Basic Linear Algebra
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Lecture - 26
Orthonormal Basis and Geometry in the Euclidean Plane- II

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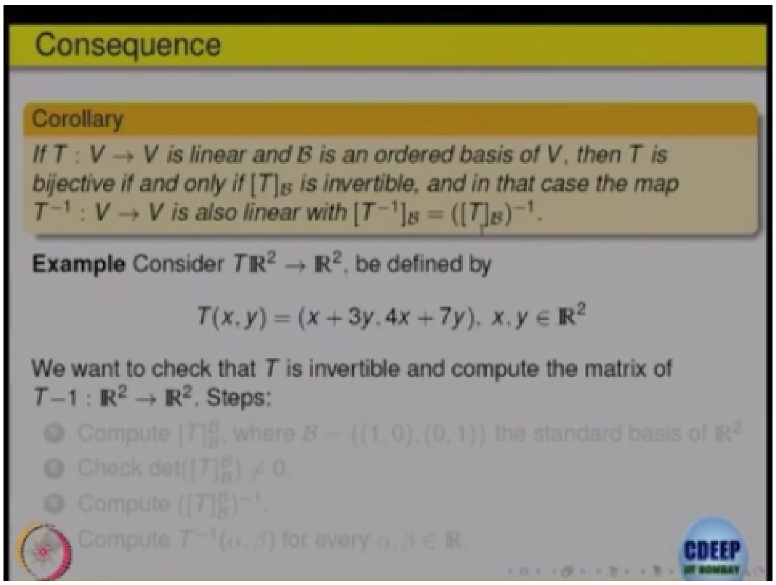

Let us try to do one example of this again to see what is happening right to understand so let us do this example okay so that do you feel comfortable in computing things?

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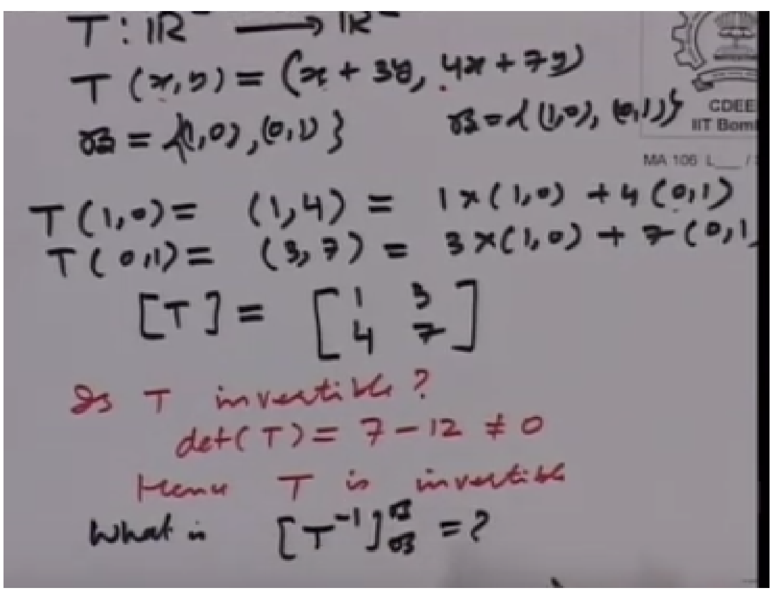

So, T is from R2 to R2 T of xy is x+3y4x+7y okay so if I want to compute the metrics what do you I have to know. You need to have a basis on the domain and basis on the co-domain. So, let us the simplicity let us take the standard basis on both okay on be on the domain the base is 1 0 01 in the same basis on the co-domain. So, 1 0 I am just taking this because computational becomes slightly simpler otherwise you have to solve the equations and so on.

Okay so what is T of 1 0 and T of 0 1 I have to find the metrics of T right? so what is T of 1 0 so 1 and 0 so the first element is 1 and second element is 4 is that okay? what is T of 01 x is 0 and y is 1 so this is 3 and 7 so what is that? I should write this as a linear combination of the basis elements there. So, that is simply 1 times 10+4 times 0 1you know this has become simpler we got to take on standard basis.

Otherwise I have to write alpha times +beta times that and solve those two equations find values of alpha and to beta right that little always solving a system of equations. It is very easy because it is just 0 1 1 0 see this is =3times 1 0+7 times 0 1 right. So, what is the metrics of T matrix of T is 1 4 then 3 7 right . So, now I want to check whether T is invertible or not. So, question is if the T invertible how do you check?

Either you can check kernel of T is this 0 right or range of is a column space that dimension 2 or here the determinant is going to be very useful is a 2/2 determinant of this is very easy. So, what is the determinant of this 7- right so that is = that is !=0. So, hence T is invertible. So, that is because of our metrics is quite 2/2 considerations even 3/2 3/3 is also okay or you guys can directly try to compute.

T of 0 0 T of xy=0 should imply x=0 and y=0 that will give T=1 1 that is good enough that will be x+3y=0 4x+7y=0 solve the equations and see that x comes out as 0 and y comes out as 0 that is T is 1 1 that is also good enough. So, whichever you find easier in a particular example you can do that so T is invertible so what is that you want to do now. So, what is our matrix so T is invertible what is T inverse the matrix of this.

You want to find the matrix of the inverse linear transformation right so what will be that by Theorem it says inverse of this matrix. So, let us write that.

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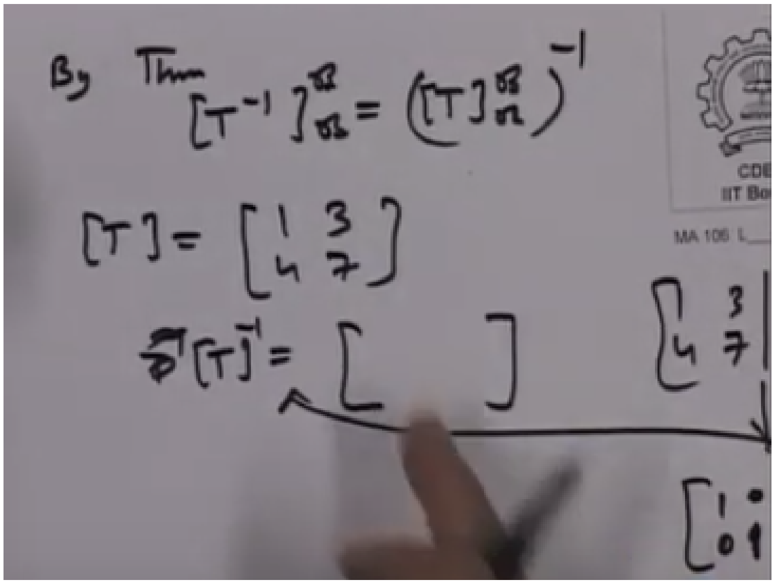

So, by theorem T inverse matrix of this = look at the matrix of T and find its inverse right. So, given the matrix T so this is the matrix T we had 1 3 4 7 you have to find what is T inverse The inverse T inverse so we will find T inverse of that because it is nice and simple you can do it by adjoined method if you like or you can do by row operations along with identity matrix you remember that process whichever is easier you can do that is it clear okay.

So, what you can do is to find this T inverse and start with the T so 1 4 3 7 and write 1 0 0 1 and transform it to using row echelon form so the first part comes out as 1 0 0 and 1 and this will come out something and that is precisely by inverse. That is another method of doing or you can do it by already you found out the determinant so if you use that method of adjoined what was the determinant -5 right. So, -1/5 adjoined method so what will be the, so first row and first column gone.

What shall I put here 7 right and then next one this goes this goes -4 and similarly this goes this goes and -3 and that goes 1 so this right, now not only I have found the inverse I can now find what is my reverse linear transformation. So, let us compute T inverse of xy what is that = if I want to find out I can do that I have already got an matrix of it. Right so what is T inverse of xy . Let us write what is T inverse of.

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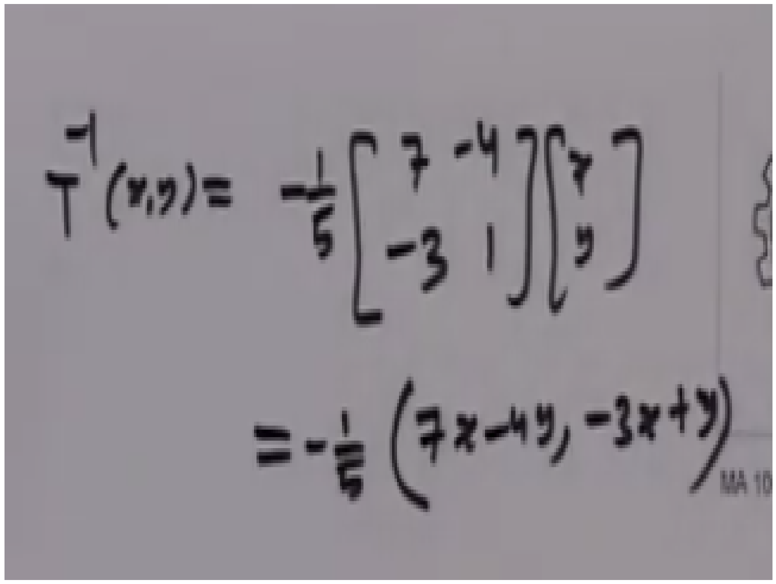

So, T inverse of xy is same as look at the matrix of this of T inverse and multiply by that vector simple so it is -1/57 -3 -4 1 multiplied by xy so what is = you can multiply out and write down vector. So, that is the expression for okay let us write -1/5 so what will be 7x -4y so I am just writing it as an element in R2 instead of writing as a matrix what in the second -3x+y right? so that will be the formula for the T inverse right we started with T was okay.

So, I am just revising T was x+3y 4x+7y with respective to that basis we found out the matrix of this. Because this is not =0 and actually =-5 determinant here so this is not the determinant is not 0 so this is invertible. Okay matrix of the inverse what we do is we just look at the formula from the theorem that the Inverse matrix of inverse of the of that original matrix. So, given this we found the inverse.

And by now using this I can find out what is my expression for the linear transformation T inverse. That also can be found okay that we found out as so is it clear to everybody? yes so basically. What we are saying is we are giving it an association between the linear transformation and matrices if T is formed V to W v is dimension n and W is dimension M and matrix comes out to be order of M cross N right from the co-domain to a domain that is order.

For that you have to fix a basis on the domain and you have to fix basis on the co-domain and how to get the matrix for the ith element of the basis element in a domain, compute the image, write it as a linear combination of elements in that co-domain. Using those basis in the co-domain the co-efficient which come there that give you the ith row of the matrix of the linear transformation.

And this association is very nice the matrix of the sum = sum of the matrices the matrix of the scalar multiple is the scalar multiple of the matrix. And matrix of the composition is product of the original matrices and as a consequences is also tells you another way of finding what is inverse linear transformation if you need B right okay. so that is about linear transformations and associate and the reason why we want to do convert linear transformation to matrices.

Is because matrices are numbers and all matrix operation can be put on machine later on at the end of this course. We also tell you something called general vector spaces which need not be sub spaces of RN. So, this concept of vector spaces is more general concept you can have objects which objects can be added for example look at all 2 by 3 matrices look at all matrices of order 2 cross 3 you can add them you can scalar multiple them.

And to behave as if they have all the properties that vectors have addition of 2 cross 3 is again a matrix of 2 cross 3 close under addition close under scalar multiplication. But this matrix addition is commutative it is associative and scalar multiplies and distributes so it has all the properties that we have for RN addition and scalar multiplication so matrices 2 cross 3 form a vector space.

Which is not of numbers its elements are matrices themselves you can have many other so we will discuss it in the end of the course there is a concept of general vector space. And whatever we have being doing almost everything works for those vector spaces also and they are important for computation point of view so we will come to that bit later so let us just come back to our thing.

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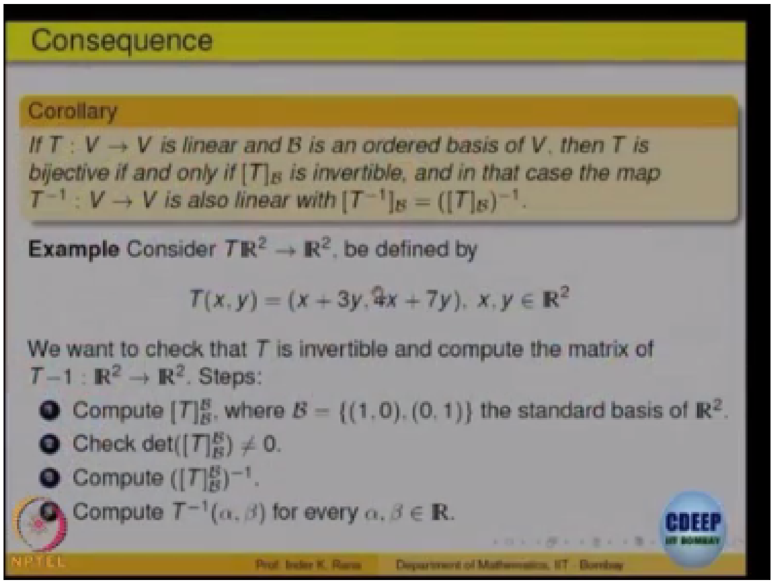

Idea is compute the matrix of T with respect to the given basis here it is B and B both you can have some different in the domain different in the co-domain check determinant is 0 or not = 0. If it is not 0 it is invertible and if it is 0 it is not invertible problem is over and if it is invertible the determinant is not 0 you can find the inverse of the matrix by using our techniques either by looking at reduced echelon form or by adjoined method whichever is.

And then using that matrix you can write down the formula for T inverse of vector by matrix multiplication as we did last time so this is matrices and linear transformations and now what we want to do next is see we have been trying to do sort of two things which in R2 R3 and Rn in a setting but our is true for R2 R3 we try to extend till Rn general settings R2 there is a basis so every vectors of sub space should have basis how to get a basis and so on.

And maps on them but on R2 and R3 there is another important thing there is a notion of not only the notion of distance and not only notion of linearity there is a notion of angle in R2 and R3 given two vectors you can find out angle between those two vectors given two vectors in R3 you can find out what is the angle between them and because of that you can do lot of geometry and lot of more geometry can be done in R2 and R3.

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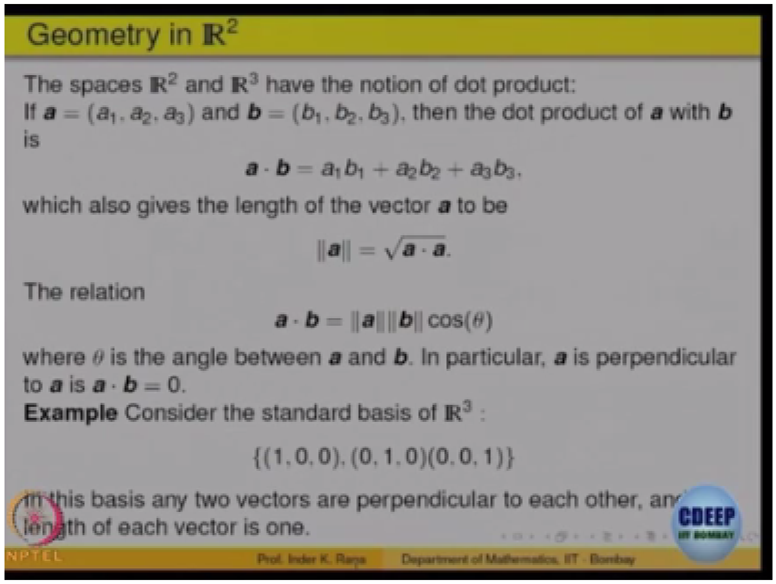

So, let us just revise that and see what we want to do so let us just revise R2 and R3 given two vectors a and b a is a1 a2 and a3 b is b1 b2 and I am just looking at R3 R2 we can remove one vector component same will work so this is what is called dot product we define in vector algebra and vector calculus whatever it is dot product a.b is a1b1 + a2b2+a3b3 advantage or in this notation it also gives you what is called the magnitude of that vector.

To treat a as a vector it has as a length, its length is nothing but a.a so that is a1 square+a2 square+a3 square square root that is usual Euclidean distance Pythagorean distance between two points in R3 so that is express in algebra now, slowly geometry is being converted into algebra that notion of distance we define a.b that is a function R2 to R2 a, b goes to a.b from R2 to scalars and magnitude is define in that way and it also gives the notion of the angle.

So what is the notion of angle a.b is magnitude of a magnitude of b in to cos of the angle between them theta so here is the definition of angle comes what is the angle between them I can define cos of that angle by this expression if there are non-zero vectors they can divide by that so cos theta = a.b divided by norm of a into norm of b so the matrix notion of angle right measurement comes out in the form of algebra now this equation.

So once that is there you can say when there is two vectors perpendicular in the angle between the image 90 that is a.b = 0 so that gives you algebraic notion of perpendicular of two vectors and dot products that is = 0 so that geometry be converted in to algebra basically keep that in mind because the reason is you can do geometry in R2 you can do geometry in R3 you cannot do geometry in R4.

You cannot visualise that so but you want to do that same things so all these concepts carry over to R2 R3R4 Rn any and so we can do everything so this is the standard basis in R3 1 0 0 010 and what is the property of that any 2 of them are perpendicular to each other and magnitude of each one of them is = 1 we have seen many other we have seen basis for R2 or R3 need not be unique. There are many other basis infinite number of basis possible actually but each will have 3 elements.

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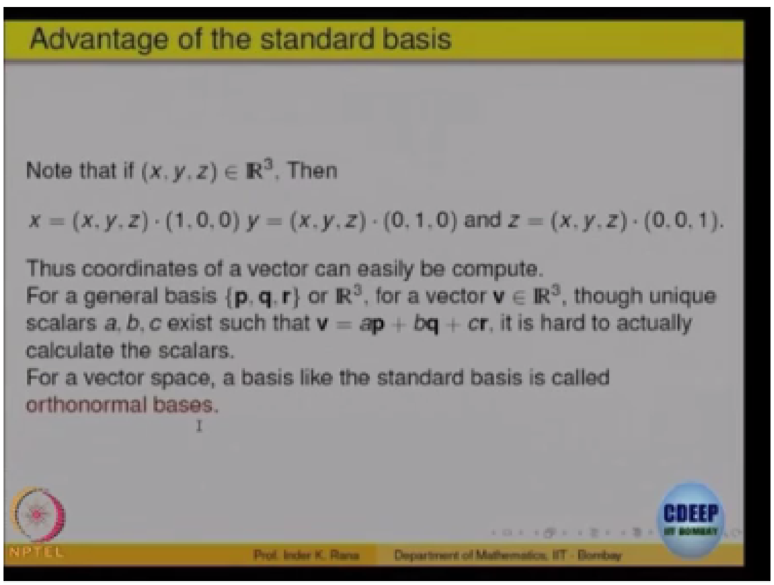

What is the advantage of these 3 vectors why did I chose in my example also let us take standard basis this is because computation from simpler for example if x is any vector then it is a co-ordinate x y and z are simply obtained as what is x? x is the vector dot 1 0 0 and what is the component y? x y z the vector dot 0 1 0 and the 3 rd one x y z is 0 0 1 if I take a dot product of each 1 of any vector.

With each 1 of them I get the co-ordinates of that is the advantage otherwise how will I get the co-ordinates if on the general basis if instead of 1 0 0 0 1 0 and if add other basis then i had to write x y z = alpha times V1 +beta times B2+ gamma times and solve those 3 equation in 3 variables to get alpha beta and gamma those will be the co-ordinates of the vector with respect to that basis. I have to do lot of work and but here I have to take simply a dot product.

And I will get it very easily so these basis which each element is perpendicular to other element have an advantage here so that is what I am saying here you have got some p q r on some other basis then writing V is actually we have to calculate them by solving system of equations so I am telling you what is the advantage of having 1 0 0 010 and 001 over other basis in R2 and R3 so this is normally given in a name such a basis normally called orthonormal bases.

Each vector is normal that means standardised and the length = 1 and orthogonal to each other so the 2 are combined as orthonormal so the standard basis of R2 or R3 is an example of a orthonormal basis we define it formally also.