**Calculus for Economics, Commerce and Management**
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**Lecture – 01**
**Introduction to the course**

Hello, my name is Professor Inder Kumar Rana. I am a meritus fellow at IIT Bombay in the department of mathematics. Welcome to this course called calculus for economics commerce and management, students. I will give you first a brief outline about the course, and then we will start discussing the main topic namely calculus and it is applications.

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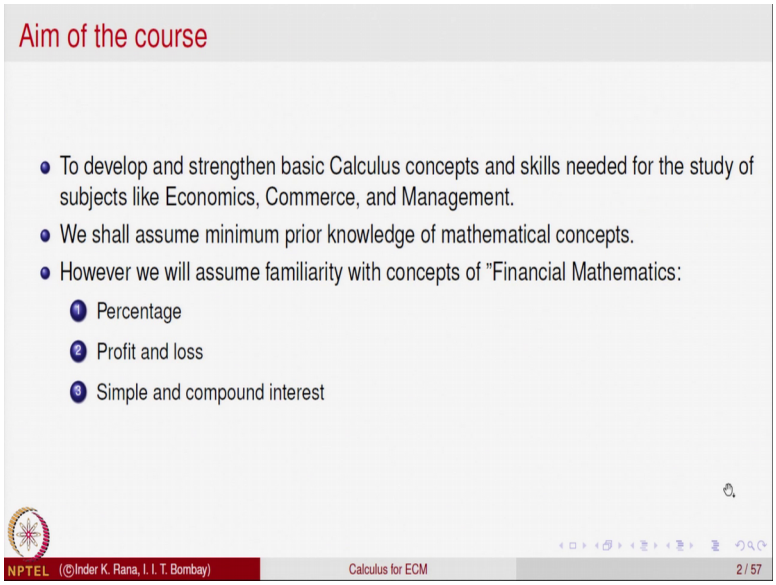


So, the aim of the course basically we would like to develop and strengthen the basic calculus concepts and skills that are needed for the study of subjects like economics commerce and management. We shall assume very little knowledge of the mathematical concepts, because many of you might have left mathematics as a subject after tenth standard onwards, and might not have studied mathematics in the later courses in your college. So, I will try to define basic concepts as an when required and some of the concept probably leave it for you to read and revise.

So, that you follow the course comfortably and learn the techniques; however, there are as a few things as I said we will assume that you are familiar with the basic concepts of

financial mathematics, namely the percentage the what is the concept of percentage, how it is calculated what it means and so on. We will also assume your knowledge about profit and loss, when things are bought and sold there is a profit there is a loss, how much percent profit how much percentage of loss and so on other things.

Then we will also assume your knowledge about simple and compound interest, because many of the our examples will deal with these concepts. And these are very commonly used in economics commerce and management. So, these concepts we will assume that you know it very well if you have forgotten these things, I will strongly say as that you look up a book of say mathematics of standard 10 or so and revise it.

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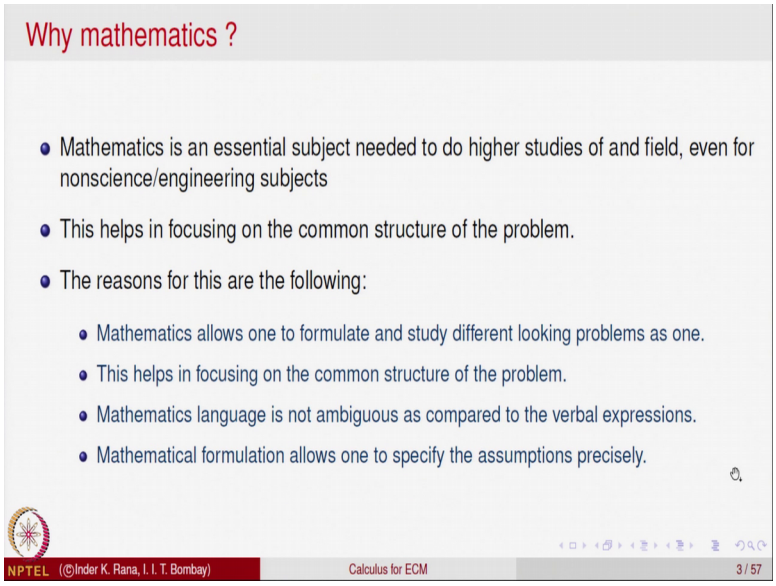


Mathematics one can wonder as students of economics commerce and management, why mathematics is required for higher studies in non-science subjects and non-engineering subjects. Yes, I agree that long back may be around 20 years back, one thought mathematics is not required by non-engineers and nonscientists. But now there is hardly any field in our day today life where mathematics is not used is not useful.

And much more so, a in economics commerce and management. So, I will try to give you a brief reason why mathematics is required and is important. So, first of all mathematics helps in focusing on the common structure of the problem, the problem may be arising in economics, it may be arising in physics, it may be arising in commerce, it may be arising in mathematics itself. There is a common structure behind all the

problems, and so one can forget the context where this problems arise solve them mathematically without any knowledge without any concepts of the other fields. And the reason for that are the following mathematics allows one to formulate and study different looking problems as one. So, one study is the mathematic problem and then applies it to the various fields will give you a examples later.

I am just trying to give you an broader view of the topic. This help in focusing on the common structure of the problem what is the problem, and how does one analyze it. And mathematics language is not ambiguous as compared to the verbal expressions. In mathematics an expression as one and only one meaning, it cannot be 2 different meanings; that means, a mathematical expression. If it is read and interpreted it can be only one interpretation possible for it. So, that is there is no ambiguity in interpreting. And in mathematics problems one can specify precisely the conditions under which the mathematical problem is going to be solved. So, the assumptions are very clear in mathematics, they do not vary as you try to analyze the problem.

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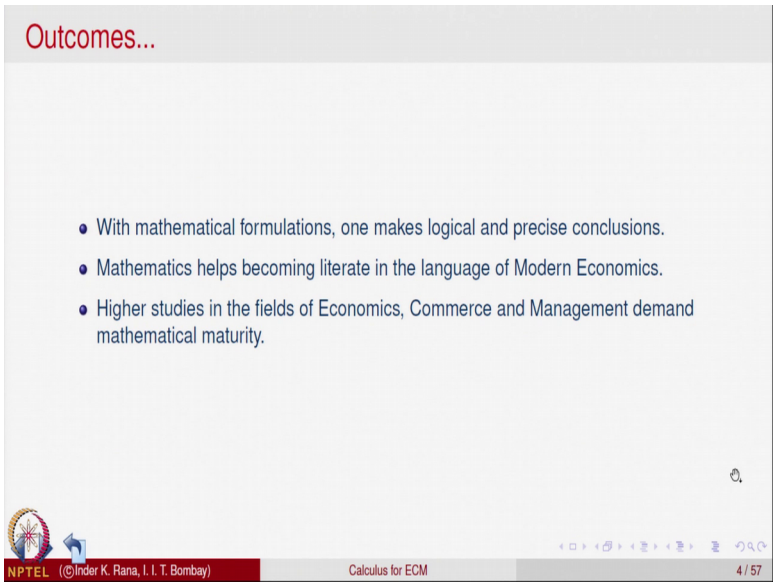


So, and with mathematical formulations once the problem is stated, the conditions are specified you make very logical conclusions, and very precise deductions can be made.

So, if some conditions are satisfied if that is a problem, then this will be the conclusion. So, that is how mathematics works and of course, as I said mathematics helps you to become literate in the language of any higher studies more. So, in modern economics

commerce and management. So, all this fields a non-even the non-science is and non-engineering fields require the mathematical maturity if you want to do some higher studies. How mathematics is really used? So, let me give you a brief outline. So, step one one observes the real word phenomena and collects data. The problem could be arising in economics could be arising in management, could be arising in physics, could be arising in engineering sciences anywhere. So, something that you want to analyze you observe that phenomena and collect the data. For example, on a roads some accidents are happening.

So, you collect the data how many accidents are happening whether they are happening in the morning or in the evening, and you one would like to find out the reasons why such things are happening.

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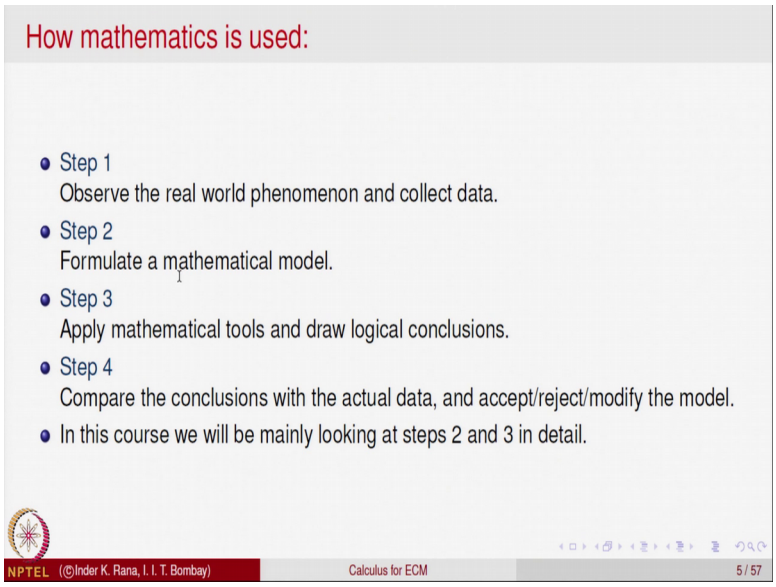


So, one collects a data about the number of accidents happening. So, this is what you observe an experiment you observe a phenomena and collect the data from that. Once a data is collected you thought to formulate a mathematical model out of it. For example, problem of accident happening on a road it could be that, there is no speed limit fixed on the road. And so, one need to formulate what could be a reasonable speed limit that should be fixed. So, that it will depend upon on the data as for example, how many whether the road is being used by heavy traffic or light traffic, what is the size of a heavy traffic truck.

And what should be the distance between 2 cars, and if they are with the speed limit then how much distance should be there between different vehicles. So, that if someone breaks, then it should not go and hit the next one. So, that kind of for mathematical formulation is required for example, how much is the breaking distance, how much is the time required to stop a car before hitting it next car sort of. So, this is the kind of mathematical formulation one has to do for example, it could be problem in predicting forecast of rains in a particular country in a particular city. So, one would like to collect the data over the year, what is the temperature what is the pressure and so on. And then one tries to formulate some mathematical equations a model which tries to say that if these are the conditions these are the weather conditions these are the temperature condition these are the wind directions and so on.

Then one tries to solve a problem. And similar things happen in almost every field. So, this is formulating those mathematical problem, which will help us to make some deductions. Apply the mathematical tools and draw the logical conclusions. So, once the data is collected and you have formulated the mathematical model put the conditions, you apply the mathematical tools to solve that problem, make the logical conclusions. Once the conclusions are available you compare the conclusions the actual data that you have to collected on the step one, whether your conclusions match with your collection of data you are observe actual observations if they match, probably your model is correct and you can generalize it. If not, you have to tweak the model a bit more and do it.

So, this is how mathematical process is this is what is called the process of mathematical modeling is about. So, in this course we will not be dealing with all this mathematical modeling, we will be seeing how in situations of economics commerce and management some problems can be formulated as mathematical problems, how mathematical tools can be applied and deductions can be made. So, mainly we will be concerned with 2 and 3 step 2 and 3, and that more precisely how calculus tools are used in in applying to a mathematical model and making conclusions. So, that is mainly going to be the crux of the course right.

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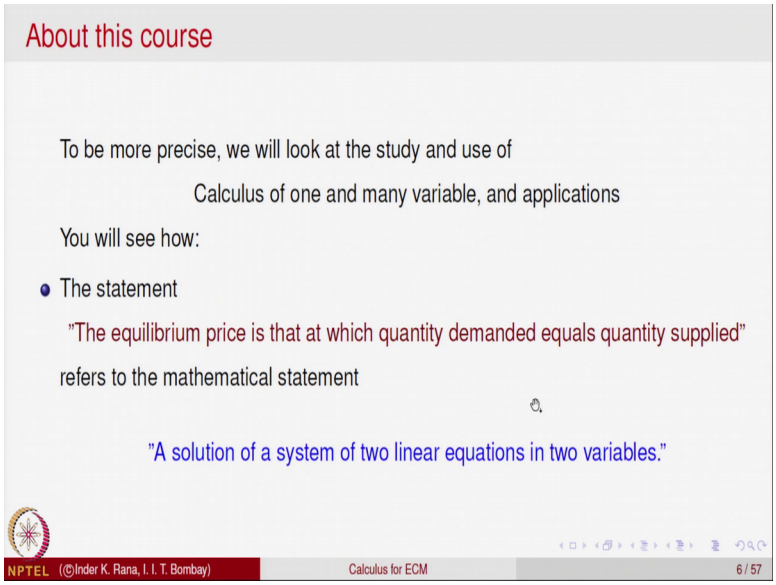


So, about this course for example, let me also say for example, how the calculus tools would be used. So, to be more precise let us look at calculus and app it is applications.

In our field of say economics. You will see how there is a statement you might have come across in your course is an economics the equilibrium price is at which quantities demanded equal to the quantity supplied. So, that is what is called equilibrium in economic models. And this refers to the mathematical statement that a solution of a system of 2 linear equations in 2 variables. So, that is how the mathematical problem will arise out of which and will be can be solved.

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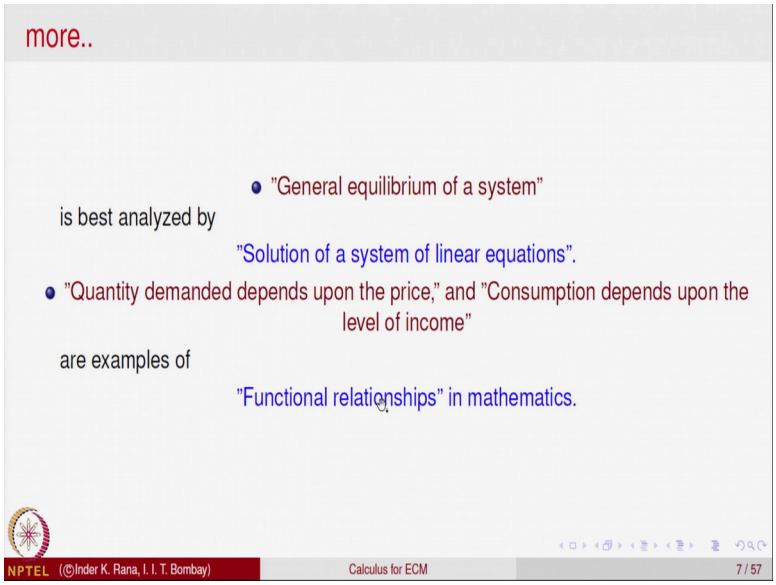


For another problem that you can look up is the following say general equilibrium of a system, what does that mean. So, mathematically this is analyzed by solution solving a system of linear equations. So, this goes in the subject of applications of linear algebra to economic models. For example, there could be quantitative demanded depends on the price and the consumption depends upon the level of income.

So, this is the common assumptions in economic models. So, these are examples of functional relationships in mathematics. So, there is a relation of function which says the quantity demanded is a function of the price, or the consumption depends on the level of income. So, is a function of the income.

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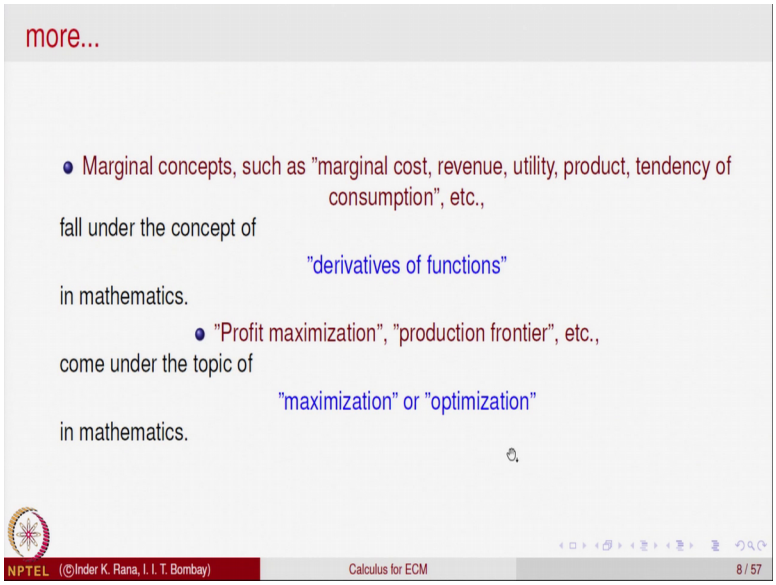


So, the functional relationships describe these things very well. Marginal concepts such as the marginal cost revenue utility product, tendency of consumption etcetera, all this all under the concept of derivative of functions in mathematics and their applications. And profit maximization production frontier etcetera, these are the topics these are the problems in economics, that are best on analyze under the maximization or optimization in calculus problems using calculus and so on. We will discuss these things in detail soon slowly as we progress the course, I just wanted to give a snapshot of how mathematics is going to be used in economics commerce and management. To get maximum benefit from this course here are my suggestions that be interactive do not just be a passive observer of lectures of, because this is a offline course.

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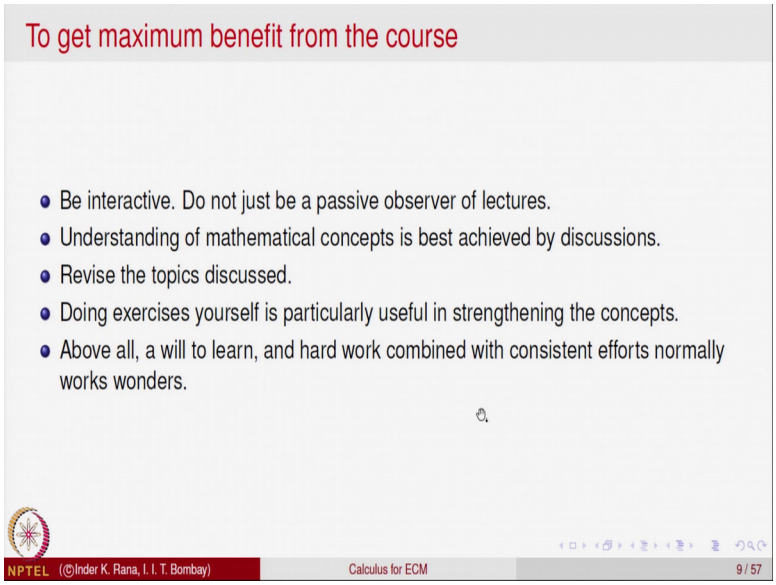


You will be given the videos for this course every week, you will listen to the videos. So, from these are not going to be face to face lectures. These are going to be video lectures, were we will be listening to lectures, trying to understand the concepts. And then so, there is a lot of responsibility on your shoulders that you have to be interactive, when something is not clear you have to probably put a question on the forum saying what is that concept about or try to analyze yourself look up the inter web and try to find out the answers. Most likely you will not have such problems, but you will have to interactive. Understanding of mathematical concept is the best achieved by discussions.

If possible try to form some groups if 2 or 3 of you are taking this course together, it is best that you form a group where you watch the video videos lecture videos together, and try to discuss with each other the topics concerned. Because we are found students learn best or even everybody learns best when there is a discussion on the topic or the problem under way. So, and of course, this is we keep on telling our students, that revise the concepts revise the topics discuss. Do not postpone it. And doing the exercises yourself is very useful thing in strengthening concepts. Whenever you listen to a lecture see a concept doing discuss, exercises are meant for you to revise and strengthen how you much you have understood that concept.

So, do the exercises yourself, that is a very and it works wonders once you get the hang of doing exercises yourself, it will help you greatly, right. This course is going to last for about 8 weeks.

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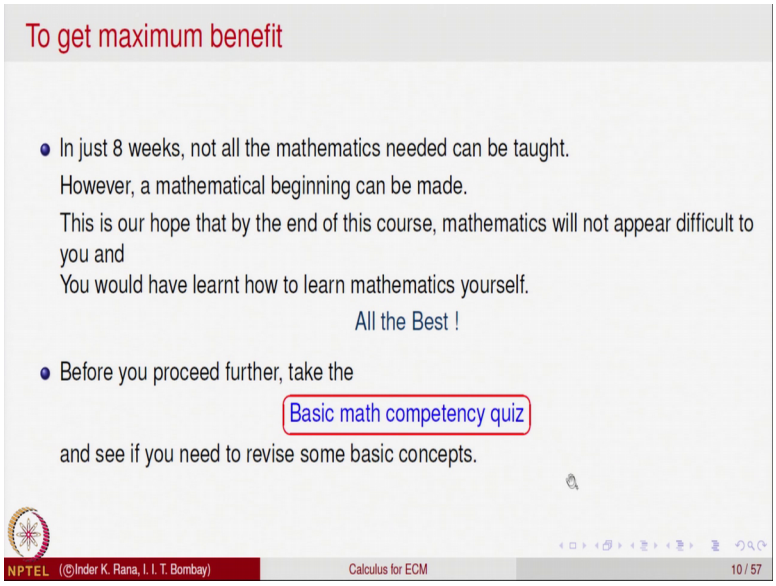


So, not all the mathematics needed or can be taught in this 8 weeks, time is short. So, however, will give you enough material to get started on trying to use mathematics to learn and to apply in your topics. So, a good mathematical beginning can be made and that is my hope. So, this is our hope and at the end of the course mathematics will not appear to be as difficult as you probably might have thought. So, and you are learnt at least how to learn mathematics yourself. So, all the best for this course. And before you go out to the next topic I strongly suggest you will be given an assignment about a quiz called basic mathematical competency quiz.

So, have a look at that quiz and try to see the solve the problems or solve the questions in that, and see whether your comfortable with the basic mathematical concepts. This quiz is more about the financial mathematics that I had talked about and if you find you are not very comfortable at some of the concepts in that. So, strongly suggest that you revise that. So, this is about the general background of the course what we are going to do and so on.

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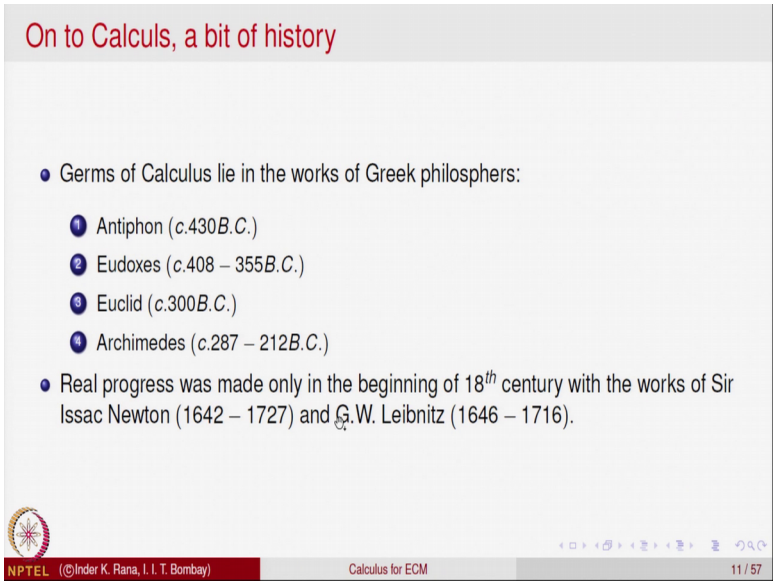


So, let us begin with our main topic for this course namely called calculus. So, I will start with a bit of history about this topic called calculus. It has a history of around 2000 years. So, germs of calculus lie in the work of the Greek philosophers.

Some of them I will name antiphon around 430 years before Christ eudoxes around 408 to 355 before Christ, and then Euclid around 300 B.C. and of course, Archimedes around 287 to 212 B.C. these are some of the philo Greek philosophers of course, they contributed a lot to lot of branches of mathematics physics and so on, but as far as our course our subject calculus is concerned these are some of the people who contributed for calculus. Let us move over to you will see that from around 200 B.C. to around 1642 no real progress was made as far as calculus is concerned. The real progress started only after the works of sir Isaac newton, his period is 1642 to 1727 and G.W. Leibnitz around 1646 and 1716.

So, both is people initiated what the modern calculus, they can be thought of as the inventors of modern calculus. Let me point out that Isaac newton was more or less a trained mathematician. While, Leibnitz was a non-mathematician. Leibnitz was is something like ambassador from France to France to U.K. in one of the diplomatic parties he met physicist Huygens you some of you might have heard the name Huygens a physicist. And while talking to in the party he got interested in mathematics, and that is a beginning of his mathematical career you can say, and Leibnitz gave his ideas gave birth

to calculus as we see it today. So, Leibnitz and newton are both credited with the invention of calculus. Newton had work on calculus, but never he did not publishes his work, while Leibnitz worked and published.

So, when Leibnitz published newton said. So, he has discovered this calculus 7 years back and so on. So, a great controversies started between newton and Leibnitz who is the real originator of calculus.

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And this controversy went on for about 6 years, there is a nice history about taking credit or giving credit who discovered it.

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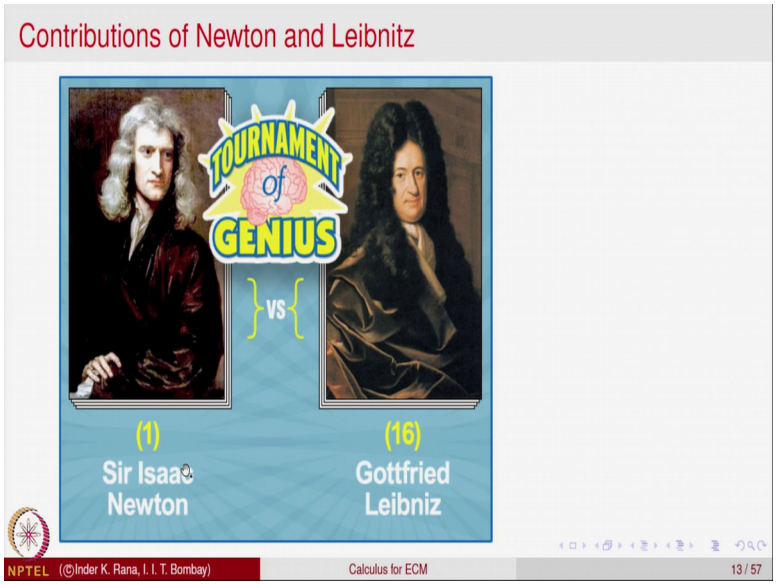


And so, there was a kind of a wrestling between Isaac newton and Leibnitz about who discovered calculus. But eventually both of were given the equal credit, and they are both added slightly different approaches, but leading to the same concepts in calculus.

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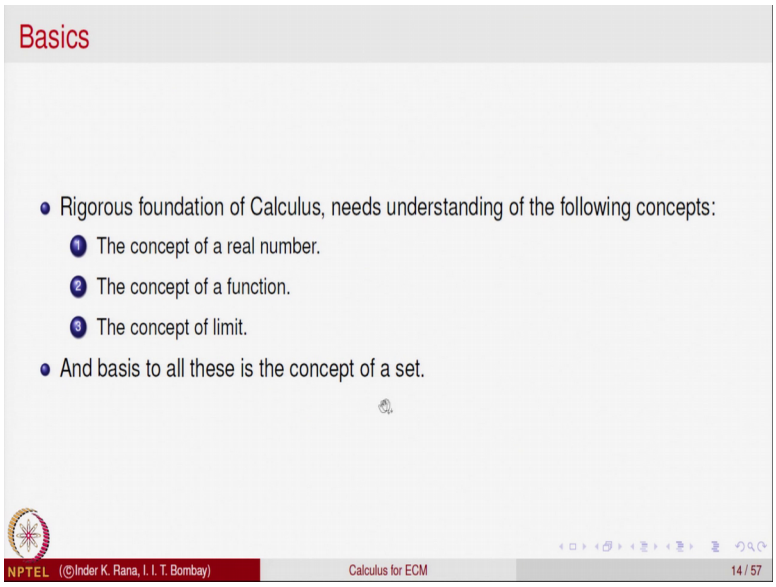


So, after their work it was discovered that though there were lot of ideas in the works of newton and Leibnitz, a rigorous foundation of calculus if it is to be done that needs understanding of the following concepts.

Namely what is the concept of a real number, in 18th century beginning of 18th century what is a real number was not clear, it was not mathematically defined not mathematically very well understood. There was the concept or a function that also was not very clear, what is a function mathematically. And the concept of limit all these 3 were basic foundational for the rigorous foundation of calculus. So, at least the works of newton and Leibnitz led to realizing what are the problems which we had to tackle. So, and to all of this there is a basic, namely the basics of all this is the concept of set and set theory. So, what we will do is we will start with the concept of set, and set theory. Basically, will not go in detail about sets and set theory will what is a minimal thing required for our calculus course. Then we will go over to the concept of real number, and describe properties of real number.

Whatever we require then we will go over to the concept of functions and then limits and then to other concepts in calculus. So, this is a route we are going to follow for our course. So, let us begin with the concept of set theory. So, why sets are required and what is a set.

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So, let us begin with the observing that in everyday life we often group objects as a single entity depending on a common sort of attribute or a property they have. And so, that things become manageable. So, let me give you some examples about this. For example, we have box of sweets. There is a box in which there are sweets of different

types. So, to manage this different type of sweets may be to gift to somebody and so on, we pack them in a nice box.

So, this is a box of sweets. So, different things for this common attribute is they are all sweets and edible. So, that is next let us look at for example, something similar a bag of toffees.

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Toffees of different types are packed in a sort of a bag. So, the bag has a collection of objects namely toffees. And they have common attribute say, they have may or may not have a common attribute in general, but here they had do have students in a class.

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So, they are all could be boys could be girl's students in a class they have grouped in a class, because we want to manage who will come and teach that collection of students, what type they will study what topic and so on. So, a students in a class that is the collection and put in a box called the class.

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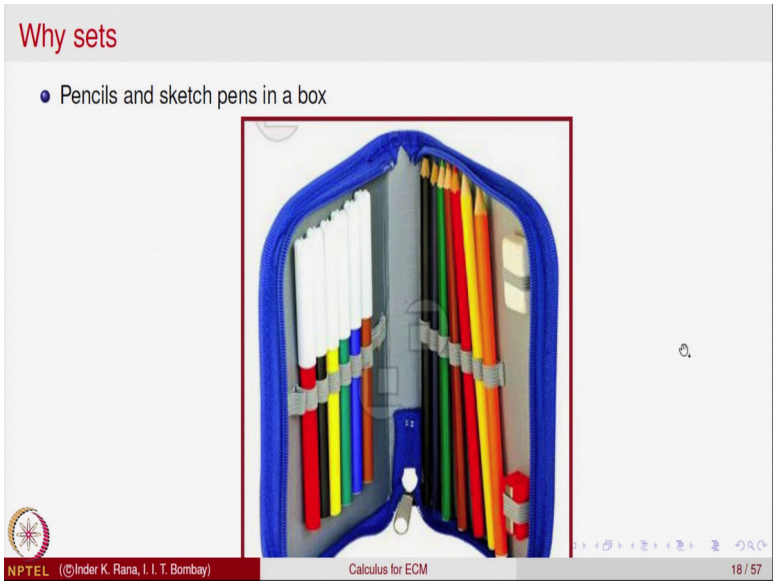


Sketch of pens and pencils that is a another collection. So, this is a box containing sketch pens and pencils. A box full of shirts of different right cloths. So, that is a bag having.

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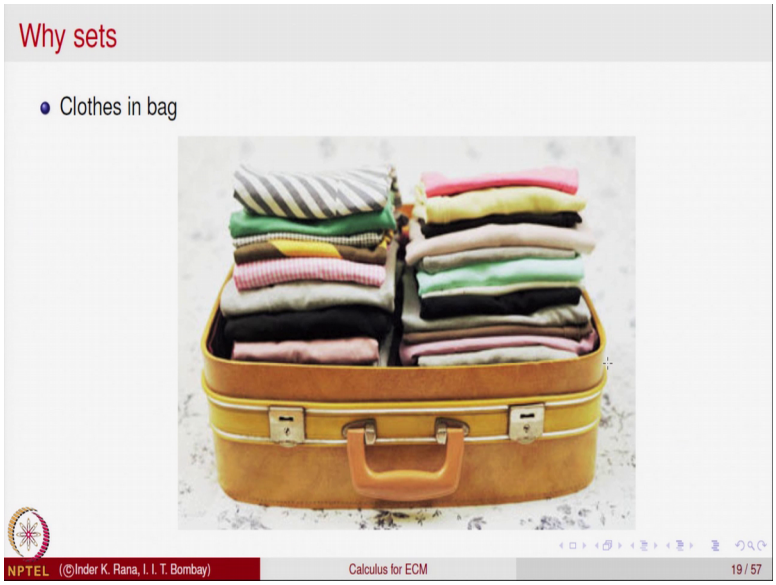


So, this is a collection of objects put inside.

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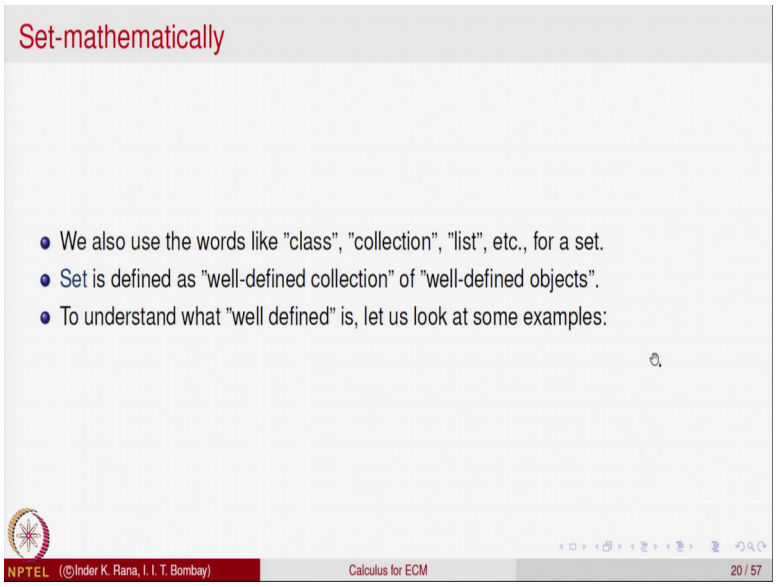


So, we are interested in finding sort of analyzing or what should be called as the class collection or a list of objects in a in a college in a coll. So, a formally we define a set is a is a well-defined collection of well-defined objects. So, a well-defined collection or well-defined objects that means, it is a collection which is well defined, and there are objects in it which are also well defined. To understand the what well defined means, we will look at some examples. So, our basic thing is we are looking at a concept called

collection of objects, and we are trying to understand, what is well defined collection and what is well defined object mean? So, we will do it in the next part of the lecture.