Calculus for Economics, Commerce and Management
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Lecture – 03
Venn diagrams, operations on sets

Welcome back. So, in the previous lecture, let us just recall, we are defined the a notion of a set.

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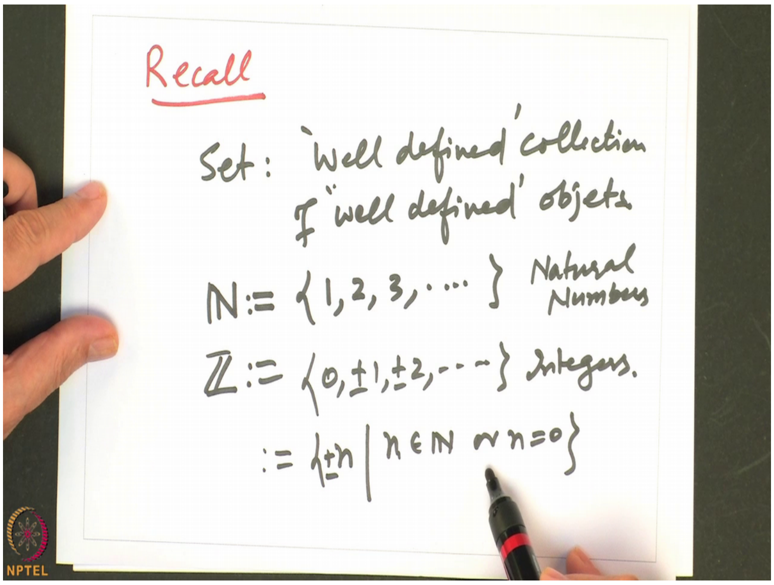

So, a set well defined collection. So, it is a well-defined collection of well-defined objects. We looked at 2 important examples from the point of view of our subject. One was a set of natural numbers which is denoted by N and that is 1 2 3 and so on. Then we looked at the set of a integers which was so, this 2 dots will mean this is a defining thing also. So, N is defined to be equal to this. So, set of integers so, these are natural numbers, the set of integers is along with 0 plus minus 1 plus minus 2 and so on.

So, that is the set of integers. So, one could also write the set of integers as N, where what is N? N is or let us write it as plus minus N, where N is a natural number or N could be equal to 0. This could be another way of writing in the integers, but so, this is what is called the explicit way of writing and this is by a rule. So, let me give you another example of describing a set By the rule method.

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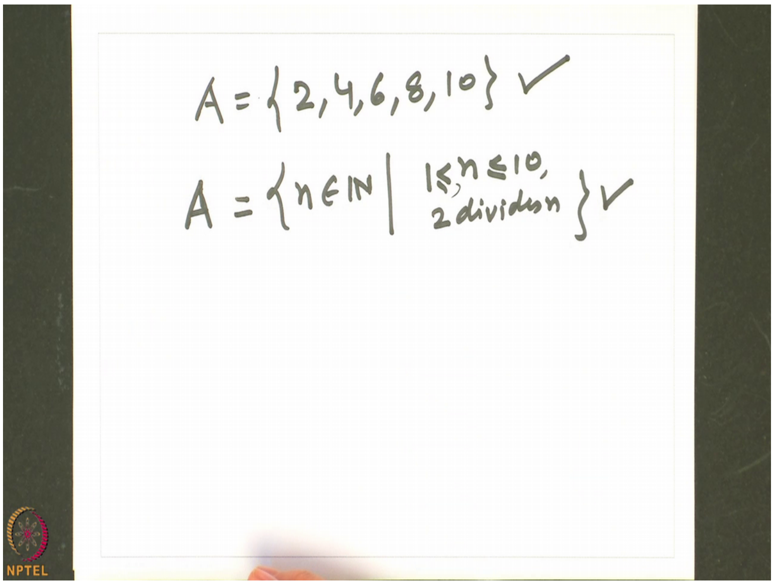

So, let us look at the set. So, let us write the set A to be equal to all-natural numbers. So, let us write 2 4 6 8 and 10. Let us look at this way. So, this is an explicit description of the set having elements 2 4 6 8 and 10. So, let me look at this let us consider try to write it as all N a natural number, such that this N is between 1 and 10.

So, less than or equal to so, this is a symbol which is used to indicate N is bigger than or equal to 1 or 1 is less than or equal to n. N is less than or equal to 10 is a natural number between 1 and 10 such that 2 divides n. So, let us look at this set. So, here I am picking up natural numbers by a rule. So, there are 2 parts of the rule namely 1, n is between 1 and 10. So, I should be considering all numbers less than or equal to 10 and bigger than equal to 1. So, all the numbers from 1 2 3 4 up to 10, and see which one of them are divisible by 2. So, 2 is divisible by 2 4 is divisible by 2, 5 is not so, 5 is not here 7 is not divisible. So, is not there 6 is 8 is divisible.

So, it is there 9 is not. So, it is not there and 10 is divisible. So, it is there. So, this a I can also write it as equal to this. So, this is explicit way of writing, the set and this is writing a by a rule. So, we have described 2 ways of writing a set, one explicitly and the other by a rule.

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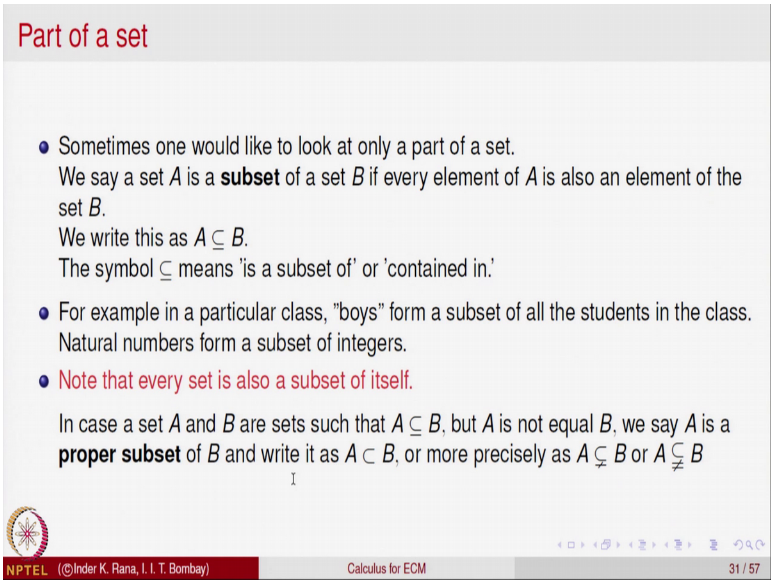

So, let us now go over some other things we can do with sets. Sometimes one would like to look at part of a set we have already been doing it. Actually, but given a set we can look at a part of it. So, such things motivate one to define, what is called a subset. So, we say a set A is a subset of a set B. So, we are given 2 sets A and B. We say A is a subset of B if every element of A is also an element of B. So, A in a sense is a part of B. So, a subset is you pick up some elements of B and form is new set called A.

So, A is a subset of B. So, this is normally written as A with this symbol this is a symbol contained in or equal to. So, this is read as this symbol is read as contained in or equal to B. So, A is inside or equal to B, A is a subset of B symbol means A is a subset of B or A is contained in. So, this is looking at a part of. For example, in a particular class boys in that class form a subset of that class. There may be girls also in the class. So, we pick up only boys. So, boys form a subset of all students in a class. Natural numbers form a subset of integers. Every natural number is also an integer. So, natural numbers form a subset of integers. Note that every subset is also a subset of itself. So, B is also a subset of B. It because actually B is equal to. So, looks a bit surprising in the beginning, but it is right.

So, these are concept of a subset A is a subset of B. In case there are some elements of B, which are not in A; that means, to form a we picked up elements of B, but some of them are not picked up in that case we say A is a proper subset of B. So, what is in case A is and B are set sets such that A is a subset of B, but A is not equal to B; that means, A is a part of it, but not whole of it. Then we say A is a proper subset of B. And write it as a contained B. We do not put that below a equality sign another bar. So, contained in or equal to that is this symbol, right. And here is a like look like C B that is the symbol contained in A is properly contained in B. Or some time to make it more precise we write A is a subset of B, but not equal to or like this. So, this this and this there are 3 different ways of writing that A is a proper subset of B.

But it should not be understood as every element of A is an element of B; that is, A is a subset of B, when you want to say a proper there are some elements at least one element in B which is not in A, that is a proper subset. Here is a unique object called empty set.

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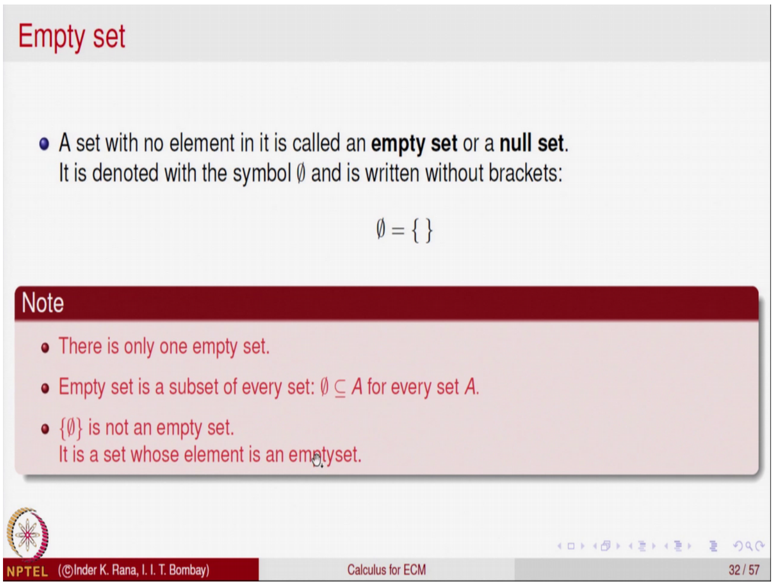

A set with elements in it is called an empty set or a null set. So, it is a bag, in which there is nothing so, that is an empty set. So, an empty set is normally written as 2 flower brackets with nothing written inside; that means, this is a set with no elements in itself. So, clearly there is only one empty set. Why there is only one empty set? Why because there is nothing in it. So, by vacuous statement it is a subset of everything say empty set is subset of every set.

So, there is nothing to check. It will check whether A is a subset of B you have taken element of A and show it is in B, but when there is nothing then there is nothing to show. So, it is obvious. So, that is how mathematics takes vacuous statements; there is, so there is only one empty set. And there is empty set is a subset of every set A, right. Note that this is not an empty set, Because this is a set whose element it is itself an empty set. So, this is not an empty set. It is this is not a bag, which is empty it is a empty bag with empty bag inside it.

So, you should think it that way. So, this is not an empty set. So, this is an empty set is this is a set whose element is an empty set, it has an element and that itself is an empty set. Here is a geometric way of representing sets, which is useful sometimes in understanding sets and subsets and various operations on sets which we going to describe. So, let us look at what is a Venn diagram.

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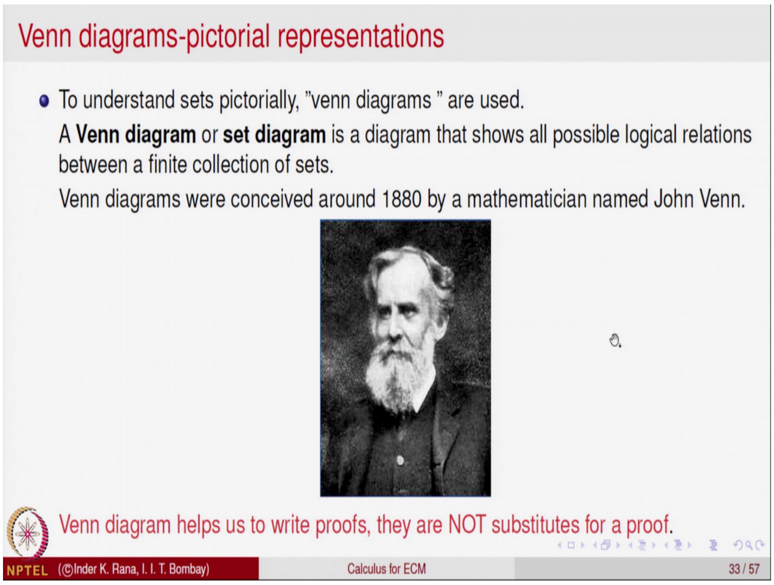

So, to Venn diagram is a diagram which shows all possible logical relation between a collection of sets, a finite collection of sets. We are given sets a b c d e and so on. And you want to show by a picture a relation be ween these sets. So, these were this method was conceived by a mathematician in 1880 called john Venn. So, Venn diagram a method of understanding relative relatively a collection of sets, but you should not think that Venn diagrams proves certain things. They are not proofs they are only way of understanding things.

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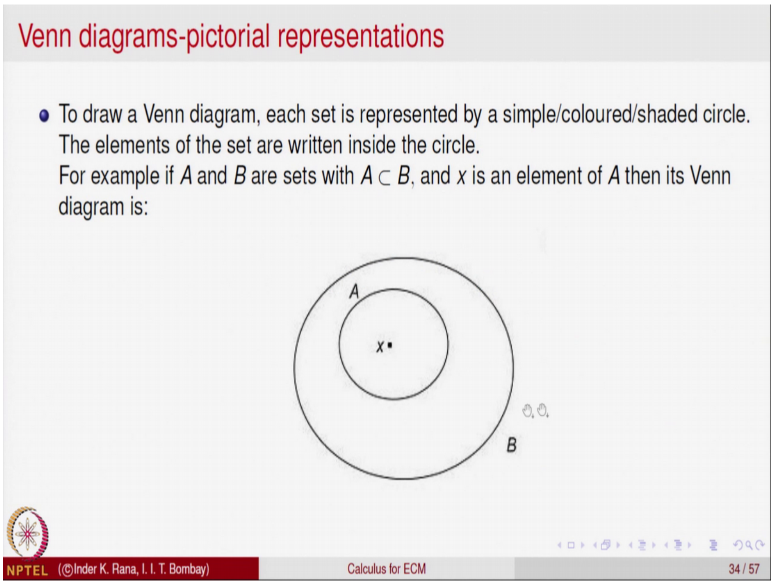

So, let us see some example and understand. To draw a Venn diagram each set is represented by a simple colored or shaded circle, right. An element of a set are written inside the circle.

For example if A and B are 2 sets and A is a subset of B, proper subset of B, and x is an element of A, then it is Venn diagram will look like. So, this will be A is a part of B. So, this is B outside circle represents B. A is a part of it. So, it is a smaller circle, smaller circle inside B. Is a proper so, it is a smaller thing, there is something left out. And x is an element so, we put a dot inside a to indicate that x is an element of a set A. So, this is for a proper subset of B, x is an element of a this will be the Venn diagram for it. So, we will see how this things are used to understand some basic operations on sets. See the operations like additions subtraction multiplication and division produce other numbers.

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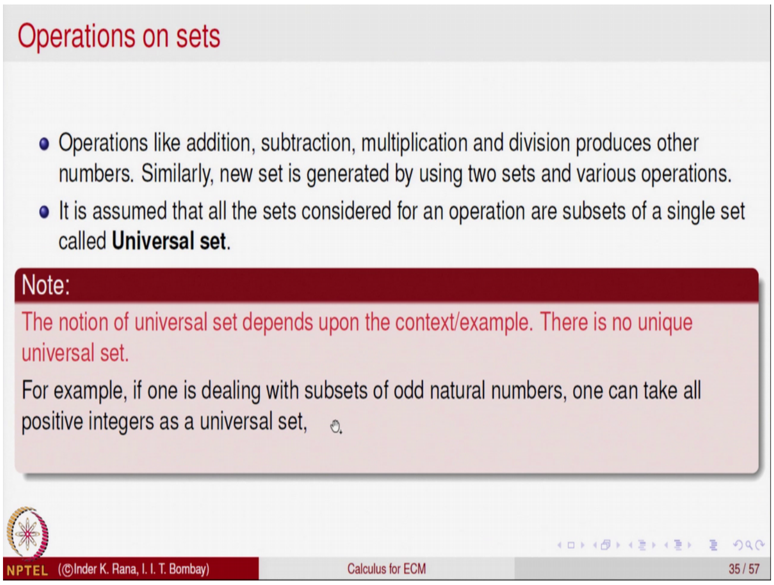

So, you take numbers add them multiply them and do such things to get more numbers, right. Similarly, new sets are generated by using 2 or more sets by various operations. So, here are some of them. So, it is assumed for all this operations that the all of the sets considered are part of a bigger set called the universal set. So, what is a universal set that will depend on the problem it may vary from problem to problem right. So, in a given context there will be a bigger biggest set that will be called as a universal set. For example, if one is dealing with subsets of all odd natural numbers, you are trying to deal with some properties, one can take all positive integers as universal set. So, there is nothing specific about universal set, that will be depend on the problem in the concept or.

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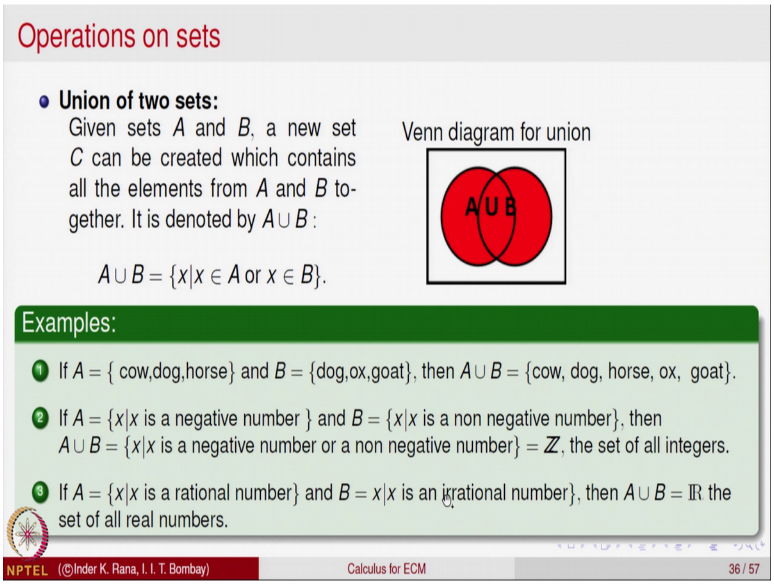

So, let us look at what is call the union of 2 sets.

So, you are given 2 sets A and B, A and B are 2 sets, using them you can form a new set C as follows. So, what is a new set C? C so, if I describe all the elements of C, I would have told you what is that set C. So, C is a set which is created which contains all elements of A and also of B together. So, take elements of A take elements of B and put them together and form a new set; that is called the union of 2 sets and it is written as A union B. A this comes from the word union up A union B. And what are all elements of it? An object x belongs to A union B if x belongs to A or x belongs to B. It can belong to both of course. So, pictorially if this circle represents A, and this circle represents B in the Venn diagram, then A union B will be the portion covered by both the circles together.

So, that is A union B. For example, A is a set of cow, dog and horse. And B is the set of dog, ox and goat. Then what will be A union B? A union B will be all x, that x is either a cow or a dog or a horse dog is already taken care of ox is not. So, x could be a ox or x could be a goat. So, what will be A union B? It is the collection of cow, dog, horse, ox and goat. So, that is A union B. All the elements of A and B put together and written in the flower brackets. You keeping those 2 things to that rule in mind that nothing should be repeated. So, that is the A union B. Let us look at a another way another example. Let us look at a to be the set of all x, x is the negative number. So, B that x is a non-negative number. Then what is A union B? X is a negative number or it is a non-negative number.

So, x is the union of both; that means, either x is a negative number or a non-negative number. Do you think this is equal to z this is a set of integers? I do not think. So, there is a problem here, why? Because negative numbers mean it is minus 1 minus 2 minus 3 and so on, non-negative numbers mean, question comes what is a number, then are we looking at natural numbers if you are looking at only natural numbers, then this union is set of integers removed from it 0 is not part of it. Because 0 is neither negative or is non-neither non-negative. So, this union is not zee what is it equal to it is equal to plus minus 1 plus minus 2 plus minus 3 and so on, 0 is not part of A. So, let us look at all x that x is a rational number and B x is a irrational number, then what is A union B? Is real line at we are not really defined here what is a rational and irrational number probably we will come back to this example later on we define real numbers.

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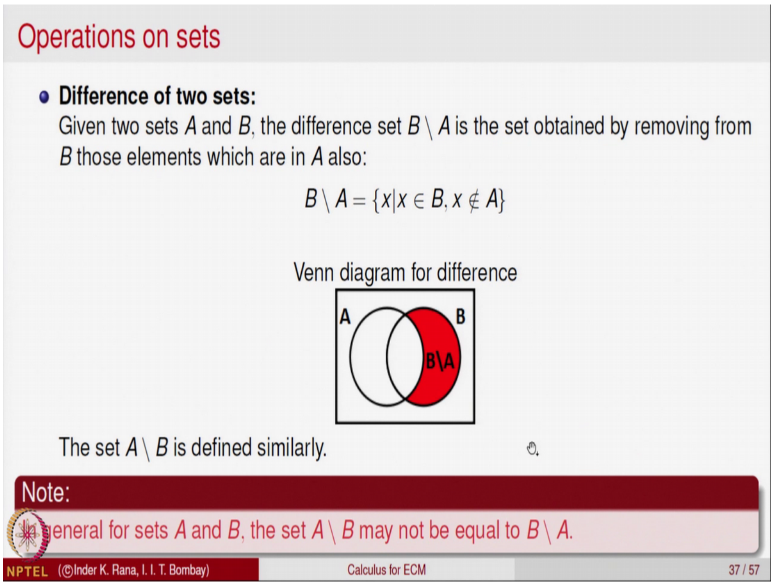

So, this is union difference of 2 sets. Given 2 sets A and B, B slash A is the set obtained by removing from B those elements which are in a also. So, from B remove elements of A. So, that means, this is a set of all elements x in B such that x is not in A. So, this is called the difference of 2 sets B and A, 2 that does not means. So, this is from B you removed a difference of B difference of A from B, because it should be clear B minus A means see there is English may cause you problem. This just means from B remove elements of A. So, this is B the, and that is a this portion a point here in between. Here is also in A, it is in B and it also in A. So, we remove it. So, what is left. So, this is red part is B minus A. So, you can define A minus B similarly. So, this part right, from A if you remove the portion which is common to both, and I will get A minus B.

In general A minus B need not be equal to B minus A, right. And more often than not they will not B.

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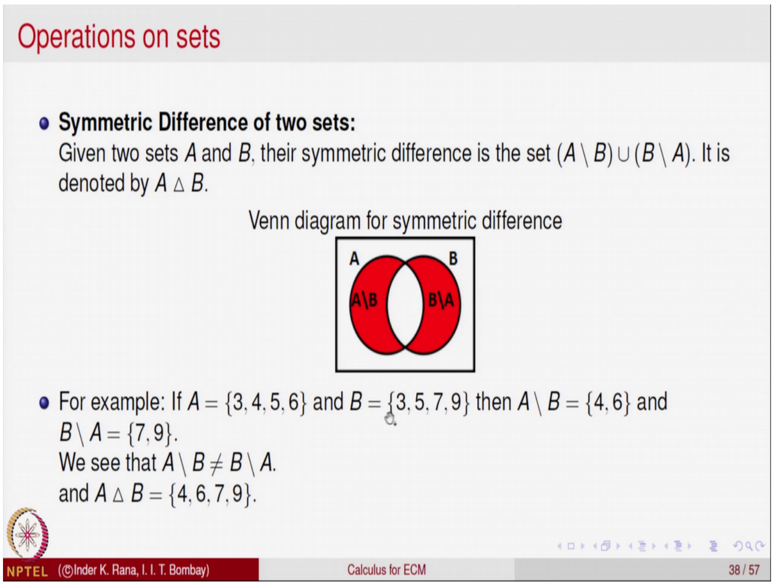

Symmetric difference so, if you take 2 sets A and B from A remove B, and from B remove a you get 2 parts, and put them together take the union of them, that is called the symmetric difference. And it is written as a delta B. So, this is A minus B union B minus 1. So, if A is 3 4 5 and 6, and B is 3 5 7 and 9, what is A minus B? From A remove elements of B. So, what are the elements that will be removing? You will be removing 3 and you will be. So, from A remove elements which are also in B. So, 3 is in both. So, remove 3 5 is in both. So, remove that. So, what is left is 5 and 6. So, A minus B is 4 and 6 only. And if you write B minus A from B remove the elements of A, from B 3 5 7 9 3 and 5 already in A.

So, remove them. So, you remove 3 and you remove 5, what is left is 7 and 9. So, B minus A is only 7 and 9, right. So, obviously, this is shows A minus B is not equal to B minus A. And what is a symmetric difference? The union of these 2. So, 4 6 7 and 9 put together. So, that is a symmetric difference.

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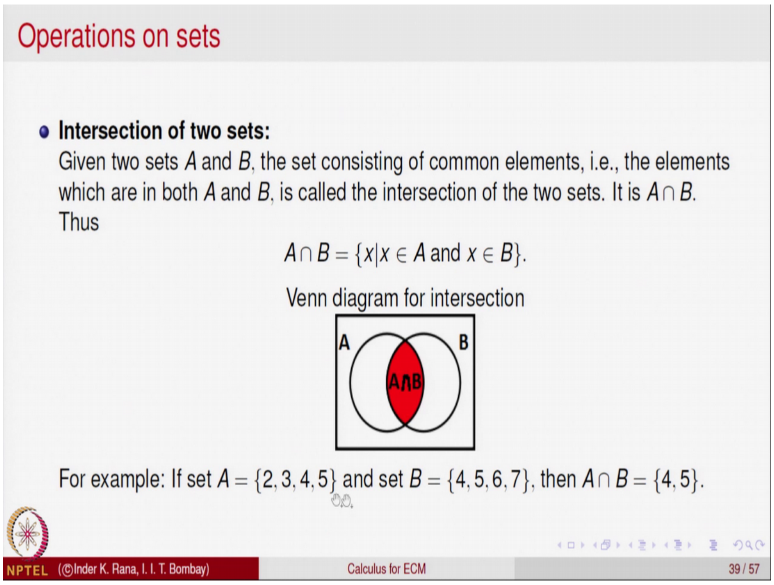

So, that is how you understand symmetric difference. Next comes intersection of 2 sets. So, given 2 sets A and B, right. We can form a new set of those elements which are common to both. So, the elements which are in both A and B is called the intersection of 2 sets. So, that is written as A intersection B. So, what is A intersection B? It is all x, x in A and x in B. So, if this circle represents A and this represents B. Then A intersection B is this common portion the red one. So, that is and this is represented by a inverted u, right. And that means and so, you should be understood as N A and B say elements which are in a also in B.

So, for example, if A is 2 3 4 and 5, and B is 4 5 6 and 7. So, what are the common? 2 is in a, but 2 is not in B. So, 2 is not counted. 3 is in a, but 3 is not in B. So, 3 is out. 4 is in a 4 is in B. So, 4 is in and 5 is in a 5 is in B. So, A intersection B is 4 and 5 2 only 2 elements.

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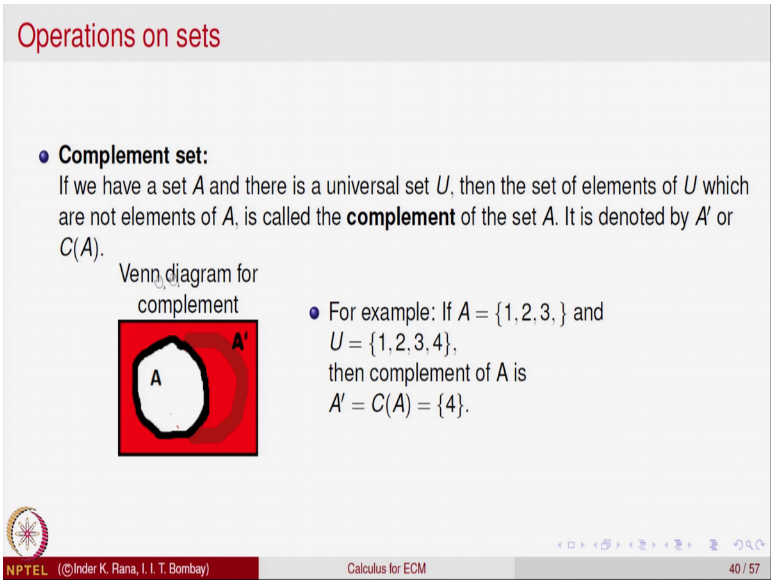

So, that is what intersection means. Next comes what is called the compliment of set. So, for the compliment you should think of set A, which is part of a universal set. So, here given A is a set, and there is a some universal set in that context U. So, then you look at what are the elements of U which are not elements of A. So, you are looking at U minus A essentially. So, elements of u which are not elements of A is called the compliment of the set A. And it is normally written as a with a bar up. Or also written as C signifying compliment A.

So, here it should be well understood that we already have a universal set in mind or in the context. So, there is A so, sometime in the Venn diagram you write this as set A, say universal set the square area and this inside thing is this is your A. So, what is a compliment? The part of the universal set which is not in A. So, which is outside. So, the red portion that is compliment. So, for example, if A is a set 1 2 3, and you take the universal set to be 1 2 3 4, then the compliment is as the element 4. 1 2 3 is removed from the universal set to get the compliment.

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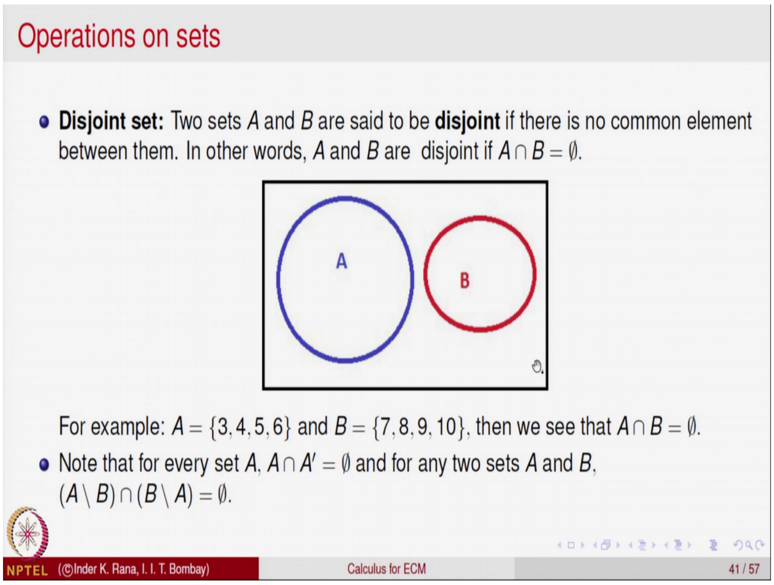

So, that is a compliment of the set. You say 2 sets are disjoint, if there is nothing common between them. There is no element which is in both A and B. That essentially means A intersection B is a null set is an empty set.

So, this is what the notion of disjointness means. This is a disjoints set. So, you represent may be there is a universal set, this is A, and this is B and there is nothing common between them. So, there is a Venn diagram representation for that. So, for example, A is 3 4 5 and 6, and B is 6 7 8 9 and 10, then there is no element which is common to both. So, that is a empty set. So, here is something which you can easily verify; that is, set and it is compliment are already always disjoint. And A minus B and B minus A are also disjoints sets there is nothing common between them. So, this is you can easily verify. So, this is disjointness there is nothing common.

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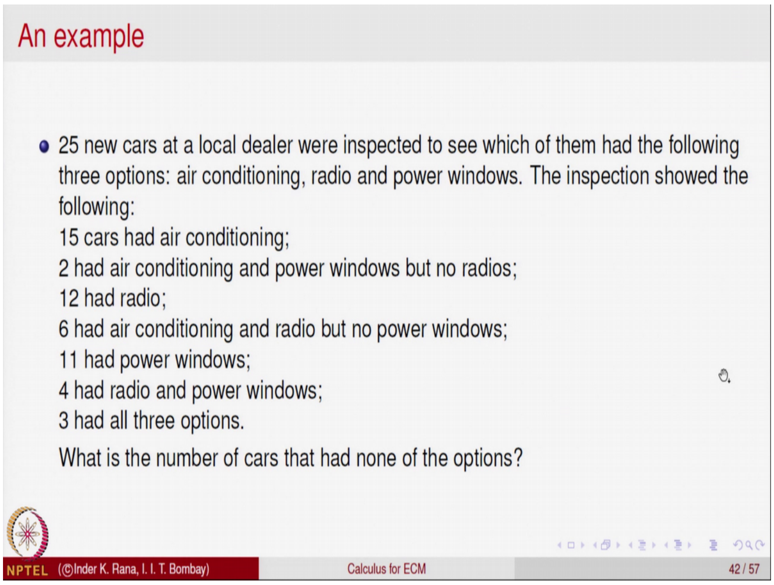

Let us look at some example to illustrate how Venn diagram can be used.

So, we are looking at an example of 20 there are 25 new cars at a local dealer, and are being in inspected to see which one of them have the following 3 options. Which car has air conditioning has a radio power windows. So, these are 3 things we are looking for in each car. And the data collected shows the following 15 cars had air conditioning. 2 had air conditioning and power windows both, but no radios. 12 had radios, 6 had air conditioning and radio, but no power windows. 11 had power windows, 4 had radio and power windows. 3 had all 3 options. They means they had 3 cars had air conditioning, as well as radio as well as power windows.

So, we want to analyze the question, what is a number of cars that had none of the options; that means, it had neither air conditioning nor radio nor power windows. So, to analyze this kind of problems, sometimes Venn diagram is useful. So, let us try to draw a Venn diagram for it, but before we doing the Venn diagram, we will try to analyze a few other things.

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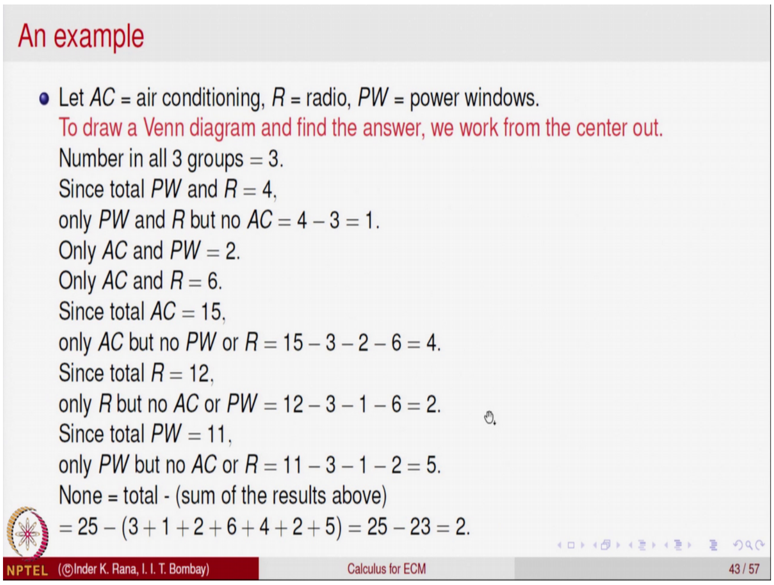

So, how do we analyze this? Here is the methodology. So, let us denote AC denote AC the letters denote air conditioning, R denotes radio, and PW denotes power windows right. So now, let us try to draw Venn diagram to be work from the center out. See there are 3 sets here, right. Cars having air conditioning cars having radio, a cars having power windows.

So, there will be 3 sets. So, among 3 sets, there is a portion where all 3 are present. So, that is the center most part. So, will try to center most out part. So, we are trying to interpret the data in that format. So, number of all 3 groups; that means, the number of cars which are all 3-options AC radio and power, we were given they were 3. See total power window and radio is equal to 4, that is a data given. So, only power windows and radio, but no AC is equal to 4 minus 3 that is 1. Only AC and power windows is 2. So, only AC and radio is 6. Since total AC is 15 so, AC but no power window and radio is 15 minus 3 minus 2 minus 6 and that is 4 right. So, this is being interpret from the data.

So, once you have obtain that since total number of radios, cars with radio is 12 it gets only radio, but no AC and power. So, 12 minus which have AC, or power window remove them you get 2. So, since total power windows is 11. So, you get power window, but no a C. So, from power window 11 remove AC and radio. So, you get 5. So now, none will be total minus the sum of the above results. So, all this results are added and you get 2.

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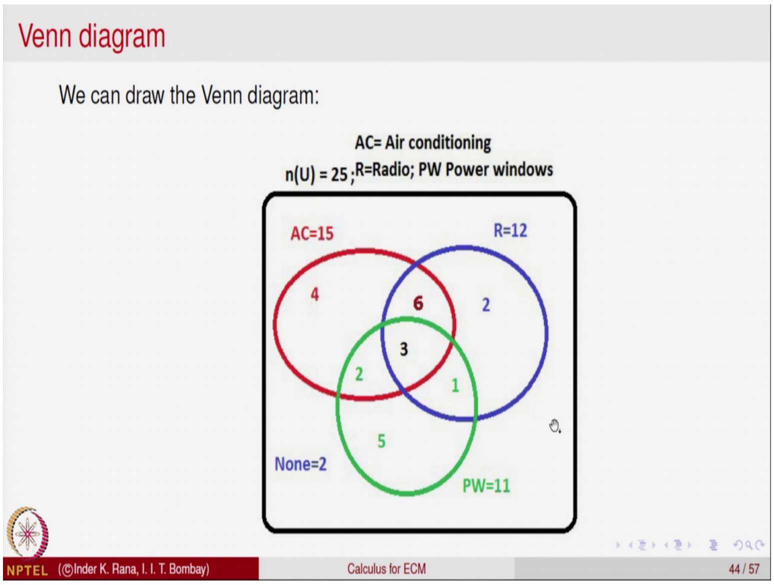

So, to interpret this diagrammatically, here is the diagram. So, AC air conditioning and right R is a radio PW is a power window N u is 25. So, this is a universal set of all the cars. Inner most red ones are AC, radio one are blue and green is the power window. So, inner most that is which we which have all 3 that is all 3 options there is number is 3, right. Those who have AC and radio, both that is 6 AC and power windows that is 2.

And power windows and radio that is 1. And all this put together remove and remaining is 2. So, this is Venn diagram for the problem. So, that is how Venn diagram is used to understand and analyze the problems. So, I will stop here for today what we have done is basically, we have try to analyze the operations on sets, right. We looked at the operations namely of subsets what is mean of subset, what is a meaning of union of 2 sets what is a meaning of intersection of 2 sets. And then we will looked at symmetric difference. And how Venn diagram can be used relatively describe some sets. And caution that Venn diagram is only way of understanding is not exactly way of proving things. So, we will stop here today and continue next time.

Thank you.