Mathematics in India: From Vedic Period To Modern Times
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Lecture-6
Decimal place value system

When this lecture will be discussing the decimal place value system, so its origin, how it has been used in the Indian mathematical and astronomical text, not necessarily mathematical astronomical. So that discussion about this decimal place value system as technology even philosophical literature. So I will be giving certain citations from the commentary of (FL) also.

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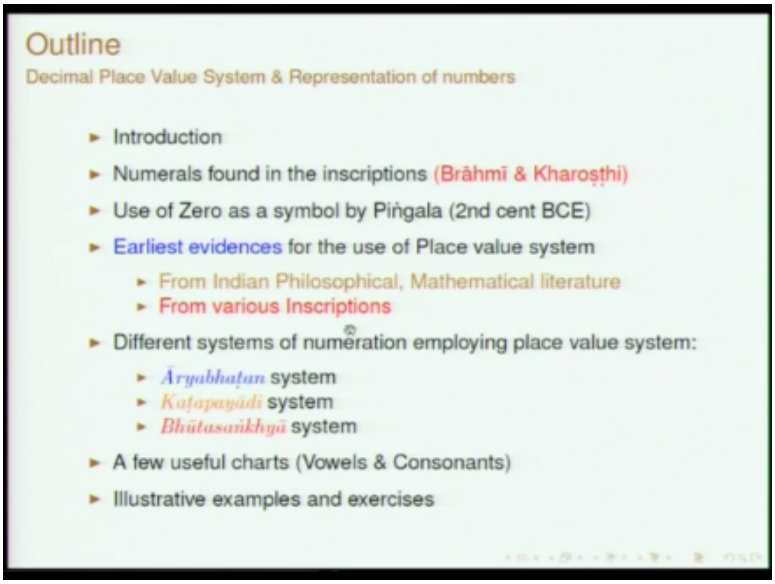

Wherein he draw sinologist from this and then tries to project a certain philosophical view point which he wants to project. a. normally we do not have opportunity to learn, so what is origin of a certain system. So because the history of science is never a part of our educational curriculum, anyway so we will try to find out so this one of the most fundamental discoveries.

And it has been fairly will recognise that Indians where the discoveries of this decimal place value system. The earliest concrete reference so in terms of the representation has to be paste from some of these are inscriptions. So we have this Brahmi and Kharosthi etc. at the earliest evidence can be traced even philosophical literature as I was mentioning after the introduction and the reference circum of philosophical as well as mathematical literature.

I will move on to describe three different representations which have been used by Indian astronomers to refer to numbers, aryabhatan system, katapatadi system and bhutasamkhya system. So these are 3 different representation which have been employed to represent huge numbers which occur in the astronomical works. So the sequence in which I have ordered them does not necessarily represent the sequence in which these they were used in the society.

But it is for certain different convenience I have used the this ordering here. So Bhutasankhya seems to be the most earlier thing and based on the evidences that are available in literature this seems to be the earliest. So katapayadi when is not very sure as to when it was discovered but certainly (FL) has a employee and (FL) wherein the longitude are represented in some sentences particularly in the occasion of what is called upakarma.

So this is (FL) also rendered as per the various other rendering which is done (FL) and so on. Anyway so these are all different representation which have been employed by people and we will show certain charts, so wherein the vowels and consonants, so how are the numbers assigned to these vowels in different representations and then we will have a few example. So this is the outline of the present talk on decimal place value system.

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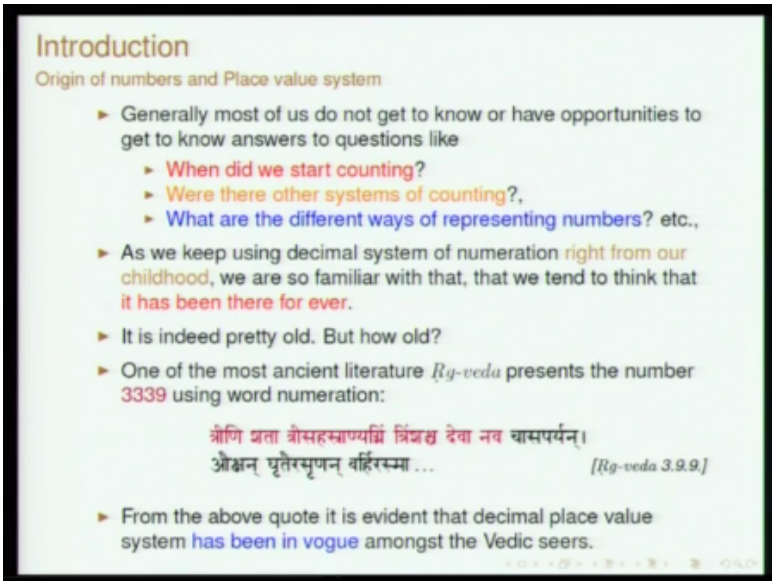

As I was mentioning so many of us do not force the question when did we start counting, what are the different systems of counting or what are the different ways of representing numbers. So we have one way of representing (FL) script is different but even the way of representation, normally two things are known, that is what I learnt, so one is this the so-

called Arabic notation and rather is Roman notation. So this are the 2 things which are generally thought.

So we are so familiar that we do not even ask this question that whether they could be other representation and when did this way of using notation started, obviously we will be initially using words to represent number. So after all notation comes much later whatever be the discipline. So initially the language, so we have the word to represent a particular number 2,3, 10, 100 and so on.

And then comes in the notation, so we will briefly see how did this particular way of representing started in the course of our lecture, so as I as mentioning obviously pretty old, but how old so the as I was coating yesterday, so this (FL) so where the number 3, 3, 9, as mean represented in word numerous ok. So we have word (FL) 300 and so on.

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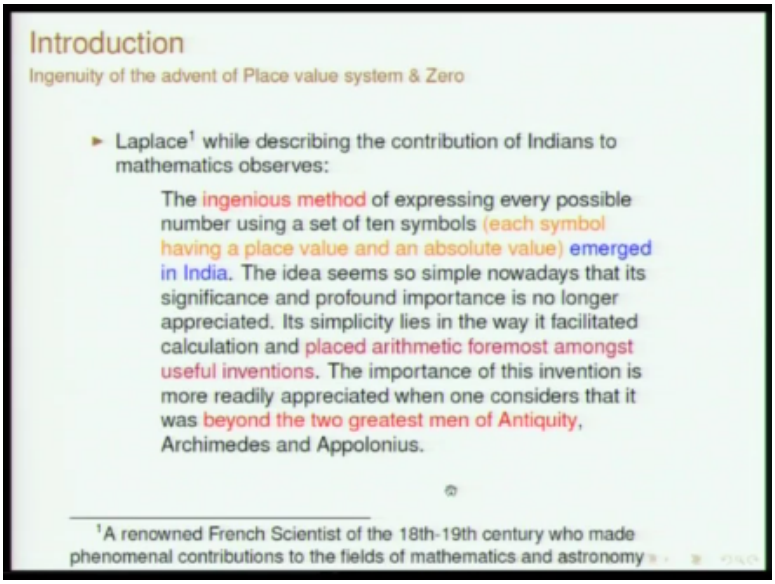

Talking about the ingenuity of the decimal place value system and mention of 0, so one of the famous scientist of recent pass the French mathematician Laplace so observes the following. The ingenious method of expressing every possible number using a set of ten symbols each symbol having a place value and an absolute value ok the place value and absolute value emerged in India.

The idea seem so simples nowadays that its significance and profound important is no longer appreciated. Its simplest lies in the way it facilitated Calculation and placed arithmetic foremost amongst useful inventions. The importance of this invention is more readily

appreciated when considered it was beyond the 2 greatest men of Antiquity Archimedes and Apploninius. So I will not described implication of the last sentence.

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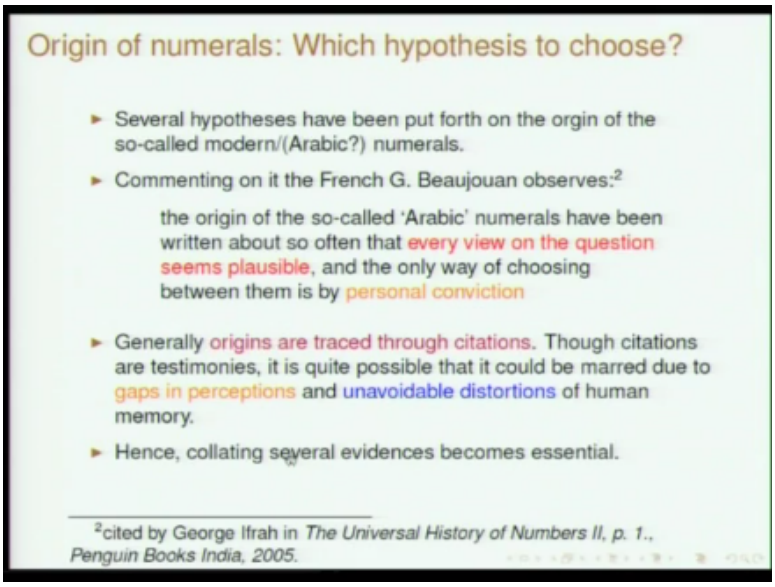

But anyway there is not much time but he is for you to founded over. So when historian try to trace back so what would be the origin and so on and so forth, obviously several hypothesis will be put forth. Many of the hypothesis so would seem to be true, so that has been very nicely heard, the origin of the so called Arabic numerals have been written about so often that every view on the question seems possible and the only way of choosing between them is by personal conviction.

So why does (()) (07:25) us so we go by certain citations and testimonies and over a period of time took loss of memory due to one soon way of understating what we convey, so it quite possible that there are gaps in process and this caution to take place and it is essential that 1 police several evidences to come to a certain conclusion and in this regard to the interesting book. So which I would recommend all of you to go through.

And this book is by Georges Ifrah. So the book title Universal History of Numbers, so I would take a couple of minutes to just to say how this book are originated. So apparently Georges Ifrah is a school teacher and he was teaching numbers, so to young kids, so one of the students asked so where did these numbers come from. So how do we understand where it got originated and the present day we of the presentation this is the only representation.

And so on and so forth, so Ifrah could not answer the question raised by the student, it is a certain conviction in his own mind and therefore he decided that the next few years tracing to find out evidences to answer this question. So he resigned the job apparently and then it is stated that this man so went around from one place to other place and he was even lying on road side not having enough resources to stay in.

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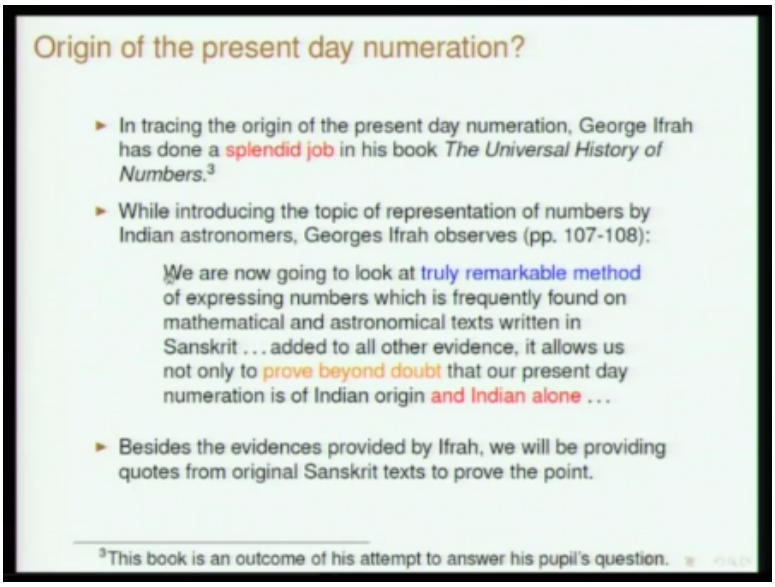

And he that narrates the seriousness with which you have gone ahead and written this book that is what has been stated, after doing research for decades, so he finally says with reference to the origin we are now going to look at it is truly remarkable method of expressing numbers which is frequently found on mathematical and astronomical texts written in Sanskrit, added to all other evidence.

It shows us not only to prove beyond doubt that our present numeration is of Indian Origin and Indian alone. So this is following on to convey through this, so beyond what Ifrah has said so Ifrah obviously is not so conversion with some the literature we will also go 8 o clock evidence of literature, we just give a flavour as to how different interpretations can be given for it origin. I just have it photographs here.

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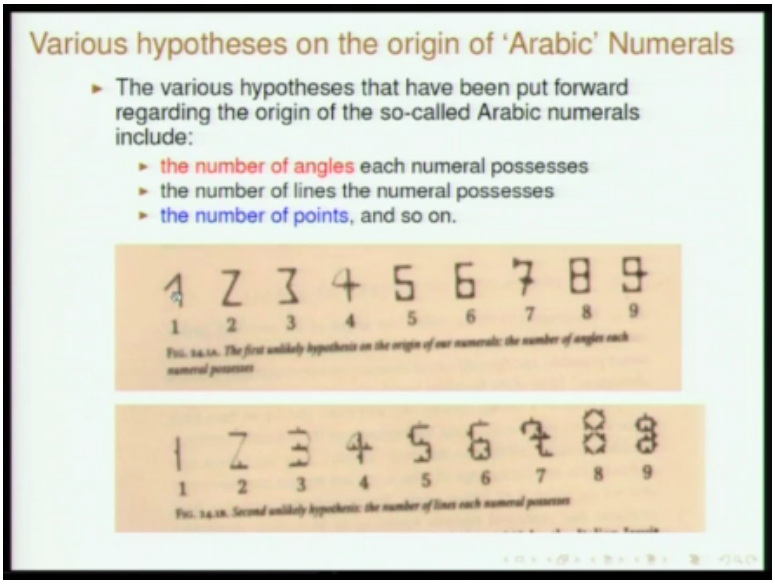

So it says see the numbers, the number of angles see if you look at 1, so only one angle, it say 2 so only 2 angles, 3 there are 3 angles, 4 and so on. So this is one, rather present says so it has nothing to do angles it has to do the lines, so if you look at this, so only one line there are 2 lines he says, there are 3 lines 1, 2, 3, 4 lines 5 lines as was the different people can obviously come up with different hypothesis.

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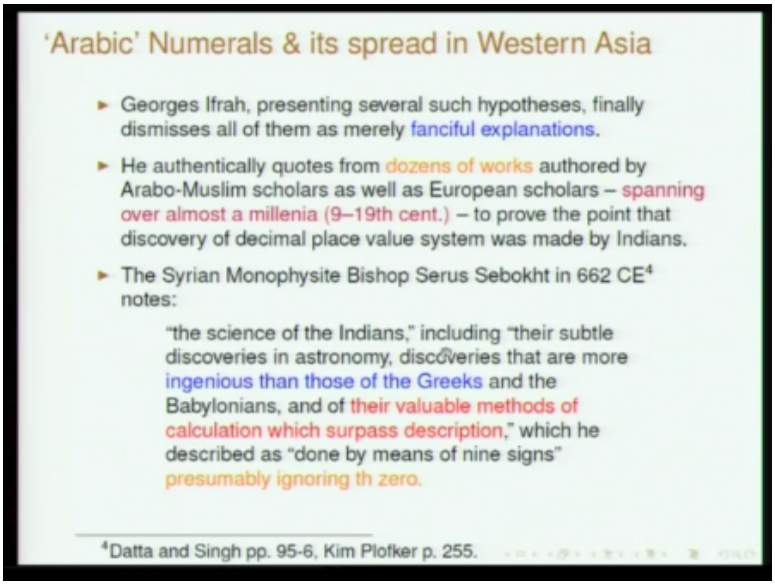

It is just for the fake, just I want to show this slide there is nothing more to read here. So finally Ifrah says that most of the presentation that find that all scientific location, and therefore we need to trace back and not to rely on these types of information. So having coated from dozens of works so both from Islamic astronomer as well as European scholars, see finally come to the conclusion as they was seeing before that is (FL) origin.

This coat from the Bishop in 7 century also founds certain evidence that the science of the Indians including the subtle discoveries in astronomy, discoveries that are more ingenious than those of Greeks and the Babylonians and of their valuable methods of calculation which surpass description, I mean he has been in high price of what has been done here which he describe as done by means of 9 science.

In fact this facility is a lot, so that is what we need to understand from this quotation, so the kind of facilitation that it offers in the present in this as we will see from the astronomical work. So if one where to make a simple comparison I mean the way this Roman script, so one has to write at the addition subtraction the kind of dislocation multiplication all the offer that is something which is really amazing.

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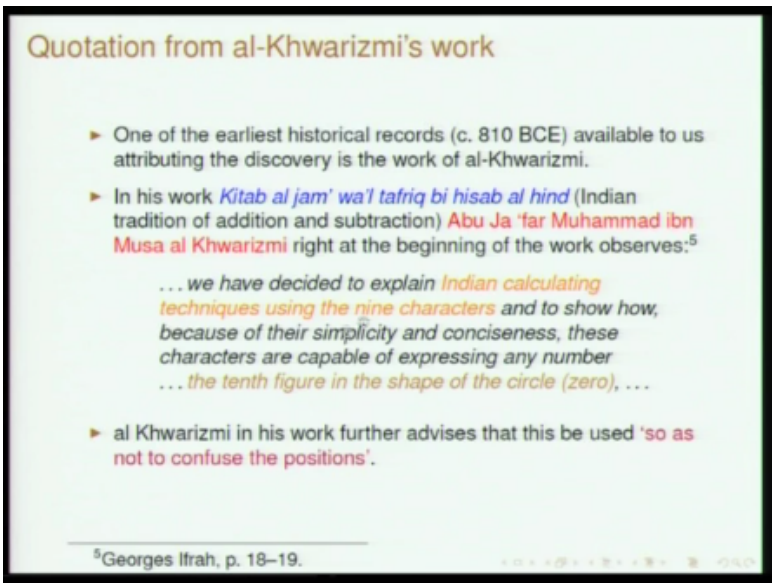

And that is why even the class 5 please with the reference to the advent of this decimal place value system. This (FL) also in his work he says we have decided to explain Indian calculating techniques using the 9 characters and to show how because our simplicity and conciseness capable, so these characters are capable of expressing any number. So be there all in high place of this decimal place value system.

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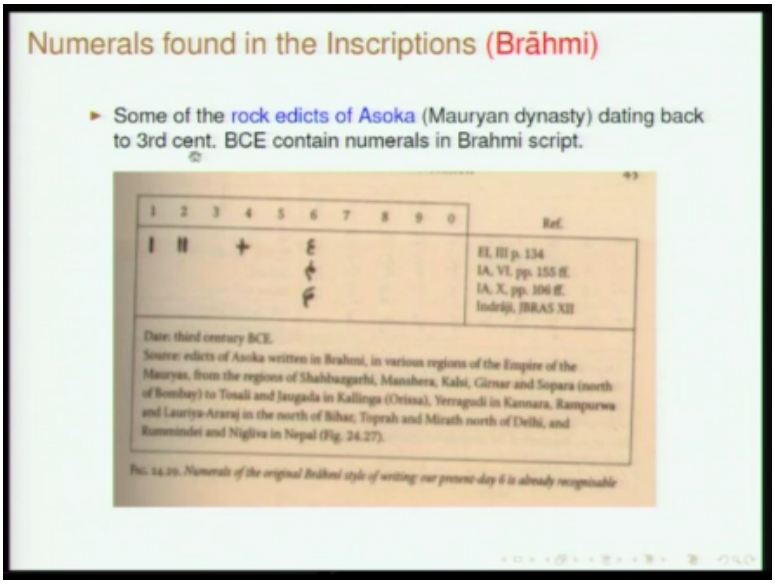

So think of multiplication which is representing so in Roman and if you had represent something like particular 20000 or something which is 1000 that, so it is all extremely confused ok. So the evidence in the form of inscriptions is the 3rd Century BC, you have something like this is the kind of representation that finds in the rock edicts of Ashoka.

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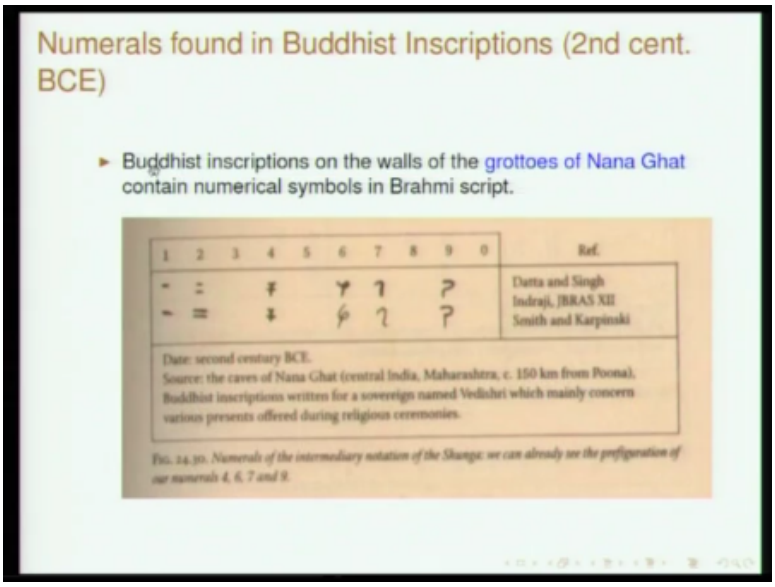

So as I then proceed further there are Buddhist inscription so about 3, 4, so second century BC so we have this representation.

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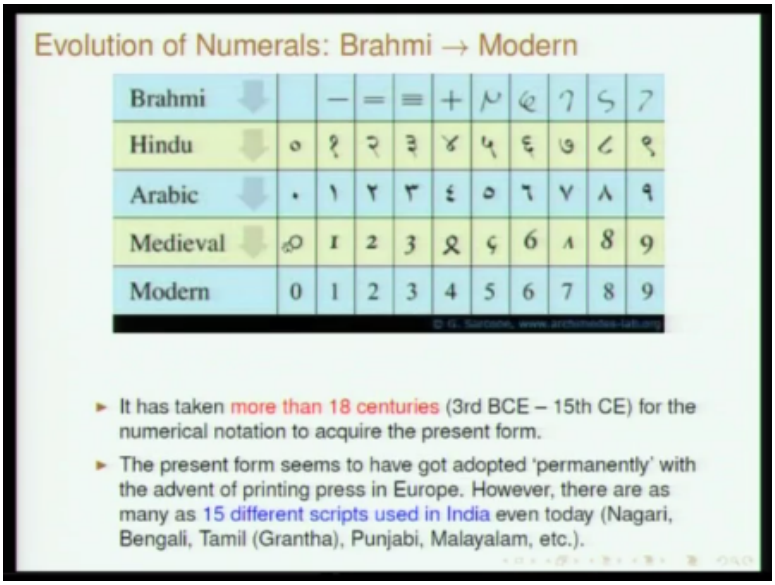

And we proceed further and in short so it seems to have started something like this and today view this. But almost taken about 18 centuries for the numeral notation which we use today. So starting from brahmi to modern. So the modern is estimated around 15 century, 16 century when the printing started. So up we have different scripts in different languages that is a different thing but the place value thing is a different things.

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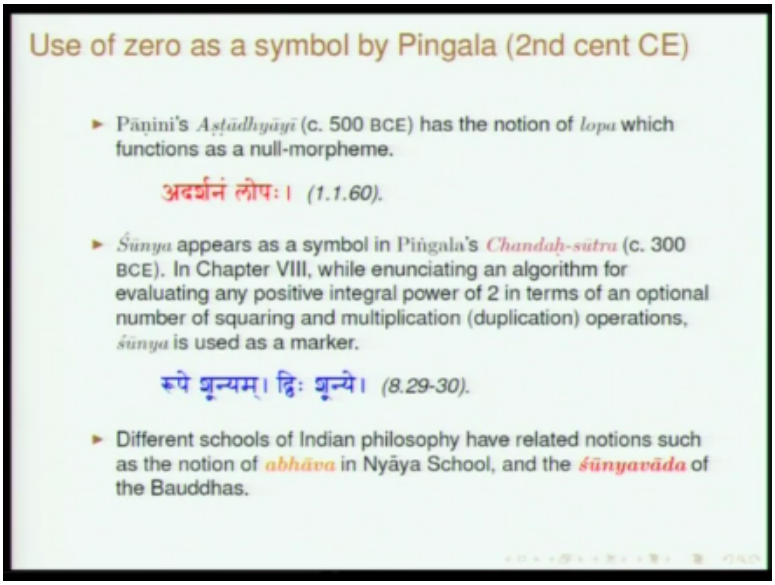

So with reference to 0 as number and how it is represented, so it is too difficult to precisely fix, but we find places of it in the Panini's work on grammar as well as Pingala from sutra, so we have (FL) so this was the word (FL) also represents certain understanding of what this physical world is with reference to do this the Philosopher's, this word (FL) is does not necessarily represent the number 0 when it is used in philosophical basis.

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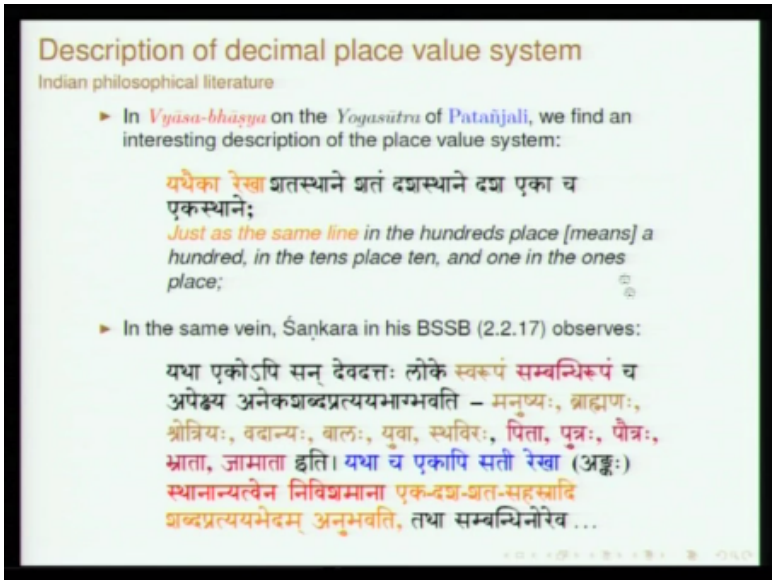

But in this chandahsutra there is nothing to do with the philosophical implications but it is use as a marker, now I give a couple of quotations from commentaries on Patanjali yoga sutra and (FL). So this form a very very concrete evidence (FL) is a certain line ok so which you draw, so he says if it is placed in (FL) so obviously (FL) a certain place value (FL) 100, if you just shifted so it represent 10.

So (FL) the same, so it has a certain place value, it has a certain value to represent a number, so both of them and this is more graphic portrayal to convey a certain philosophy Shankara says (FL) we call him a boy, we call husband, will call directory, we call him grandfather whatever it is, so depending up on the relation that you want to associate with another individual, it is describe by different terms.

(FL) similarly he says (FL) the same line, so (FL) depending upon where it is placed (FL) by Shankar as an example to convey something else in philosophy, but the point is so we have a very very concrete evidence by Shankar science, so it has gained and such a understanding in the society, that examples are something which should be known for everybody, so the way did you not going to be certain example to convey something else which is more perform ok.

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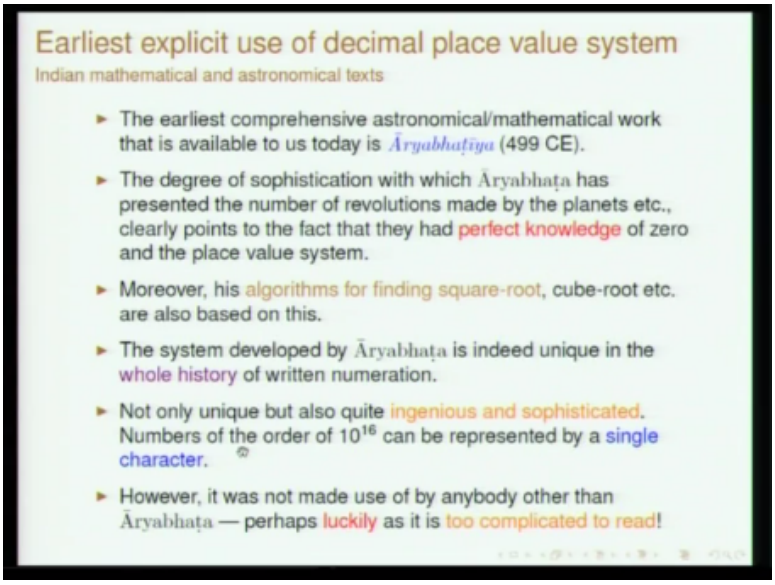

The explicit use of decimal place value system obviously is found in the mathematical and astronomical literature. So the earliest text that is available for us today is aryabhatiya or Aryabhata. So this was composed in 499 degree. So the degree of sophistication with which Aryabhata have been able to handle is something which is quite amazing. He has introduced his own way of representing numbers.

Obviously it is a decimal place value that will be very very evident not only from the representation of numbers but also some the procedures which Aryabhatta has described to obtain square root, cube root etc. So the algorithms which will be discuss later we will make it extremely clear to you as to how they have been able to use this place value system in order to do lot of mathematical computation.

In fact quite ingenious and sophisticated that a single syllable can represent a number order of magnitude 10 to the power of 16. This is quite convenient to represent huge numbers but the same time this extremely difficult to pronounce as you will see with a few examples which I will show little later.

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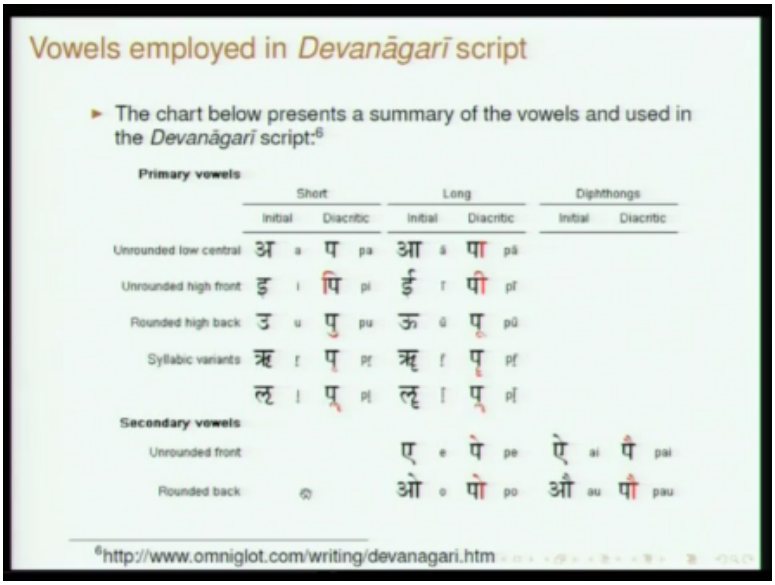

For instance this 4320000 years is represent by (FL) by Aryabhatta (FL) represent this particular number and what Aryabhatta has done is so he has chosen the vowels to represent the place value, so we have a system of representing vowels and ingenuity lies in Aryabhatta making use of the vowels to in other words I mean he has thought of vocalise to the place value I would say. So vowels are essential for pronouncing consonants.

And there is a will teach (FL) we do not say (FL) so what are essential for pronouncing consonants and Aryabhatta has smartly using these vowels to pack a certain place value to that and the consonant will the person the number. So for those who are not familiar with vowels so I just thought I will just display this chart. So (FL) so this is all had a longer one (FL) so that is how sutra also (FL). So this refers to Panini, so give the set of vowels (FL).

So in Aryabhatta systems once we reach (FL) with a certain function had tagged to that, so will be multiplying that consonant by 10 to the power of 16. So Aryabhatta has further classified the consonant into 2 books.

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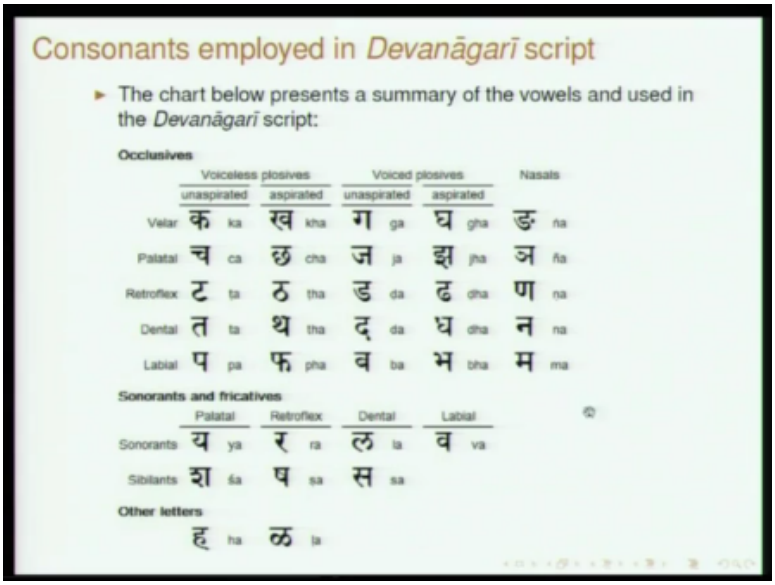

So this start to help in understanding this kind of classification, so the matrix that you see here (FL) so this 5/5, so 25 he calls them as varga letters, so varga and (FL) the term has also the meaning of square, so the operation of square, squaring. So this is also referred to as varga, so but as far as this is concerned this set 5*5 I mean this 25 may be it is in this sense is called vargs, it falls into a certain class ok.

So then we have (FL) I do not think is employed by Aryabhatta, but up to this certainly so he has associated certain numbers to all this consonants and the place values will be decided by the vowels ok. So this is extremely important because when a string is given to decide for one has to have a very clear understanding whether this false in (FL) so when we place these consonants with the vowels.

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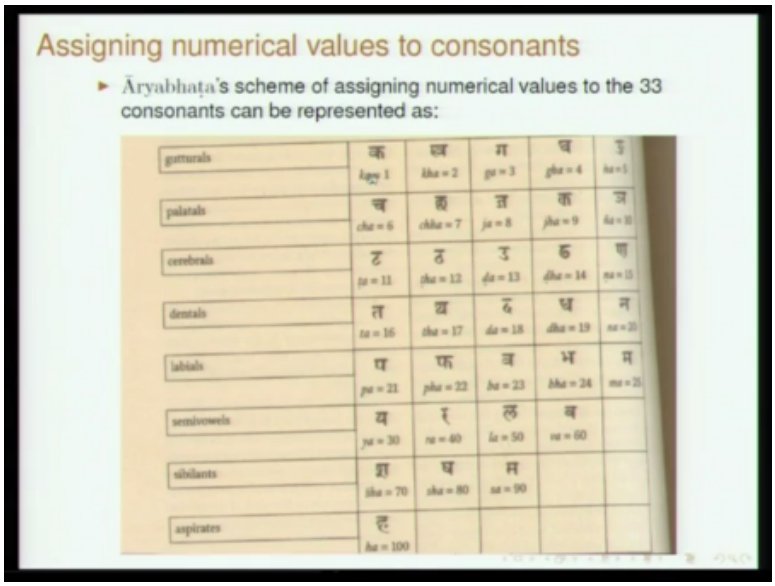

So we will classify depending upon whether it is (FL) has to be placed under, so if you make a mistake will be multiplying it by a power of 10 which will go wrong. So in (FL) he assign values 1-25 (FL) so on, so we come to 25. So 1-25 is associated with the consonant which forms (FL). So with reference to (FL) so it goes from 30 to 100.

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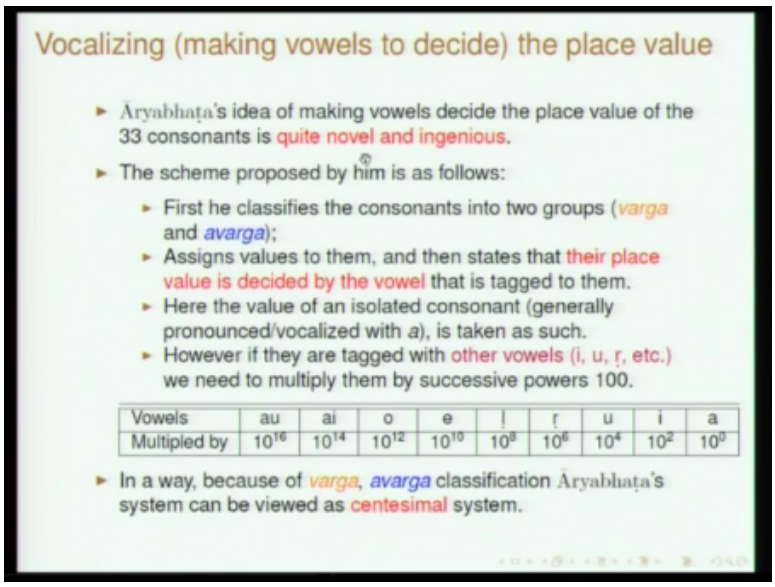

So as I said is quite novel and ingenious in a way Aryabhatta system can be thought of as the (FL). The vowels as I said are used for denoting place value but since we have classified the consonants into two groups varga and avarga and man can associate as an power of 10 with the varga and another with our avarga. In fact he has described his system by means of a (FL) but all that you need to remember is so every vowel so as we move from (FL).

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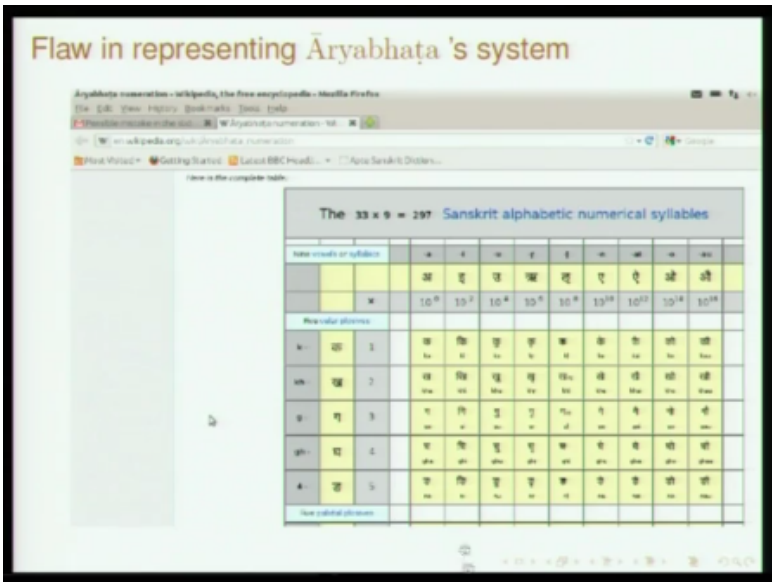

So it goes as 10 to the power of 2 in this sense it can be called as 10 decimal system. So here I would also like to show a chart wherein some error in fact I offered this course on mathematics in India to our student at IIT, I thought something and then for the student send me send me a mail. So looking at something in the web and then said so there is he seems to be discrepancy between what you said and what is there in the web.

So I said there is an error there, so I thought I will just show this to you also to see that so this year is not committed ok. So we have to be very careful when we take materials from the web. So the error is the following so (FL) stopped this and therefore there will be serious error in deciphering the number when we take this stop to be the (FL) ok.

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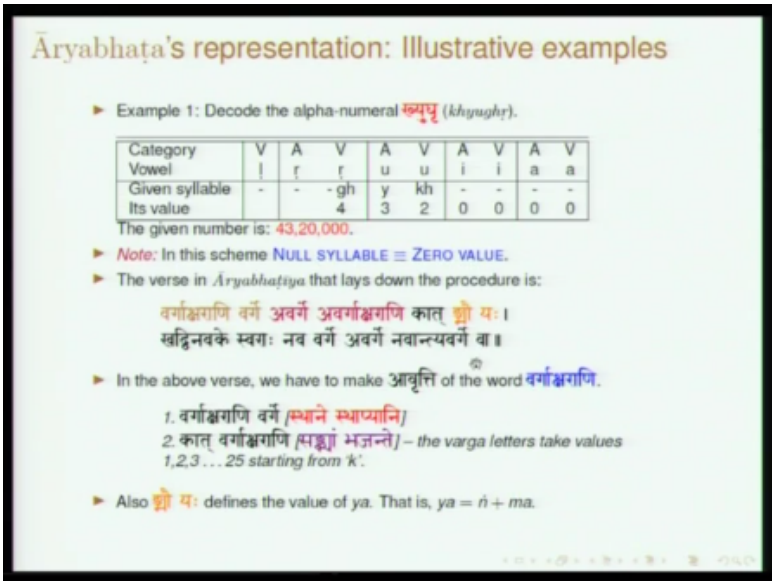

So let us see a couple of examples, so cute root, so this is a very important number and that is why I decided that we should stop with this example, so what is to be done before getting into this table. So you please say look at this word, so Aryabhatta says (FL) so here we use so term varga in 2 senses, when he uses the compound (FL) it refers to the varga letters, but when he uses the word (FL) separately.

So it means you have to place it in the (FL) we mean 10 to the power 0, 10 to the power of 2, 10 to the power of 4 and so on. So here whenever you are given a string, so when you find a varga letter, so then we place in the (FL) and if you encounter (FL) you have to place in the (FL). So now it will become pretty evident you, so you have to create a certain table, so it will be very convenient in the initial stages for us to decide for the number.

For every vowel you have to create two places one is (FL) so for all we have 2 for (FL) and so on and so forth for every vowel will create two places and whenever we get a varga letter we place in the varga and encounter avaraga consonant you play it under varga. So this is what the (FL) essentially detail and if we look at the last part of the first half of the word it says (FL) so the number starts ok (FL) assigning numbers to various functions 1-25.

And then he defines what (FL) represent, so very clearly he states (FL) so if you look at this table, so (FL) put together, so whatever be the value that is the value which is assign (FL) ok. So that is what it means (FL) so this incidentally will give you a flavour of how Aryabhatta has written his work. So that is this style of composition in those days and the entire Aryabhatta is just 108 versus, so which deals with all mathematics, all astronomy and so which will be just take in A4 sheet ok.

So that is how it is this and so this is what it is (FL) basically refers to vowels, (FL) arranges student want means, so (FL) so now we look into this example (FL) 3 vowels and 3 consonants that define here, and vowels are (FL) 2 vowels and 3 consonants, so (FL) is the second thing ok third, so the value of (FL) is 2 and we find (FL) also there. So that has the value 4 (FL) and we find (FL) now when we try to arrange it here so since (FL) does not have a vowels on its own.

So when you have this kind of a combination so then the vowel that is tagged to the consonant with succeeding it, so will be the vowel which is tagged with this previous consonant with vowel also. So this is the annotation and (FL) place this in varga place (FL) set below varga, fine then we move on to the next syllable. So the next syllable is (FL) is 4 and it has to be placed below (FL), so because the vowel tagged with (FL).

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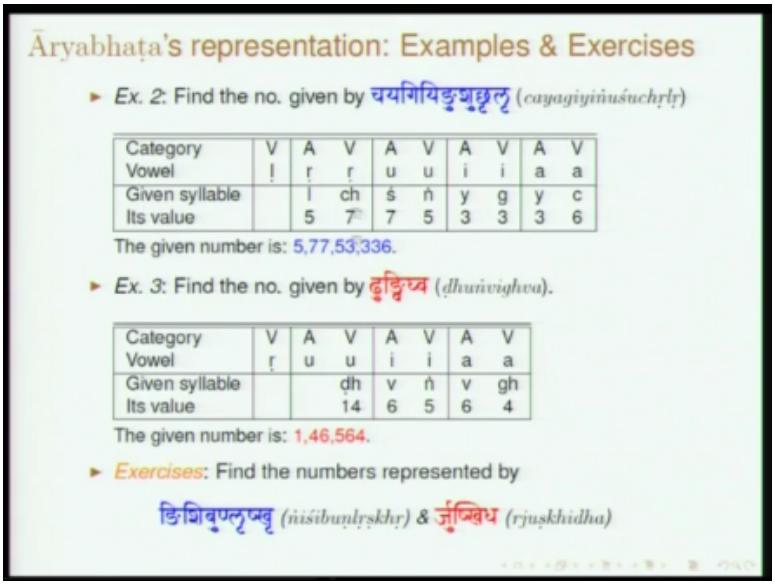

So this actually represents 4 zeros followed here, so 4320000 so this is the representation which has been given by Aryabhatta. Aryabhatta give reprehensive numbers. So we will see one more example (FL) so that is why I said it has an advantage of representing huge numbers in very short form, but it is bit difficult to read (FL). So here we again notice that there are several vowels which have been employed in this string.

We have (FL) so if you look at this table (FL) so this basically represents the number of revolutions made by moon, so in fact to give you the significance of this number so this gives the number of revolutions made by the sun in a large period which is called (FL) so under this representation the number of revolutions of sun and this represents the number of revolutions made by the moon in the same period of 4320000 years.

So this are strings which have been provided by Aryabhatta in order to tell you the number of revolutions made by the planets in a large period called (FL) and one more example (FL) so here we have (FL) without vowel and the following vowel is (FL) and therefore (FL) represent 4 and (FL) represent 6 ok. These things (FL) it is very very difficult to pronounce, but these all basically the revolution numbers which has been stated by Aryabhatta.

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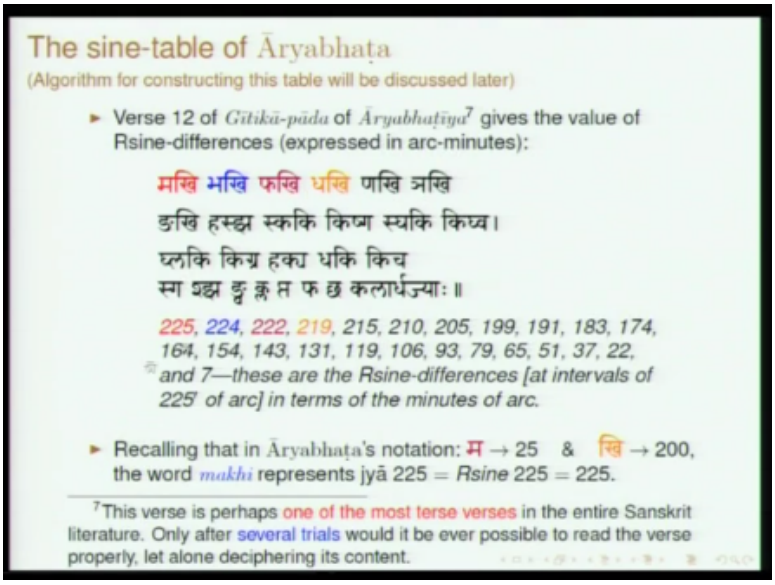

Much more interesting thing is so Aryabhatta actually present the entire time table in numbers. So you may remember that in school days you would have referred to sin table class cable one page to page all that will be there, but this man has presented in simple words (FL) so this is how the, so all these basically represent this values of sin difference in fact we say sin table.

So the quadrant is divided into 24 parts will see later as to how Indian astronomers and mathematicians have found very efficient ways of computing accurate sin values for a certain period of time. so this Aryabhatta what he has done is he has basically presented this sin table in one words (FL) because it is tagged with E, so you have two 25 25. So this two 25 basically represents the sin, so sin theta suppose theta is 3 degree and 45 minutes.

So sin theta is theta you know right, so it is presented in minutes and therefore so 225, so if you take 3 degree 25 minutes if you represent in minutes 225, so sin theta is theta, so what is the kind of thing, so what has increases the next value of 225 it chose so it will reduce will keep on reducing. So basically what Aryabhatta has given is sin table in the form of differences, suppose you want to find out the sign of 10 degrees.

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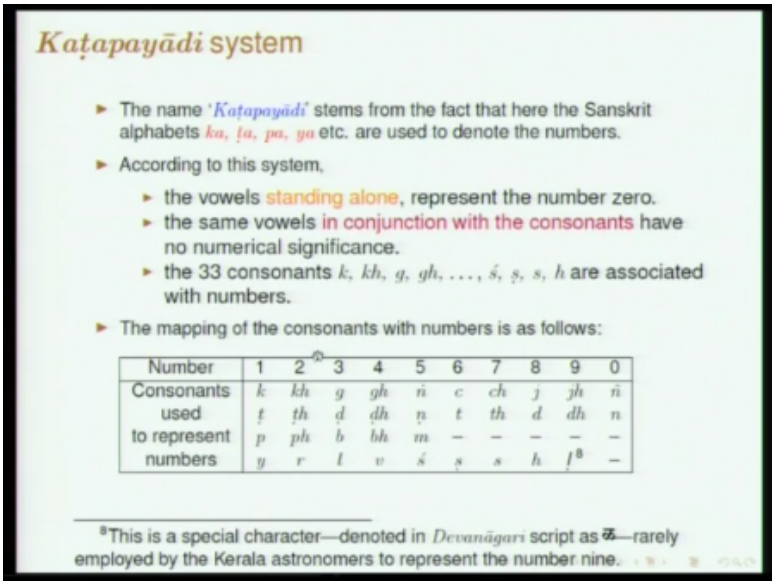

So you have to add this+this and then you had use some interpellation to get the value, so how this has represented ok. Now we move on the other system called which is called kattapayadi system. So the name kattapayadi stems from the fact that in this system (FL) so all of them represent 1. See if you look at this table so below so we have representation from 1-0. So here also it is mapping of consonant to a certain number.

But since we have 25 of them so in kattapayadi what has been done is (FL) and so on and (FL) so Aryabhatta not assign the value to (FL) and that represents number 9, here there is no classification as varga and avarga that you have many to one mapping or other one to many mapping. So if you want to represent number 1 you can choose any of the 4 here ok. So 2 you can choose many of the 4 and so on.

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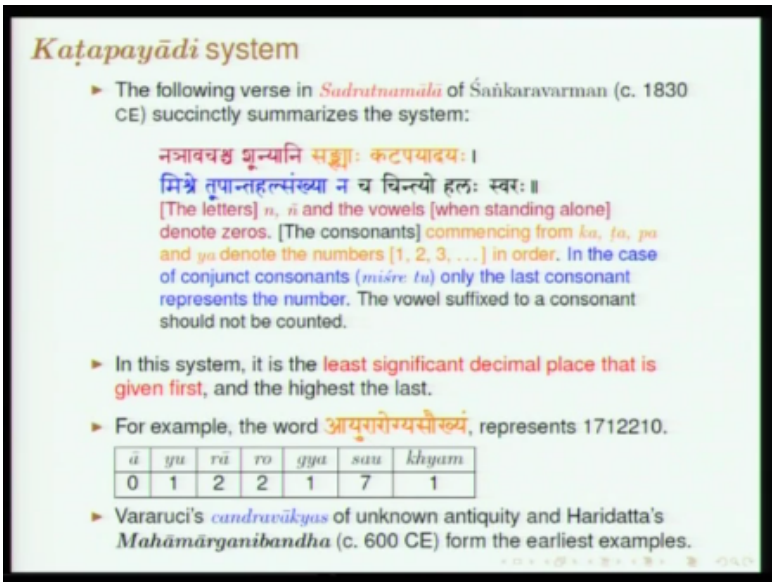

So this is what basically katapayadi system is all about. What has been done here has been beautifully summarised in one single words by (FL) so the entire systems has been captured in this. So it says (FL) then it comes to a conjunct form of a syllable then how do you decide to (FL) here in Aryabhatta system since the vowel was used to tag the place value, so we have problem here it is not that way.

So you find the string and you keep on it always start from so the least the place value and then you proceed to the higher, so when you have a certain string fun instances you look at this (FL) so in fact a famous work (FL) ends with string (FL) represent this number, so when you look at the definition which has been provided in this. So it says (FL) actually represents consonant ok.

So (FL) means any vowel tagged to the consonant once should not considered. So why is he stating this, so if a vowel occurs on its own without a consonant activate then it represents a number, but if it is tagged along with so only consonant has to be considered. So that is what (FL) means that which is close to so for instance in this string which has been given here (FL) is a vowel so all vowels where present number 0.

So (FL) so if you look at the definition so (FL) is a certain pneumonic which has been used to refer to all the vowels, so it has to prove Panini sutra (FL) which is used to refer to all the vowels. Similarly (FL) in fact if you look at the maheshwara sutras (FL) something which is used to refer to all the consonants. (FL) so we can see this example (FL) consonant, so what to do is you drop (FL) and you take the consonant which immediately preceded the vowel.

So it can be a consonant 2 consonants and 3 consonant fed with all those cases, the one which immediately proceed to vowel has to be considered, has to be counted and therefore you count only (FL) what is drop only is taken. So basically it is represent number 1,7, 1,2,2,1,0, commentator as said this actually represents the date on which this work (FL) got completed. So this is the another way of coding.

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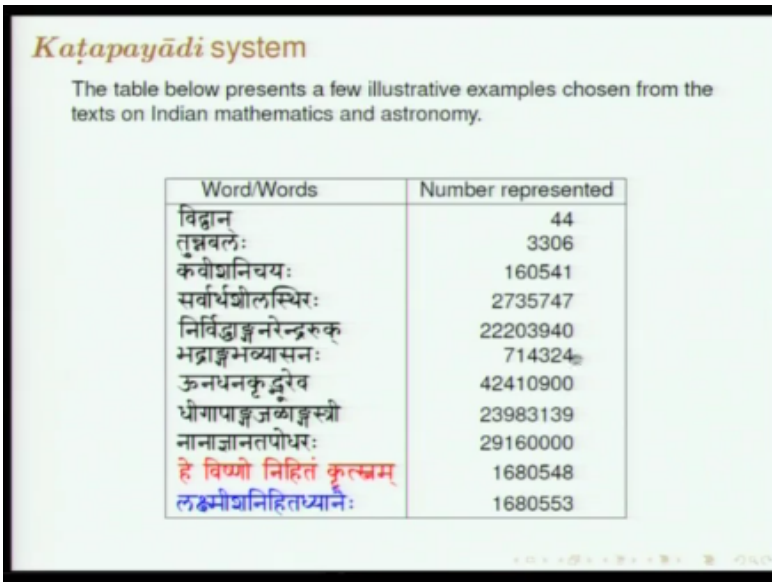

In fact there are several examples like this (FL) also it starts with the (FL) it represents date and (FL) appears in the last verses text and determiners this number if you see this the difference is only 5 days, apparently this person Nilakantha has composed the commentator say this entire work something about 430 verses and all in just 5 days. So this all exam.

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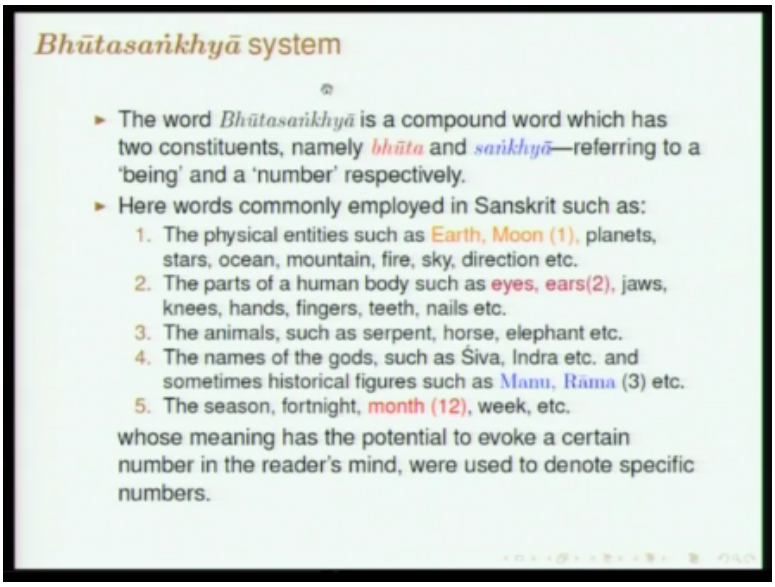

Quickly I will also mention about this bhutasankhya, in bhutasankhya system as the term itself gives an idiom, so bhuta is something which is existing, so sankhya is number. So you choose a certain elements in nature, so for instance eye, when you say eyes so they represent 2 ok. So when you say fingers it represent 10, when you say veda it represents 4. So based on the familiarity so with the bhutas.

Bhutas not necessarily the physical elements, we could be physical element it could be certain mythological think it could be referring to some literature which is quite common. So for instance when they say (FL) so moon the moon associated with the earth. So they all represent number 1. So any synonym of earth to represent 1, of moon will the present. So then for representing 0 the use (FL) space looks like a shadow empty in the sense.

So space is used to represent number 0, any synonym of (FL) is 0. So as I said eyes, ears, jaws, knees, hands, finger system, hands is 2. So sometimes they use (FL) there are 14 manmantras and therefore (FL) represents 14 ok and rama Ayodhya Rama, Balarama, Parasurama, so it represents 3. So guna when they used (FL) all this things, so this is how the bhutasankhya system is.

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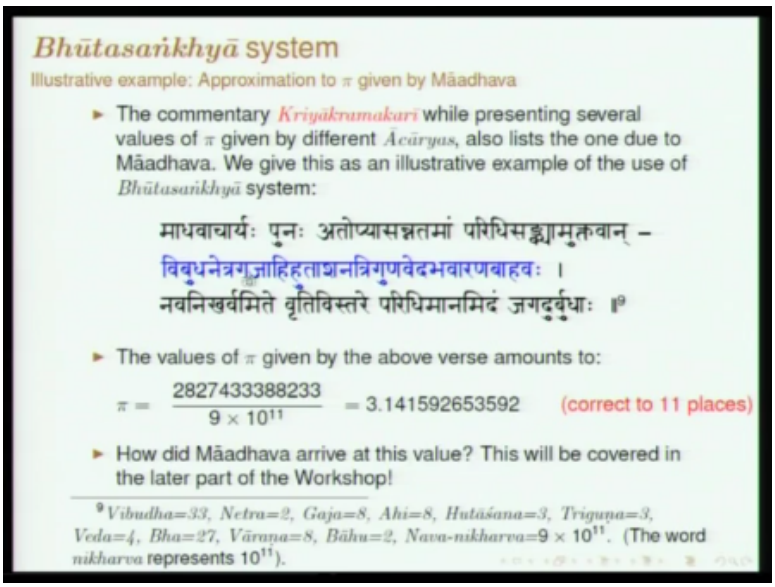

When they use month there are 12 month, so one interesting example is Maathavas value of Pi, so what I thought just used herr, so (FL) so you look from here so (FL) basically represent (FL), so therefore it is 33, so it has to do with the description found in Puranas and (FL) is 2 then (FL) and we have 3 here, 3 also represents 3, represents 333 consecutively ok. (FL) 3 and then we have (FL) and so this first line basically represents this number.

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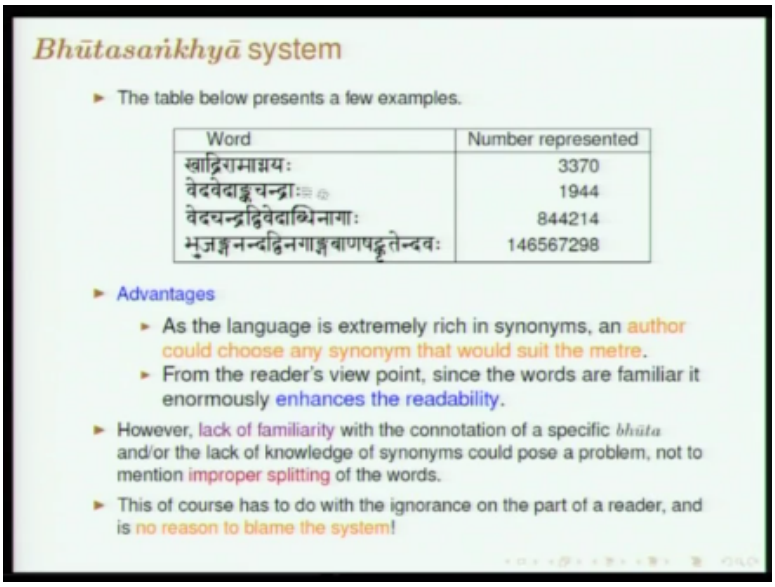

And the denominator is represent (FL) this actually gives you the value of pi which is correct to 11 decimal places. (FL) will discuss later. So these are all few other examples. So (FL) so to conclude, so Aryabhatta system though quite ingenious is a bit difficult to use that systems to represent because it is too difficult to pronounce and therefore not many people in fact we do not know of any (FL) Aryabhatta using his system to represent the number.

So it is very profound in its own way, as far as (FL) is familiar with the saturn system it is in fact it is very convenient to use both (FL) system for composition, so which is also see if you are familiar with certain terms it is quite convenient to read and therefore mostly astronomers have resorted to (FL) system. In fact that seems to have been the most deadliest system but then Aryabhatta device own and (FL) had own format.

And in fact as I told you (FL) something else and it also represent the number in fact the entire (FL) system of astronomy is based on this (FL) when you decode it gives you if anything conclusively. So all these (FL) proved the facility with which people have been using decimal place value system. So with this we conclude here, thank you.