Calculus for Economics, Commerce and Management
Prof. Inder K. Rana
Department of Mathematics
Indian Institute of Technology, Bombay

Lecture – 09
Functions, graph of a functions, function formulas

So, welcome to today's lecture. If you recall; in the previous lecture, we had started looking at geometric representation of the plane. So, let me explain that a bit more.

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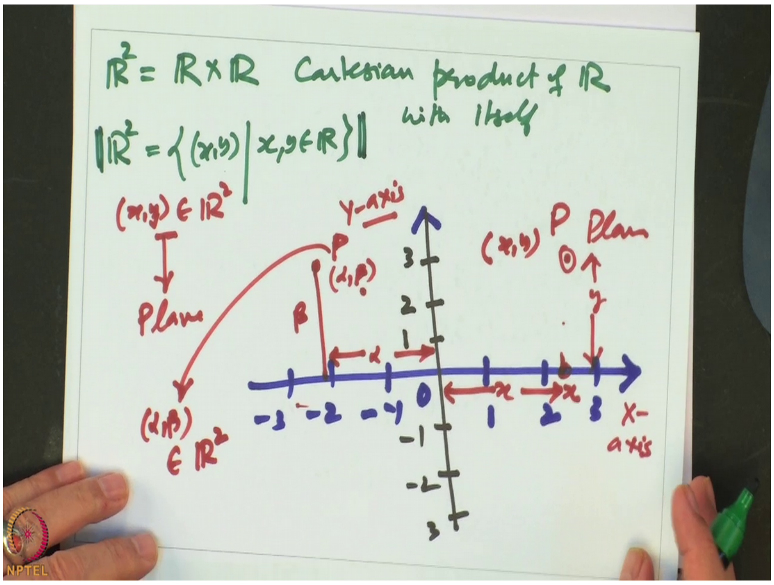

So, that we have a good idea about; so, here is R cross R. So, R 2. So, this is the Cartesian product of the set of real numbers with itself. So, as a set, R 2 is nothing, but x comma y where x and y both belong to R. So, these are set of all ordered pairs; what is a geometric representation of this; we take this as the plane; this paper as the plane and on the plane, we draw 2 lines 1 vertical and other horizontal.

So, this we mark it with arrows; you can think of this as a copy of the real line starting at 0. So, on this side, it will be 1, 2, 3 and so on; on this side minus 1, minus 2, minus 3 and so on. Vertical also is a copy of the real line where upside is positive. So, this is 1, this is 2, this is 3 and so on and the bottom side is minus 1 minus 2 minus 3 and so on. So, we have made 2 copies of the real line; one is horizontal, another is vertical and both are perpendicular to each other what we want to do is every point every ordered pair x comma y belonging to R 2, we want to associate to it a point P in this plane. So, this is my plane right a point P. So, how do we will locate the point P when we have the ordered pair x comma y, we would look at x and then move x units along the horizontal axis.

So, this is what is called the x axis horizontal and the vertical one is called the y axis. So, x units along the horizontal line, we go if x is positive will go this side if x is negative, we will go this side. So, somewhere x comes x units. So, this is x right in the ordered pair for a y, we have got the number y. So, what we will do is again look at whether y is positive or negative if it is positive we go up if y is negative, we go down and that many units. So, let us say y is positive. So, we go up right and reach a point. So, this is y units. So, we reach a point and this point is called P. So, P we say P has got coordinates x comma y; that means, x units you move on the x axis; along the x axis and then go parallel to y axis and distance is y.

So, that is; so, from here R 2 to plane, we have got a map this map is a correspondence actually why it is a correspondence. Now suppose I am given any point P right, it has got coordinates alpha beta say. So, what does that mean; that means, if I move alpha; alpha is long axis beta is a long y axis. So, this distance will be beta and this distance will be alpha right. So, to reach this point I have to move alpha units and then beta units I get.

So, every this; so, this points gets associated with alpha comma beta ordered pair belonging to R 2. So, what we are saying is every point in the plane gets associated with an ordered pair and every ordered pair gives us a point. So, this is the set erratic representation as a set and this is the geometric representation of the plane as set of points. So, we will be using this to represent the graph of a function.

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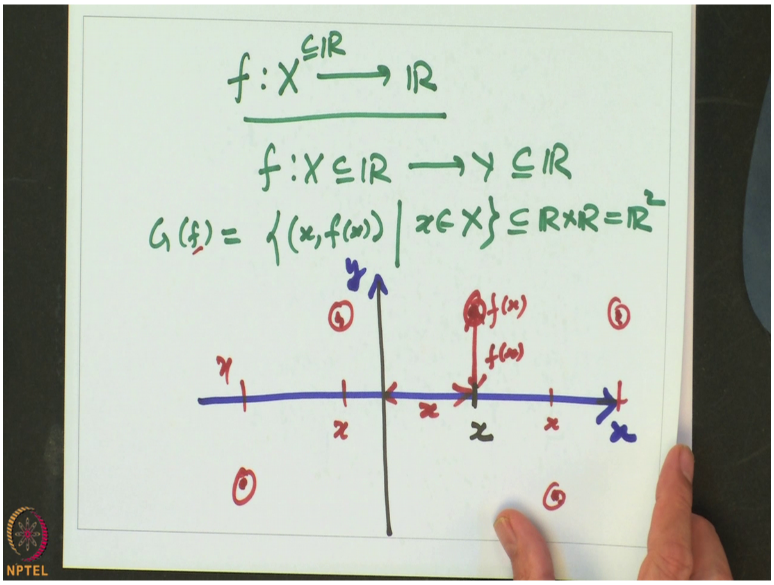

So, let us take graph of a function. So, f is a function from a set x which is a to a set y right here we have got x is a subset of R. So, let us be more precise f is from x a subset of R to y which is also a subset of R. So, what is graph of f? Graph of f is nothing, but the set of ordered pairs x comma f of x; x belonging to x right that is a subset of R cross R that is R 2; how do we represent it graphically. So, as before we take our graphic representation. So, this is x axis this is y axis this is x axis. So, what we do you take any point x comma f x. So, so take x, this is x units and what we do from we now we move how many units y y is f of x. So, go parallel to say go parallel you will get f of x.

So, this is f of x this height and this is x. So, you get a point. So, look at all such points. So, given any point x look at may be f of x is here may be f of x is for this point x axis for this point x f of x may be somewhere here, right. So, this dots we will represent the graph of the function g of graph of the function f. So, these dots are the; for this x f of x could be probably here. So, this dots represent the graph of the function f. So, that is what geometric representation of the graph means.

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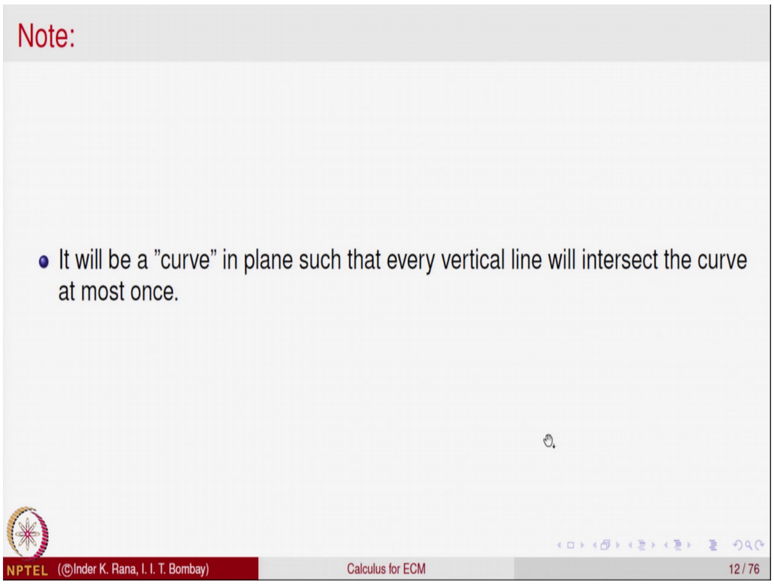

So, here is an observation that a graph normally will give you some kind of a curve in the plane a graph will give you a some kind of a curve in the plane, right, but every curve in the plane need not be the graph of a function this is lightly settle point, we will see it soon what does that mean.

So, for example, let us let us look at a picture and try to see.

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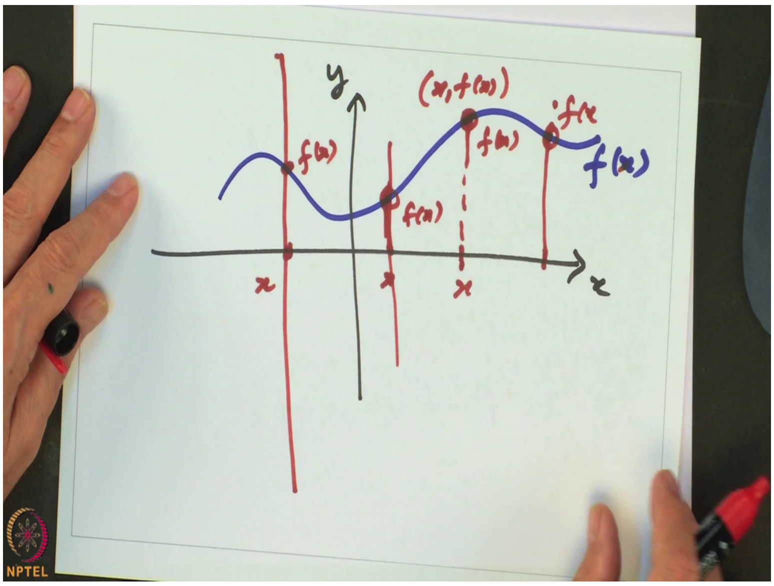

Let us say this is the x axis and this is the graph f of x. So, that is my graph. So, here is y here is x. So, for a point for a point x; what is f of x. So, this is f of x because what are the coordinates of this point x comma f of x. So, this is also I think this is the right time to also say that how do you interpret a graph. So, we are given some graph what does that mean; that means, if I take at any point x a vertical line then wherever it cuts it that is a value of x at every x this is f of x at every point x this is this point is f of x right and you see in this picture whenever I am drawing a line at every point x a horizontal line that cuts the graph only at one point.

So, this line that drawn is the graph of a function it is a relation which is a function let me draw another graph another curve and look at what is happening.

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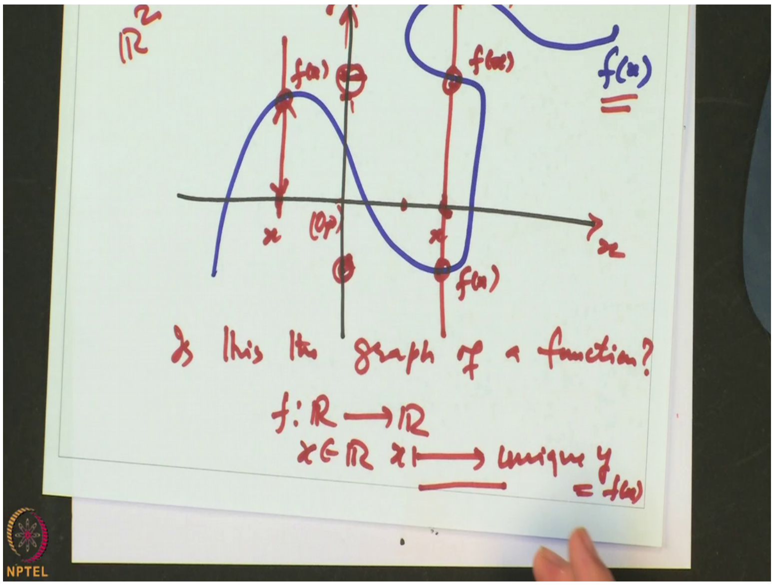

So, let us draw another picture and let us look at f of x can I say that this curve is the graph of a function f. So, let us call this curve as some this curve. So, let us is this the graph of a function. So, this is my x this is my y, right. So, if it is the graph of a function then it is from R to R, right because this is how I have looked at this is the real line this is 0 0 and so on. Now for a graph what is the; it is a subset of plane this is R 2. So, it is relation. So, this points x y on this blue curve are related to each other because they are on the curve.

Now, the question is can I say this is the graph of a function well to be the graph of a function what is the property we want for every x belonging to R x should go to unique y equal to f of x and how do we find for any point x what is f of x for x. So, I draw vertical line. So, this is my f of x this height right. So, this is my f of x, but if I go for example, here take a point x and draw a vertical line it intersect at more than one point so; that means, what at this point possibly this could be the value of the function this could be the value of the function this could be the value of the function; that means, these 3 points right if you look at corresponding points in y. So, this is one point; this is the second point and this is the third point.

So, these 3 points on y are let it with the same point x; that means, this condition is violated x is not associated with unique point y it associated with more than one point. So, this blue curve is not the graph of a function. So, a test for a graph test for a curve to be the graph of a function is that every vertical line should intersect at the most at one point. So, this is what we are saying that it a curve in the plane such that every vertical line will intersect the curve at the most one will be in the graph of a function for example, here.

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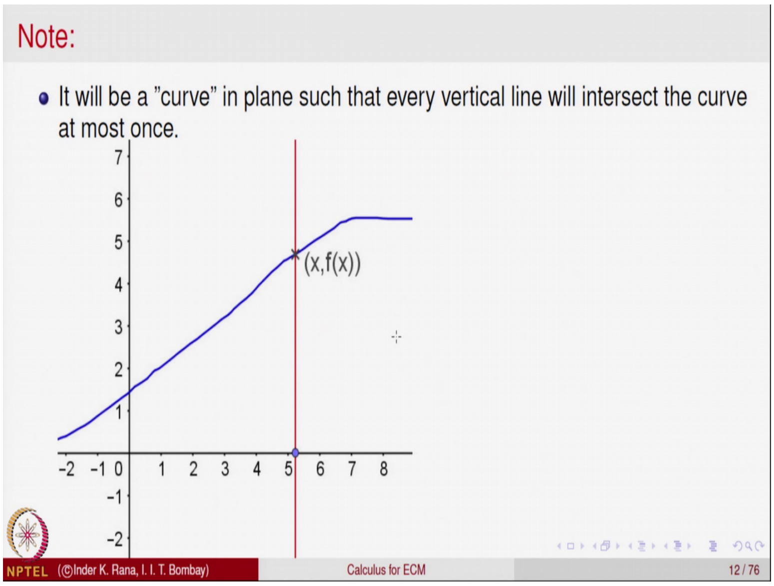

So, this if I take any point x and move it this vertical line around then it intersects only once. So, this is the; this blue line is the graph.

But what we saw is just now one paper that is what the graph of the function. So, not every picture in a plane is the graph of a function right. So, the test is the horizontal; vertical line test every vertical line should not intersect in more than one point then it is a graph of a function I think this is a right time also to look at what is called 1, 1 and onto function.

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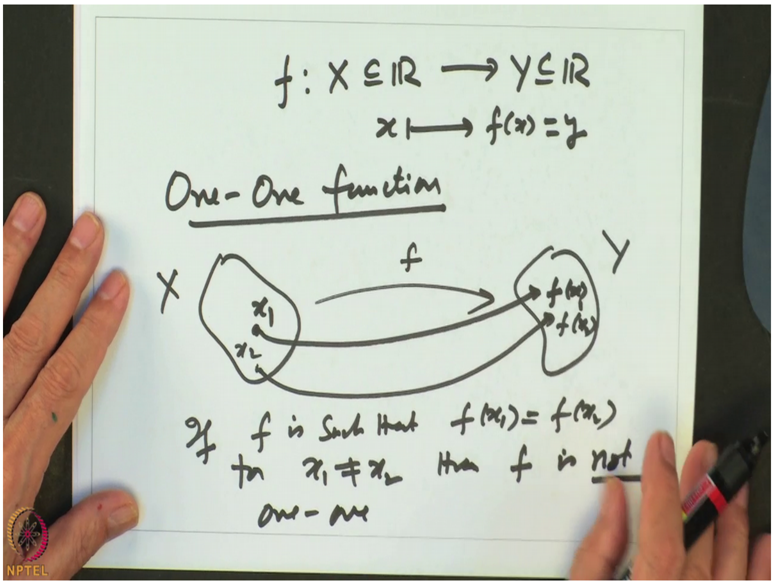

So, let us look at something. So, let us say f is a function from x which is again a subset of R to y which is a subset of R x goes to n f of x which is normally denoted by y what is called a one; one function intuitively the real line, but let us imagine this is a x and this is y as such that is a part of real line, but anyway; let us for the understanding let us say this is x possibly a point x goes to a value f of x, right. So, let us call x 1 goes to value of x 1 and another x 2 also goes to some value f of x 2.

But it is; so, happens x 1 are equal to x 2. So, if f is such that f of x 1 is equal to f of x 2 for x 1 not equal to x 2; that means, different points go to same value then we say f is not 1 1, it is not 1 1 because more than one value goes into the same value, right. So, let us look at some examples of this.

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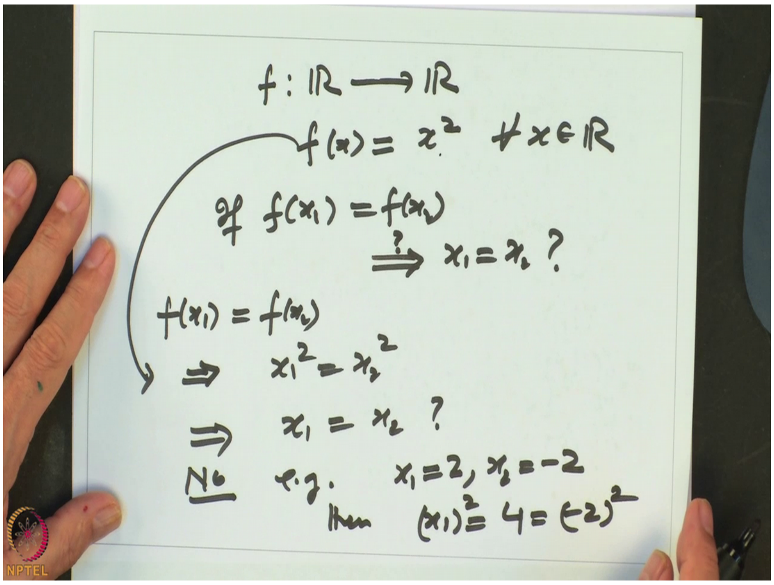

So, let us look at function f from real line to real line where f of x is equal to x square right for every x belonging to R. Now can you say that if f of x 1 is equal to f of x 2 does this imply that x 1 is equal to x 2 well; let us analyze f of x 1 equal to f of x 2; what does it imply this implies by the real formula from here x 1 square is equal to x 2 square.

Does this imply x 1 is equal to x 2 well, we all know no in general for example, if I look at x 1 is equal to 2 x 2 equal to minus 2, then x 1 square is equal to four which is same as minus 2 square. So, what does this mean; that means, that this function is not 1 1.

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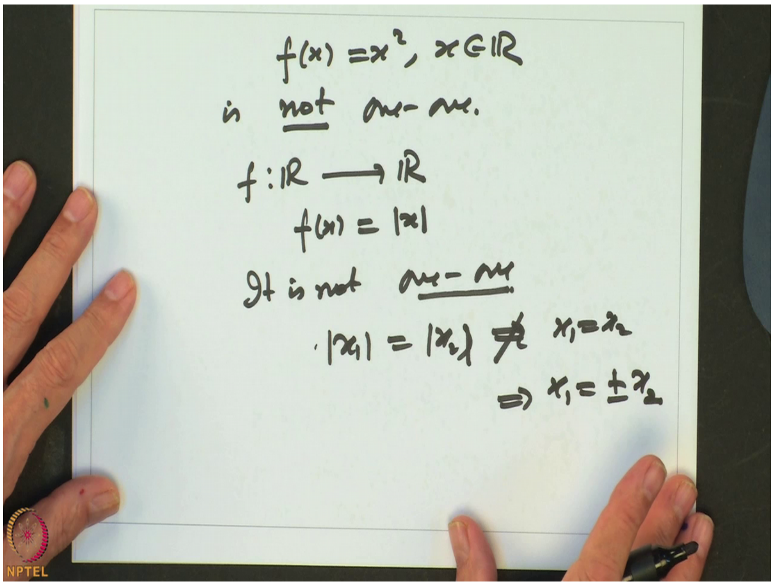

So, let us write this as an example of. So, f of x equal to x square x belonging to R is not 1 1; si not a 1 1 function; let us look at geometrically what does it mean let us look at a function f from R to R which you have already come across f of x is equal to mod x once again it is not 1 1; why it is not 1 1; for example, mod of x 1 equal to mod of x 2 does not imply does not imply x 1 is equal to x 2 it only implies that x 1 is equal to plus minus of x 2.

So, this is not a 1 1 function right let us also define what is called another property functions.

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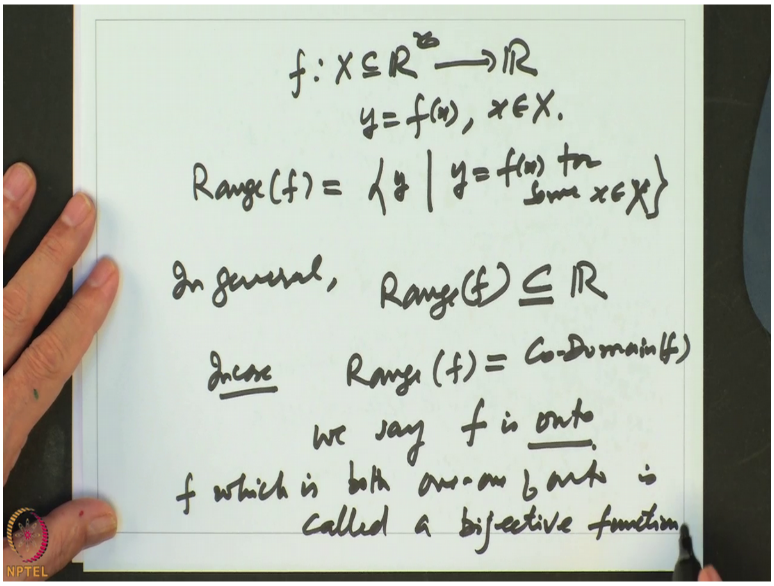

So, let us say f is a function from x from a R to R. So, y is equal to f of x right now we recall we define what is called the range of the function to be equal to all y such that y is equal to f of x for some x belonging to the domain. So, in general we saw examples range f is a subset of the co domain here it is R that is a co domain right. So, in case range f is equal to co domain f we say f is onto f is a onto function a function f which is both 1 1 and onto is called a bijective function.

Before we give examples of 1 1 onto functions more examples of it which you are able to visualize let us draw graph of some easy functions. So, let us look at a function which is called the constant function normally a functions are specified by formulas. So, let us look at a function say for example, from 1 to 2.

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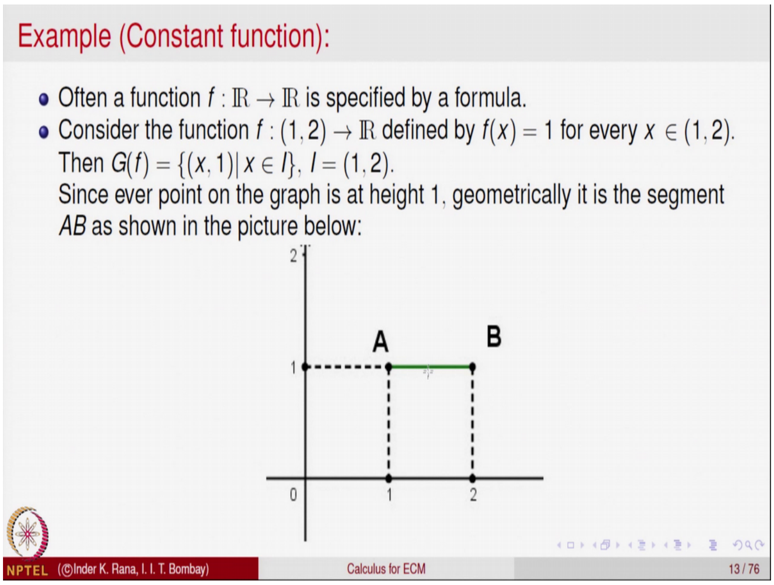

So, this is a interval which is the part of the real line and the co domain is the real line. So, this is the function given by the formula f of x is equal to 1; for every x in the domain 1 2. Let us try to draw a graph of this function how would you draw a graph of this function for every x, I should look at the ordered pair x comma f of x; f of x is equal to one; that means, I should look at all ordered pairs x comma 1; x belonging to 1 to 2, but with a association the ordered pairs x comma one what are these.

So, these are points on the plane which lie on a line on a horizontal line. So, what is that? So, a point between 1 and 2 if I take a point x here what is f of x; f of x is a height one. So, this is a point. So, this line segment this line segment is the graph of this constant function f 1 to 2 to R by given by f of x is equal to 1. So, this is a constant function right taking value one everywhere on the domain which is 1 to 2. So, for this function the domain is the interval one to 2 it is a open interval 1 to 2. So, what is the range; range is single point one right range is the single point one can we say this is 1 1; obviously, not this is not 1 1 because all the points in the domain all get mapped onto same value one. So, this is a function which is not 1 1; right and co domain is R, what is the range the single point 1.

So, this is also not onto; so, this is neither 1 1 nor onto, but it is a nice very simple function with a nice graph. So, the constant function f x is equal to 1; in this domain; 1 to 2, this is the graph of it let us look at slightly more general example.

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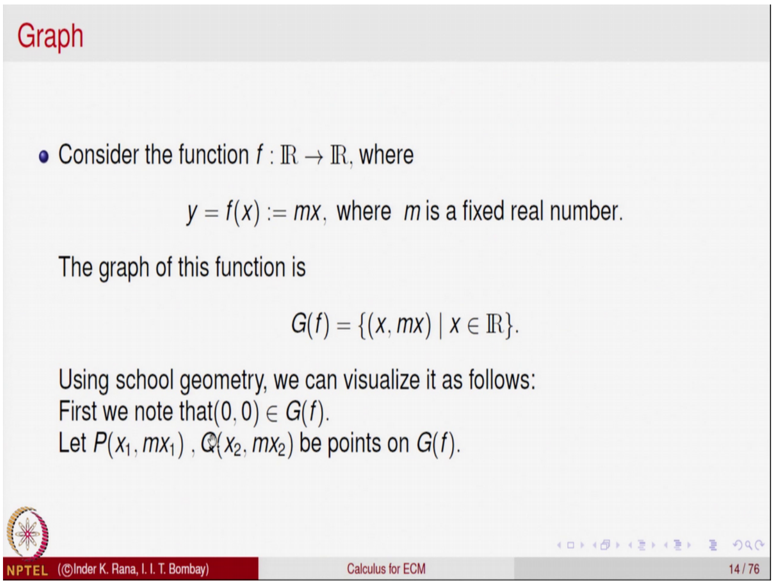

Let us look at a function R to R given by the formula y is equal to f of x. It is m times x where m is a fixed real number. So, what we are saying every point x is related to y where y is equal to m of x right we want to look at the graph of this function what is the pictorial representation of this function. So, graph of this is the set x comma m of x x comma y where y is equal to f of x. So, the graph is x comma m of x; x belonging to real line let us see these ordered pairs how they get represented on the plane in terms of the coordinates that we have defined, right.

So, using school geometry we can visualize it as follows first let us note that the point 0 0 belongs to the graph if x is 0 x is 0 in the domain then y is also 0 0. So, the point with coordinates 0 0 is on the graph of the function right next let us take 2 points P and Q on the graph. So, the first point is with x coordinate x 1. So, what will be the y coordinate m of x 1 Q is x 2. So, the y coordinate will be m of x 2 because that is a function f of x. So, these given these 2 points.

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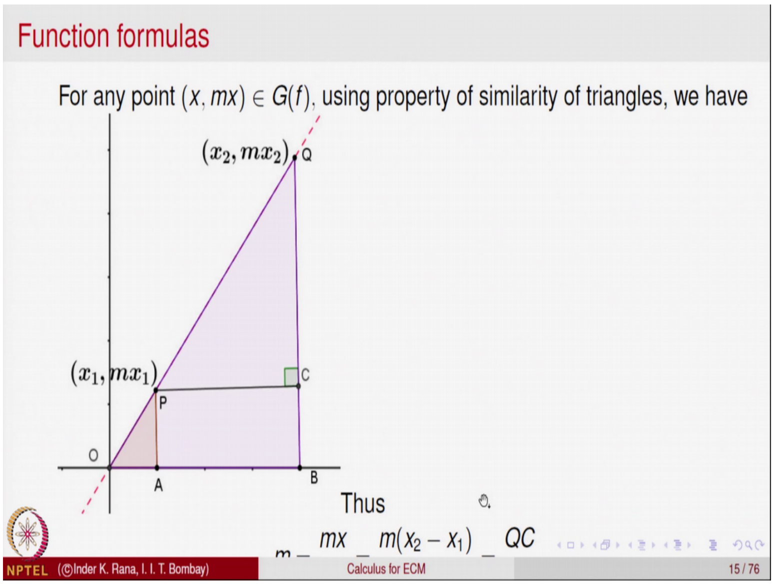

So, these are 2 points let us take any general point x comma m x on the graph of the function. So, now, this is the point P which is x 1 m x 1 this is the point Q which is m 2 x 2 m of x 2 y is m of x 2. So, and 0 0 is already on the graph.

So, let us look at this 2 triangles O P A and O Q B or you can also look at that triangle P Q C sop; let us look at this triangle P O A and Q P C, these 2 triangles are similar triangles because this angle is 90. This angle is 90. This angle is equal to this angle because this is horizontal line; I have drawn this is a vertical line. So, these 2 angles are equal by the property of parallel lines and it transverse intersecting these 2 angle. So, these 2 triangle are similar what does similarity of triangles means similarity of triangle says that the ratio of the corresponding sides are equal. So, P A by O A is same as Q C by P C, right. So, what is P A by O A? So, that is m x 1 divided by x 1 that is m right and m x 2; this point is m x 1. So, what is this height that is m times x 2 minus x 1 this height is this base is equal to m times x 2 minus x 1.

So, that gives us the relations. So, that is not visible. So, let me write that.

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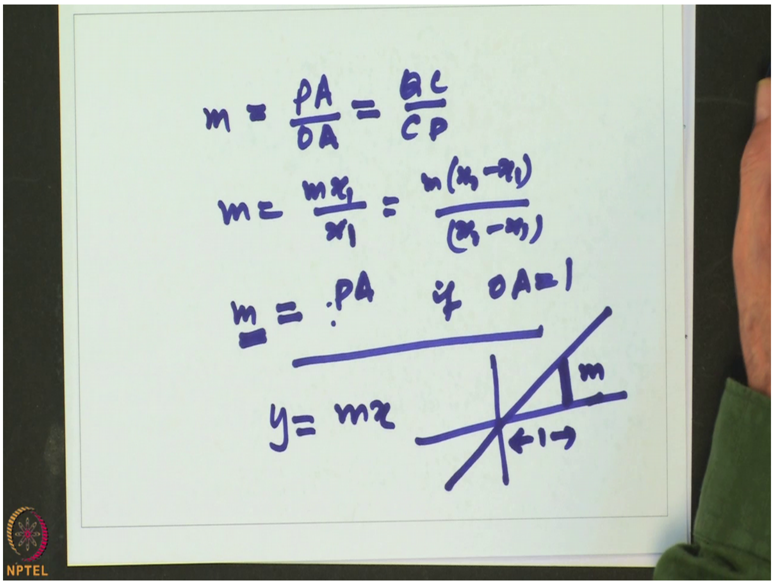

So, that gives us the relations m is equal to P A divided by O A that is also equal to Q A divided by C P; this one is equal to m times x by x 1 by x 1 and this is equal to m times x 2 minus x 1 this is y; x 2 minus x 1. So, all these are equal right. So, what is m? M; if I take x 1 equal to 1; then it is equal to P A if O A is equal to 1; right. So, that essentially what we are saying is if I take any 2 points right, then they are going to be on the same line that is the red dotted line, the graph of this because if I take any 2 points on that line then the ratio is same.

So, for every point x if I take a point here that is going to be ratio is going to be same. So, that triangle is going to be same as similar to this triangle.

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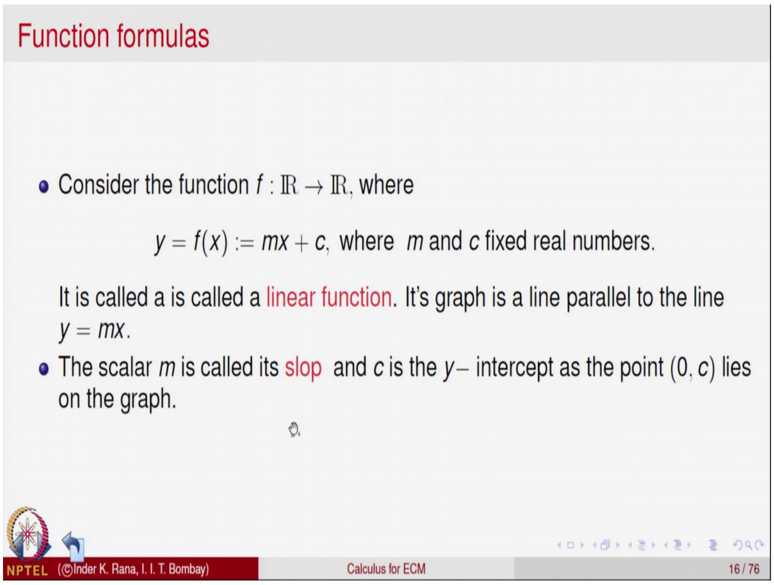

So, all this points are going to lie on the line so; that means, that the equation of the so; that means, the line y equal to m x will have graph as a line where if I take this distance as one right this height will be equal to m. So, that is the property. So, that is the graph of in general if I take a function y equal to m x plus c where m and c are fixed then if c is 0 it is original graph, then only what we are doing is we changing the y value of y by adding c to it. So, it is just moving translating moving the graph of the original function by c units it if x is equal to 0 then y is equal to c. So, it passes through the point 0 comma c and m remains the same that inclination in a sense remains the same it is parallel.

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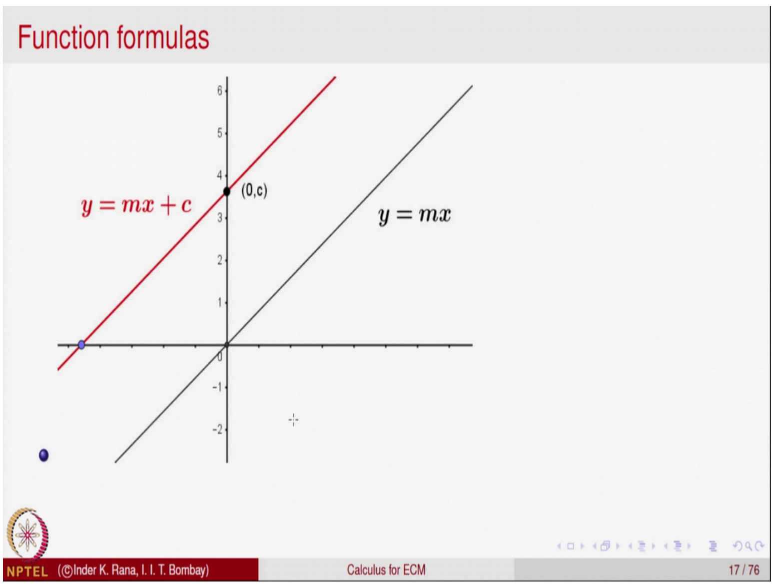

So; that means, if this is the line y equal to m x plus c and if we just translate it and draw a parallel line then this is the graph of the function y equal to m x plus c this is. So, such functions are called linear functions. So, what we have done is we have looked at the concept of a relation look at the concept of a function we have try to discuss when is a relation called a function functions are normally given by formulas we have tried to look at the graph of a function as a subset of the Cartesian product and we have drawn the graph of a linear function y equal to m x plus c and that turns out to be a line and conversely one can easily show probably, we will do it in the next lecture that every line is the graph of a linear function so.

Thank you.