Mathematics in India: From Vedic Period to Modern Times

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Lecture - 35
Trigonometry and Spherical Trigonometry 3

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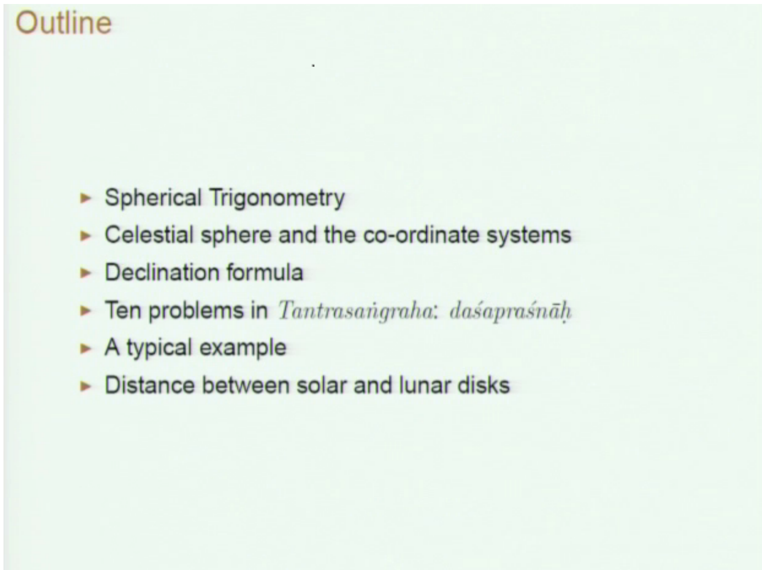

Okay, so the last of the three lectures on Trigonometry and Spherical Trigonometry, so here we will be exclusively dealing with spherical trigonometry which had started in the previous lecture. So we will consider the celestial sphere in the co-ordinate systems, so then the declination formula which is a very important formula, then what are known as 10 problems in Tantrasangraha (FL), and a typical example and then distance between solar and lunar disks okay.

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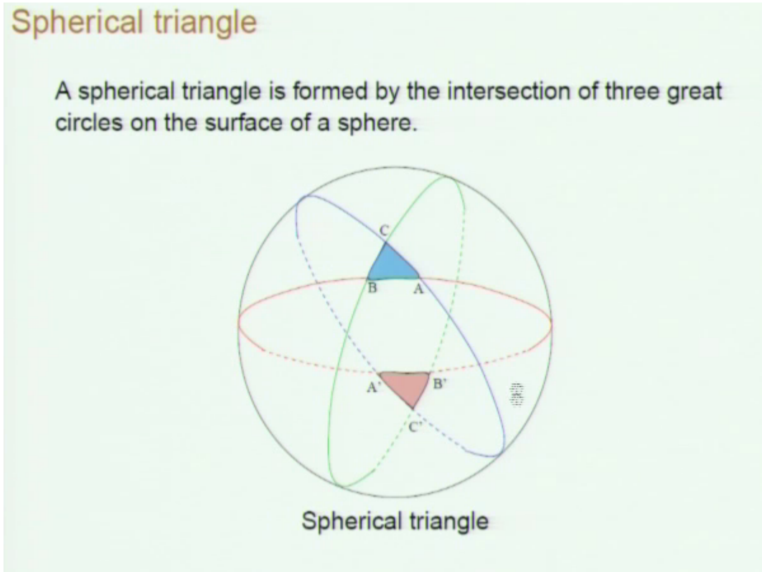

So a spherical triangle is formed by the intersection of 3 great circles on the surface of the sphere, I had mentioned it in the previous lecture, so this is one great circle passing through a C, so this is another great circle, and this is another great circle. So great circle means a circle whose center is the center of the sphere, so there may be circles whose center is not like if you consider a circle somewhere here its center is not a center sphere, so that is called a small circle.

So in the various relations it is only these arcs which are you know coming from great circles arcs, they are you know important and they give useful relations, so that is why we consider that so we had written like this also yesterday.

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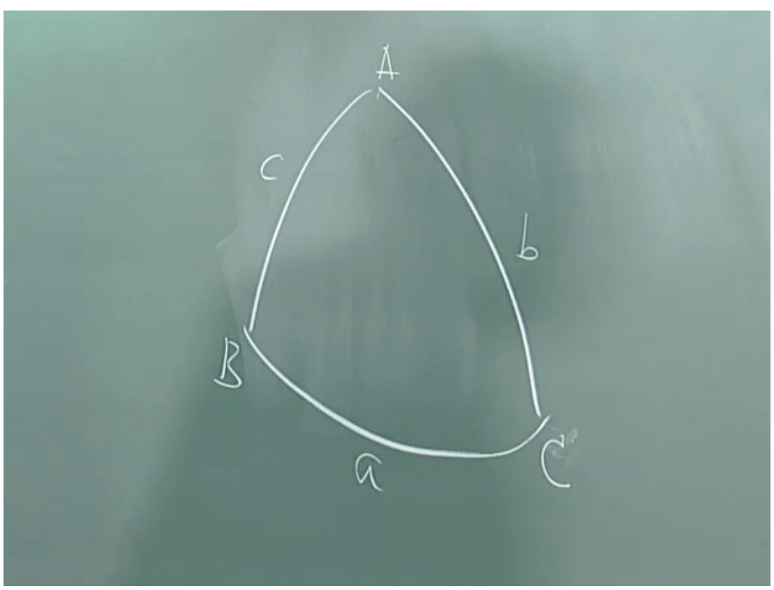

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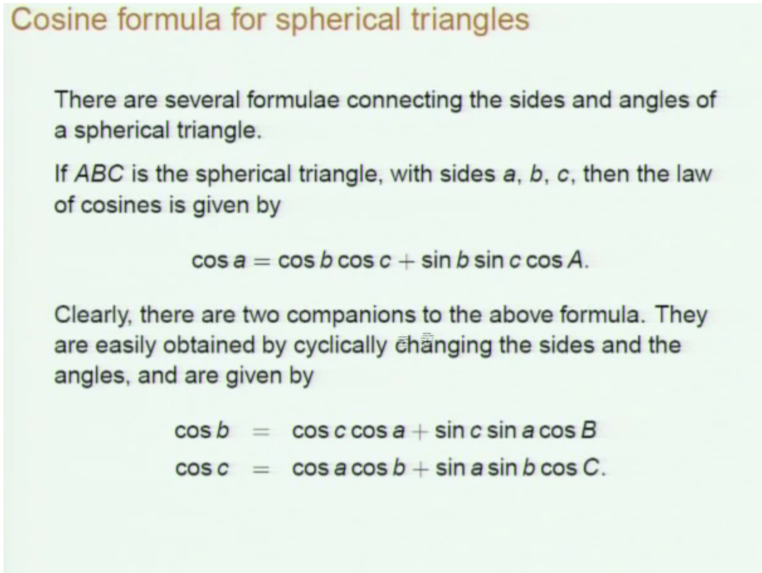

And there are several formulae connecting the sides and angles of a spherical triangle A B C, suppose if A B C is spherical triangle, see here in a spherical triangle both are almost in a same status, see in plane triangle the remember the sides are lengths, and these are angles. Whereas here both the sides and distinct angles are angles kind of thing roughly, because they are arcs we are always discussing with arcs.

So this circular arc BC is circular arc it is not a straight line, and it is a part of a circle whose arc length is small a, and the angle between these you know tangent to this AB you know this arc, and tangent to AC, so that angle is a spherical angle on of the spherical triangle, so similarly you can from the 3 spherical angles and 3 sides A B C. Then there is a famous law called cosine law of cosines, so cos of a=cos b cos c+ sin b sin c cos A.

And clearly there are 2 companions to the above formula because cyclic symmetry you know a goes to b, b goes to c, c goes to a, so then you will have 2 more relations against the cosine formula okay.

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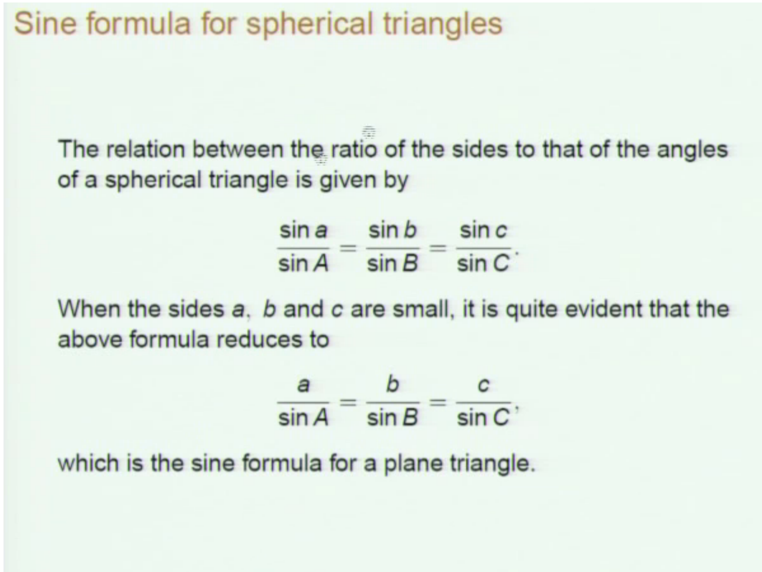

And similarly, next there is a very important cosine formula is used all the time in modern spherical trigonometry, and a sin formula also relation between ratio of the sides to that of the angles of a spherical triangle sin a/sin a, so here remember that in triangle it is like this here in a spherical triangle sin small a/sin capital A= sin small b/sin capital B= sin small c/sin capital C. So when the sides a, b, c are small it is quite evident that the above formula reduces to this which is the sine formula for a plane triangle okay.

So this sine formula is used in Indian spherical trigonometry various times, but cosine formula is never used I mean it is never you know used as such of course what are equivalents of cosine formula will be there obviously they needed that, so they will get the same result as what is got from the cosine formula but they do it in a different way okay. So these are simple things about spherical triangle but those simple they are you know they can get you very interesting results.

Even about 40 to 50 years back there were Pancharatnam using the properties of these things he could you know get some interesting very important result in optics by analyzing this you know this light waves and all that, and which are related to a spherical triangle in some way, so he got a new result and new geometrical phase called Pancharatnam phase, they were quite simple result but very interesting, and very geometrical and has got very nice significance and very elegant results you can get even now if you understand okay.

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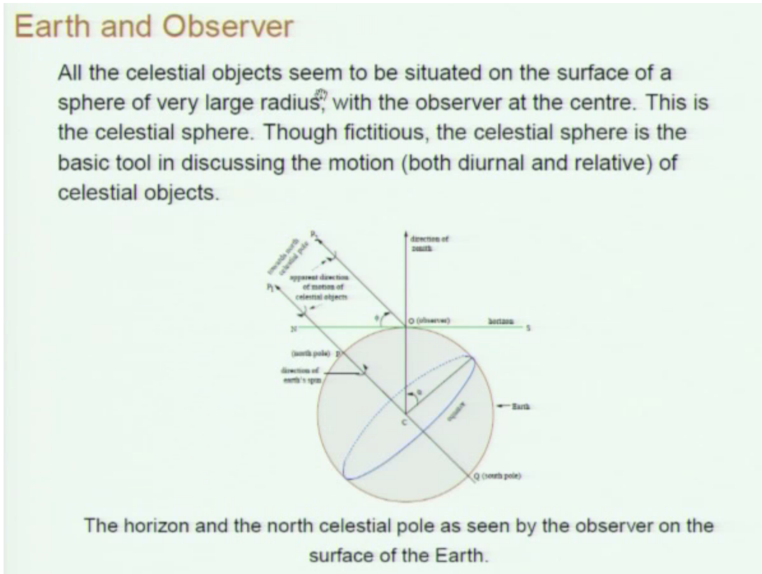

So now we are of course not interested in those things, we are interested in astronomy. So we how do I mean how does one particular individual view the celestial sphere, so that is important so that the observer and we are situated on the earth, so we have to this relation we must be clear. So all the celestial objects seem to be situated on the surface of a sphere a very large radius with the observer at the center that is what we observe right.

When we observe the sky so we see that all this sun, moon, planets and stars, so they are all moving in some large sphere which is blue okay in on which hemisphere and they are all moving in that, and this is the celestial sphere, though fictitious the celestial sphere is the basic tool in discussing the motion and both the diurnal and relative of celestial objects. So what is diurnal? The daily motion of the each of these objects.

And what is relative? Relative motion among them so that is the sun and moon etc. moving the background of the stars so that relative motion also is very important. So in this figure this is the of course the earth and this is the equator of the earth, and this is observer. So observer you see when you see the sky obviously he will see only the upper part right, because the lower part is hidden you know because he cannot see here because earth will obstruct him.

So only half the number of stars, half the number but half the part of the sky he will see, so that is this hemisphere I will show the figure corresponding to that, and this is the axis of rotation of the earth okay, so PQ so these are directions of earth spin we know that it is a rotating okay. And you can from the observer you can draw a line parallel to this, so that is the apparent earth, this is towards the celestial pole, there will be hardly any difference even OP2 and CP1.

Because this is the order of about 6000 kilometers, whereas the stars etc. are millions of miles away I mean so this will be almost you know this will be you can take it as parallel. So your this suppose this is the latitude of the place, if this is the latitude of the place so that is the person is at an angle phi with respect to the equator, so that is a latitude the important thing is the altitude of the north pole is equal to the latitude of the place is a fundamental result very simple result in Astronomy okay.

So that is how much this north pole is above the horizon okay that is the lower part of the sky, so that will be equal to the latitude of the place, so for Chennai it will be only 11 degrees, and for Delhi it will be nearly 25 or 26 or whatever degree, so the altitude of the north pole at a place is equal to the latitude of the place okay.

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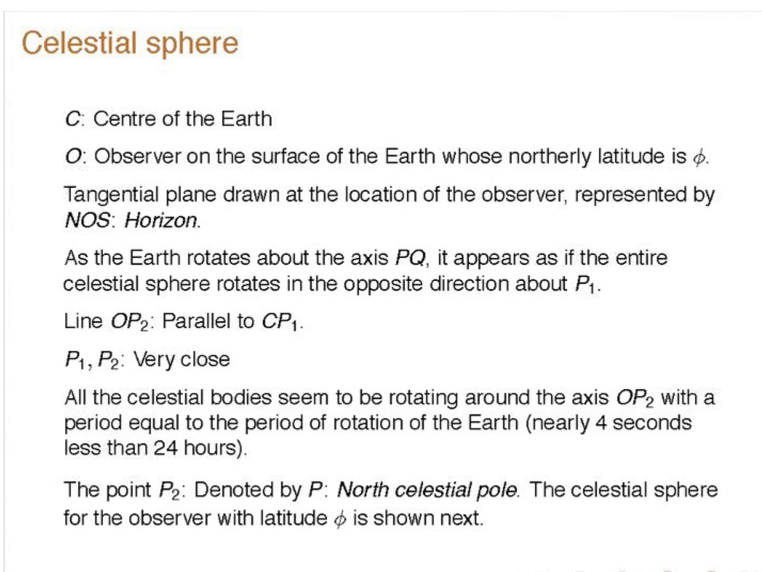

So in the celestial sphere this C is the I come to this, here C is the center of the earth, then O is the observer and surface of the earth whose northerly latitude is phi, and we had drawn a tangential plane at the location of the observer and this is a horizons, this called a horizon you know this horizon is written as a line, but it will be actually a big circle where the sky and earth seem to meet at a far away distance, so that is the horizon called (FL) in the Indian astronomy.

And as Earth rotates about the axis it appears as if the entire celestial sphere rotates in the opposite direction about P1 okay, line OP2 is parallel to CP1, so it looks like as if the whole thing is and everything is rotating around this axis, so that is what it was clearly stated by Aryabhatta in his Aryabhattia, he says that you know just like you know people in a boat they you know they will see the objects on the back move backwards.

But actually you are moving forward in a boat, so similarly, the all the celestial objects they seem to be rotating from eastern portion to western portion, but is actually due to the rotation of the earth and stars are fixed essentially and the motion is due to the motion of the earth rotation of the earth okay. So celestial sphere, celestial pole and the latitude I written that.

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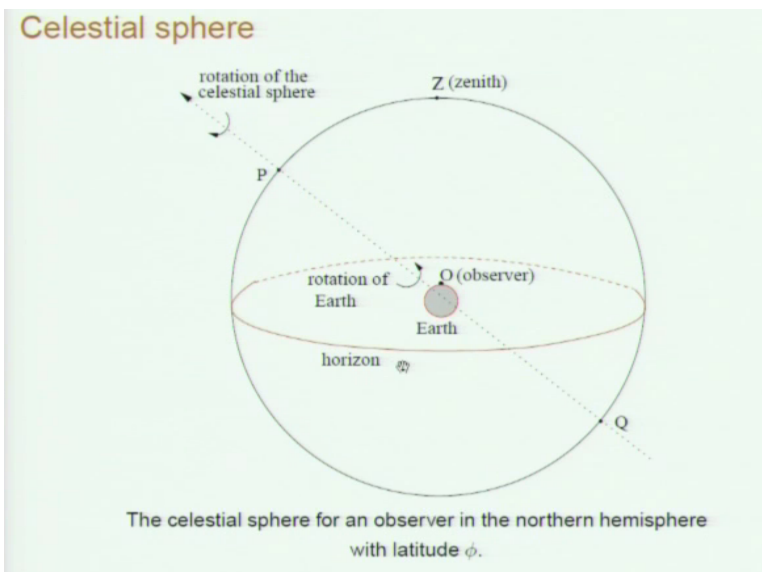

So this how we know this is the fundamental figure which comes in all the calculations. So this is the observer O, and this is the horizon okay, and this is the axis of rotation the direction of the axis of rotation of the earth, the topmost point is called is zenith and the lower most point is called nadir, nadir is results in the terminology used in modern these days okay, so this is how so this is the hemisphere which you see. So this is the celestial sphere for an observer in the northern hemisphere with the latitude phi.

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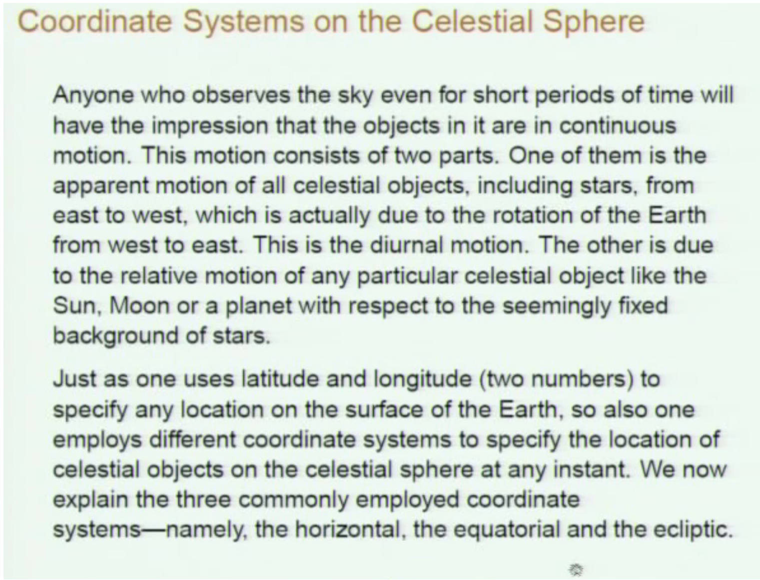

So now to specify a point you know on this thing for any kind of accurate description you should have some accurate ways of describing it okay, so how do you locate the position of a point on the surface of the earth okay, so essentially we will specify it by we can know the co-ordinate of a place on the earth by specifying its latitude and longitude right.

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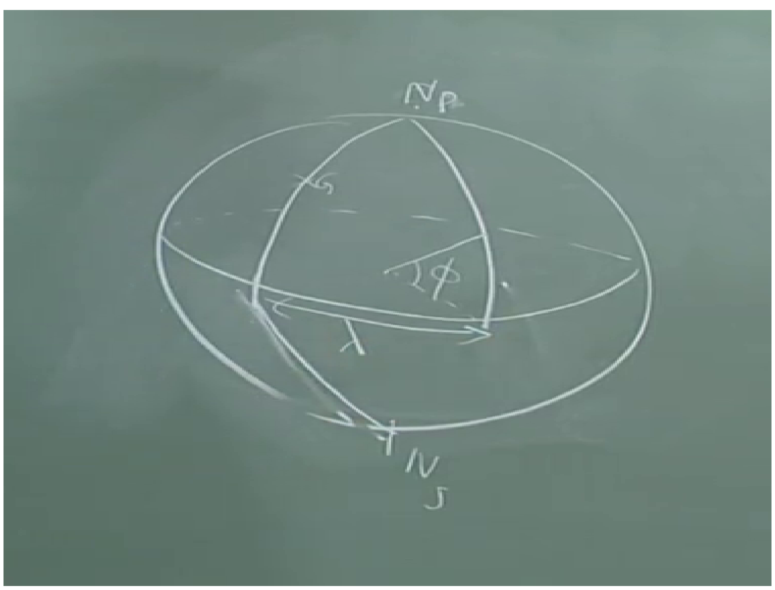

So this is the earth okay and suppose so these are north pole, this is the south pole okay, so then suppose this is the these are reference plane celestial equator, so this phi this angle so that is called a latitude of the place, and longitude so that will depend upon you have to give some reference line you see, so there is no absolute significance to that. Nowadays, we take you know this the great circle passing through Greenwich from the north pole to south pole is it take the great circle arc.

This great circle in a semicircle between north pole and south pole which is passing through Greenwich okay so that is called Standard Meridian node is, and this angle so this is called the longitude right, so the longitude of earth is sorry Chennai will be around 80 degrees or something like that, these are longitude and latitude is around 11 degrees, so latitude and longitude you see.

You can do it in some other ways, see you can take this as a reference plane okay and you do you can point it and do it in some other way but they will not be useful, this is what is useful okay. But in celestial sphere when you describe the motion of celestial objects, there are many they are not one but 2, 3 which are quite useful okay. So just as one uses latitude and longitude 2 numbers to specify any location on the surface of the earth.

So also one employs different co-ordinate system to specify the location of the celestial objects on the celestial sphere at any instant, so we now explain the 3 commonly employed co-ordinate system the horizontal system, the equatorial system and the ecliptic system.

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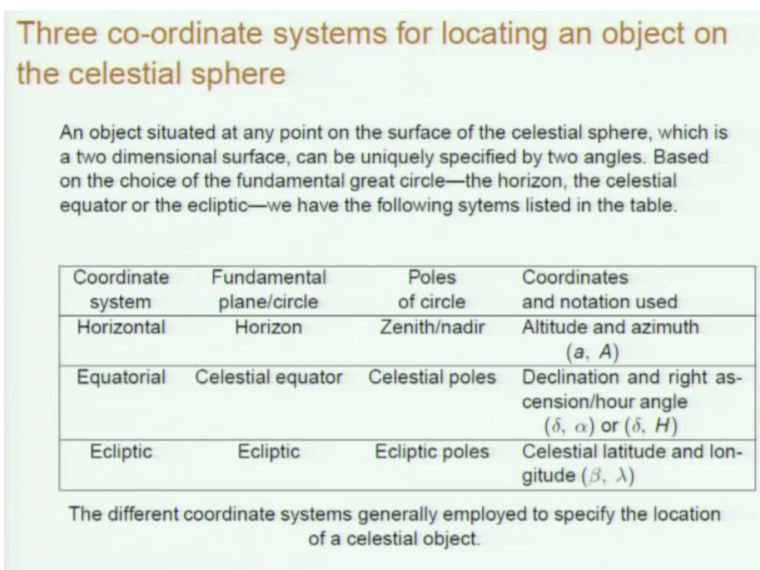

Three co-ordinate systems for locating an object on the celestial sphere

An object situated at any point on the surface of the celestial sphere, which is a two dimensional surface, can be uniquely specified by two angles. Based on the choice of the fundamental great circle—the horizon, the celestial equator or the ecliptic—we have the following sytems listed in the table.

Coordinate system	Fundamental plane/circle	Poles of circle	Coordinates and notation used
Horizontal	Horizon	Zenith/nadir	Altitude and azimuth (a, A)
Equatorial	Celestial equator	Celestial poles	Declination and right as-cension/hour angle (δ, α) or (δ, H)
Ecliptic	Ecliptic	Ecliptic poles	Celestial latitude and lon-gitude (β, λ)

The different coordinate systems generally employed to specify the location of a celestial object.

So this is the here the horizontal system is also called Alt-azimuth system, the fundamental plane is a circle is the horizon, so that is a fundamental this thing, and pole of the circle is zenith and nadir, and the co-ordinate that are used is altitude and azimuth okay. So that is supposed this is the fundamental circle for this earth you know position on surface of the earth, so this is the equator if the reference plane, and this north pole and south pole are the reference poles okay.

And the coordinates are the and this latitude and longitude, so it is equivalent of this in the celestial sphere is altitude and azimuth, so I will show the figure. And this another thing called equatorial co-ordinate system for which the fundamental planes the celestial equator, and the poles are celestial poles, and the coordinates are declination, and right ascension or hour angle.

And similarly, for the motion of planets and moon and all that it is more convenient to use some what is called ecliptic co-ordinate system where the fundamental plane is ecliptic, and the poles are the ecliptic poles, and the coordinates are celestial latitude and longitude. So essentially same idea but used in different ways and used for motion of celestial objects.

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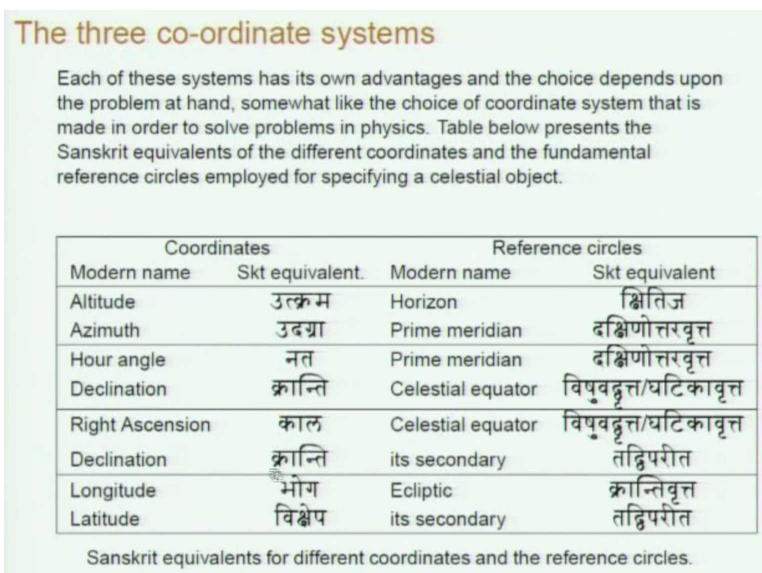

The three co-ordinate systems

Each of these systems has its own advantages and the choice depends upon the problem at hand, somewhat like the choice of coordinate system that is made in order to solve problems in physics. Table below presents the Sanskrit equivalents of the different coordinates and the fundamental reference circles employed for specifying a celestial object.

Coordinates		Reference circles	
Modern name	Skt equivalent.	Modern name	Skt equivalent
Altitude	उत्क्रम	Horizon	क्षितिज
Azimuth	उदग्रा	Prime meridian	दक्षिणोत्तरवृत्त
Hour angle	नत	Prime meridian	दक्षिणोत्तरवृत्त
Declination	क्रान्ति	Celestial equator	विषुवद्वृत्त/घटिकावृत्त
Right Ascension	काल	Celestial equator	विषुवद्वृत्त/घटिकावृत्त
Declination	क्रान्ति	its secondary	तद्विपरीत
Longitude	भोग	Ecliptic	क्रान्तिवृत्त
Latitude	विक्षेप	its secondary	तद्विपरीत

Sanskrit equivalents for different coordinates and the reference circles.

And these are the Indian names for that they also use this kind of co-ordinate systems, and altitude is called (FL), Azimuth is called (FL), hour angle is called (FL), declination is called (FL), then what is known as right ascension I will soon show you (FL), declination is again is same thing but (FL), this first thing is different, and then for the ecliptic co-ordinate longitude is called the various name (FL), and all that latitude is (FL).

So the reference circle (FL) so for the horizon it is called (FL), Prime Meridian is called (FL), and for the hour angle equatorial system is (FL) the prime meridian, and reference plane is (FL), and similarly, for the right ascension and declination is (FL), and declination is the (FL), same actually is essentially (FL), only is that and the okay, longitude for the ecliptic the reference circle is (FL), and the latitude is (FL).

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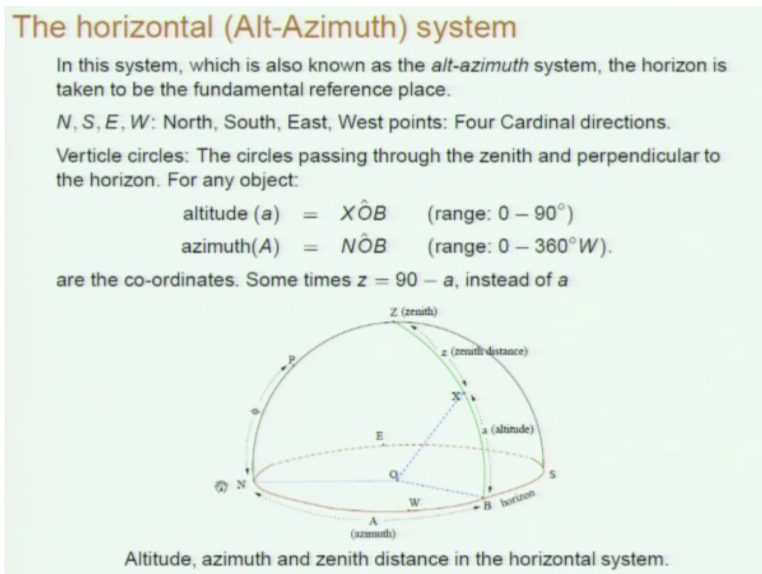

Altitude, azimuth and zenith distance in the horizontal system.

So far the horizontal system or alt-azimuth system sometimes it is called, so essentially this is your celestial sphere this is your object, so this how much it is elevated above the horizon so that is the altitude. And suppose it takes some point northern point let us say, then north to this point you see you draw the great circle passing through the object under zenith, so that is called a vertical circle, so this angle where it hits the horizon to point B intercept it at B.

So then this is called as azimuth, so this is the horizontal angle is called as azimuth in nowadays even now in physics but it was earlier used for astronomy, so this is altitude and is called zenith distance altitude + zenith distance is 90 degrees. And you can use any units for that you can use it degrees, you can use minutes as in Indian astronomy, sometimes you can use the radiance also, but normally in Indian astronomy it is expressed in terms of minutes and sometimes in degrees also (FL) also used okay.

So this is the horizontal or alt azimuth system, we met you know some people who are handle telescopes they will know that you know if you go and ask for a telescope so they will say is it do you want alt azimuth mount or equatorial mount okay, so that is you know how you will observe, I mean what are the ways you can rotate the thing and all that, so these are the alt-azimuth system.

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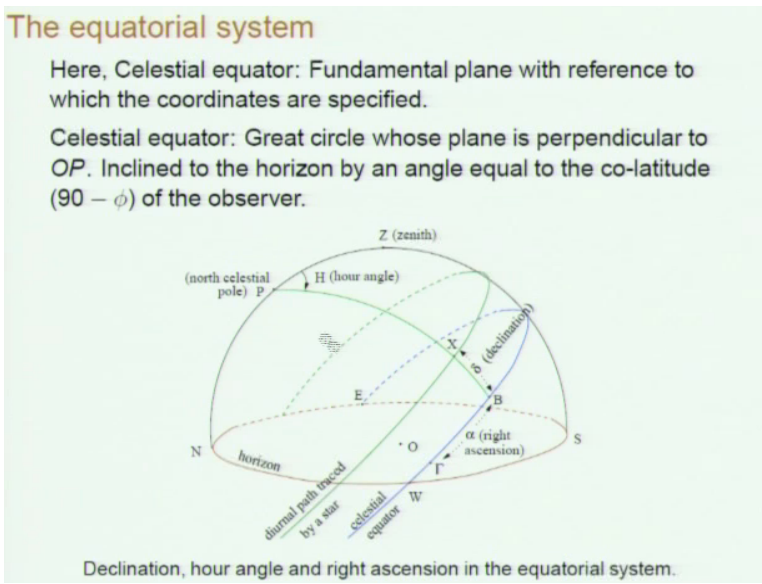

And similarly, equatorial system is you know now this is your celestial sphere and this is your north celestial pole, now draw a great circle through east west and which is perpendicular to this OP, draw a circle which is perpendicular to the celestial I meant this axis OP, and which is intersecting the horizon E and V that is by definition that will be east and west points, so that is called the celestial equator which will be an imaginary circle in the sky which is parallel to the terrestrial equator.

So that is this thing, and then from this you drop a perpendicular arc from P to this, so then suppose your object is this, what is the distance between this object and the equator in angular measure, so that is called declination, and in this angle you know how much it is moved from this is called a Prime Meridian, so how much it has moved so it has come from here and would move like this by how much angle it has moved that is called hour angle.

So that we will tell you how many angles it will tell you how much it has moved in the sky after crossing the Meridian, so Meridian is you know that uppermost to the semicircle see this vertical circle, so that how much has it moved how many degrees has it moved okay, so that is that 15 degrees is essentially equal to one hour right, because 360 degrees will correspond to 24 hours. So from that you can find out how much time has elapsed after the Meridian transit you see, so this is equatorial system.

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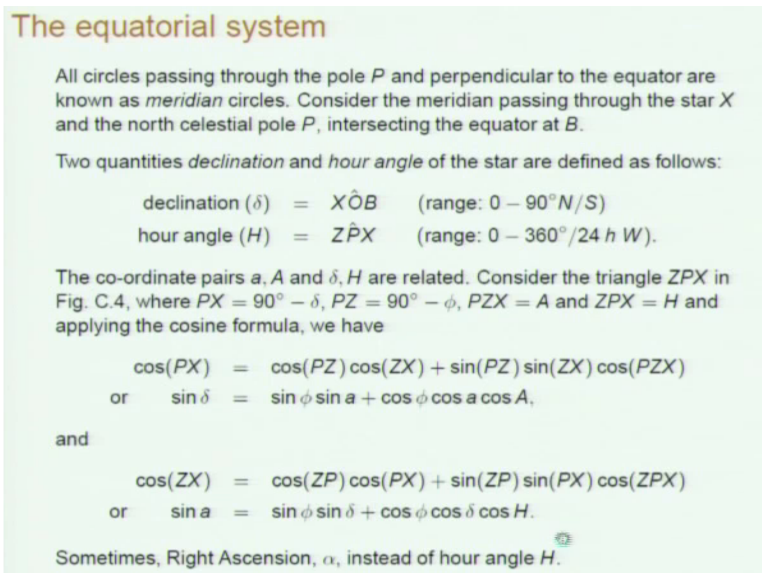

And in this also there is a declination and hour angle so that are these things, sometimes it may be more convenient to what is the right ascension, so there is a point called equinox which is intersection point between equator and ecliptic, so the equinox and this so drop the perpendicular from X to this celestial equator, the angle between this equinox and this point that is called a right ascension.

So that is also used sometimes, and so there is an angular measure along the equator, anyway so this you had the altitude and azimuth or declination and hour angle, so they will be related to this is spherical trigonometry relation, so these are the things we get related.

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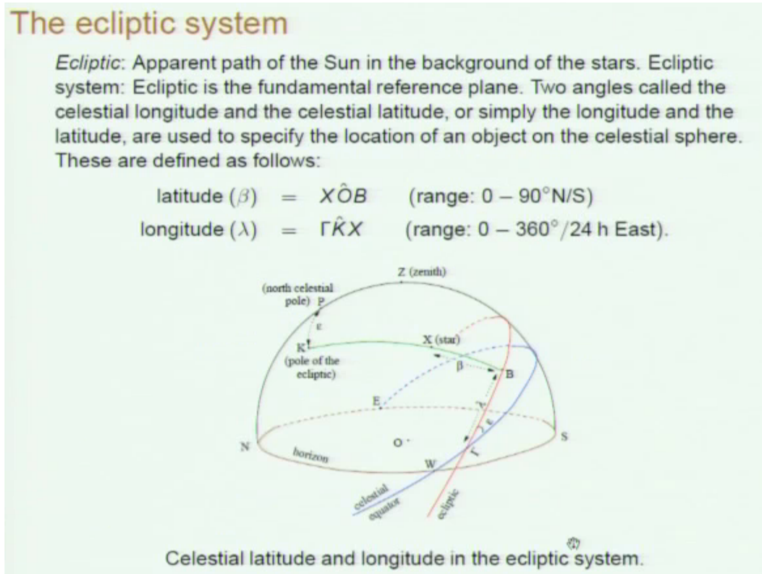

And this ecliptic system especially for measuring the longitude of the position of planets and all that you had to use this, even actually for sun also so you have to sun and moon also, so there you know so you are considering the circle which is apparent path of the sun so that intercept the equator at point this like this, and then suppose this your star or any object so draw a perpendicular like this, so this angle that is called longitude celestial longitude.

And this is called the latitude at (FL), (FL) in Indian thing and lambda is called sometimes it is called just (FL) or sometimes sin of that is called (FL) from the contest it is clear, sometimes we just called (FL), so this is this. And everywhere it had 2 angles, one angle along the reference circle, and one angle for perpendicular to that, so that is the and all this coordinates are related.

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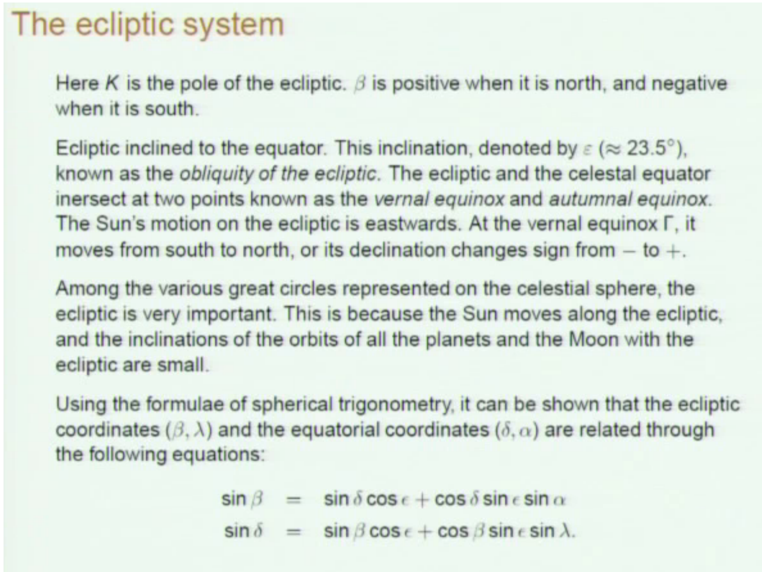

Beta is related to all this delta and all that ecliptic system, so this ecliptic system, so this is the equatorial system, and this is alt-azimuth system. So all of them will be used in.

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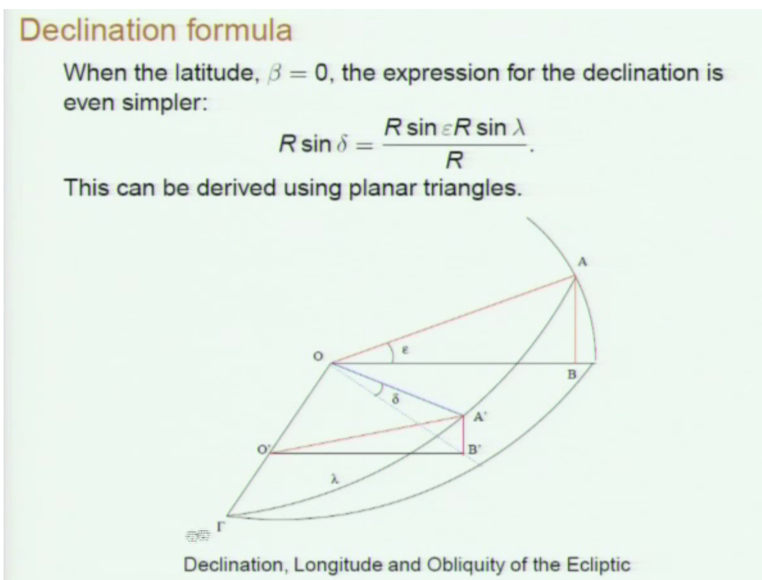

So declination formula, so remember what is declination so essentially this is the angular measure between the object and the equator that is called declination, and suppose it is moving you know it is actually situated in the ecliptic also okay, so then in that case so that longitude celestial longitude will come, so how it is related to longitude delta, so that we can find out so for instance I have written here in this diagram.

So this is the equator celestial equator, this is ecliptic there is an apparent path of the sun, and planets also moon they also will be very close to this ecliptic, so suppose if the object is situated here so then the angle between these planes you see this plane is ecliptic plane O, this is gamma, O, A, so that is the ecliptic plane and O, gamma, B you know this is the equatorial plane, the angle between them is this epsilon or the called obliquity of ecliptic.

"Professor - student conversation starts" (FL) not the terrestrial latitude this is the celestial latitude this is (FL) no (FL) this is the motion this is not (FL), this is the you know describing coordinates of the celestial object not the shadow, and there is nothing to do with shadows, no (FL) but is how to describe the coordinates of the object you see that is what we are discussing and the declination how much it is above the equator, so that is what we are trying to see. **"Professor - student conversation ends."**

And I am saying there for instance for sun we write it in terms of longitude, longitude is what is given, so what is its declination you see the declination is nearly 23 and 1/2 degrees at the June 21^{st}, and it is 0 on March 21st, on any other day because the declination will be the 90 degrees for June 21st, so in between March 21st this will be in between some values know, so that is lambda and how is declination related to lambda that is what we are trying to see.

And then in that case you can drop a perpendicular from here to this, and here to the plane of the equator both, so then this O, A, B and this O prime, A prime, B prime both planes will be perpendicular to the equator, and this will be clearly R sin delta, and by similarity of triangles you see this will be R, this angle will be lambda you know this angle, gamma, O, A prime is lambda, and this line will be O prime, A prime that is sin lambda.

So this is R, this is R sin lambda, this is epsilon, and this is R sin epsilon, and this is R sin delta, by proportionality of the triangles you get this, so this can be derived using planar triangles. So this is a well-known result you know in Indian in fact any astronomy we will have to know this, otherwise they cannot perform any calculations they cannot give any get any useful result, so this is the declination formula.

All these things are very clearly explained in Yuktibhasa, so as I told you most of the text will give only the results and how to use the results they are given, but explanation of all these things you know or you should go for some commentary, and Yuktibhasa is more or less commentary especially astronomy part it is really a commentary on the Tantrasangraha where all kinds of results are given in a systematic manner, so there they will explain all this things you know, I will come to that typically what is given in Yuktibhasa.

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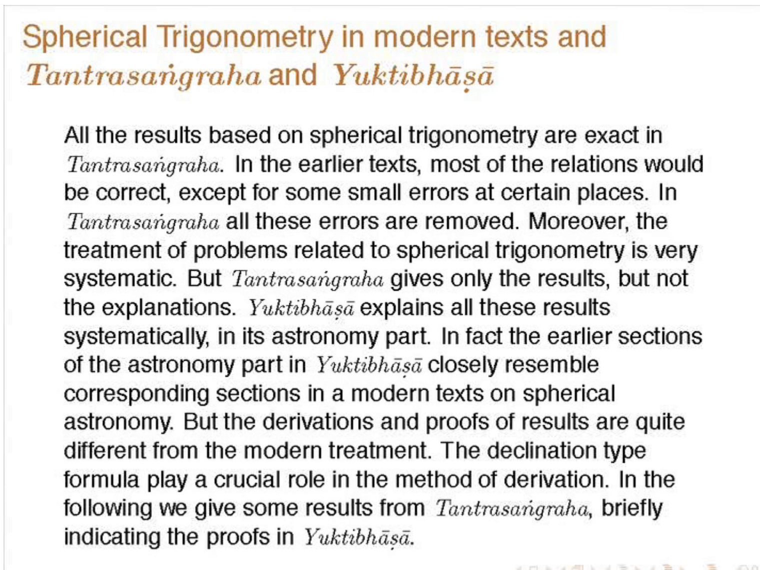

So a spherical trigonometry in modern text and Tantrasangraha and Yuktibhasa, so all the results based on spherical trigonometry are exact in Tantrasangraha okay. So and in the earlier texts, most of the relations were correct, but for small errors at certain places especially some things which will come in the concept of eclipses called (()) (27:06) and all, I do not want to go to technique, really there are some side things which are you know not correct but not very inaccurate actually numerically they are not very bad but they are not correct anyway.

And Tantrasangraha all these errors are removed, and all the results are exact and correct, the spherical astronomy results so there are correct, of course planetary motion there may be some parameters maybe you know not coincide with modern parameters, but the model is all again correct we are not going into that now, but what I mean is saying is as for a spherical trigonometry is concerned all the relations are absolutely correct.

And moreover the treatment of problems related to spherical trigonometry is very systematic, so they were there so called (FL) all these things will be there, and here the treatment is more systematic and he will go from we can see one example you will take how it is more systematic, and but Tantrasangraha gives only results but not the explanation. So Yuktibhasa explains all these results systematically in astronomy part.

In fact, the earlier sections of the astronomy part in Yuktibhasa closely resemble corresponding sections in a modern text on spherical astronomy, so if the earlier people had astronomy in BSC some of them I think still have, so they will start with celestial sphere, they will start with horizon, they will talk about the cardinal points, zenith, and celestial equator all these system of coordinates and all that.

Exactly similar things are there in Yuktibhasa but in a different language, and all these things the figures are not written they are explained okay, they are you know described, so but if somebody knows some astronomy one can easily recreate the figures, so we had done that when we wrote the explanatory notes for Yuktibhasa. So all there is no ambiguity always you know one can clearly see what he is trying to see.

So all these things are very clear you know clearly and systematically explained, so it is really a textbook actually, and there is goes in a way similar to modern astronomy part. But derivations and proofs of results are quite different from the modern treatment, because in the modern this thing you will go to cosine formula and then immediately do various results, cosine formula is not dead but something equivalent to that is there okay.

So but the treatment of problems is you know you will get the eventually the same result, but using slightly different principles, so that is what I am going to say. You know the declination type formula play a crucial role in the method of derivation and in the following we give some results from Tantrasangraha only indicating the proof in Yuktibhasa. So basically they will use this kind of formula declination kind of formula and then find the rule of proportion, declination and formula and rule of proportion so that is what is used.

So essentially what I am trying to say is what they considered? How it is post in Yuktibhasa? Of course it was understood in earlier text also but they might not have stated it like Yuktibhasa which is a textbook.

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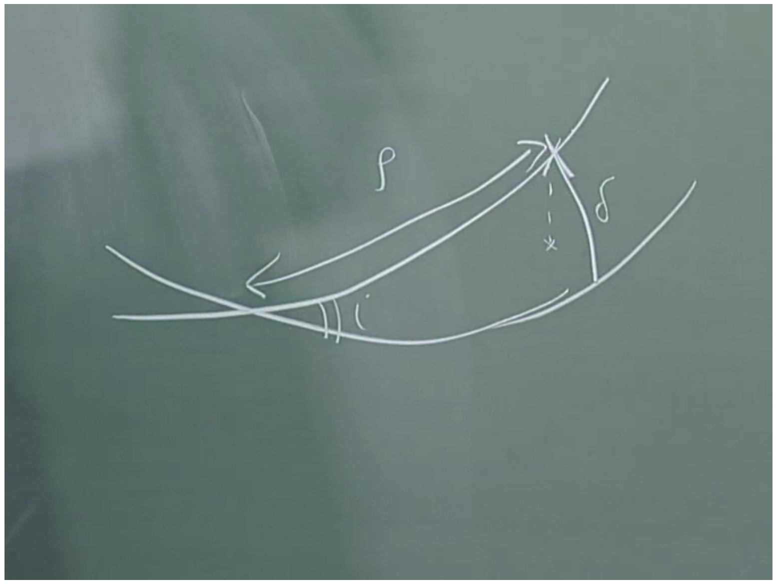

So what how he suppose you got 2 great circle, suppose I have 2 great circles intersecting at some point, and suppose the inclination is known, so then suppose you consider a point which is some anything some angle some arc on one circles, then drop a perpendicular to the plane of the other circle, what is the length of the perpendicular? Okay, so that is how you poses the problem and that is what he says to the declination formula.

Because this will be essentially some kind of ecliptic this is declination, and this is perpendicular R sin delta, how it is related to this arc and this. And if we have say for some this is a simple problem, but for some complicated problems you have to use this several times you know you have to consider I will give an example various kinds of circles have to be considered, so then it becomes a little more this thing detail.

And but principle is the same if it becomes tedious but not difficult, you have to be patience you see, anybody who wants to goes to astronomy celestial astronomy even very elementary thing is how to be patience, because you have to draw the figures and all that and see you know it will be very confusing, only patience is the most important thing.

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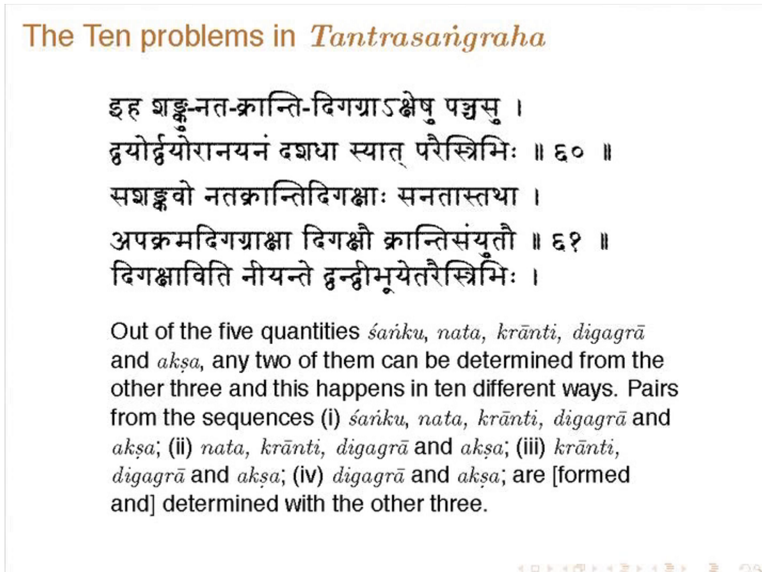

So 10 problems in Tantrasangraha is a (FL), so here you know out of the 5 quantities sanku, nata, kranti, digagra and aksa. Sanku is you know related to the zenith distance, Nata is related to the hour angle, kranti is related to the declination, digagra is related to the so-called azimuth and aksa is related to the latitude okay. So here (FL) really 5 of them are there, (FL), so 2 of them can be determined (FL) 2 of them can be determined from the 3 others.

And this can be done in 10 different ways okay, so it is 5C2 or 5C3 which is equal to 10, and this can happen in 10 different ways. So you form the pair systematically sanku, nata, kranti, digagra and aksa, then nata you will leave that, because all those involving these some nata, kranti, digagra so like that, so they are all determined.

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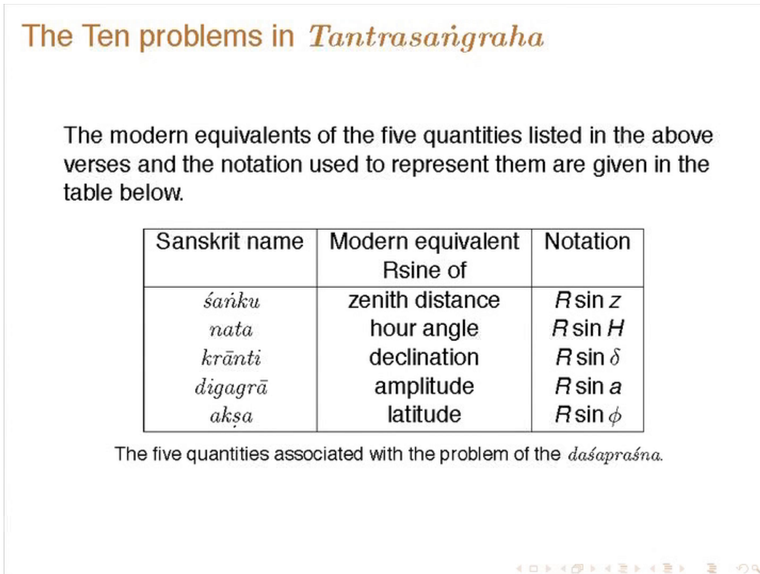

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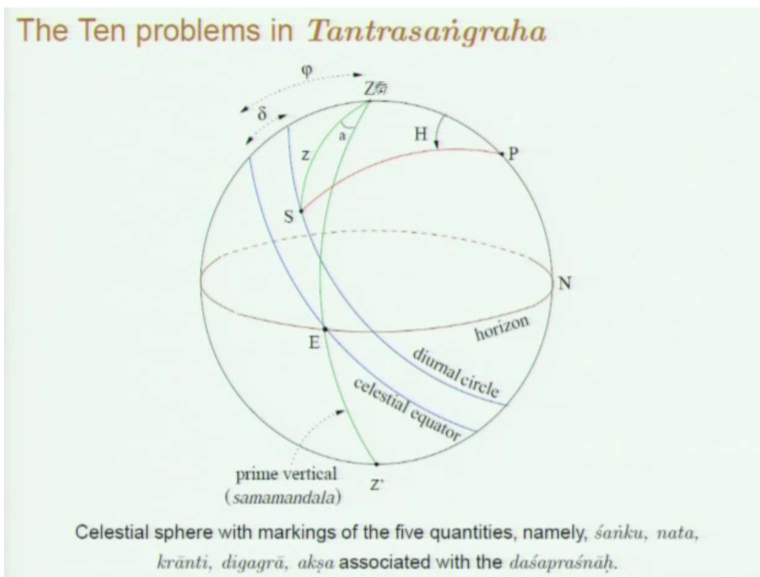

So sanku is zenith distance I will show this is what is talking about, so this is celestial sphere, this the diurnal circle, so this hour angle, and this is the so-called azimuth. So here the azimuth is measured from this is called the prime vertical, the vertical circle which is goes from zenith to this z is (FL), so this is east this is the lower portion, so because angle between this you know to draw this to this, so this angle is called digagra is so called as azimuth.

So this is the declination kranti, this is the aksa you know latitude this angle is latitude, hour angle is called nata it will tell you how much time it is taken sun is any object which is moving like this okay, so you draw imaginary line this is you know line is rotating like this, so this angle is called as H that is nata, so sanku is cos Z as we will see sanku is cos Z basically, nata, kranti is this, digagra, aksa okay, the 5 quantities. So one can determine 2 from the other 3.

So this is a table, sanku is zenith distance R sin z, nata is hour angle is R sin H, so normally sin itself is you know is more you should call it as you know sin itself you give the name. So kranti is R sin delta, but some time rarely may mean declination arc one also, but one has to be little careful you know, so people who are working in area will normally what is correct. So digagra is called amplitude R sin, and aksa is called latitude R sin phi, phi is a latitude of the place.

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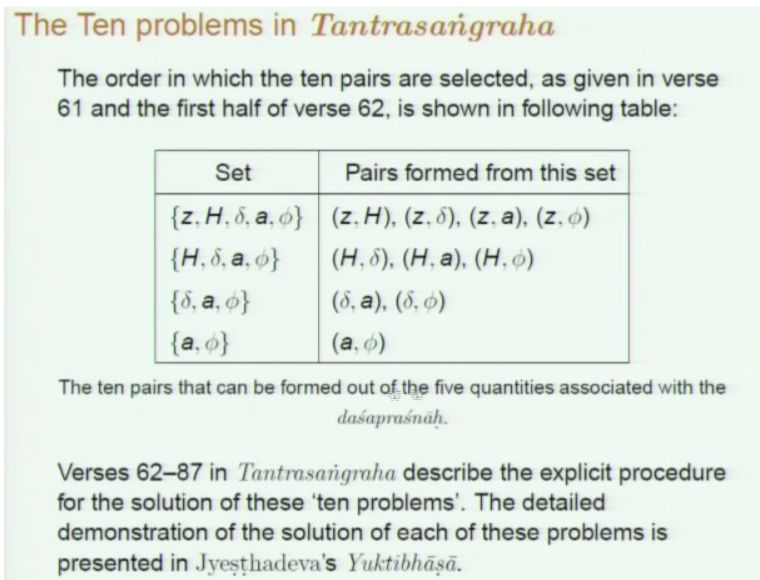

So these are the how to determine 2 from the other 3 out of these 5 qualities, so that the 10 dasaprasnah that is what saying, so you have got z, delta, H, a, phi. So you can form these pairs, first you have this z, with z you form all the pairs the 4 of them really, so now z is exhausted. Now we will take H with delta, H with a, H with phi, that is 3 of them. Then delta a and delta phi, and a and phi.

So this is how it is actually ordering 10 pairs that can be formed out of the 5 on quantities associated with the dasaprasnah. So versus 62 to 87 in Tantrasangraha describe the explicit procedure for the solution of this 10 problems. The detailed demonstration of the solution of each of these problems is presented in Jyesthadeva's Yuktibhasa.

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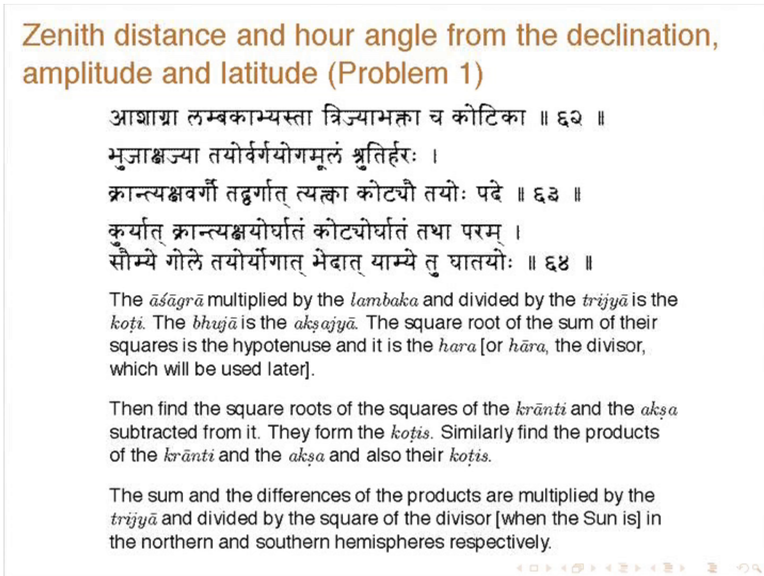

So this will tell you I mean how it is you will be interested to know, how the results are presented okay this sloga's that is the thing you know, there are no equations how do you describe it. So the equations are implicitly given by this (FL), the asagra multiplied by the lambaka and divided by trijya is the kotis etc., so this is what is saying, we will see what is the result.

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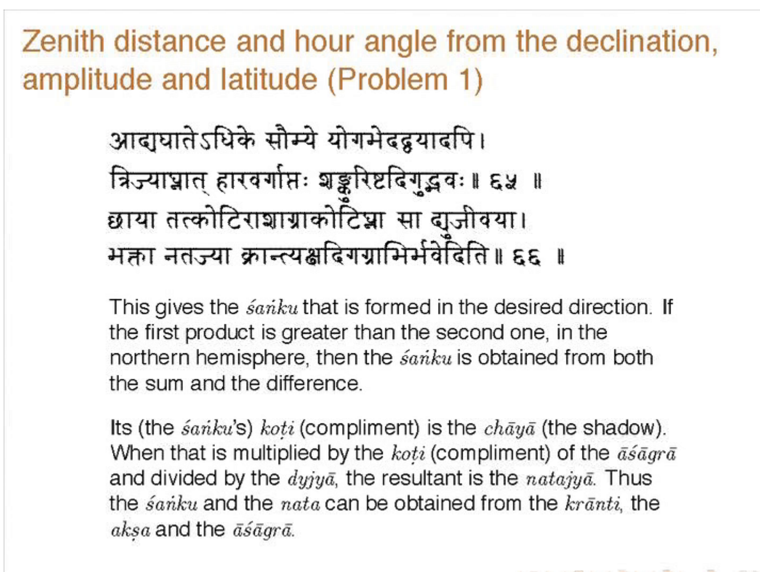

This gives the sanku that is formed in the desired direction, and when it is northern part of the celestial sphere, and southern part is some difference so is telling all this here.

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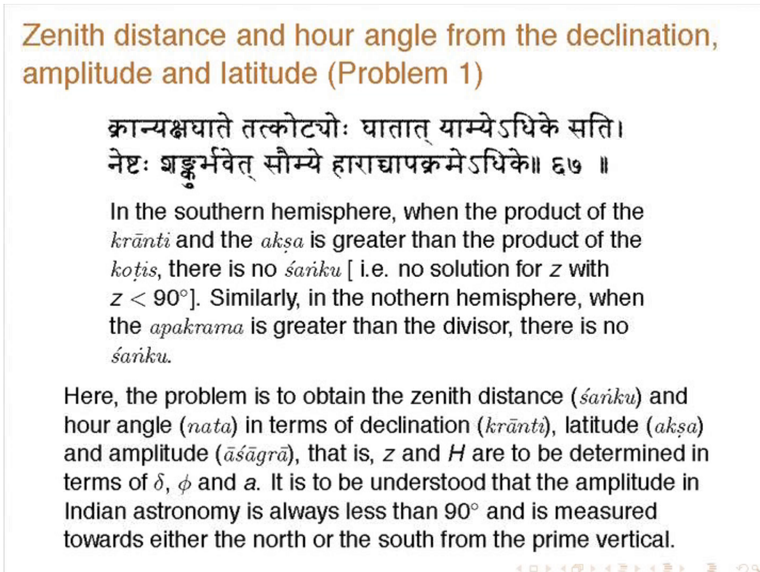

Tantrasangraha, versus in Tantrasangraha and its explanation is in Yuktibhasa. So Tantrasangraha is so called Tantra's texts, so they will not give much theory they will give all the results, results are there explanations are to be found in the format, Yuktibhasa use the. Tantrasangraha by Nilakantha somayaji or Nilakantha, so lot of reference was there to Nilakantha you know, there is infinite series and Aryabhattia so on so forth.

Nilakantha somayaji, so he was this work was composed in 1500 C and Yuktibhasa came little maybe 25-30 years later, so Jyesthadeva wrote Yuktibhasa which is commentary on this thing, but actually only the astronomy part it is commentary on Tantrasangraha, the mathematics part it takes off you know all no results in Kelallur mathematics, so there is more or less an independent thing.

Though, he is humble enough to say that is you know whatever is in Tantrasangraha I am explaining, whatever results have been in Tantrasangraha to know this you know I am giving all this that is how it goes. So finally the result is and similarly, there are some conditions, when there is a product of the kranti and aksa is greater than, there is no sanku, there is no solution like you know.

So here the problem is to obtain the zenith distance and hour angle in terms of declination, latitude and amplitude, so that z and H are to be determined in terms of delta phi and a.

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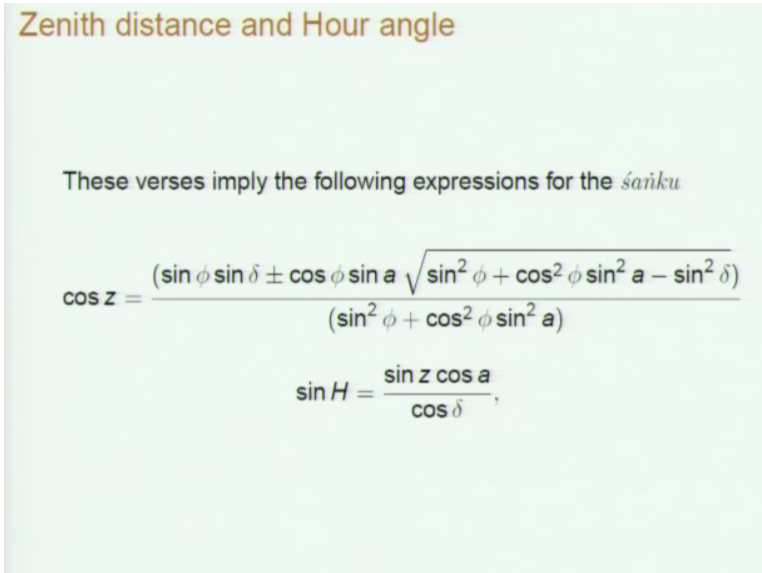

So this result is giving cos z=sin phi sin delta+-cos phi sin a*this, so that is what he is saying you know take aksa take it square, take it kotis take it square and multiplied by the square digagra subtract the declination from this, and all the whole of this you take the square root and multiplies by the lambaka which is this thing that is how it is expressed, so this is a very there is no ambiguity I mean there may be some places where it may be interpretation required.

But most of the places it is very clear you know this is the expression which is implied in that, so z and H, so phi delta in here and there, so z and H are determined as this, and then once you determined the z you can find out H. So the solution of this problem, so what is given so a is given, the declination is given, the latitude of the place is given, so what are you determining we are finding this zenith distance or the altitude is the alt-azimuth of that zenith and Hour angle you are finding.

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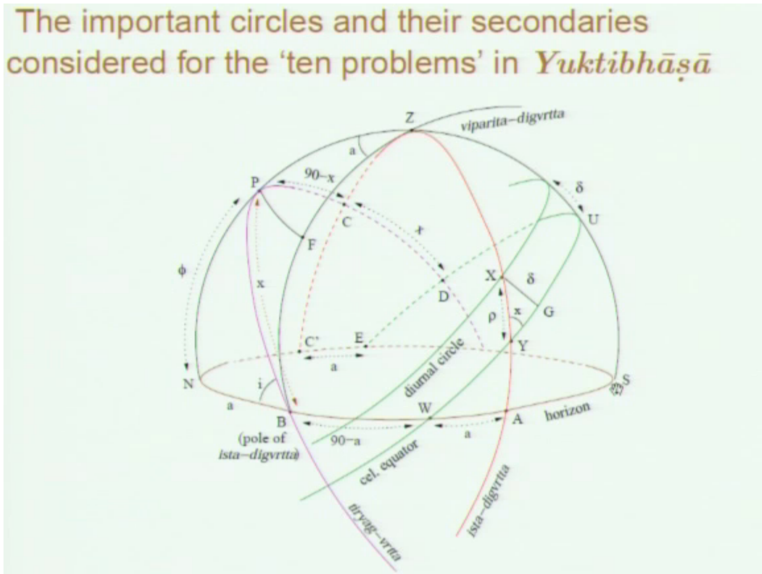

So for that of course Nilakantha gives only the results, so similarly, he will give explicit results for all the 10 problems, and Yuktibhasa gives the proof of these things, so I will just write I am just giving the basic method and how they describe various circles and all that. So you consider a celestial sphere, he says you take the celestial equator write the you know diurnal circle is called (FL), I mean that is the daily circle associated with the sun okay.

So then what you do is you know suppose this is your object here okay, so then this is in here, so this is called with the corresponding amplitude is called (FL), to write a circle which is you know at this angle RK from this is called (FL), and then you say called a pole of (FL), this is that which is 90 degrees from here okay, and then from this you know you are write a this thing called (FL), which is you know a perpendicular to this.

So various circles are the same and then you draw a various circles okay, so all the circles you have to find and then he will tell you how to, he will done in 2 to 3 pages so patiently you have to plot through that and you will get the result, so please you can have a look at Yuktibhasa, so we are given all the. So as I told this is all given in the it is not verse it is not sloga in Yuktibhasa it is only prose only, in prose he given all these things.

So this we are translated into this figure okay, so and then you know work with that and show that, so that is what one has to you know the research scholars here working in mathematics in astronomy they had to do some such things you see, you have to draw the figure and all that from what is stated in Hindi work, so this is the thing. So that is one of the, all the others are also patiently they are explained okay in the work Yuktibhasa all the other results also all the 9 also okay.

So there is no need to show them you see it will be overwhelming to say the list not difficult but you know lot of details are there (FL), intersect this all that there is angle find this angle use this formula so like that, so lot of detail only but principal is quite simple.

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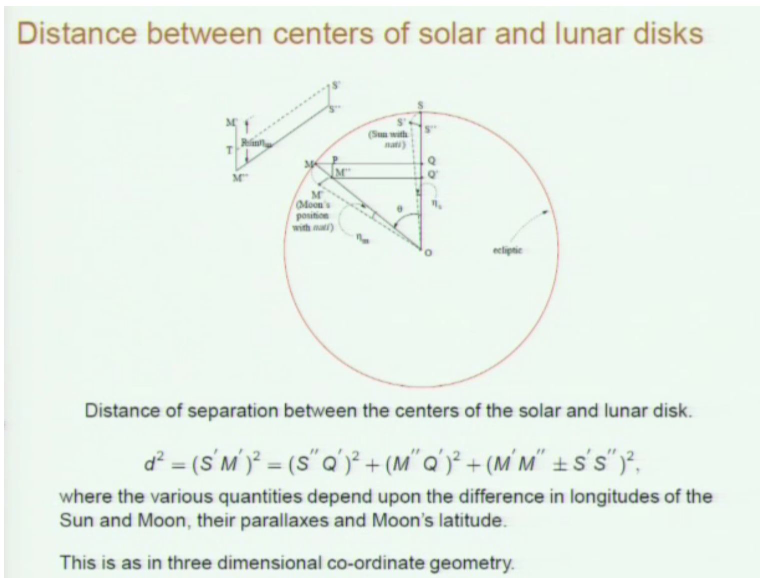

So then I will slightly different thing I will talk about it, so in this work Tantrasangraha it also talks about distance between centers of solar and lunar disks, it is related to spherical trigonometry. So what is this thing is you know sun is there and moon is there first we think that you know they are relative position is the difference between the longitude of sun and moon okay, now but that will not give you the separation really because moon has a latitude.

So moon is away from the ecliptic and even sun because of so-called parallax, you know all these things are explained or described with reference to the center of the earth, for the observer you have to this angle will be slightly different, so that is called the effect of parallax.

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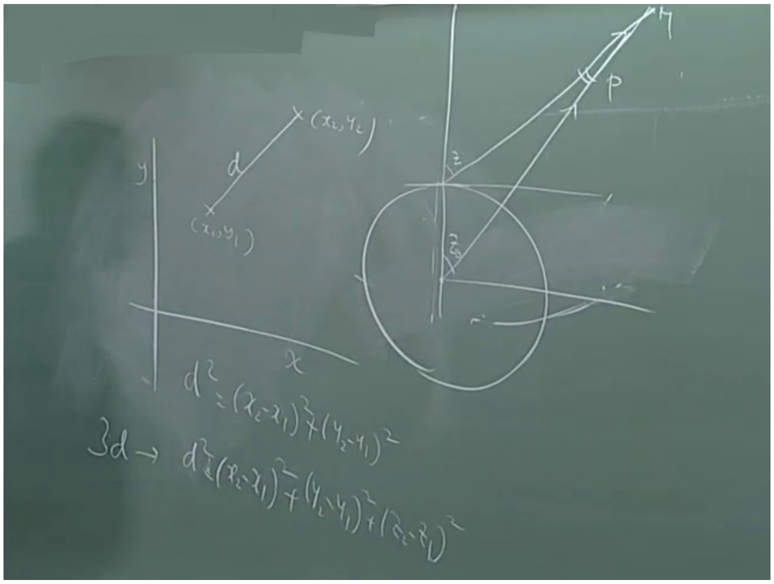

So what I am saying is if you are if you suppose you are observing some object there is a moon okay, so you are observing from here okay, so there is some reference line let us say and suppose this is your celestial sphere, so you are observing from here okay so then this is the direction what you are observing from the surface of the earth, so then this is the direction under the difference between this and this.

So the zenith distance for this is z, and zenith distance for this is z0, I mean imagine a person sitting on the center of the earth and observing it with the same kind of sphere celestial sphere, so he will observe it zenith distance is z0, and this one is this. Anyway so this angle is called parallax, so it plays an important role in eclipse calculation and all that, so because that the sun also will be slightly deflected.

So now in that case sun also will be deflected this is ecliptic circle, so moon has come out of that so it is M prime and sun is in S prime, so then what the essentially the procedure says you know draw a perpendicular arc from this to the ecliptic plane similarly, from this to the ecliptic plane, so then this is the S double prime and M double prime. So now there are projections, but what you want to know is the real distance angular separation between sun and moon.

What is observed? The observed locations are sun and moon the angular separation between them, so for that so that distance you see but one has to find that distance, essentially I have to find that chord in the sphere chord this thing, angular separation of course corrected angular separation will come later, so this chord you know essentially gives the distance between these points actually observed points as S prime M prime.

So he writes it as you know so that you drop this perpendicular S prime to on the plane, and then he says that you know there are 3 things, so this is the separation you see S double prime Q prime is this separation along this line you see perpendicular line, and M prime M double prime Q prime is separation along this line this horizontal line, separation along this separation along this, and then it is separation is the plane of the ecliptic.

So one has come out this way and one has come out that way or both might have come out of the same way, so that separation the 3 separations are there you have to square them add them and take the square root that is the distance between the 2 quantities. So these essential is same as in 3 dimensional co-ordinate geometry okay, so because suppose you have in 2 dimensional co-ordinate geometry what do you have, the distance between 2 points.

So these are the co-ordinate axis x and y let us say, suppose one point is there x1, y1 are the coordinates and other point is there with coordinates x2, y2 so joint these things, so then the distance d squared=x2-x1 whole square +y2-y1 whole square, so this is x2-x1 the separation along the x-axis, y2-y1 is the separation along the y axis. So the total separation is this you have to take this squares of them and then take the square root add them and take this that is the distance.

And if it is 3 dimensional you see one is here and let us say one is here, so 3 coordinates are needed x1, y1, z1, so in 3 dimension the generalization of this is d square= x2-x1 whole square +y2-y1 whole square+z2-z1 whole square, so this will be distance square. So this is a separation along x-axis, this is separation along y-axis and this is the separation along z axis, so these 3 are taken they are squared and take the sum and then take the square root.

Precisely the thing is being said you know, so they do not say coordinate axis and all that they will say that you know separation along this thing, that is 1 Rashi they say that S double prime Q prime is 1 rashi, rashi does mean not necessarily a zodiacal sign quantity kind of thing, one rashi then this is another rashi along a perpendicular line, and then this one this along the you know perpendicular to the plane of this slide perpendicular to plane of this slide is another rashi that is the separation along that.

So take the squares of all these things sum them and take the square root, so that the distance between centers of solar and lunar disks, so that is the result. And of course this will depend upon these various things will depend upon the difference in longitude of sun and moon, they are parallax and moves latitude another. All those details are of course given, so these are some of the things which are discussed.

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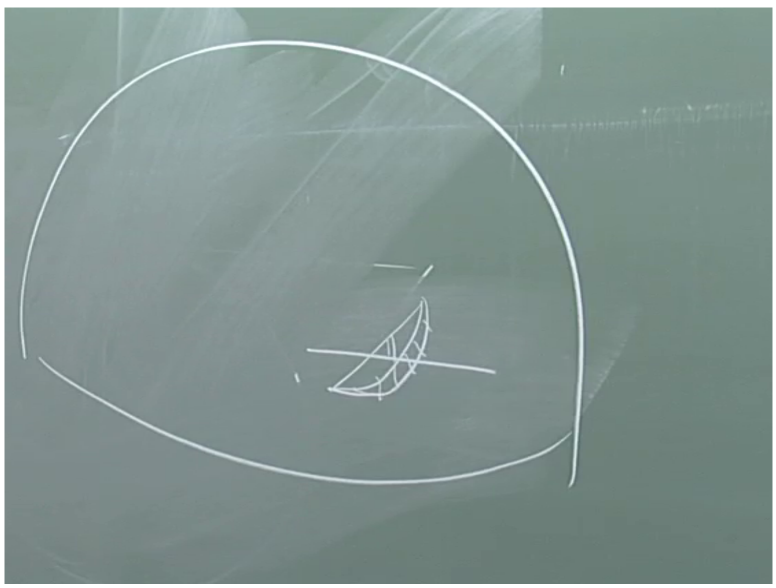

And there are somethings which are really interesting and rather sophisticated things which they do using spherical trigonometry say for instance (FL) they call it you know in the celestial sphere some moon, suppose moon is like this, so this is the crescent moon okay the crescent moon is there suppose this is the shape of this, then how in what angle it is inclined to the horizontal plane kind of a thing, so that is called (FL) okay the horns of the moon how much it is thing.

So that also uses in nice should have cleverly you should do the all these calculations so they are there in the, so I will just give you some player of you know the kind of things that are done in complicated reasonably advanced for that time you see the 500 years or 600 years back or so for that time it is fairly somewhat advanced and sophisticated. So they could handle it in a systematic manner and all results are.

Because I really apply you useful for daily problems for finding the positions of the planet and eclipses and all that, they are all in all these things will be very useful so that is what they are doing spherical trigonometry. So these are normally all these things are discussing this thing called (FL) 3 problems (FL) okay, or sometimes also called (FL) the chapter on shadows okay that also is all these things. There in Tantrasangraha actually it is nearly 172 versus out of 420 or so on this.

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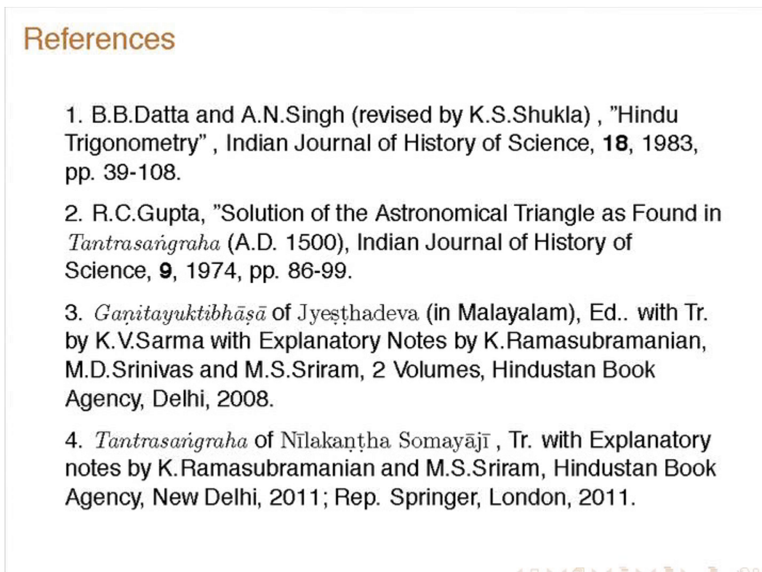

The references are given here okay; I think I will stop here.