Calculus for Economics, Commerce and Management
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Lecture – 08
Relations and functions

Welcome to this lecture. In the previous lectures, we had seen the concept basic concepts of set theory, then we looked at the concepts in real numbers and then we looked at the sequences of real numbers and important property of real numbers was the completeness property of real numbers. Today, we start looking at the notions of relations and functions in mathematics. So, let us start with looking at an example of a scenario is that of a stock exchange, we know that on a stock exchange shares are bought and sold and the persons who buy the shares are called traders and the activity of buying and selling is called trading of shares.

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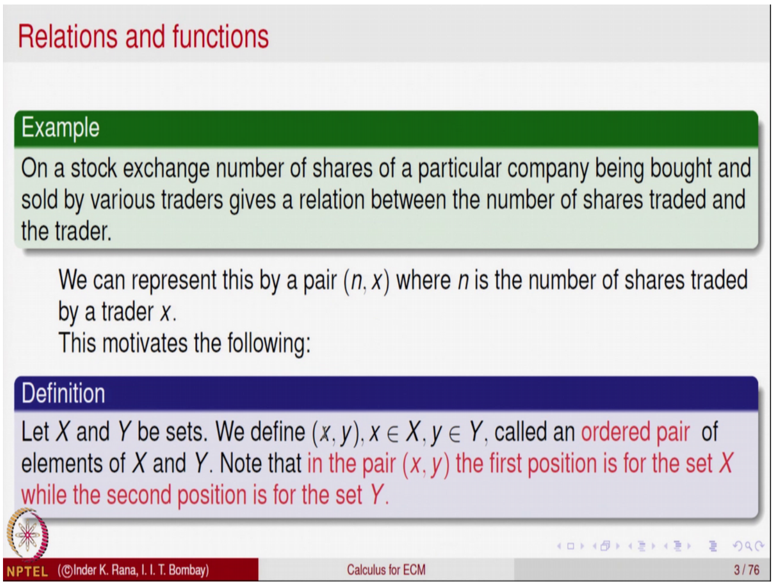

So, the scenario is that on particular stock exchange shares of a particular company are being bought and sold by a certain set of traders and we want to buy the end of the day we would like to represent this data the activity what has been done on that day. So, when a particular trader buys or sells a share that will give you number. So, let us say X is a trader and n is the number of shares he has bought or sold.

So, that will give us the observations are summarized in a pair called n comma x where n denotes the number of shares been traded by a particular trader X. So, this motivates one to define the concept of a ordered pair. So, let us define mathematically x and y be 2 sets, then we define x comma y in round brackets x comma y where x belongs to an x and y is an element of y and such a object will be called a ordered pair of elements of x and y note that in this pair this x is coming from the set x and y is coming from the set y.

So, there is a the positions of appearance of x and y in this pair is being ordered as the first position comes from x and the second position comes from y that is why this is called a ordered its a pair in which the order is fixed the first one comes from x the second one comes from y.

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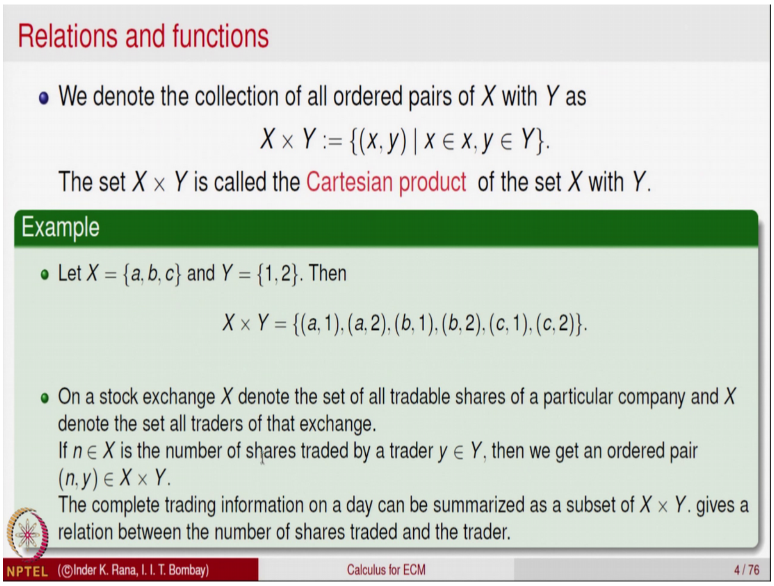

So, such a pair is called an ordered pair of elements of X and Y; the set of all ordered pairs is denoted by this set x. This is called times or cross x cross y. So, x cross y is the set of all ordered pairs of elements of x and of y this set is called the Cartesian product of the set x and y or better. It is called the Cartesian product of the set x with y because there is a order. So, we should be careful right. So, let us look at some examples of this things let us say x is the set of English alphabet a b and c and y is the set of numbers one.

And 2 then what are the ordered pairs possible we take one element a of x and we can pair it with either 1 or 2. So, when we pair it is one we get a ordered pair a comma one similarly when we pair it with 2 we get a ordered pair a comma 2 next we take the element b of the set x and pair it with the elements of y. So, we get b comma one ordered pair and another ordered pair as b comma 2 and finally, we take the element c from x and construct ordered pairs with one and 2 of y. So, that gives us ordered pair c comma one and c comma 2. So, for the set x a b and c and the set y 1 and of elements one and 2 the set of all ordered pairs x cross y is consisting of these elements a one a comma 2 b comma one b comma 2 and c one and c comma 2.

So, there are six elements in the set of Cartesian product of x with y for example, in our example of the stock exchange if x denotes the set of all tradable shares of a particular company and I think; let us call it as the set y y; denote this x denotes the set of all tradable shares of the company and y let us change this x to y because otherwise there will be a confusion and y denote the set of all traders of that exchange then for number of shares x n belonging to x number of shares traded by a trader y in y will give us an ordered pair n comma y. So, that is an element of x cross y. So, that is how the ordered pairs and the Cartesian products come into come into existence, but keep in mind x cross y is not same as y cross x this is Cartesian product of x cross y let us look at this example a bit more explicitly.

So, we had the set x which was equal to a b and c and y was equal to 1 and 2.

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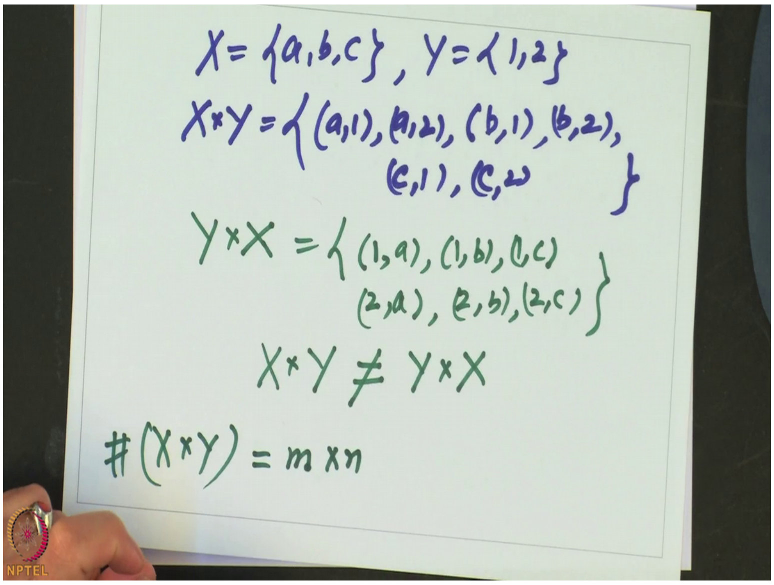

So, we had X cross Y as the set of all ordered pairs. So, we had a comma one a comma 2 then we had the ordered pair b comma 1 b comma 2 and then we had the ordered pair c comma 1 and c comma 2. So, this our ordered pairs. So, this is the Cartesian product of X cross Y, let us look at what will be the set Y cross X. So, that will be equal to now we have to pick up elements of Y first and then form ordered pairs with elements of X. So, Y has got 2 elements let us take one. So, this is one and it is pairing with a elements of X gives you 1 a next gives 1 b and next gives 1 c and similarly with 2 element of y that gives a pair with a as 2 a and 2 with b and 2 with c.

So, y cross x is the set of all ordered pairs of y with elements of x and that is one comma a 1 comma b 1 comma c 2 comma a 2 comma b 2 comma c. So, that clearly exhibits that x cross y in general is not equal to y cross X because X and Y could be of totally different, but keep in mind is a nice observation that X and cross y has got six elements and same is the number of elements of y cross X that happens very more generally if X has got m elements and Y has got n elements then x cross y the number of elements will be equal to m cross n, but we will not have opportunity of using this too much. So, let us come back to our study of ordered pairs of elements of X and elements of the idea is that the complete trading information on a day can be summarized as a subset of X cross Y right.

Here X is the number of shares being traded and y is the set of all traders. So, that is a importance of the Cartesian product X cross Y. So, it signifies. So, subsets of x cross y signify some kind of relation activity happening between elements of X and elements of Y.

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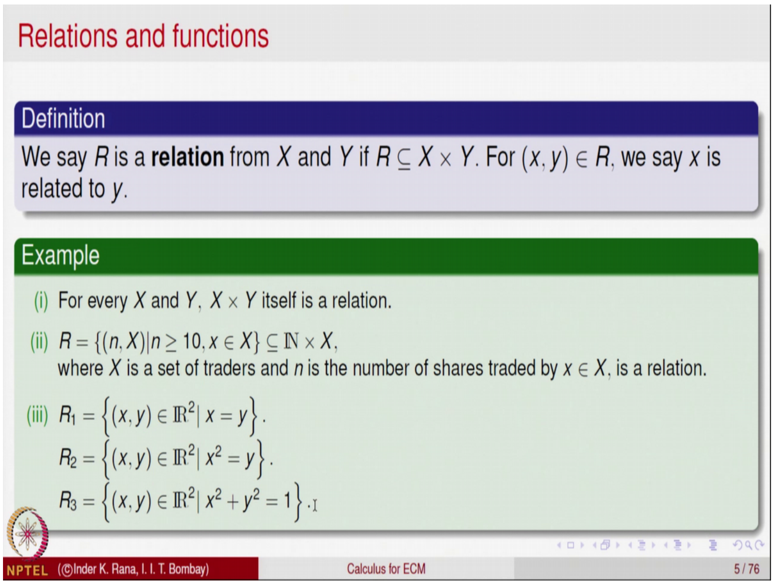

So, this motivates one to define the concept of what is called the relation we say R is the relation from X to Y or X with Y if R is a subset of X cross Y. This is not a very good words, X and Y relation from X, it should be better word is 2 Y. So, the relation from X to Y that is between elements of x and elements of y the order is there is a subset of X cross Y. So, all subsets of X cross Y will be called as relations if we if a element X comma Y belongs to R. So, that is a ordered pair we will say X is related to Y. So, that is that is how we read and understand it.

So, let us look at various examples of relations if X is any set and Y is any other set, then x cross Y itself is a relation the Cartesian product itself; that means, every element of X is related to some element of Y right because all ordered pair is taken into account let us look at the relation n comma x because n is bigger than or equal to 10 and x belongs to x right; that means, we are looking at a subset of the natural numbers cross x if you would like to understand this example a bit more; let us say x is the traders right who are buying shares only. So, n will be positive and. So, this n comma x will signify. So, this relation will signify all those traders who have bought at least 10 shares on a particular day of that company. So, that is the relation that all data will be captured in this set.

So, x is the set of traders n is the number of shares and traded that day and if that is bigger than or equal to 10 another relation possible between numbers real numbers could be x comma y belonging to R 2 where x is equal to y right this example does not have does not really say anything much towards some activity relating to economics or. So, on, but that is relation right. So, x is related to y if x is equal to y right for example, 10 will be related to 10 or 10 will not be related to five right let us look at another relation x comma y in R 2; that means, in this this sets x is equal to y and equal to r. So, x and y are same sets.

So, we looking at the Cartesian product of R with itself this is also Cartesian product of R with itself and the relation is x is related to y right x is comma y belongs to R 2 if x square is equal to square of that number is equal to y, right, another relation possible could be x and y in R 2 such that x square plus y square equal to one this has some geometric significance namely the points on a circle right x is related to y if x coordinate y coordinate are points on the circle, but that is mathematically it is just some relation between x and y. So, this is how you can construct more and more relations between sets.

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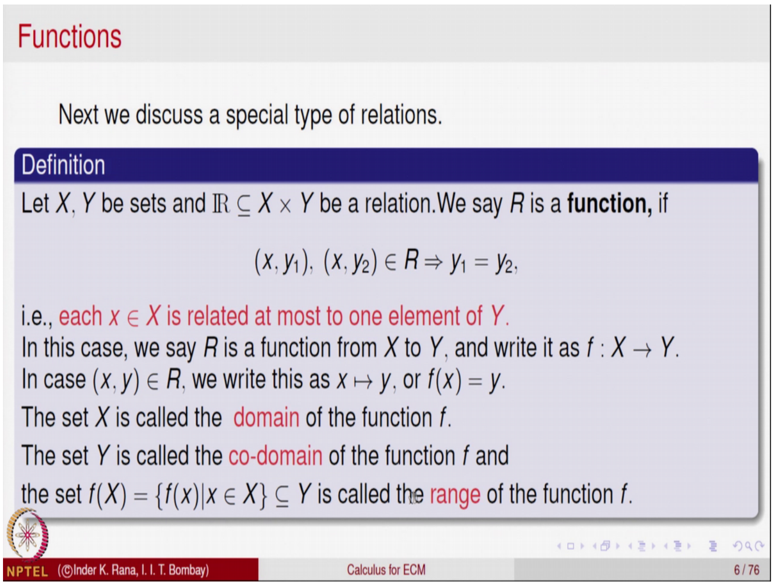

Next we are going to look at some special kind of a relations a let us have 2 sets x and y and let us say R is a relation from x to y then we say x y 1 and x y 2 if they belong to R if it implies y 1 is equal to y 2 and this this property has this relation has then we say it is a function.

So, here a function is a special type of relation with the property that whenever x is related to y 1 and x is related to y 2, then y 1 must be equal to y 2; that means, every element of x is related to one element of y it cannot be related to more than one element of y so; that means, each x is related to at the most one element of y. So, if x is related to 2 elements, it may not be related to any element, but if it is related then y 1 must be equal to y 2 such a relation is called a function in this case R is called as a function from x to y. So, is a relation between elements of x and y and it has a special property that each element of x of x is related to 1 element of y, then we say R is a function such a relation is called a function from x to y and we write it as f colon x, right arrow, this is a arrow pointing towards the right y ok.

So, in case x is related to y; that means, if this relation x is related to y since then in this terminology we will write it as x with a bar right arrow right; that means, x is related to y or we also write f of x is equal to y. So, this is how slowly we are developing the notion of a function and the language in which the functions are written. So, the set x is called the domain. So, what is the domain that is the first part of the function or first part of the relation it is coming from x. So, x is called the domain of the function and y is called the co domain of the function f and the subset f of x right look at f of x namely all the points elements of y which are related to some element of x that is called the range of the function. So, a relation is a special type of function a function is a special type of relation in which every element of x is related to some element of y right and it is related to only one element of y.

It cannot be related to more than one element this x is called the domain and y is called the co domain of the function the range is all the points of y which are related to some point of x are called the range of the function, let us try to understand a bit more. So, that the concept of function is quite clear intuitively, let us try to represent geometrically a bit this is the set x and this is the set.

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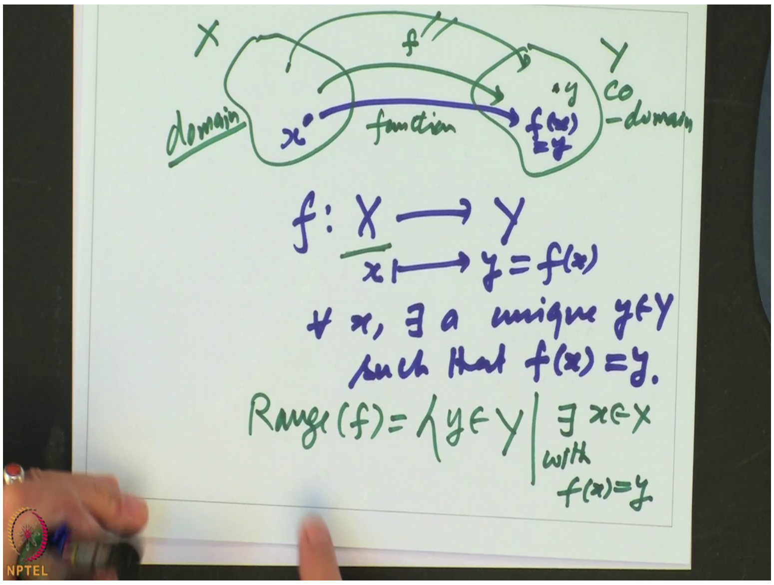

Y you can think of a function as some kind of a association f what it does it takes point x and takes it to a point f of x that is y. So, f from x to y, it takes a point x and send it to y which is nothing, but f of x and a point x cannot go to 2 different points in y it should go only to one point.

So, for every x there exist a unique y belonging to y such that f of x is equal to y. So, this x which is on this side is called the domain. So, this x is called the domain of this function and y is called the co domain. So, not every point of y will be coming from a point x that may not happen right, some points of y come from points of x. So, what is the range of this function f is nothing, but all those y belonging to y such that there exist some x belonging to x with f of x equal to y such a thing this set is called the range of the function let us let me give you very simple example.

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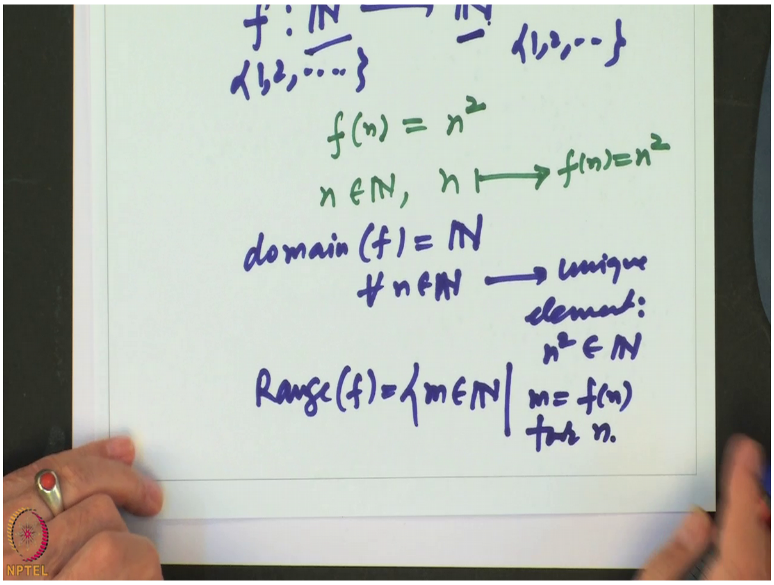

So, that you understand let us look at a function f from natural numbers 2 natural numbers. So, what are natural numbers recall this is a numbers 1, 2 and this is also 1 2 and so on.

So, what is the function let me define the function to be f of n is equal to let us say n square; that means, what; that means, if I pick up a point n in the domain then this n goes to f of n which is n square right. So, what is the domain of the function? So, clearly every point in the domain, the domain is of f is n and for every n point in the domain it goes to some unique element and that element is n square belonging to n. So, this is the function its domain is f co domain is n. So, what is the range; range of f is equal to all numbers in the co domain that is n such that m is equal to f of n for some n what does that mean f of n Equal to m square mean f of n equal to n.

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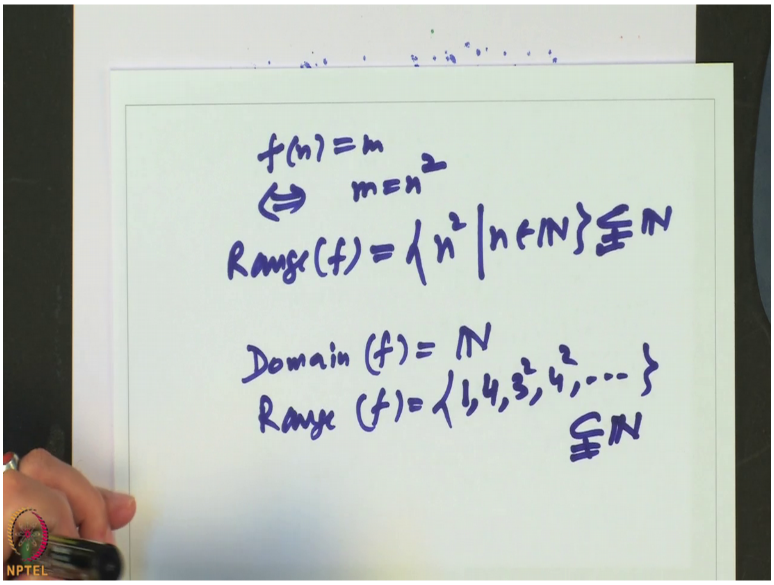

Means this will happen only m is equal to n square. So, what is the range of f is equal to n square n belonging to n which is note it is a proper subset of n. So, the domain of this function is equal to n and the range of f is equal to all numbers they are one right one square is 1 2 square is 4 3 square 4 square and so on which is a proper subset of n. So, co domain is something which can be bigger than the range of the function range is a part of the domain it can be equal we will see later on when these 2 are equal we will look at some properties later on. So, let us continue the study of functions the definition of a function.

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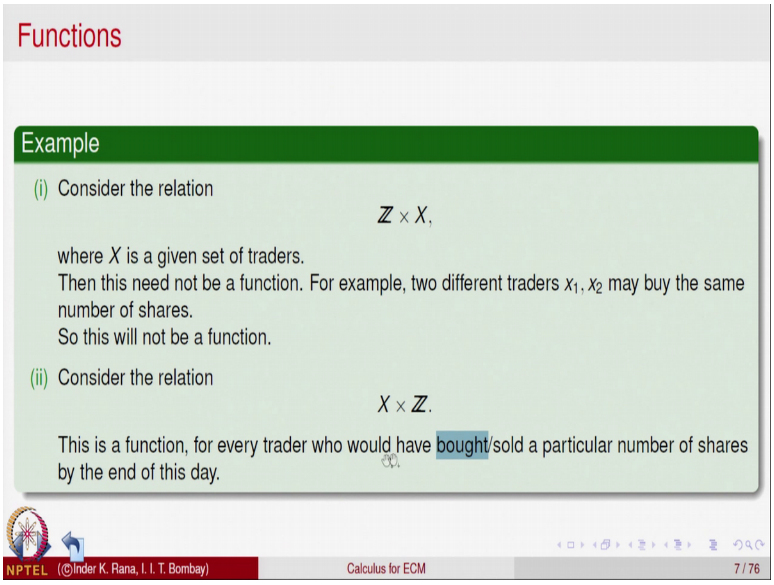

So, consider the relation integers cross x relation where x is the given set of traders right and then we will looked at the pairs right n comma x that is; that means, the whole set n comma x; z comma z cross x the Cartesian product of integer is x is this not a function because we can have 2 different traders say x 1 and x 2 buying the same number of shares say 10. So, for, if this is the relation, then 10 comma x 1 is possible and 10 comma x 2 is possible and x 1 is not equal to x 2; that means, a number 10 in the domain can get associated with 2 different numbers in x. So, that is not as such this is not a function. So, this is not a function, but let us just change look at x cross z. So, what does that mean; now here x is the number the traders and z is the number of shares they buy.

So, each trader x either he buys some shares. So, that will be some positive number or it does not do anything no activity the number will be 0 or he sell something. So, is number will be minus ten. So, then this set x cross z which is itself can be treated as a relation is also a function right because for every trader will either buy or non activity or will sell. So, for every trade there will be unique number associated with it. So, that gives us a function at the end of the day right. So, so these are some examples of functions.

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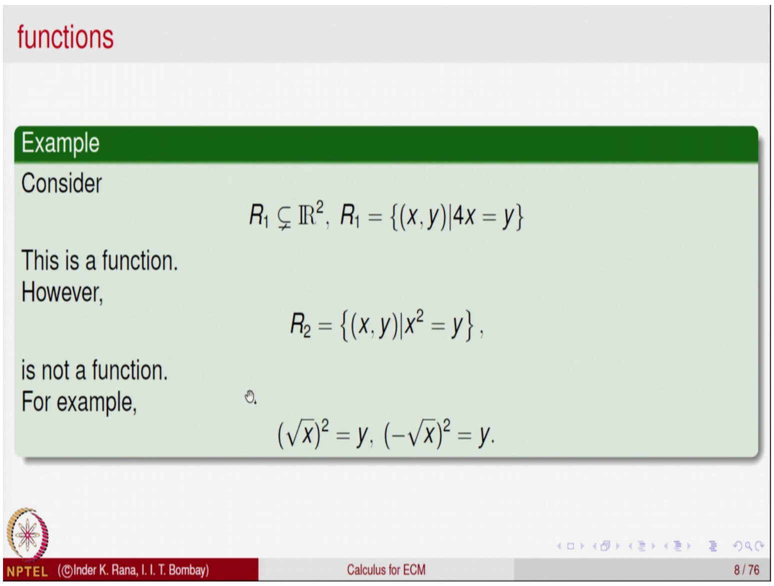

Let us look at a relation R 1 in R 2 where the relation is x y is equal to. So, what is the relation x is related to y if 4 x is equal to y is this a function is this is a function because for every x what is the y is related to x is 4 x is equal to y; that means, x is related to y.

So, there is only one possibility right if 4 x is equal to y 1 and also 4 x is equal to y 2 then of course, y 1 has to be equal to y 2.

So, there is a function let us look at the following another relation x y. So, that x square is equal to y. So, is this a function can we say that if x square is equal to y 1 and x 2 square also is equal to y can we say that x 1 if sorry if x 1 square is equal to if x square is equal to y 1 and x 2 square is equal to sorry, I am sorry, let us to check whether it is a function or not each element x must be related to unique element in y. So, let us say x square is equal to y 1 is it possible that some x square is also equal to some y 2 right that is not possible. So, let us just look at this slightly more carefully.

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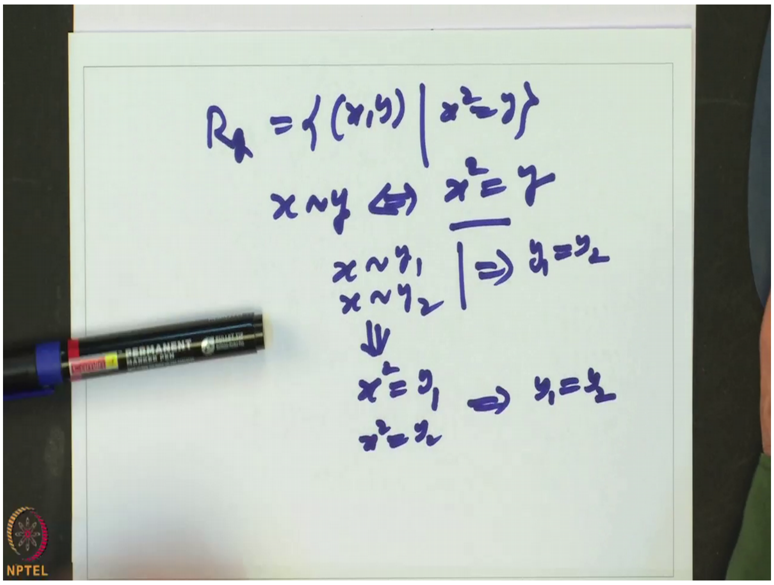

So, the relation R 1 R 2 is x y such that x square is equal to y; that means, x is related to y.

Let us just indicate this way if and only if x square is equal to y right. So, it will be a function if x related to y 1 x related to y 2 should imply y 1 is equal to y 2, but this implies from here x square is equal to y 1 is x is also related to y 2, then x square is equal to y 2. So, that implies y 1 is equal to y two. So, this actually a function, here is a type of that x is not a function is not true this actually is a function let us right. So, this is actually a function actually, it is example of a function which is not 1. So, we will deal with it what is happening here is there are 2 different x 1 and x 2 can be get related to same element of y. So, that is that is fine.

So, I think we will do it soon what is a concept of one function. So, this is a function this is a type of here this is a function.

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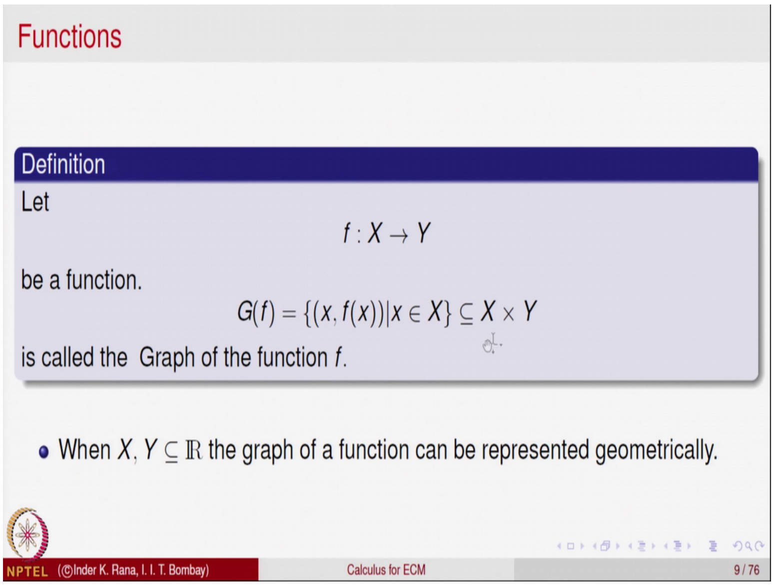

So, let us look at the next thing that we want to relate to a function is what is called the graph of a function we want to represent functions as pictures. So, we want to give a geometric representation of functions. So, let us take a function f x to y is a domain is x co domain is y its graph of the function is the set of all ordered pairs. So, x comma y where y what is y? Y is equal to f of x right f is a function. So, x will go to y. So, f of x goes to y right f of x is equal to y. So, graph is a set of all ordered pairs x comma f of x; x belonging to x. So, it is the subset of x cross y. So, graph is a subset of x cross y ok.

So, we what want to do is we want to represent this set this is a subset of x cross y geometrically and this is done as follows if x and y are both subsets of real.

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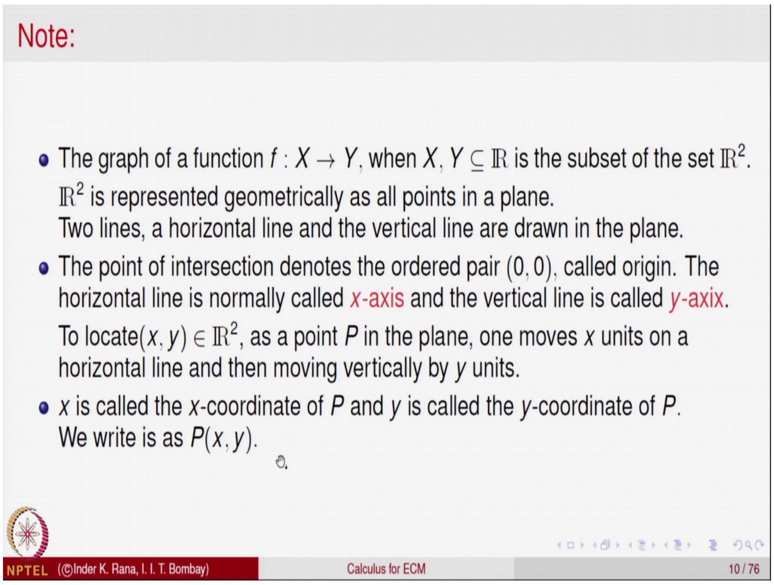

Line then that gets a very good geometric representation namely as follows what we do is x is equal to y is equal to R. So, the graph is going to be a subset of R 2. So, we will represent R 2 geometrically as points in the plane. So, R 2 is represented as points in the plane and what is the representation the representation is we take a plane you draw 2 lines one horizontal line another vertical line in the plane and wherever these 2 lines because they are one is horizontal other is vertical. So, they are perpendicular to each other.

So, they will intersects somewhere at a point then point is called the origin the horizontal line is called the x axis and the vertical line is called the y axis to locate a point x y on R 2 in the. So, given a ordered pair in R 2, how is the point located in the point plane what we wonder is one moves on x units on the horizontal line and y units on the vertical line. So, the; that means, on x axis you move x units on the y axis you move y units and you locate a point. So, that is a point which is associated with a ordered pair x comma y. So, x is called the; this distance the units you move is called the x coordinate and the other one is called the y coordinate. So, one write is as P x y.

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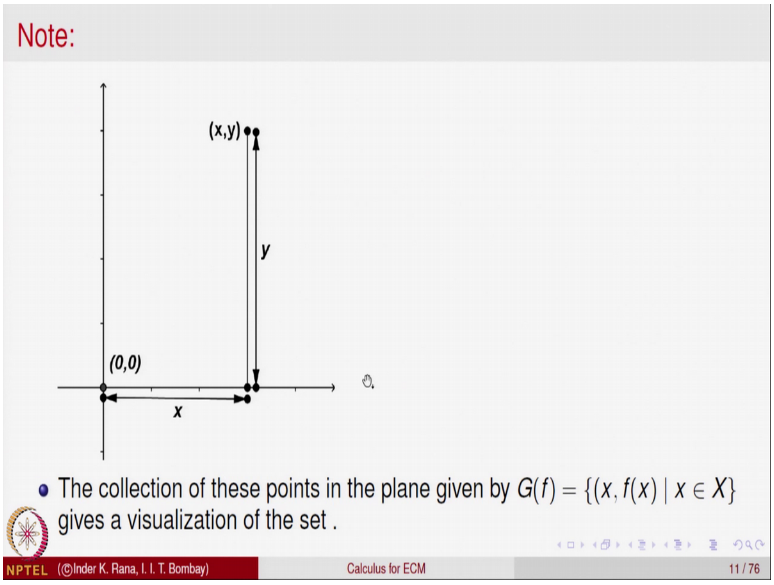

So, here is the pictorial representation in the plane you take a vertical line and you take a horizontal line. So, this horizontal line is called the x axis.

This vertical line is called the y axis. So, the horizontal line is marked with units, right. So, this point origin where the intersect is taken as 0 equidistance point one 2 3 and. So, on right on the negative side on this side so that is a line that is basically the real line and similarly the real line here. So, here the real line is put as vertical line. So, up is positive going down is negative. So, if you take a point P. So, if the coordinates if the point ordered pair is x comma y to locate it you go x units along x axis and y units along y axis you reach a point. So, that is a point p and conversely every point gets a unique ordered pair by moving down that is the y coordinate a moving along x axis is the x coordinate. So, R 2 is given a geometric representation at.

So, a points in the plane y x axis and y axis and the collection of these points in a plane given by for a function the graph right if you locate all this points using the graph of the function all this ordered pairs that gives a visualization of the graph of the function. So, we will continue in the next lecture.