Calculus for Economics, Commerce & Management
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Lecture – 30
Monopoly market, revenue and elasticity

Welcome to today's lecture. Just recall that in the previous lecture, we had been looking at the properties of functions how to optimize functions looking at the maxima minima absolute maxima absolute minima and how the tools of calculus namely properties of the derivative help us to look at the points what are called critical points and analyze them for to be the point of maxima or minima. We are started looking at some examples in economics scenario some models in economics and we had come to a; we had derived a relation namely that at a point of maximum say if Q naught is a point of maximum, then the marginal of revenue at that point is equal to marginal of the cost at that price.

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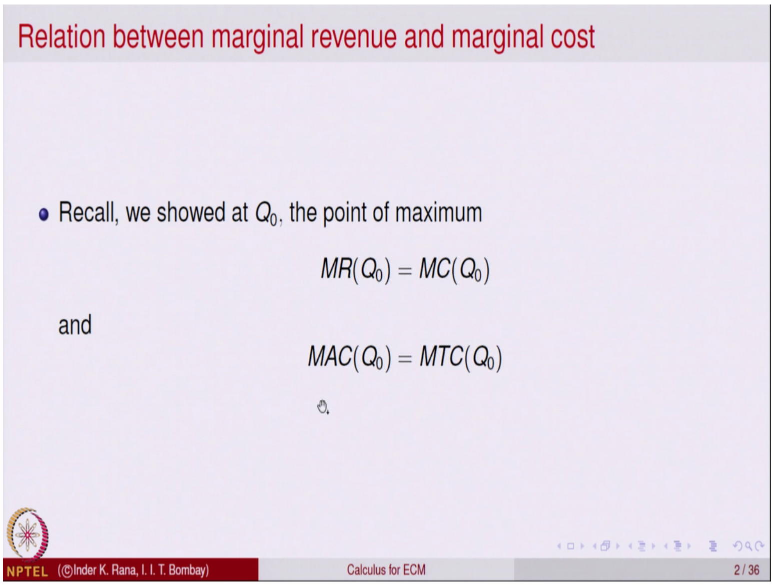

At that Q 0 the demand or production and similarly the marginal of the average cost at that point is equal to marginal of the total cost this is same as a marginal of the cost sometime it is written as a marginal of cost some time, it is written as marginal of total cost depending on what is given. So, these 2 relations we had developed and deduced using calculus tools in our previous lecture, but would like to give a caution today.

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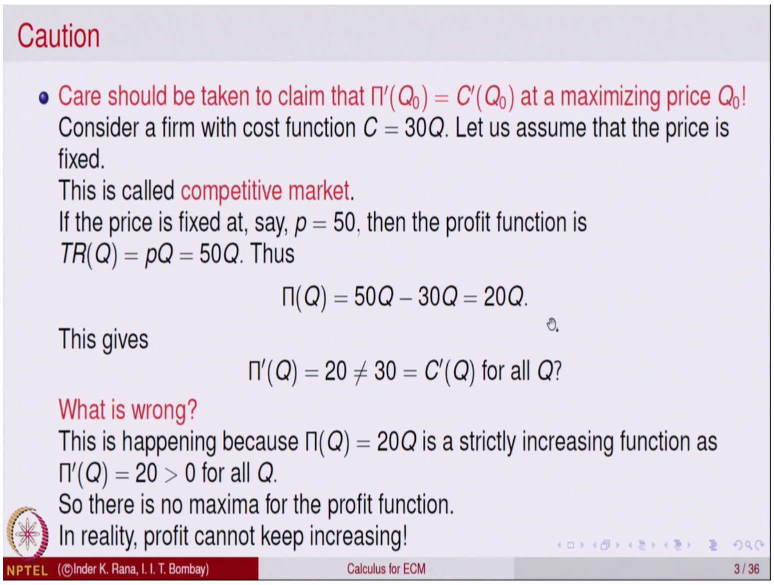

That care should be taken to claim that say for example, the derivative of the profit that is a marginal of the profit is equal to marginal of the cost at a maximizing price say Q 0 before 1; decides about verifying such a relation that such a relation is true one should ensure that Q 0 is indeed a point of maxima for the function.

Otherwise this may not be true. So, let us look at an example to caution you about this concept that considered a firm with cost function C is equal to 30 Q and let us assume that the price of this quantity is fixed the in the market the price is fix of the product that is being manufactured and sold this in economics scenario is normally called as the competitive market the price is not changed if the price is fixed say at p equal to 50, then the cost and the profit will be given as follows then the profit function total revenue is equal to p into Q. So, that is equal to 50 Q.

So, that is the total revenue. So, the profit total revenue minus the total cost. So, profit will be fifty Q minus 30 Q that is 20 Q. So, that is the profit function. So, if you look at the derivative of this that will be equal to 20. So, marginal of the revenue is equal to 20 whereas the marginal of the cost C is equal to 30 Q. So, marginal of the cost is equal to 30. So, 20 is; obviously, not equal to 30. So, at no value of Q the quantity being produce the marginal of the profit will be equal to the marginal of the cost why is that happening what is the reason for that why the reason could only be that the there is no point there is no value of Q for which there is a maximum.

So, let us ascertain that. So, so what is wrong is that if we look at the derivative that is equal to 20 that is positive so; that means, what our calculus result say that when the derivative of a function is positive the function must be strictly increasing. So, this means that the profit function which is equal to 20 Q is a strictly increasing function. So, it will not have any maximum value at all because for. So, it looks like that the more number of if you increase the production the profit should increase of course, this profit cannot increase in definitely. So, there is no maximum no maxima for the profit function has such mathematically, but in the reality profit cannot keep on increasing in definitely.

So, this is not a very good model of economics for in the competitive market. So, the point of caution was that this equality that the marginal of production not equal to the marginal of the cost fails because at no point at no value of Q the profit function is maximum. So, there is no wonder that this relation is not true. So, what we are saying is if Q 0 is a point of maximum then this relation holds. So, if there is no point of maximum then the relation need not hold. So, let us look at another example which is normally called the monopoly market. So, let us consider a monopoly firm with demand and price been given by p into Q is equal to 20.

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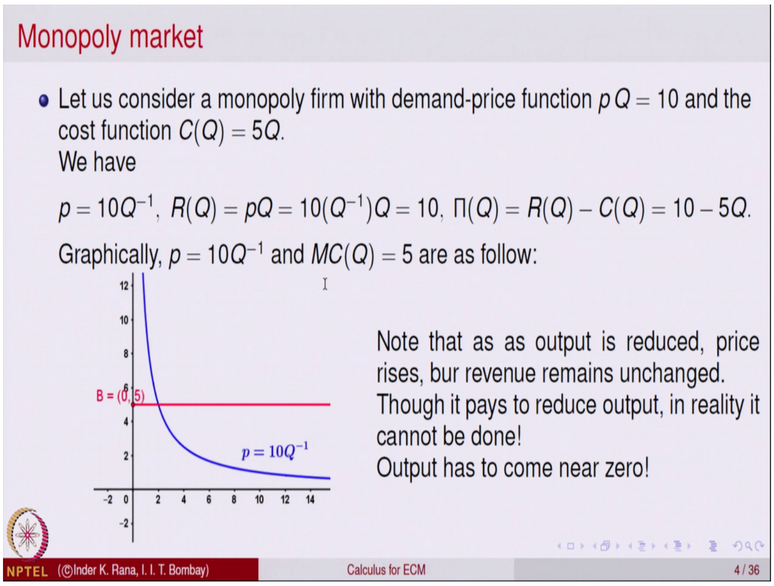

So, p is the price Q is the quantity produced that is equal to 10 and the cost function is equal to 5Q. So, in this scenario we would like to analyze what is the profit. So, what is the price and demand relation p Q was equal to 1.

So, p as a function of Q, we can write as p equal to we divide by Q on both sides. So, we get p is equal to 10 into Q raise to power minus 1 we can also write as 10 by Q if you like of course, Q is always going to be positive and what will be the revenue function the revenue function is p into Q the quantities produce into the price function. So, p into q. So, p is equal to 10 Q to the power minus 1 and Q is itself. So, this power balances out. So, we get equal to ten. So, revenue is always equal to 10 and the profit function pi Q is revenue minus the cost. So, revenue is equal to 10 and the cost is 5 Q. So, 10 minus 5 q. So, using this given data we have written down all the functional relationships be of p as a function of Q the total revenue or the revenue as a function of Q and the profit as a function of Q.

So, graphically this means p will be equal to 10 Q it will minus 1 and MC marginal of consumption. So, here is the marginal now here is the consumption function sorry cost function C Q. So, marginal of cost from here looking at the derivative will be equal to five. So, marginal of cost is 5 and the product function price is equal to 10 Q to the minus 1 and the graph of these 2 functions is given as follows. So, 10 to the power 10 into Q to the power minus 1 is this blue graph and marginal of cost is equal to 5.

So, that is going to be in straight line it is a constant function this function p equal to 10 Q minus 1 is really a interesting function in the sense that as here Q is in the denominator. So, as Q comes to closer and closer to 0 the value of p is going to increase. So, it looks like that output if we reduce the output. So, Q is the output. So, if you reduce the output as per this relation the price must rise right. So, as you reduce the price goes up is raises, but here the revenue function is equal to 10. So, revenue remains constant the price is increasing, but the revenue remains unchanged.

So, it may look like that one should reduce the production to increase the price, but in reality this cannot be done because the price cannot be because Q the quantity produce cannot be made equal to 0. So, this is a kind of scenario which is also not very advisable also; let us note that.

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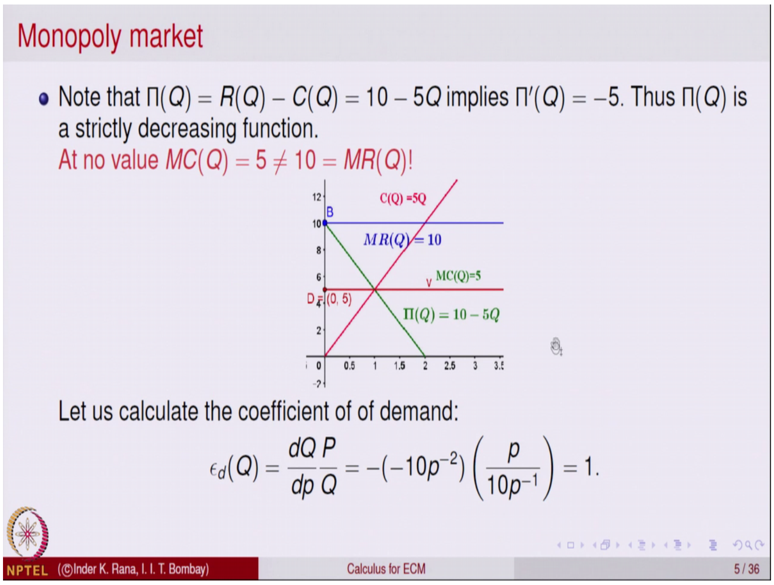

Te profit function which was given by the revenue function minus the cost function 10 minus Q that implies that the marginal of the profit is equal to minus 5 so; that means, what the derivative is negative; that means, the property is actually strictly decreasing function. So, even when you reduce the price will go out, but your profit will be strictly decreasing function. So, here is the scenario. So, marginal we can drop draw all curves this is the cost function CQ equal to 5 Q the red line the marginal of the revenue. So, revenue was equal to 10 revenue was equal to 10 q.

So, this is the marginal of the revenue and the profit. So, I MC marginal of the cost is equal to 5 and marginal of the revenue is equal to ten. So, these are not going to be really ever equal never going to be equal right. So, this is a graphical view of saying the same all the things that we want it to say. So, let us calculate in this scenario let us calculate the coefficient of demand for this economic model. So, recall coefficient of demand at a point Q was defined as the derivative of Q with respect to p into p by q. So, if we put the values derivative of Q with respect to p is minus 10 p to the power minus 2 right. Let us change our demand price relationship from p Q.

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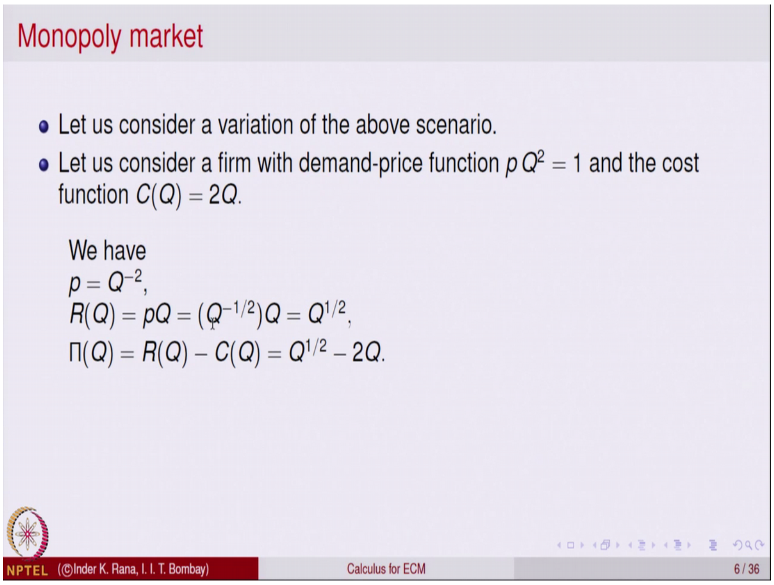

Equal to 10 to pQ square equal to one and the cost function is C Q equal to 2 times Q. So, let us write down all the functions relevant functions in this model.

So, we have p is equal to when you divide by Q square p is equal to Q into Q raise to power minus 2 one over Q square. So, that is p equal to Q rise to power minus 2 what is the income revenue function. So, revenue function is equal to p in to Q. So, value of p is Q square. So, put the value of Q square into Q raise to the power minus 2 into 2 here is here is problem. So, that is p raise to power minus two. So, it is p into q. So, this should be corrected. So, p into Q means p into Q raise to the power minus two. So, this needs a correction.

So, we will just stop here and make the correction; we had looked at some examples of how maxima minima and relations between marginals are deduced let us now just look at deduction of general formulas for various things. So, let us look at a relation between the revenue and the elasticity of economic model. So, let us consider a firm with inverse demand function p equal to p is a the price this is the function of Q.

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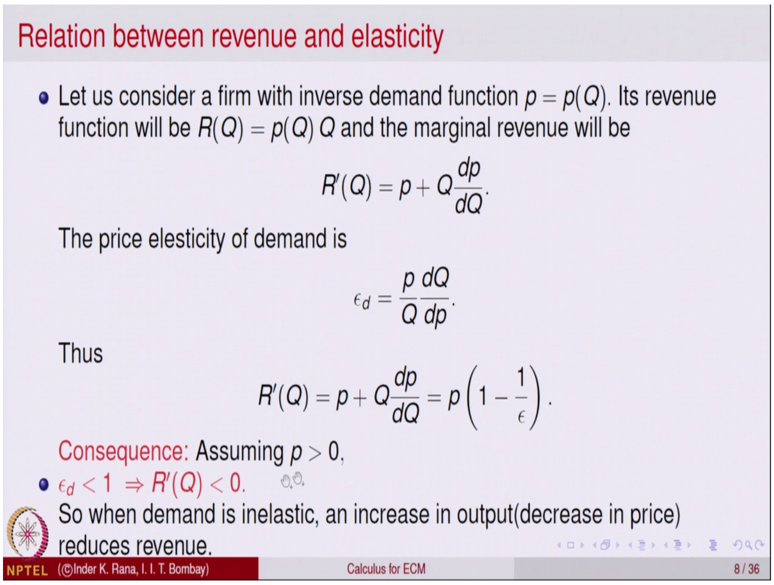

So, p is a function of Q given by p equal to p q. So, and. So, revenue of such a model will be given by p Q into Q that is the revenue because price into the quantity produce or the product produced. So, p the function of Q p as a function of Q into Q, the marginal of revenue from here we can deduce what is the marginal of revenue. So, we differentiate both sides of the equation and find out the marginal of revenue. So, derivative of R prime R prime being the derivative is equal to here is a product rule. So, that will give you p times the derivative of Q and derivative of Q is one. So, derivatives only p plus derivative Q into derivative of the first function, Q into dp by dq now this value can be put in the coefficient of elasticity of demand. So, we know by definition the coefficient of elasticity of demand was equal to p b Q into the derivative dq by dp. So, let us put this values from the given data.

So, here we have got to dp by dq. So, from here we knew; know that p and Q are inverse from p even find out the value of q. So, they are interchangeable as dependent functional relationships. So, derivative of dq by dp also can be found out and. So, derivative dq by dp is my one over my derivative of dp by dq. So, that we have seen in the notion of differentiation. So, let us use that and put the value. So, R prime Q was given by this relation. So, p plus q. So, we take out p common here. So, that is one plus Q by p into dp by dq and that inside the bracket is precisely one over of the coefficient of elasticity epsilon. So, this value can be put here by taking p out common here and using the fact by fact that dq by dp is equal to 1 over dp by dq from that the inverse of derivative of the inverse function relationship.

So, R prime Q. So, R prime Q is normally written as the marginal of revenue. So, as a consequence we get the marginal of revenue or R prime Q is equal to this. So, from here we can deduce the consequences namely if the coefficient of elasticity is less than one right if this quantity is less than 1, then this quantity will be negative. So, R prime Q will be a negative quantity so; that means, when and we have remember we had called the coefficient of elasticity less than 1 demand being in elastic. So, demand being in elastic increases the output or decreases the price and hence output reduce reduced the revenue so; that means, R prime Q negative means what; that means, the demand is inelastic. So, that mean increase in output will decrease in price the revenue will decrease revenue will decrease, right.

So, R prime Q less than 0 means R Q is decreasing function. So, epsilon d less than one will imply that revenue will decrease.

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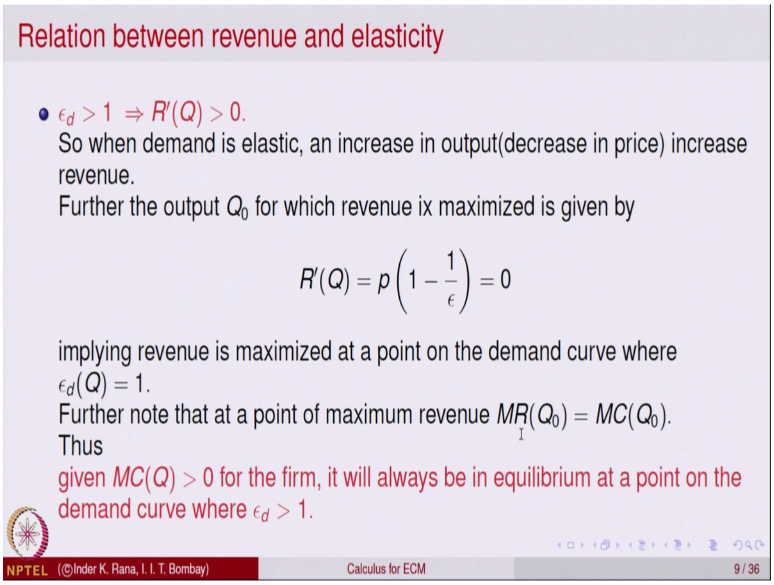

So, if is bigger than one then dash Q the derivative is positive which also we can be written as the marginal of Q is positive so; that means, demand is very elastic in that case and the increase in output. So, if we increase the output, then revenue should increase because R prime Q is positive; that means, R Q is a increasing function. So, as Q increases R Q will decrease. So, this is our relation between the elasticity of demand and the revenue that we get from the using derivative. So, further the output Q 0 at which point it is maximized. So, supposing at a point this is maximize; that means, R prime with Q at that point will be equal to 0.

So, that will give us implying that the revenue is maximize at a point of demand curve where epsilon dq is equal to 1. So, for this to be equal to 0 this epsilon should coefficient of elasticity should be equal to 1. So, when the point the price the value of Q, where epsilon d coefficient of elasticity is equals to 1 the revenues will be maximize. So, that is the output that will maximize the revenue that is another consequence of this. So, further we from here we note that marginal of revenue at Q 0 is equal to marginal of cost at Q 0. So, thus that also gives that marginal of revenue business. So, with remember that we have proved already. So, if you want to say that R prime Q is positive that is same as the marginal of revenue is positive. So, that is same as MCQ is equal to positive.

So, the marginal of the cost function if it is positive the; that means, what; that means, will always be the firm will always be in equilibrium at a point where the demand curve point on the demand curve where epsilon d becomes bigger than one right, actually equal to one from here right. So, that is another consequence of this right. So, let us summarize what we have done today is looked at some of the examples and the relations between various economic scenarios relations between the various quantities like first of all we looked at the question that namely you should have ascertain that there is a point of maxima for profit function or the revenue function to say that the marginal of revenue will be equal to marginal of cost at that point. So, that was our starting point and then we looked at the relation between elasticity and the revenue. So, we will continue this analysis in the next lecture.

Thank you.