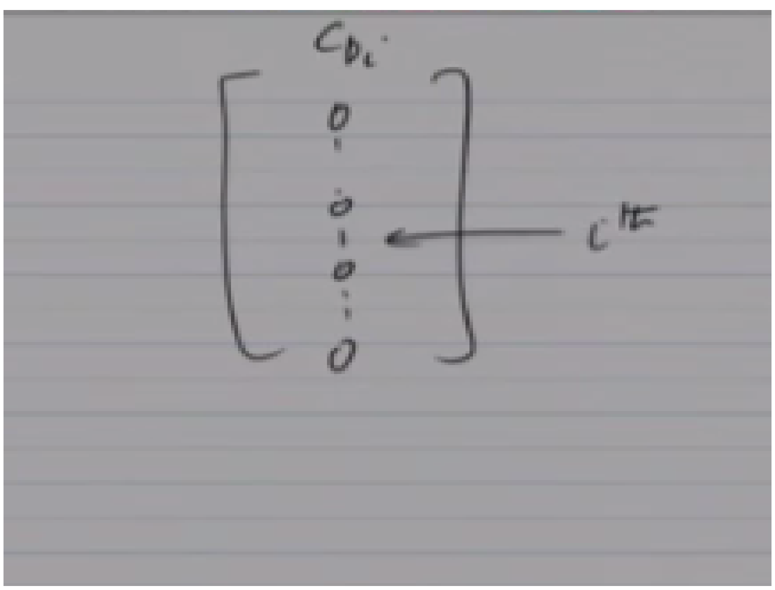
There is a number r, okay, so this is m cross n, there is a number r between 1 and the minimum once again, I am going to there, okay and there are numbers p1, p2, pr, right as many as, okay.

(Refer Slide Time: 02:03)



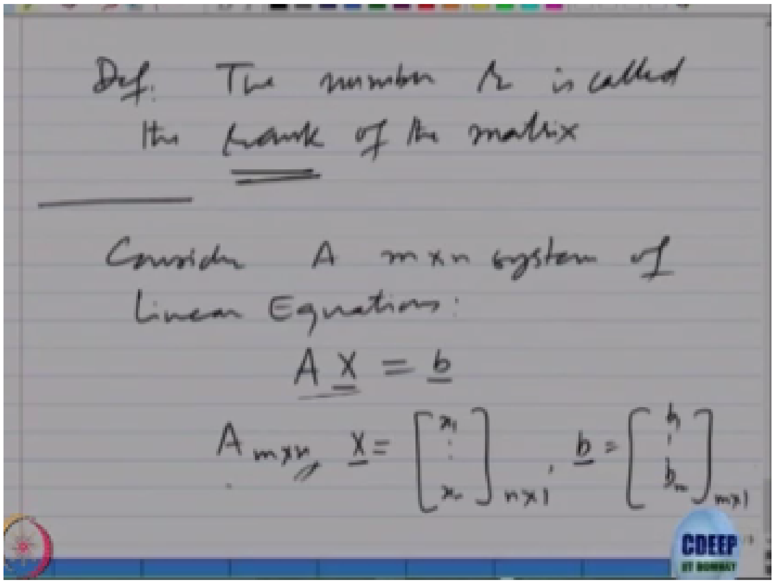
So, there exist this such that the matrix looks like R1, Rr and 0, 0 everything, so this is m cross n, so how many zeros will be there? These are m - r rows are all 0, what does the row one look like; so row 1 that looks like 0 1 and something, right and similarly, the other, so let us write Ri, so this is p ith column, pivot will be 1 right, reduced row echelon form, right and in the column Cpi; so Cp ith column, all entries are 0 except other than; which one; ith column, okay. So, entry; what shall we call it; for in the column, the Cij = 0, for j0 = pi, means;

(Refer Slide Time: 03:53)



The columns looks like this, that was this just write that so, this is the column Cpi, it is 0 here 0 1 0 0 and what is this place; these are ith, right, these are ith entry in that; ith row, ith row is nonzero, were describing the non-zero rows, so ith row is non-zero, the non-zero entry is; first non zero entry is 1, everything else is = 0.

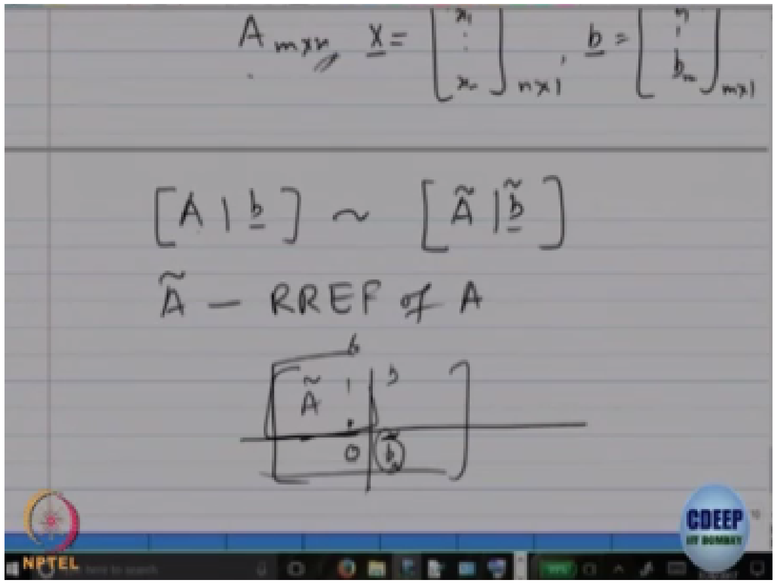
(Refer Slide Time: 04:29)



So, let us give this R a name; so let us put a definition, the number r is called the rank of the matrix, so what is r; r is the number which is coming as the non-zero rows, r rows are non-zero in the reduced row echelon form of the matrix, right, so that number r is called rank of the matrix, right, r is called the rank of the matrix and now let us see; so let us try to apply to a system of equations, what does it mean in the new terminology, right?

So, consider A m cross n system of linear equations, so that is Ax = b, so the m equation in n variables, right, so A is m cross n, okay, what is x? that is the variables right, so x1, xn let us write that so that is n rows and 1 column, right and what is b? b is the vector coming on the right hand side constants, so we saw that so, what is b = b1 up to bm, right, m equations, so that is m cross 1, so m cross n multiplied by n cross 1 = m cross 1.

(Refer Slide Time: 06:49)



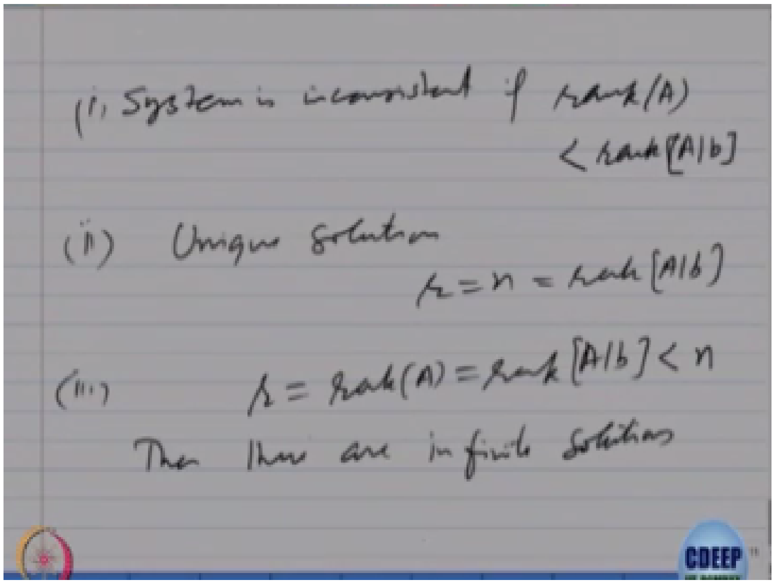
Ax = b that is the matrix notation for a system of linear equations, right and how did we said we will see whether a solution exist or not, so we said we will look at the matrix A and B that is augmented matrix, we said we look at that right and reduce A to row echelon form or reduced row echelon form, okay, so when you do that this will become A tilde and this will become b tilde, where A tilde is the reduced row echelon form of the matrix, right.

So, A tilde reduced row echelon form of A, okay and we said the system is inconsistent if what happens; if there is a row in the part of A, which is 0 and there is a nonzero constant coming in b, right, now in terms of; if I look at the bigger matrix A and B, A has already been reduced to row echelon form or reduced row echelon form and b has changed to something, so if I look at the bigger matrix A tilde b tilde compared to Ab, then the rank, the number of nonzero rows in A will be strictly < the number of nonzero rows in the bigger one.

If it is a 0 here and a nonzero there in b, clear to everybody, yes or no, so the matrix A tilde b tiled will look at; it is 0 here and some nonzero entry coming here, some br tilde let us say, right, so this is the rth, okay, this is the nonzero entry here, so that part is of A that is A tilde, bottom rows are all 0 in A right, somewhere nonzero, so in the; but in the zero thing in b, something nonzero is coming and that column corresponding to b, if a non-zero comes, that means, you have got an equation like 0 = nonzero, right.

But look interpret this in terms of the rank; rank of A is this much but the rank of the bigger one there is a nonzero entry here, so it will be bigger than the rank of A, where the number of nonzero rows; at least one more that inconsistent equation, right.

(Refer Slide Time: 09:26)



So, we write this as that system is inconsistent, if rank of A is strictly < rank of Ab, so we are writing that result in terms of the rank now, okay, so that is 1. Unique solution; that was the system have unique solution r = n and that should also be = rank of Ab, right, consistent only when the ranks are same, otherwise inconsistent, right, so when consistent and rank is equal solution like this and there is only one solution possible.

And the third; r = rank A which is also = rank Ab, system is still consistent but this is strictly < n, it is strictly < n, right, so then there are infinite solutions and there are; so this result is interpreted in terms of the rank, right and how do we get those infinite solutions; the variables, so one makes a convention that the pivotal variables will be computed in terms of the non-pivotal variables.

In equations, right somewhere pivot is coming that is corresponding to the variable and that gives the relation that = something, right, so in that you have to give arbitrary values to some variables to compute 1 in terms of the other, so what you do; you always compute the pivotal variable by giving the arbitrary values to the non-pivotal variables right, compute in terms of them, so that is the convention.

Because we have to give it a machine, we cannot say randomly you do compute, whichever you like, whatever arbitrary value you like, so for a definiteness we say that whenever the number of solutions is infinite, right that means, there are n - r variables which are going to get arbitrary values, r is the rank of the matrix A, n is the number of variables, so n - r variables are going to get arbitrary values.

So, we give those arbitrary values to non-pivotal variables and the pivotal variables are computed in terms of non-pivoted variables to get all possible solutions and later on will see, this also has some other usefulness, when we want to describe the solution space, we want to describe also infinite solutions in some finite number of them, so we will look at what is called the basis of the solution space, so this will be useful there in there, so this rule will be useful there, so we will see examples next time of this, okay, right.