Mathematics in India: From Vedic Period To Modern Times
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Lecture-8
Aryabhatiya of Aryabhata-Part 2

So this is the second part of our lecture on Aryabhatiya, so in the earlier part so we saw so the algorithm presented by Aryabhatta for extracting square root, cube root and so on.

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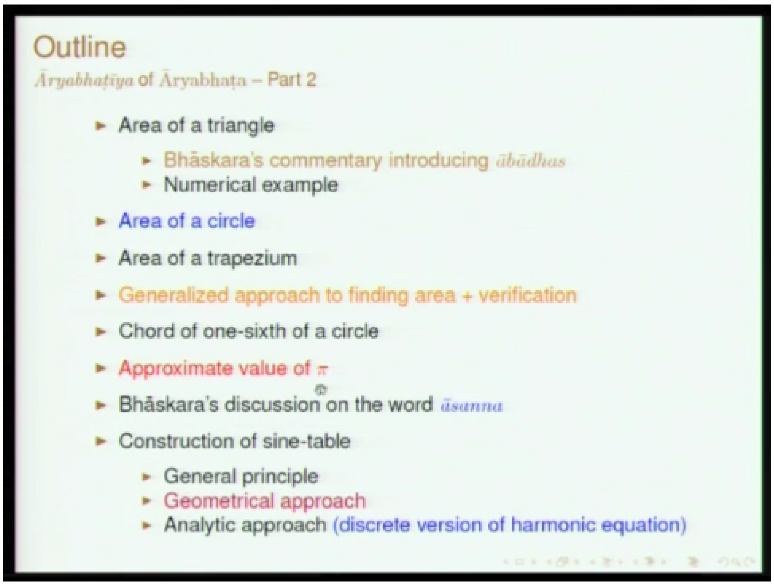

So in this lecture we will be starting with the formula given by Aryabhatta of finding the area of various geometrical objects. So starting with triangle then proceeding towards trapezium circle and so on. So in the latter part, so we will see the approximation which has been given by Aryabhatta for the value of Pie and the method which has been by him for generating the sin table. So in fact Aryabhatta has given 2 different approaches for constructing sin table.

So one is the geometrical approach, the other is the analytic approach. So we will see both of them and then during our discussion on the verse which gives the value of Pi, so we also see an interesting note which has been given by the commentator Bhaskara on the use of the word asana. So this has been discussed at great length given by Nilakantha Somayaji which will be covered later.

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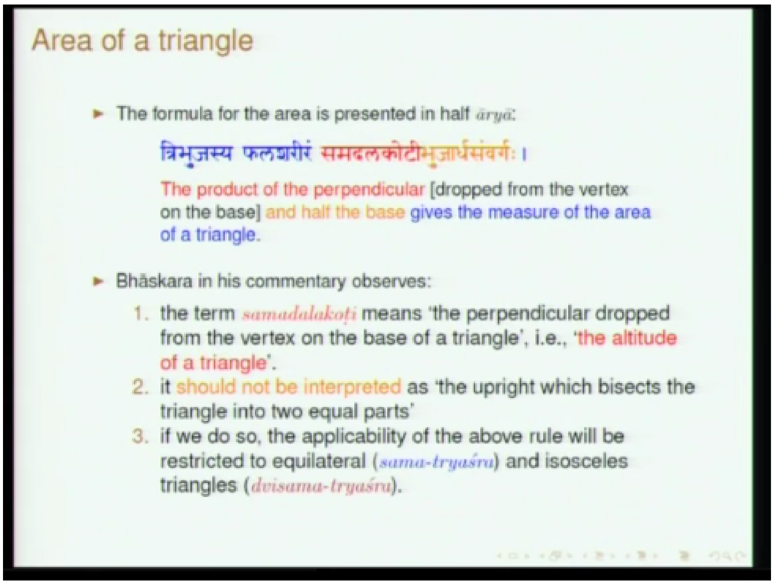

But here we will see the note which has been given by Bhaskara on used the word asana. So let us start with the verse which presents the formula for the area of a triangle. So Aryabhatta says (FL) so normally he say half into this into height. So it is essentially same expression which has been presented (FL) is the word which has been employed by people to refer to the area. So here the word (FL) should be understood as (FL) the measure of the area.

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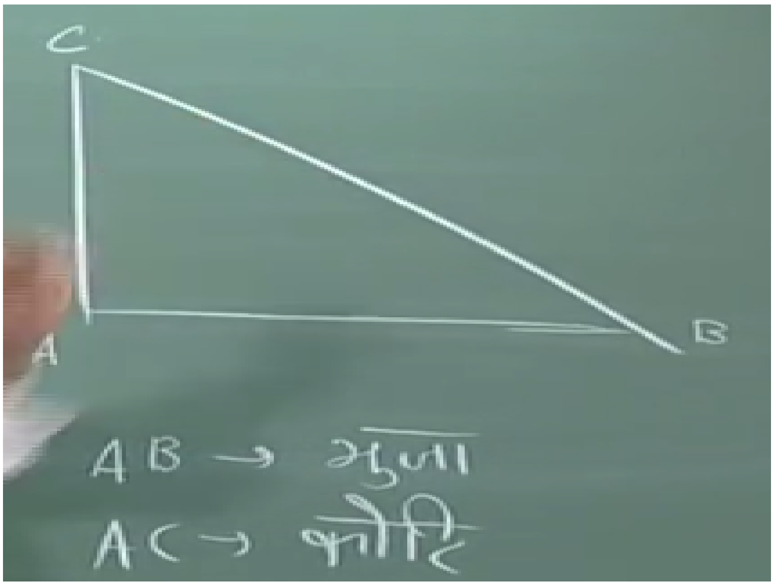

(FL) so normally we are familiar with half into base into height, so here this (FL) can be taken to be this site is base and (FL) is the height ok. So in fact in the word (FL) has been analysed by the commentator Bhaskara, see in a triangle usually if you have a right angle triangle so let us say ABC, so if AB is (FL), then AC is (FL), and of course this is called karna. So if this (FL) can we interchange dividing up on the size of the angle.

So one is (FL) and the third is karna, now if we consider a triangle, so this is the part which we are calling as (FL). So (FL) and this is (FL) so this is the base, this is height half of it base is the area of the triangle. In fact the word gala is generally used to refer to half splitting ok. So (FL) means so (FL) so generally is taken to be equal division ok, so when you thought of divide so this is how the meaning as such comes from that.

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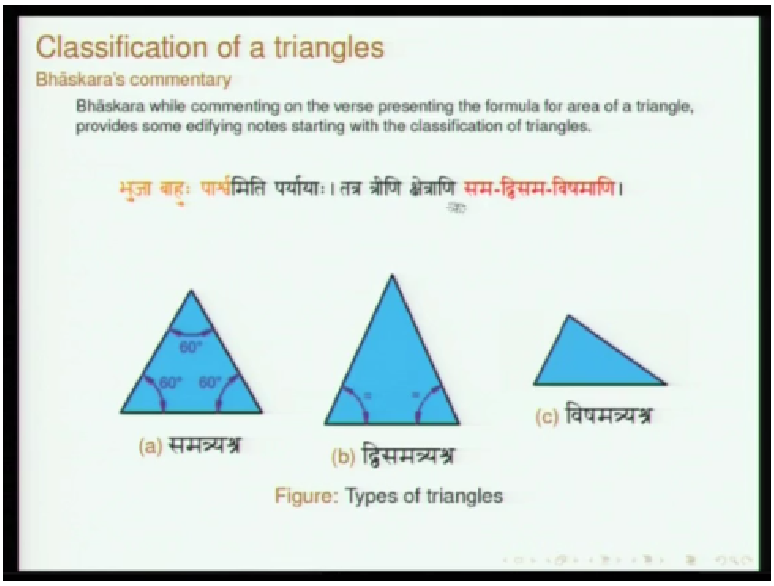

But then if we use this then if not be applicable for all the triangles in fact Bhaskara, so comment is discussion by classifying the triangles into three types, he says (FL) they are all synonyms and there are 3 type of triangles.

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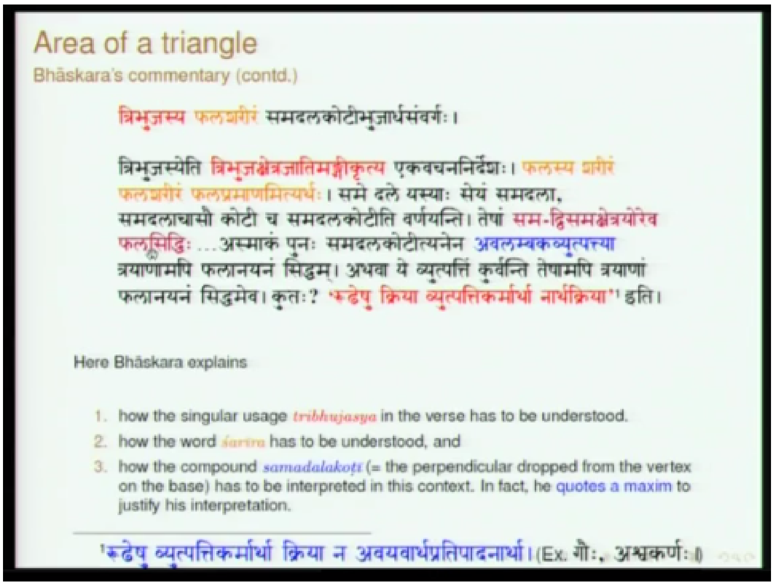

Further continuing the discussion he says (FL) in fact we should have 3 types of triangles (FL) this is applicable to all the triangle or is it something which is specific to one of the three types or any of the 3 types. So he says this (FL) means so the group of all the class, so it is applicable across the class of the triangles (FL) so then he goes on to discuss about the word (FL). So if you say (FL) so if you use this kind of derivation of the word.

Then it is confined to only these two types ok. So (FL) so if you draw perpendicular then the perpendicular device the base into two equal half only in these two cases, (FL) scaling type. But this formula which has been presented for area is applicable to all types, then how do you understand the word (FL). So this is the question which has been raised by Bhaskar and he actually quotes very interesting matching I would say (FL).

So the word (FL) should be considered as a (FL), so (FL) in the sense, so if you not so try to find the derived meaning of the word and it has to be just for instance, so in the case of the word (FL) in Sanskrit, So (FL) is generally referred to as a goat. But if you go to the derived meaning that which is not known, so it never applicable to goat ok. So when you consider this (FL) so you just have to accept whatever is the meaning which has been accepted by the society at that point.

That is what he is trying to convey, so (FL) if at all you want to provide some kind of a (FL) is not for the purposes of (FL), but this is the very interesting matching, so which can be applied in various disciplines various contacts. Here the categorically states (FL) and therefore this derivation should not be taken in the literal sense here. So (FL) should be simply understood as the perpendicular that is dropped on the base from the vertex.,

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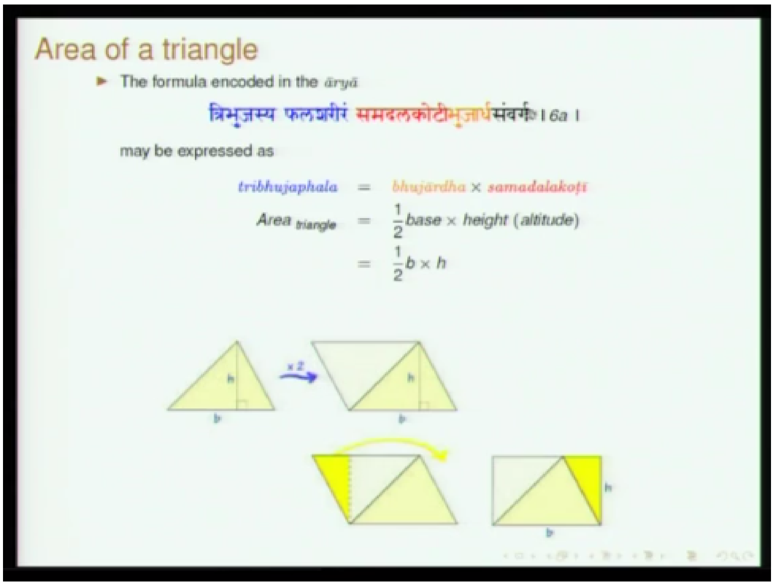

So this is what the sutra Aryabhatta sutra means (FL) so the area of the triangle is (FL) so half of the base into (FL) height, so this is all it is, so it is applicable to all the three types of triangles. So this verse basically tells you that the area can be calculated using this formula. But then if you do not know the height, so you only know the sides of the triangle, then how do you go about finding the area.

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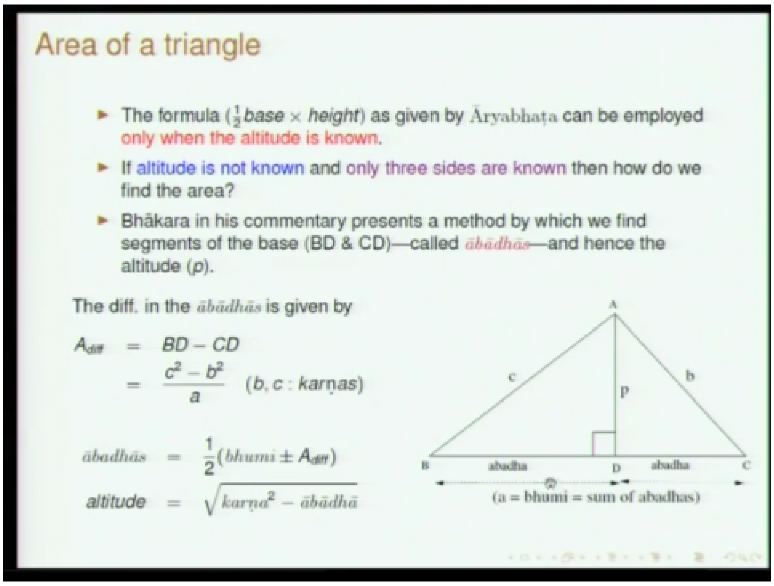

So then there should be a way to find out the height of the triangle. So that is what is discussed by Bhaskara in his commentary. So the word (FL) has been employed to refer to the two parts which are generated by dropping the perpendicular from the vertex, ok to drop the perpendicular which is referred to as (FL) so Bhaskar essentially gives a certain method by which you force to calculate the (FL).

And then from that you find out the (FL), so if you know only the sides, so you know the base, so that is taken for granted and for finding height so based on the 3 size for the presence of formula. This is how it goes, so first he gives an expression for the difference in the Abacus and the difference in (FL) so BD-CD, BD is (FL) CD is (FL), so the difference is c square-b square/Armstrong

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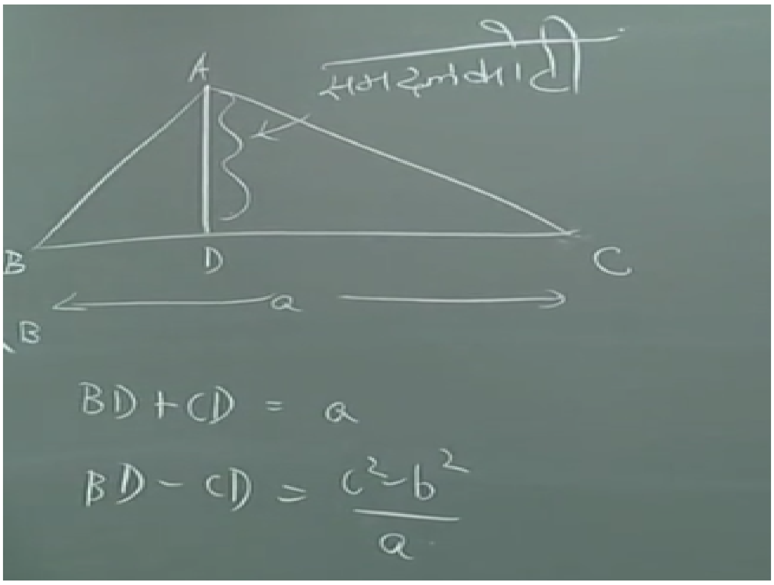

So this can be easily seen and we have an expression for the difference in (FL) in terms of the sides of this ok. So if you have this basically, so ABC, so this is B, so BD+CD=A which is known and BD-CD if given to be c square-b square in this. So once we have these 2 we have the expression for so just add this and subtract this. So you have the expression for the 2 (FL). So this is stated to be half of (FL) is the word which is used to refer to the base.

So (FL) difference so this gives you the expression for both the (FL), so having known this (FL) so it since you see you can calculate P either from this (FL) or from the other (FL) so this is basically (FL), so the karna can be either c or b depending upon either of the triangle that we choose. SO this is what ((FL) .

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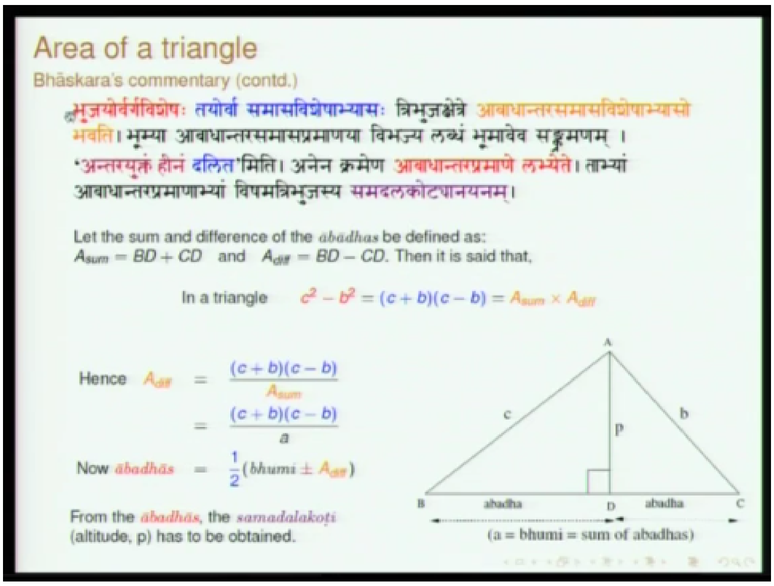

(FL) in fact is a very interesting to read the way he present this (FL) c square-b square, (FL) is putting together essentially means sum and (FL) is difference, so he says bsquare-c square+c=b and there is also equal to (FL) so the sum of the two (FL) in the difference to the 2 (FL). So this is straight away seen from the conservation of these two triangles ok. So you get this, so this is the first relation at he says.

So then so a from this expression we see that so a difference is so csquare-b square, so which what he stated in the earliest slide and therefore (FL) is this. So this is how Bhaskara proceeds. In fact the sides verse (FL) so whenever you have something of this form this is called (FL) and this is called (FL), so you have you have to just sub this then find the difference.

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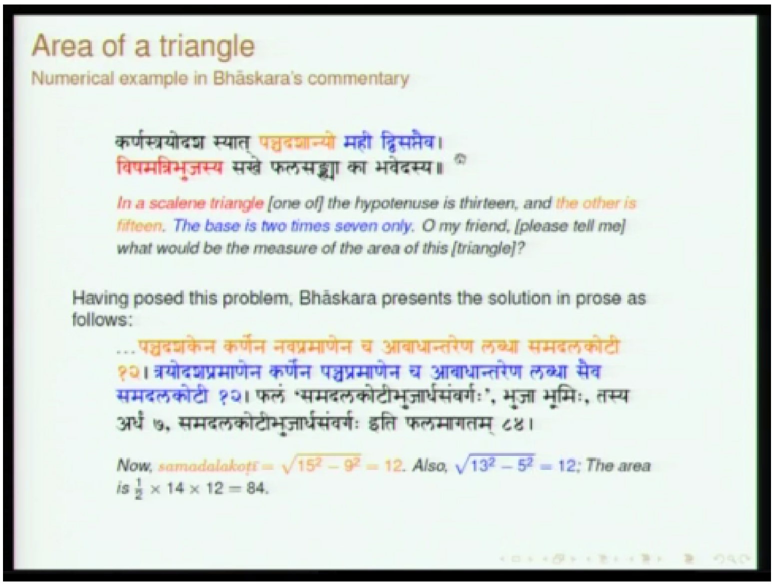

In fact this is how demonstrated with the formula example (FL) so this is the Bhaskara from several examples on various occasions wherever he has to illustrate the use of a particular formula. So I will skip this basically the presents how one finds the area of a triangle. So given the three sides and you have to calculate the height and from (FL) you have to get the areas.

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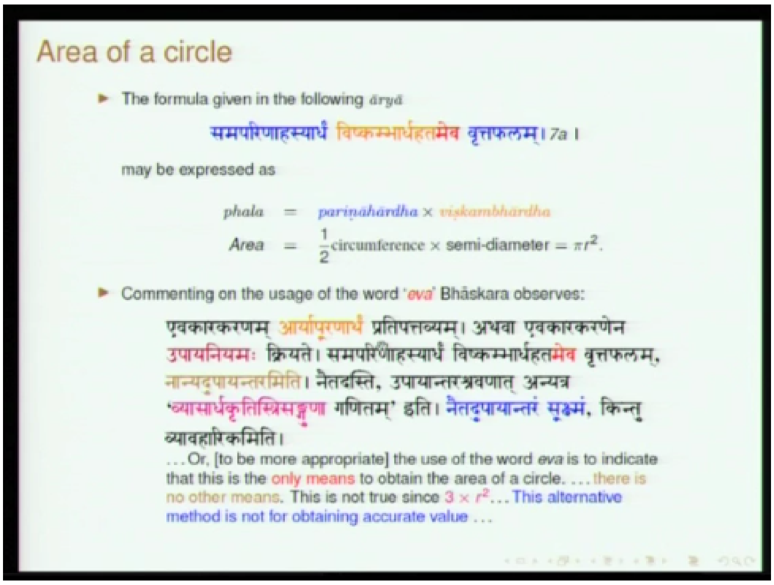

So now I move on to the area of circle, so which has been presented by Aryabhatta. This is the very interesting, so in fact it is half verse wherein he has used the word (FL) has lot of significant. (FL) refers to circumference, so (FL) is diameter, so (FL) is half of it, so (FL) so it refers to half the circumference. So the arc has to be absolutely significant thus far it means, so (FL) means multiply by the radius semi diameter.

So this is the (FL) fine, so this (FL) pi*r square fine, so (FL) commenting on the usage of the word (FL) so which is quite edifying, so he says (FL) so why did Aryabhatta used the word (FL) so this is meaningless or can be assign from sense to that. So this is the analysis which it has. (FL) taking a very simplistic one can say, so in certain cases so we have to fulfil the metrical compulsions.

And therefore so one add some word which need not be necessarily conveying something which is very meaningful. So this is one way of saying that (FL) has been used in just to feel the metre of the bus. So (FL) putting a certain constraint on that means which you can involve, so finding the area, so is there any other means **no** no there is no that you only this way. So that is the way this has to be understood (FL).

So (FL) there is no other means, so this is the only mean to obtain the exact area of a circle. So (FL) what should he say, so if there are different path you reach the place then you can say choose only this path, there is only one path then there is no point in saying that you have to go only with path. So then he stars he ask this question no no this does not seem to be appropriate (FL) that also means.

So what is (FL) so somewhere it stated so (FL) radius, (FL) is square , so (FL) r square multiplied by 3. So which means the value of pi is approximately getting to be 3 and for various practical purposes people having employing this. So in order to see that those things are not taken to be the right formula to be employed for finding the area of a triangle. So this (FL) Aryabhatta has given a certain guide line that this is the (FL) to obtain the exact area.

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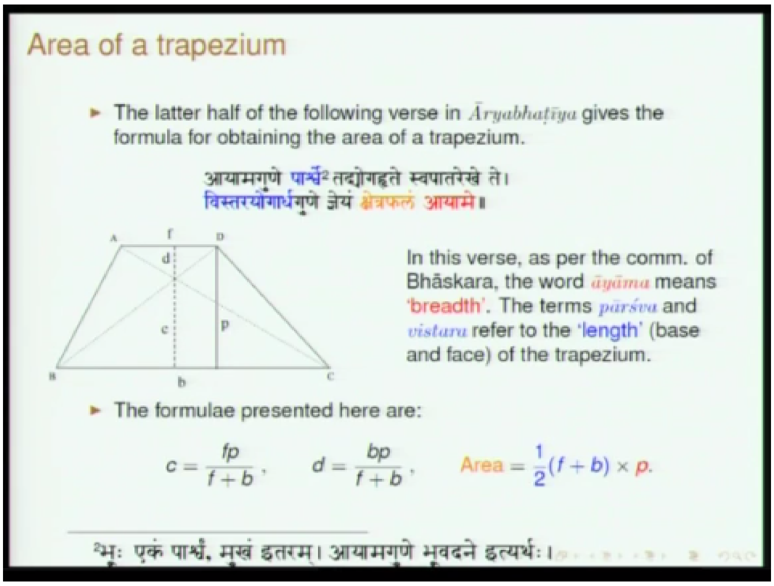

So (FL) where you do not require that kind of accuracy. So next we move on to the area of trapezium (FL) see in fact the latter half of the word actually presents the area of a trapezium and the earlier half of the verse has been used to define something else. (FL) is a certain geometrical figure, so that is the sense which has been used. This (FL) is the area of this khetra.

(FL) here is used to refer to the base and the face ok. In this figure BC is base and AD is the face, (FL) is multiplying, so (FL) what is the other term (FL) ok so (FL) is used to refer to this perpendicular distance the distance of separation between the base and the face okay. So the area of trapezium is halftime (FL). So this is the formula which is used, so what is the earlier part of the verse.

So here he says (FL) distance I said, so which is mark as P here (FL) is multiplied by the (FL) so (FL) is also basically the site ok. So (FL) so this BC and AD are also referred to as 2 part, so (FL) so this F*P and d*p, so (FL) yoga is again (FL) refers to the some of the base and face that 2 part was (FL) refers to c and b, so it consider 2 diagonals drawn. So then this is the formula that has been presented.

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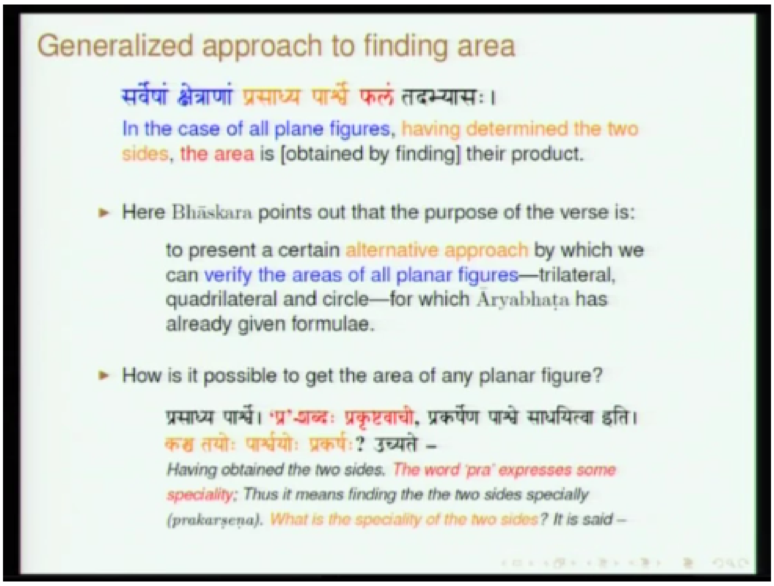

In fact the next was a very interesting verse wherein a analysis has been done and these analysis has been done by Bhaskara which is quite verifying when we look at his commentary. He says (FL) generalise approach to finding area. So initially started triangle anymore on trapezium, when it discussion about circle. So when he says either a way general way of understanding how we compute area.

When we see some kind of a similarity or can we do something so there can be general formula prescription can be given and you can fit in different cases into this by and considered as a special case then only thing that is kind of analysis which has been done by Bhaskara while coming up on this verse. So (FL) geometrical figure (FL) geometrical figure (FL) so you have to just find the side of appropriate value of side take a product of this.

(FL) so area is product of two things, so with this in mind this (FL) has to be different appropriately for various objects. So that is the import of this is what is conveyed by Bhaskara. So (FL) is area (FL) is the product, after all having given expression independently for all this. So why is Aryabhatta give one more word, so this seem redundant. So do not need, so this is the kind of question that he rises.

And then he says, so this can be considered as a different way of arriving at the same result, sort of cross check one with the other ok. So that is the purpose of this, and if the commentary goes like this (FL) basically computing, so (FL) so this (FL) depending upon the case which you are parentally handling depending on the circle, trapezium, this whatever, so you have to appropriately find the size.

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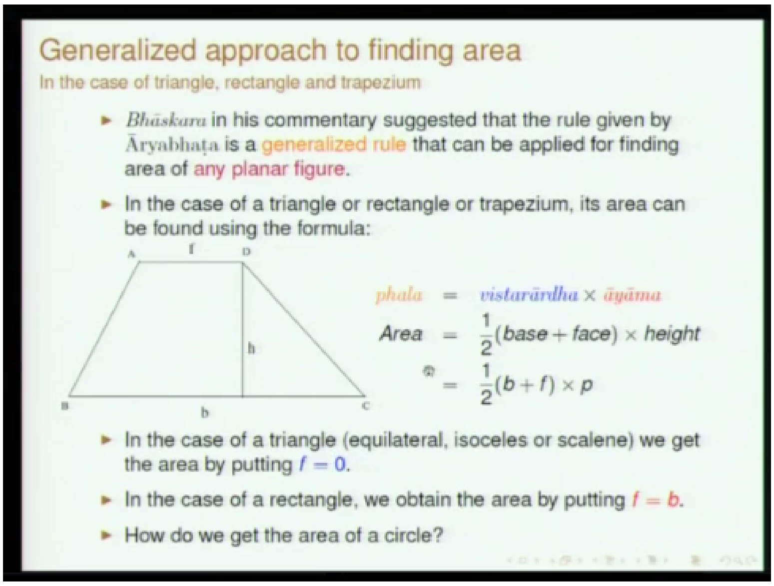

(FL) in fact he goes on, let us consider this figure, so in this (FL) ok (FL) is basically can be defined as product of 3 things, so what is that, (FL) half of this. So in the case of the triangle so what happens to the face so I just collapses to 0 right, so (FL) 0 in the case of rectangle, so (FL) same as b ok and how do we go about for circle.

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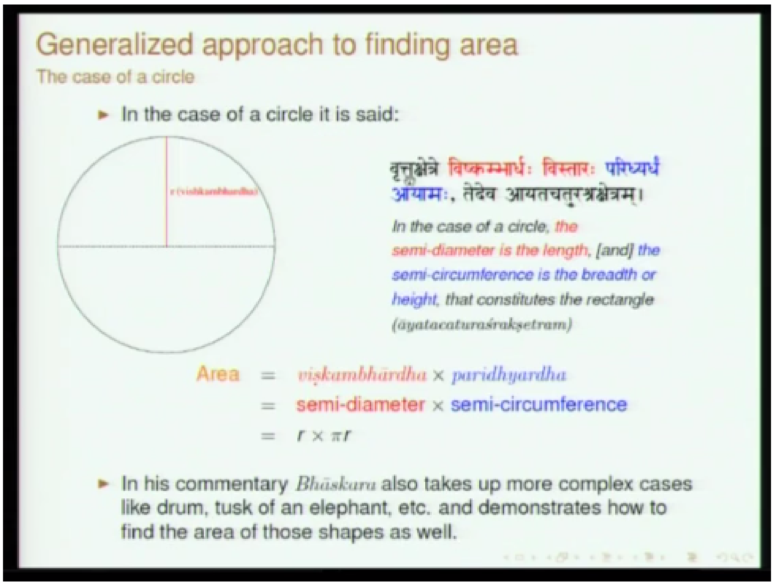

He says (FL) is the kind of distance separation between the faces, so in the case of a circle what you have to conceive is (FL) is to be taken as (FL) so this is the kind of (FL) visually fit to be a rectangle even this circle can be conceived of (FL) this was has been explained by Bhaskara.

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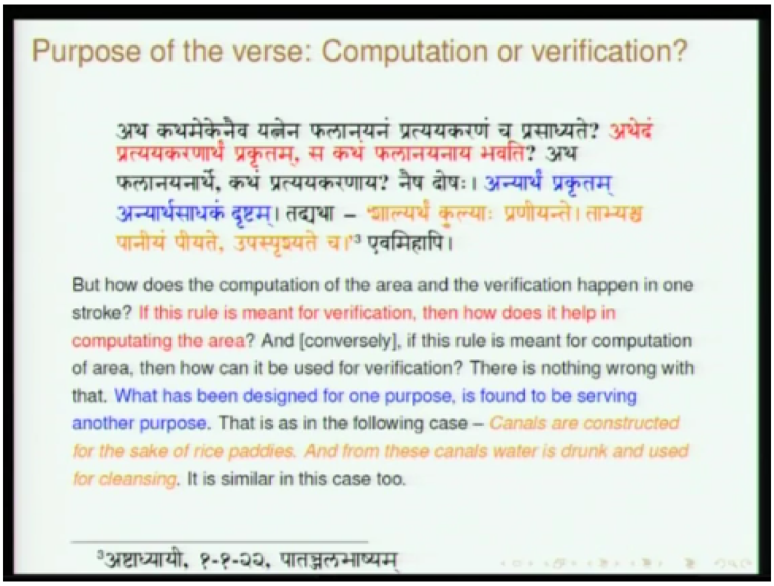

In fact further there is a very interesting discussion, so I want to just code this a certain maximus which has been employed by Bhaskara to justify the presence of this verse here ok. So (FL) in Tamil they say if you throw one stone you get 2 mangoes, oru kalula rendu manga, so something like this, so (FL) in one effort.

(FL) (FL) finding the area and prathikaram is verification ok (FL) for a particular purpose in mind, but it can also serve certain other purposes. So this quotation is quite interesting (FL) means water canal ok (FL) so it is a is basically form of rice, ok (FL) water canals are created to water the fields (FL) so incidentally somebody can go and use the water for some other purpose so nothing is lost.

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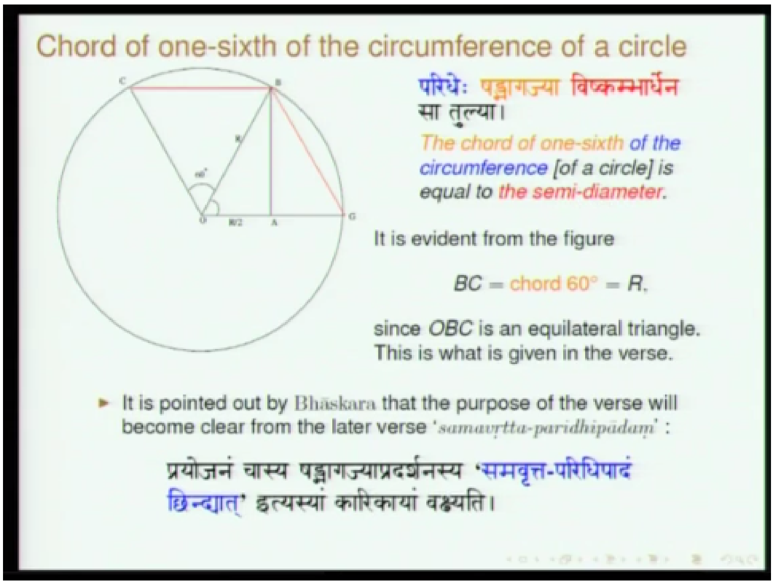

So it is in this sense so we can understand that this sloga, so which has been composed by Aryabhatta to cancel several purposes. So now I move on to get another interesting topic which actually forms the basis for one of the construction methods which has been described by Aryabhatta for finding the time table. So this half verse (FL) this is the statement (FL) is circumference (FL) half of the circumference.

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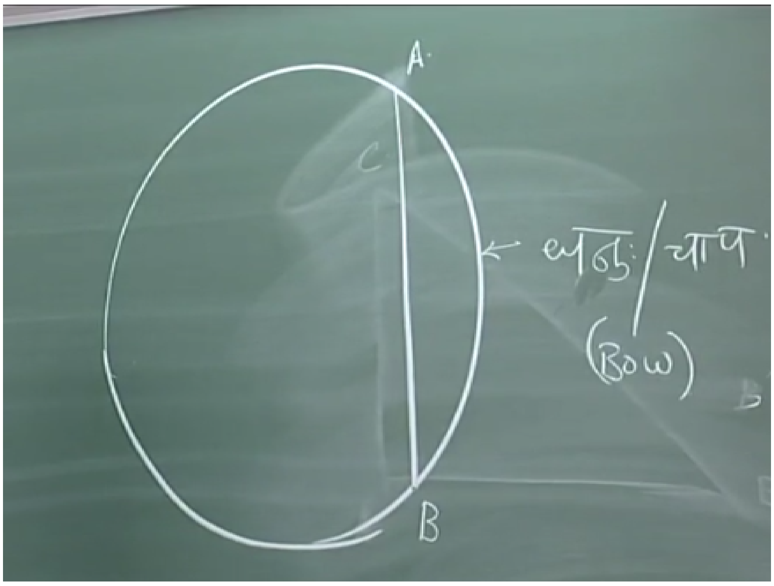

(FL) refers to the god ok, so in fact (FL) suppose you consider circle AB is a part, so which is referred to as (FL). So this portion of the circle, so this is referred to as (FL) which actually means bow, so this looks like arch looks like bow and any arc length is referred to as (FL) or any synonym for that. So this card A B is refer to as (FL) various terms which have been used. So this is out of conceived as bow and this is considered as string and they all mean string.

So what is stated in this work is (FL) so one sixth ok so if you conceive a circle so divided into 6 equal parts, so the card corresponding to one sixth of the circumference is referred to as a (FL) and he just states (FL) so this is same radius. So this is pretty evident so if you think of this triangle OBC, so which is an equilateral triangle, so we have this cards which is same sa the radius fine. So card of 60 degree is same as the radius.

So what is the purpose of stating this, in fact he says (FL) state, so in fact Aryabhatta is going to make use of this in yet another wherein he is going to present the sin table. So this is what (FL) the purpose of designing this can be understood with the verse beginning with (FL) so will come to that in a minute.

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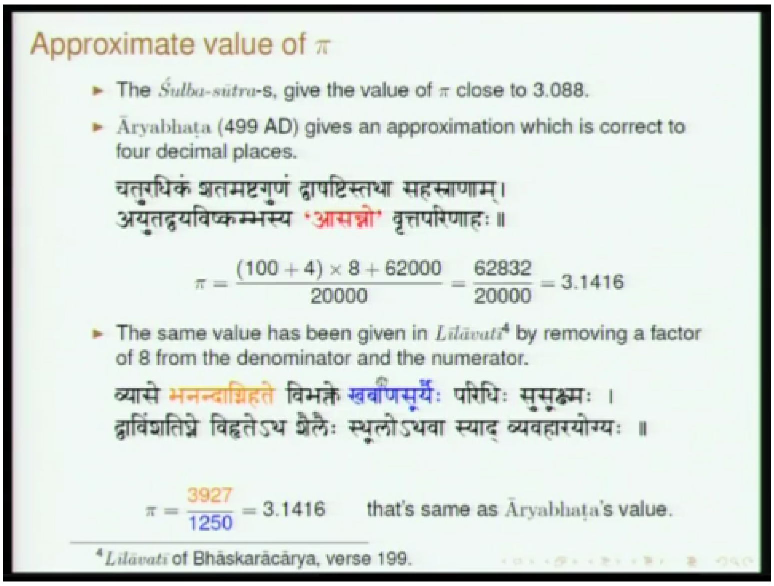

But before going to be procedure which has been delineated by Aryabhatta for finding this time table. So we will discuss this very important verse which has been quoted in various forums in various ways and it is also been misinterpreted in various places. So (FL) see the first half of the verse basically presence the value 52832. So how does it go about (FL) so 4+100 (FL) multiply by 8 and (FL) is 1000 52000.

So this refers to this number and what is stated in the later half of the verse is (FL) so what are these numbers correspond to, so this is the number which has been employed in order to obtain something. So (FL) is 10000 (FL) is 20000. So if the (FL) diameter happens to be 20000 then this number which was stated happens to be the circumference. So what is stated is the ration of a circumference to be diameter.

The use of the word (FL) has been discussed in great length, so why did Aryabhatta use the word (FL) you should simply said (FL) is something which is nearby close by which can be understood to be approximate. So we will see how this has been analysed, but before that I also wanted to give this verse as an example of the use of (FL) and also to say that this same value has been given by Bhaskara in a slightly different way.

So (FL) so it represent 27, (FL) are famous, so it refers to 9, agni is 3, so 3, 9, 27, so this is what it is, so (FL) divided, so divided by (FL) sum is 0, (FL) is 5, ok, the word (FL) has been employed to refer to number 5. So why is (FL) very good, so the names you have got it, but then the significant is the following. So (FL) arrow all the sense will be simultaneously attack, he will lose yourself completely.

So that is why he called (FL) ok, so (FL) this is fairly, this is (FL) in the sense there are various approximation which has been given, this is a far better approximations that is what we need to understand.

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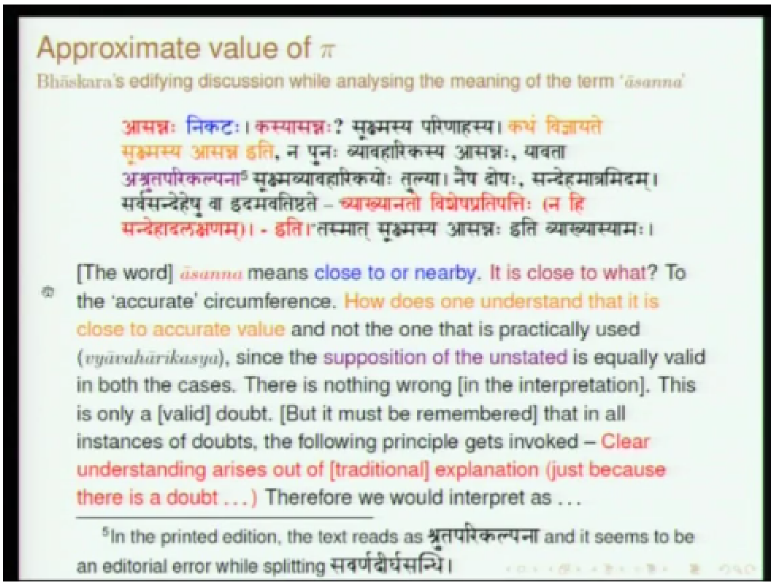

So now I present the discussion which has been presented by Bhaskara to you on the use of the word asana, (FL) so close to what, so there are 2, 3 things which are used one is (FL) this one way of saying, so there is a (FL) which is very (FL) very accurate. So this is close to the very accurate value. This is one way of saying, but then he says (FL) is taken as 3, so this can be close to this or close to that also.

So (FL) is something which is common to both (FL) in fact the edition it was a problem so the (FL) is what is found in edition. So I was just breaking my head so how this (FL) something with you try to create a in your own mind (FL) is something which is your imagination. So (FL) is does not make much sense, when something is hurt then you have to imagine. So that is why we should be.

So this (FL) is just I believe because of this (FL) so this is split it as (FL) that is my guess. So (FL) then he says (FL) it is true that your objection is valid then he says (FL) general maths in which is very important to understand, so (FL) this will be coded very frequently in this study of philosophy (FL) all that. So he says (FL) whenever there is a sort of doubt which arises then you have to resort to the (FL).

So how it has been analyse, how it has been understood, so (FL) means so a certain explanation which has been upward in the tradition, so from that we understand, we understand certain special meaning (FL) therefore we say (FL) edition says that this is close to the accurate value and therefore we will say (FL) means only this in this context. (FL) he says (FL) let not take the word (FL) could be something which is close by.

(FL) residing close by, (FL) this the different way of look at that, is it if you let us not debate on that. So if you say (FL) already a gross value (FL) is closer to that, so which means (FL) we will get a much gross value for the circumference (FL) is going to make some effort in specifying some values saying that this si going to be much loser and therefore use this. So this does not make much sense.

And therefore why purely logical reasoning so we can arrive at the computer (FL) ahs to be understood only as something which is very close to the exact value. Then he asked question (FL) so this is the similar question which Nelakanta also ask and then beautifully explain what means by irrational and that which I think professor Srinivas will discuss the quotation later. So here Bhaskara says (FL) by which you will be able to exactly state the perimeter.

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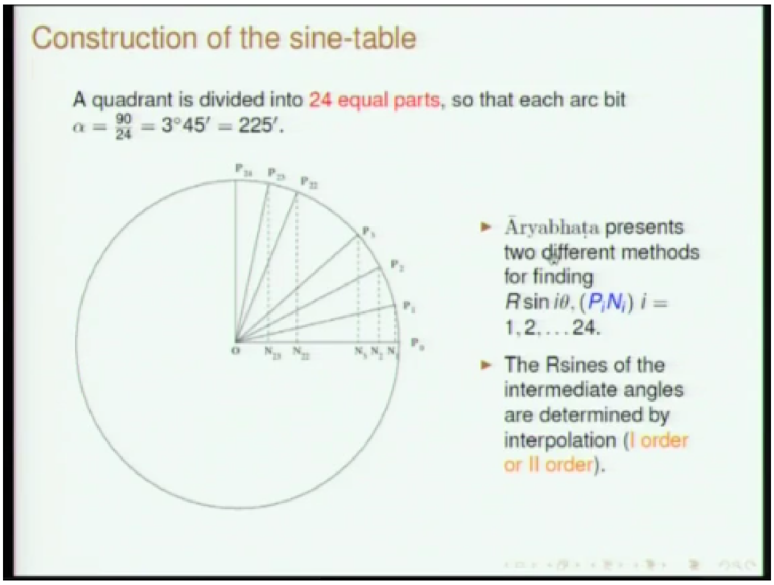

So here is out of conclude the discussion on the use of the word of asana the method which has been described by Aryabhatta to find out the tabular or science. So generally in almost every astronomical work on mathematical work we will see that a certain method has been described in which typically they will divide a quadrant into 24 parts. So as shown in the diagram, so you take the circle, if you take this quadrant (FL) p24 and divide this into 24 equal parts and the points are marked there P1 P2 P3 on so on.

So what is it that we are interested in. So we are interested in the card links, so which is basically fine (FL) so the projections which have been shown here P1 and P2 are basically the science. We consider this triangle On2P2, so P2n2 is basically sin if you consider this as angle P2ON2 is angle and the sign is PQ. So these are refer to as (FL) and we denote them as R sin of i times theta. So theta is 90/24, so which is 3 degree and 45 minutes to 25 minutes.

Here so the purpose is to determine all the PiNi, ok so this 24, the last value obviously is going to be OP24 which is same as the radius of the circle ok and once these values are known PiNi which are referred to as in fact this could be precise they should be refer to as (FL) total card length ok, as I showed here so this is (FL) half of it which is what is sin, but for the process of convenient, so people have started using jaw itself to refer to (FL).

So this understood from the context ok, if we know Rsin values corresponding to the multiples of theta so frmo 0-90 degree, then for any intermediate value so we use the position and usually this first order interpellation employed and if more precise values are required to later second order interpolation formula has also been stated I think maybe by Sriram or myself will do it later.

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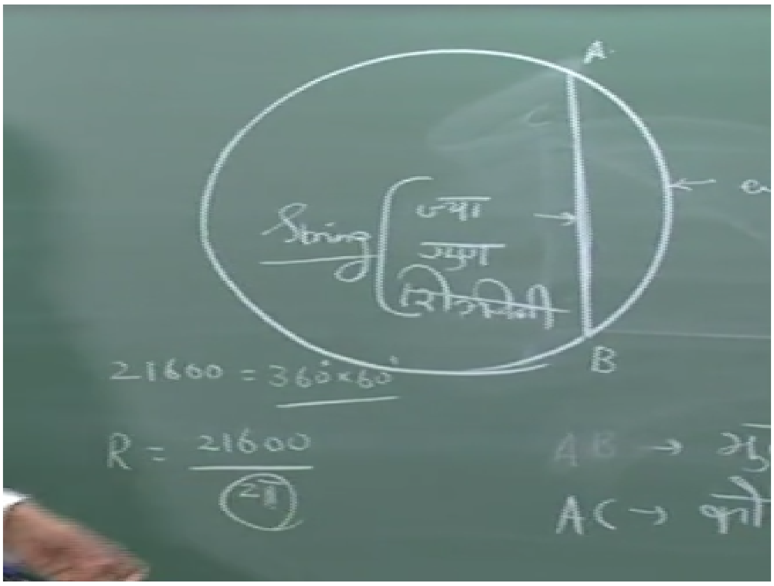

So what is the verse, so in fact I refers to eelier (FL) so this specification of radius as equal to the value of cot 60 was in connection with this work. So (FL) so this si the geometrical approach to construction of the sin table given by Aryabhatta. (FL) is the circumference ok, so the circumference of the (FL) circle, (FL) refer to one fourth. So (FL) refer to one quadrant of a circle (FL) means may you divide may you split.

(FL) so if you conceive it as triangle and rectangle (FL) ok, so (FL) without the detailed commentary given by Bhaskara. So this verse as follows the next verse which has been presented by Aryabhatta which is analytic approach is going to be almost impossible for us to understand. So the value of cos 60 was said to be R and this value is basically 3438, so this is pretty evident. So because usually what people do is in almost all the astronomical there.

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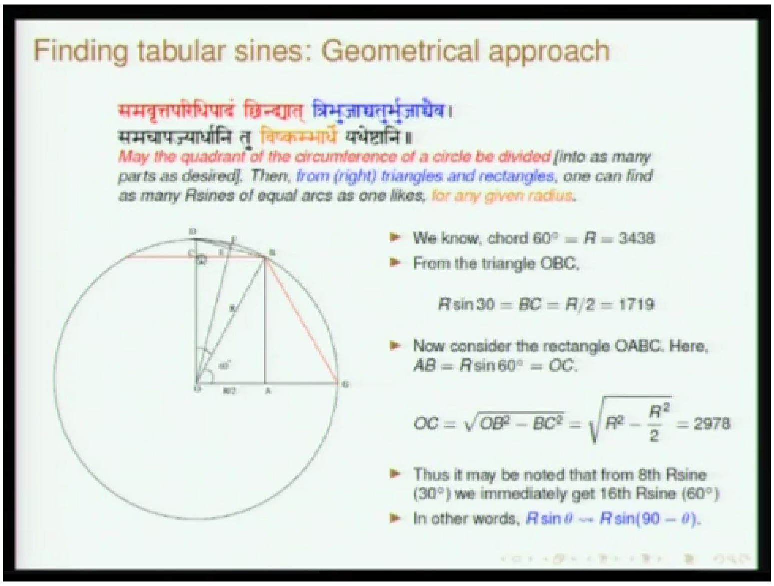

But for some so this circumference is taken to be 21600 which is basically 360x60, so 360 degrees*60 minutes, so the number of minutes in the circumference or rather the number of units in a circle is taken to be 21000 etc. Now that Aryabhatta in his verse (FL) he has given the ration of the circumference the diameter which is essentially 5. Once you know that then you know what radius is. Radius is 21600/2pi.

So this was sort of stated by Aryabhatta and this will be very close to 3438, this value, in fact Bhaskara so gives all this value, so while explaining in this verse in detail. So how do we proceed, so by geometrical construction, so all that require is so this value initially you know. So you have to just take with 3434 it is known, so since you 21600, the value of 5 is 1 and therefore the radius is known.

Ok we will start with them, then if you look at carefully so since this was stated to be radius consider this triangle OBC, so this is redius, so BC this angle is 30, and therefore the radius is known BC is known, so sin 30 degree is also known, so sin 30 degree is basically the 8 sin, if you conceive it to the 24 part, so 30 degree is going to be 8, now you know since you know the radius, you also know sin 30 ok r/2.

So this is basically r/2, this is basically r/BC, r/2 and therefore this is also known, no you consider this, OABC, so this is the kind of (FL) which can think of, so sin 60 is AB and is also equal to OC, but you know r you know r/2, so you do r square-r square 2, so you basically get this value OC, so you know sin 90, you know sin30, you know sin60, so what is to be understood in general is if you know sin theta.

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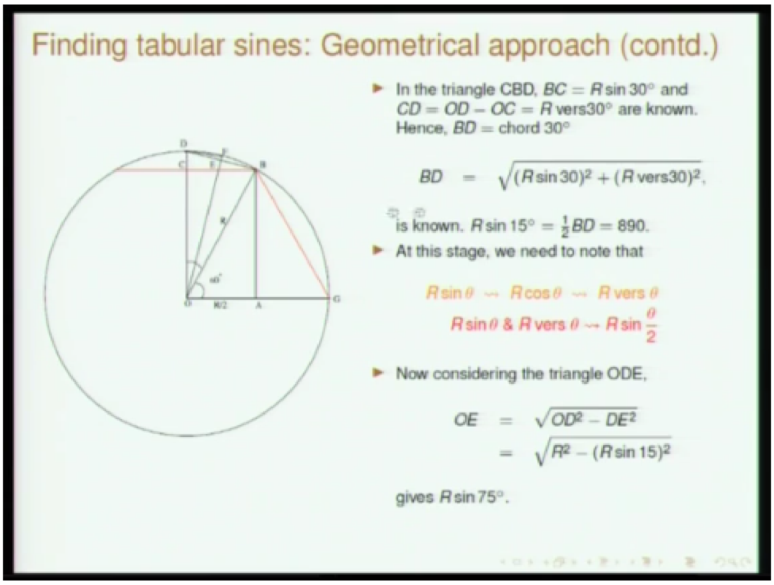

Then obviously you can get sin90-theta ok, so this si known, ok then since 30 is known so you also know verse 30, so verse is basically so r-rcos theta, so sin 90-theta is basically cos theta, so r-rcos t in this diagram so OC is basically cos theta (FL) and CD is the (FL) ok, so OD is r, so OC is r cos theta, so r-rcos theta is basically CD, so this is also known. So once it is known so you have sin30 and verse 30.

You know rcos theta, so r-rcos theta was 1 and so what 30 is known, we just do this, so BD BD if you see you know BC, you know CD, so now you can in this triangle BCD you know BD also, so what is BD, BD is is basically the card of 30 degree, we started with card of 60 degree which is basically radius. So now you have come to card of 30 degree, so half of it going to be rsin50 (FL) so you can keep on expanding this kind of an argument.

And you will be able to construct the entire sin table, so the principle is simple so rsin theta givs r cos theta, and that gives (FL) which is r-rcos theta, and from sin theta and vers theta you will be able to this is what account here, we will be able to get theta/2. So once theta is known 90-theta is known and theta/2 is also known. So with this we will able to construct the entire sin table.

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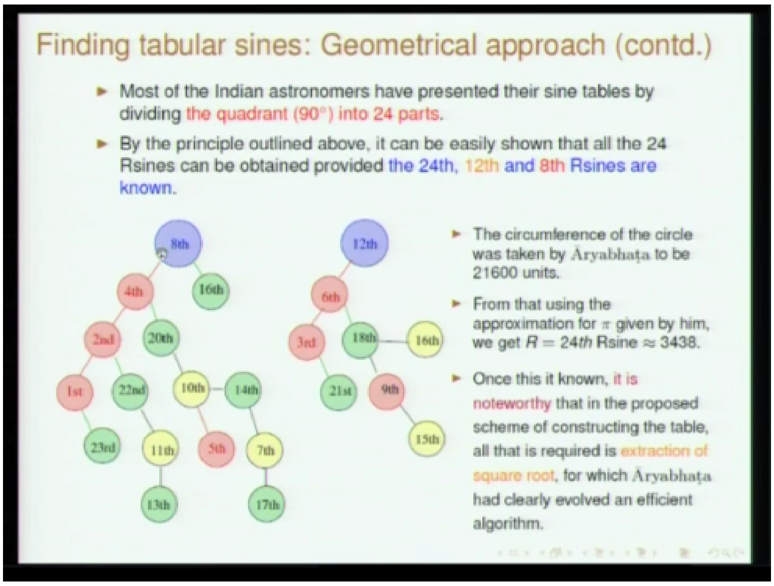

I am just going to show this how this seem to be work. So you know 8 sin, so once you know8 sin it is 30, so it can get the 4 sin and the theta/2 and 90-theta, so 16 sin is known, from 4 we can go to second and on other side we can go to 90-theta is 20th, so you can follow this 3, and this will give you the most of the sin values, and you could start with again 12, so 45 degree, so 45 degree all that going to do is we have to just draw line GD.

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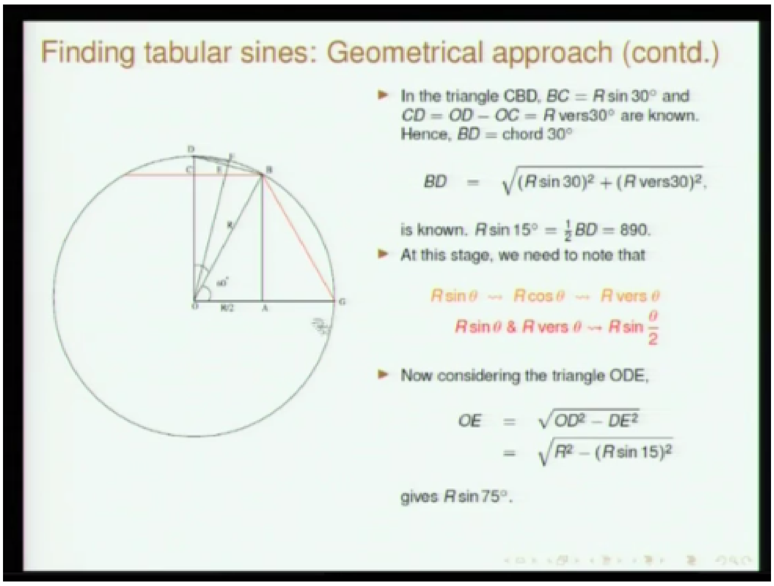

So for 45 degree all that you need to do is you have to draw line DG, see this GD gives you the card of 90, so half of it going to give you, so this GD how do you know, so you know r, so you find r as r root 2 r, so root 2 r is known. So then you know 12 sin, and then you follow the same 3, so 6 sin. So here if you go with 24.

There is not square, so from 6 we get 13 or 18 and so on. So you will see that in all the 24 sin values can be easily obtained by this geometrical approach. Ok, so with this now I stop the discussion on Aryabhattiya, so we will continue in the next lecture, thank u.

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