Basic Linear Algebra
Prof. Inder K. Rana
Department of Mathematics
Indian Institute of Technology - Bombay

Lecture – 03
Introduction III

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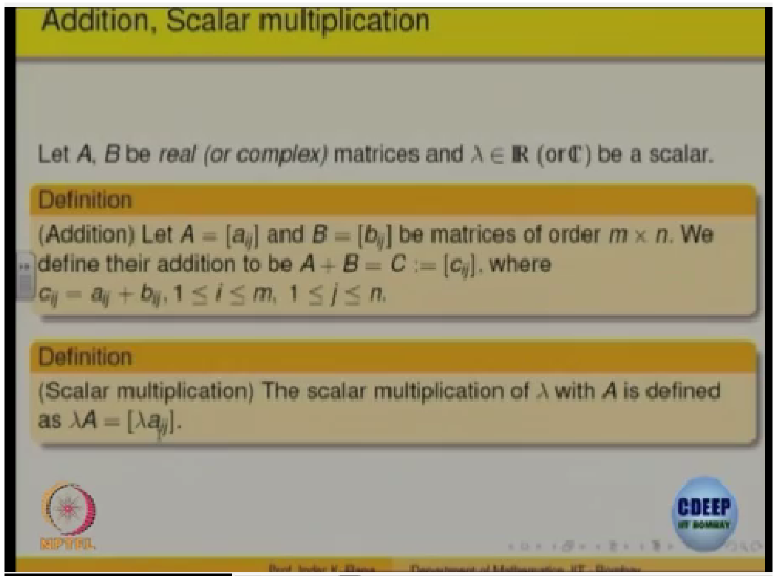

You can also define addition. So what will be addition? Let us revise addition. Given 2 matrices A and B. If A is aij, B is bij, order is same, m*n. So addition will be aij+bij. So ijth entry of the sum is the sum of the ith jth entries of the corresponding matrices. So multiplication of matrices, for example here when you do a+b, a+b is same as b+a. The matrix a+the matrix b=b+a, right. This is a nice property like numbers, right. a+b+c is same as a+b+c, associative property.

Commutative property, associative property and so on. For example, you can take the matrix to be all entries of 0, then what is a+, you can call that as 0 matrix. So what is a+0 matrix? It is again a. So 0x is like an identity. So properties similar to that of number addition are appearing with this kind of addition, right. So we will like to define some multiplication where some nice properties come out, okay.

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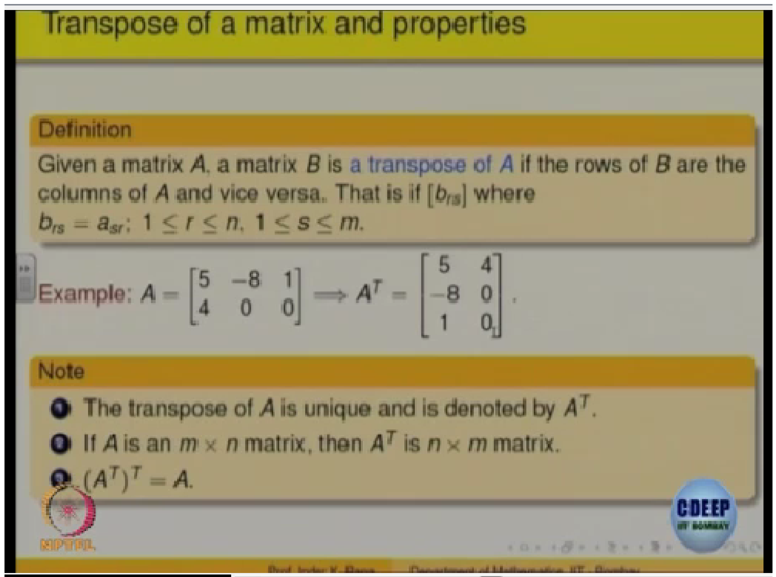

Let us just look at the one simple operation called the transpose of a matrix, okay. So what is a transpose of a matrix? If the matrix is m*n, transpose is just interchange the rows with the columns. Whatever was the first row, make it the first column of another matrix. So define a new matrix, first row from where becomes the first column and similarly other are replaced. All the rows become columns and all columns naturally will become rows, right.

So if it was m*n, what is the new matrix? n*m, right, rows have been. So where you can write the new matrix if you want to write the rs'th element that is sr, interchange the row with the column. So that is the transpose. The simple example for this, this is the transpose where first row is 5 -8 1 4 0 0. So that is, so this row has become a column where this row has become column here.

So here are some simple observations. Transpose is a well-defined matrix, right. We are given all the entries of it. If A is m*n, obviously AT is n*m and what is transpose of transpose? We are getting back the matrix. You interchange rows with columns, you again interchange, right, rows of that with columns, you will get back the matrix. So ATT=A, right. So these are elementary properties of matrices.

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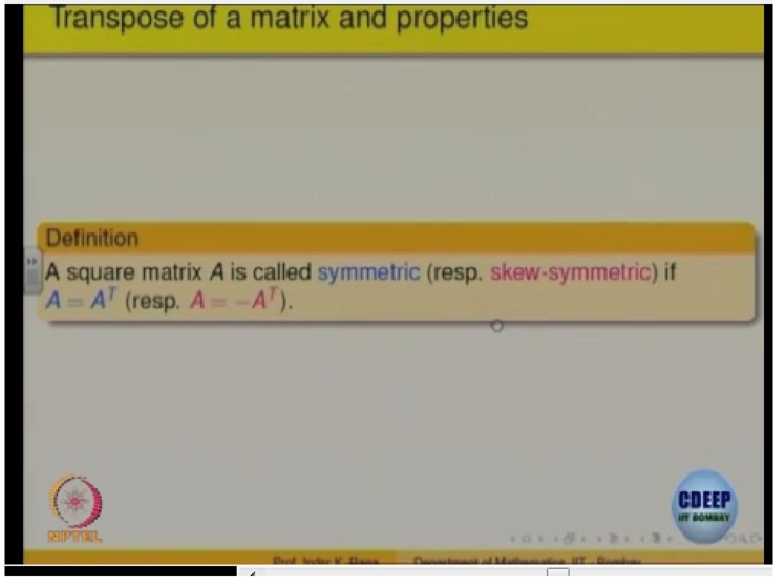

So we looked at what is defined as a transpose. So we will call a matrix to be a symmetric if A=AT. If the interchange of rows and columns does not change the matrix, right and we will call it as anti-symmetric or normally call skew-symmetric if it is -AT. Does it put any condition on the matrix A? If I say a matrix A is symmetric, does it put any condition?

Yes, because transpose will change rows to columns. So if A was m*n, transpose becomes n*m and if you have to be equal, that means m has to be equal to n; otherwise, they are not equal from the equality of matrices. So the concept of symmetric or skew-symmetric matrices is defined only for square matrices, okay. So that is 1.

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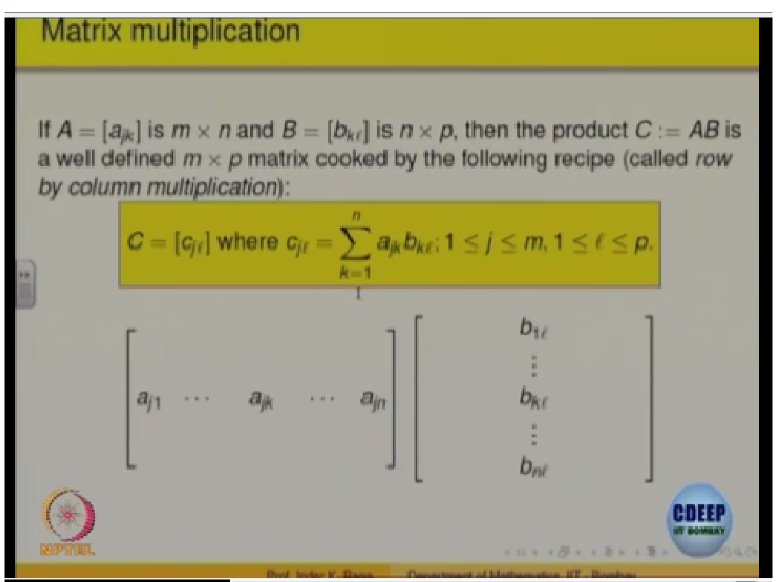

So here is now the matrix multiplication. So we have got 2 matrices ajk and another matrix is bkl. So here observe A is m*n and B is n*p, okay. Then the product of these 2 matrices AB, right, is a new matrix, let us call it as C. So how do I describe C? So I should tell you what are the entries of C, right. So let us look at the entry cjl, okay. jth row, lth column. So it is defined as, you take ajk, you take bkl.

So when you take ajk, j is fixed, k is varying. So what is happening? What is changing here in ajk? When k varies, jth row kth column. So you are taking that jth row, entries of that, right. And what is bkl? l is fixed, k is varying. So what is that? That is the lth column, right. So multiply the 2, add up, you get cjl. So row of the first one, column of the second, the corresponding entries you multiply and add.

And that naturally puts the condition that the number of columns here should be same as number of rows in the other. That is why we have put m*n and n*p, the number of, right. The columns here is same as the number of rows there. So m*n. So what will be the resulting matrix? The resulting matrix, this is summed up. So cjl, j varies 1 to m, l varies from 1 to p, so that gives you a matrix of the order is m*n, A; n*p, that is matrix B.

This n, each entry is getting multiplied, then add it up, you will get n*p order of the product, right. It looks a very artificial way of defining the multiplication. But later on, we will see that this is the way it should be defined to have nice properties, okay. We will link it with what are called linear transformations. Like in functions, we have got composition of functions. Later on we will define what is called a linear transformation.

And composition of linear transformation will correspond to matrix multiplications. So we will tie up a bit later, okay. But at present, we have to just remember that way, right. So this is the matrix multiplication, okay. So for the matrix A, take the jth row, take the corresponding column, lth column. You are looking at jl. So jth row, lth column, multiply. So this multiplied by b1l+ so on, add up, you get cjl, the jth row, lth column entry for the product.

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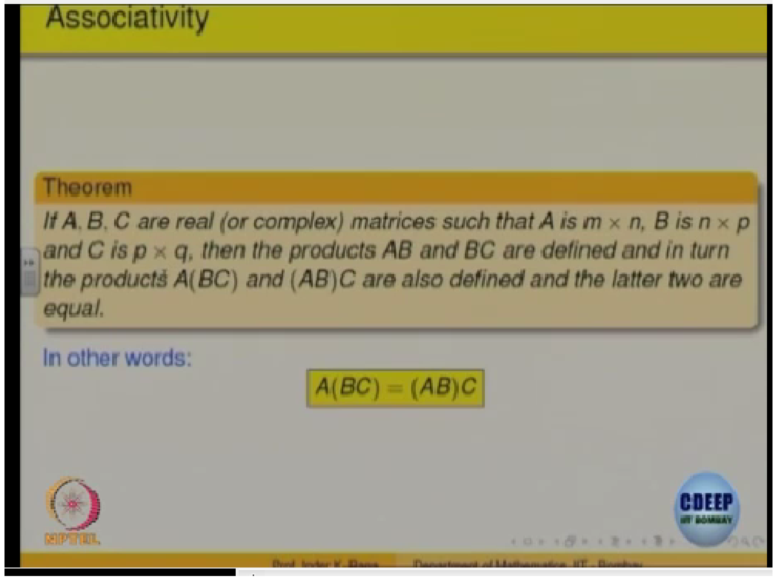

One obvious way of saying that is a nice property is that it is associative. A*B*C, A*BC is same as AB*C. This way defining of product has got a nice property again, associative, okay. It is not commutative. A*B need not be equal to B*A, right. One can give examples for that. So if you want to define A B C, right, what should be A? The order of A B should match; number of rows here should match with number of columns.

There the number of rows with B should match with number of columns, so that is why you see the condition is put A is m*n, B is n*p and C is p*q. So n is match here and p match is here. So the resulting matrix will be of the order of m*q, right. That will be the order of the matrix. So this is the property.

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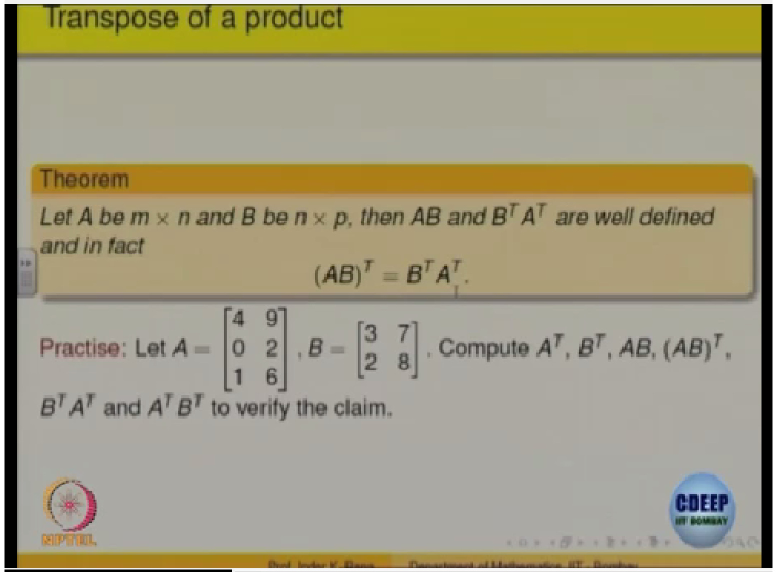

This we will not prove it but you can try to write a proof yourself, right. And you can try to verify it with some simple examples. Product transpose is same as transpose inverted product, right. ABT is same as BT*AT. One reason that this should be so, because if you take AB and transpose. The rows are going to be interchanged with columns. So similarly if you have BT, already rows are interchanged with columns, rows are interchanged with columns.

So the remaining thing product will have rows and columns already matching, okay. So but try to verify it. Try to write a proof of this if possible. Those of you who are very keen, take A to be aij, B to be some bkl, so that A*B is defined, now right. If A*B is defined, then BTAT is automatically defined. Is that so? Check it, okay. So for example here in this example you can look at, want to verify this? Okay.

Write down as an exercise and verify for this, okay at your leisure that 4 0 1 9 2 6 and this, so that means you will have to calculate AB, calculate ABT, calculate BT, calculate AT, product of that, that should come out to be equal, okay, right.

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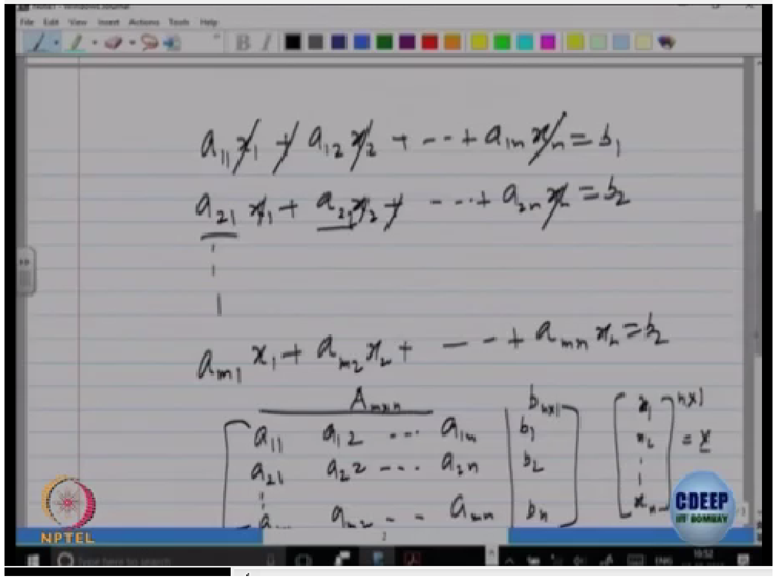

So this is a system, I have written it on, so look at the system. So as I said, the coefficients are not really important, right. So let us forget the coefficients, right or forget the variables and the operations on them. So what I will get? I will get a matrix which looks like a11 a12 a1n. Second will be a21 a22. From here, a22, a2n and we will have am1 am2 amn. And the right hand side, to write that one, we will put it as b1 b2 and bn.

So all the data about the system is captured in these numbers, right. Let us try to write this slightly more neatly, okay so that we get our system. So let us, this part of the data, I will call it as a matrix A. I will call this as b, okay. So what is the order of A? That is m*n, right. What is the order of b? n*1. And what was the variables? x1 x2 xn. So let us write them as a column vector. So what is the order of that? n*1, right.

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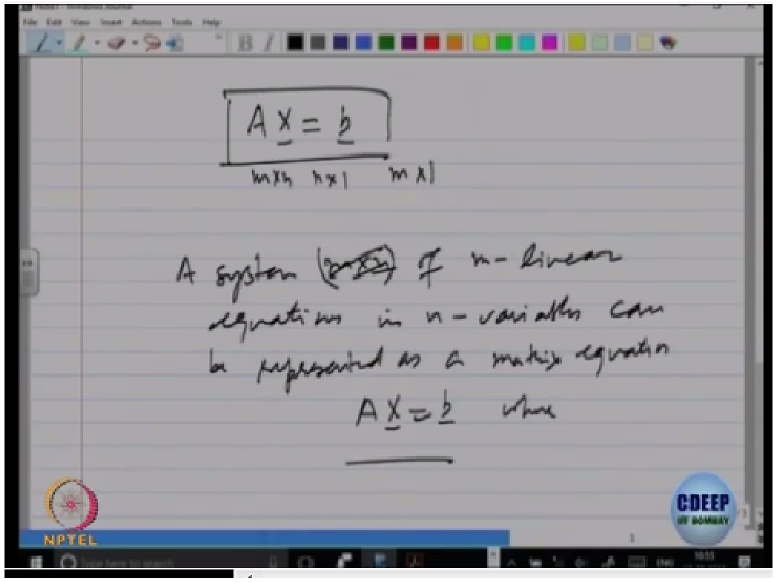

So to get this equation, let us compute what is AX=b where this is b and this is called X, that is multiplication, right. So what we are doing is? We are trying to calculate that. So when I multiply, what will be $a_{11}x_1+a_{12}x_2\dots$ will be equal to b_1. So that will be my first equation, right. Is it clear? So when I write this as a matrix equation, this is m*n. A is m*n. X is, right, n*1.

I will get m*1, okay. So all the data, the equation can be represented as AX=b. So this is the matrix representation of a system of linear equation. So we will call the system m*n, right, a system instead of writing m*n of m linear equations in n variables, can be represented as a matrix equation AX=b, where what is A? A is the matrix which are the coefficients coming in front of the variables, right.

X is the variable, unknown quantity we want to solve and b is the vector on the right hand side, right. So that is the matrix equation. And this is actually equality because when you multiply, right, when you multiply, you get the first equation, second entry as the second equation, third entry as the third equation. Equality of matrices give you all the system, right. So the problem is how to solve this system of linear equations.

And the Gauss elimination method that we had just now seen, it says you can do those 3 operations that does not change the system of equations, right. Solution for the system. What was our goals? You can interchange, you can replace the position of the equation. That means what?

In this matrix multiplication, what are you going to do? Changing the order of equation. That means interchanging the position of one of the rows, interchange of rows, that should not affect the solution, right.

That does not actually affect the solution. What was the second? You can multiply any row by a non-0 scalar. That does not change the solution, right. And third you can add any 2 rows if you like, right. The solution does not change. So you will get a new system which is equivalent to the earlier one and the solution does not change.

And the idea is that this matrix which is representing, right, it should have lesser number of coefficients coming, more 0's coming so that you are able to solve the system very easily, right. So we will see it next time on this Gauss elimination substitution method for a general system of linear equations, right.