Calculus for Economics, Commerce and Management

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Lecture – 02
Concept of a set, ways of representing sets

In the previous lecture, we had started looking at the concept of what is a set mathematically. So, we had looked at some examples, and we said that to manage things we put objects in collection objects could be of different nature different type and so on. But we said mathematically if you want to analyze this collection we should have it is a well-defined collection of well-defined objects. So, the question comes what does the mean well defined mean, and what does well well-defined collection means, and what does well defined objects in that collection mean. So, to understand this 2 the word well defined let me illustrate this by some examples.

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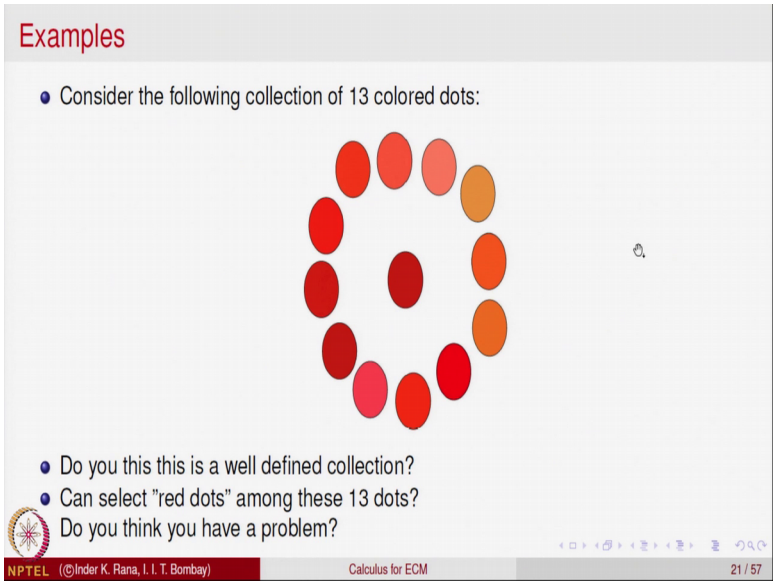

So, that we are able to go ahead. So, let us look at. So, consider the following collection of certain colored dots. So, they are colored dots given and they are 13 in number I hope they are 13 in number let us count yes they are 13 in number. So, we are given 13 dots, which are all colored and question is do you think this is a well-defined collection well. You are given the objects right. So, it is a well-defined collection, there is no doubt about it, right. So, this is a well-defined collection of 13 dots. They are again imagine they are

physically given you we have you are given a physically a box containing this 13 painted card boards you can imagine, right. So, it is a well-defined collection. Now the next problem that I am going to ask you is the following, can we select red dots among these 13 dots.

So, among this collection of 13 dots which are colored, cards you can think of circular cards which are colored, if I ask you to pick up the cards which are red, and separately put them in as a separate box. So, think it about a minute and try to write down on a piece of paper your solution. You can if you like you can just count for example, I may think this is red this is red 2 red 3 and 4. I do not think this is red. I will according to my selection your selection may be different right. So, if you have if you are watching this video at home, and you are in a class where you have some other persons also watching. You can play this game jointly, and ask each one of them to pick up what is how many red dots are there in this. And you will be surprised to find you will find different answers from different. People hardly any 2 may agree what is which are the what are the number of red dots in this.

So, there is a problem. So, we have a problem of selecting red dots among the 13 dots. So, what does that mean? That means, the term red or the color red means different to different people it is not well defined, if it is well defined means it should mean same to everybody, but different persons visualize what is red. So, among this 13 dots, how many of them are visualize as red depends on the person what he can sees red to be. So, the word red is not defined. So, it is difficult to find out the number of red dots in this. So, this is what not well-defined means.

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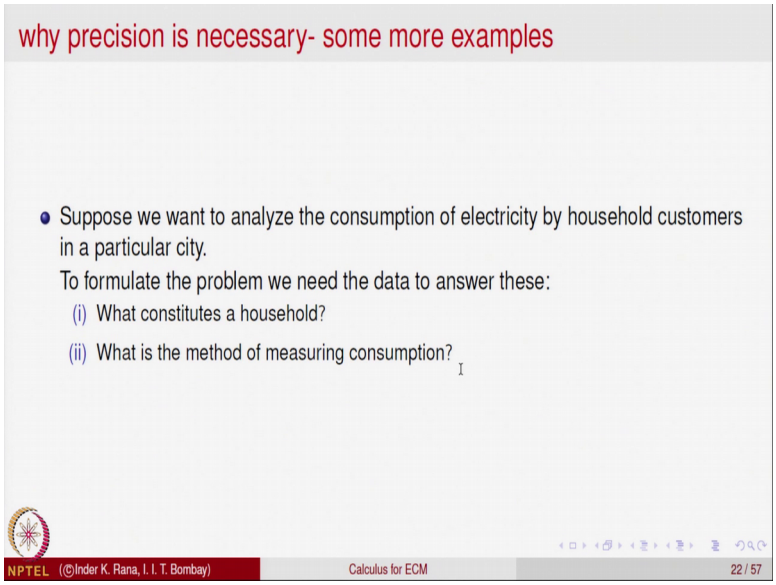

So, this is what is called precision. So, for example, we if you want to analyze the consumption of electricity by household customers in a particular city, you have a problem you want to analyze how much electricity is being consumed by household customers in a particular city. So, of course, a particular city; that means, you has to first of all define what is a city.

So, probably you will it that is does not cause much of a problem, because a city means a well-defined boundary as per the corporation or the municipal corporation of that city society is well defined, or by the electricity department. What is a household customer? So, we have to define what is a household customer. So, probably that is also well defined namely if the electricity consumer that the line or the meter electricity meter is in a house, where a family or somebody is staying that is a household, and it is not in a place where there is a business being done or a shop or something. And then third thing is what is analyzing the consumption. So, we should be very well aware of what is how do you measure the consumption of electricity. And we have now very well devised instruments meters which the calibrated meters which measure how much somebody is using.

So, here all the everything seems to be well defined to formulate the problem we need the data. So, as I said, we will what constitutes a household that we have to make it precise what is a method of measuring consumption, that we have to make it precise and

we have to. So, that means what? That means, they nobody should be able to doubt whether what is a household if we are said this is a household and there should be any controversy or ambiguity in saying of measuring of the consumption of electricity. So, that is what called defined means.

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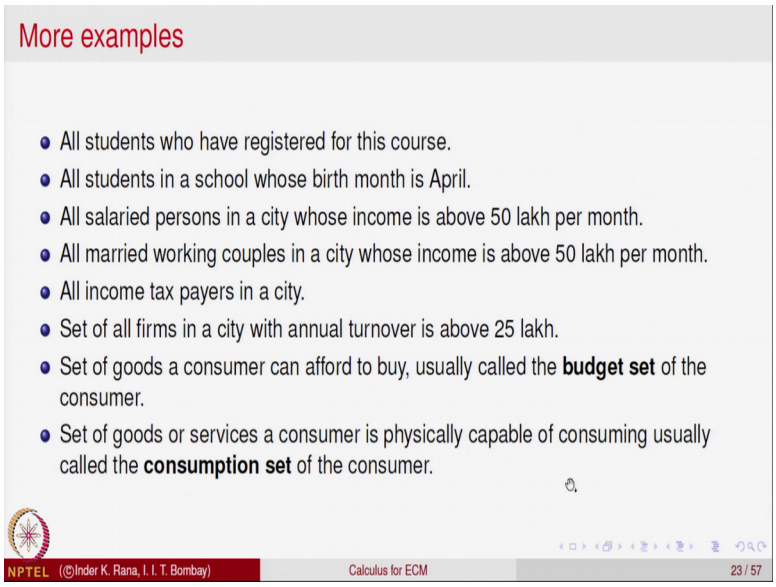

So, let us look at some more example. All students who have registered for this particular course, a course that is a very well-defined collection.

Because we have their names, we have their numbers email ID's and everything, and the office managing this courses they have the records. So, this is well defined collection of all students who are registered for the course. All students in a school whose birth month is April, that is we have a record of all students were registering in a school. We have their birth records, and we can find out which one of them are born in the month of April. Well defined all salaried persons in a city whose income is above 50 lakh per month. Of course, here we have to understand what is this salaried person. So, who is employed somewhere and getting a remuneration every month or every day or something like that. So, salary should be well defined, and income should be well defined, what is the meaning of income?

So, we should be very well able, we should be able to say without any ambiguity what does income mean and what does salary mean for a person. Then only we can do any analysis of such data. All married working couples in a city whose income is above 50

lakh. So, that also is seems to be a well-defined collection. All income tax payers in the city well. So, all income tax payers probably will be classified who file their income tax returns they are income tax payers. So, has a very well-defined collection. Set a firms in a city whose annual turnover is above 25 lakh. So, this in this maybe we have difficult to say which are the firms. So, this will only go by some official records, when they have their audited this statements of the firm yearly audited statements where they which is way this disclosed, how much is the turnover.

So, if you go by that method of picking up the firms then it is well defined. Set of goods a consumer can afford to buy normally it is called the budget set. Meaning, this is in the this of this objects this goods the consumer will be able to afford to buy, because of his income were was. So, this is normally a economic term it is called the budget set of the consumer. The set of goods or services a consumer is physically capable of consuming is usually called the consumption set for the consumer. So, these are the terms coming from economics, and we want to analyze them make any study of this things, we should know clearly what is called the budget set, and what is called the consumption set. So, what I am trying to show is how the nature of collection of objects in a different context can arise to analyze different kind of problems.

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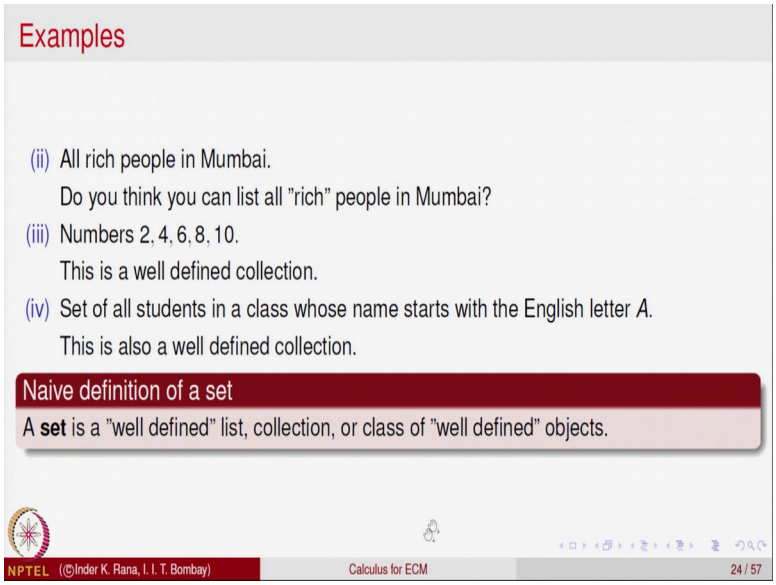

Let us look at some more examples, we want to analyze who are all the rich people in Mumbai, well can be say what is who? Who is rich? Who is not rich? Seems to be a

problem in deciding what does rich mean. Can we say can we go by the bank balance of that person, or can we say how much a property he owns. And so, it is not a very well-defined word that rich, what is rich mean it is not a very well defined. Because given a person we cannot categorize it as rich or non-rich. So, this is not a very well-defined collection. So, rich in Mumbai is not verified. Let us look at the numbers 2, 4, 6, 8 and 10, here we have actually given the collection where given is box having the numbers 2 4. So, this is a well-defined where actually given hold of those objects. We do not have to pick up right.

So, normally such when the actual things are given it is well defined. This is well defined collection. Set of all students in a class whose names starts with the English alpha English letter a yes that is very well defined, because we know how many which are the students in a class we have the roll list we can pick up according to the name which starts with a or not, this is well defined. So, naively I hope we have understood what is well defined means. So, a set mathematically very naive definition of a set is, a set is a well-defined list or collection of objects of well-defined objects. So, the collection should be well defined and the objects in each of them should be well defined. So, well defined we are not really given a mathematical definition of what.

Well defined means, we are only try to give you some examples which illustrate what well defined is. So, actually this is a very deep routed philosophical problem in mathematics in sets theory itself, what is set. And we will not go into it. In fact, in whole of mathematics one does not define a set. Set is something that is given to you, only you manipulate things in a set. So, I will not going to it. So, intuitively we will understand what well defined means. So, a well-defined collection of well-defined objects will call them as set.

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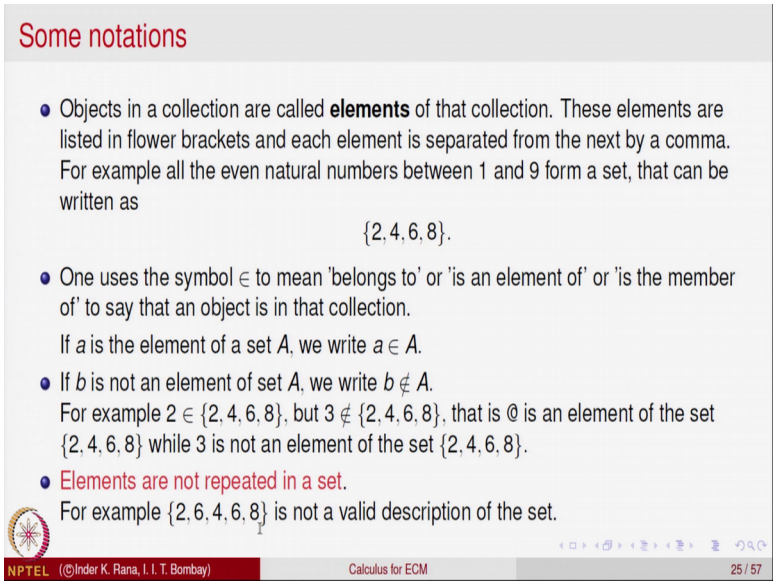

So, well once we are defined what is a set it is a collection of objects. So, objects in a collection are called elements in that collection. So, that is a collection of things. So, elements their objects in that collection. So, each object in that is called an element. So, the elements this elements are listed in so, how is set written we use the brackets the flower brackets, and write down elements inside we separate it with commas.

For example, let us we have natural numbers between 1 and 9. We want to look at all even natural numbers between 1 and 9. So, we know that 2 is a even number, 4 is even, 6 is even, and 8 is even between 1 and 9. So, these form a collection and this collection mathematically is written by putting this bracket. This is called a left flower bracket, this is called the right flower bracket. So, we put these 2 brackets opening bracket and the closing bracket, and the elements the objects inside 2 separated from 4 by a comma 4 separated objects by a comma and 8. So, this is how the set of all even natural numbers between 1 and 9 will be written mathematically. One uses the symbol belongs to so, when we want to say 2 is an element of this. So, we will write this as 2 belongs to be that symbol in this set. For example, if a is an element of a set we will write it as a belongs to A right.

So, this is we are trying to now develop the language of set theory, and mathematics we will find this coming becoming use becoming more and more common and English will become less and less used in mathematics, because mathematical languages very precise

and it has only one view of interpreting. So, a set is written by this way putting in bracket, and saying an element something belongs to the set a is written as a belongs to A, right. And if something does not belong some object does not belong then you put a cross against not against belonging. So, be this means does not belong to A. So, b is not an element of A is written as b with this belongs by the cross does not belong to A. So, a belongs to A, b does not belong to A. So, for example, in that 2 4 6 8 2 belongs to that set because here is 2 setting inside. So, 2 belongs, but 3 does not belong because 3 is not in this collection.

So, we will write 3 as not belong to it. So, this is a type where it should be saying that 2 is an element of the set while 3 is not an element of that set right. So, it is very clear. So, this symbol is used a belongs to A, the element a belongs to A, but the object b does not belong to A. And one thing more elements are not repeated in as set for example, when we are represented a set this set as 2 4 6 and 8, an element is listed only once is prurience is shown in that collection only once not again and again. For example, we would not write 2 comma 2 again, right. Only once is enough to indicate for example, this 2 6 4 6 8 is not a valid description of the same set. Because here we are repeated 6 in this representation. So, that should be avoided that should not be written. So, elements when described in a set are not repeated, right.

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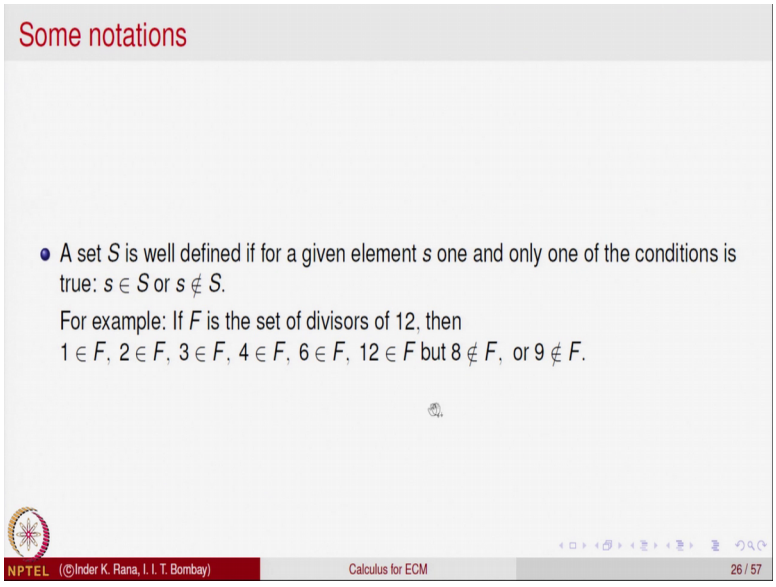

So, let us go ahead a bit more and try to understand. So, in a sense saying a set is well defined, right. You take up an element s there should be only one and one a possibility of saying either s is inside, that set s are that element s is not inside that set. For example, in that example of rich people in Mumbai if I pick up a human being right, I should be able to decide conclusively whether he is rich or not, I can not say he may be rich or he may not be rich. So, that is what the well-defined s amounts to be. So, for example, if f is a set of all divisors of 12. So, here you should understand what is divisor of something that divides 12. So, one divides 12 2 divides 12. So, one belongs to F, 2 belongs to F, 3 belongs to F and so on, 6 belongs to F 12 belongs to F.

So, these are all divisors of 12, say all of them belong to F, but 8 does not divide 12. So, 8 is not an element of F, similarly 9 does not divide 12. So, 9 is not an element of F. So, this is how the language of set theory is getting evolved a set is a well-defined collection of well-defined objects, and even object belongs to it we denoted by that symbol belongs. And if it does not belong it is not in that collection we write it as it by a slash does not belong.

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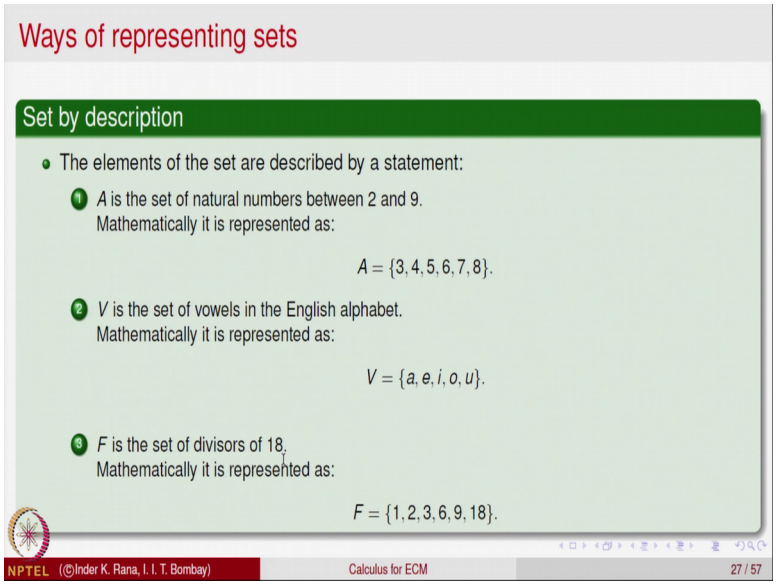

So, this is set by description right. So, let us look at some more examples. A set is let us consider the set a is a set of all-natural numbers between 2 and 9. So, what are all the natural numbers between 2 and 9 3 4 5 6 7 8. So, we have listed all of them. So, we write

a is the set which is equal to which is in the brackets, opening bracket, closing bracket, 3 4 5 6. And so, I described all the elements.

So, this is a set. Let us look at another set. V is set of all vowels in the English alphabet. So, what is that? So, we know the English alphabet a b c d so on. In that if you have the knowledge what is a vowel. So, you can pick up we know there are 5 vowels and they are a e I o u. So, we know precisely how to pick up a vowel from the alphabet collect them together put them in the bracket, and that is a set of all vowels from the English alphabet. So, that is a well-defined set. So, this is a set by describing, we are describing all the elements of the set explicitly, we are showing you all the elements. The set of all divisors of 18 something we are done similar to 12. So, one is a divisor of 18, 2 is a divisor of 18, 3 is a divisor of 18, 6 9 and 18 all are divisors are 18. So, f is the set right. So, when we say consider the set f of all divisors of 18 we mean f equal to bracket 1 2 3 6 9 and 18 bracket close.

So, this is how we write the set of all divisors of 18.

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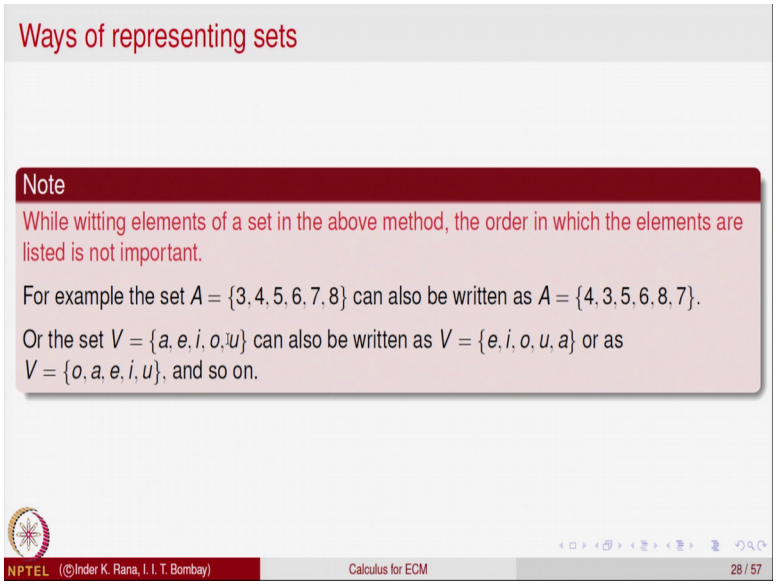

So, this is called the set by description. Let us note while writing elements in a set in our method, the order in which the elements are listed is not important. So, one we said and we are listing all the elements of a set you should not be repeating them, a element should not be repeated it should be listed only once. And secondly, it is not necessary that you should be writing in the same order which you think is a natural one. For

example, the set 3 4 5 6 7 8 this is set, which member are this all members are equal, we are not saying this is a first member this is a second member 5 is a third member or so on or element or the set you got have written this set also as 4 3 5 6 7 8 or 8 7 whichever way.

So, order of the elements when you are describing it is not important in which order they are written. They all should be written and only once they should be written, that is what it says, right. So, for example, the vowels a e I o u. Here we have written vowels as they occur in the alphabet from looking at a b c d so on, but I could have written it also e I o u a or o a and so on. So, different ways of writing the same set, they all mean the same set. So, the order in which the elements are listed is not really important. So, this is one method of writing a set, where we are explicitly able to exhibit all the elements of that set. There is a another method where we specify a rule for picking up those elements of a set.

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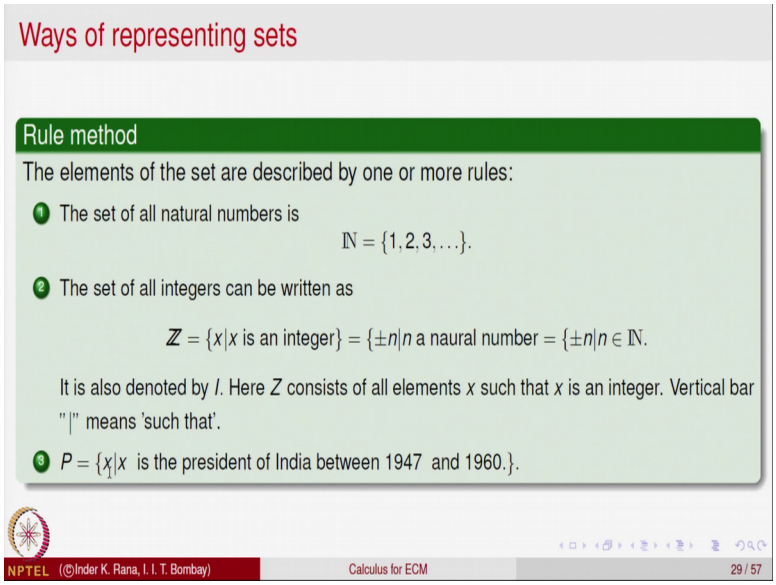

So, let me give some examples before we right. The elements of a set are described one or more rules the rule could be one or more.

For example, the set of all-natural numbers. So, one way is I write n. So, this is by the way this is the symbol we will be using throughout our course for the set of natural numbers. This is n with a additional line on the side. So, this is called script n. So, this is called script n, and this will signify whenever we use you this this will mean the set of

all-natural numbers. So, what is a set of all-natural numbers 1 2 3 of course, I have written 4 or I could have written 5 and so on, but we know that we cannot write all-natural numbers describe explicitly. So, one writes 3 of them 1 2 and 3, and then puts dots to indicate that to we go on doing that. So, this is one way of writing the set of natural numbers. Now let us look at the set of all integers, right. What are this set of all integers? This is normally denoted by zee z capital Z with a additional line in between, also denoted sometimes by the letter I for integers. The symbols z comes from the German word called Zahren calculation.

And because German mathematicians gave it first this name the symbol. So, it is used very often. It is used mostly actually I would say, but some are the books in school they use I for integers. So, what are integers? integers are you look at all the natural numbers here is a typo here, along with that we should also have 0 inside it. So, what is the set of all integers? This is a correction here set of all integers is 0 plus 1 minus 1 plus 2 minus 2 plus 3 minus 3 and so on. So, that is a set of all integers. So, we can write x, x is an integer or write as plus minus and n a natural number, and it should be I should be closing the bracket. So, there is a mistake here in typing.

So, what is a mistake? The mistake is first is z is the collection of all integers, and then I should be writing as opening bracket plus minus n, this line indicates such that n is an natural number, and or it should be 0. N could be 0, 0 is also a part of the integers. So, that is way of writing the integers right. So, let me probably show it to you writing on a piece of paper. What does integers mean?

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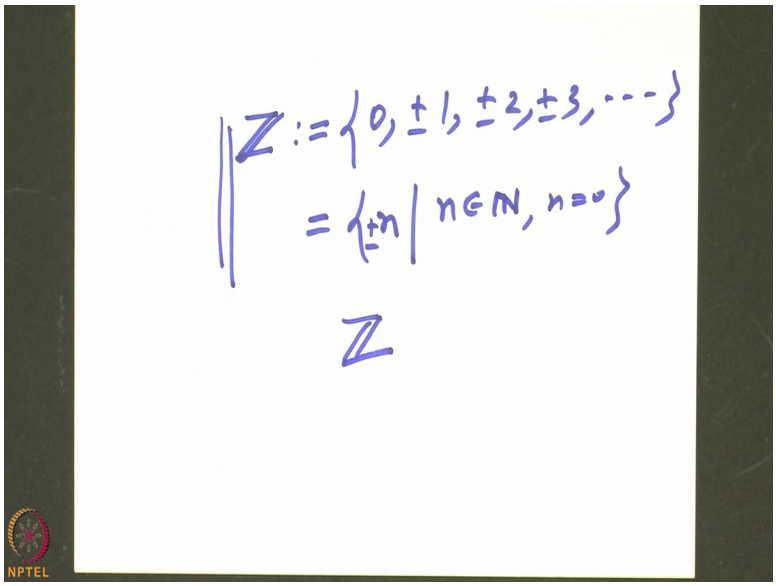

So, integers z denoted by z is equal to it is 0 plus minus 1 plus minus 2 plus minus 3 and so on. Or you can also write it as all n such that n belongs to n, plus minus n or n equal to 0 this is another way of writing integers. So, either is good enough. So, this z is the letter which is used to indicate integers. So, let us go over to next one.

So, as I said this when I write this way this vertical line means x such that that is how you should read it and understand. So, let us look at p for example, p could be the set of all x objects x such that. So, what is x? X is a president of India between 1947 and 1960. So, this is one way and if you know precisely who are these people one can put their names in it and write it explicitly also. So, in this method the elements of a set so, what is the roster method or the rule method of representing a set?

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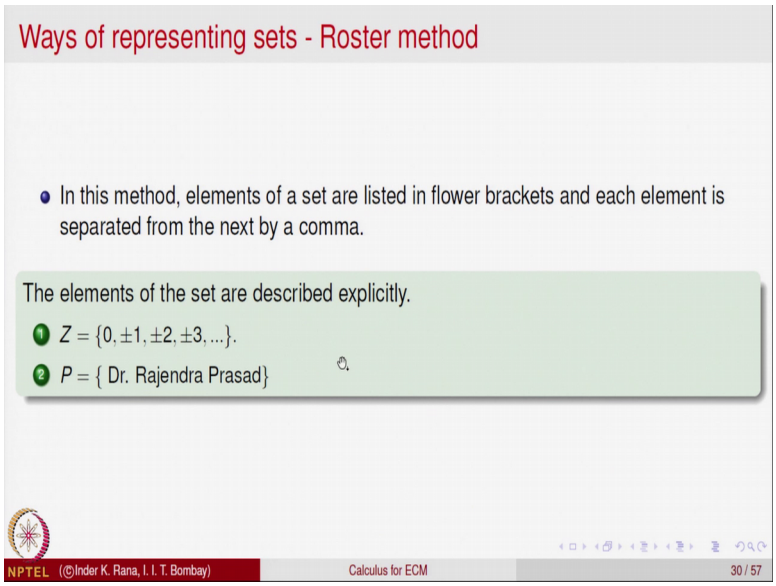

In this method the elements of a set are listed in flower brackets. And each element is separated from the next by a comma that is the explicit method of listing right these are described explicitly. So, this is explicit method of description of integers right. So, the earlier one when we said presidents in India between this year and this year.

So, if we remember correctly there is only one president that president doctor Rajendra prasad. So, what we are saying is a set is a well-defined collection of objects, one. And there are 2 ways of representing a set. One by putting a flower bracket, and listing all the elements of the set or describing the objects by a property. So, there are 2 different ways of writing whichever is convenient we should use them. So, we will stop here. And continue our discussion of set theory.