Calculus for Economics, Commerce and Management

Prof. Inder K. Rana
Department of Mathematics
Indian Institute of Technology, Bombay

Lecture – 10
Function formulas, linear models

So, welcome to today's lecture. In the previous lecture we had started looking at the graphs of functions, we looked at the graph of a constant function. Let us go to the next step of looking at a function defined by y equal to m x, or f of x is equal to m x. So, f of x is denoted by y. So, one normally write it as y equal to m of x where m is a fixed real number.

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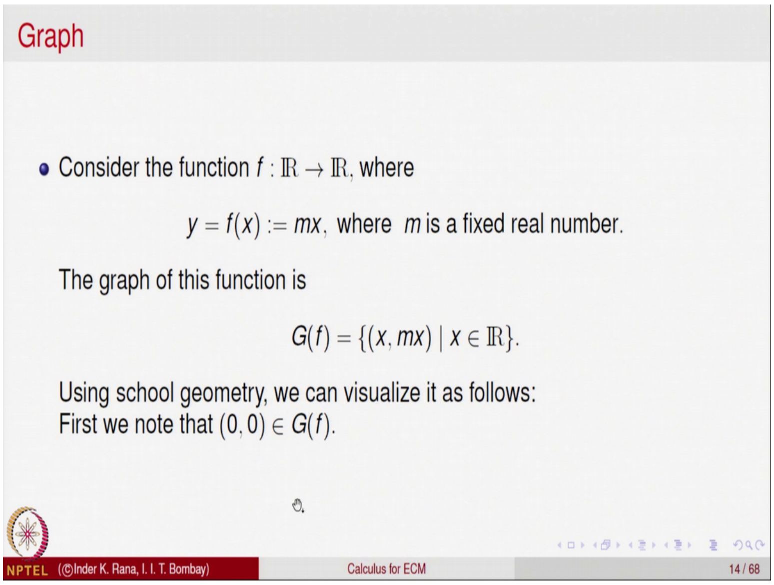

So, one would like to know what is the graph of this function, how do you visualize the graph of this function as a set of points on the plane. Remember we had done a association every ordered pair x comma y was identified with a point on the plane with x axis y axis as a coordinates as x comma y. So, let us try to identify what is this set a graph of the function f as set of points on the plane R 2.

So, for this let us first observe that for x is equal to 0 m x is equal to 0. So, the point 0, 0 belongs to the graph of the function. So, the graph of the function includes the origin 0.

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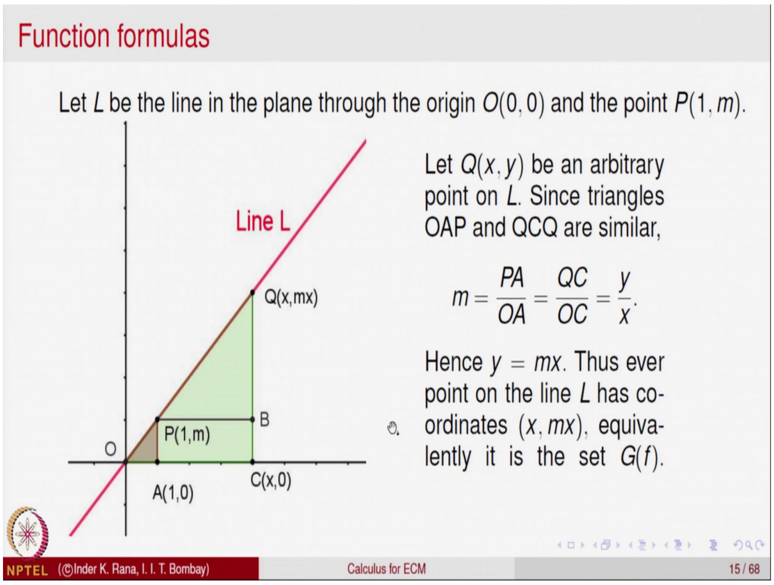

So, let us look at any point any line in the plane which passes through the origin because the origin is on the graph and if in our equation in our function x is equal to 1 then 1 comma m also is a point of the graph. So, our graph is going to pass through 2 points, let us test the possibility that our graph is aligned passing through the point 0 0 and the point P which is 1 comma m. So, let us call this line as m and let us take any general point Q on this line and find out its coordinates if Q 1 is a point then on this line, this line. So, let us take Q if the Q coordinate, let us say the coordinates of Q are x comma y we want to compute y in terms of x. So, let us look at these 2 triangles QCA and PAO.

So, what I have done is I have drawn a line here parallel to the x axis and this is the line which is parallel to the y axis and they meet at the point B. So, let us look at the triangles QCO and the triangle PAO, this small brown triangle and this big triangle. So, these two triangles claim is that these two triangles are similar because this angle is common to them and this angle is 90, this angle is 90, so these two triangles are similar. Since these two triangles are similar the sides must be in the proportion so; that means, if I look at the ratio PA by AO that ratio is m. So, that must be equal to the ratio QC divided by CO and that ratio is y by x because Q has coordinates x y. So, from this we get that y must be equal to m x. So, y must be equal to m of x thus every point on the line has got coordinates x comma m x or equivalently we can say that this is also this identifies with the graph of the function because of the unique association of every point the ordered pair x comma y with the point with coordinates x comma y in the plane. So, this shows that the line L is the graph of the function y equal to m of x.

We can go slightly ahead and look at the function which is given by y equal to m x plus c.

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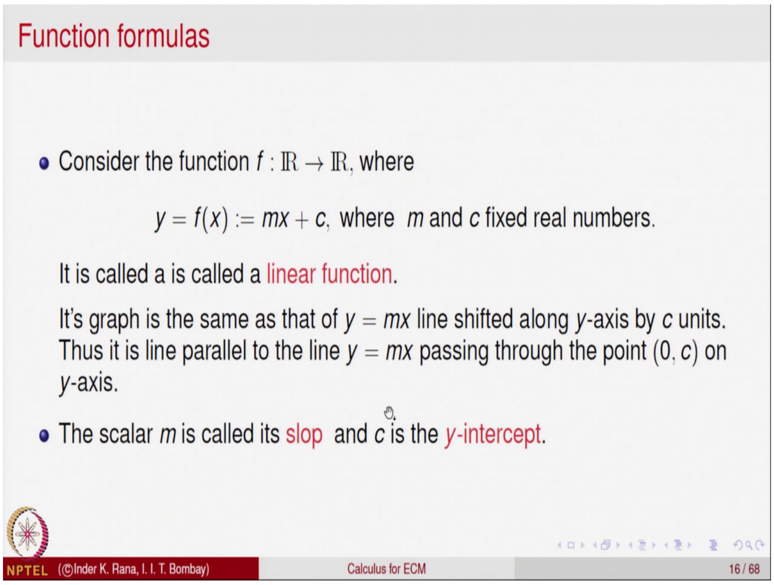

So, if you compare it with the previous function this is just y equal to m x plus c so that means, what for every value of x whatever was m x c is added to it; that means, y is the y coordinate is being translated by c units. So, we can just look at the dynamic aspect of geometric and say that the graph of this function, this function is also called linear function. So, the graph of this function is nothing, but it is shift the graph of the line y equal to m x shifted along the y axis by c units. So, let us just will have a look at the graph.

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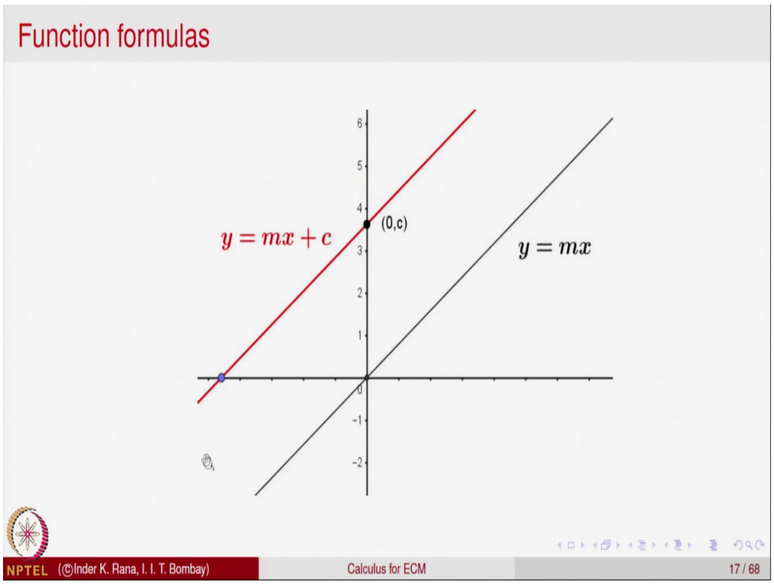

This is the line y equal to m x plus c. So, if you drop c this is the line y equal to m x this is going to be parallel to this line and what we are done is we just lifted up, we are translated this graph up by c units. So, depending on c is positive or negative this will go up or down. So, this is the graph of the function y equal to m x plus c and this graph is this function is called a linear function, so it is given by y equal to m x plus c. So, as you have realize there are two important numbers in this one is m other is c.

This number m is called the slop of the linear equation or geometrically the slop of the line y equal to m x plus c and c is the y intercept; that means, the graph passes through the point 0 comma c. So, this is the graph of that function y equal to m x plus c. Why this m is called the slop? If you recall in the previous thing in the previous picture we will looked at the point P right. So, this will the point P, for this line which is y equal to m x m x if this height is equal to m and this distance is equal to 1 then this is that line. So, as you can see if m increases, if m increases this line will tilde towards more towards y axis. So, the inclination of this line or 1 there is more slop for this line. That is why this number m indicates what is called the slop of the linear equation and c is called the y intercept. So, this is called the slop intercept form of the linear equation because m denotes the slop and c denotes the intercept.

One can also geometrically if you look at a line then one knows that there is one and only one line passing through 2 points in the plane. So, that fact we can utilize. So, let us take 2 points on this line y equal to m x plus c and try to see what we can say.

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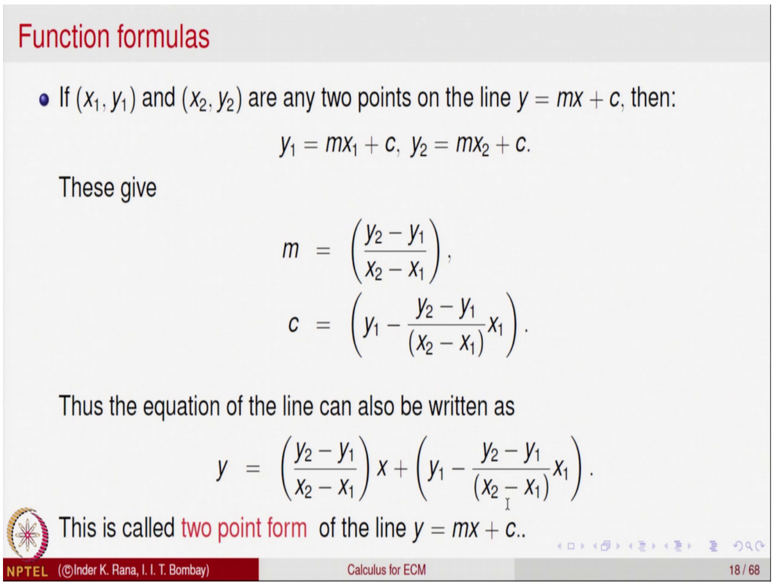

So, let us take x 1 y 1 and x 2 y 2 to be 2 points on the line y equal to m x plus c. Then because these 2 points lie on the equation y equal to m x plus c we must have y 1 is equal to m x 1 plus c. So, x 1 y 1 satisfies the equation y equal to m x plus c and similarly x 2 y 2 also satisfies the same equations. So, y 2 is equal to m x 2 plus c. So, from these two equations we can calculate the value of m and c. One way is we can subtract this equation from this equation. So, c will vanish and you can calculate the value of m and put the value of m in anyone of the equations and get the value of c. So, we leave it for you to simplify and see that from these two equations if you calculate m that comes out to be y 2 minus y 1 divided by x 2 minus x 1 and in the value of c in terms of x 2 y 2 x 1 y 1 is comes out to this.

So, one can write down the equation of the line y equal to m x in the form y is equal to y 2 minus y 1 divided by x 2 minus x 1 multiplied by x plus c and see the value of c as calculated above is y 1 minus y 2 minus y 1 divided by x 2 minus x 1 into x 1. So, this form of the linear equation, this form of the line is also called the two point form of the line y equal to m x plus c. The two point form tells you immediately what is the which are the two points through which it is passes or it can be use to write down the equation of a line passing through two points either way and you can also calculate the slop and the intercept from the values of the points x 1 y 1 and x 2 y 2. So, this is normally very useful and writing down equations of lines. So, what we have done is we have drawn a graph of a general linear equation y equal to m x plus c.

Let us see how linear equations or linear functions arise in the study of economics commerce and management. So, we will look at some examples in the language of economics and so on. So, let us look at the first example it says a manufacturer of music players like to know the following.

There is a manufacturer who is making music players and he interested in knowing that what will be the effect of 5 percent increase in the prize on the demand, if we increases the prize of the music player by 5 percent what will be the effect of this on the demand when the current music player prize is 9000.

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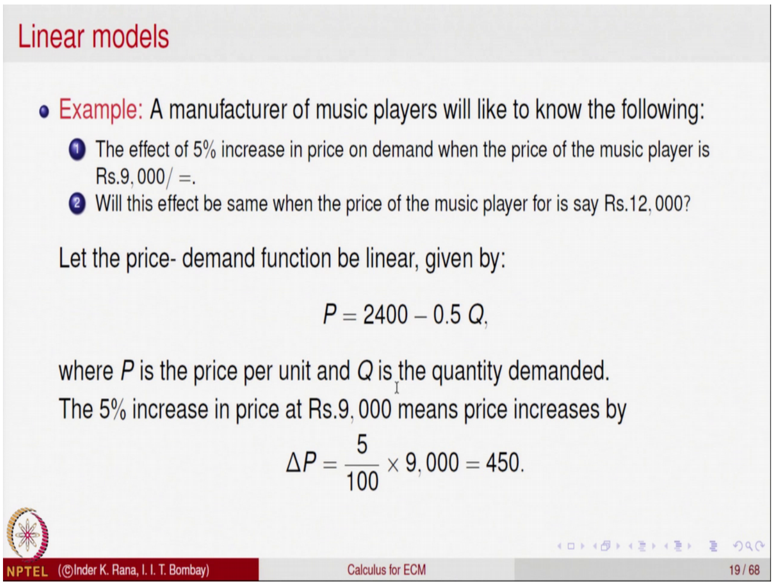

So, at present when the music player is selling for 9000 rupees per music player if you increases the prize by 5 percent how much will it affect the demand.

So, you would like to know that will the effect be same if the price of the music player say is 12000 see. So, what he is interested in is when the music player is selling for 9000 another is a increase of 5 percent whether the demand will go up or go down that is one question. And the second is when the music player is selling for 12000 at that price if we increases the price by 5 percent what will be the effect of this on the demand of the music player in the market. So, to answer this questions we have to first know how is the demand and supply are related to each other. So, let us say we know that the price and demand function is related by this equation. So, that is linear equation. So, P is the price and Q is the quantity demanded. So, this is related by this. So, this is the equation.

How is that equation coming? We are not really at present interested in modeling we know that the manufacturer says that this is relation with it. So, P is the price per unit and Q is the quantity demanded. So, this is the linear equation in the variable P and Q. So, the price is a function of the demand Q.

The 5 percent increase in the price at in the price is 9000 means. So, what will be the increase in the price? 9000 is at present the price of the music player 5 percent of that is 450. So, that is the increase in price delta P. So, that that means and also we know that delta P by delta Q right y 2 minus y 1 divided by x 2 minus x 1 is the slop and for this, the slop is here is a linear equation is slop as minus 5.

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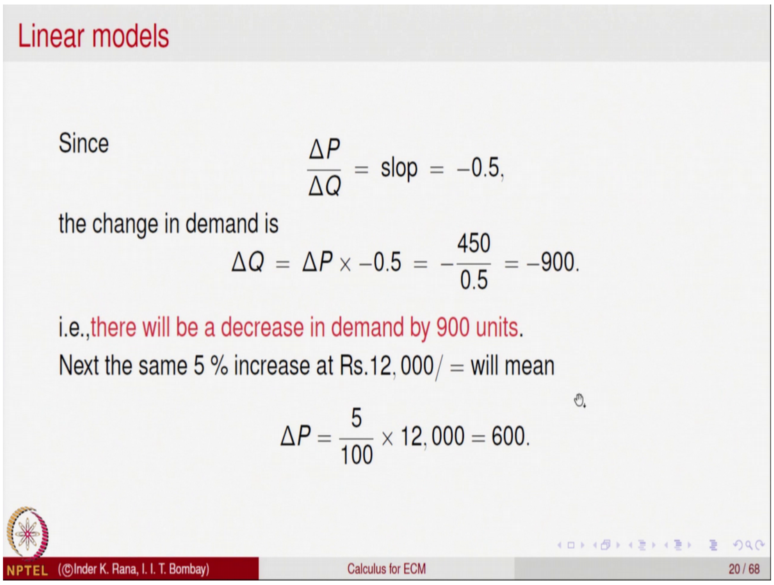

So that means, the change in price divided by the change in demand which is equal to mathematically equal to the slop is given by minus 0.5 so; that means, the change if you take it on the other side, that means, delta Q sorry this is there is a mistake here the slop is y 2 minus y 1 right that is ok. So, there is a mistake here. That means, we should be dividing by. So, delta Q is delta P divided by minus 0.5. So, when you do that, increases delta P is 500. So, here is the type of it should be division. So, that should be division. So, forget this type. So, from here delta Q is delta, P divided by minus 0.5. So, that is here so; that means, this is equal to minus 900.

That means, if when the music player is selling for 9000 rupees there will be a decrease in demand because delta Q is negative; that means, it will be a decreasing demand by 900 units. So, now let us calculate the 5 percent increase when then music player is selling for 12000 and that case delta P is 5 percent of 12000 it is 600.

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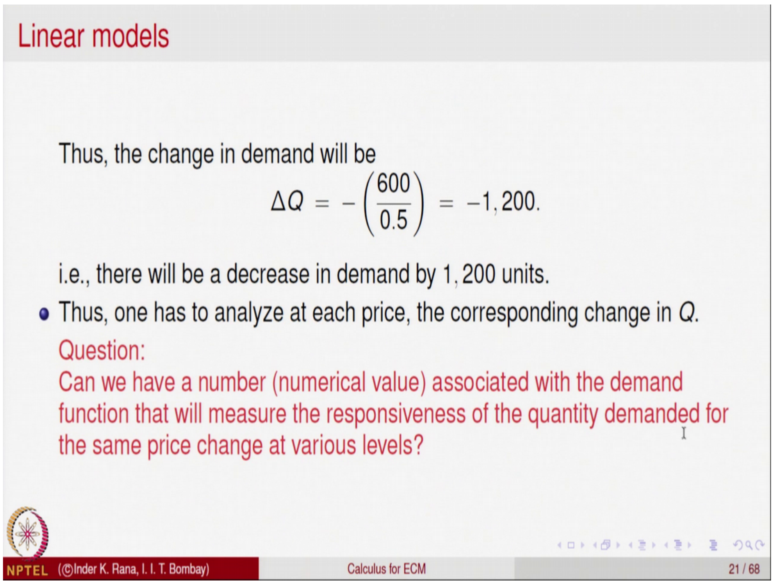

So, we calculate now delta Q it is delta P divided by that percentage. So, that is minus of 600 by 0.5 and that as 1200 so that means, there is a decrease in demand by 12000; 1200 units. That means, in this scenario as the price of the music player goes up any in increase will make the demand will make a decrease in the demand. So, thus in this model what we are saying is one has to analyze at each price the corresponding change in Q, at each point P value for P if we even if the price increase or decreases same the value of the effect on the demand is different.

So, corresponds to at every point P we have to calculate what is the change in Q? So, the question arises can we have a number some numerical value associated with the demand function that will be measure the responsiveness of the quantity demanded for the same price change at various levels. So, there what we are saying is we are given the demand and supply function. We are looking for some characteristics some something that we can say will indicate how much with the demand will change at different price levels. So, to answer this question naturally we have said that the slop of the linear function is not a suitable measure for this, so if slop is not the suitable measure what it could be.

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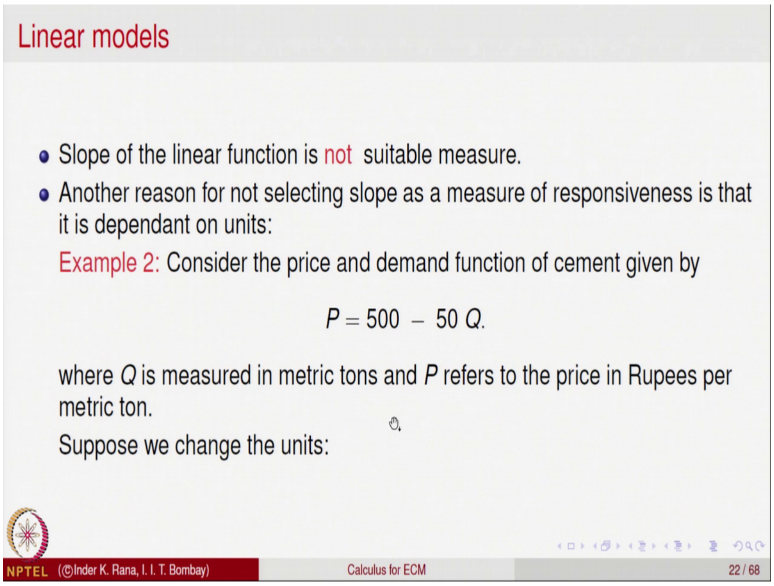

Another reason why the slop is not a useful measure of responsiveness is that the equation demand and supply equation depends upon the units used for the quantities that item being produced or demanded.

Let us look at an example and illustrate this. Consider a price and demand function of say cement which is given by the price is equal to 500 minus 50 times Q. This is again a linear price demand relation where Q is measured it is the cement being sold. So, cement is sold in units let us say it is metric tons and P is a price in rupees per metric ton. So, P is the price per metric ton of the cement. So, unit for the quantity is metric tons and the price is per metric ton. So, this is a relation.

Now, let us suppose we change this units from metric tons to something else. So, here is a change suppose we change the units.

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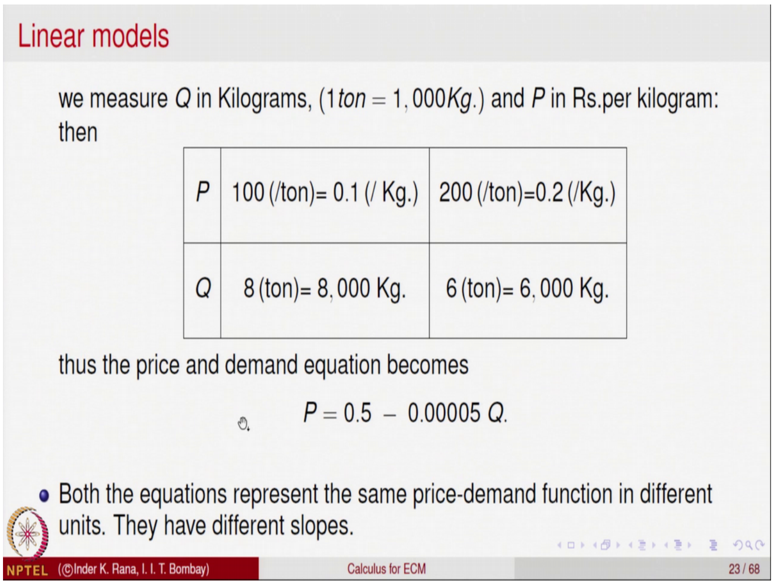

So, we measure Q instead of metric tons we measure in kilograms. So, we in case you do not remember 1 ton is equal to 1000 kg and the price is measured in per kilogram. So, we want to have a relation between P and Q, where P is in price per kilogram and Q is in per kg.

So, let us see the conversion. So, when P is 100 per ton. So, it is point 1 k rupees per kg for 200 ton it is point 2 kg 8 it will be. So, Q quantity it is 8 ton it is 800 kg and for example, 6 ton it will be 6000 kg because 1 ton is equal to 1000 kg. So, once you do that we convert the relation. So, thus the price and demand equation becomes P is equal to 0.5 minus 0.00005 Q, here Q is in kilograms and P is price per kilogram. So, this is what the equation becomes.

And now you see the slop here is changed to this because the units have change the slop has changed. Now, both the equation represents the same price and demand function this also represent the same price and demand, the original also represents the same price and demand, but in different units. That means, now the slop is different. So, what I am trying to emphasize is that for linear function slop cannot be taken as the measure of responsiveness, how the price and demand will change if one is change another is observed.

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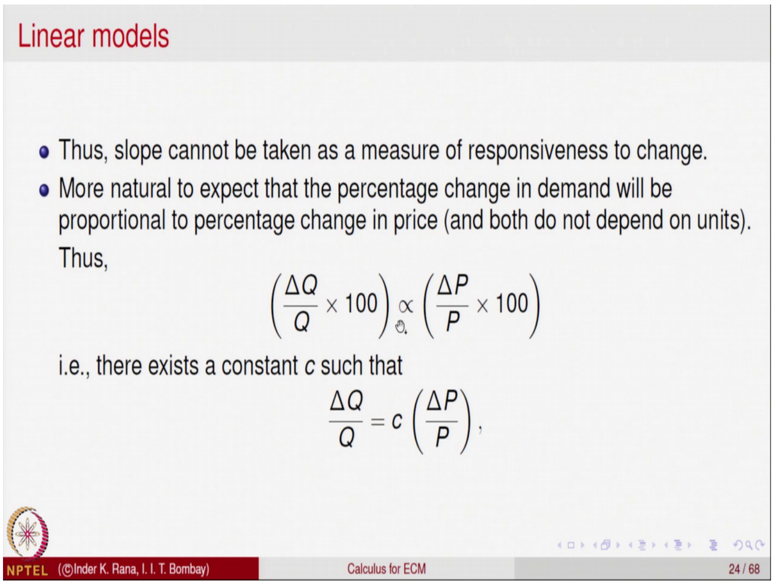

So, if slop is not the measure of responsiveness for change what could be? So, is more natural to expect that the percentage change in demand will be proportional to the percentage in price. In that case there is a unit is gone from the scenario.

Once if you look at the percentage change in demand a price changes from P 1 to P 2 and we could see the percentage demand percentage change in the price and correspondingly compute the percentage change in the price of the demand. So, then the 2 are independent the 2 quantities percentage does not depend upon the units, and is more natural to assume that in a price demand relationship the percentage change in demand is the proportional to the percentage change in price. So, this is represented mathematically you are saying delta Q by Q right, if at a price Q delta Q is the change right price is change by. So, delta Q by Q is the proportional to change right. So, when the units will cancel out each other. So, claim is that this should be proportional to delta P by P multiplied by 100.

So, this is the percentage change in the price, this is the percentage change in the demand and we are putting a hypothesis that this should be so, this should be proportional to this. So that means what, so that means we can, if you want to remove this proportionality; that means, mathematics says the proportionality constant of proportionality if it is c then delta Q by Q is equal to delta P by P then that relation will hold for that constant c and; that means, we can calculate c from here. So, c is equal to delta Q by Q divided by delta

Q by P. So, that is equal to delta Q by delta P right and this goes up, into P by Q. So, the constant of proportionality is the change in Q the change in demand divided by the change in price multiplied the price at that stage and the demand at that point.

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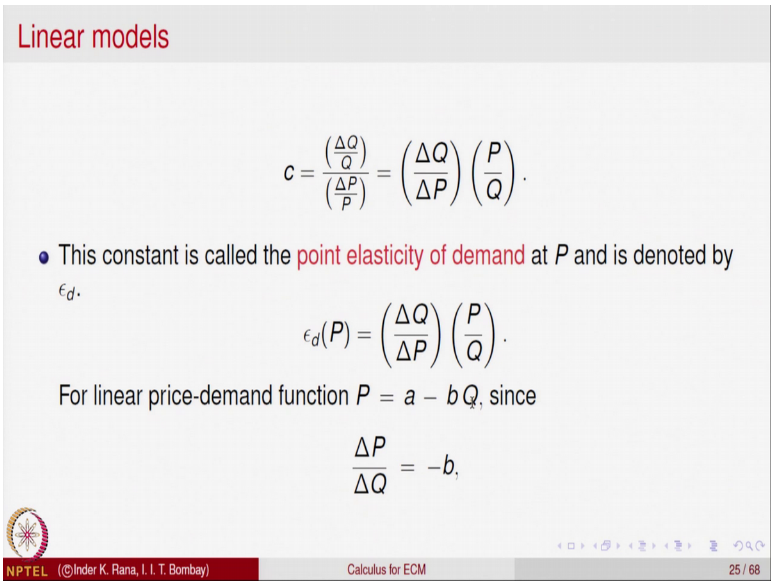

This is the constant which is independent of units this constant is called the point elasticity of demand at the price P. So, this number c which is given by at a price P we know the demand price demand function we know Q and if there is a change delta Q will give you the change delta P. So, this is called the cause point elasticity of demand at the point P and is denoted by the Greek letter epsilon with a lower d that may epsilon indicates the elasticity and d denotes the demand. So, this is the price elasticity of demand defined by delta Q by delta P into P by Q. So, for linear function if the demand function, price demand function looks like P is equal to minus a minus b Q right, so that is the price demand function then we know that delta P by delta Q is nothing but minus b that is a slop actually.

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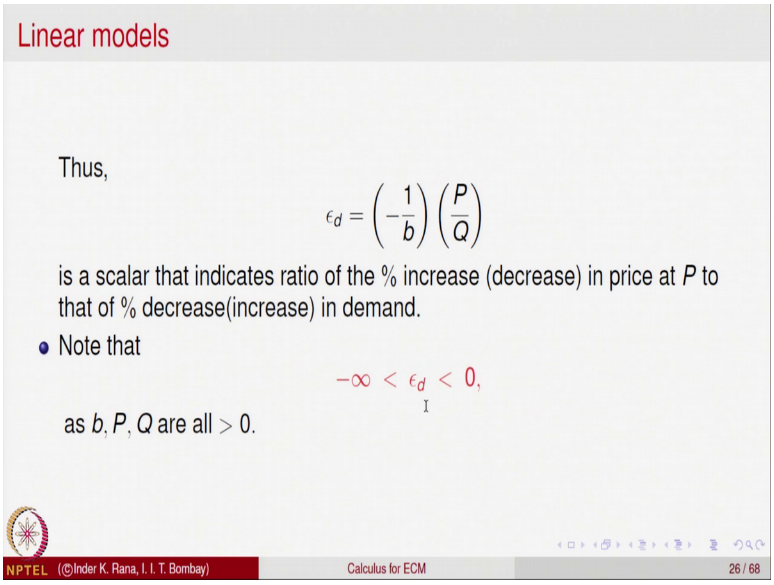

So, that gives for a linear function the coefficient of elasticity is nothing, but minus 1 over b P by Q right, which is a scalar that indicates ratio of percentage increase or decrease in price at price P right.

So, that indicates that if there is, this is the ratio of the percentage increase or decrease at a price P to the percentage and decrease in demand at that price. So, this is the coefficient of elasticity or a linear model of price and demand linear model. So, where the price and demand is this P is equal to a minus b Q right. So, let us make a observation that this number P is positive price is always positive quantity is demanded is positive the number b in the equation is positive. So, when the linear equation demand and supply equation b is positive. So, all this quantity being positive epsilon d is a negative quantity, so it lies between minus infinity and 0. So, this is only says that epsilon d coefficient of constant of elasticity is always negative. So, depending upon what is a value of epsilon d is it will.

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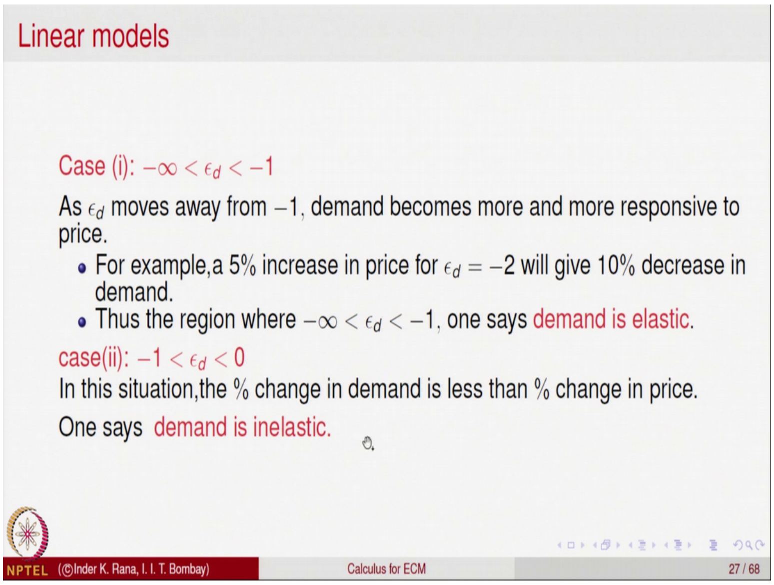

So, we will let us just look at a few cases let us look at case 1 when epsilon d is between minus infinity and minus 1 it is negative.

So, as epsilon d moves away from minus 1 what will happen? As it moves away the demand becomes more and more responsive to the price right because it is equal to delta d by. So, you have to keep in mind what is the coefficient of elasticity it is minus 1 over b P by Q. So, as it becomes more and more towards minus 1; that means what; that means, b is fixed right if it has to become towards more and more negative quantity; that means, Q should become smaller than P. So, demand, so that indicates that Q should become bigger than P at a rate much faster. So, it becomes demand becomes more and more responsive to the price. So, for example, 5 percent increase in a price at minus 2 will be will give you 10 percent decrease in the demand, because if have this is 2 so that ratio is equal to minus 2, so it is double.

Thus the region where in minus 1 to 1 one says demand is very elastic it is more responsive, so one says demand is very elastic. When it is between minus 1 and 0 this is a situation when the price change is less than percentage change in the price because it is between minus 1 and 0. So that means, the demand is inelastic it does not really effect the price much.

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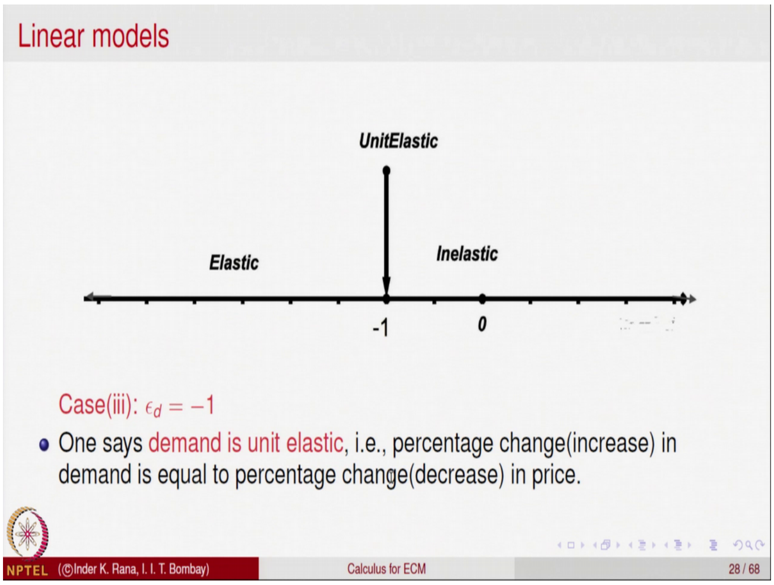

So, this is the situation when it is on this side right, then it is very responsive and it is not responsive at all this is called the this point equal to minus 1 is called the unit elastic. Finally, when it is equal to minus 1 as we had said it is unit elastic change in price.

So, for a linear model of price and demand we have try to define a notion called coefficient of elasticity or constant of elasticity for price and demand. And as it turns out for any linear demand and supply function this is always a negative quantity, so when it the value is less than minus 1 it is very elastic, when it is between minus 1 and 0 it is highly in elastic and at the minus 1 it is unit elastic.

So, we will continue the study of such things more in the next lecture.