Mathematics in India: From Vedic Period to Modern Times
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Lecture - 32
Jyanayanam: Computation of Rsines

So in this lecture on computation of sines, which is referred to as Jyanayanam so I will be trying to cover as to how various approximations and various techniques have been evolved over a period of almost 2000 years. So as we see that this is a very, very important topic because almost all kinds of calculations are crucially dependent upon accurate computation of sines.

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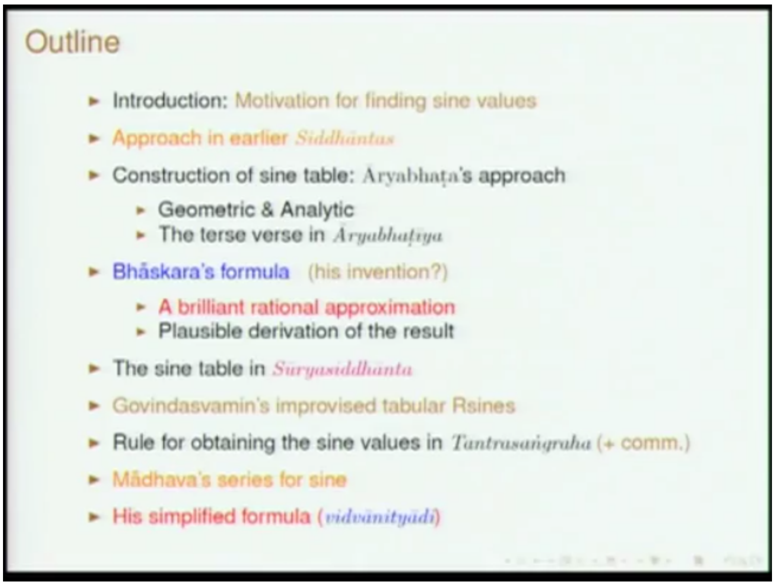

First, we will try to give a brief introduction as to why this sine values are so important and then we will see the approach taken by the earlier Siddhantas. The earlier Siddhantas I mean the Siddhantas that have been compiled by Varahamihira in his text called Panchasiddhantika, so pancha is 5, so he has compiled so the 5 Siddhantas, which were prevalent so around his time so which is around 505 AD.

And whatever Varahamihira has noted down with reference to this sine computation, we will be just quickly tracing that. Then we will quickly recall so whatever we discussed when we discussed the Ganithapatha of Aryabhatiya so the 2 approaches, which we have taken and then we will spend quite a bit of time on an interesting rational approximation, which has been presented by Bhaskara.

So in his commentary on Aryabhatiya-bhashya as well as his work called Mahabhaskariam so he has cited a couple of verses which gives a very interesting rational approximation for finding sine. So we will also see how Bhaskara has approached this particular rational approximation. So there are 2 or 3 ways which has been attempted by various scholars.

So we will discuss one such method so which has been presented by K.S. Shukla in his translation of Mahabhaskariam. Then we will quickly see the table which has been presented in Suryasiddhanta and then I will also touch up on this Govindasvamin's table of this Rsines. So he has also presented a table, which is a sort of improvised version of what Aryabhata has given.

And it is a very interesting table in the sense that it gives very accurate value so even around 9th century so and then we will discuss the verses which are available in Tantrasangraha, which present the technique for computation of tabular Rsines and then we will quickly go through the Madhava's verses as well as the infinite series and how he has handled the infinite series to compute sines.

So this is the idea, so as I was mentioning it is so crucial because the computation of time so in fact the reckoning of time whether it is masa, varusha, dina, tithi whatever it is so it is crucially dependent upon the planetary positions. So we say 1 year is over because sun has returned back to the same position, 1 month is over because the moon has come back once again in conjunction with son and so on.

So all the yes crucially depends upon the computation of planetary positions accurately and as you will see even professor Sriram will be touching upon this so as to how the sine function appears in the computation of planetary calculations and therefore the sine function is extremely important if one has to even determine time so which is something indispensible.
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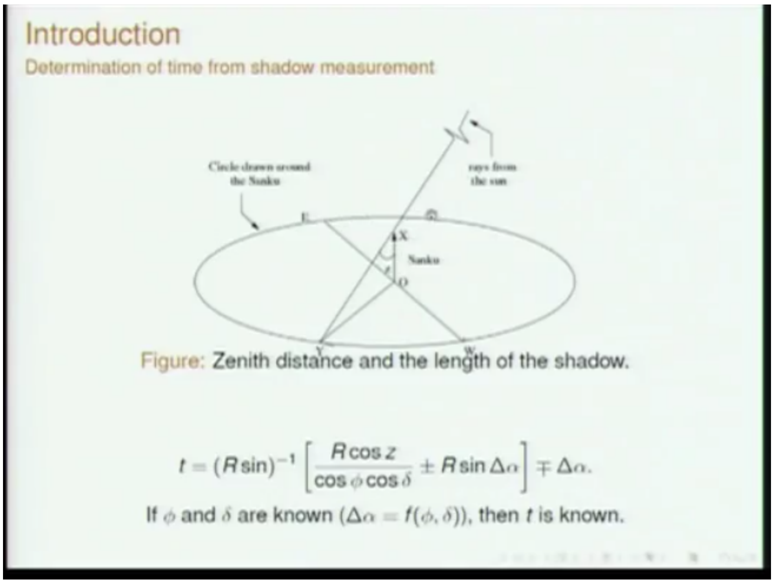

So I also showed this earlier so when we discussed this Sulvasutras that even the precise computation of time so can be had so using this kind of a formula which is presented so as you can easily see that sine and cosine functions appear so in this particular calculation as well as the inverse functions.

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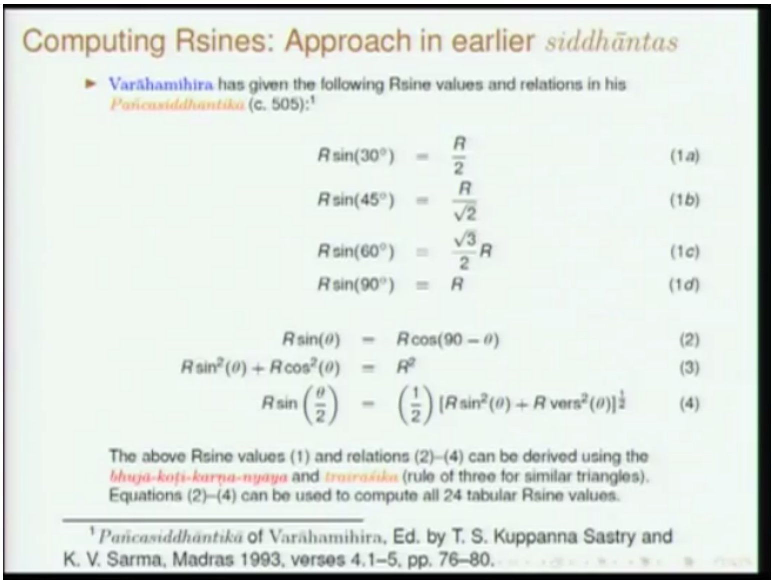

In Panchasiddhantika, we have these values of sine so very clearly stated. So sin 30, sin 45, sin 60 all that, so this we saw that easily by constructing a certain triangle so 1 will be able to see all these values can be easily (()) (05:02). Based on these values of sines and the sort of equations given below 2 to 4, 1 will be able to generate the 24 tabular Rsines, which we discuss while dealing with Aryabhatiyam.

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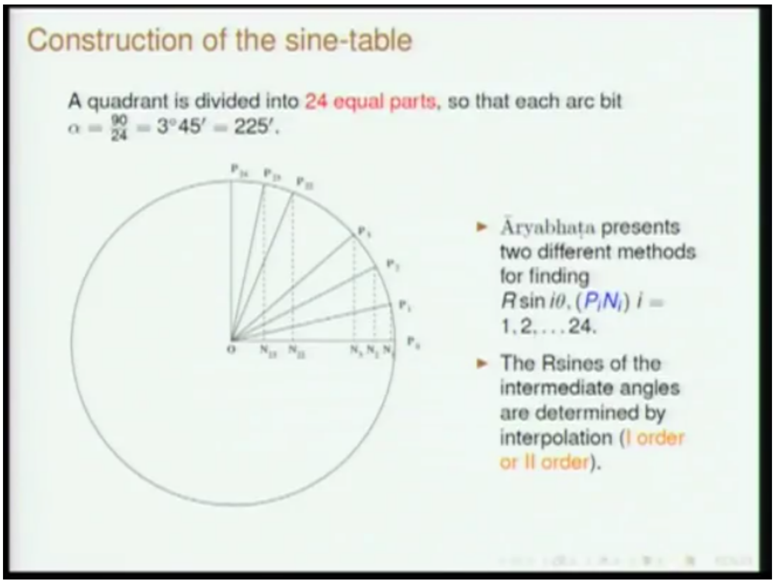

Usually so a quadrant is divided into 24 parts, so p0, P1, P1, P2 and so on so each of them is taken to be 3 degree and 45 minutes 225 minutes so the idea is to obtain the sine values so at these intervals so this is what people have attempted. This is what Aryabhata has given in his table so which is referred to as (FL) so that is how it is. So this is the approach so which is generally taken.

And you will also see in other lectures that people have attempted to give the sine values for 1 degree also, so they have reduced the interval to see that the accuracy can be improved upon. See if you know the value at p1 and if you know the value at p2, then if you want to find out the sine at some value in between p1 and p2, then generally we use first order approximation.

Second order approximation has also been given to see that we can have much better values but if you can reduce the interval itself and the table is available then it will be far more convenient and attempt in that direction has also been done by astronomers and mathematicians. So Aryabhata so has given this (FL). So in this verse which we explained in our Aryabhatiya lecture.

So we clearly showed that if you know the value of sine precisely say R 30 degrees and 1 more value.

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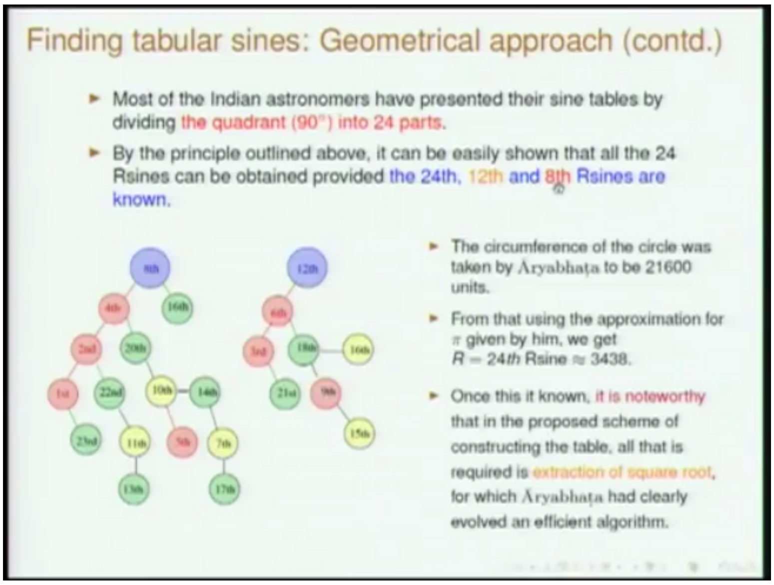

See for instance if you know 24th, 12th and 8th so then you will be able to compute the entire sine table and this was explained. So I will not spend much time here. So once you know this value so 8th sine is basically 30 degrees and 12th sine is basically 45 degrees. So both of them can be obtained by simply looking at the geometrical construction, so one will be basically R/2. So once you know that so you will be able to compute the entire sine table.

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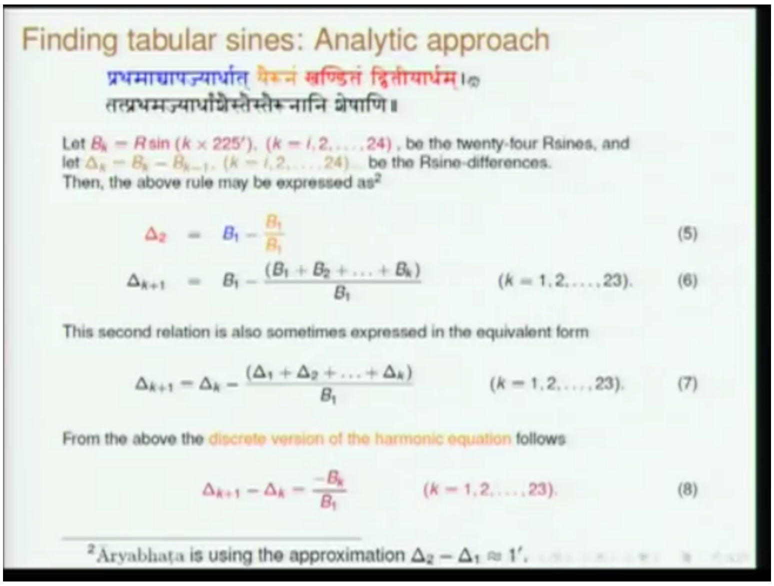

The other approach which has been taken by Aryabhata so this one can call as geometrical approach in the sense that you start with this geometrical figure, you make certain observations and then you will be able to compute the entire sine table. The other approach which has been taken by Aryabhata is the analytic approach. See here Bk represents the kth bhuja.

So the terminology for Rsine is bhuja and cosine is koti. So here so our idea is to obtain so all the values for B1 to B24. So this verse which you find in Aryabhatiya essentially translates to this kind of an equation. See equation 8 so wherein delta represent the first order sine difference, delta k+1=delta k represents the second order sine difference and what you note here is so it is proportional to the sine value at that point.

The second order sine difference is proportional to the sine value so which essentially is the discrete version of the harmonic equation. So here when you say it is proportional and what you find as a proportionality constant is 1/B1 okay. So B1 represents the jya of the first sine so generally it is taken to be 225 I mean so the value has been improved upon as we will see during the course of the lecture.

So how this constant has been improved upon over the period of almost 1000 years so people have sort of done and we will see that the value which has been presented by Kerala astronomers is astonishingly accurate okay. So that has to do with the kind of invention which has been done so in finding out infinite series for sine so that we will see as we progress during this lecture.

So this is about the Aryabhata approach to finding sine values so 1 is geometric the other is analytic.

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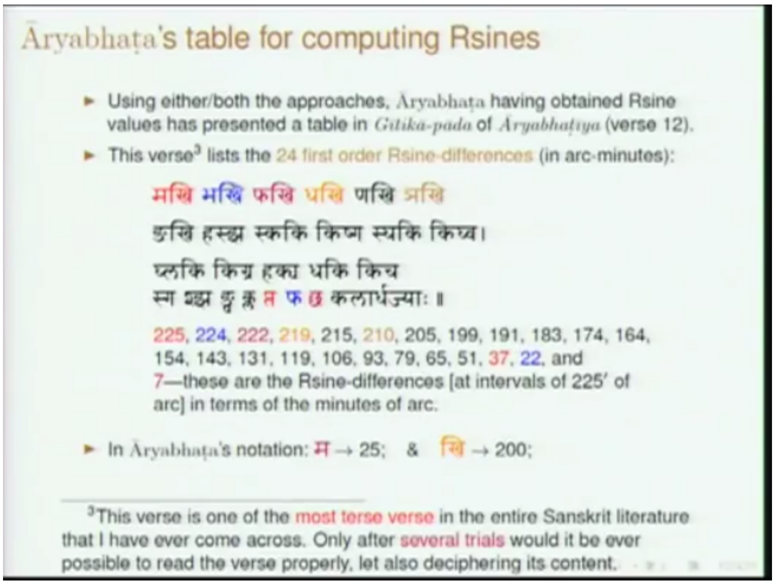

So both ways you basically construct the sine table. The sine table which has been presented by Aryabhata is in the form of a single verse, which essentially gives the Rsine differences, the first order sine differences and as you can easily see the first order sine differences will keep decreasing. It starts with (FL) and then it ends with (FL) so this is how he has presented the entire sine table in one single Arya.

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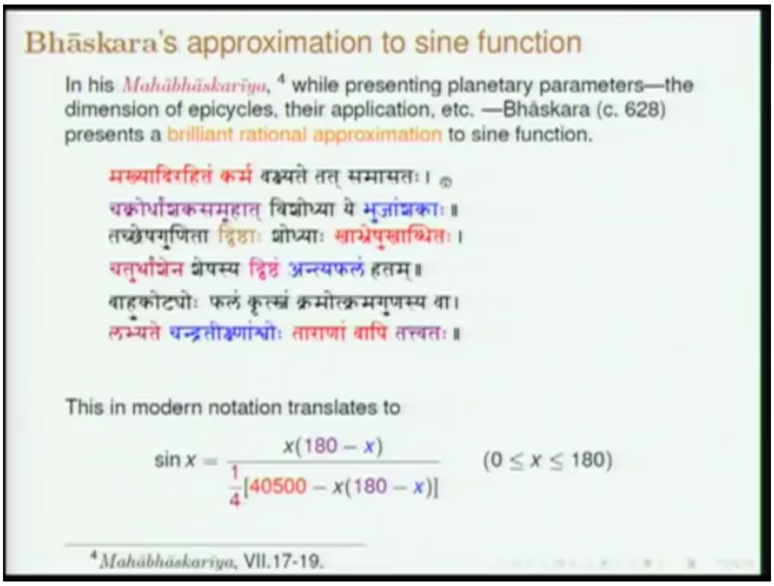

So now I come to the interesting approximation which we find in Bhaskara's work. So before I proceed to explain the verse given by Bhaskara, I just wanted to say couple of things so Bhaskara starts his approximation to sine by saying (FL) so here the word (FL) refers to the verse of Aryabhata so which starts with (FL).

So (FL) so means so a certain process by which you can avoid the (FL) means the table the table of Rsines can be avoided. So the idea is if you want to get a certain value of sine so you look at the table and then choose that particular value and then find out the difference and then do that. So either it is first order, second order and then you get the actual value of sine. So he is saying (FL) means you can completely avoid the table.

So which means he is presenting a certain other procedure by which you will be not requiring the sine table at all. So what he presents is a certain approximate formula so this is not necessarily the invention of Bhaskara. So this is what one comes to understand from 2 factors, 1 is so this verse if you look at it is present in 2 of his works, 1 is this Aryabhatiya-bhashya.

So wherein he actually explains a certain verse in Gitika-pada and he says (FL) so if you want to do computation of this so without employing the table so he just goes on (FL) that is what he says that is a certain hint wherein he says Aryabhata himself might have known. So this is what one can guess from that.

The other thing is see this Bhaskara in his own work Mahabhaskariya, so when he presents this verse so this is presented in such a way as if it is quite well known. So it is not presented as if it is something new which he is presenting. So it looks as if this approximation might have been available even during the period of Aryabhata. So one is not very sure whether this is the invention of Bhaskara at all, but anyway so this is the very interesting approximation.

He says (FL) so this is where it is over. So what does he is saying? (FL) chakra basically refers to 360 degrees, chakra is a circle and it is used to refer to 360 degrees. Chakra ardha is half of it so (FL) is degrees so (FL) means to be subtracted so (FL) so the degrees corresponding to the sine value, which you want to compute. In fact, this is applicable for not only sine, cosine also as well as (FL).

In fact, that is what Bhaskara towards the end he says (FL) so all that he says. This is a very interesting approximation. So you can easily follow that this particular line basically refers to 180-x, then he says (FL) so (FL) means remainder. So remainder that is obtained by subtracting x from 180 (FL) multiplied so (FL) means it has to be placed in 2 places.

So whenever you have the same quantity appearing in our computation they will always say so keep it in 2 places, so do not get lost. So like we store in memory in computer, so that is what they say keep in 2 places. (FL) means they have to be subtracted, subtracted from what so (FL) so this (FL) refers to 0 (FL) is also 0 (FL) refers to 5 (FL) again 0 and (FL) is 4 so this has to be subtracted from this number.

And this has to be divided (FL) so (FL) is one fourth see that is why we divided this by 4 okay, 1/4th of this quantity so this (FL) here basically whenever you find in most. They sometimes explicitly state that it has to be divided so when you say divided you always use by so by is (FL) itself in many places will indicate the process of division. So here (FL) that is how it has to be understood.

Then so (FL) means wherever you have placed this so that has to be divided. So (FL) so then he says (FL) so it is actually referring to a certain difference process. (FL) actually refers to the radius of the epicycle which perhaps will be made clear in the other lectures so (FL) so he actually describes a certain process of computing the planetary position and incidentally he gives this kind of approximation which can be employed to compute the sine function okay.

So (FL) see so whether it is sun whether it is moon (FL) means sun so it is really true see (FL) the one who has scorching rays okay. So (FL) okay. So this is something which is applicable over the range 0 to 180 okay. So usually the table is presented only up to 0, but this formula as you can easily see also captures the symmetry of the sine function.

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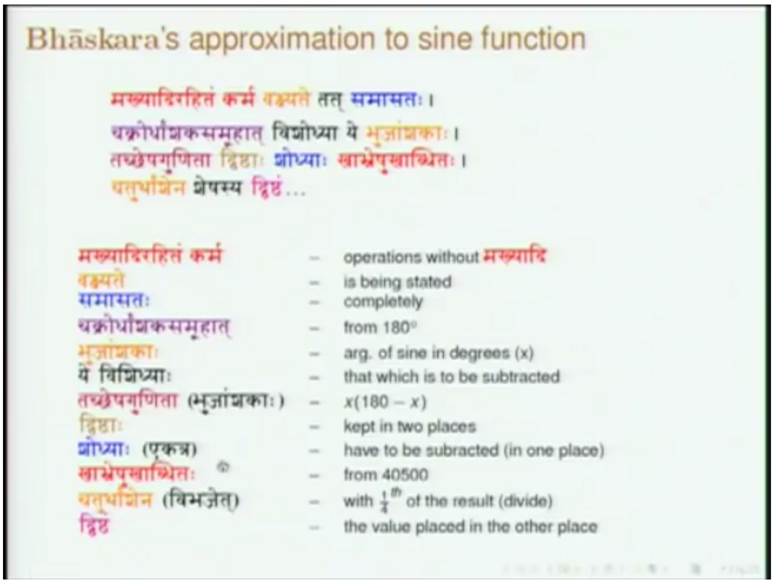

So this I have almost explained so all that so **(FL)** okay.

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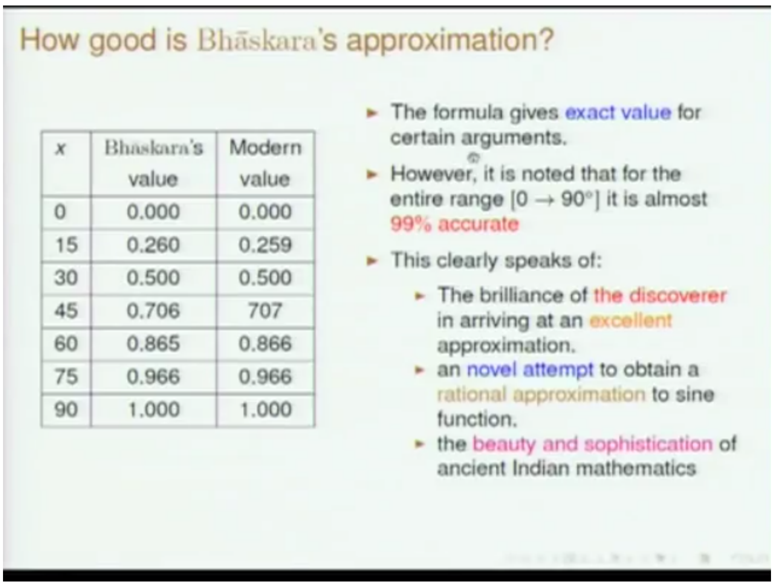

So how good is the rational approximation, which has been presented by Bhaskara? This formula which has been given by Bhaskara so if you put x is 0 so you will get 0. So when you put x is 180 also, it is going to become so if you see that closely so for x=90 you will get 1. So even in the intermediate values you can see that it is exact for certain values but in the intermediate values also it is almost 99% accurate.

So it is a very, very good approximation and so whether Bhaskara founded or it has been available in the tradition whatever it is whoever be the discoverer so it is indeed an excellent approximation for sine and there has been some attempt which has made so I remember to see that this formula can be taken so to compute the sine value in a much more efficient way so even in the modern devices.

Anyway and it also actually speaks off a certain attempt, a normal attempt to capture this function. So the sine function is something which is ubiquitous and so one needs to capture its value in some form or other form so which will be as easy as possible so whenever we want to compute this values.

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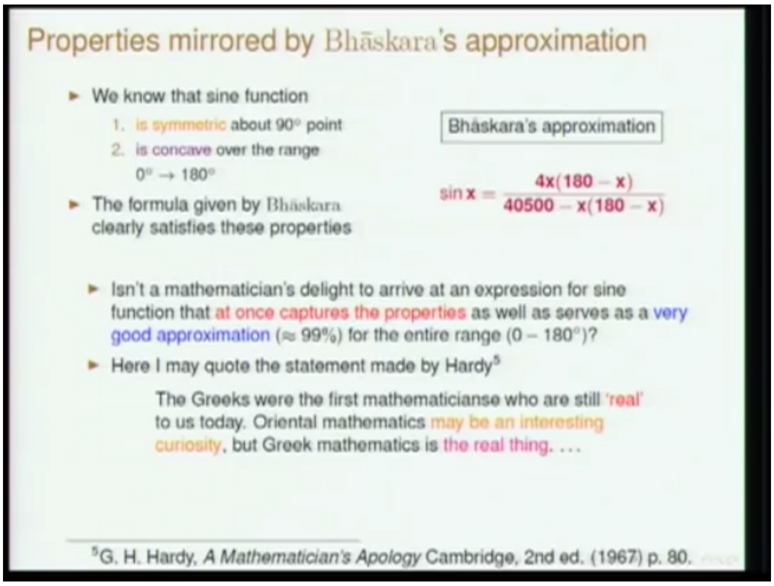

So this formula is also a very beautiful formula so in the sense that it actually captures this symmetry so which is there in the sine function and it also so since it is applicable for the range 0 to 180 so it also captures the concavity of this sine function and at this stage I would like to quote Hardy, he says the Greeks were first mathematicians so who are still real to us today.

Oriental mathematics maybe an interesting curiosity but Greek mathematics is the real thing. So this is the kind of statement which Hardy makes so in the early part of 20th century so in spite of knowing these kinds of brilliant discoveries. By brilliant discoveries we mean the infinite series for pi, its various fast convergent approximations as well as the series called trigonometric function such as sine, cosine and tan inverse functions.

In fact, it is not that by the beginning of 20th century, Hardy was not aware of some of these significant contributions made by Indians but maybe it has to do with what people call as the Eurocentric view.

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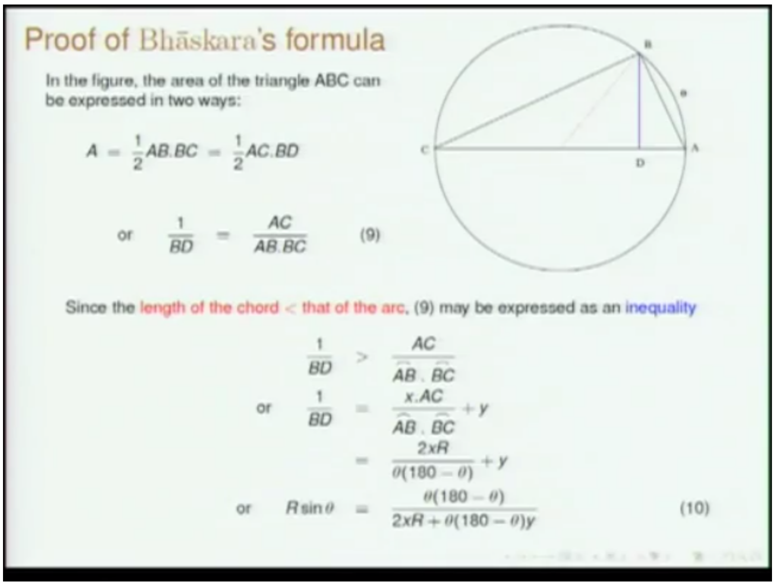

So now I give you the approach to arrive at this interesting rational approximation, which has been given by Bhaskara so this you can easily follow so there are 2 or 3 approaches so which has been presented by scholars. So here I am presenting an approach which is based on the geometry and the kind of analysis, which has been done around that period by most of which is familiar to you by now.

So here just consider this triangle, so A, B, C so the area of this triangle can be represented in 2 ways, so one is half times AB*BC fine, so AB*BC base times the other is AC*BD fine. So from this 1/BD is AC/AB*BC. So you know that the length of the chord is always<that of the arc fine. So the sine function is basically finding the relation between the chord length and the arc length. So this is all the sine function is all about.

So now you can write this equation 9 as in any quality in this form so what has been done is so in the denominator so AB and BC have been replaced with arc lengths. So since the arc length is much bigger than the chord length, so this becomes the any quality so we wrote this way this. This by introducing x and y, I write it as an equation 1/BD=x times AC+y okay. AB we just take it as theta and therefore the other part BC becomes 180-theta.

So that forms the denominator so we have this equation. So you can easily see that so BD is nothing but Rsine theta fine. So BD is Rsine theta and therefore we have equation 10.

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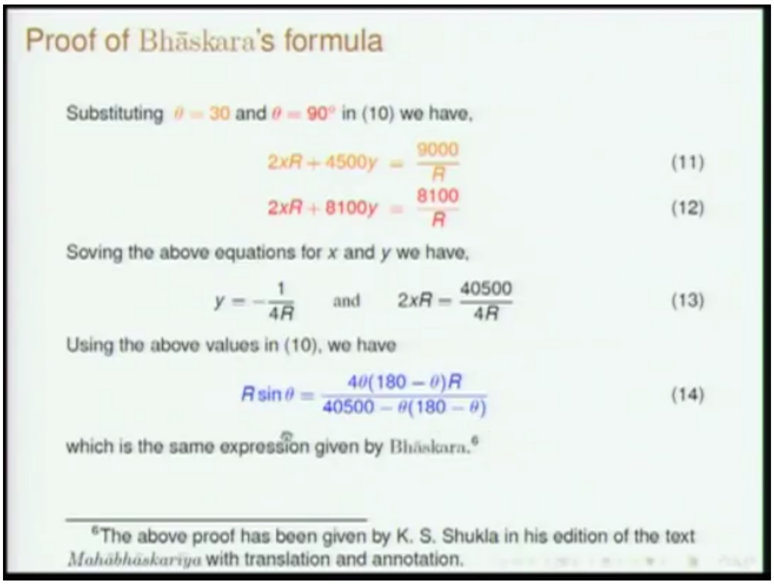

If you substitute theta=30 and theta=90 so in the above equation so this is the equation so by simple substitution, you will be getting these 2 equations 2xR+4500 is this and this. So by solving this for x and y, you will get y is 1/-4R and once you plug in this value into any of this equation, you will get x=this now you have the Bhaskara expression.

It is very simple. So this is something which might have been used by whoever who discovered this interesting approximation for sine. So this is basically the Bhaskara formula. So this derivation is presented by Shukla in his Mahabhaskariya.

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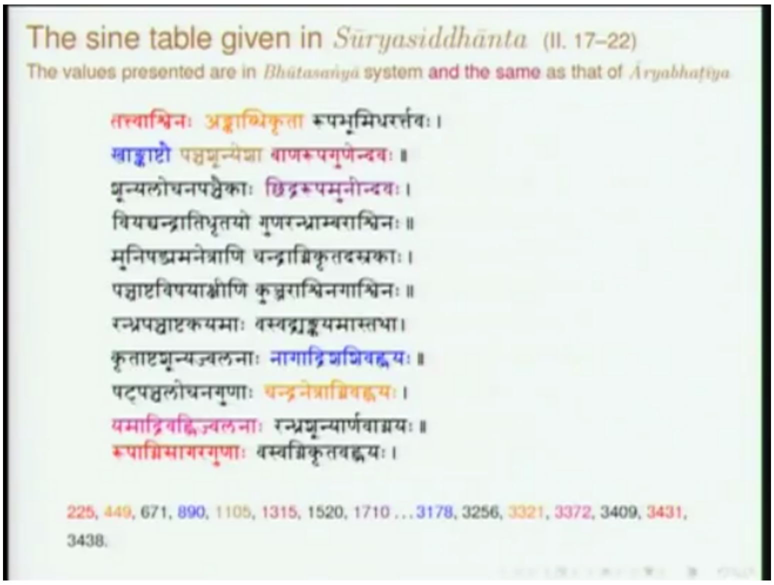

He is also giving some other algebraic derivation. So now I move on to the sine table which has been presented in Suryasiddhanta. So one is not very sure as to when this Suryasiddhanta was really composed so because we have the (FL) so in Varahamihira's Panchasiddhantika and Suryasiddhanta seems to be an improvised version of that. So anyway Suryasiddhanta presents this sine table in these verses.

And what has been used is Bhutasankya system so if you recall this Bhutasankya system some of these words see (FL) so (FL) is the number of (FL) so 25, (FL) refers to 2 it is 225, see (FL) is 9 so the number leaving out 0 (FL) is 9 (FL) is 4 (FL) refers to the number of (FL) and therefore this represents 449 and see (FL) the forum is unique for every entity and therefore (FL) refers to 1 (FL) is mountain so that refers to number 7.

So (FL) that is 7 and (FL) seasons number 6 so it is 671. This table which has been presented in Suryasiddhanta is identical with the table which has been given by Aryabhata, which is (FL) so the only difference is Aryabhata has given the differences the first order sine differences so here it is presenting the value of sines themselves. For instance, the last value so if you look at see (FL) refers to 8, agni refers to 3, (FL) refers to 4 and (FL) refers to 3 okay.

So this table is identical with the table that has been presented by Aryabhata but it only presents in the form of pinda-jya's okay so whereas Aryabhata gives the value of (FL) the differences in sines okay.

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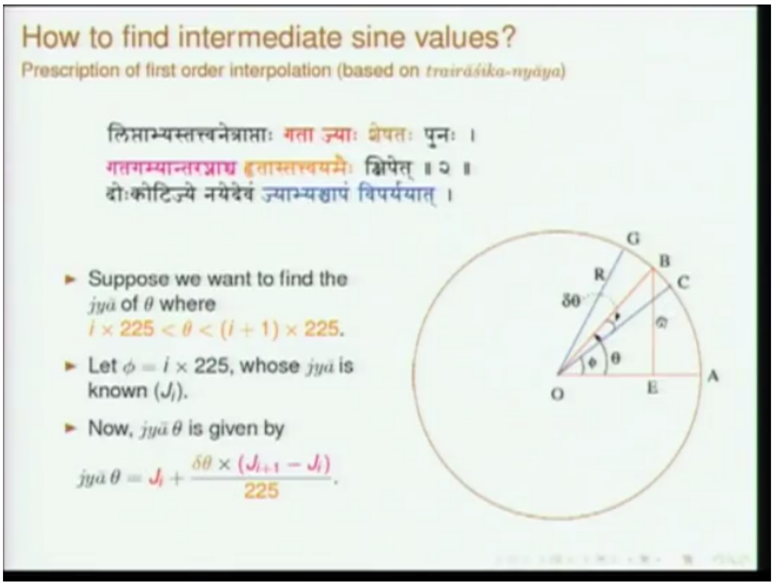

How do we obtain the intermediate sine values? So this is the general prescription so which is based on this (FL) or the first order interpolation. So it is clearly states (FL) for instance suppose AOC so represents an integral multiple of 225 so they give the values for every 225 so if you know the angle AOC and if you want to find out see AOB the sine corresponding to AOB.

So then all that you do is so you take the value corresponding to AOC which we write as Ji this can be simply pulled out from the table and for this so this COB which I have marked as delta theta which is an increment so you use this (FL) rule so if for 225 minutes so which is GC so this is going to be the difference. Then for delta theta what is going to be the difference? So by rule of 3 you add this.

So this is what has been stated in this verses see (FL) see (FL) is already (()) (28:32) (FL) is that which is going to be the next value. So (FL) is difference multiplied and divided by 225 so this gives you the value for any intermediate sine argument.

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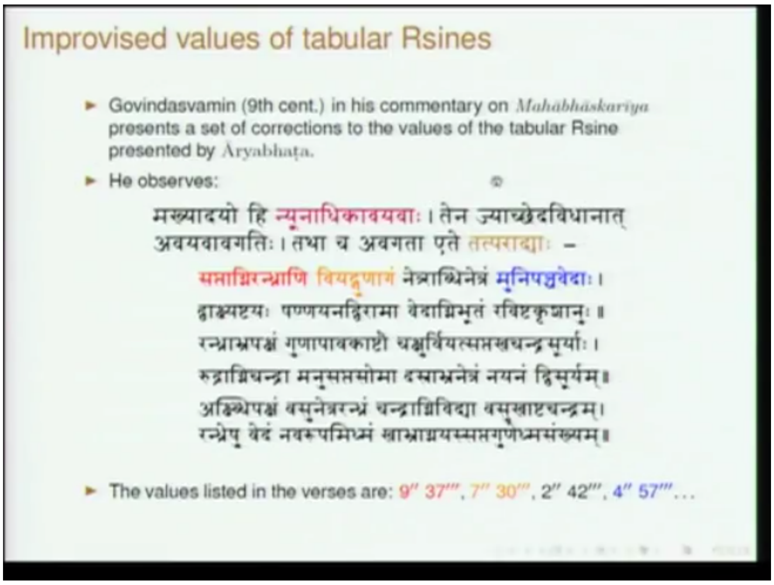

Now I move on to the sine table so which has been presented in Govindasvamin's vyakhyana of Mahabhaskariya. This is a very elaborate commentary which has been written by Govindasvamin and here he presents a set of verses which primarily give the correction to the Aryabhata values. So in fact it starts like this (FL) see (FL) means something which is less something which can be greater.

So the values which have been presented in the (FL) table so this (FL) which has been given by Aryabhata. (FL) so sometimes it can be so it is basically the approximation so rounding off has been done. So it could be slightly less, it could be slightly greater. So by specifying the measure by which it is less or more, so we will be able to get the exact value so that is what he is trying to hint here.

And he gives (FL) the word (FL) is used to refer to thirds so we know degree, we know minutes, we know seconds so if you go one more step 160th of a second is referred to as (FL) so here he says so (FL) so the verses in which I am going to list the numbers are basically in (FL) and then it will be in (FL). So here he says (FL) see (FL) is 7, agni is 3 (FL) means hole so why does it represent 9? In the body, (FL) okay so in the body there are 9 holes, 7 in the face and 2 below okay so that is how it is.

So he gives basically the correction which has to be done to the Aryabhatan values so to get the accurate values.

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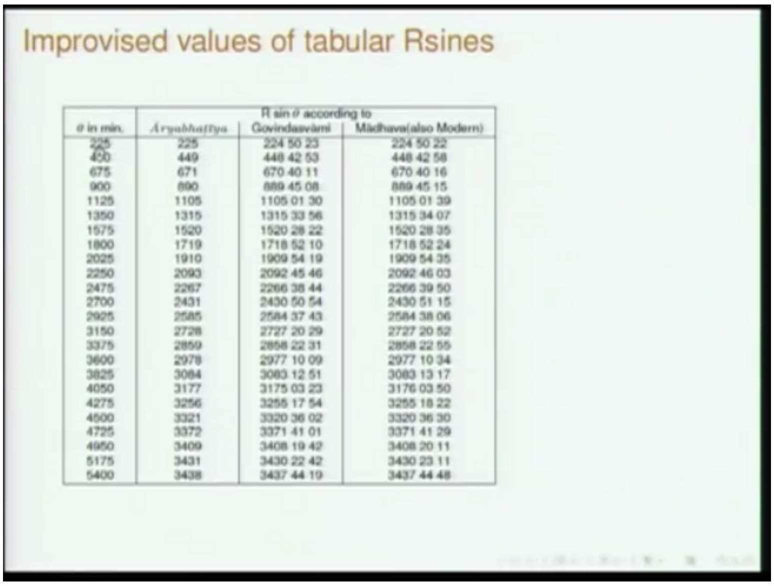

Improvised values of tabular Rsines

θ in min.	R sin θ according to *Āryabhaṭīya*	Govindasvāmi	Mādhava(also Modern)
225	225	224 50 23	224 50 22
450	449	448 42 53	448 42 58
675	671	670 40 11	670 40 16
900	890	889 45 08	889 45 15
1125	1105	1105 01 30	1105 01 39
1350	1315	1315 33 56	1315 34 07
1575	1520	1520 28 22	1520 28 35
1800	1719	1718 52 10	1718 52 24
2025	1910	1909 54 19	1909 54 35
2250	2093	2092 45 46	2092 46 03
2475	2267	2266 38 44	2266 39 50
2700	2431	2430 50 54	2430 51 15
2925	2585	2584 37 43	2584 38 06
3150	2728	2727 20 29	2727 20 52
3375	2859	2858 22 31	2858 22 55
3600	2978	2977 10 09	2977 10 34
3825	3084	3083 12 51	3083 13 17
4050	3177	3175 03 23	3176 03 50
4275	3256	3255 17 54	3255 18 22
4500	3321	3320 36 02	3320 36 30
4725	3372	3371 41 01	3371 41 29
4950	3409	3408 19 42	3408 20 11
5175	3431	3430 22 42	3430 23 11
5400	3438	3437 44 19	3437 44 48

So this is the table, so Aryabhata has taken for 225 as 225 and Govindasvamin actually says so 224, 50, 23 so how do you get this see you can see that see so in this you have to remove 9 37, 37 (FL) and 9 (FL) so you will get this value 224, 50, 23. So what is interesting to note is these values which have been given by Govindasvamin are very close to the values which are given by Madhava having obtained the infinite series.

And then giving the accurate values so once you have the infinite series in place so obviously you can get very, very accurate values depending upon the number of terms which you consider so it is a very fast converging series. So what Govindasvamin so even without that has so this also is sort of indicative as to how they have been trying to improvise the sine values so meticulously.

So this Govindasvamin is in 9th century and Madhava is 14th century and Aryabhata 5th century.

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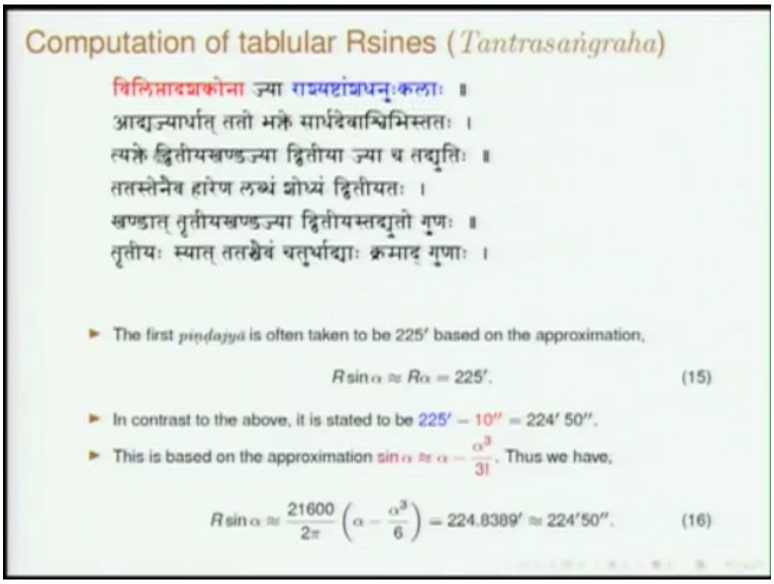

So that has been a certain effort continuously going on to improve upon the values. So coming to this Tantrasangraha so which is composed in 1500 so Nilakantha presents the following verses. So he says (FL) in the first line is of importance the more or less the procedure is same as that of Aryabhata so I will skip to the rest okay. So (FL) means reduced by less so jya here refers to the first jya.

So jya of (FL) is 30 degree (FL) is 1/8th of that so 1/8th of that corresponds to so which is 225 minutes so he says so 225-10 seconds so gives you the first jya. So (FL) so one can easily see that so this is based upon this approximation so Rsine alpha is simply taken as R alpha, sin alpha is alpha so by Aryabhata and therefore 225 is taken as 225 so if you use this next level of approximation, so alpha-alpha 2/3 factorial one can easily see that so you get this so 224.83 and this is what it is.

So this is what has been given. It is quite clear that he starts with this approximation instead of taking sine alpha as alpha.

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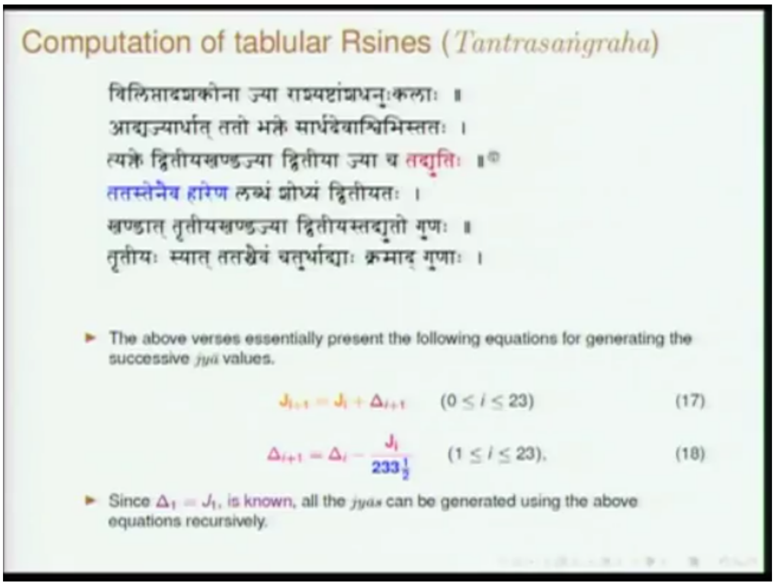

So then what you said in the later lines of these set of verses is basically this relation so which is same as the relation which has been given by recursion relation given by Aryabhata but for this constant factor which appears here. So this relation can be employed to get the entire sine table once you know the first value once J1 is in place the rest of the things can be easily obtained from this recursion relation.

So in fact (FL) in fact in this verses (FL) see (FL) is one half, deva refers to number 33 (FL) refers to 2 so 233 and a half see so this is the divisor. So this is what has been given.

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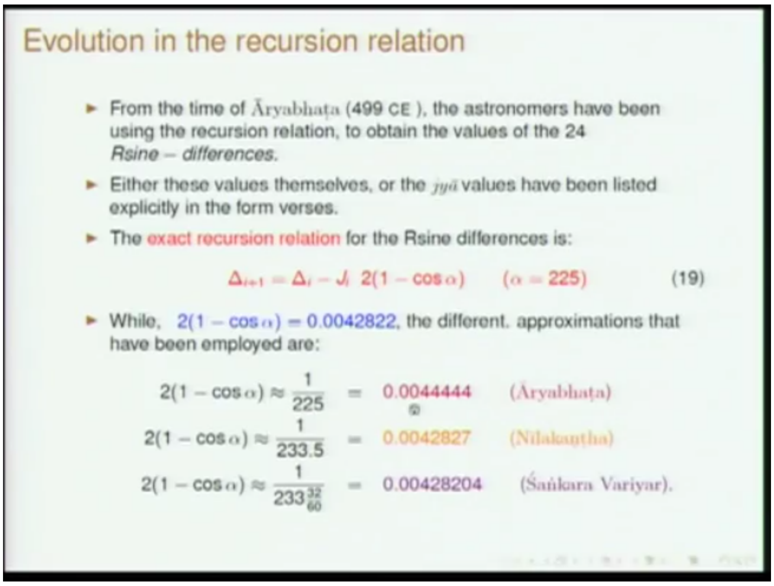

To quickly summarize we use the recursion relation to obtain the sine table and one can easily see that exact recursion relation will have this factor. The proportionality constant will be 2 times 1-cos alpha okay and this 2 times 1-cos alpha has been written as 1/225 by Aryabhata and Nilakantha actually gives this factor to be 1/223 and a half see that is what we saw (FL) and in fact Shankara Variyar so further improves upon.

And then he says 223 and 32 (FL) so that is what he says so you can easily see that the modern value is so 42822 and so this is a much better approximation than 233 and a half. So this is how they have attempted to improve upon.

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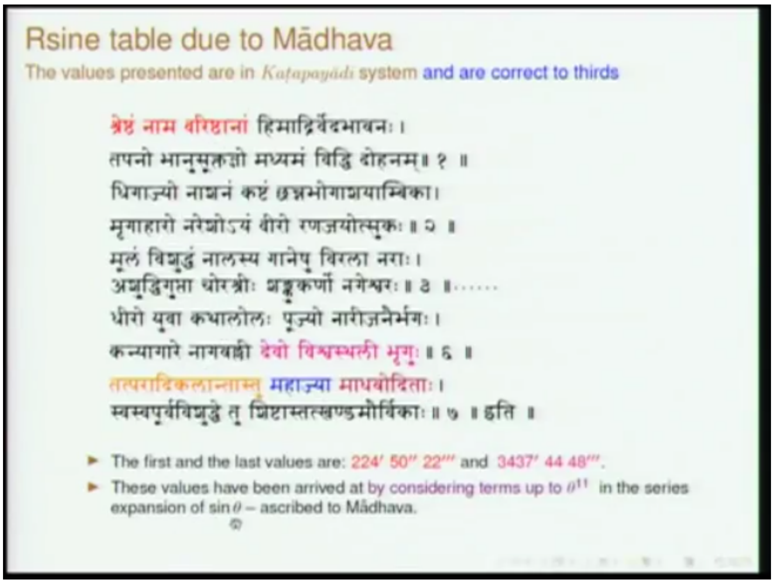

And Madhava actually gives a set of verses so which not only gives up to seconds but also up to thirds. In fact, the last line if you note here (FL) so (FL) is starting from thirds then you move on to (FL) then you move on to (FL) so (FL) see Aryabhata has taken so this to be 3438 so 3437 so 44 and 48 so that is what it commence to see so here this is given in Katapayadi notation.

So we saw that so in Katapayadi so (FL) see so both of them refer to 212 (FL) refers to 0 ma is 5 (FL) again 0 so this is what you get so similarly here (FL) the last word so (FL) see (FL) so that is how you construct the sine table which has been given by Madhava and one could see that Madhava would have obtained so this kind of a table by using the sine series.

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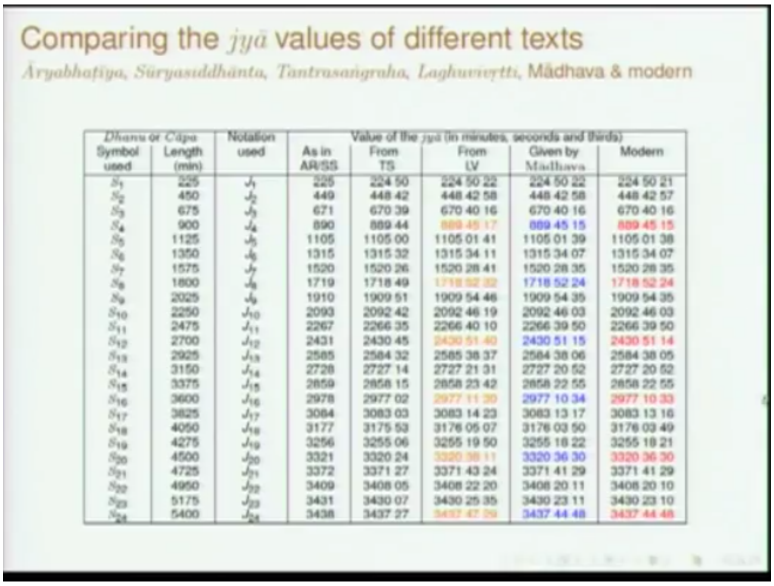

Dhanu or Cāpa: Symbol used	Length (min)	Notation used	Value of the jyā (in minutes, seconds and thirds): As in AR/SS	From TS	From LV	Given by Mādhava	Modern
S_1	225	J_1	225	224 50	224 50 22	224 50 22	224 50 21
S_2	450	J_2	449	448 42	448 42 58	448 42 58	448 42 57
S_3	675	J_3	671	670 39	670 40 16	670 40 16	670 40 16
S_4	900	J_4	890	889 44	889 45 17	889 45 15	889 45 15
S_5	1125	J_5	1105	1105 00	1105 01 41	1105 01 39	1105 01 38
S_6	1350	J_6	1315	1315 32	1315 34 11	1315 34 07	1315 34 07
S_7	1575	J_7	1520	1520 26	1520 28 41	1520 28 35	1520 28 35
S_8	1800	J_8	1719	1718 49	1718 52 32	1718 52 24	1718 52 24
S_9	2025	J_9	1910	1909 51	1909 54 46	1909 54 35	1909 54 35
S_{10}	2250	J_{10}	2093	2092 42	2092 46 19	2092 46 03	2092 46 03
S_{11}	2475	J_{11}	2267	2266 35	2266 40 10	2266 39 50	2266 39 50
S_{12}	2700	J_{12}	2431	2430 45	2430 51 40	2430 51 15	2430 51 14
S_{13}	2925	J_{13}	2585	2584 32	2585 38 37	2584 38 06	2584 38 05
S_{14}	3150	J_{14}	2728	2727 14	2727 21 31	2727 20 52	2727 20 52
S_{15}	3375	J_{15}	2859	2858 15	2858 23 42	2858 22 55	2858 22 55
S_{16}	3600	J_{16}	2978	2977 02	2977 11 30	2977 10 34	2977 10 33
S_{17}	3825	J_{17}	3084	3083 03	3083 14 23	3083 13 17	3083 13 16
S_{18}	4050	J_{18}	3177	3175 53	3176 05 07	3176 03 50	3176 03 49
S_{19}	4275	J_{19}	3256	3255 06	3255 19 50	3255 18 22	3255 18 21
S_{20}	4500	J_{20}	3321	3320 24	3320 38 11	3320 36 30	3320 36 30
S_{21}	4725	J_{21}	3372	3371 27	3371 43 24	3371 41 29	3371 41 29
S_{22}	4950	J_{22}	3409	3408 05	3408 22 20	3408 20 11	3408 20 10
S_{23}	5175	J_{23}	3431	3430 07	3430 25 35	3430 23 11	3430 23 10
S_{24}	5400	J_{24}	3438	3437 27	3437 47 29	3437 44 48	3437 44 48

So to have a comparison so here is where we present and what is noted is the modern value and the value which has been given by Madhava are almost identical so up to thirds so you see that so 3437 44 48 3437 44 48 okay so up to thirds so they almost coincide and the kind of accuracy to which it has been computed has to do with the accurate value which they have been able to find out using the series so that I will explain now shortly.

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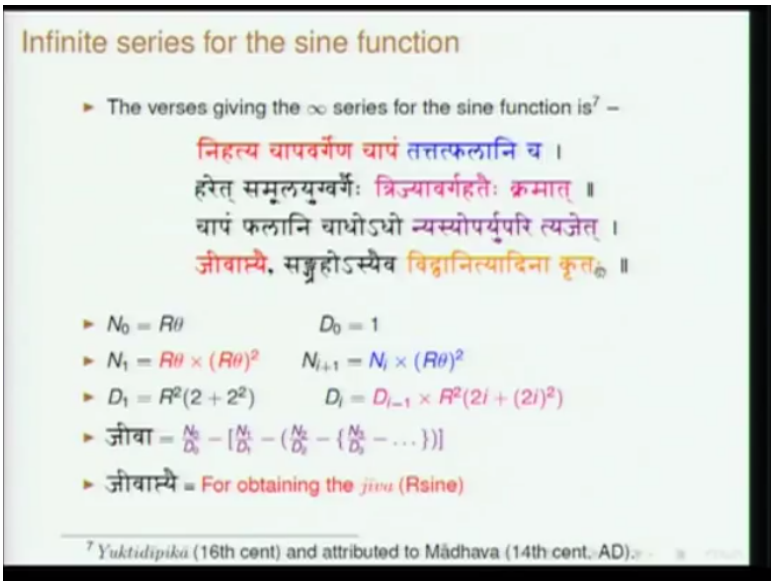

So Madhava so this has to do with the initial value of R in fact it is usually see even without this sine series so Govindasvamin has been able to obtain the table which is fairly close to the Madhava table which is what I showed you so it depends on the value of R that to take so if you take R to be 3438 and then do your computation then the accuracy that you will be able to achieve will be far less than the value of R that R is basically (FL).

So in all your computations so you will have this (FL) which is the radius and if you are able to compute the radius accurately then I think you will be able to improve upon significantly okay. So the verses which present the sine series go like this (FL) in fact there are 2 things which I have to explain. So this was cited earlier, 1 is the terms which actually represents the various terms in the series in the verse.

And finally this (FL) so (FL) capa refers to the arc, so which you have been repeatedly saying (FL) always refers to the arc, jya refers to the chord so (FL) refers to the square of this. So (FL) is having multiplied so (FL) so you have to multiple this by this (FL) means see once you multiply R theta square by R theta so you get R theta 3 so this also has to be multiplied by R theta square that is why he is saying (FL) means the results that you obtain.

So (FL) we have to do this multiplication so (FL) so basically what you will get is so R theta so R theta cube R theta to the power 5 and so on. Then having obtained them he says (FL) means divide so what should be the divisor (FL) refers to even number okay so (FL) refers to square of the even number so (FL) see (FL) means the base has to be added, squaring is an operation so it can be done on any number.

So (FL) means the base so 2+2 square 4+4 square so this is what is refer to so basically this quantity 2i is 2i square so (FL) and so on. So basically it amounts to this kind of a series and he says finally (FL) is same as jya if you want to find out jya so then you do this computation so this amounts to the infinite series for sine function okay and finally he says (FL) so having given a certain infinite series for this jya so he says (FL).

So a shorter form, a convenient form okay so (FL) is a shorter version okay so (FL) see we saw (FL) so (FL) means the verse which has (FL) so similarly (FL) means the verse which has vidvan as the first term (FL).

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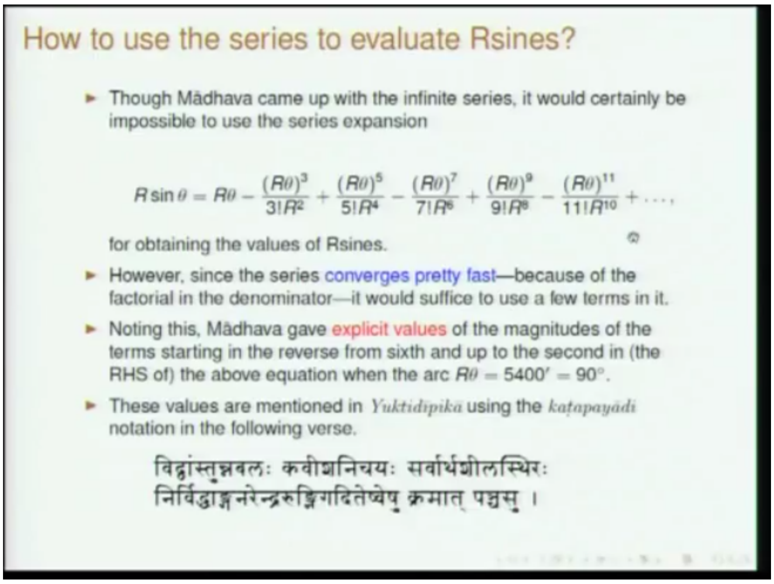

So we will see that vidvan verse now okay. So the series which has been used by Madhava so though it is the very, very important discovery from the mathematical view point so think of so using some infinite series to obtain something, it is almost impossible. So similarly so this you will see even with reference to pi so you have some series.

See pi will be approximately equal to various other rational approximations but equality will be holding good only when you have infinite series so though the terms will be decreasing but then the equality is valid only if you consider all the terms but you cannot do that. It is almost impossible for anyone to compute or any device to compute what one has to do is one has to have a good approximation for that so which will be discussed later.

And here so this series is actually a very fast converging series so you need not consider so many terms so Madhava has considered up to 6 terms in the series and so having done that so he has composed another verse so which can be conveniently used so you have some mark value and plugging in every time and then using the series is going to be very, very cumbersome.

So we have R square appearing so R is that 34, 37, 38 so whatever the number of minutes and all that. So you cannot be doing every time and he has given a certain convenient way of doing and so basically the coefficient which are present here so have been handled and computed and some numbers have been given so those numbers can be easily plugged in along with the argument value and you will be able to find the sine values.

So **(FL)** so **(FL)** so 5 terms have been specified here. So each of them actually represent a certain number vidvan means normally a scholar.

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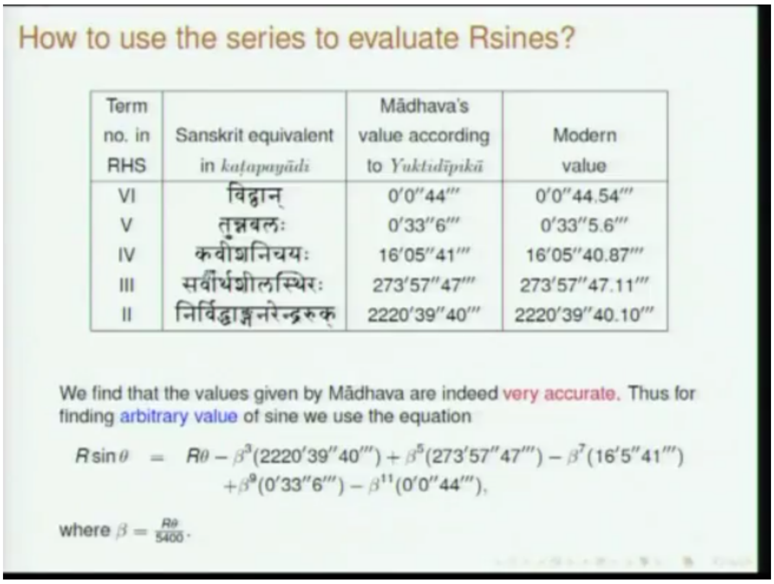

But vidvan in this context refers to number 44 say (FL) so in Katapayadi notation vidvan similarly (FL) refers to this number (FL) 0 cha is 6, ya is 1 see in Katapayadi so basically he has listed so 5 numbers (FL) so this basically correspond to see so (FL) 1, 2, 3, 4. 5 leaving out the first so he has given the values of the coefficients so in this particular forum.

So as you can easily see the last 6th will be much smaller so because you have a 11 factorial appearing here okay. So this is the order he has given in the reverse order so (FL) so one can now rewrite that equation see so wherein all this are taken out so 3 factorial R square etc so all this I mean pulled out and basically a set of numbers have been given and with this one will be able to compute the sine value.

Since it is a very fast converging series the 6 terms are more than sufficient to get very accurate values. So that is how he has composed this verse. In fact, the later half of the verse gives how it has to be done. As we saw it is quite interesting to note how? So various techniques have been evolved over a period of time in order improve the accuracy of the computation of sines.

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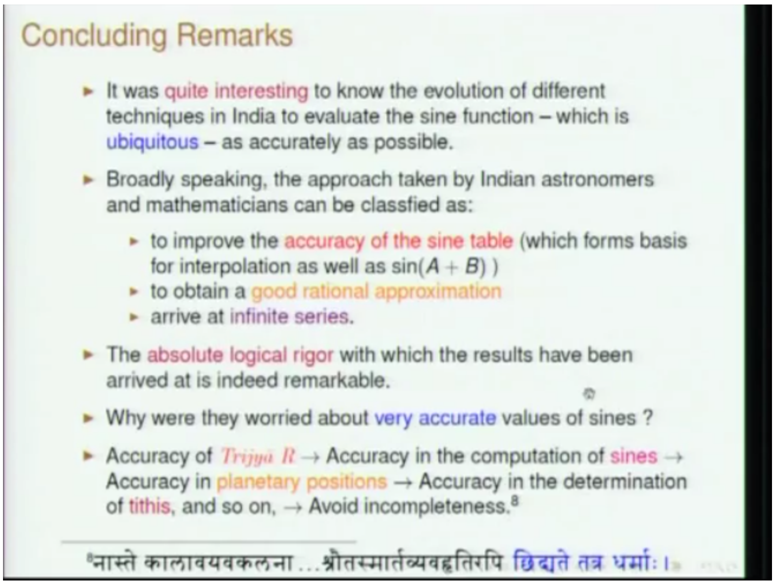

So if one were to look at there are basically 3 approaches, 1 is to improve the accuracy of the table which has been presented for finding the sine values, which will be used for doing either second order approximation or even if you want to use so perhaps professor Sriram will be explaining in this lecture on (FL) so sin of A+B etc even there you need a very accurate sine table which forms the basis.

So that has been done, then another way is to use the rational approximation where you do not have to have the table at all, you can just plug in the value and you will be able to get the sine value so this is another way and third of course is infinite series and this has also been tamed to have a nice forum so by (FL) and we will see in the lectures on proof the logical rigor so with which we have approached this problem to arrive at is also quite remarkable.

And why were they interested in very accurate values? If you look at whether it is geometrical approach or it is other things so the value of (FL) plays a very crucial role. So (FL) basically refers to the radius so you need to determine the radius so radius once it is precisely known, then you find out root of R square-something whatever you want to do so that will also be quite accurate.

And that was required to compute the sines and which was requited to get the accurate planetary positions, which was required to determine the tithis etc precisely so when this tithi ending so when is this eclipse occurring so all that and this in turn was required in order to know the precise occurrence of a certain event okay. So that is what we call as say for instance we say that this lagna that lagna from when does this lagna begin.

So all that is computation of time, in fact people were really concerned about the precise determination of ending moment of tithi so whether we should celebrate Deepavali today or we should celebrate Deepavali tomorrow, whether we have to ekadashi today or tomorrow. So that depends on precise computation of sunrise time and the precise ending moment of tithi.

So in fact one of the astronomers of recent times so (FL) at the beginning of his work, he quotes this verse see (FL) their concern is (FL) so all that whether it is (FL) or (FL) or our transactional thing so all of them will get shattered (FL) therefore he says (FL) so therefore I engage myself to compose this work, which will give you much better position of this planets so which in turn can be used to determine the precise moments of occurrence of various series.

So thus we see that the sine plays a very crucial role in various things. So with this we conclude our talk on sine function. Thank you.