Basic Linear Algebra
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Lecture - 01
Introduction - I

Okay so our basic course is this course linear algebra, I call it as basic linear algebra.

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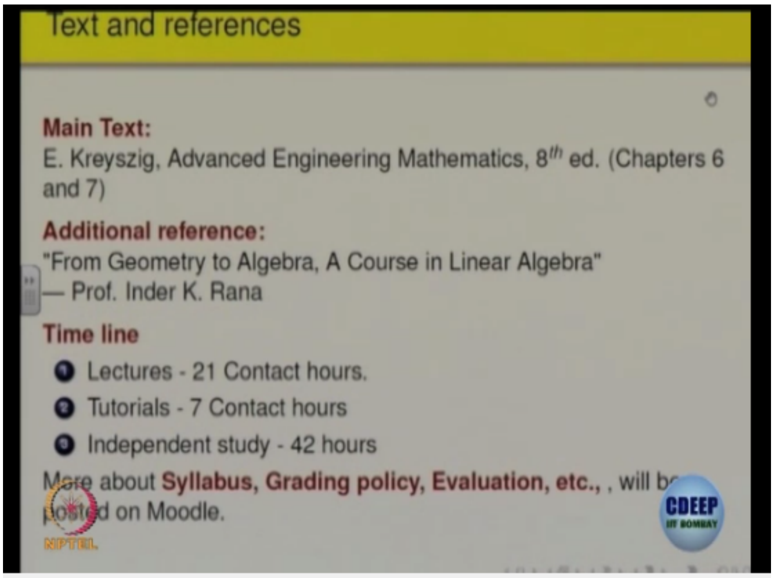

For this course, the main text as referred in the bulletin is Kreyszig, Advanced Engineering Mathematics, 8th edition Chapter 6 and 7. So those are the basic two chapters that will cover roughly. The additional reference you can also refer the book from Geometry to Algebra, A course in Linear Algebra available in our library as well as outside. This is a half semester course, so roughly there will be 21 contact hours.

Every week will have a tutorial hour, so that will be around 7 weeks, so 7 contact hours for the tutorials and you are expected to put in about 3 hours per week okay. So that means 2 hours per week so 42 hours. Other things like about syllabus, grading policy and evaluation, I will be putting it on the Moodle, so access your Moodle sites and get from there okay.

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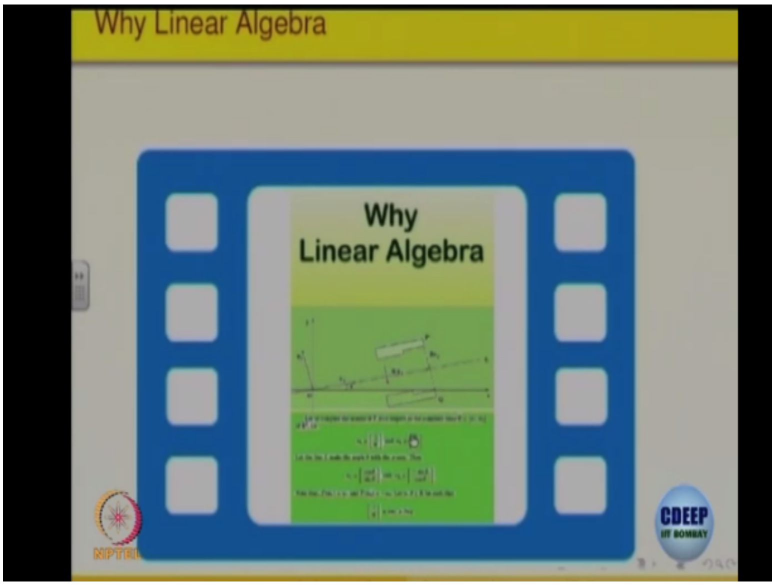

Let us look at why one studies linear algebra right. So let me show you a small.

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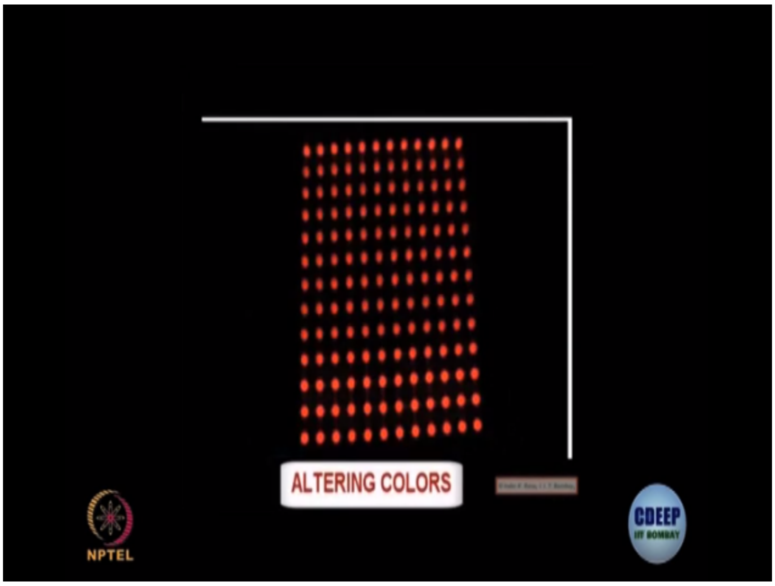

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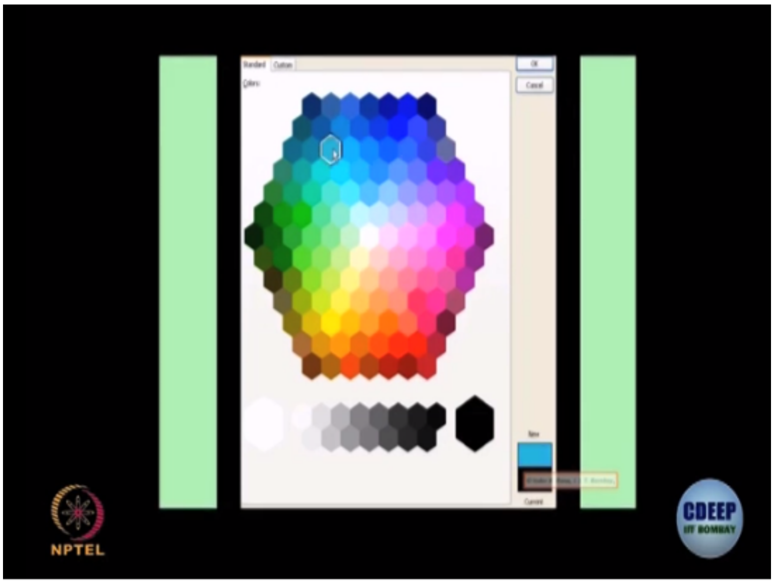

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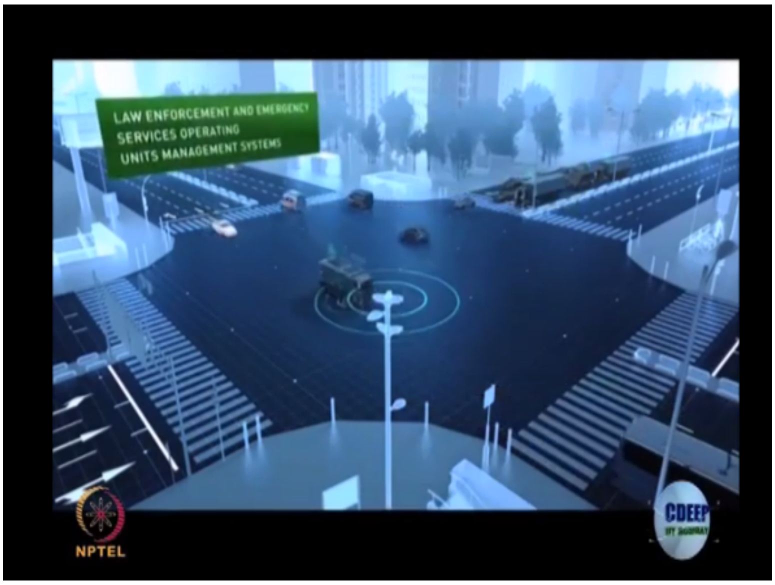

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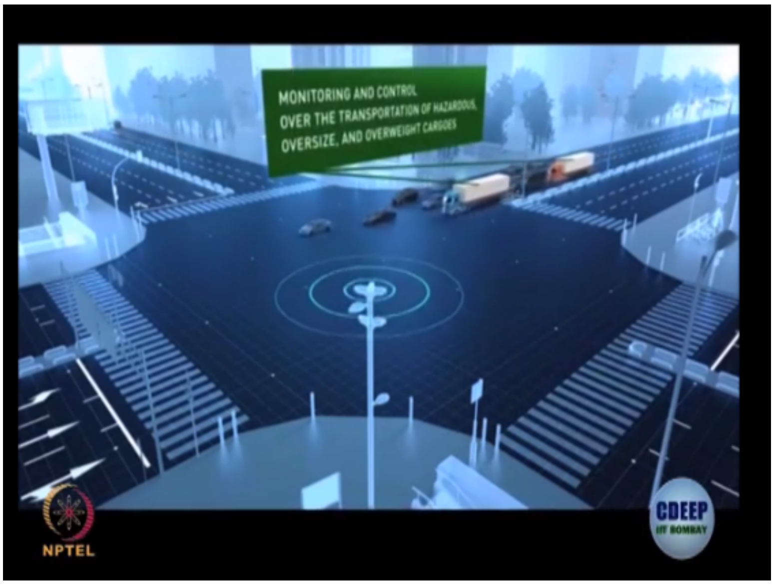

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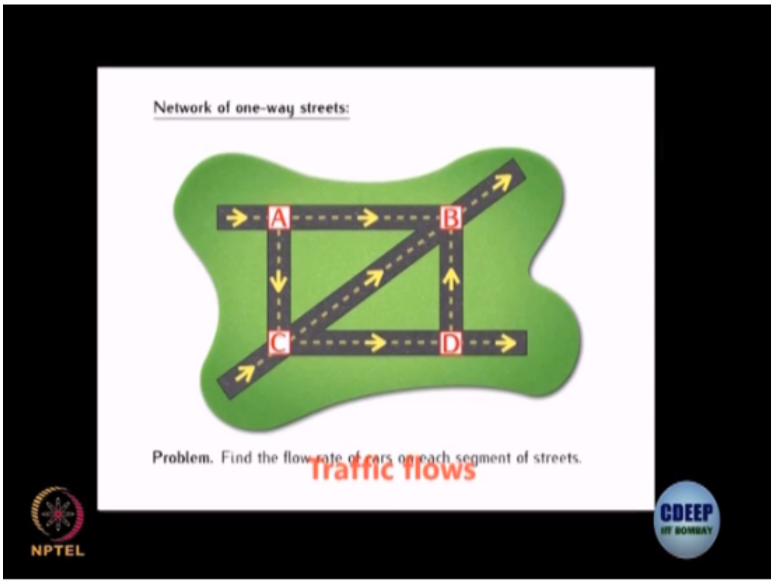

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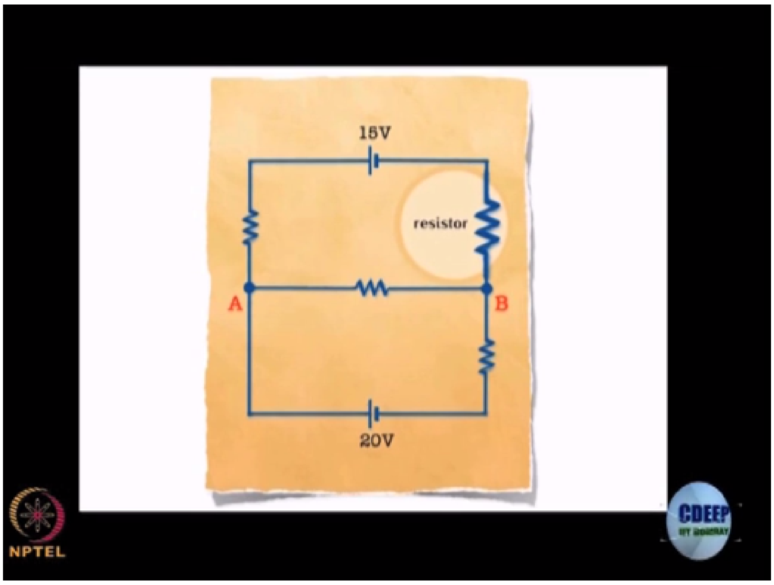

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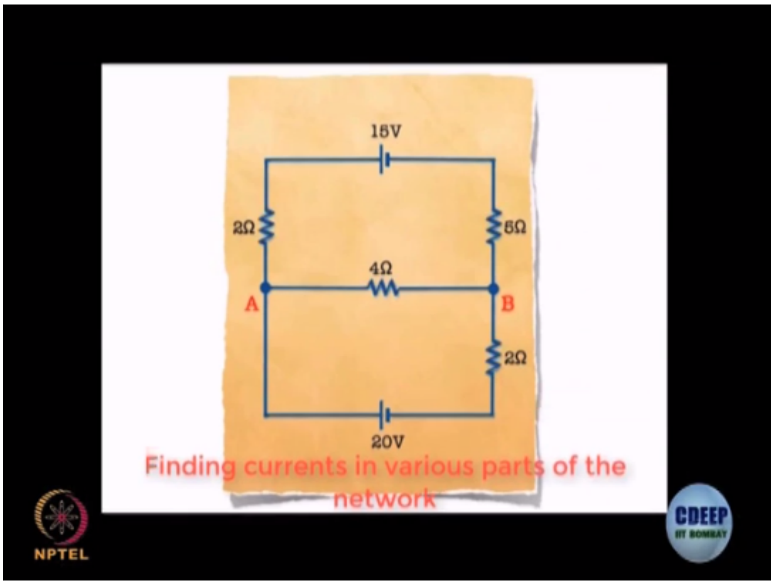

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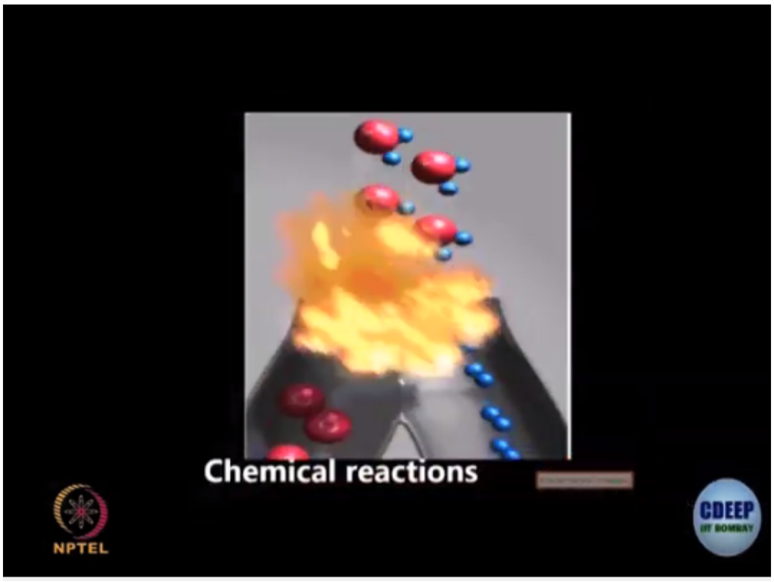

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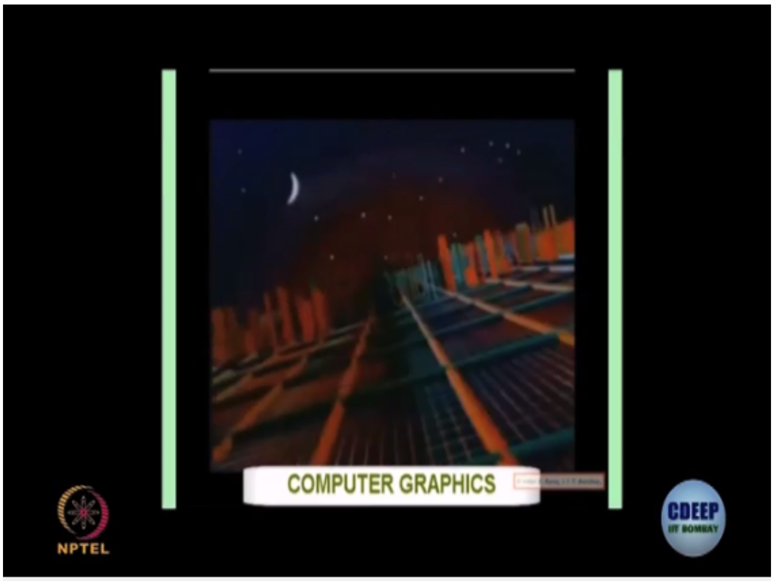

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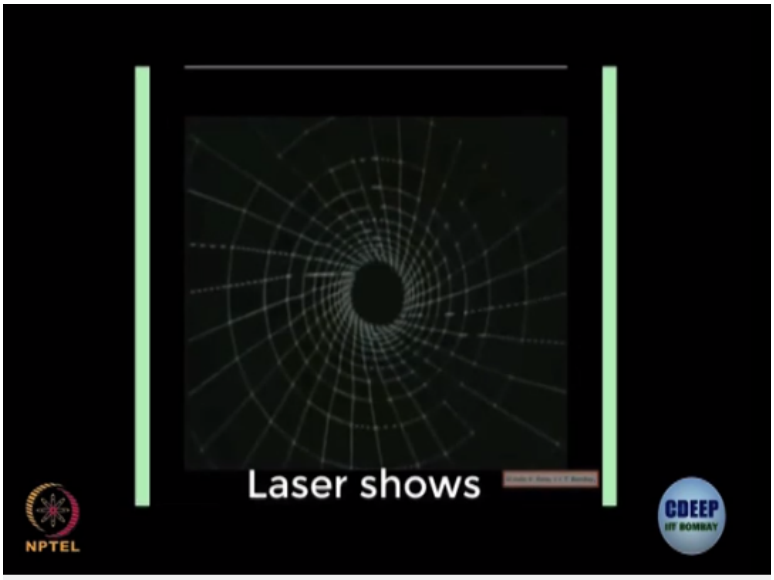

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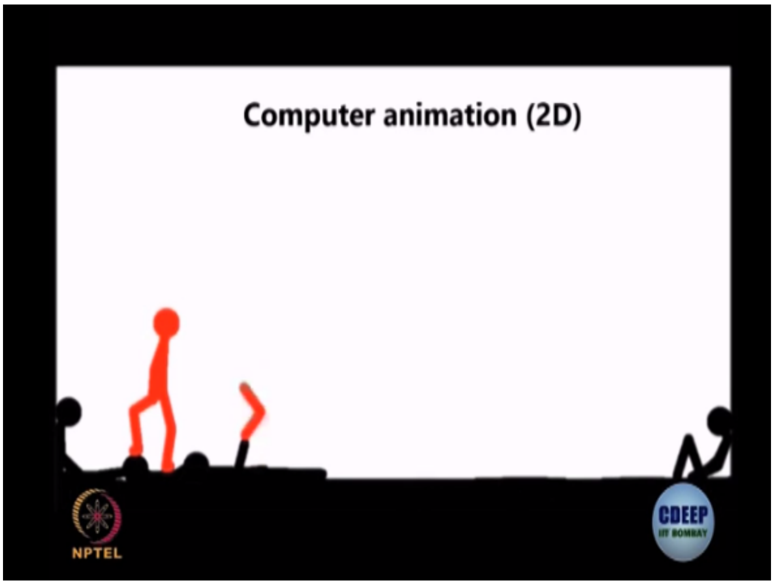

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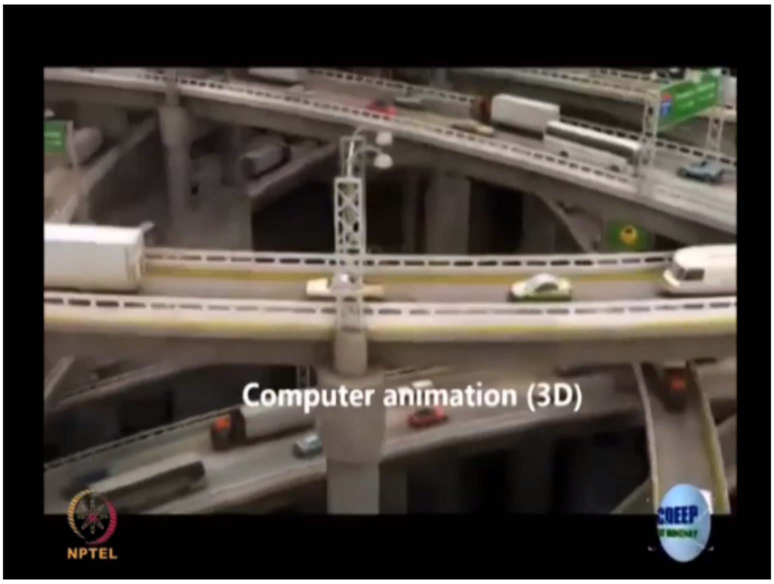

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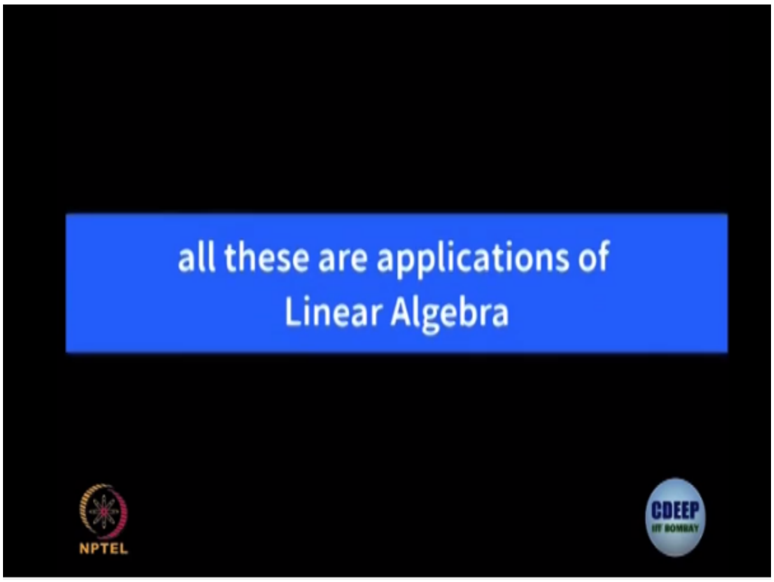

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Basically, these are all applications of linear algebra and if you are going to work in any one of these fields later on, there are many more actually, list some of them, you need linear algebra. So that is why linear algebra is put as a basic course for all engineers as well as mathematic students, economic students and so on.

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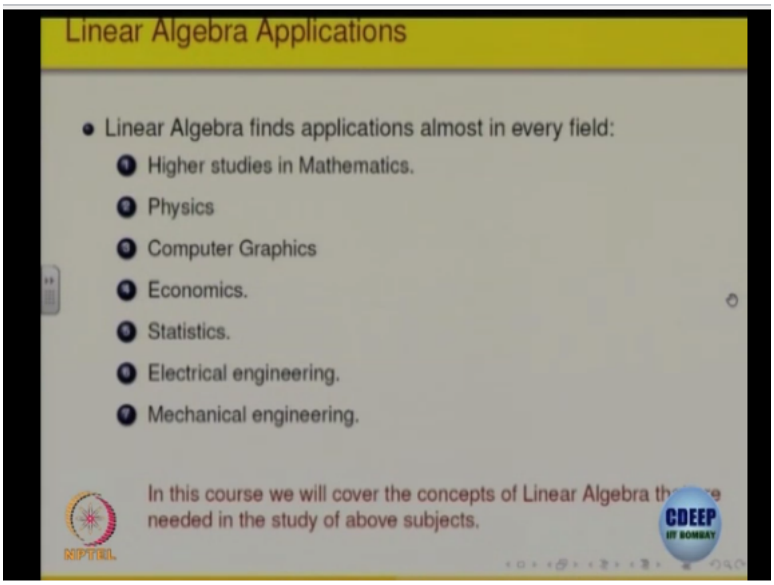

So let us look at some of the other topics. So higher studies in mathematics will lead positively linear algebra, physics needs linear algebra, computer graphics yes, economics so all modern economics is mathematical economics, statistics, electrical engineering and mechanical engineering and many more. So there are many more fields where this will be linear algebra is useful.

So let us begin with what we are going to do in this course is basically cover some of the basic concepts that will be required later on by you.

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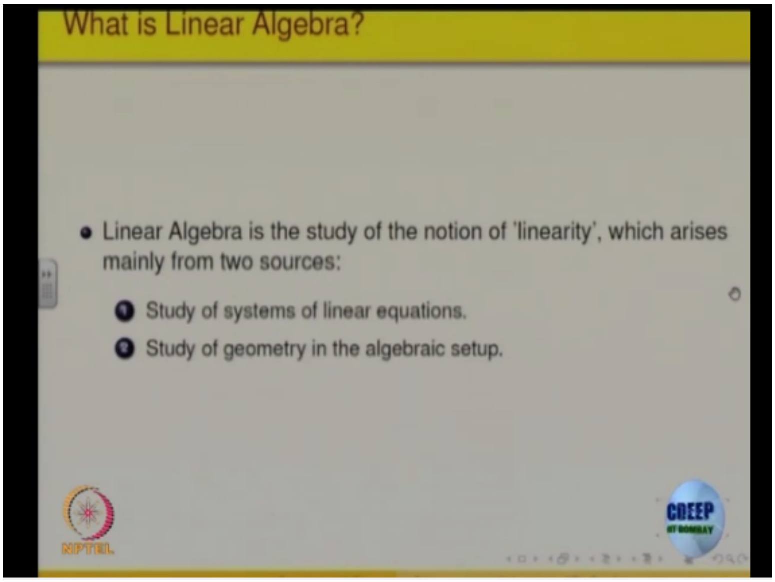

So basically linear algebra is a study of linearity which arises in two situations. One is study of linear equations in one and more variables and secondly it also is study of geometry in the algebraic setup. So that is where the linearity comes into picture. So geometry can be studied only in one dimension or two dimensions or three dimensions but you cannot visualize geometry in fourth dimension.

So how do you do geometry in fourth dimension and high dimensions? That is done by linear algebra okay, making it abstract. Also, there are two components of linear algebra.

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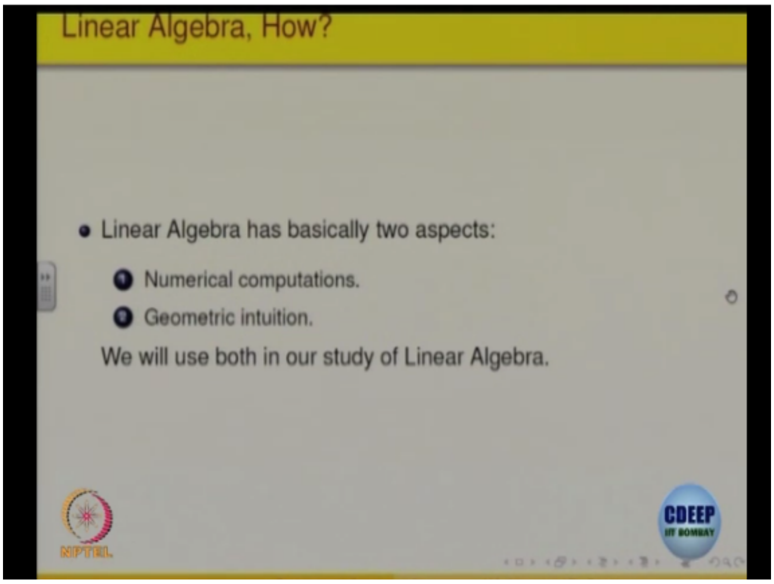

One is the computational aspect that is called the numerical computations and other is geometric visualization or intuition. How geometry can be transmitted into. So we will be using both these aspects in our course to study the linear algebra okay, so that you get a feel for the subject.

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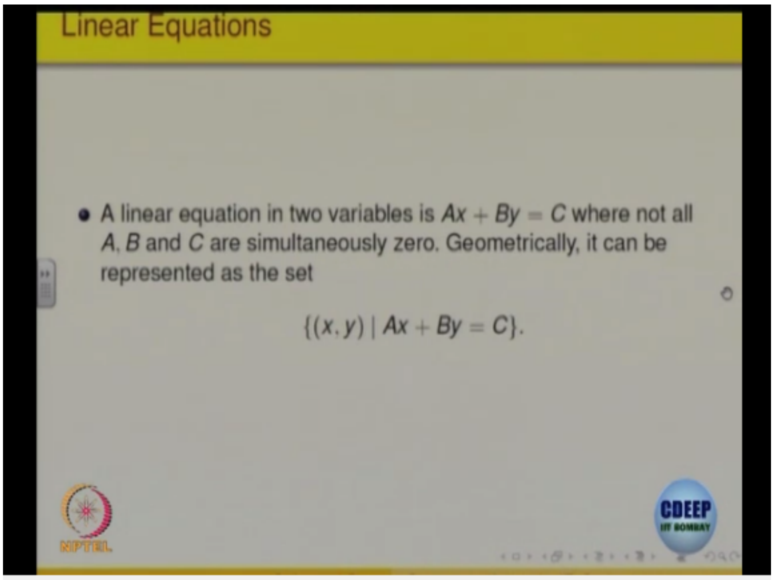

So let us I am just going to start with calling some of the basic facts, you might have studied in your school or last year in some of the courses. So what is linear equation in two variables? So equation of the type Ax+By=C. Their x and y are variables and A, B and C are scalars, which are fixed. The variables are the quantities which can take any values. So Ax+By=C is called a linear equation in two variables.

And not all A, B and C should be=0 because if all are 0 then there is nothing to be done right okay. So as a set you can write this as ordered pair x, y such that Ax+By=C, so as ordered pair so you can write this as a set.

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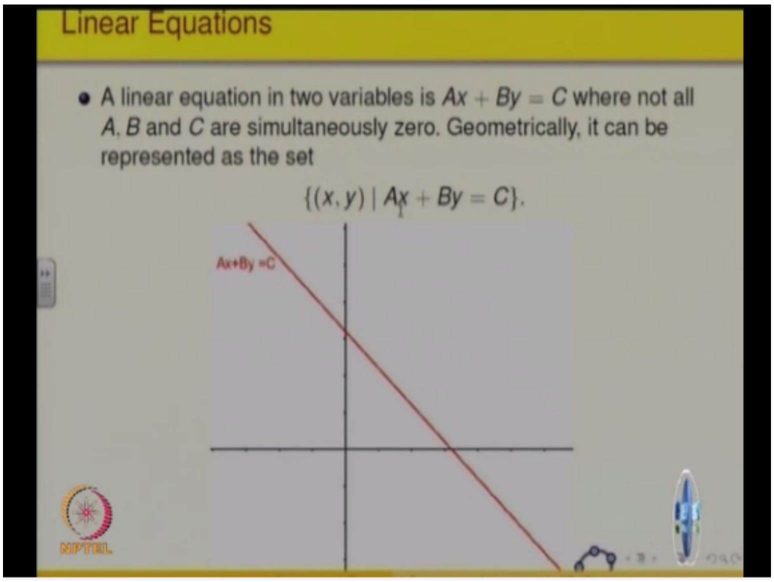

So now once you write it as a set geometrically you can think of plotting it right. You can plot this. So this is what is called the graph of line, it is a straight line and it indicates all the pairs x, y which satisfy this equation. So you can call the graph as the solution of the linear equation right. Those are the values which satisfy. It is interesting to see whether what happens when A, B and C change.

How does this line changes right? So let me just show you something, you might not have seen it earlier, how does things change when A, B and C.

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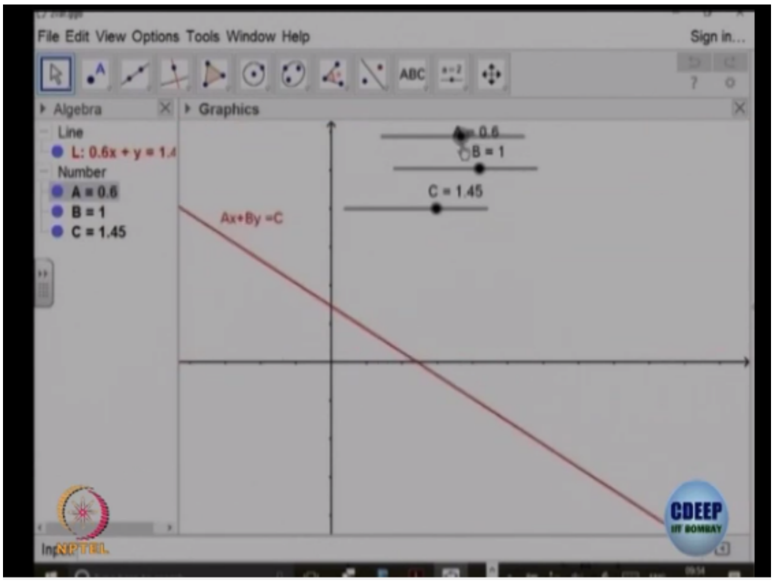

For example, this is a line and you can see the value of A=1, value of B=1 and value of C is 1.5 and you can change this. If I change the value of A right, you see what is happening to the equation, that inclination is changing that something is remaining fixed and that is a point where it is cutting the axis, y axis. If I change B again, I am changing B to various values again right, the inclination changes.

And if I change C right, you can see what happens. It is interesting to see that many times this is not observed that if A is positive, this is an inclination and when A becomes negative that is an inclination right and that is the position horizontal when A is 0 okay. So negative 0 to positive. So that is geometry of this solution set of linear equation in two variables.

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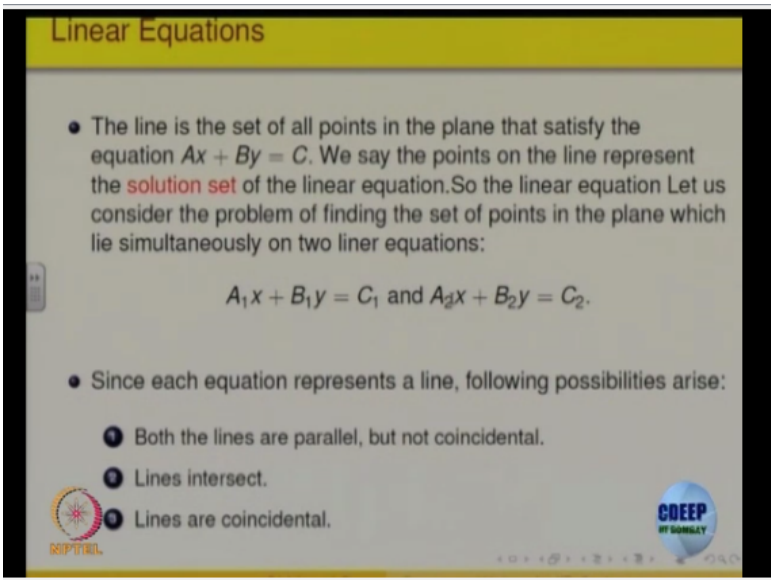

Let us slightly go bit further and let us look at two equations in two variables. So the two equation A1x+B1y=C1 and the another equation A2x+B2y=C2 and we want to find, see for Ax the first equation the solution set is the line geometrically. The second one, the solution set is another straight line. So when you want to say that you want to find simultaneously solve this right, that means we want to find those points x and y in the plane which satisfy both the equations.

And that essentially means that is where the lines will intersect. So both lines are parallel okay but not coincidental, so they will never intersect. So there is no solution possible geometrically. Lines intersect and there is one of the axioms of geometry that the two lines always intersect only at one point if the two intersect if they are not coincidental. That means (()) (12:30) unique solution for the two system of two linear equations right.

If at all they are non-coincidental then there will be one solution and lines are coincidental that means the two lines coincide, so there are infinite number of points where the solutions are there. So geometrically we get this information from geometry that for the system of two equations in two variables 3 possibilities arise. Both the lines are parallel but not coincidental. That means they never intersect, no solution and unique solution and infinite number of solutions.

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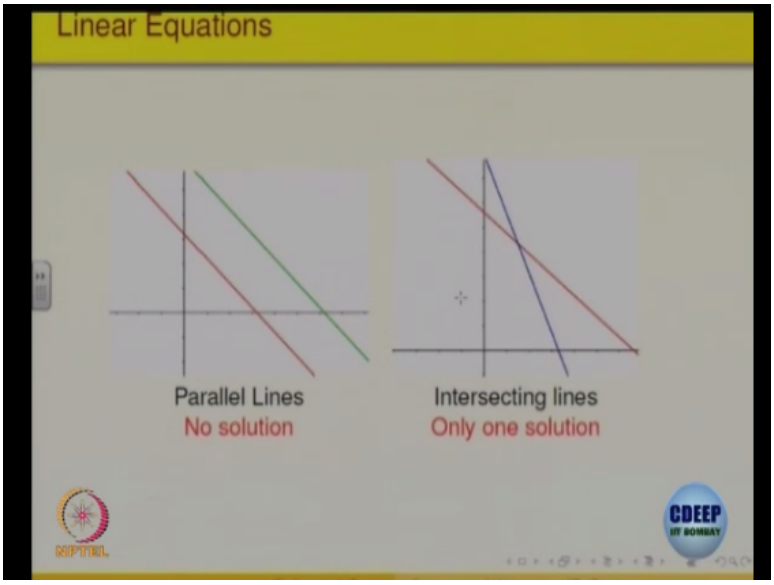

So these are the geometrical possibilities for linear equations.

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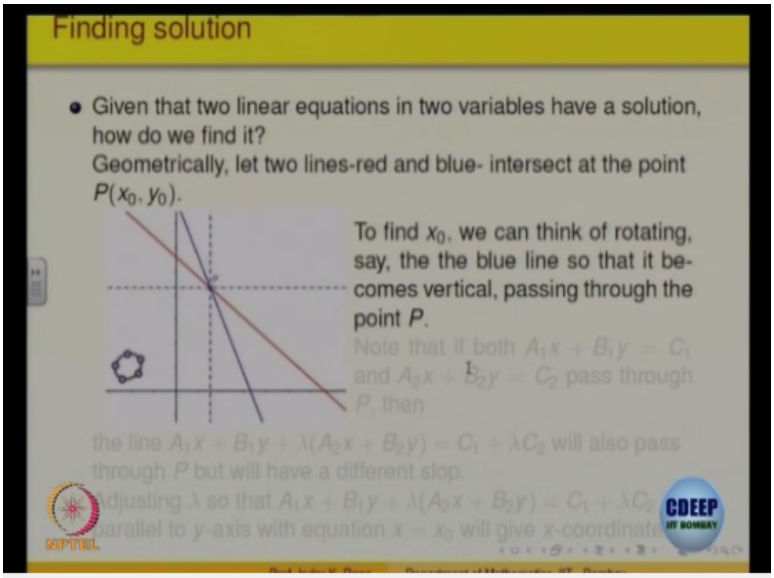

One can try to solve these things geometrically. How do you find the point of intersection geometrically? That is a question one would like to answer. So the idea is if this P is the point where these two lines intersect, if you can rotate one of the lines so that it becomes horizontal or vertical, that will give me the x coordinate or the y coordinate of that point right. So given a line, one of the (()) (13:48) there is a point of intersection, I rotate the other line okay.

So that when it becomes horizontal right when it becomes horizontal what will be the line, y=something, so that is the y coordinate of the point of intersection. So basically geometrically that is what you want to do and one can do that. I just want to show you that how it is possible to rotate and that gives the hint how to solve the equations algebraically okay. So let us look at what the (()) (14:24) mean?

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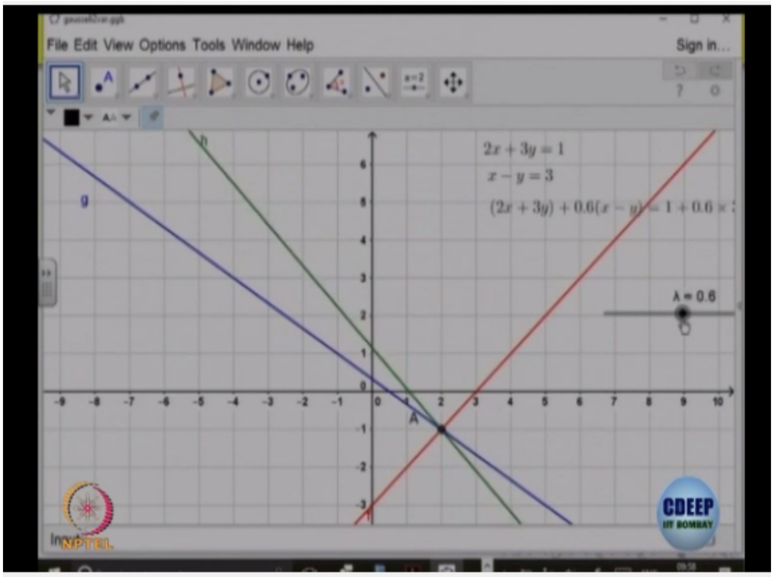

So these are the two lines, blue and the red one which intersect this point okay. So 2x I just taken an example 2x+3y=1 x-y=3 okay. Now what we have done is have taken a combination of these two lines. So 2x+3y+3 times, the 3 is the value which is I am going to change okay and let us see what happens. So what we are discovering is that if you take a combination of the two lines, the linear combination of the two lines then what happens?

See point of intersection remains the same when you take a linear combination but what is happening is the slope of the line is changing right when it becomes horizontal that will give me the value of the x coordinate and when it becomes horizontal that will give me the value of sorry right so that will give me the value of the y coordinate. So geometrically that is how you can solve a system of two linear equations in two variables.

So basic idea is if I take a linear combination of the two equations right, idea should be that one of the variable should vanish, either it should become x is=x0 or y=y0, that will give me one of the coordinates. So I can put back the value in the equation either of the equation will get back the other. So geometrically that is how you solve, so it says that we should be doing something all change of variation, sorry we should be doing elimination of variables for that equations.

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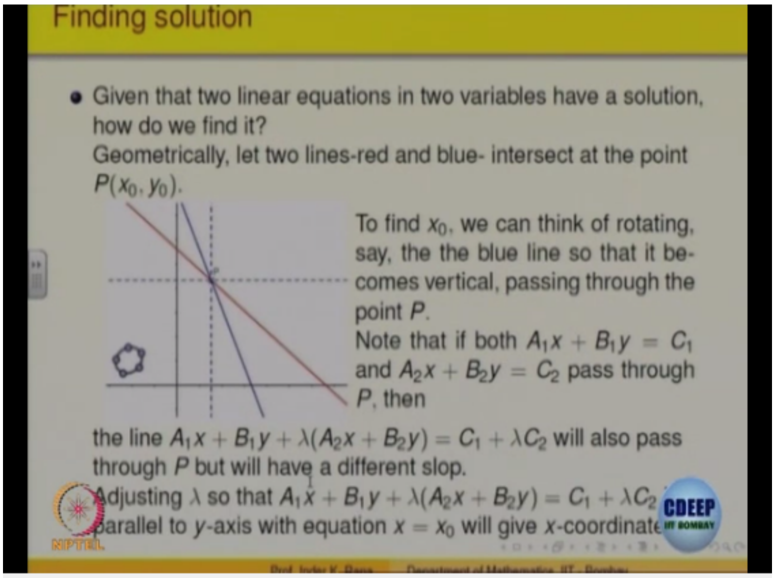

So let us just so that is what is explained here that if I take a linear combination, the slope will be different right, the linear combination will have a different slope because A and B would have changed for the new equation but the point of intersection remains the same. Why does the point of intersection remains the same, whether the x0, y0 satisfies the first equation right.

Then, Ax0+By0 A1x0+B1y0=C1 and similarly for the other equation, when you add they will remain the same right. So point of intersection will remain the same, the only slope changes so it rotates.

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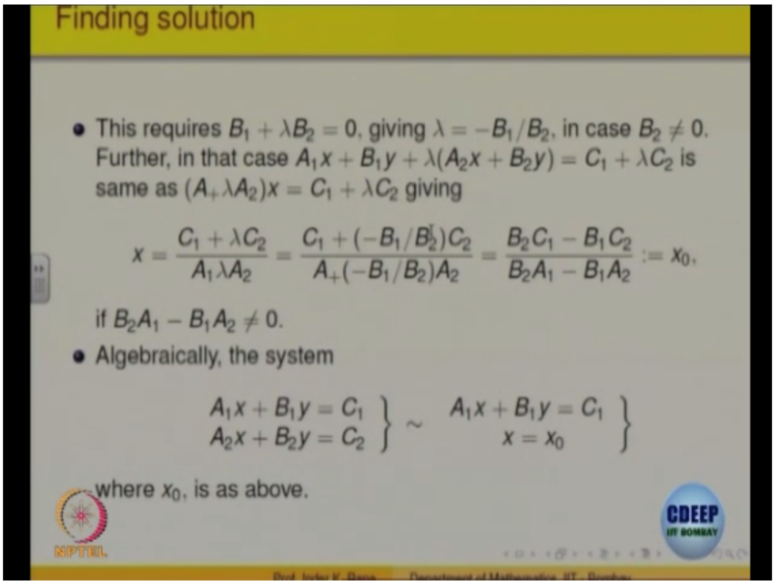

So that is the geometric way of solving but algebraically we can solve it okay. So algebraically the idea is try to eliminate one of the variables so that you get the value of the right, eliminate one of the variables you get the value of the one coordinate of one and find out the other. So for two, you do this in the school, so given and you want the coordinate of x, so you want the y component to be 0, so put that equal to 0 and solve okay.

So you can do that algebraically also. So the basic idea that we are getting from here is that to find a solution of two linear equations in two variables, you try to eliminate one variable right from that by taking the linear combination of the two equations okay.

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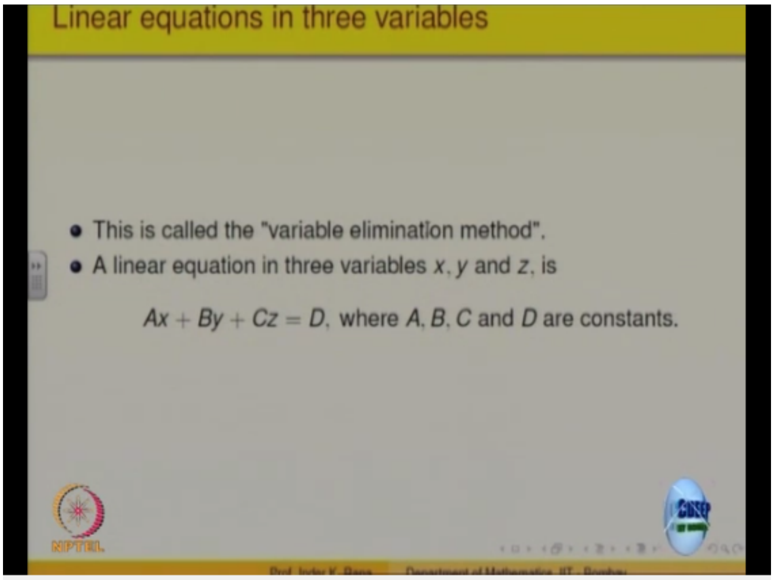

So this is called the variable elimination method for the two variables. Let us try to go a step higher and see what happens when we have got equation in 3 variables. So Ax+By+Cz=D where A, B, C and D are constants and x, y, z are scalars. So what is this equation, what does it geometrically represent? So that represent x, y and z, so it is a triplet x, y, z right that will be a point in R3 right. So let us see how what does this represent?

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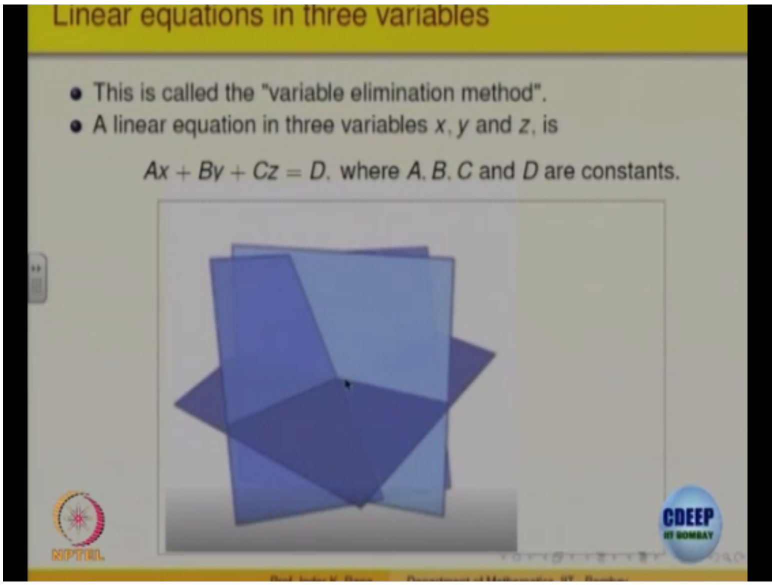

So let us just try to see.

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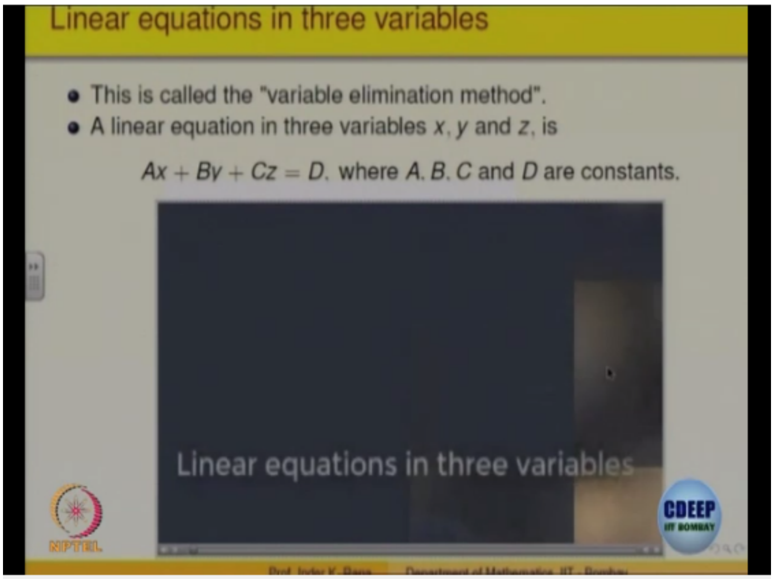

What does the equation in 3 variables? (()) (18:48).

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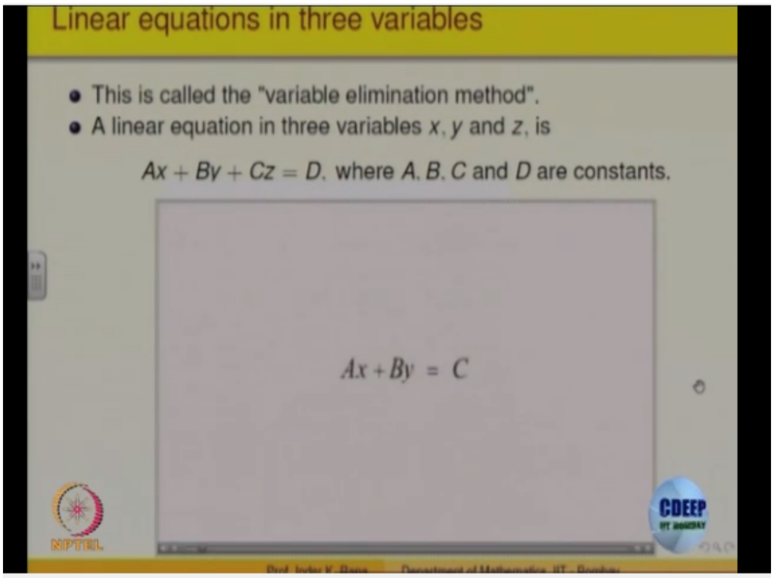

Transform of your equation in two variables x and y is written as Ax+By=C.

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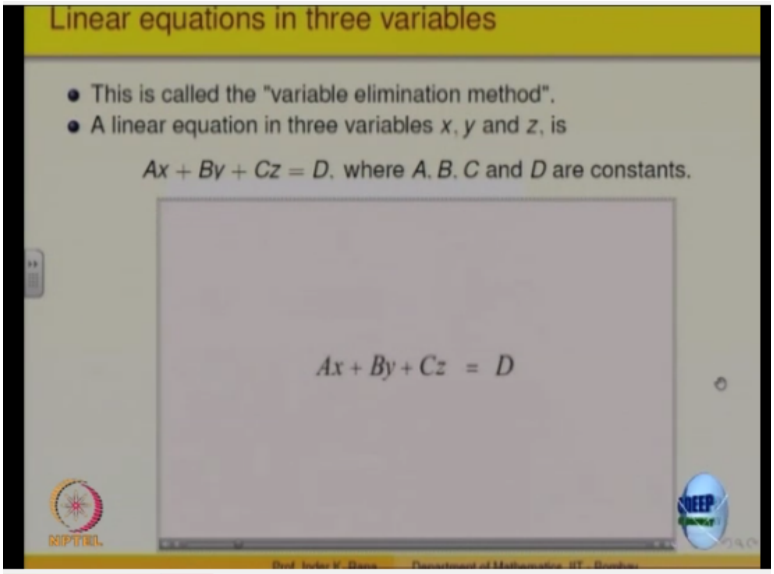

The standard form of your equation in 3 variables x, y, z is written as Ax+By+Cz=D where at least one of the coefficients A, B or C must be nonzero. Since the equation contains 3 variables, it must be plotted using a 3-dimensional coordinate system.

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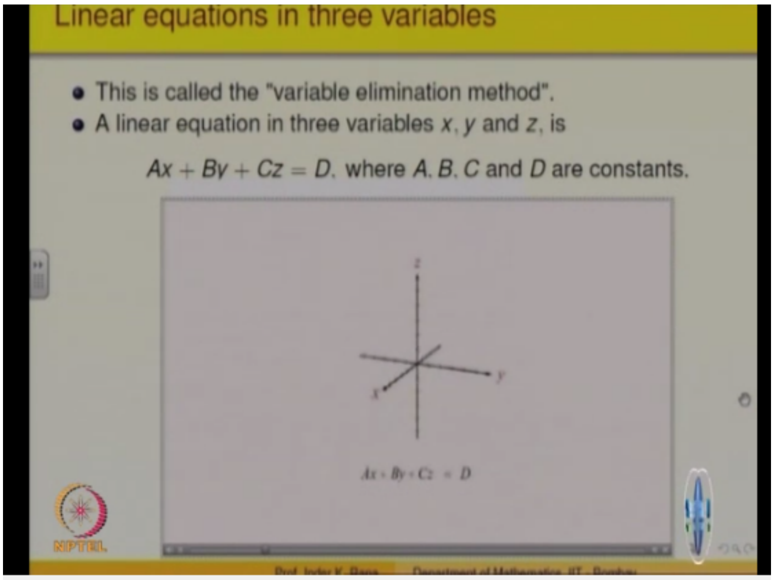

Start by setting A and B to 0 and C to 1, this eliminates all the variables but z, if we set the value of z as 3, we get the linear equation as z=3.

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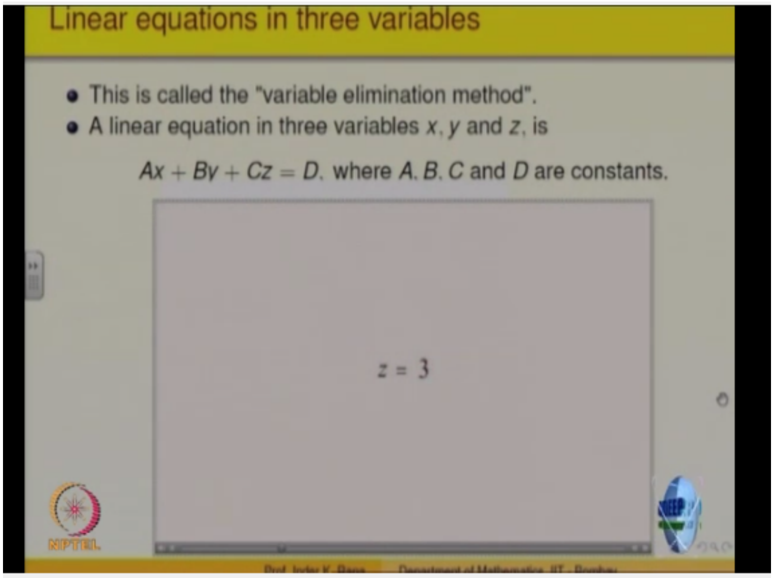

Since x and y are free to take any values, the graph of this equation consist of every point in 3-dimensional space, the z coordinate is 3 and x and y coordinates are in real numbers.

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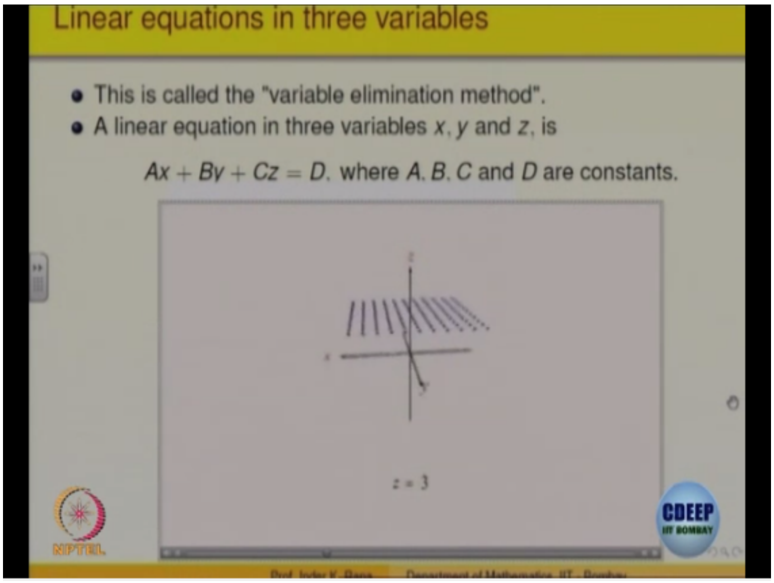

The graph of this equation is therefore (()) (20:10). If we set A and C to 0 and B to 1, all variables but y are eliminated from the equation.

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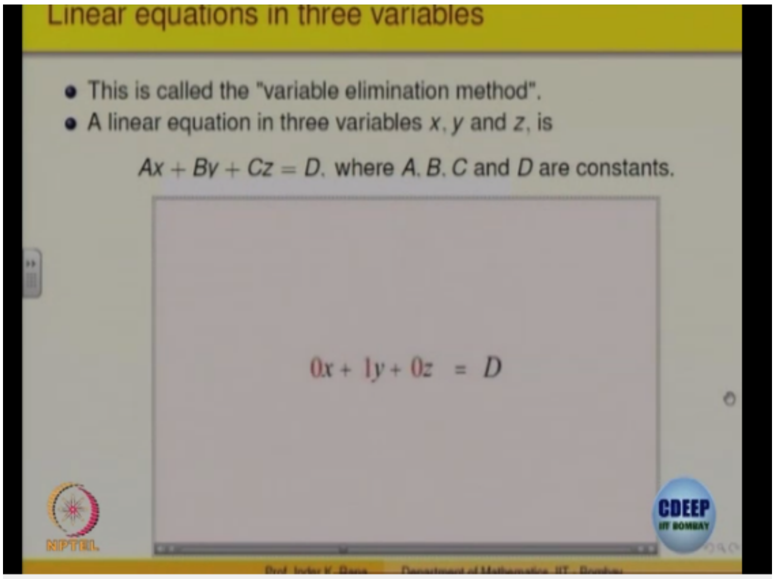

And setting D into 3, we get the equation y=3. Now since x and C are free to take on any value, the graph of this equation consist of every point in y coordinate is 3.

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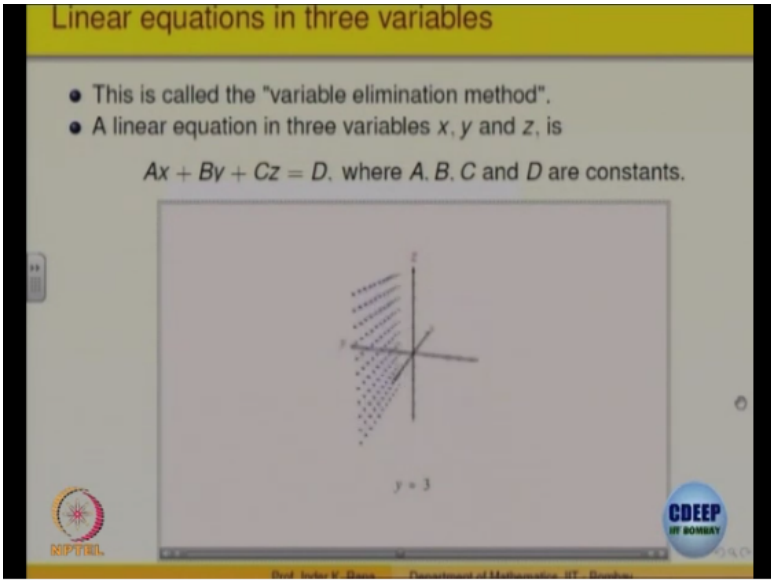

The graph of this equation is therefore (()) (20:55). If we set A, B, C and D to 1 0 0 and 3, all variables but x are eliminated. This graph is consistent of point with the x coordinate of 3.

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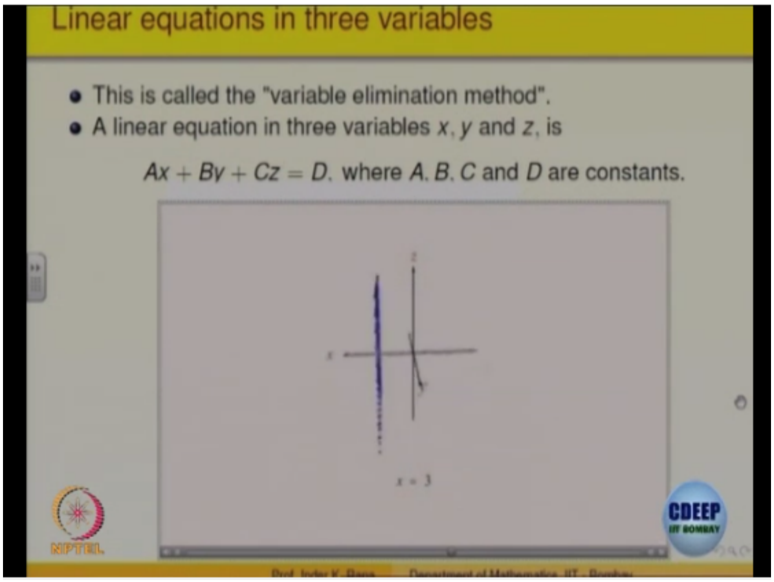

The graph of this equation (()) (21:28) to 0 and the x and y coordinates are 1, this eliminates z from the equation but leaves the variables x and y. Now if we set the value of D as 3, we get the linear equation as x+y=3. Let us ignore z (()) (22:00) and plot this equation on x, y plane. This gives us a line which intersects the x and y axis. Every point on this line is a solution to the equation x+y=3 where the z coordinate of every point is 0.

So there are more solutions than this. All points on any line for x+y=3 are solutions to this equation (()) (22:27) value of z.

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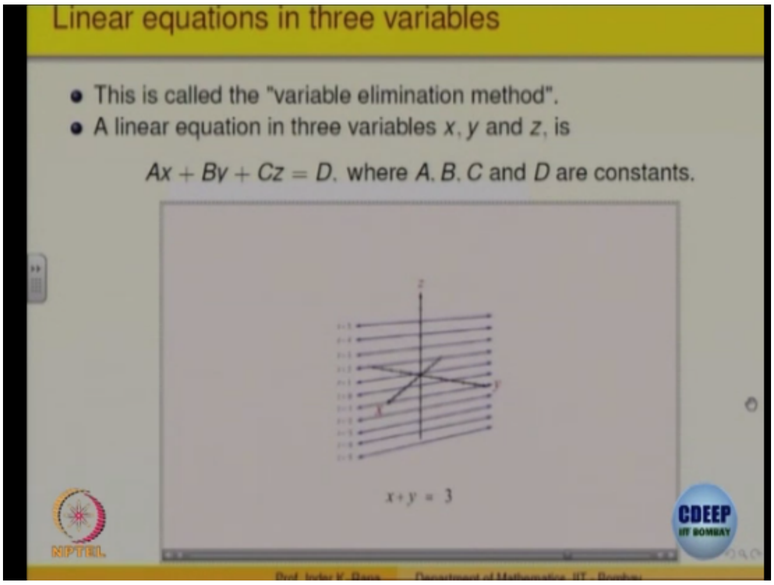

There are an infinite number of lines for this, one line for every possible value of z. Since z can be any zero number, all these lines together create a plane. Each point in this plane corresponds to the solution of the linear equation x+y=3. The graph of the linear equation in 3 variables will always be (()) (22:56) graph in Cartesian coordinate assuming that the constants A, B, and C are not all 0.

Any plane which corresponds to the solutions extends infinitely in all direction. Depending upon the values of A, B, C and D, the point may lie in any position (()) (23:19).

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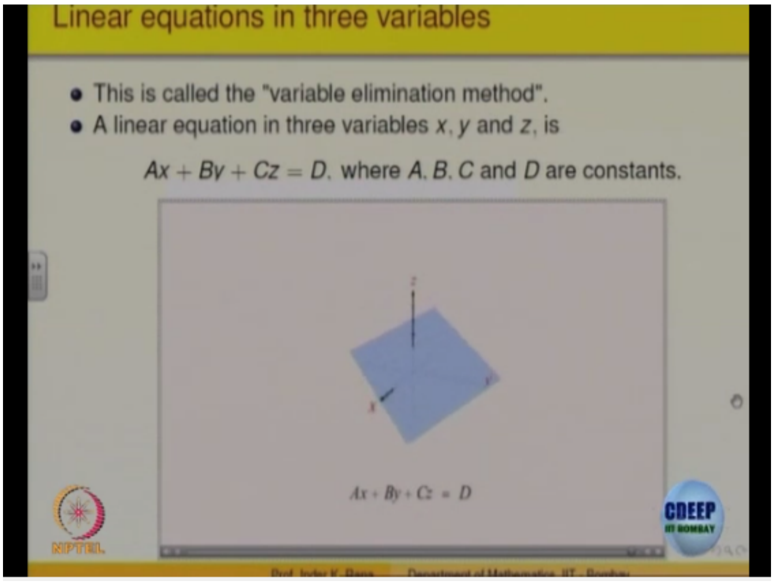

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So equation in 3 variables represents a plane geometrically and one would like to know that given so that all points in the plane are solutions of that linear equation of 3 variables.