Calculus for Economics, Commerce & Management
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Lecture – 11
Linear models, elasticity, linear functions, nonlinear models, quadratic functions

Welcome to today's lecture. In this lecture, we will continue the study of coefficient of elasticity. In the previous lecture, we had seen how to measure the responsiveness of change in demand with respect to change in price and we came to the definition of coefficient of elasticity and for a linear price and demand function, we saw that this constant is always negative and between the values between up less than minus 1 for the negative numbers if the coefficient is elasticity is negative number less than number one then is very responsive. So, it is very elastic in the sense that a small change reflects in a big change in the price.

And demand the small change in the price will result in a significant change in the demand when it is equal to one this coefficient of elasticity is one there is equal change in. So, this is why it is called the unit elastic; see that price and less than between minus 1 and 0 there is not much significance change that is going to come if you change the price with no not much change comes in the demand for that coefficient of elasticity. So, all that depends upon the model; how is the model; what is the slop of the linear model, but it does not depend on the units being used. So, that is. So, so let us look at some more examples for this.

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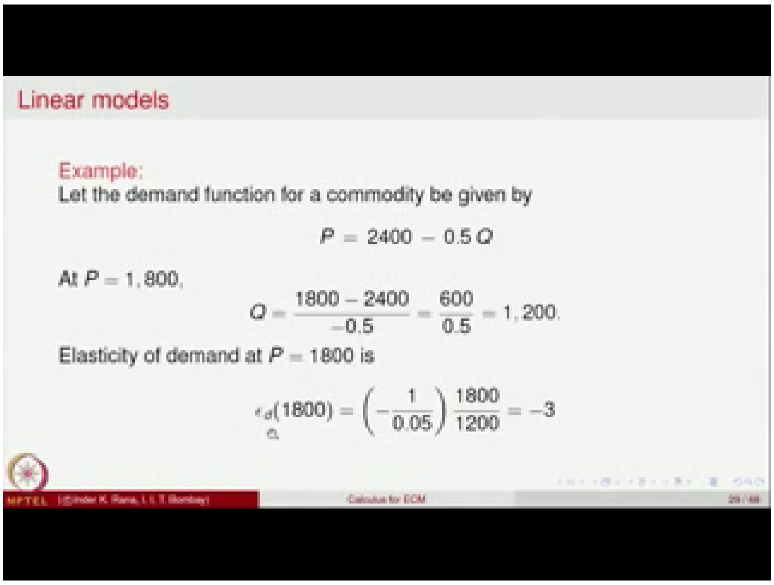

Let us. So, look at the demand and price; price and demand function given by P is equal to 2400 minus 0.5 Q ok.

So, for this linear demand function minus 0.5 is the slop of minus 0.5 is a slop of this linear demand function. So, at a price 1800 when the price is 1800, we can calculate; what is a change; what is the Q. So, when it is 1800; we can calculate; what is Q and from this equation, we calculate Q to be 1200; that means, when the price is 1800 the quantity demanded is 1200 units. So, the elasticity at the point P equal to 1800. So, this is indicated elasticity of demand at 1800 is minus 1 over b. So, here b is 0.5. So, minus 1 over b multiplied with the price P by Q. So, 1800 divided by 1200 that is minus 3. So, the coefficient of elasticity for this function at the price 1800 is minus 3.

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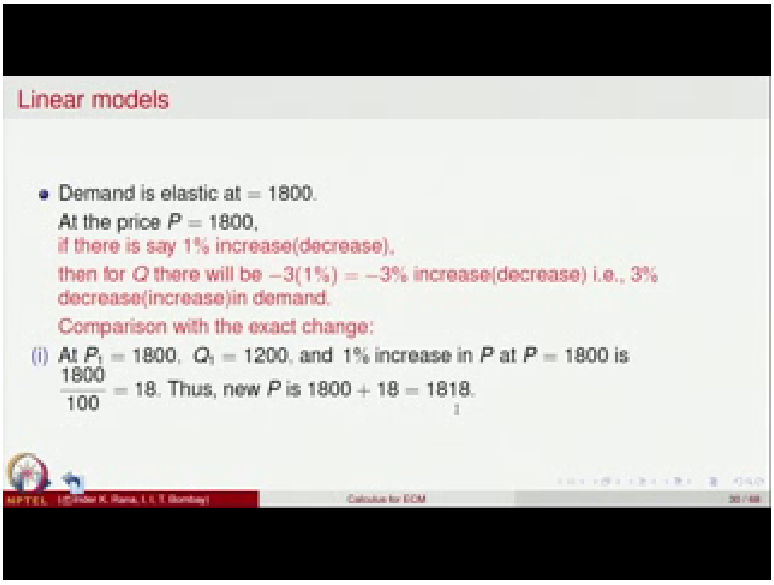

So, let us see how this is effect the scenario. So, demand is elastic at 1800, it is less than minus 1. So, at the price 1800, if there is a change; say if there is a 1 percent increase or decrease in the price then what is the change in the demand. So, then Q there will be a minus 3 percent increase because coefficient of elasticity is minus 3. So, if there is a 1 percent increase, then there will be one percent decrease or increase in demand. So, if there is a increase in the price there will be decrease in the demand 3 percent of that. So, let us compare it with the exact change that happens at in the scenario. So, just to see how much the coefficient of elasticity is a good measure of responsiveness at the price P 1.

Which is 1800 Q 1 is 1200 and the 1 percent increase in the price gives you the price as 1800 plus 18. So, the new price that is increase. So, the new price will be 18 right. So, now, let us look at.

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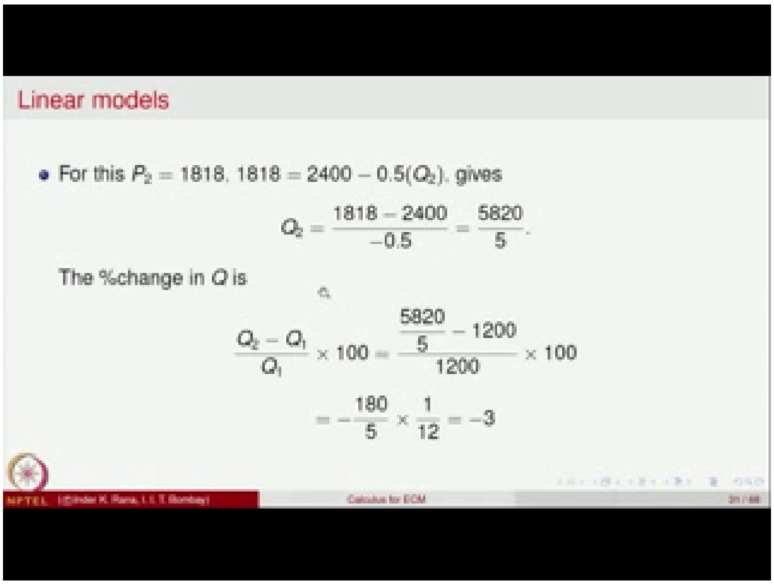

What is the; for this new price P 2 what is the demand Q 2. So, we can calculate that. So, Q 2 from that linear relation we can compute Q 2 and that is equal to this. So, what is a percentage change? So, we are calculating the actual thing. So, the percentage change Q 2 minus Q 1 divided by Q 1 multiplied by hundred that is the percentages in Q 2 minus Q 1 that is equal to this simplifies to be equal to minus 3 ok.

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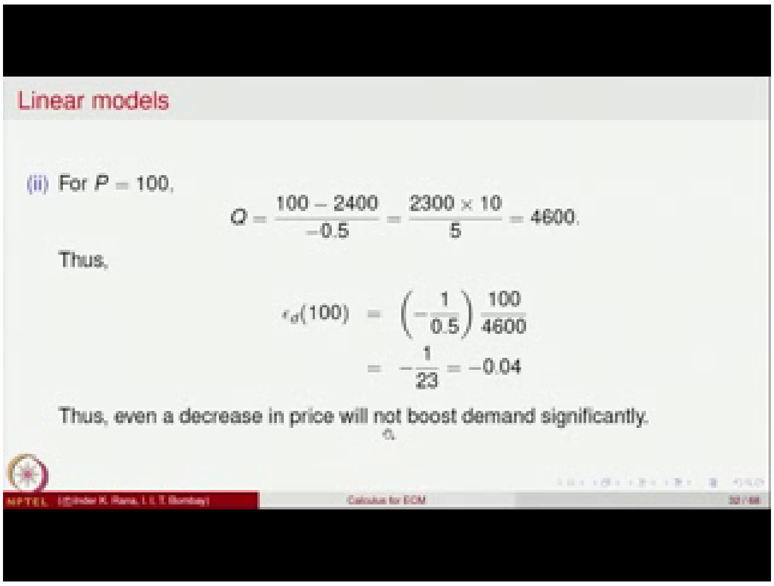

For P equal to 100; the Q corresponding Q which change will come out to be fourteen right from the same formula from the demand price demand we can calculate Q.

So, actually what is happening in epsilon d at eighteen at one hundred price comes out to be equal to right one minus 1 over b P by Q. So, that is equal to minus 0.04. So, that says thus even if there is a decrease in price it will not demand boost the demand significantly right or them because the coefficient of elasticity 1800 is just 0.04 very low which is very near.

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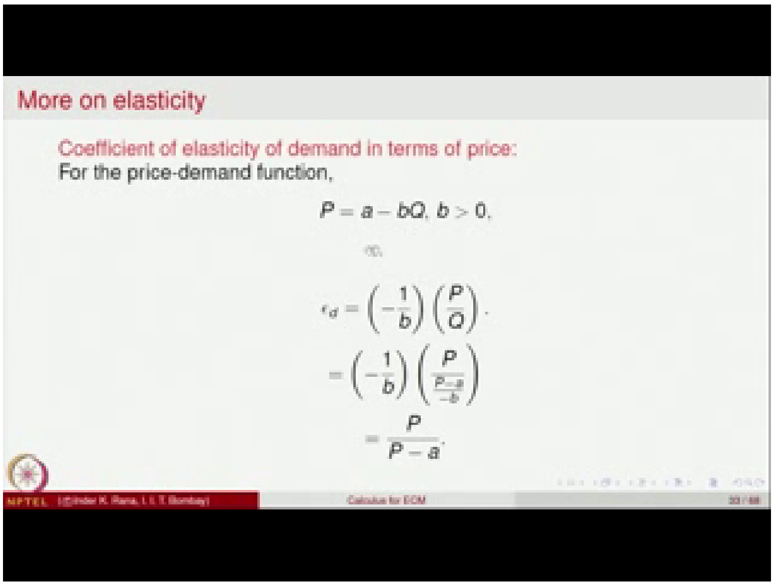

0. So, this is how we analyze the effect of change in price and change in demand in terms of coefficient of elasticity now next let us look at interpreting coefficient of elasticity in terms of purely in terms of demand or in terms of the price. So, we know that the demand price demand function is P is equal to a minus b Q where b is a positive quantity.

So, by the formula epsilon d was equal to minus 1 over b P by Q and what do we do here is we put the value of Q in terms of p. So, Q is equal to P minus a divided by minus b. So, we put that value here. So, that b minus b minus b cancels it; it is equal to P divided by P minus a; a is that constant in the linear equation that is a intercept y intercept. So, coefficient of elasticity for demand epsilon d can also be interpreted in terms of the price right. So, that is P divided by P minus a. So, here you see that the slop is not entering into the picture at all.

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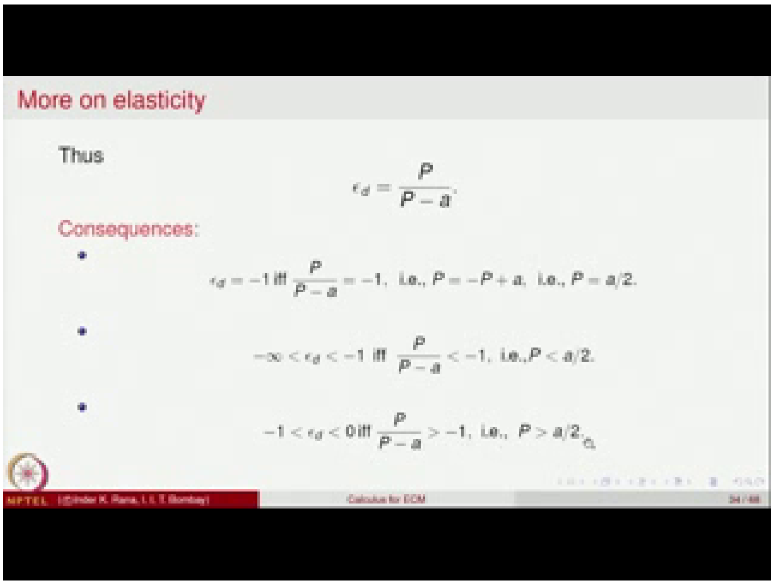

So, this is a formula for coefficient of elasticity of demand at a price P. So, what are the consequences of this formula for example, we said that if epsilon d is equal to minus 1 is a number between minus infinity and it is a negative number.

So, if it is equal to minus 1 this will happen if and only if this is equal to minus 1 right and when we solve that equation that says P is equal to a by 2 if P is equal to a by 2 then coefficient of elasticity is equal to minus one; that means, there is a unit elasticity at that point next possibility when epsilon d the coefficient of elasticity is between minus infinity and minus 1.

It is less than minus 1 and we saw it is very responsive and this in terms of the price is a related with if you put the value P divided by P minus a is minus 1 so; that means, P is less than a 2. So, when the price is less than the number a by 2 the price and demand are related with each other and the coefficient of elasticity is very responsive a small change in price will result in a much bigger change in the quantity demanded and finally, when it is between minus 1 and 0.

So, that gives the relation P is bigger than a by two. So, coefficient of elasticity the values when it is highly responsible in terms of the price when P is equal to a by 2 that is unit elastic when the price P is less than a by 2 that is very responsive highly elastic and this P is bigger than a by 2 it is non elastic. So, this remember coefficient of this a is the constant which appears in the linear equation P is equal to a minus b by b times P. So, right. So, that is geometrically that was a intercept answer.

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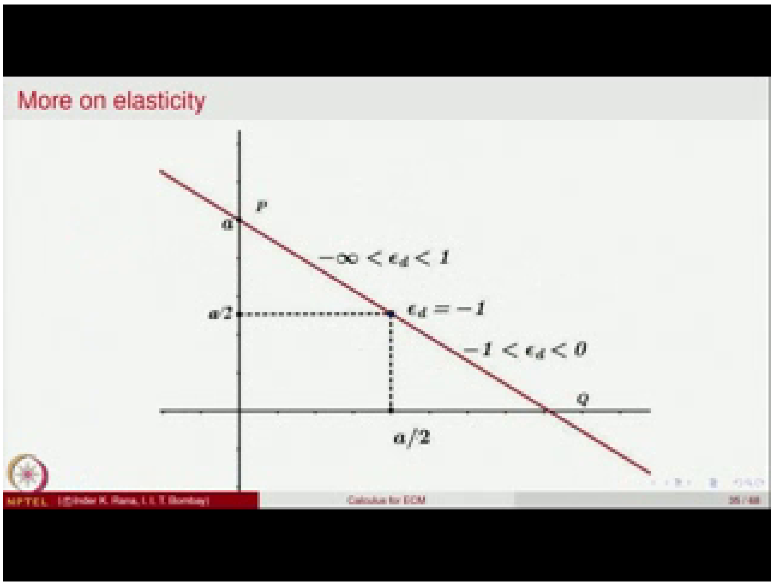

So, we can represent it geometrically as follows. So, if this is the price demand function here is the price. So, as you see as the price increases demand is decreasing.

So, that is the price demand function it says unit elasticity occurs when the price is a by 2 when the price is bigger than a by 2 it is very elastic here it should be minus one. So, that is a. So, this this is type of here that should be minus 1 and when the price is between a by 2 and 0 of course, price is never going to go 0 it is highly in elastic. So, that is the portion. So, that is a relation between elasticity and the price in terms of the coefficient a and the earlier definition in the definition we had use the constant b that is a slop. So, both formula are useful in interpreting results.

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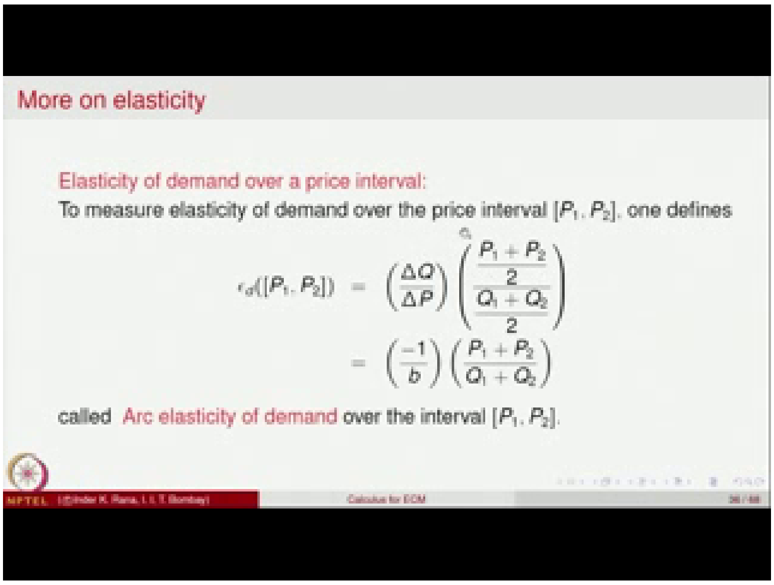

This is till now we have defined coefficient of elasticity of demand at a particular price P we can also look at coefficient of elasticity of demand in a price range in a interval form P 1 P two.

So, to do that what we do is the following to measure the elasticity of demand over a price interval P 1 P 2 right which is denoted by coefficient of demand coefficient of elasticity of demand in the interval P 1 to P 2 is defined as this is the change delta Q by delta P that also is nothing, but the relative change proportionate change in Q by P and here it is the average instead of taking P by Q we take the average P 1 plus P 2 by 2 and Q 1 plus Q 2 by 2 we take the average of that interval. So, we get minus 1 by b P 1 plus P 2 divided by Q 1 plus Q two. So, normally coefficient of elasticity of demand in the interval P 1 to P 2 is also called elasticity of demand over the interval P 1 to P 2.

So, that is also the name given to this number.

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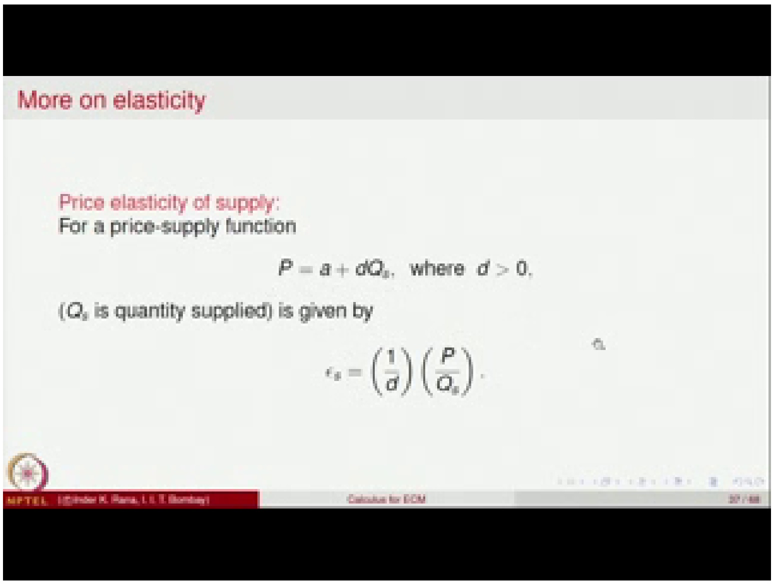

One can also interpret see we had looked at in terms of price and demand we can also look at a equation where price and supply relationship comes normally a price and quantity supplied is given by say a linear relation of the type normally it is P is equal to a plus d Q s; Q s is indicating this is the quantity supply d is a constant which is bigger than 0 and a is bigger than 0 this is a linear equation what does that mean; that means, if d is positive as Q increases as the supply increases quantity is supply. So, this is the relation between the slop here d is positive. So, as the supply increases the price will drop. So, here the coefficient of elasticity of supply is one over d this is d because we have put in terms of minus.

So, that is minus. So, that cancels. So, it is one over d P by Q s price elasticity of supply for the linear equation. So, one can interpreted accordingly in situations.

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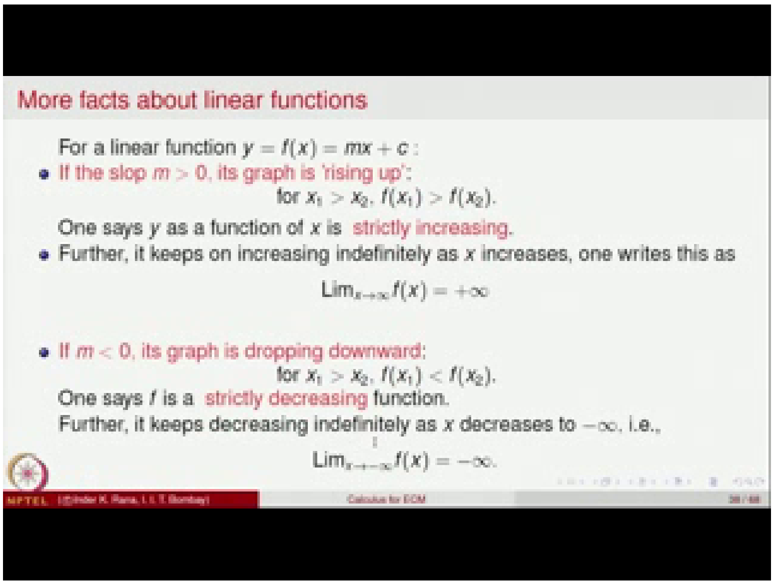

Let us look at some more facts about the linear functions which are going to be useful first of all this slop which is m right if it is positive; that means, the graph is rising so; that means, what as you move from left to right; right as you move from left to right it will increase y increases. So, if this is y this is x; x is a horizontal y is vertical. So, the graph will look like this way right from the left to right it is increasing it is rising up that is why it is geometrically it is rising up mathematically one will write it as for x one bigger than x 2 f of x 1 is bigger than f of x 2.

So, that is called increasing or strictly increasing such a function normally is called a strictly increasing function. So, if the slop is positive in the linear function is strictly increasing and how much it can increase a linear function will increase to any value you like? So, one writes it as limit x going to infinity as x increases f x keeps on increasing as x becomes very large f x also becomes very very large. So, that is interpreted a saying limit x going to 0 of f x is equal to plus infinity similarly if m is negative then the graph is dropping downwards then the as you move from left to right the horizontal that line that is a graph should be coming down. So, it should be sloping down move dropping downwards.

So, that is mathematics terms we say the function is strictly decreasing for x one bigger than x 2 f of x 1 is strictly less than f of x 2 x 2 is smaller, but f of x 2 is bigger than f of x one. So, that is called a strictly decreasing function and how far it can decrease it can decrease to the value minus infinity it can become as small as you wanted. So, these are some properties of, but keep in mind this is only a symbolic way of saying limit x going to infinity f x is equal to plus infinity is only a symbolic way of saying that the function keeps on increasing to any large value you want and similarly limit f x x going to minus infinity is minus infinity is saying that the function should decreased right is decreasing n can become as small as you want it.

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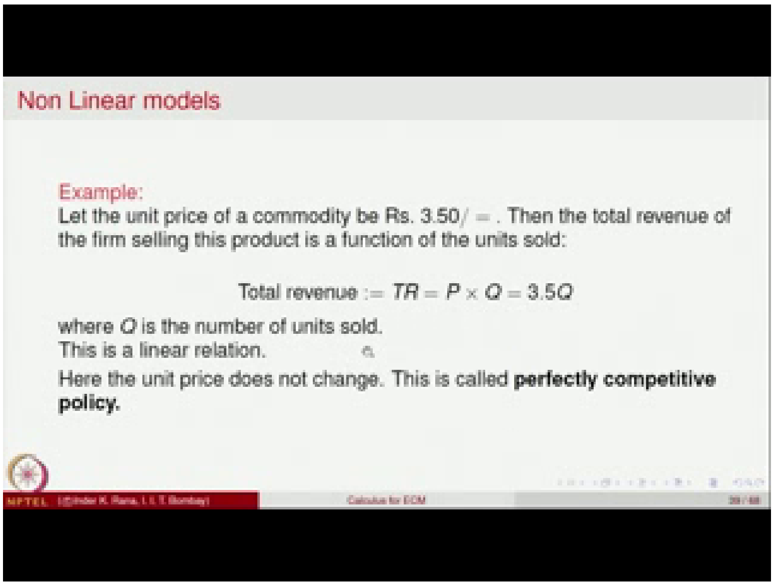

Let us look at some non median models till.

Now, we are looked at a very simple models of price and demand there are some other equations which appear in study of economics commerce and management. So, let us look at an example, let us look at the price of a commodity which is price per unit is 3.50 then the total revenue of the firm selling its product is a function of the unit sold say P units are sold and each unit is sold at a price 3.50. So, we get the total revenue this is the price per unit if Q units are sold and P is the price of per unit that is given. So, total revenue will be P times Q.

So, price per unit is equal to 3.50. So, total revenue is 2.5 Q where Q is the number of units sold that is a total revenue of that particular company. So, this is a linear function; obviously, it is the line it is graph of this is a line passing through the origin.

So, here what we are assuming is that the price is not changing now and then right P is fixed the price is fixed. So, this is what normally called a perfectly competitive policy for the company supposing this policy is change.

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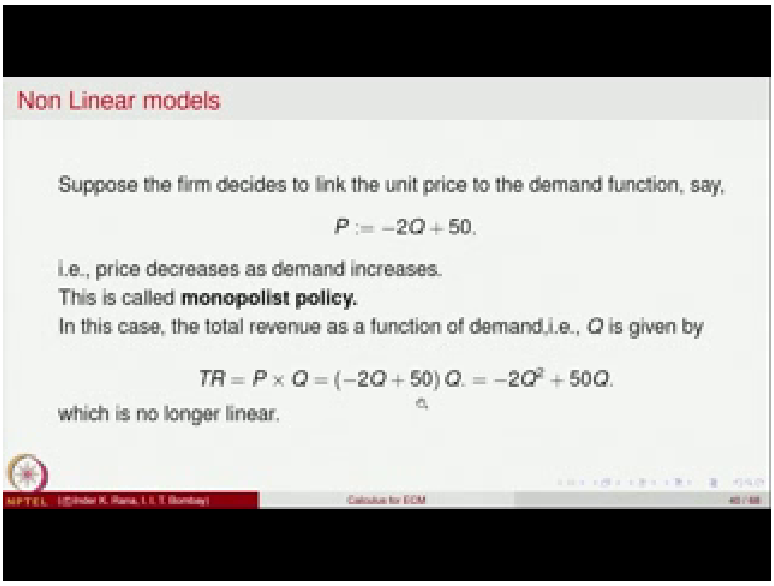

And suppose a firm decides to link the unit price to the demand function by this relation by a linear relation. So, P is equal to minus 2 Q plus 50. So, price is not fixed it says depending on the demand our price will change. So, this is a variation in the price and, but this is a linear equation. So, here the price decreases at the demand increases right if Q increases P is decreasing demand decreases. So, this is perfectly that depends on the company what price relationship they want.

So, this is called a monopolist policy right in which the price is linked with the demand in this case let us calculate what is the total revenue of the company again total revenue is P into Q the price into the quantity being sold. So, P here is this function. So, this gives you minus 2 Q plus 50 multiplied by Q. So, that is minus 2 Q square plus 50 Q and this total revenue is a function of Q, but it is no longer a linear function the power 2 is appearing here. So, this is no longer linear.

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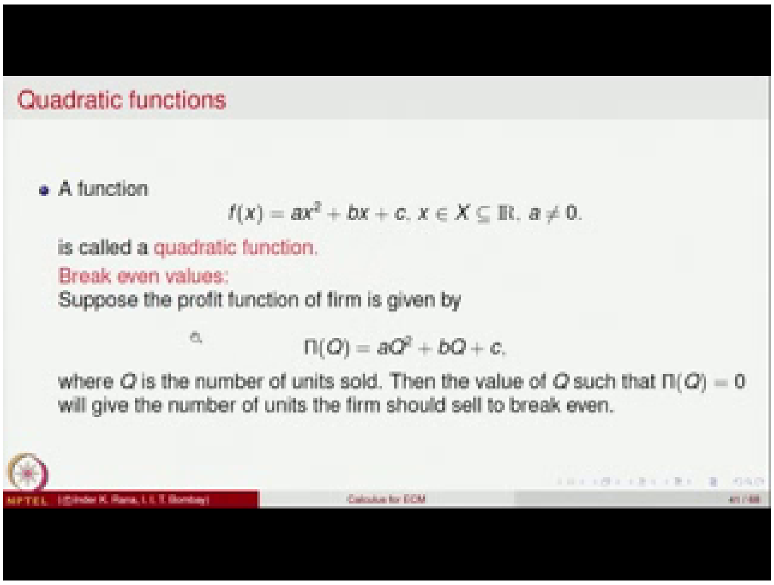

So, study of this functions is called the study of quadratic functions. So, we define a new type of functions now a function f x given by the formula f x is equal to e x square plus b x plus c for x belonging to a subset of real line where a at least should not be equal to 0 because if a is equal to 0 this just becomes a linear function is called a quadratic function.

So, a quadratic function f x is equal to a x square plus b x plus c has got 3 constants a which is a coefficient of the power 2 x square b x where b is the coefficient of the power one and c is with a constant term. So, one says c is constant term b is the coefficient of power 1 and a is the coefficient of power 2. So, this is called a quadratic function and of course, we are putting the condition that a is not equal to 0. So, let us see what is a use of this kind of equations or such kind of functions suppose the profit function of a company as we saw in the previous example is given by a quadratic function. So, the profit function is called this is symbol called pi pi of Q right this is capital pi the Greek letter instead of writing P; P may look like the price.

So, one uses a symbol pi; pi Q the profit function is a function of Q the number of units being sold is quadratic function a Q square plus b Q plus c. So, where Q is the number of units being sold. So, if you want to say this is a profit of the profit is positive the company is making a profit when pi Q is positive when pi Q is negative the company is going in a loss. So, what is pi Q equal to 0 that is the values Q where pi Q is equal to 0 is the are the values where company will break even where it is neither making profit nor making any loss. So, such values such Q the number of units being sold is called a breakeven point for the company the break even values. So, for a to find out the break even points for a company whose profit function is given as a quadratic function one has to solve a quadratic equation.

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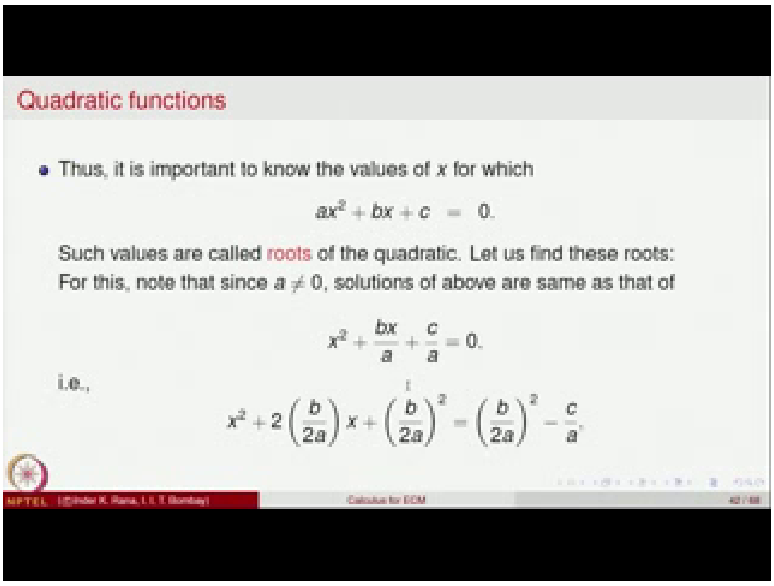

So, this is important. So, it becomes important to know given what are the values of x when a quadratic equation a x square plus b x plus c will be equal to 0. So, here a b and c are constants and the value x for which this is satisfied is called a root of the quadratic equation. So, our next aim is going to be given a quadratic equation; that means, given if we know the coefficients a b and c what are the values of x for which this equation will be satisfied. So, to solve this let us proceed as follows since a is a quadratic equation a is not equal to 0. So, we can divide by a. So, let us divide by a and we get the equation namely x square plus b by a x plus c by a equal to 0. So, what we are trying to do is we are trying to form a complete square by using a method in mathematics called completing a square. So, this is x square.

So, what this term we multiply and divide by two. So, this can be written as 2 multiplied by P by a into 2 that 2 in the denominator is a commodity here. So, it is 2 times this term in the bracket x let us add square of this term b by 2 a both on the left side and right side. So, when you added on this side this becomes this equation on the left side on the right hand side you have added b by 2 a to the power 2 and c by a we have taken it on the other side. So, that is minus c by a. So, we get x square plus 2 b by 2 a x plus b by 2 a square is equal to this constant on the right hand side. So, this reminds one of a formula in school algebra namely x square plus 2 something into x plus something square equal to something. So, left hand side is a perfect square it is square of x plus b by 2 a.

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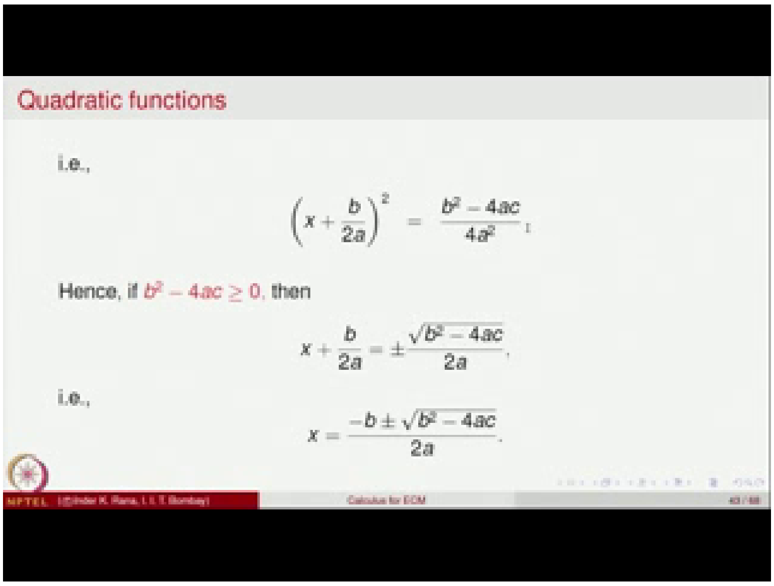

So, using algebra formula one writes a left hand side as x plus b x plus b by 2 a whole square is equal to b square minus four a c divided by four a square. So, once there is now we got x square we want to find a value of x. So, we take square root on both sides. So, this implies namely if this quantity is bigger than or equal to 0 we can take square root on both sides because for negative quantity this negative real numbers square root does not make sense. So, that gives you that this is x by plus b by 2 a; this is a square root this side we will be either plus or minus square root of this quantity and square root of this quantity is square root of b square minus four a c and divided by 2 a 4 a square; square is 2 a plus minus we already accommodated here. So, that gives you the value x is equal to minus b by 2 a; this term can be taken on the other side.

So, minus b by 2 a plus minus b square minus four a c. So, this gives you 2 values of x x is equal to minus b plus minus b square minus four a c divided by 2 a.

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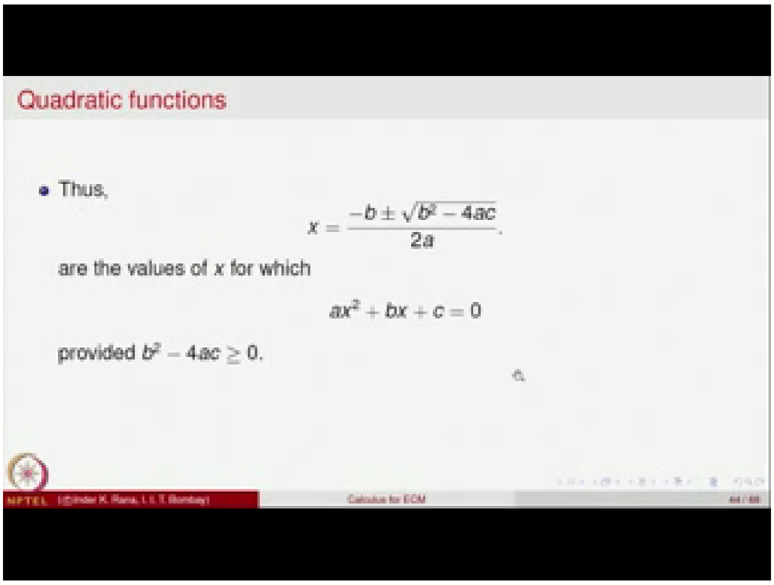

So we get 2 possible values of the quadratic. So, if the quadratic is a x square plus b x plus c equal to 0 and b square minus four a c is not equal is this number is not equal to 0 then there are 2 roots possible x is equal to minus b plus minus b square minus four a c divided by 2 this quantity normally is called the discriminant of the quadratic because it becomes important to know whether the quadratic will have solutions or not. So, what we had done is we have try to solve a quadratic and find its roots.

So, the roots 2 roots exist the roots are 2 roots exist if b square minus four a c is bigger than 0. So, we will continue this study of quadratic equations in our next lecture.

Thank you.