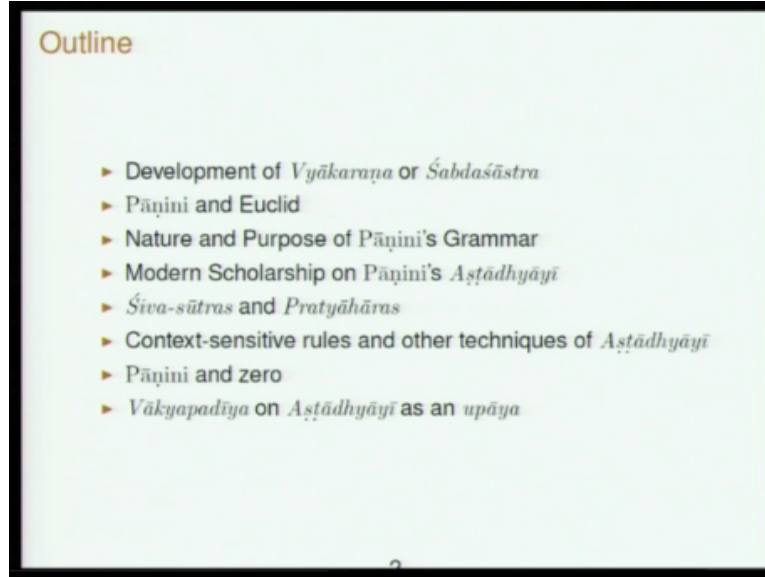


**Mathematics in India:
From Vedic Period To Modern Times
Prof. M.D. Srinivas
Centre for Policy Studies, Chennai**

**Lecture-4
Panini's Astadhyayi**

In this lecture we shall deal with the work of Panini.

(Refer Slide Time: 00:21)



And other grammarians in India and the kind of influence they have had on the development of mathematics. So in this lecture I will say something about the development of vyakarana sabdasastra grammar, something about the centrality of Panini in Indian tradition, some discussion of the nature and purpose of Paninis's grammar, few quotes on model scholarship as understood Panini Astadhyayi.

We will in particular discussion something about the sulbasutras and the kind of techniques that arise from them, the nature of context sensitive rules and other techniques in (FL), something about the idea of zero and Panini and I will say something about the (FL) how it says that Panini's grammar is (FL) we will explain what it is (FL).

(Refer Slide Time: 01:12)

Development of *Vyākaraṇa* or *Śabdaśāstra*

- ▶ Pre-Pāṇinian: Yāska's *Nirukta*, *Prātiśākhya* Texts, *Āpiśali*, *Indra*, *Kāśakṛtsna*, *Śākaṭāyana*, *Vyādi*, etc
- ▶ Pāṇini (c.500 BCE): *Aṣṭādhyāyī Sūtrapāṭha*, *Dhātupāṭha*, *Gaṇapāṭha*
- ▶ Kātyāyana: *Vārttika*, *Pāli-vyākaraṇa*
- ▶ Patañjali (c.100 BCE): *Mahābhāṣya*
- ▶ Śarvavarman: *Kātantra-vyākaraṇa*
- ▶ Candragomin (c.450 CE): *Cāndra-vyākaraṇa*
- ▶ Devanandin (c.450): *Jainendra-vyākaraṇa*
- ▶ Bhartṛhari (c.450): *Vākyapadīya*, *Mahābhāṣya-dīpikā*

There are many pre-paninian text, Panini (FL) is not the first the grammar of Sanskrit, there are many pre-paninian grammarians (FL) we all know there were several (FL) text in various (FL) of the Vedas, Panini himself refers to officially (FL) text with 8 chapters, (FL) that is the collection of sutras, there is a there are 2 alike text with the (FL) on Panini Astadhyayi was written by katyayana (FL) Patanjali.

(Refer Slide Time: 02:16)

Development of *Vyākaraṇa* or *Śabdaśāstra*

- ▶ Jayāditya, Vāmana (c.600): *Kāśikāvṛtti*
- ▶ Jinendrabuddhi (c.900): *Kāśikāvivaraṇa-panjikā* or *Nyāsa*
- ▶ Kaiyaṭa (c.900): *Mahābhāṣya-pradīpa*
- ▶ Haradatta (c.1000): *Padamañjarī*
- ▶ Dharmakīrti (c.1000): *Rūpāvatāra*
- ▶ Hemacandra (c.1100): *Siddhahaimacandra*, etc
- ▶ Vopadeva (c.1250): *Muḡdhabodha*

There were other systems of grammar also in the post-Panini era (FL) is a very famous philosopher, he wrote a very important text (FL) he also wrote a commentary on mahabhasya, he all know that he wrote that the famous (FL) Jayaditya Vamana wrote the first simple commentary on Panini (FL) this was commentary upon Jinendrabuddhi, Kaiyata wrote a commentary on Mahabhashya.

Dharmakirti is a Buddhist grammarian wrote the text book Avatar, Hemaandra was a scholar of several disciplines in Gujarat, he wrote grammar of (FL) also. Vopadeva passed book about the (FL) standard text like (FL) north India. A new tradition of grammarians the using the tradition called (FL) tradition start at in 1415 century Ramacandra Prakriyakaumudi.

(Refer Slide Time: 03:26)

Development of *Vyākaraṇa* or *Śabdaśāstra*

- ▶ Rāmacandra (c.1350): *Prakriyākaumudī*
- ▶ Nārāyaṇā Bhaṭṭātīri (c.1600): *Prakriyāsarvasva*
- ▶ Bhaṭṭoji Dikṣita (c.1625): *Siddhāntakaumudī*,
Praṣṭhāmanoramā, *Śabdakaustubha*
- ▶ Kaunḍabhaṭṭa (c.1650): *Vaiyākaraṇabhūṣaṇa*
- ▶ Varadarāja (c.1650): *Laghu-siddhāntakaumudī*,
Sāra-siddhāntakaumudī
- ▶ Nāgeśabhaṭṭa (c.1700): *Mahābhāṣya-pradīpoddyota*,
Bṛhacchabdenduśekhara, *Vaiyākaraṇa-siddhāntamañjūṣā*,
Paramalaghumañjūṣā, *Paribhāṣenduśekhara*

5

Narayana Bhattatire a well-known author of Narayana he was a student of the great astronomers (FL) can the standard text which was learnt by students Panini's grammar was hardly studied in the original format after 17 century, people studies Bhattoji Diksita Sihhantakaumudi or several which causes inflammation which is what is study today, Varadaraja Laghu-siddhantakaumudi sara-siddhantakaumudi.

Discussion of philosophy of grammar was equally important in this modern period (FL) then of course most of you are aware of the great grammarian and philosopher Nagesabhatta in 18 century who wrote several very important text Paramalaghumanjusa, Praibhasendusekhara many many other important.

(Refer Slide Time: 04:26)

Development of *Vyākaraṇa* or *Śabdaśāstra*

Grammars of Other Languages

- ▶ Tamil: *Tolkāppiyam* (c.200 BCE), *Vīrasolīyam* (c.1200), *Nannūl* (c.1300)
- ▶ Kannada: *Karnāṭaka-bhāṣābhūṣaṇa* (c.1100), *Śabdamaṇidarpaṇa* (c.1200), *Karnāṭaka-śabdānuśāsana* (c.1600)
- ▶ Telugu: *Āndhra-śabdacintāmaṇi* (c.1100), *Āndhrabhāṣābhūṣaṇa* (c.1250), *Trilinga-śabdānuśāsana* (c.1300)
- ▶ Pali: *Kaccāyana-vyākaraṇa*, *Saddalakkhaṇa* (c.1150)
- ▶ Prakṛita: *Prākṛita-prakāśa*, *Prākṛita-śabdānuśāsana* (c.1200)
- ▶ Persian: *Pārsīprakāśa* (c.1575)

Parallely there was an influence on grammars in other languages also, Tamil had Tolkappiyam in very ancient time, then Virasoliyam Nannul, Kannada had Karnataka- bhasabhusana, sabdamanidarpana, karnataka-sabdanusanana, Telugu had Andhra-sabdacintamani, Andhabhasabhusana, Trilinga-sabanusasana, Pali is a very important from ancient kaccayana (FL) there are later grammar like saddalakkhana.

(Refer Slide Time: 05:13)

Śāstras: Present Systematic Procedures

Most of the canonical texts on different disciplines (*śāstras*) in Indian tradition do not present a series of propositions; instead they present a series of rules, which serve to characterize and carry out systematic procedures to accomplish various ends.

These systematic procedures are generally referred to as *vidhi*, *kriyā* or *prakriyā*, *sādhana*, *karma* or *parikarma*, *karaṇa*, *upāya* etc., in different disciplines.

The rules are often formulated in the form of *sūtras*.

As explained in the (FL) also that sastras in India do not initiate a set of propositions, they explain as a procedures, and these procedures are in different sastras then I could have remain like (FL) etc. and the basic text of various sastras actually and for these procedures in the form of sutra.

(Refer Slide Time: 05:38)

Śāstras: Present Systematic Procedures

According to *Viṣṇudharmottarapurāṇa* (3.5.1): A *sūtra* has to be concise, unambiguous, pithy, comprehensive, shorn of irrelevancies and blemish-less.

अल्पाक्षरमसन्दिग्धं सारवद् विश्वतोमुखम्।
अस्तोभमनवदाद्य सूत्रं सूत्रविदो विदुः ॥

Pāṇini's *Aṣṭādhyāyī* is acknowledged to be the paradigmatic example of a canonical text in Indian tradition. All other disciplines, especially mathematics, have been deeply influenced by its ingenious symbolic and technical devices, recursive and generative formalism and the system of conventions governing rule application and rule interaction.

There is very ancient saying of found (FL) or what a sutra is, it just explain the major characteristic of sutra (FL) that it has to be concise, unambiguous, pithy, comprehensive, shorn of irrelevances, blemish-less etc. This generally acknowledge that Panini's Astadhyayi is the paradigmatic example of such a sutra text.

(Refer Slide Time: 06:20)

Pāṇini and Euclid

"In Euclid's geometry, propositions are derived from axioms with the help of logical rules which are accepted as true. In Pāṇini's grammar, linguistic forms are derived from grammatical elements with the help of rules which were framed ad hoc (i.e. *sūtras*)..."

Historically speaking, Pāṇini's method has occupied a place comparable to that held by Euclid's method in Western thought. Scientific developments have therefore taken different directions in India and in the West...

In India, Pāṇini's perfection and ingenuity have rarely been matched outside the realm of linguistics. Just as Plato reserved admission to his Academy for geometers, Indian scholars and philosophers are expected to have first undergone a training in scientific linguistics..."¹

Note: The word "derived" means "demonstrated" in the case of Euclidean Geometry; it means "generated" in the case of Pāṇini's Grammar (*upapatti* and *niṣpatti*)

¹J. F. Staal, *Euclid and Pāṇini*, *Philosophy East and West*, 15, 1965, 99-116.

As explained again in the introductory top Panini holds the same kind of position in Indian tradition as a Euclid, Euclid has in the (FL) European tradition.

(Refer Slide Time: 06:45)

Śabdānuśāsana: Pāṇini's Aṣṭādhyāyī

अथ शब्दानुशासनम्।

अनुशासनं प्रकृतिप्रत्ययविभागेन व्युत्पादनं तद्वाकरणेन साक्षात्क्रियत इति साक्षात्प्रयोजनम्।

[अन्नम्भट्टीय-प्रदीपोद्घोतव्याख्या]

अथैतस्मिन्शब्दोपदेशे सति किं शब्दानां प्रतिपत्तौ प्रतिपदपाठः कर्तव्यः गौरश्वः पुरुषो हस्तौ शकुनिर्मृगो ब्राह्मण इत्येवमादयः शब्दाः पठितव्याः। नेत्याह। अनभ्युपाय एव शब्दानां प्रतिपत्तौ प्रतिपदपाठः। एवं हि श्रूयते बृहस्पतिरिन्द्राय दिव्यं वर्षसहस्रं प्रतिपदोक्तानां शब्दानां शब्दपाठायणं प्रोवाच नान्तं जगाम।

[पातञ्जलमहाभाष्यम् पस्पशाह्निकम्]

10

Now what is the basic purpose of Panini's grammar which are in purpose of Panini's grammar (FL) is the first (FL) it is the first (FL).

(Refer Slide Time: 07:15)

Śabdānuśāsana: Pāṇini's Aṣṭādhyāyī

Now, the instruction of utterances

Instruction, namely generation (of utterances) by using *prakṛti*, *pratyaya* and other components, this is done by grammar, and that it is its direct purpose.

[Annambhaṭṭīya-Pradīpoddhyotavyākhyā]

Valid utterances cannot be taught by *pratipada-pāṭha* (stating each of them individually). Bṛhaspati tried to teach Indra valid utterances by *pratipada-pāṭha* for thousand divine years, but reached nowhere near the end.

[Mahābhāṣya of Patañjali, Paspasāhnikā]

11

He has written a (FL) which is written on commentary on (FL).

(Refer Slide Time: 07:30)

Śabdānuśāsana: Pāṇini's Aṣṭādhyāyī

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[*Mahābhāṣya* of Patañjali, *Paspaśāhnikā*]

11

Instruction namely generation of seconds (FL) by using prakṛti, pratyaya in other component. So the words of Sanskrit are generating or produce like you have sort of book you have various basic unit then you sort of the so that is the nature of the Panini's grammar. That you have the basic units prakṛti, pratyaya and things like that in other component and from that you produce valid utterance, valid (FL) that is the valid use of Panini's grammar.

(Refer Slide Time: 08:06)

Śabdānuśāsana: Pāṇini's Aṣṭādhyāyī

अथ शब्दानुशासनम्।

अनुशासनं प्रकृतिप्रत्ययविभागेन व्युत्पादनं तद्वाकरणेन साक्षात्क्रियत इति साक्षात्प्रयोजनम्।

[अन्नम्भट्टीय-प्रदीपोद्घोतव्याख्या]

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[पातञ्जलमहाभाष्यम् पस्पशाह्निकम्]

12

And the same thing is explained in a different way by Patanjali who is explaining that what is an ideal way to teach correct sentences, correct word. So the ideal ways who list all of them and then he says the (FL) tried to follow this method to inspect Indra and he took 1000 divine years near each divine is supposed to be 360 years. So it took 1000 divine years to teach Indra, a list all valid the word, sentences etc. and he could not list it at end.

But of course that is the ultimate method that you really want to say without any doubt that you know the correct sentences, you have to be able to have a list with you, but that is not even be possible. So we have to do something we have to take records to a different methodology.

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Śabdānuśāsana: Pāṇini's Aṣṭādhyāyī

कथं तर्हीमे शब्दाः प्रतिपत्तव्याः। किञ्चित्सामान्यविशेषवल्लक्षणं प्रवर्त्यम्। येनाल्पेन यत्नेन महतो महतः शब्दौघान् प्रतिपदोर्न्।

किं पुनस्तत्। उत्सर्गापवादौ। कश्चिदुत्सर्गः कर्तव्यः कश्चिदपवादः। कथंजातीयकः पुनरुत्सर्गः कर्तव्यः कथंजातीयकोऽपवादः। सामान्येनोत्सर्गः कर्तव्यः। तदुथा। कर्मण्यण् (३.२.१)। तस्य विशेषेणापवादः। तदुथा। आतोऽनुपसर्गं कः (३.२.३)।

[पातञ्जलमहाभाष्यम् पस्पशाह्निकम्]

12

(FL) instruct the valid words and attendances of the languages (FL) ideal Sanskrit project used to be thought to students till 56 years ago to learn how to learn Sanskrit, (FL) what is that method, that by a small effort you are able to understand large number of correct valid occurrences in Sanskrit that method is (FL) means general rule (FL) means exceptional rule.

(Refer Slide Time: 09:58)

Śabdānuśāsana: Pāṇini's Aṣṭādhyāyī

How are these utterances to be known?

Some characterisation with what is general and particular is to be provided, by which, with little effort, great amount of utterances are known.

What is that characterisation? *Utsarga* (general) and *Apavāda* (special/exceptional) rules...

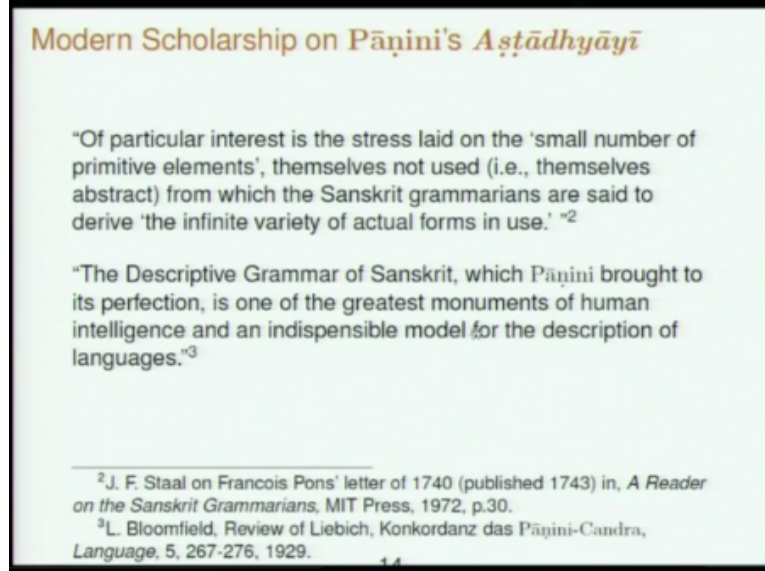
[*Mahābhāṣya* of Patañjali, *Paspaśāhnika*]

12

So by a set of general and exceptional rule we tried to approximate and reach all possible valid occurrences of the Sanskrit language in that is the method that Patanjali is formulated

and remember right at the formulation of what is grammar, (FL) is important is not more important than the general rule (FL) he has given Pathanjali there is giving two examples of a (FL) these are the 2 sutras which tell you the first one is the general rule.

(Refer Slide Time: 10:56)



(FL) the second one is (FL) to be used that, we will not going to such technicalities, so we will see an example of (FL) while discussing find grammar. So quickly let us see how modern scholarship has understood Panini grammar how they have been salt of bewildered surprised by its depth in detail.

So even as early in 18 century (FL) became aware that Sanskrit grammar are doing something very different from the grammar text that was not believe, they are trying to use primitive element to derive the infinite variety of actual foreignsic use, the grammar that are in general user what are called as paradigmatic grammar. Paradigmatic grammar are where you learn finish for identity as it is.

We do, do that even in Sankrit today, when we install of due remember the (FL) this is an example of paradigmatic instruction. So you remember (FL) both this ways, but Panini's grammar is to tell you to how to derive (FL) and it will tell you derive all chapters and all values and all things. So it is different from the paradigmatic grammar that was generally known in Europe.

(Refer Slide Time: 12:26)

Modern Scholarship on Pāṇini's Aṣṭādhyāyī

"The idea that a language is based on a system of rules determining the interpretation of its infinitely many sentences is by no means novel. Well over a century ago, it was expressed with reasonable clarity by Wilhelm von Humboldt in his famous but rarely studied introduction to general linguistics (Humboldt 1836). His view that a language 'makes infinite use of finite means' and that a grammar must describe the process that makes this possible.. Pāṇini's grammar can be interpreted as a fragment of such a 'generative grammar' in essentially the contemporary sense of this term. " ⁴

∞

"Modern linguistics acknowledges it as the most complete generative grammar of any language yet written and continues to adopt technical ideas from it". ⁵

⁴N. Chomsky, *Aspects of the Theory of Syntax*, MIT Press, 1964, p.v.
⁵P. Kiparsky, Pāṇinian Linguistics, in *Encyclopaedia of Language and Linguistics*, VI, 1994.

And (FL) field is explaining Panini's grammar as the best descriptive grammar of Sanskrit (FL) in 1930 descriptive grammar for the technique that grammarians are very high, then this new thing called generative theory of Syntax started in 1964 (FL) is writing that this tradition in Europe goes back to Humboldt in 1836 and then he also says Panini's grammar can be interpreted as a fragment of such a generative grammar in essentially the contemporary sense of the term.

And it has one of the leaders of the generative grammar tradition in modern times says the modern English acknowledge modern linguistics acknowledges it has the most complete generative grammar of any language yet returned and continues to adopt technical ideas from it.

(Refer Slide Time: 13:16)

Modern Scholarship on Pāṇini's Aṣṭādhyāyī

The algebraic formulation of Pāṇini's rules was not appreciated by the first Western students; they regarded the work as abstruse or artificial. ... The Western critique was muted and eventually turned into praise when modern schools of linguistics developed sophisticated notation systems of their own. Grammars that derive words and sentences from basic elements by a string of rules are obviously in greater need of symbolic code than paradigmatic or direct method practical grammars....

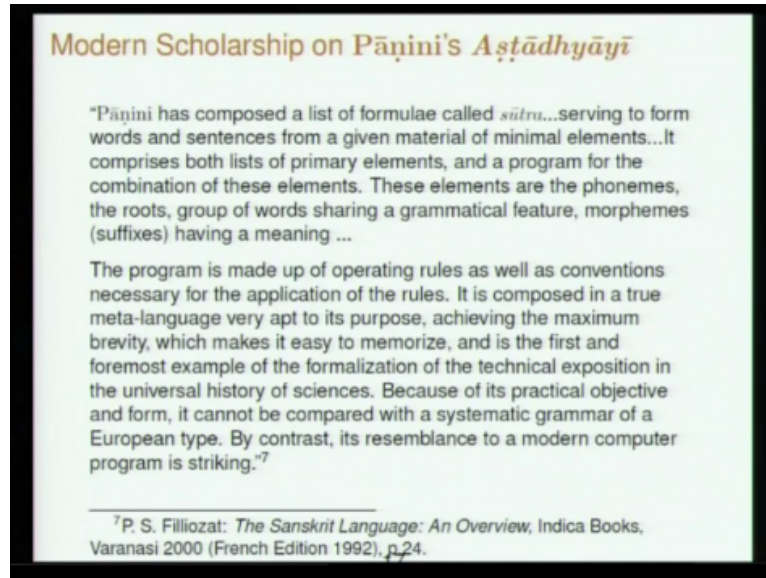
It is a sad observation that we did not learn more from Pāṇini than we did, that we recognised the value and the spirit of his "artificial" and "abstruse" formulations only when we had independently constructed comparable systems. The Indian New Logic (*navya-nyāya*) had the same fate: only after Western mathematicians had developed a formal logic of their own and after this knowledge had reached a few Indologists, did the attitude towards the *navya-nyāya* school change from ridicule to respect. ⁶

⁶H. Scharfe, *Grammatical Literature*, Wiesbaden 1977, pp.112, 115.

There is another scholar who is trying to explain that when modern scholarship confronted itself with Panini's grammar much of it sounded very true and unnecessary and somewhat confusing and later on when similar techniques did get developed in Europe, some of them by under the influence of Panini's grammar itself, it became very clear that Panini is doing something very interesting if not very something very sophisticated.

So then nothing has happened to the Indian tradition of Navya Nair which people thought was coming later utterances till similar techniques became known in Europe and they were then understood to be something philosophically significant.

(Refer Slide Time: 14:06)



And another very important scholar of Sanskrit saying to explain how the structure of Panini's grammar is different from all other grammar that it uses basic elements such as phonemes, the roots, group of words, morphemes and then produced by a set of roots a meaningful sentences and he also says that in the universal history of sciences Panini's grammar is a first and foremost example of formalisation of technical exposition.

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Modern Scholarship on Pāṇini's *Aṣṭādhyāyī*

"Pāṇini's grammar is universally admired for its insightful analysis of Sanskrit...Generative linguists for their part have marvelled especially at its ingenious technical devices, and at intricate system of conventions governing rule application and rule interaction that it presupposes, which seem to uncannily anticipate ideas of modern linguistic theory (if only because many of them were originally borrowed from Pāṇini in the first place.)..."

The grammar has four distinct components:

1. *Aṣṭādhyāyī*: a system of about 4,000 grammatical rules
2. *Śivasūtras*: the inventory of phonological segments
3. *Dhātupāṭha*: a list of about 2,000 verbal roots...
4. *Gaṇapāṭha*: a list of 261 lists of lexical items...

The grammar is a device that starts from meaning information... and incrementally builds up a completely interpreted sentence."⁸

⁸P. Kiparsky, On the Architecture of Pāṇini's Grammar, 2002.

And it looks very different from the paradigmatic grammar but it is something like a set of computer programs or something like that. So another modern college explain how generate this modern technology Panini's grammar mainly because many techniques have been not understood based upon it at various time. So the distinct components of Panini's grammar of the following (FL) which itself has about 4000 sutras.

Sulbasutras which is a collection of phonological sound segments (FL) which has about 2000 verbal roots, then ganapatha which is a specific list of items there 261 as a list, and grammar is a device that takes you from a certain meaning that you formulate in your mind to the final expression of what you want to stay in the form of a sequence of sounds.

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Śiva-Sūtras and Pratyāhāras

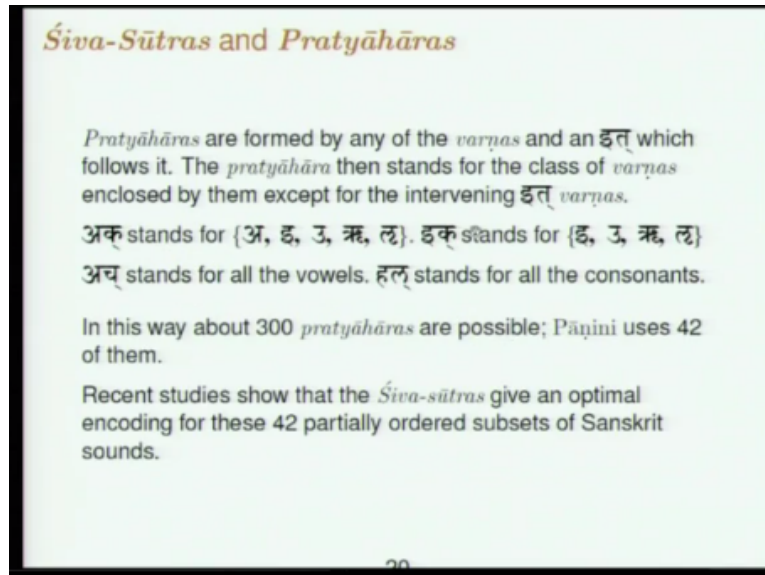
१ अ इ उ ण् । २ ऋ लृ क् । ३ ए ओ ङ् ।
४ ऐ औ च् । ५ ह य व र ट् । ६ ल ण् ।
७ ञ म ङ ण न म् । ८ झ भ ञ् । ९ घ ढ ध ष् ।
१० ज ब ग ड द श् । ११ ख फ च ठ थ च ट त व् ।
१२ क प य् । १३ श ष स र् । १४ ह ल् ॥

Each *sūtra* has a set of *varṇas* followed by a marker (ण्, क्, ङ्, च्, ट्, etc) called the *इत् varṇa*.

एषाम् अन्त्या इतः

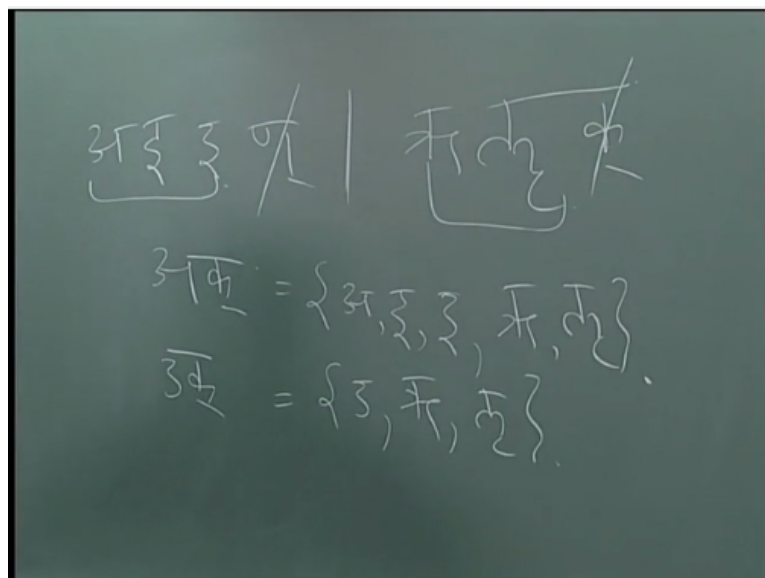
So we will first look at the sulvasutras, these are called (FL) there are 14 of them (FL) etc. mostly classified based upon the way the sounds origin in when speak them. So each of these formulae has one symbol at the end which is called (FL) is to produce a subsector sounds that we need in the grammar, whenever you want to say this sound instead of listing them we will make a simple formula and bring that list by that formula.

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So this technique is known as pratyahara technique, so when we say (FL) the first two sutras of Panini.

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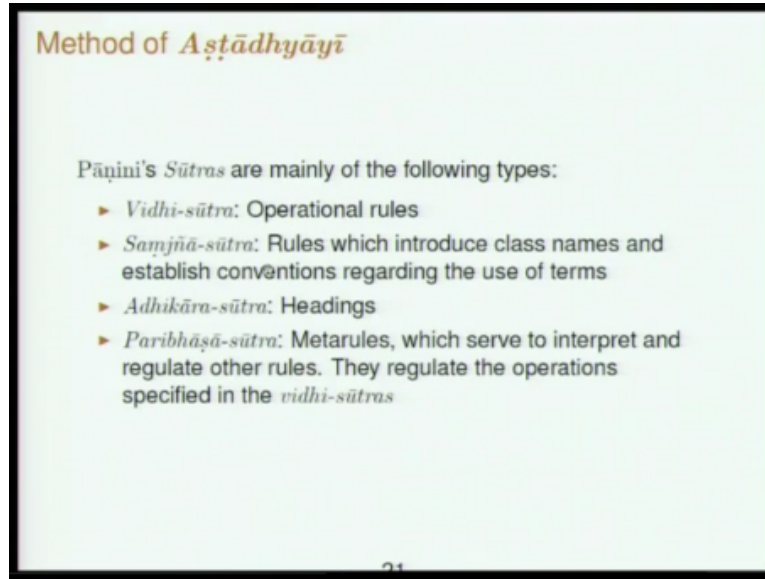


So when you say (FL) we are using (FL) we omit this and we omit this, so the formula (FL) stands for the high sounds, if we say (FL) it only stands for 3 sounds (FL) so by now it should be clear to you the kind of technology Panini is using. (FL) will stand for all the above (FL) is

the pratyahara (FL) and of course you can see that this 14 formula and the last letters of the (FL) there to be omitted and mathematically you can form 300 such pratyaharas like (FL).

You can for 300 pratyaharas, Panini in his Astadhyayi is using only 42 of them and this is always been a question because whenever you discuss Panini the Indian grammarians also want to prove that he has used the shortest means or the simplest means to do something and so the question has been asked that could we have listed this 42 subjects that Panini is using by a different set of pratyaharas which is simpler and this question has been answered.

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That it is not possible that this is in fact the optimal way of encoding 42 orders subjects of Sanskrit sounds this 14 pratyaharas sutras are the best of the optimal way of doing that. So as we saw the 4000 sutras of Panini there are various kinds of sutras. There are vishi-sutras which tell you what is to be done, what rule for either changing a sound or including (FL) certain context or that doing something in for some obtaining some meaning.

There are Samjna-sutras which introduced groups of entities or establishes some conventions in use of terms. There are adhikara-sutras which are heading (FL) we will be discussing this and this is this points will keep following, what are the techniques used in Panini's grammar is something called Anuprati that you take the same word in earlier sutras and they use them assume them in the latter sutras also.

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Method of *Aṣṭādhyāyī*

Examples of *Paribhāṣā-sūtras*

- ▶ **Saṣṭhī sthāne-yogā** (1.1.49): Genitive designates 'in place of'.
- ▶ **Tasminnitinirdiṣṭe pūrvasya** (1.1.66): Locative defines the right context.
- ▶ **Tasmādityuttarasya** (1.1.67): Ablative defines the left context.
- ▶ **Yathāsamkhyamanudeśaḥ samānām** (1.3.10): For groups with the same number of elements, the corresponding elements are to be related in order.
- ▶ **Pūrvatrāsiddham** (8.2.1): (From now on every rule is regarded as) not having taken effect with reference to preceding ones.

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Most important of paribhasha sutras they tell you did you like the other rules they regulate operation of the other rules. So let us see some examples of Paribhasha-sutras (FL) yoga the saṣṭhi when it appears it will mean in place of so if you want to replace one sound to another sound you use the sound that is to be replaced, you use it in the genitive case. Tasminnitinirdiṣṭe purvasya locative defines the right context.

So if you want to say before something a certain change has to work at the later point the later sound is used in the Tasminnitinirdiṣṭe, Tasmādityuttarasya, so something which follows a given sounds in operation is to be done you indicated by the Panchami vibhakti are the ablative case. The other rule is yathasamkhyamunadesah samanam, if there is a set of transformation in 2 sets of entities are given you correspondingly take one to the corresponding thing in the other side.

That is the same number you just follow a simple rule 1-1 corresponding. These are simple paribhasha-sutras and there are very complex (FL) in fact one of the most interesting paribhasha-sutra Panini is called the (FL). Purvatrasiddham this occurs in the beginning of the second padha of the 8th (FL). So this says that all those sutras of Panini which start with the first (FL) up to the first part of a (FL) or taken to be (FL) with respect to the later rules.

That means the later rules do not exist as far as the earlier rules are concerned. So the later rules come in a linear sequence there called the (FL) so each of them also the same (FL) will follow, so in operation that you do in using rule of (FL) 8th chapter then you cannot bringing an operation of the second chapter of Panini following that. So all of them are (FL).

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Context Sensitive Rules of *Aṣṭādhyāyī*

Phonological rules are typically of the form "sounds of class *A* are replaced by sounds of class *B* if they are preceded by sounds of class *C* and followed by sounds of class *D*", which in modern phonology is usually denoted as

$$A \rightarrow B / C-D$$

Pāṇini formulates the above rule as follows:

A + genitive, *B* + nominative, *C* + ablative, *D* + locative.

Example: **Ikoyāñaci** (6.1.77)

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So Panini grammar as you can see as many many complex ideas like we will just examine one kind of rule that is used in Panini, just to see the operation of the previous (FL) that I discussed (FL) etc.

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Ikah Yañ Aci
Shroshthi Saptarwi

$$\begin{matrix} \overline{\text{I}} + \text{Vowel} \rightarrow \overline{\text{I}} + \text{Vowel} \\ \overline{\text{I}} + \text{Vowel} \rightarrow \overline{\text{I}} + \text{Vowel} \end{matrix}$$

So a context sensitive rule, what I mean by that is something like this, I written it like this which means that if this A is standing between C and D then it will go to D, this is the nature of the rule, such a rule is called a context sensitive rule. That A goes to B, then C is before that and D is after that. Now in Panini's grammar such a rule is formulated by on A you put the (FL) and on B you put the (FL), on C you put the (FL).

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Context Sensitive Rules of *Aṣṭādhyāyī*

ikoyaṇaci (6.1.77)
iK stands for { i, u, ṛ, ḷ },
yaṆ stands for { y, v, r, l }
aC stands for all the vowels.

From 6.1.72, **saṃhitāyām** is carried forward. Thus the *sūtra* provides that:

i, u, ṛ, ḷ → y, v, r, l before a vowel, in close contact

This gives

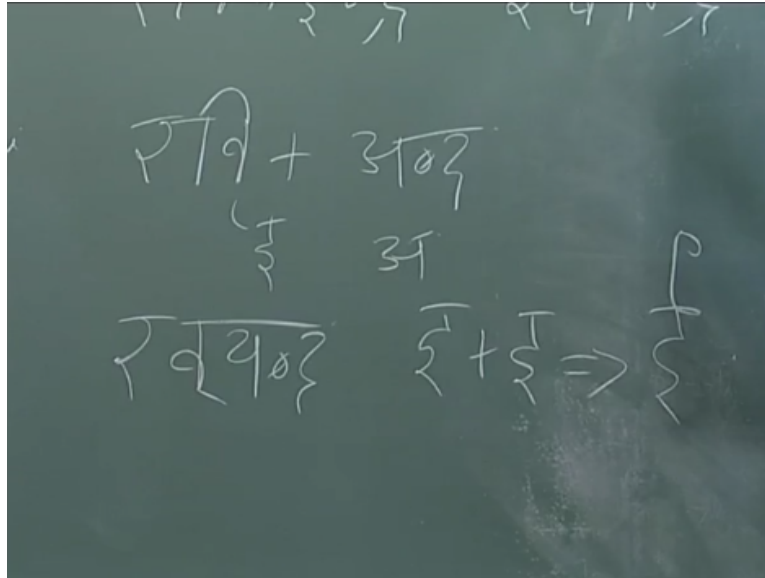
i + a → y + a, u + a → v + a and so on.

Akaḥ savarṇe dīrghaḥ (6.1.101) is an *apāvāda-sūtra* to the above, and gives:

i + i = ī, u + u = ū and so on.

And on D you put the (FL) and you state your rule then it means that C is before D is like that in the context A goes to B, let us see this with the well known what is called (FL) so when a vowel follows this set of (FL) which is in the (FL) this will become YaN, so C+one of the (FL) E+vowel goes to (FL) the vowel follows E, E will become YaN, similarly (FL) so let us write it in Sanskrit itself E will become (FL).

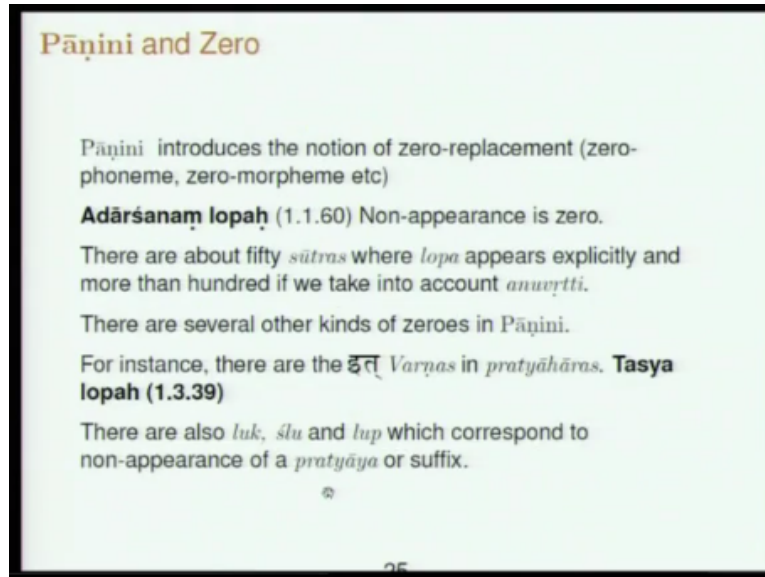
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Similarly so by the (FL) rule Panini is doing what is called the Yan and the so something a very simple thing let us take (FL) so here is the (FL) and A is R which is following so it immediately become (FL) E+R will become (FL) this is called the YaN and the so Panini is saying that this is completely characterised by this formula (FL) that will become YaN. Now we also know that (FL) so that is an exception.

So that exception is called samhitayam (FL) this is sutra called 1.101 on Panini and that (FL) when (FL) when the same sounds follows (FL) this is how the rules of Panini.

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Pāṇini and Zero

Pāṇini introduces the notion of zero-replacement (zero-phoneme, zero-morpheme etc)

Adārsanam lopah (1.1.60) Non-appearance is zero.

There are about fifty *sūtras* where *lopa* appears explicitly and more than hundred if we take into account *anuvṛtti*.

There are several other kinds of zeroes in Pāṇini.

For instance, there are the इत् *Varṇas* in *pratyāhāras*. **Tasya lopah** (1.3.39)

There are also *luk*, *slu* and *lup* which correspond to non-appearance of a *pratyāya* or suffix.

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So scholars are speculated that some of the important influence of Panini on Indian mathematic could have been the invention of zero itself. Now Panini introduces the idea of Lopa and even in the first (FL) Panini ashtadhyayi of chapter, the Sudarshan (FL) there about 50 sutras where (FL) this number will be more than 100 if you take into account and that is taking the earlier world in related sutras.

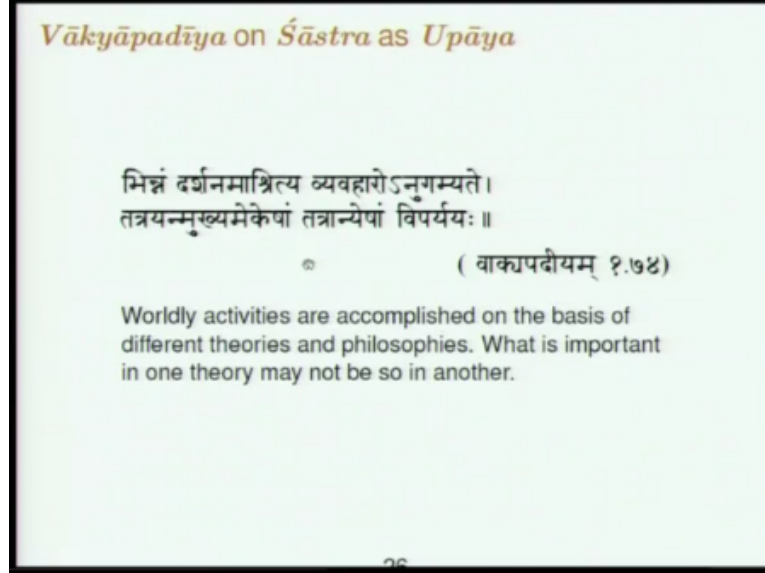
In fact to later examples is a case of anu anuvṛtti noted it in (FL) you take word samhitayam, samhita means sounds occurring here to each other that is when you are doing what is called give for the combination and (FL) as we know in Indian languages. So when (FL) are close to each other samhita (FL) when they are very close to each other that is when these changes occur.

So the word samhita is obtained as Anuvṛtti in understanding the Sutra (FL) this is the kind of way in which sounds are the words by anuvṛtti Panini Sutra. So the idea is that when some (FL) is removed that is sounding like a zero because like that point what is to be done and we told by another one. So this is like when we say when you want to multiply if the 0 was not that the multiplication will go one way.

If the 0 is that the there is a multiplication will go another way, in fact the cancellation in Panini will also be going similarly when the 0 mark is there when it is not there. So that is the

way 0 appears as a linguistic element in Panini ashtadhyayi, there are also other versions of 0 which are known as (FL) so anyway this is a very technical subject and I will not be saying more on that.

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Is this example clear or is there any doubt ok, now we come to how the Indian philosophers understood on Panini's grammar, as you all know that in Delhi around 4000 rules are sutras party is able to formulate all rules for formulating correct words in Sanskrit and this it has to aid or aid of other this like (FL) of course the sulbasutras also. And it was very successful, there is (FL) was considered wrong.

But even then Indian philosophers (FL) like Panini's grammar is only an approach to language, it is not a substitution for the reality of language itself in fact in Pathanjali (FL) like at the beginning there is a discussion that when we want words we do not go to a grammarian's and ask them or when we want sentences or when he want to say something we do not go to a grammarian and say give macro environment a few sentences or give macro environment 2 words.

Unlike when we want parts or when we want vessels we go to a metallurgies metal worker or a potter to give us pots and vessels. So a grammarian is working with the language which is already established in the world, the reality of the language lies in the world, the grammarian task is to give an approach to that reality and therefore (FL) is here explaining the philosophy of Sanskrit grammarian.

This is impact the philosophy of most others (FL) also in Sanskrit (FL) activities are accomplished on the basis of different theories and philosophises. What is important in one theory may not be so in another and both commonly known (FL) is a very big work comment with this very difficult to understand without looking up its commentaries. So this (FL) is widely quoted by Indian astronomers also.

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Vākyāpadīya on Śāstra as Upāya

उपादायापि ये हेया तानुपायान् प्रचक्षते ।
उपायानाम् नियमो नावश्यमवतिष्ठते ॥
अर्थं कथञ्चिद् पुरुषः कथञ्चित्प्रतिपद्यते ।
(वाक्यपदीयम् २.३८-३९)

Upāyas (procedures taught in *śāstras*) are to be discarded, even though they are to be used for accomplishing an objective. There is no necessary limitation on such means. One accomplishes objectives by one means or the other.

As noted by the commentator Puṅgyarāja :

कश्चिदाचार्यः पाणिनिविरचितेन लक्षणशास्त्रेण शब्दानधिगच्छति
कश्चिदन्येनेति न नियमः ।

Bhaskara 1 in his commentary of Aryabhatiya codes this words, (FL) commentary of Aryabhata (FL) the procedures got in sastras are only means and (FL) to be discarded even though if (FL) we have to use that is which purpose is for accomplishing this object they are they have no higher status than that (FL) to accomplish the object (FL) are uses and there is no other imitation also, that there is no they can spend that you put on the methods that are to be used.

And in impact as regards the meaning most people do know what the meaning without having to go through the grammarians exercise, now the way this is understood is from the commentary of (FL) basically saying (FL) so one teachers may like to teach such grammar to Panini's grammar and another teacher may would like to use another system of grammar, if all of them but equally well there is really no limitation on which is the system to be called.

There is rarely sophisticated understanding that you almost come up with fairly complete theory of mechanism for characterizing all valid worlds of sentences of a language you also say that this is only an inch and if others are coming up with other formulations which can be equally successful or at least which can which are use you are free to go ahead and use it.

This is the kind of philosophical principle that the grammar is only a means for understanding valid occurrences.

The valid occurrences are the out in the world is something that is very important even in other sciences when discussing the relation between theory and observation in Astronomy. This kind of a principal files say more directly, so I have site at this (FL) but on the entire Panini's sastras (FL) is a means for understanding and this was the kind of understanding that Indian grammarians had in setting up the grammatical formulation.

This is I think we have summarise some essential features of Panini's grammar which are of importance in understanding the development of mathematics in India, Thank you.