Mathematics in India: From Vedic Period To Modern Times
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Lecture-5
Pingala's Chandahsastra

Like Panini Pingala is another great deal in ancient India. Pingala is the person who systematized the Chandahsastra (FL) he wrote the Chandahsastra with Chandahsutras which give the theory of prosody both of Vedic metres as also of the classical Sanskrit we touch.

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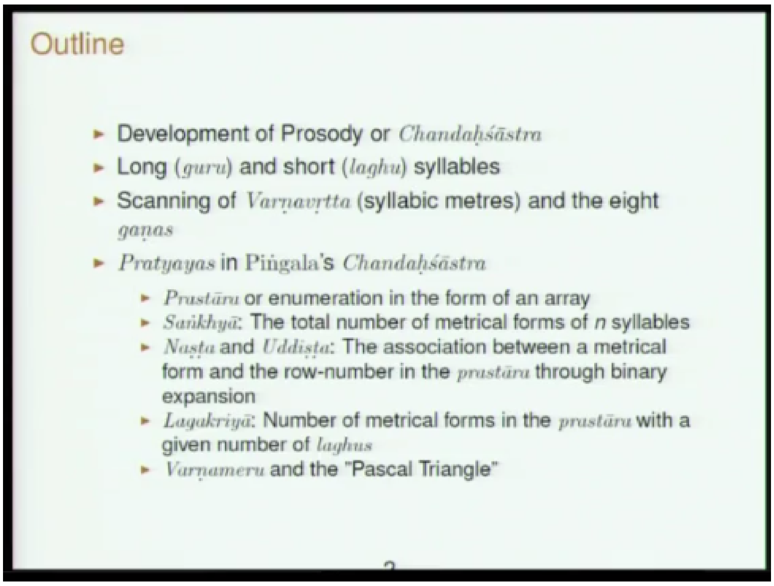

Again do not know like in the case of Panini exact date of Pingala generally scholars place him around 300 before BC. So in this talk I will give a brief overview of the development of Chandahsastra, then we will going to the interiority of what is meant by (FL) how do we scan a syllabus metre of varvrtta in terms of eight ganas, and then we going to the combinatorics ideas that well developed by Pingala in the last chapter of chandahsutra.

This are known as the Pratyayas, so Pingala discusses 6 pratyayas which are combinatorial techniques or combinatorial tools for studying the Sanskrit metres (FL) and these are the basic (FL) this at the 6 Pratyayas in Pingala we will discuss.

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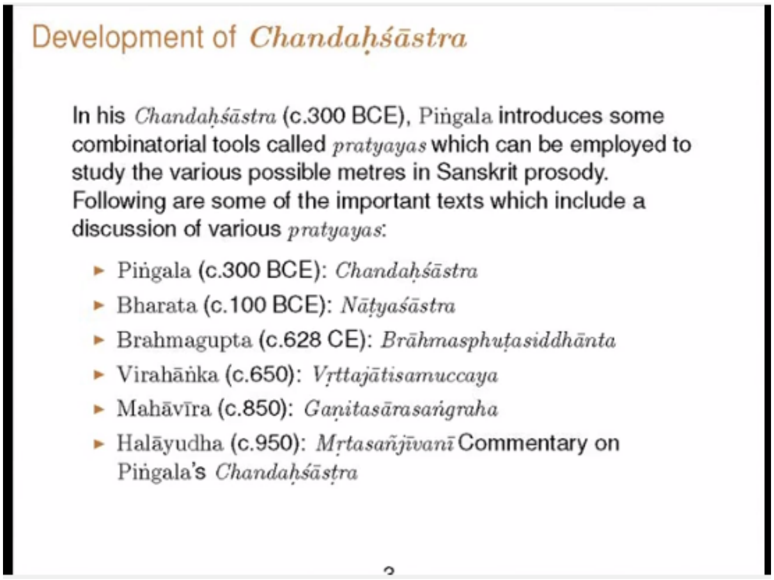

Chandahsastra again has a continues history (FL) is a classic word (FL) is a chapter on Chandha, then mathematical words like brahmagupta's Brahmasphutasiddhanta, what are called Matra British were introduced in discuss more in greater detail in (FL) very interesting commentary on Pingala Chandahsastra.

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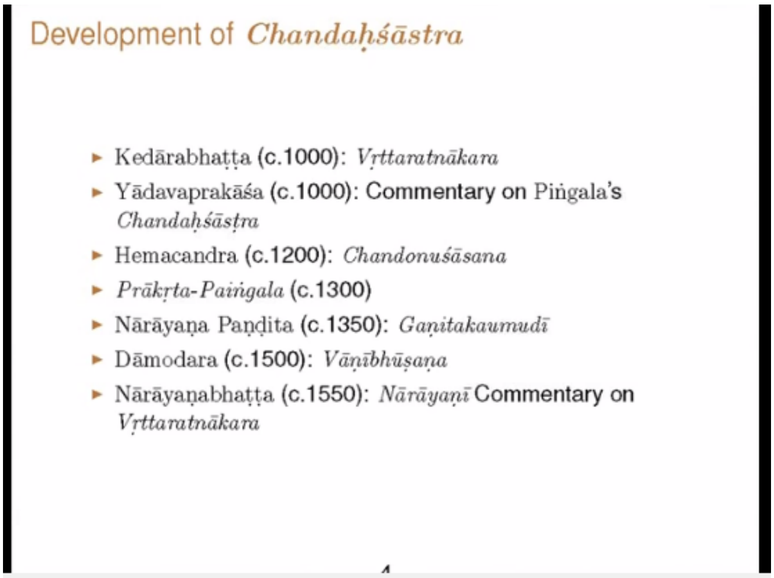

The text that is most commonly study by students of Sanskrit is a book written around 1080 by Kedararabhata is called Vrttaratnakara. Hemacandra wrote again on (FL) is another text. The mathematics books like (FL) also discuss the problems related to the combinatorial problems related to Sanskrit, macro environment touch then Damodara Vanibhusana in the very interesting commentary on (FL) by Narayana Bhatta.

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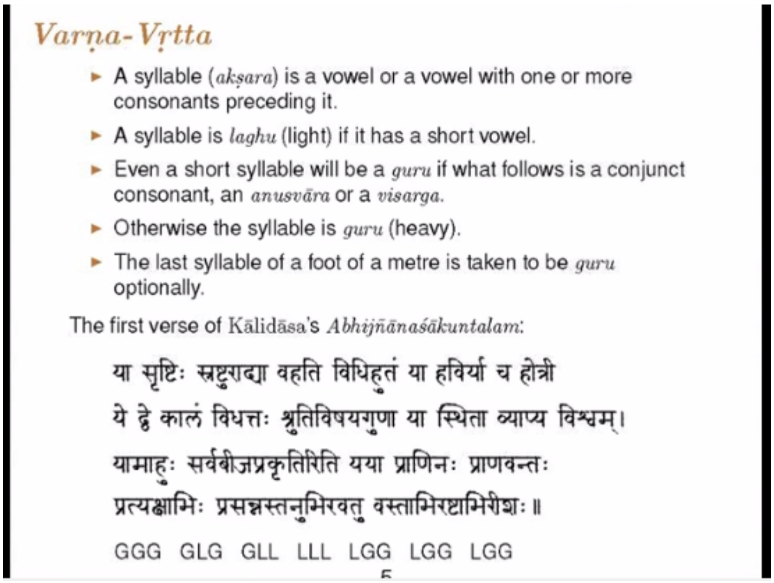

So we will straight away going to what is the way in which Sanskrit metre are understood, the basic building blocks of studying Sanskrit meters are the characterization of syllables by (FL). So what is a syllable so you take anything so Srishti that is this consists of two (FL).

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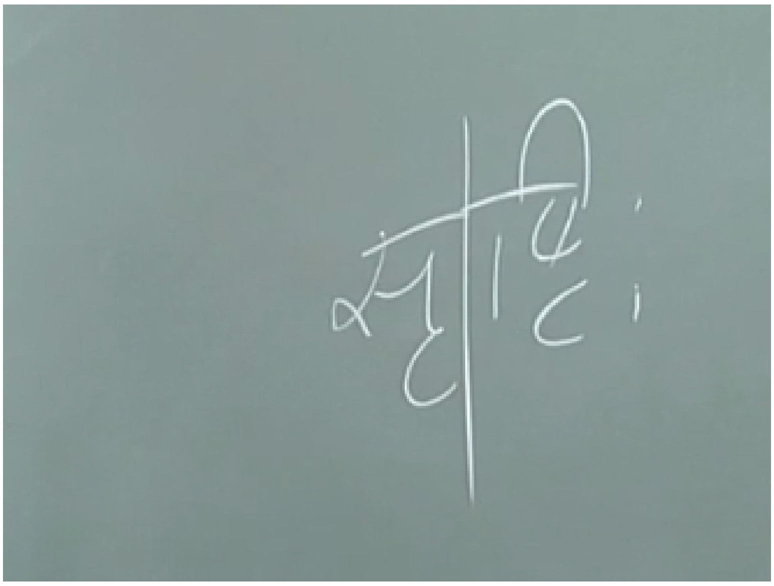

(FL) now this syllable are of 2 types, laghu the short syllable, guru the long syllable, a consonant with a short vowel is a laghu, unless it is followed by (FL) a consonant with long vowel is always a good, in the end of each foot of a metre the last syllable can be optionally taken to be good. This is the definition obviously it not be clear we nearly state it, so let us take some very interesting world the invocatory works of Kalidasa abhijnanasakuntalam.

(FL) so this is one foot of this meter, it has 21 syllables, now let us understand what are the Laghu and Guru in this. So first (FL) so first one is (FL) that should be a Guru, second civil now look it should be a laghu, but it is followed by a conjunct consonants (FL) so followed by (FL) that possible becomes Guru (FL) by again itself should be a Laghu it is a short vowel but it is followed by (FL).

(FL) it is again followed by (FL) therefore it is a guru (FL) is the first laghu in this so (FL) it is very complicated (FL) followed by a short vowel that is a laghu (FL) all these 3 are Laghu (FL) so till you come to (FL) are followed by (FL) therefore it is a guru, (FL) but it is followed by (FL) that followed by Kanchan consonant that is a Guru, (FL) now the important thing is all the 4 pathas have the same lughu guru structure.

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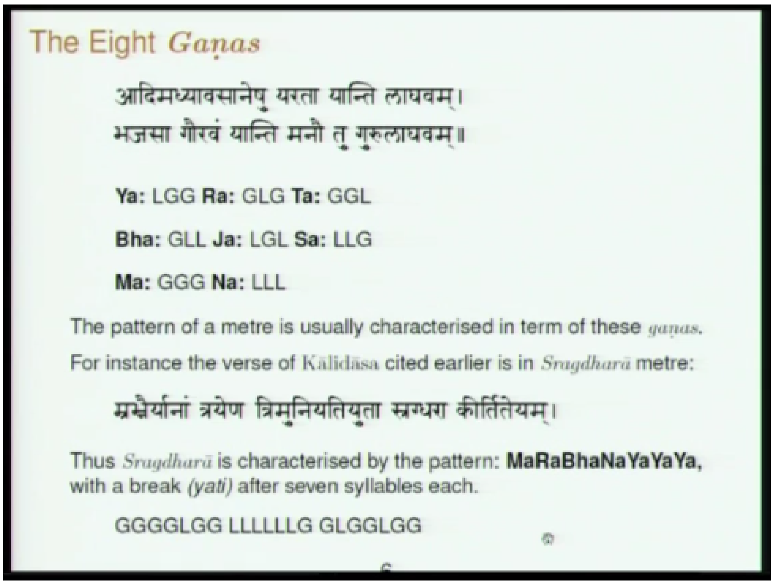

That is the basic point of what is called the (FL) now the way to read it actually we will see it in a minute, the points where you have to pass is also define in the definition of a metre eventually something called this (FL) very beautiful meet in Sanskrit languages which also used in several sound Indian languages in specification other languages also. So this (FL) its definition itself is given in terms of some units called ganas.

Instead of saying (FL) is given by GGGGLGG LLLLLLG GLGGLGG the way to define this gana meter in terms of what are called ganas, ganas are unit of 3 syllables each with a particular structure of laghu guru. So (FL) so the second part is very simple (FL) number 7, so there is a (FL), there is a pause after each unit of 7, after each unit of 7 syllable you should pause, so (FL) is characterised by pause that is a meter of 21 syllable.

There is a pause occur every 7 syllable and it has (FL) it has these ganas (FL) now what are these ganas, so 3 syllable each syllable can be laghu or guru therefore there will be 8 possibilities, so there are these 8 ganas and they have been given these things (FL) and how to remember that one way is to remember these words (FL) laghu at the beginning meet it at end, so a gana with laghu at the beginning which means other 2 are guru.

A gana with laghu at the middle, a gana with laghu at end, so they are called (FL) will have guru at the beginning, middle and end. (FL) so these are the 8 ganas in terms of these 8 ganas all classical meet us are characterised and of course it is not divisible by 3 you will say so many of these ganas and followed by a guru or laghu (FL) and the beauty is this definition is also (FL) the meter which is define.

(FL) so this definition of the meters (FL) is also classified in the (FL) it has MaRaBha Ma is GGG, Ra is GLG, Bha is GLL, Na is LLL, Ya as you can see is LGG then the triplet of LGG, LGGG, LGG (FL) so this si the correct (FL) and so that is the (FL) that is the way this (FL) is constituted.

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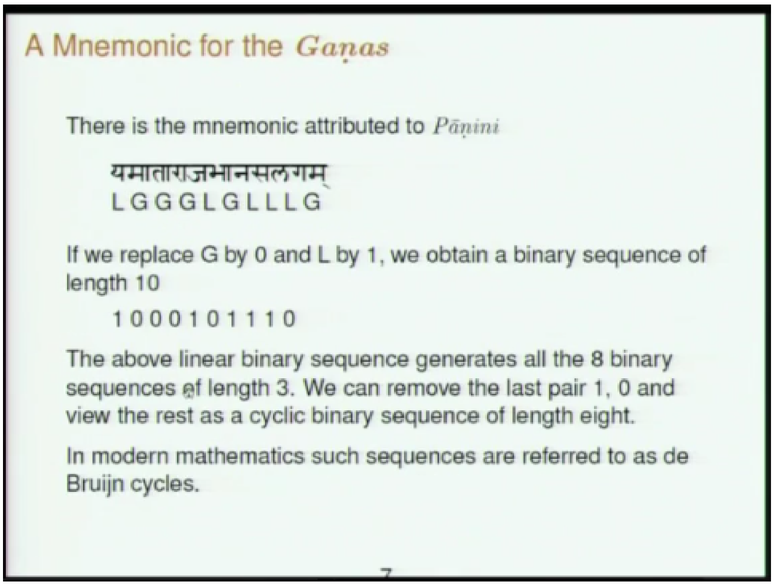

Now there is another very nice mnemonic to remember the ganas, this si the formula which is attributed to Panini. This is the formula which will not find in any classical Sanskrit work on prosody, so but sighting 4,5 years ago I think Donald Duck computer programming which are he wanted to know here it is (FL) various people searched. So there is a book on Telugu prosody by a man call Charles Brown written in 1843 where he has coated this.

But almost all students of Sanskrit know this is thought orally by everybody and general it said that this goes back to Panini. So what is this (FL) in this all the ganas are encoded linearly (FL) characterization of the (FL) Guru Guru Guru (FL) characterization of Jagannath that is lugu guru (FL) if you remember that you know what is (FL) and if you write replace guru by 0 laghu by 1 you have a binary sequence of 10 increase.

If you remove the last 210 which is same as the beginning 10, you can put them on a circle you have a binary sequence of length 8, now this binary sequence of length 8 is a special sequence this 100 1 001011. So each triplet here is a binary sequence of length 3, if you put this on circle give you put this on a circle you will find that it generates all possible triplets binary triplets of length 3 and such a cycle in today world called in communication cycle.

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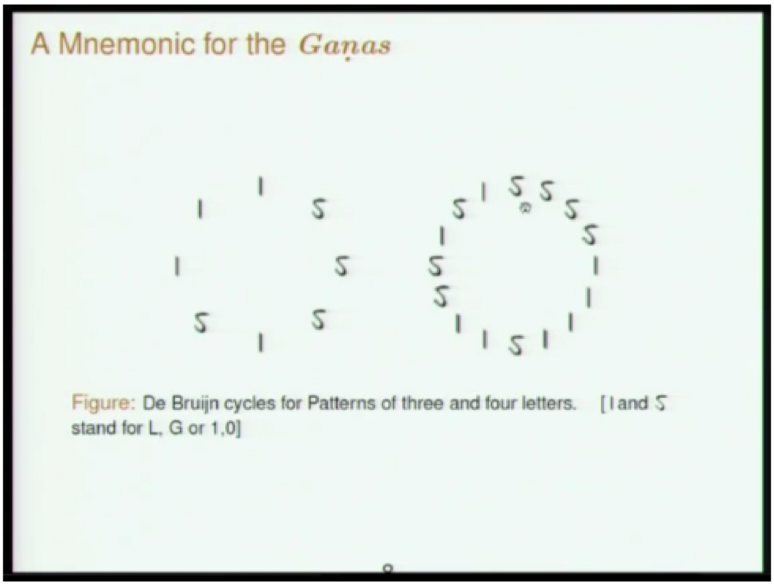

So (FL) is the oldest example of such a cycle, so you can have such cycles for binary sequence of length 4, there are 16 of them, again you can put them on a circle and generate all possible binary sequences of length 4 by a chord of length 16 like that in general.

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Pratyayas in Piṅgala's *Chandaḥśāstra*

In chapter eight of *Chandaḥśāstra*, Piṅgala introduces the following six *pratyayas*:

Prastāra: A procedure by which all the possible metrical patterns with a given number of syllables are laid out sequentially as an array.

Saṅkhyā: The process of finding total number of metrical patterns (or rows) in the *prastāra*.

Naṣṭa: The process of finding for any row, with a given number, the corresponding metrical pattern in the *prastāra*.

Uddiṣṭa: The process for finding, for any given metrical pattern, the corresponding row number in the *prastāra*.

Lagakriyā: The process of finding the number of metrical forms with a given number of *laghus* (or *gurus*).

Adhvayoga: The process of finding the space occupied by the *prastāra*.

But as well as concern the ganas are 8 and we need to know (FL) now we come to the (FL) all these ganas are defined in (FL) also. So in 8th chapter the last chapter of Chandahsastra I think like Panini Ashtadhyayi, Pingala Chandahsastra is also a as 8th chapter, he introduces 6 pratyayas of Chandahsastra. The first one is called prastara. Now (FL) had a particular structure in terms of Guru and Laghu.

It has 21 syllables, so at each place you can have Guru or laghu the particular choice has been made and you obtain the metres (FL) but you have to know like a mathematician that Pingala was so what are the possible meters with 21 syllables can you write them down, can you understand something about them. So these (FL) are basically dealing with these questions are called questions combinatorics.

And Indians were one of the greatest specialist in combinatorics, so whenever they had various things the first question is how to classify them, how to put them in a sequence and how to understand them, what follows what. So this is what Pingala does of all this made meters. So (FL) is a rule by which you can write down all the possible metres of a particular length. So if the length is only 3 there are only 8 possibilities these are 8 gana.

So you have an array of 8 rows, but if have 4 syllables an array of 16 rows which have 5 syllable 11 array, 32 rows in obviously when you reached (FL) it will be an array of very very large number 2 to the power 31. So (FL) is the rule by which you can write down, now one you have written down then the question is the next mathematical questions is suppose I tell you have certain role number then you tell me what is the metrical pattern that appears there.

And suppose I give you a metrical pattern then you tell me which row of the (FL) it belongs, so these 2 are mathematical questions, one is called (FL) another is called uddishta, it is called (FL) if you have written down the (FL) on the ground and then wind has blown and then (FL) gone away but then I want to know what is 20th row in the (FL) 5 syllabus (FL) 32 rows. So I want to know what is that is the 20th row.

So (FL) is the problem without doing the (FL) once again I should just go and write down what is the metrical form, what is the (FL) it appear in the prastara. Then there is (FL) which already have told you, it is 2 to the power n each slot each level can be laghu or guru, so if you have n syllable the number of possible need is 2 to the power n that is the (FL). Then (FL) deals with how many meters are there of a 7 syllable of which there are 3 gurus.

So how many 3 guru meters are there in 7 syllable prastara, so obviously this will lead us to the binomial coefficients which we saw in the introductory lecture that is (FL). So now we will going to each of them and as you can see the crucial mathematic that is laghu and guru and binary sequences. So the crucial mathematics that will appear now is binary mathematics and Pingala was the great originator of all the binary mathematics.

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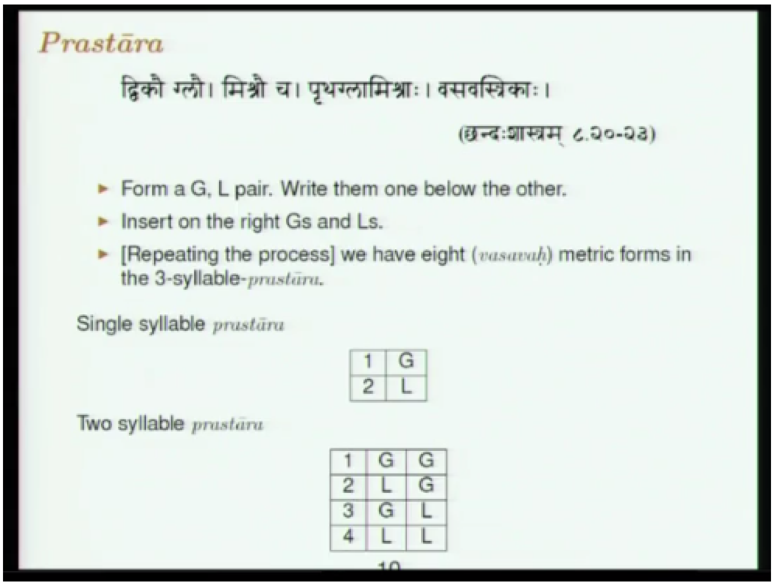

And this was of course he discovered in 1990. So first is prastara, so how do I list all (FL) is meter of the particular length, so if you have 1 syllable there only 2 G and L. If you have 2 syllable we have GG LG GL and LL. So Pingala is giving a rule in a very very qualified form is basically saying to form a pair G below L right that below each other and fill up the right with 2 Gs and 2 Ls.

So Pingala rule is take the prastara of the previous put it below itself, and in the right fill it up with equal number of Gs, and Ls, so let us see how we get the prastara of 3 syllable from the prastara 2 syllables, this is the prastara 2 syllable.

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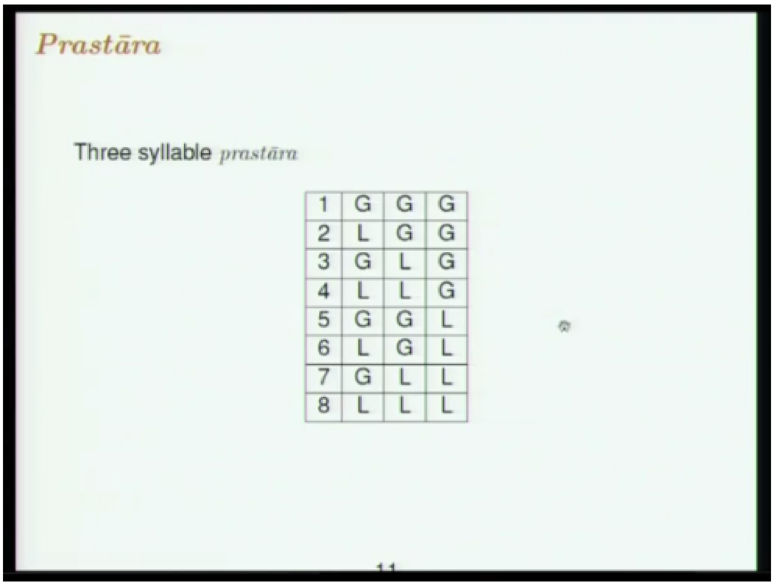

Let us go, so now this portion if you see this is the same as the prastara 2 syllable here, GG LG Gl, LL, GG, LG, GL, LL. So this portion is prastara 2 syllabus, this portion is prastara 2 syllables, in the right you write 4 G and 4 Ls have obtained the prastara 3 syllabus. So Prastara should have each and every metrical form of that when should appear one and only one and it should be a rule by which you generate all of them so that is the ruler of the prastara.

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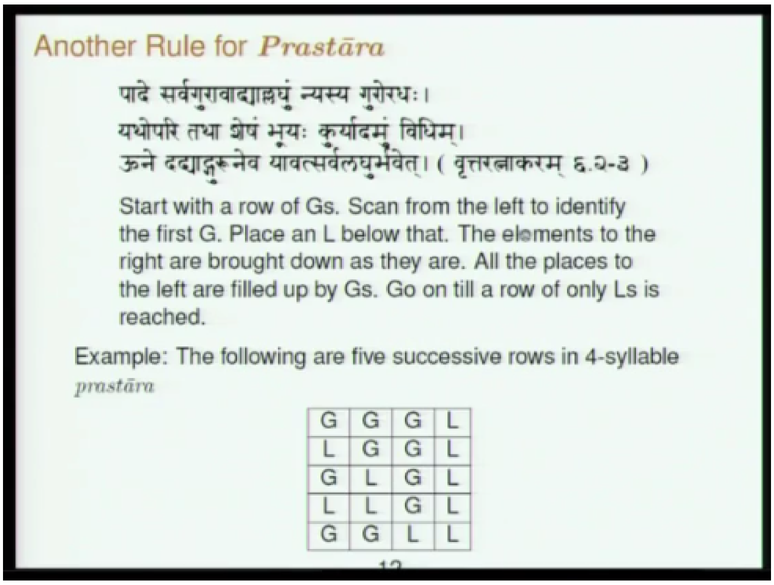

Today the word prastara will be a mostly in (FL) I will combinatorics I will tell you how the mathematical theory of prastara in music is discussed by (FL), the rule for doing prastara, this is a different kind of a rule supposing you start with some row of the prastara in 4 syllabus and you want to know the next row without having to start from the beginning and doing all that. So this rule is found in (FL).

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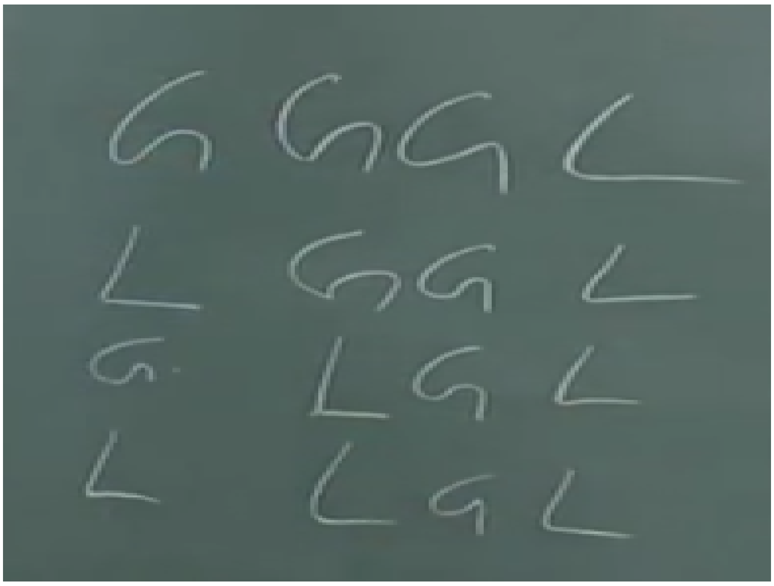

So going from one row to next row the rule is the following, start scanning from the left, once you encounter a G put a L below, bring down whatever it to the right as it is, if there is something else to be written in the left fill that up with the Gs, so let us do the next row, start from the left first time G is encounter put the L there, bring down whatever L is to the right as it is, in the left fill it up with Gs.

Next row start from the left below that G put an L, bring down what is there to the right as it is start from the left to identify the first G, out the L below that, bring down whatever is there to the right as it is, fill up the left with Gs, like that you can write, this is transforming one binary sequence to another in such a way that you cover all of them actually we know something transformation of all binary sequences from 0 to 1 today.

Something like that is what you meant here, so in this prastara you can see that are you can see that are you can see it here. So this is the four-syllable prastara, start with GGGG and then go down, there is one more way of doing this prastara, there all the first row is the first column is GL GL GL GL, second column is GG LL GG LL GG LL, third column is GGGG LLLL GGGG LLLL.

Fourth column is GGGG LLLLL, so that is what is called the (FL) super fast algorithm of Pingala to do the prastara. So now we understood prastara this is to list all possible metrical form of the given length. Now we can do a small thing and she is like the relation of this with the binary numbers, suppose I put wherever G of S0 and wherever L as I put 1, let macro environment take the 6th row here.

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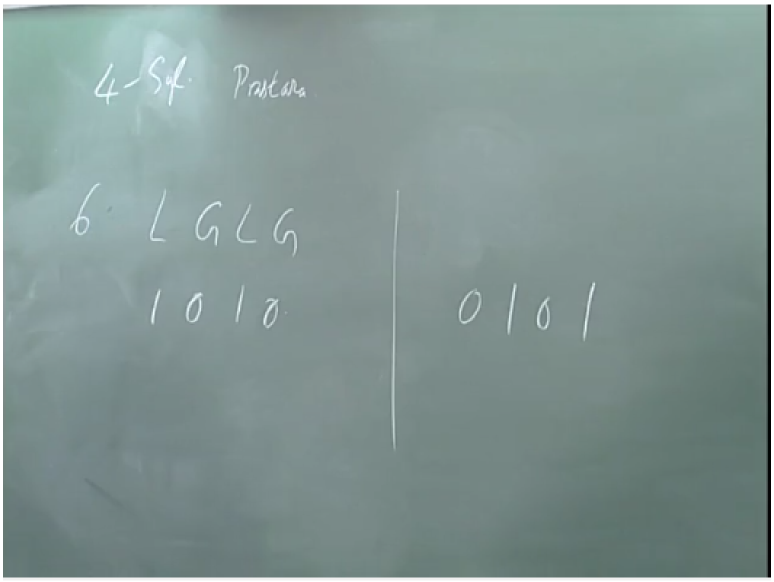

The 6th row of the four syllable (FL) prastara suppose I want to 6th row so the 6th row is LGLG, I put 1 where L as and the 0 the G as S, I take the mirror of this mirror image of this and you now view it as a ordinary binary number today during the design, you take the mirror image of this and understand the ordinary binary number, so as a binary number this will be +1 as a binary number this is 4+0+1, you have to add 1 to it.

Because this prastara is starting with the first row with GGGG is the binary number 0, so should add 1 to it, so if he said G=0, L=l is good to remember this to understand what everything like that because we are familiar with binary numbers we do not know Pingala therefore this is a good way of and then we see that each metric pattern is the mirror reflection of the binary representation of the associated road number -1.

So this is a representation of number 5, having understood prastara let us go to sankhya, today we know that this number is 2 to the power n now prastara does not say the number of metres of n syllables is 2 to the power n, he gives an algorithm for calculating it and the algorithm being an ancient algorithm happens to be one of the most efficient algorithm for calculation the n power of a number.

So the he is saying take this power so if you are looking for the prastara or n syllable take me number n, now start dividing by 2, if the number can we have you just write 2 somewhere just put a mark 2 somewhere. If the number cannot be half you detect 1 and instead of putting 0 at that point you put a symbol 0. So this is where the symbol 0 of S in Pingala (FL) it can be hard put it to 2 (FL) if you are to remove number 1 (FL) put mark 0 there.

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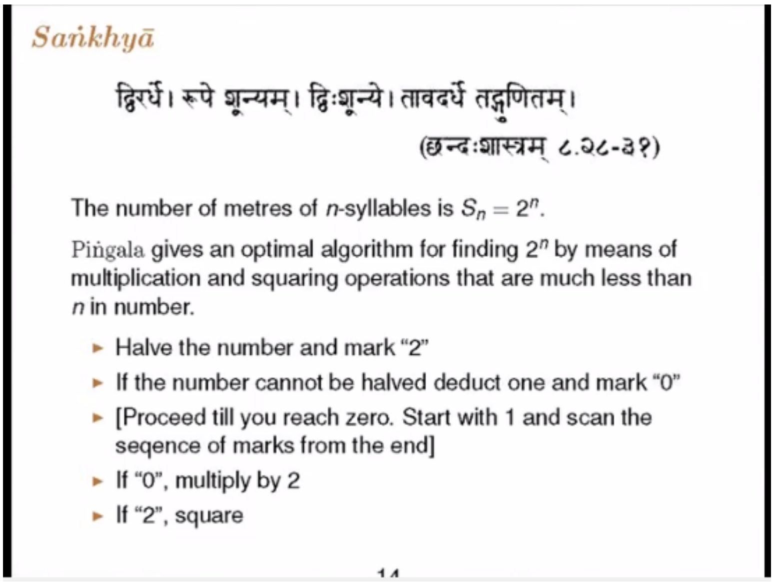

Then after you have mark all these 2 set 0 suppose you have something like this, start from the left (FL) wherever 2 of the wherever (FL) wherever 0 of S multiplied by 2, (FL) wherever 2 of S to square of the number. So ultimately to calculate 2 to the power n Pingala is giving you a sequence of operation which involves multiplying by 2 and squaring obviously 2 to the power n is multiplying 2 by itself n time.

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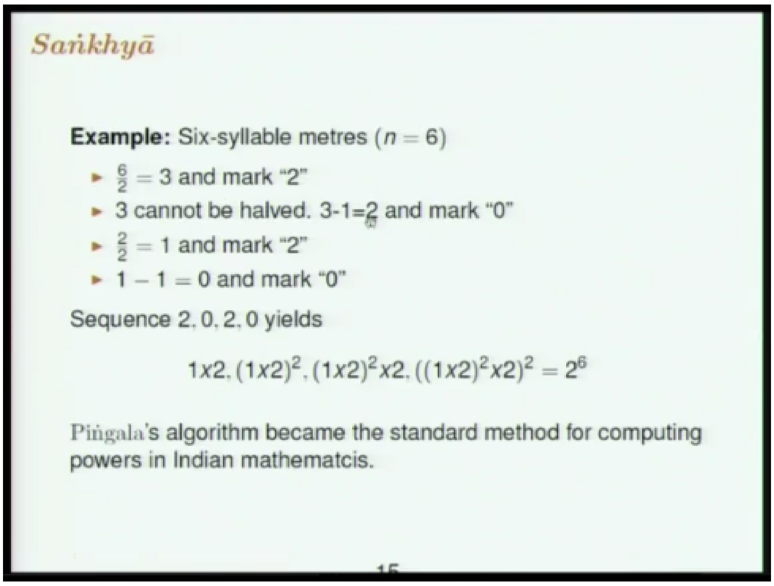

But what Pingala is giving you a algorithm which is much smaller number of set, so let us look at it with a simple example and this is equal to 6, so 6 is divisible by 2, so you mark 2, 3 cannot be half, so subtract 1 by 3 mark 0, 2 can be half you mark 1 you have reach subject 1 I mean mark 0. So you have a sequence 2 0, 20 and now from the right wherever 2 appears you multiply by 2. So corresponding to this 0 you start with the number 1.

Zero is there multiply by 2, 2 appear square the resulting thing, again 0 of S multiply by 2, 2 appears square that you get to you can justify this straight away by going back to the power whenever you can have it you can see you can go back by squaring it, wherever you cannot you can see it in also single multiplication and that is how this algorithm is ok. In fact Pingala became the standard way of calculating the n power of a number.

Normally the n power of a number of S in simple mathematics when we calculate the sum of a geometrical series. So in any book like I need to Sara sangraha Lilavati etc, the some of the geometrical series is given in terms of this algorithm and so modern scholars when they started looking at it they were totally dump found it, they did not know what has been, where is the geometric series and you know you have to calculate the n power of the each factor that appears in each term of the geometrical sequence.

And here we all have something divide by 2, but it appears that this actually is much faster way supposing you are a geometric sequence and submit up to 845 terms or something, instead of 845 operations you are something like log n instead of doing n operations after using the binary sequence of the number n. This reduces sequence something like log n, of course is not the most optimal algorithm taking the other data most powerful today.

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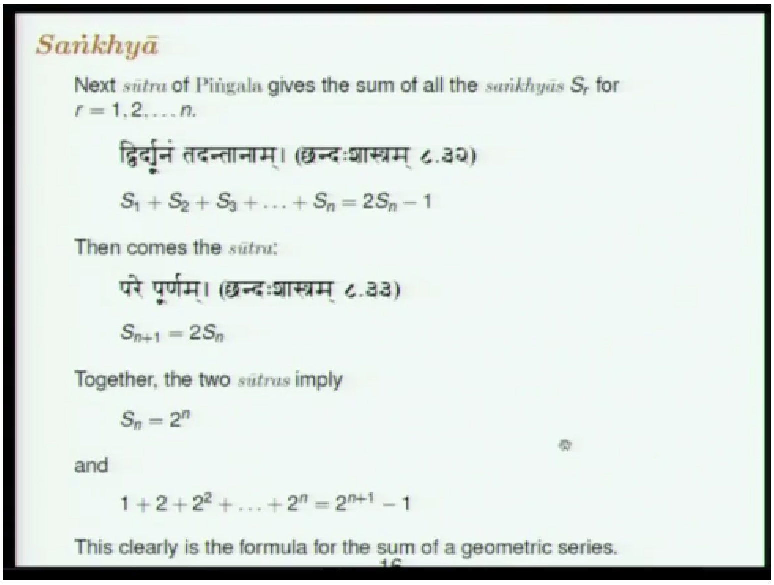

But today need much more complex devices. So Pingala then discusses some few other relations which are like some geometric series and things like that number of meters up to n syllables, then what is the relation between the number of meters in n+1 syllable with number m syllable. So clearly is very clearly away that 2 to the power n, but he never tells you that Sn is the number of metres of length n is 2 to the power n in the way we would do straight away.

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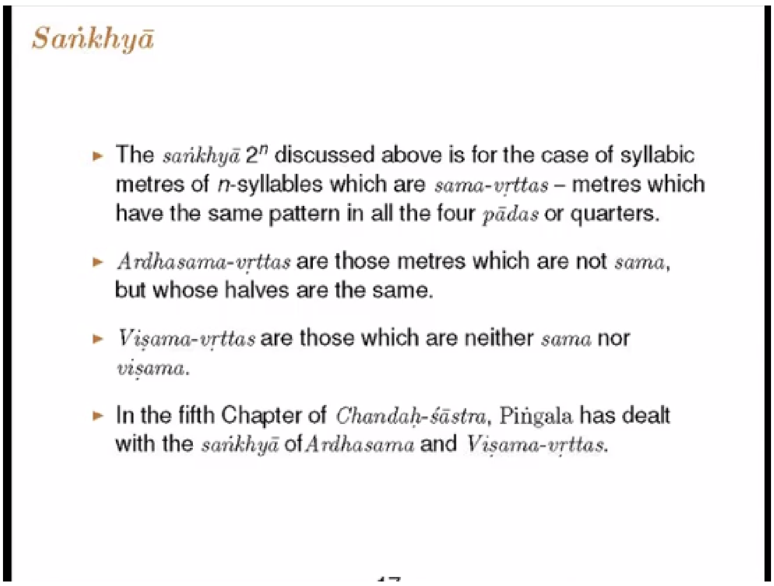

There is another thing Pingala also does, these meters that we have been discussing are what are called (FL) that is all the 4 metre is supposed to 4 feet, 4 pathas, so the Kalidas was that is at 4 padhas, all the 4 padhas if you have the same structure of lughu guru it is called a (FL). If the first and third has the same structure second and fourth have the same structure it is called (FL) and if it is not a (FL) it will be called (FL) and so next mathematical problem is how many (FL) of length L.

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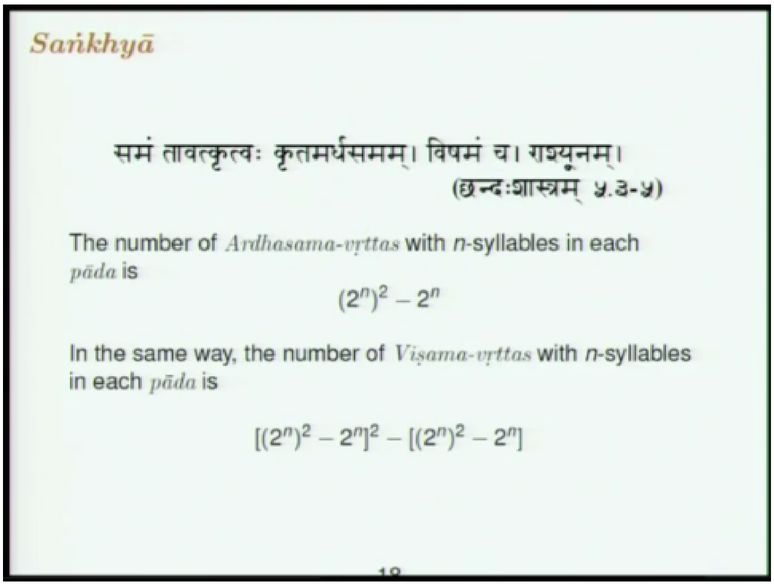

So Pingala in this chapter of this chandahsutra, he seems to be a very general mathematician as you can see his discussion on mathematical questions that arises not but these are the sum of those were very common (FL) causing the problem and giving a solution of it, (FL) so we know the number of (FL) length n is 2 to the power n. Now (FL) obviously will have n n n n the first 2 ns are all different.

Therefore it is 2n syllables, but it is 2 to the power n square if you want to understand it 2 to the power 2n, but (FL) meter are different from each other, so you should subtract the sum of the (FL) and therefore you get the actual at the sum of the (FL) the number is 2 to the power n square-2 to the power n. If you reflect on it for a minute you will see how this year. So (FL) where the first line of the meter and the third line of the meter structure.

Second line and fourth line are the same structure, so it is equal to having look all possibilities 2n, but we have to subtract the possibilities with the first and second are the same and therefore 2 to the power n squared-2 to the power n, in the same argument will lead to the number of (FL) of n syllable to be 2nsquare-2n whole things square-2n square-2n. All these getting having these 3 (FL) remove the number which was query.

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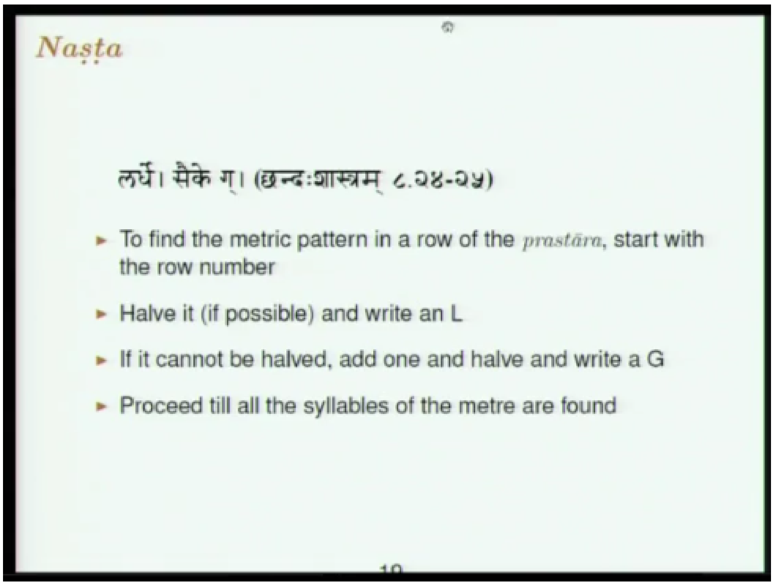

Now we go to the (FL) and basically as you can see the prastara is essentially table of converting this pattern in two binary numbers (FL) essentially involve knowing a number, how to find its binary representation and given a binary representation how to know the number essentially it is the basic binary mathematics. So (FL) Pingala has 2 rules (FL) you want to find out the metric pattern in a particular row.

So row k of a prastara of n syllables, if you start with k if it can be halved (FL) if you cannot be hard at 1 to it halve it (FL) and go on and fill up as many numbers as the, prastara is supposed to have and you will get the metrical pattern.

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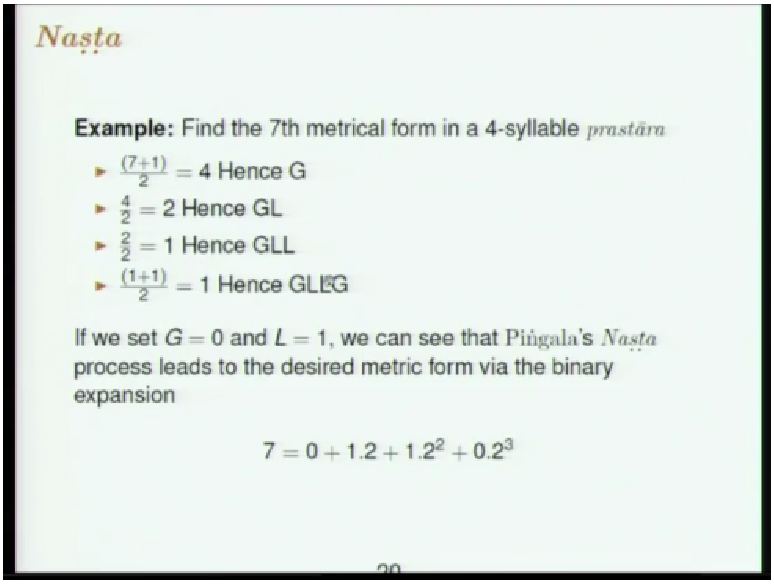

So we will quickly see the example, so we had this prastara four syllables, this is the prastara four syllabus right, way metres which have 4 syllable that is 16 in number, question is being so we are asking various question which with respect to this what is the 7 what is 8 meter and given any metre which row it appears, so it is good to remember that prastara (FL) find the seven metrical column in the four syllable prastara.

So we go back to the rule that he stated just before (FL). So 7 cannot be have add ½ you get a guru or you can so you get a guru followed by Laguna 2 can we have you get guru laghu laugh. Now you cannot stop here because you are looking for meters or four syllable. So do this operation once more you will get a Guru on the right.

So GL LG is the form which is in the rows 7 of that prastara obviously you know that you to be there but just to demonstrate that there are row 7 of that prastara. Suppose if you look simple for the case 4 but prastara of like (FL) something, it will be an interesting mathematical problem.

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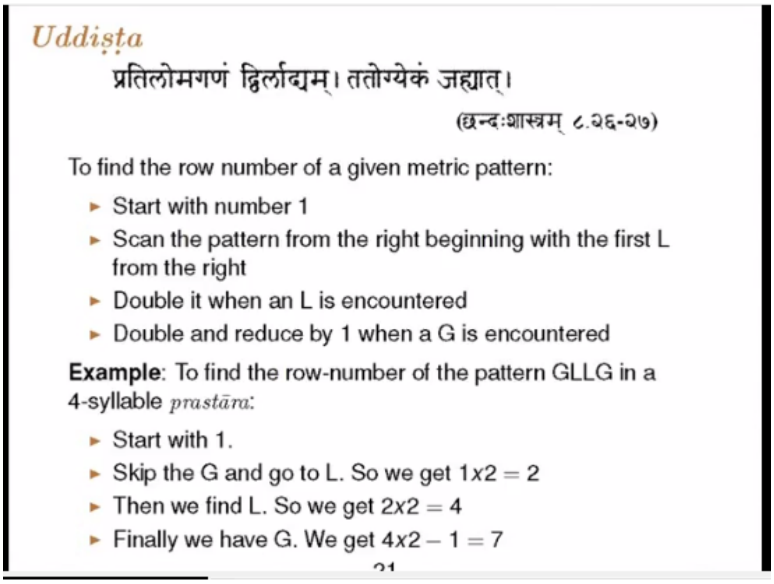

Now we go to uddista, so uddista is given a metrical pattern what is the row in which appears in the prastara. So again Pingala is only 2 sutras for this, (FL) you start from the opposite direction, scan the material pattern from the right, so GL GL LG here start scan it from right end start with number 1, whenever you find a laghu multiply it by 2, whenever you find a Guru you multiply by 2 and remove 1.

(FL) you multiply by 2 and remove 1, so this is the rule of Pingala start from the right, so GL LG we have start with 1 that is just starting number, so whenever you beginning you go to a g nothing will happen 1*2-1 is 1. So you have to really come to the first laghu that is why he saying (FL) you can just go up to the first laghu that you will find here, so the first laghu is here. So when we find the laghu you multiply by 2, 1*2 is 2.

Then again you find another laghu you multiply that by 2 your get 4, finally you have a guru so this 4 has to be multiplied by 2 and 1 subtracted you again come back with 7, in that this we have seen just a minute ago that this was the seventh metrical pattern have just given us the process. Now there is a much simpler way calculation this is much simpler what Pingala is given is optimal algorithm.

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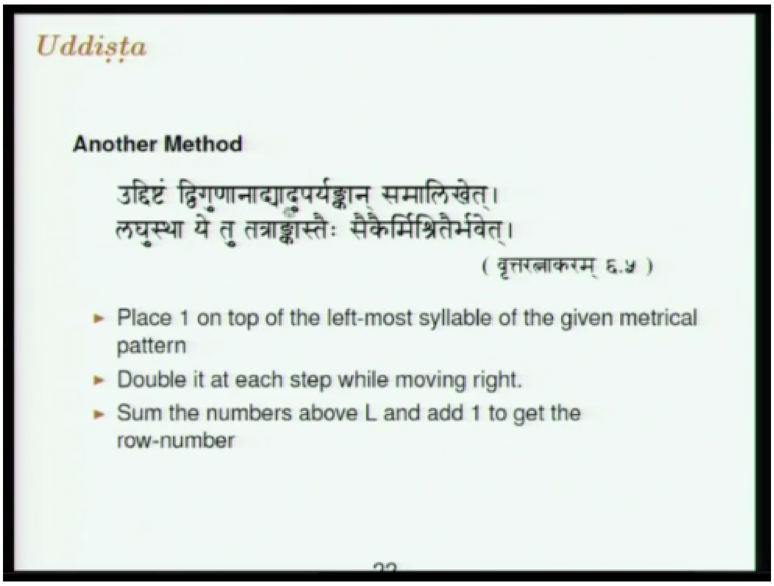

But people like us would like to know it in relation to binary numbers be there is a much simpler way of doing the (FL) process. So this si described in (FL) so to do this uddisata process so let us take some (FL) GLLGL so this is a or you can put one more so this is a 6 level prastara you want to do from the left start with 1 and keep multiplying it by 2 at each state.

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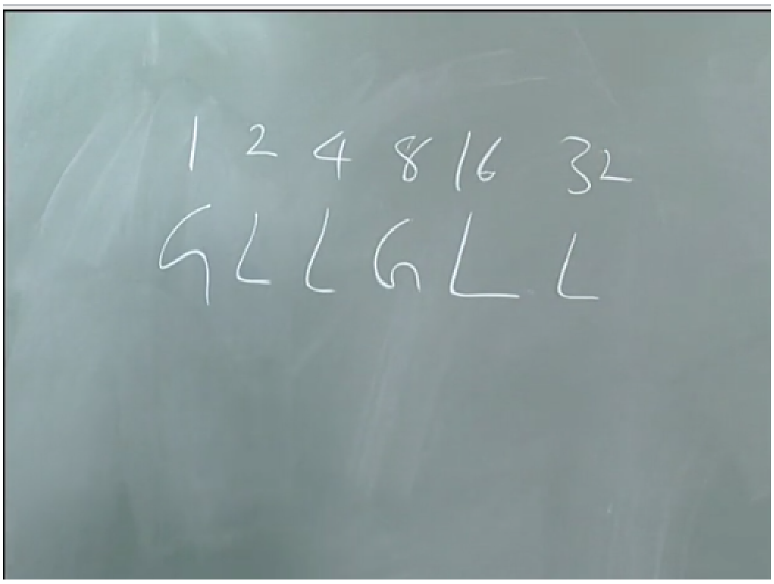

And add all the numbers above L and add 1 to it, so add all these numbers so this is 2+4+16+32+1. So that is the row in which this pattern will appear in the 6 syllable prastara. So basically it is 2 to the power 1, 2 square, 2 cube, 2 to the power 4, 3 to the power 5, so you have 1*2 to the power 1, + 1*2 square+1* 2 to the power 4+1*2 to the power 5+1 and remember if I had written this number in terms of if I had written this number in terms of binary code putting 0 for G and 1 for L.

For this is how this will appear and if I take a mirror image of this while doing a mirror image of this, this is 110, 110 now you will see this rule stated that teach place value in binary will correspond to the power that we are multiplayer G of s is all 0, so you do not add only the L have to be added with the appropriate binary place value and that is the rule Pingala does not want to give this rule is rule is much more simply because you do not have to go on calculating all this powers of 2 which you do not need.

So whenever G of s multiply by 2 and subtract 1 whenever laghu of the laghu of s is double and keep going, that is much optimal algorithm that he gives us (FL) for GLLG I have written down the (FL) process from the left write 1, 2 to 2 square 2 cube, wherever L of s add those numbers that will be the, so both (FL) processes of Pingala are essentially based on the fact that every number has a unique binary representation.

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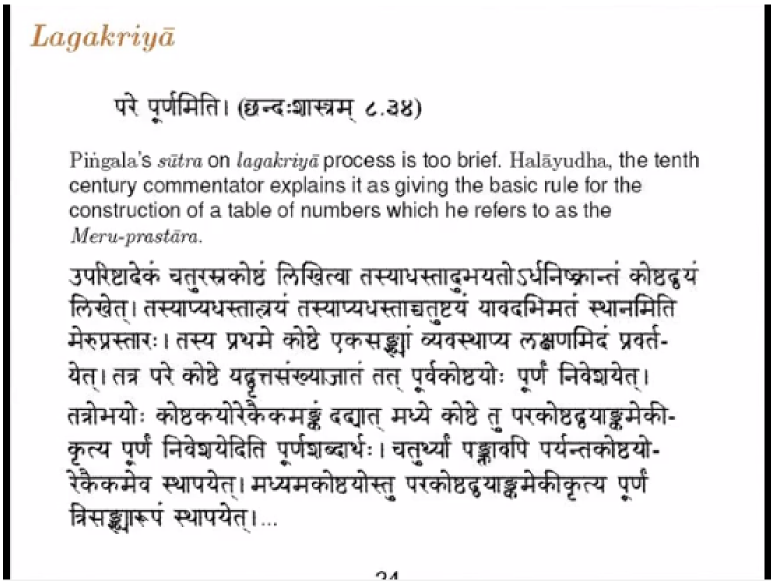

That is every number can be written unique clear the summer powers of 2 to the power n. Now we come to the last (FL) which is called the lugakriya, so given in the en syllable prastara, how many metres are there which have 5 Gurus or how many metres are there which have 7 laghus. So that kind of a question Pingala has given just one sutra at this array (FL) which is the last sutra Pingala Chandahsastra.

And what was the previous sutra the previous sutra, the previous sutra was (FL) and then that is followed with the sutra (FL) in that the sum convention hall so that you repeat the sutra when you reach the end of text. So many people said that this is nothing to do with (FL) just the statute repetition of the sutra in the end. But the commentators edit edition does not think so the commentator (FL) is very clear that Pingala is explaining how to calculate.

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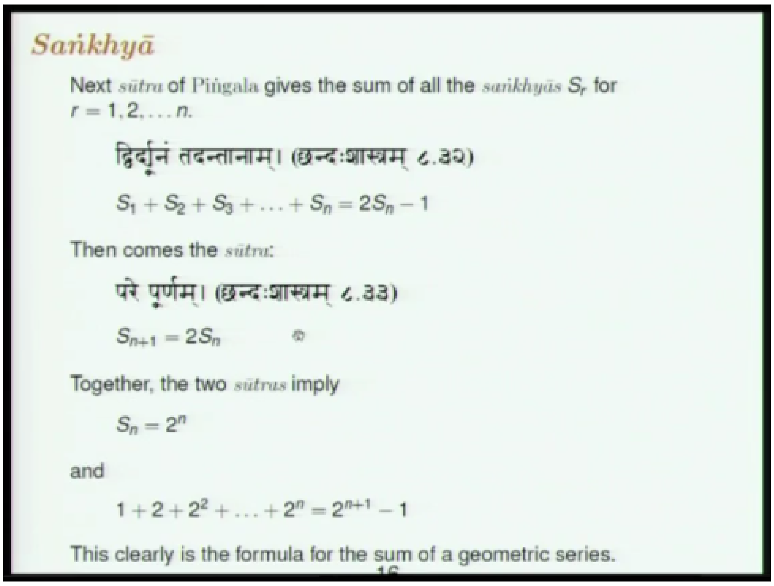

How to do the (FL) process with this sutra. So he is saying in that that previous (FL) I did not explain that sutra 8, 33 was (FL) there what Pingala was saying is the following, that is the previous (FL) which is the number of meters of all syllables up to number n is equal to twice the number of metres which are length n+/1 (FL) all all metres of length 1,2,3,4 up to n then the number of possibilities is twice the number of metres of length n-1.

And then the next (FL) Sn+1 the number of meters of length n+1 is just twice the previous number without the reduction of 1, so (FL) subtracting 1 (FL) the number of metres of length n+1 is just Poonam the whole of double of the number of meters of the previous length Sn, Sn+1 is 2Sn. This for the meanings of the previous 2 sutras of Pingala, but now then he comes with one more sutra here (FL) this will give us the (FL).

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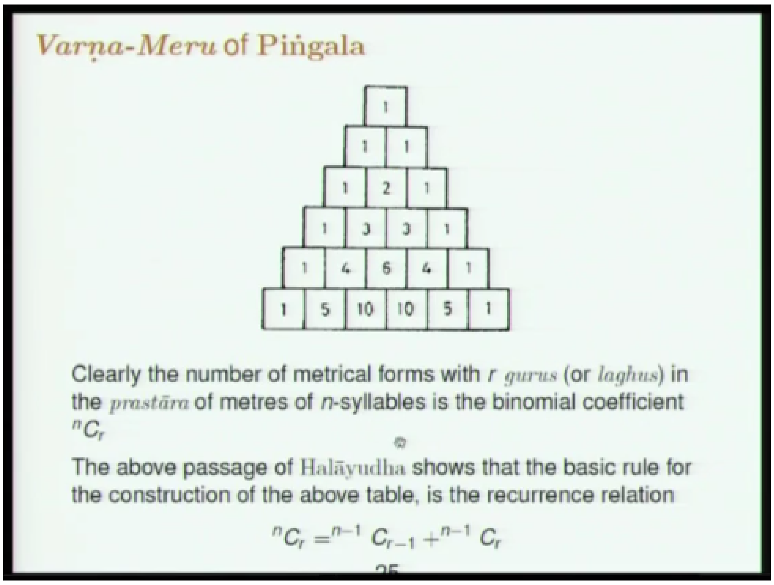

So what is (FL) we will come to this quotation in a minute let us just see the way how will explain, (FL) says you form this figure, how do you form this figure, first you write the square with 1 entry below that you write 2 square, below that you write 3 squares, below that you write 4 squares like that you go on, in the first square you put 1, in the next 2 squares you put 1. Then below that in any square you put the sum of the numbers which is above it.

So here you only put 1, in 2 you put both of them and here it is 1 so (FL) the total sum of the total sum of the 2 cells that are above it is to be entered. So that is the meaning of (FL) so then in the next row 1, 3 is sum of 1*3 sum of 11, 1 here 4 in sum of 1 in 3, 6 is sum of 3 and 3, 4 is the sum of 3 and 1, 1 and by now also know what these are these are nothing but the terms which will appear when we do a+b to the power n or 1+1 to the power n.,

These are the numbers which appear as the various terms and so each of them is a binomial coefficient, MCR stands for (FL) is a number of combination or object n object and this rule that each number is the sum of the two numbers above is essentially a recurrence relation for this binomial coefficient, it is a well-known reconciliation for the binomial coefficient that is the Pingalas recurrence relation which is coming in (FL).

So we can now read this I will just to feel happy that are you there is actually saying it (FL) written 1 square about below that half going outside right to space that is the square root touch in the middle of the first square. So that is how is to write (FL) below that right 3 squares, (FL) below that right 4 squares, (FL) till the number of syllables that you are considering or considering syllables (FL) 7 syllable have to go to 7 throw.

This is called f (FL) so put number 1 in the first (FL) put into effect the following recurrence relation (FL) so whatever number that has come up in the upper 2 cell you put the sum of the whole of it in the next row, that (FL) so then he explains what will be the first row, then what will be the second row etc. etc. that is how (FL) is explain (FL). Now this si the way the (FL) portion also studied in modern times.

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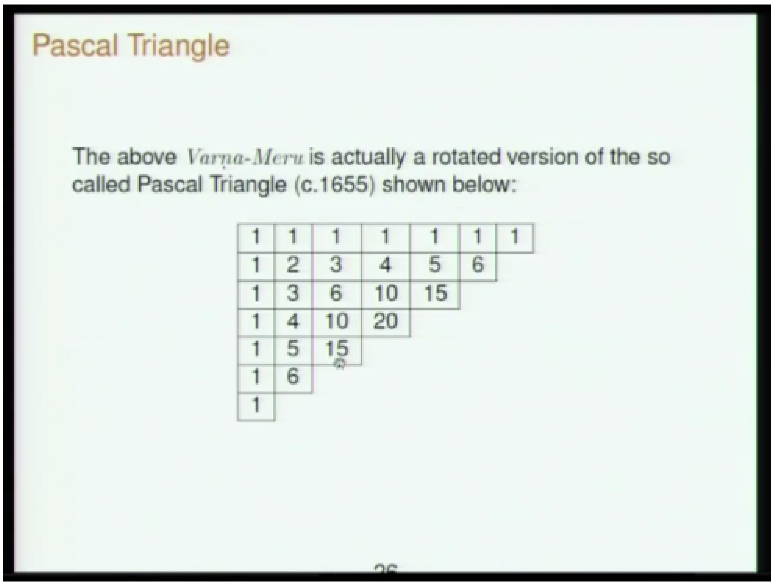

And this figure has a very very famous name in modern time it is called the Pascal triangle slightly rotated (FL) you rotate it slightly it is what is called Pascal Triangle. Pascal is a very famous mathematician wrote a tract on this and he discuss the properties of all the binomial coefficient coming out of it. Of course the history of this goes back to Pingala.

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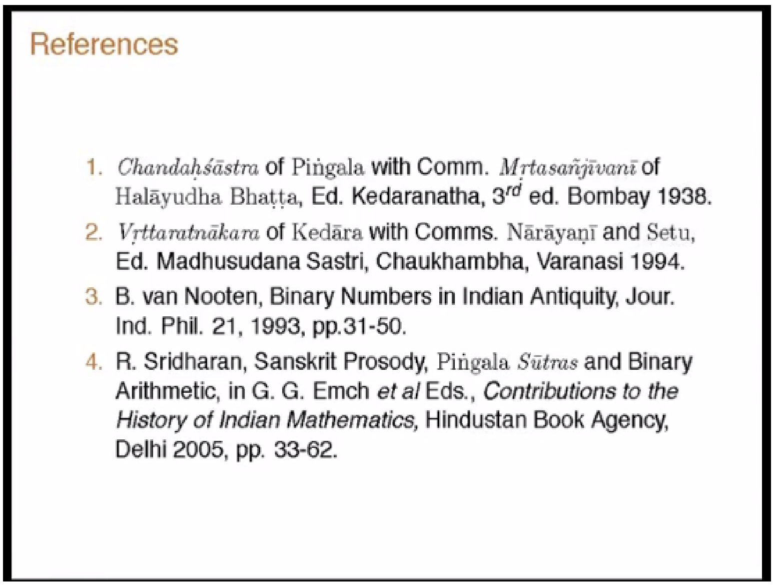

But there are many many other cultures with Chinese and Islami also have this Pascal triangle much before as well obtained, it is an unfortunate thing that all our terminology refers to events that happened in the past 2, 3 centuries and we really are not aware of the contributors to development of all our ideas. So with this we come to a end of the study of the combinatorial techniques that Pingala started.

But he started really a host of problems now whenever once you are set the protagonist of the study salary come whenever you have a list of things immediately question is can I give a rule by which all this can be put in some order and then once I put them in some order, what place is of them can I do (FL) is not a very very common problem in today's computer science also, its called permutation generation combination generation.

Everywhere generation is there, you have what is called ranking unranking, so that is such that (FL) so that sort of the virus that Pingala started his infected entire history of combinatorics and everywhere this question of yours and at each place it gives rise to some very very interesting mathematical properties here you saw entire binary arithmetic coming out of the theory of the (FL) it is very interesting topic.

And Pingala just started and this will be seen later in prosody other aspects of prosody in music, if you can see in medicine, if you can see in astrology, if you can see in architecture in several subjects we will discuss some of them later, thank you very much.