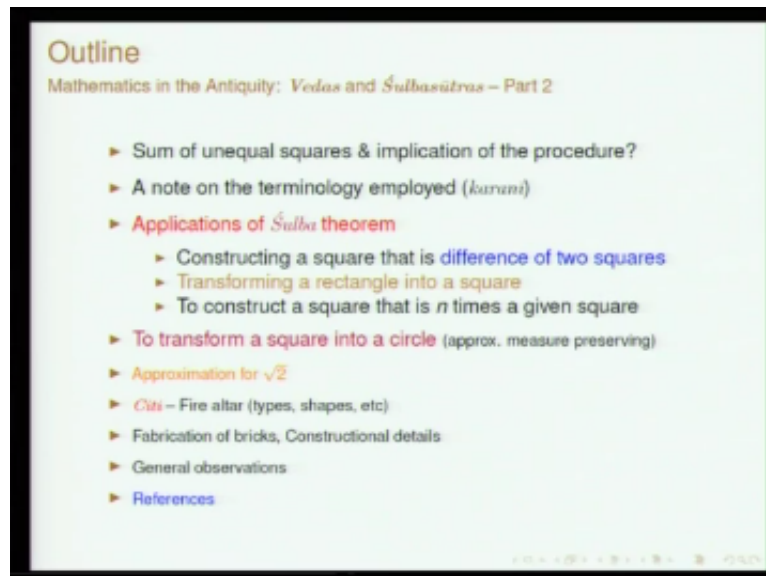


Mathematics in India: From Vedic Period to Modern Times
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Lecture-3
Vedas and Sulbasutras-Part 2

So this is the second part of a lecture on Vedas and sulbasutras. Yesterday we discussed what are sulvasutra text, so what are the characteristics of sulbakara which are defined in this text. Then we introduced the sulba theorem which is more popularly known as Pythagorean theorem. So then we came to the applications of this sulba theorem. So we also saw the kind of rational that can be seen behind the triplets that are given in the baudhayana sulbasutra.

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(FL) and so on. In today's talk we will start with the transformation of geometrical objects. For instance so we will start with supposed there are 2 square, if u have to construct a square so whose area will be this sum of these 2 square or u can think of 2 square and I want to find out a square. So which will be the difference of these two squares and if you have to construct a circle whose area is more or less the same as that of the square and so on so forth.

So these are the kinds of problems which will discuss today and we also see so in connection with this so the expression for some of the search, see this is the very common problem which one will be able to encounter, suppose I construct a (FL) has been constructed. So of a certain area and I want to construct another (FL) so which will have twice the area of this. So then we should be able to find a way by which you will be find out the value of root 2.

So if it 3 times then root 3 and so on. So these are common things, so one way is of course geometrical find out the value of root 2, so they have given certain expressions for route 2, so which is in the form of a sum of rational numbers. So all those things will be able to see today how this sulbasutra kara arrive at the value approximation for root 2 then to at the end of the talk, so I will be discussing what are known as citis.

So citi as they was mentioning is basically collecting things together putting things together. So we have several citis which are listed (FL) and so on. So these names are derived from the shape of the altar in which is constructed. So (FL) so this is how the name of various types of citis and the purpose of citis are also stated. So we will discuss all these topics today.

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Constructing a square that is sum of unequal squares

An application of the *Sūtra*-theorem

नानाधतुर्वन्ने समस्यन् कनोपसः कण्ठा वर्यीयसो वृध्मन्निचेत् । वृध्मस्य
अध्वन्यारतुः समस्यतोः पार्श्वमानो भवति । (BSS 1.50)

Desirous of combining different squares, may you mark the rectangular portion of the larger [square] with a side (karasū) of the smaller one (kanyasah). The diagonal of this rectangle (vṛdāhṛa) is the side of the sum of the two [squares].

- ▶ The term *vṛdāhṛa* in the above *sūtra* refers to the rectangle ABEF.
- ▶ Asking us to mark this rectangle, all that the text says is the cord AE अध्वन्यारतुः gives the side of the sum of the squares.
- ▶ In other words,

$$\begin{aligned}
 AE^2 &= ABCD + CGHI \\
 &= AB^2 + CG^2 \\
 &= AB^2 + BE^2.
 \end{aligned}$$

This is where I stopped yesterday, so (FL) as I said is square (FL) means desire of putting together, (FL) means putting together, so (FL) refer to smaller one and (FL) refers to the larger one. SO what he says is if you have 2 squares say A, B, C, D and then C, G, H, I. So you want to construct a square, so which will be the sum of these 2 squares basically the area of the largest square.

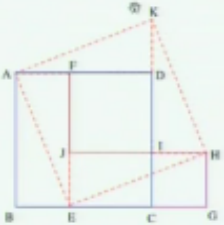
So this is a simple instruction which tells you how to go about, so without even thinking of the arithmetic which is involved in that. So you have a square, you have another smallest square, you just do a certain trick here and you should be able to get the value of the largest square without doing any calculation numerical calculation. So all that it says is so you think of this smaller square C,G (FL) here refers to the side CG for instance.

So (FL) so you just think of placing this square there or marking C, G in the largest square B, E (FL) can be understood to be a sort of rectangle here. So this rectangle is B. E, F, A (FL) so then it says (FL) is A, E, so the diagram. All that sutra says is the (FL) will give you this side of the square that you desire. So this all the sutra is, so fine. So now if you look at the A, E square is basically A B square+C, G square ok, A B square+C, G square.

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Implication of the above construction ?

- Scholars trained in the Euclidean tradition, puzzled by the mere statement of theorem, without the so called 'proofs' always wondered whether the Śulbakāras knew the proof of *Śulba*-theorem, or was it purely based on empirical guess work?
- Though *Śulbakāras* do not give explicit proofs, it is quite implicit in the procedures described by them. In fact, the previous description of construction clearly forms an example of that.



- In the figure, ABCD and CGHI are the two squares to be combined. E is a point on BC such that $CG = BE$.
- ABEF is the rectangle that is formed. Now the sum of the two squares may be expressed as

$$\begin{aligned}
 ABCD + CGHI &= ABE + AEF + EHI + HEG + FDI \\
 &= ADK + AEF + EHI + HKI + FDI \\
 &= ABEFK,
 \end{aligned}$$

which unambiguously proves the theorem.

So there is something which is interesting that emerges out of it. So I wanted to spend a couple of minutes on that. So generally all the text just state the role, so this is how the structure go, so somewhere scholars are actually puzzles whether these people knew the truth or they do not knew the truth, this has seen a question which has been discussed at great length, and what we can easily is the sulbakara though they did not explicitly give proof of this various procedures.

It is quite interested in the procedure itself, for instance in the previous thing, so if you look at the sum of these two areas can be consumed like this A, B, E, it can be consumed of various triangles A, B, E, A, E, F+ and so on. Now if you look at this, so this triangle A, B, E, if it is sort of stopped off and rotated, so that A, D, K it turns into A, D, K. So you have to just rotate it at A.

And then think of this triangle H, E, G and if you rotate it round H, so this will go and occupit the space H, K, I. So this is essentially the proof of Pythagoras theorem, this was called Pythagoras theorem, in fact this proof has been discussed in (FL) part 2. So the procedure

which has been given in sulbasutra so announce to the proof or rather I would say the proof is implicit in the procedure that has been described.

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A note on the terminology employed

- ▶ Before introducing *Sulva*-theorem, Kātyāyana has **exclusively devoted one *sūtra*** to clarify the different terminologies that would be employed to refer to cords in different contexts.
करणी, तत्करणी, त्रिर्गुणनी, पार्श्वमानी, अक्षय्या चेति रज्जवः।
karani, tatkarani ... all refer to cords. (KSS 2.3)
- ▶ The commentary by Mahidhara (c. 1589 CE)—explaining the origin of the five names given in the above *sūtra*—is quite edifying.
करणी क्रियते क्षेत्रपरिच्छेदः अनयेति करणी।
That which limits or produces the length or area is **karani (producer).**
तत्करणी तत्क्षेत्रं द्विगुण्यादि क्रियते अनया सा तत्करणी, द्विकरणी, त्रिकरणी, चतुःकरण्यादिः।
That which produces an area that is twice etc. is called **tatkarani (that-producer)**; For example, *dvikarani, trikaranī, catuhkarani*, and so on.

Otherwise I am in that used to that inspire with this I will proceed further to discuss the other interesting things. So before proceeding to that I will introduce you 2 certain terms which will be frequently occurring in sulbasutras which have slightly different conversations in different context. So that should be clarified to the coming to the latest model of sulbasutra, these become quite clear to you.

The term karani has been used in different sensors in different contexts, so you suppose it is a compound word it is likely means different things. Here I wanted to clarify taking a sutra itself from the katyayana sulbasutra. In katyayana sulbasutra we have karani (FL) so all the sutras says these, these are 5 names which have assigned to the god that you keep you thing in different context ok. So karani sometimes (FL).

So all that refers to sutras. So in fact the commentary to Mahindra clarifies so what do these terms mean (FL) so you will find in various places the terms like (FL) actually means route 2, (FL) means root 3, the term has been defined as the (FL) it is different, see (FL) if you want to find out twice the area of this, so size how root 2 obviously so we call the term (FL) similarly root 3 and so on.

(FL) suppose you consider line so the perpendicular line so that is called (FL) so that is how even baudhayana sulbasutra one is called (FL) interesting presented for the word action yeah

(FL) so this is a interesting derivation which has been presented for the word (FL) so suppose you think of a rectangle and the diagonal is referred to as (FL). So these diagonals split this into 2 halves, so (FL) means eye, so splits the geometrical object into 2 half like 2 eyes and of derivation that has been presented for the world (FL) basically (FL) 2 opposite corners that connects 2 opposite corners (FL).

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Different connotations of the word *karani*

1. **करणी = side of a square**
 कनेपमः करण्य वर्यसो वृध्मद्विखेत्।
By the side of the smaller [square]... (BSS 2.1)
2. **करणी = square root**
 पदं तिर्यङ्माने त्रिपदा पार्श्वमाने तस्य अक्षयखरवुः दक्षकरणी।
[In a rectangle] with upright one pada and base three padas, the diagonal-rope is $\sqrt{10}$. (KSS 2.4)
3. **करणी = a certain unit of measure**
 करणी तृतेयेन वर्धयेत्, तद्यत्थेन, आत्मचतुस्त्रिंशन्नेन, सविशेषः
 इति विज्ञेयः। (KSS 2.9)

Note: Though *karani* seems to have 'different' connotations, on taking a closer look, it becomes evident that some of these meanings converge to the same thing—*that which makes a square of area a*. Obviously *that* = \sqrt{a} .
 Examples *dikarani*, *trikarani*, *dashakarani*, and so on.

So karani as I was mentioning is very frequently encountered, so (FL) refers to the side, so area is A obviously karani will be the root A, ok so it is in Datsun. So similarly karani is a square root (FL) means root n ok, (FL) it is used in this sense, so this root 10 will produce a square which has an area 10, so that is the (FL) so it is in this sense in this is used. Karani is also used in the sense of a certain measure.

For instance in this sutra which we will discuss one more little later in greater detail (FL) this is the sutra which present to the value of root 2, ok so there the work karani is used in sense of a certain unit of measurement ok.

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Constructing a square that is difference of two squares

चतुरस्राद्यत्तरं निजिहोर्णं यावज्जिहोर्णं तस्य करण्यं वर्योयसो
 वृध्मद्विखेत्। वृध्मस्य पार्श्वमानीं अक्षय्या इतरत् पार्श्वं उपसंहरत्। सा यत्र
 निपतत् तदपच्छिन्द्यात्। छिन्नया निरस्तम्। [BSS 2.2]

Desirous of subtracting a square from another square, may you mark the rectangular portion of the larger [square] with a side (*karanya*) of the smaller one that you want to remove. With the [cord corresponding to the larger] side of the rectangle turned into a diagonal (*akshaya*) touch the other side. Wherever that intersects, chop off that portion. Whatever remains after chopping, gives the measure of the difference.

- **Problem:** Find the side of square which is the difference of the squares ABCD and AEGH.
- **Solution:** Obtain the *vyadhira* (rectangle) AEGH, and with radius EF mark a point P on AD. AP gives the desired measure.
- It is evident from the figure

$$\begin{aligned}
 AP^2 &= EP^2 - AE^2 \\
 &= AD^2 - AE^2. \quad (EP = AD) \\
 &= ABCD - AEGH
 \end{aligned}$$

Now I move on to discuss the sutra which gives you the procedure by which you will be able to find out the side of a square with you going to be the difference of two squares and square. So earlier we saw sutra to clear the procedure for finding the sum of 2 square feet, here is a difference. So (FL) so this is how the word is derived, so one who is desirous of removing (FL) see you have a square, you want to remove square from it.

So then he says (FL) whatever be the measure that you want to remove, the measure of the square you want to remove from this, so (FL) measure the measure of that you mark something in the largest square, for instance in this diagram suppose you have the square A,B,C,D, you want to remove a certain area. So which is given by A, E, G, H, y axis, that x is refers to us karani.

(FL) mark that A, E and then draw a line ok, (FL) it is a very clear prescription, so all that I say is so from here you just make an arch, so this (FL) other side, ok from one side you just drag it and take it to the other side, (FL) so where every it fall (FL) so once you do that so what you will get is basically the area of the square which is the difference of the two square if you construct. So A, P is the measure so which gives the side of the difference of two squares.

So how does it work out we can easily see this, see A, P square so in this just consider this triangle A, E, P. So A,P square is EPsquare-A, E square. Now this EP is same as a A, D, by construction, right all that you see here is just an application of this sulva theorem right, so this AP is directly gives you the side of the difference of two squares, so this is the

prescription for baudhayana sulvasutra for constructing a square which is the difference of two squares.

So this is a very important thing in fact this principle is invoke in doing certain another kind of transformation which I will be showing in the next slide. So this will be quite clear, all that we need to do is we had just mark the largest square. So whatever be the side of the smaller square and then take (FL) and then do then we will be able to get the other square.

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Transforming a rectangle into a square
 Sequel to finding the sum and difference of squares

दीर्घचतुरत्रं समचतुरत्रं चिकीर्षन् तिर्यञ्चान्ने कर्णो कृत्वा द्वेषं द्विधा विभज्य,
 पार्श्वयोरुपदभ्यात्। खण्डम् आवापेन तत्संपूरयेत्, तस्य निर्हास उक्तः।
 [BSS 2.5]

terms in <i>sutra</i>	correspondence with figure
दीर्घचतुरत्रं	rectangle ABCD
तिर्यञ्चान्ने	east-west cord (AB)
द्वेषम्	the portion XYCB
खण्डम्	square RSNM
आवापेन	by placing

It is evident from the figure

$$\begin{aligned}
 DP^2 &= EP^2 - DE^2 \\
 &= AE^2 - RS^2 \\
 &= AENF - RSNM \\
 &= HIJK
 \end{aligned}$$

Transforming a rectangle into a square, so this is the next problem. So the sutra goes like this (FL) you want to transform into a square (FL) you want to do that ok (FL) as I mention earlier one is called parshwamani the other is called Priyamani ok, so perpendicular to that. So all that he says is take priyamani as a karani and then (FL) so let us consider this diagram and understand the sutra.

So we have A,B,C,D is rectangle, now we want to transform this rectangle into a square. So there is a certain prescription which is given in sutra. So it says so this has 2 side which are unequal so one is AB and the other is BC, so AB is (FL) all the sutra says is (FL) so take the measure of (FL) which is AD and then mark it ok. So this will be the XY line, so you take this AB, so AB and then you mark y axis and Fy (FL).

So the remaining portion XY, BC, so that should be split into 2 half (FL) the side of the 2 space, so all that we do is take one of them and then place it here, ok.. So (FL) so there is a small portion which is remaining here. So this (FL) seen in previous sutra yeah (FL) you saw

that right so it is basically so how is it to be done is something which has been stated before so far is a look at the sutra number, so this is 2.2.

So 2.5 (FL) has been stated before. So basically the procedure which was adopted before has to be understood and so you have to apply that procedure here in order to get this. So what was the procedure adopted there ok, if you look at so we basically took this and then (FL) so he said you have to draw this line and then take it to the other side say (FL) so that is all we need to here. So we have to just take this line and then take it there and then drop it.

So what we will get is basically this DP ok, so that is going to be the side of this, and that is what the square is, so by simply taking this you just hit at this point and the script which is found here is basically occupied area. So that is the procedure for that is what is referred to as (FL) see this problem can visualise other way also. So if you think of this square all that I need is so this is what is extra here.

So we construct the square, you can think of removing this, so removing this what is the square that is going to obtain by me where is essentially this so that is what is refer to (FL) the procedure to get that. So this is the procedure for transforming a rectangle into square.

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To construct a square that is n times a given square

► Kātyāyana gives an ingenious method to construct a square whose area is n times the area of a given square.

$BD = \frac{1}{2}BC = \left(\frac{n-1}{2}\right)a$. Considering $\triangle ABD$,

$$AD^2 = AB^2 - BD^2 = \left(\frac{n+1}{2}\right)^2 a^2 - \left(\frac{n-1}{2}\right)^2 a^2$$

$$= \frac{a^2}{4}[(n+1)^2 - (n-1)^2] = \frac{a^2}{4} \times 4n = na^2$$

यावत्प्रमाणानि सम्यत्तराणि एकीकर्तुं
 चिकीर्षत् एकोनानि तानि भवन्ति तिर्यक्
 द्विगुणान्येकत एकाधिकानि त्र्यस्त्रिभवति।
 तस्येषुः तत्करोति। [KSS 6.7]

As much ... one less than that forms the base ... the arrow of that [triangle] makes that (gives the required number \sqrt{n}).

So to construct a square that is n times a given square, this is a very interesting problem. So which has been discussed and which incidentally gives you a certain way by which you can find out the value of root 10 and the value of shirt, so whatever n can be, it is a very simple and very inspective procedure, so which one can find in the sulbasutra katyayana sulvasutra.

In fact yesterday I refer to one of this I will recall that quickly now. So the problem is this you want to basically find out the value of root n, how do you go about.

The sutra says (FL) slightly different way, suppose you have n squares (FL) as much as you want ok, so (FL) so you want to find out the area which will be given by these n squares. So how do you construct a square which will have the area of some of the n square which is the most general way of getting the problem (FL) as much so much ok so (FL) means so remove one from that ok. Here we let us look at this diagram.

So he basically tells that you have to conceive of your certain triangle wherein the side of the triangle one of the side so which we can consider the base, so is $n-1$ times (FL) ok so then the other 2 sides so have to be of this measure $n+1 \cdot a/2$ and $n+1 \cdot a/2$. So what would be the perpendicular draw from A will be root n times A and you put it in a form of algebraic equation. So in this figure $BD = 1/2$ of BC that is $n-1/2 \cdot A$.

We consider this triangle A, B, D ok, so then the equation is this, so AD square ok, AD square is difference of these 2 and AB is $n+1/2 \cdot A$ the whole square and BD is $n-1/2 \cdot A$ whole square. So the difference of these two square basically is nA square which is AD square. So from this what you get is basically the value of root n. So the problem has been posted as problem of constructing a square, so who area will be n times the area of smaller square.

Whatever be the dimension is what we are going to get, see in all these things what yesterday was mentioning this sulva theorem or baudhayana theorem which is generally referred to as Pythagorean theorem, so what is under operation. We all these transformation so if I did this whether you want to submit have whether you want to find the difference so the underlying principle happens to be the baudhayana theorem ok.

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To transform a square into a circle

चतुरश्रं मण्डलं चिकीर्षन् अङ्गयार्धं मध्यात् प्राचोम्
अभ्युपातयेत् यदादतिजिष्यते तस्य सह तृतीयेन मण्डलं
परिलिखेत्। [BSS 2.9]

अङ्गयार्धं = semi-diagonal (OD)
मध्यात् प्राचोम् = from centre to the east
यदादतिजिष्यते = whatever [portion] remains
तस्य सह तृतीयेन = with one-third of that

As per the prescription given,

Radius $OP = r = a + \frac{1}{3}ME$
 $= a \left[1 + \frac{1}{3}(\sqrt{2} - 1) \right]$
 $= \frac{a}{3}(2 + \sqrt{2}).$

How to find $\sqrt{2}$?

$AB = 2a$ (given)
 $OP = r$ (to find)
 $OD = \sqrt{2} a$
 $ME = OE - OM$
 $= \sqrt{2} a - a$
 $= a(\sqrt{2} - 1)$

Now we would move on to another problem which has been the most difficult problem for which many scholars from all civilizations have been taking a head. So how was you just say I want to have a square whose area is same as that of a circle or I want to have a circle whose area is a square. So some kind of prescription which is found in sulbasutra is what we are going to discuss now.

So (FL) so this in every sutra we will find this ok (FL) so you want to transform a square into a circle, so what is to be done. So he says (FL) as I was repeatedly telling is the diagonal line (FL) means half of it. Let us look into this diagram ABCD, so this is a square and I want to get a circle. So whose area will be more or less same as with this square ok. So (FL) usually the conceive of the diagram constructed with the direction mark on one edge.

So this is how we do in all the plans elevations whatever we do you mark with. So similarly we can think of (FL) ok so the sutra goes like this (FL) is half of the diagonal which is OD, so this if says (FL) just take it this line, so it you can think of it to be OE now, so I have rotated it and brought it there. (FL) which is remaining above see when OD become OE, so sometimes protruding out of it. So that is what is referred to as (FL) here.

So (FL) whatever you mean so here it is ME (FL) that portion ME, (FL) one third of it ok, (FL) so what does this prescription amount to, so let us look into the details which have been dotted down here. See AB let us say is 2A the side of the square is 2A, so OP is the radius of the circle whose area is going to be (FL) square. So $OD = \sqrt{2}A$ obviously. So then ME is $\sqrt{2}A - A$. So that is the portion that is (FL) which is exceeding.

So the sutra said one third of this see one third of it has to be added to that say one third which is basically PM, so PM is one third of this, so what it amounts to is the radius is A/3 so half the side is A, so A/3R or 2+root, so this what is the prescription which has been given in baudhayana sulbasutra for transforming a square into a circle, so accurate this, so let us see little later ok. So in this we find root 2 of it.

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How did *Sulvakāras* specify the value of $\sqrt{2}$?

► The following *sūtra* gives an approximation to $\sqrt{2}$:

प्रमाणं तृतीयेन वर्धयेत्, तद्यत्तुर्धन, आत्मचतुस्त्रिंशोनेन,
सविशेषः । [BSS 2.12]

$$\sqrt{2} \approx 1 + \frac{1}{3} + \frac{1}{3 \times 4} \left(1 - \frac{1}{34}\right) \quad (1)$$

$$= \frac{577}{408}$$

$$= 1.414215686$$

► What is noteworthy here is the use of the word सविशेषः in the *sūtra*, which literally means 'that which has some speciality' (speciality = being approximate)

► How did the *Sulvakāras* arrive at (1)?

► Several explanations have been offered over the last centuries. Here we will discuss the geometrical construction approach.

So you know the value of A, so that you do not know what root 2 is, rupees have you been find out who so then you will be immediately able to compute this. See if you look so this sutra is 2.9 the couple of sutras later baudhayana presents another sutra, so which gives the value of root 2. So (FL) is basically some unit (FL) some unit measure, (FL) so means one third of it you have to add (FL) which is immediately preceding one fourth of that.

So one fourth or one third (FL) then it says (FL) subtraction okay negative kind of so you have to subtract 134 of that ok (FL) here refers to 1/3*4 so (FL) ok you have to subtract this and the word (FL) as I was mentioning so it means it is not exact value so (FL) approximate value ok. So this amounts to 1.414215 and this I think is correct to 6 decimal places ok, this is what is the sutra which gives the value of root 2 in baudhayana sulbasutra.

We see soon how baudhayana might have arrived at this expression for root 2. So I mention this (FL) has been studied in the more greater detail and further requirements have been presented in one of the commentaries, so there are several explanations which have been offered by various scholars to study this sutra and what we will be presenting here is certain

geometrical way of arriving at this expression for root 2 which we find in baudhayana sulbasutra.

In fact later Mr. Srinivas may be telling you how this can we obtain so from different kind of a problem which is called (FL) ok so that we touch up on later as you go through the course.

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Approximation for $\sqrt{2}$
 Rationale for the expression $\sqrt{2} = 1 + \frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^2 \cdot 3^2}$ by Geometrical Construction

- ▶ Consider two squares $ABCD$ and $BEFC$ (sides of unit length).
- ▶ The second square $BEFC$ is divided into three strips.
- ▶ The third strip is further divided into many parts, and these parts are rearranged (as shown) with a void at Q .
- ▶ Now, each side of the new square $APQR = 1 + \frac{1}{3} + \frac{1}{3^2}$.

So this geometrical construction is quite instructive and in fact recently one of the article I think Henderson I think from the Columbia University also he studied this problem when he visited India and he came up with a very interesting paper. So wherein he mentioned he also points out it is not that others have not, he also points out in his own way as to how this expression for instance. So this meaning of the word (FL) as I said is only approximate.

So with me some other term which can be added, so it is never going to end, so it is never going to end, so even from geometry can see that so to just keep on doing this process so you will be ending up with the smaller and smaller square let me describe this first. So we want to find the side of the square which will be the sum of two squares, so $ABCD$ and BEF are the two squares we consider.

So the second square $BEFC$ is first split into three parts that is what one third to understand and the third part you further divided into 3 ok. So you this is further divided into 4 parts $1/3 \cdot 4$ ok. So you just place all this in 4 and these 4 you just place here, so this goes there and occupies. So in this square $APQR$ this is void here at this point. So if you say that the side of

the square is going to be so $1 + \frac{1}{3} + \frac{1}{3} * 4$. So there is a certain void and that has to be sort of subtracted.

And how is this value corresponding to the void here that I will show in the next slide. This portion is $\frac{1}{3} * 4$ because one third of it and this is divided into 4 that is going to be this distance. So the area of this void is $\frac{1}{3} * 4$ square. So I remove a small strip from this, so that the small strip corresponds to this area. So this is how I post the problem. Suppose a strip of breath b is consider.

So two strips basically one strip along the side and the another is this is this strip which I mark here, so $2b * 1 + \frac{1}{23} + \text{this-}b^2$. So this will be the left hand side and if equate so if ignore these square so you can easily see so b is $\frac{1}{3} * 4 * 34$. So this again an approximation because I have ignored b square, so getting this estimate and this can be extended at all levels ok, so now this b happens to be $\frac{1}{3} * 4$.

So at the next level it is $\frac{1}{3} * 4 * 34$ square kind of thing and so on and so forth so in this can be extended. So the expression that is given in the sulbasutras, so very interesting expression in the sense that you will find 3 hear the same 3 appearing added with 4 and then $3, 4 * 34$ and next term I think will be $3 * 4 * 34 * 1154$ or something like that and this can go on and on ok. So this is sort of rational approximation for the shirt so which will be recurring.

(Refer Slide Time: 35:25)

Approximate value of π
 An estimate of the value of π used by *Sulvakaras*

- ▶ If $2a$ is the side of the square, then we saw that the prescription given in the text amounts to taking the radius of the circle to be

$$r = a \left[1 + \frac{1}{3}(\sqrt{2} - 1) \right] \quad (2)$$
- ▶ If we were to **impose the constraint** that the transformation has to be **measure preserving**, then it translates to the condition

$$\pi r^2 = 4a^2.$$
- ▶ From the relation given above we have,

$$\pi \left[\frac{1}{3}(2 + \sqrt{2}) \right]^2 \approx 4. \quad (3)$$
- ▶ Using the value of $\sqrt{2}$ given in the text we get

$$\pi \approx 3.0883, \quad (4)$$

which is correct only to one decimal place.

So we saw in the previous slides, so when this problem of transforming a square into a circle we had the expression for the radius to be radius is $A/3 * 2 + \text{root}2$ and how did they find out

root 2 that also we saw. So this is what we saw the radius is this, so if you sort of impose the constraint as mentioned earlier that this circle has to have the same area of the square so which was transformed then we have the equation $\pi r^2 = 4A$ square right.

So we took the side to be $2A$ and that should be πr^2 as we understand today, so what has been given as r is this expression. So if we use the value of root 2 which has been shown by the (FL) himself then the value of π do not have to be approximately this ok in this prescription which has been given. So I discussed about $2\sqrt{a}$ a similar thing can be done for root 3 also. So we can have a similar geometrical construction.

(Refer Slide Time: 37:10)

Value of $\sqrt{3}$ (*trikaranī*)
Geometrical construction described by Datta

- ▶ Each side of the new larger square $APQR = 1 + \frac{2}{3} + \frac{1}{3.5}$
- ▶ So for closer approximation, let the side of the new square be diminished by an amount y , such that

$$2y \left(1 + \frac{2}{3} + \frac{1}{3.5} \right) - y^2 = \left(\frac{1}{3.5} \right)^2$$

Neglecting y^2 as too small, we get $y = \frac{1}{3.5 \cdot 3.5}$, nearly.

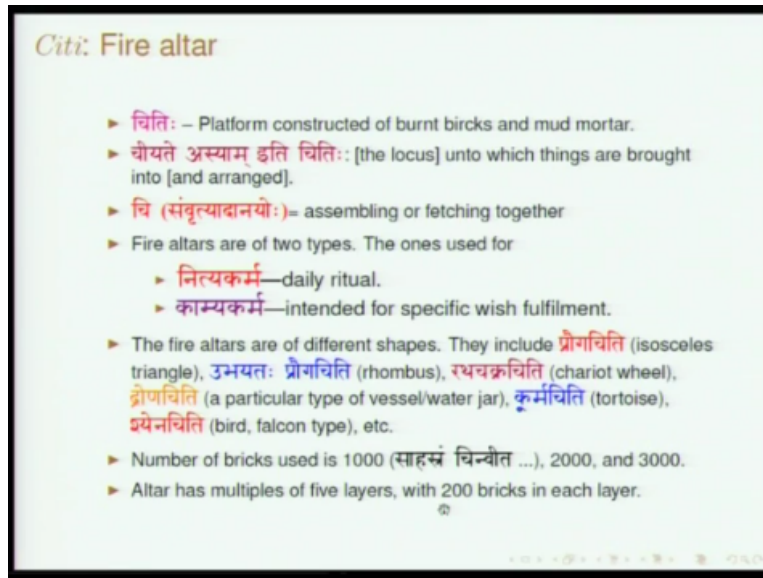
- ▶ Thus we get $\sqrt{3} = 1 + \frac{2}{3} + \frac{1}{3.5} - \frac{1}{3.5 \cdot 3.5}$

So the expression will be something $1 + \frac{2}{3} + \frac{1}{3 \cdot 5} - \frac{1}{3 \cdot 5 \cdot 5 \cdot 2}$ and so on so forth. I think this should be quite clear from the description that was given for root 2 it is very similar diagram so where in you have three squares considered see ABCD, BEHC, and then EFGH, we have to add two thirds of that on both side and then that is why we have $\frac{2}{4}$ and then one fifth of its so that this create such void and by formulating which is similar to the equation.

Because describe you will get 2,3. So this has been stated by in fact (FL) so we have the inverse problem so earlier we discuss the problem of transforming a square into circle if we invite the problem for you have a circle so that has been transformed into a square and for that we have an express of this form which is given in the sulbasutra (FL) all of them refer to diameter.

So (FL) so make it into A divide by a so (FL) 29 so you divide further by 129, once can see that that this is exactly the inverse of that and all the numbers will become evident. So I will just leave this and I will proceed to other topics, so the form is something which is very interesting that is what I want to tell once more 1/8, 1/8 and 29 and this very interesting form I think you can see the rational perhaps once you study this (FL).

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One does not know whether they (FL) but anyway one of the one of the ways is geometrical way the other is other way, all that we find is this interesting expression in this sulbasutras. So now I move on to another topic for which so all this mathematical rules have been invented by the sulbakara. So this is what is called citi as I was mentioning repeatedly. So this citi is basically a alter actually say altar ok.

So where in lot of bricks etc. are put together and a certain platform is created, so cit (FL) so this is how the (FL) derivation the word citi goes, in fact for those who are more interested in knowing the details with citi (FL) word has been defined in the (FL) ok. So this sacrificial alter are primarily fro 2 purposes, one is for (FL) the other is part of (FL) ok. Both in (FL) where in you have various (FL).

So one will be in the form of circle, the other is in the form of semicircle the other will be in the form of square. So the area of all the three has to be same and that is how it is sort of constructor any way, and there are various (FL) means that which is desired and action which is performed to fulfil a certain desire. So all these (FL) have been prescribed to be performed in citis of difference shapes.

The (FL) it will be in the form of isosceles triangle (FL) of rhombus (FL) citi in the form of a (FL) certain kind of vessel ok. So in the form of water jar, (FL) so that is one thing the other interesting part which one finds even the Vedas is so a particular person so this Vedic please performs the certain (FL) on a particular year. Suppose you want to performs in next year so then they say so the height of the alter has to increase.

See we have this mantra (FL) so in this thithi basically how 5 layers so the numbers of bricks in a particular layer will be 200. So this is one constrain which is set, the second constrain will be the area of this the whatever be whether it is (FL) so you have a certain area and the area is basically measured (FL) the person height. So that also will be fixed. SO this si the one constrain of the area, the second constrain is 200 bricks.

And the third thing which is prescribed is so 1000 bricks have to be there, so which means automatically there will be 5 layers ok so 200 in each layers. So far it have stability of these 3 should have bricks are arranged in alternatively. So the same kind of bricks which is arrange all of them will collapse. So therefore that we will see little later. Now what he says is (FL) means you have to perform it with 1000 bricks in a first year.

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Types of Fire altars (representative list)

► Different types of wish-fulfilling fire-altars are described in Vedas.

छन्दश्चितं चिन्वीत पशुकामः पशवो वै छन्दांसि पशुमानेव भवति, श्येनचितं चिन्वीत स्वर्गकामः श्येनो वै वयसां प्रतिष्ठा श्येन एव भूत्वा स्वर्गं पतति ... प्रौगचितं चिन्वीत भ्रातृव्यवान् प्रैव भ्रातृव्यान् नृदते, ... रथचक्रचितं चिन्वीत ग्रामकामः ...

► The table below presents a list some of them, along with the shapes and the purpose as stated in the text.

Name of the <i>citi</i>	Its shape	Who has to perform
छन्दश्चिति	Form of a bird	Desirous of cattle
श्येनचिति, कङ्कचिति	Form of bird	Desirous of heaven
प्रौगचिति	Isosceles triangle	Annihilation of rivals
रथचक्रचिति	Chariot wheel	Desirous of region
द्रोणचिति	Form of a trough	Abundance in food

Table: Different *citis*, their shapes and purpose.

So next time if you want to perform then it says (FL) third time if you want (FL) so it goes like that. This is also found in Vedas as 2 the purpose for which a particular citi has to be done, for instance (FL) if you desire a (FL) large number of cattles then you perform this,

(FL) you cannot conclude with (FL) that is a different thing but this (FL) means enemy ok, so not well wisher (FL) so these are these are various prescription.

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On the height and the shape of *citis*
Measurements were case-based (based on the performer) and not 'standardized'

► *Taittirīya-saṃhitā*, prescribing the height of the *citi* observes:⁴

जानुदं चिन्वीत प्रथमं चिन्वानः, गायत्रियैवेमं लोकमभ्यारोहति,
नाभिदं चिन्वीत द्वितीयं चिन्वानः त्रिष्टुभैवान्तरिक्षमभ्यारोहति,
श्रीवादं चिन्वीत तृतीयं चिन्वानः, जगत्यैवामं लोकमभ्यारोहति।

Knee-deep should he pile when he piles for the first time, and indeed he mounts this world with gāyatri, naval-deep should he pile when he piles the second time, ... neck-deep should he pile when he piles the third time ...

► Elsewhere (5.5.3) observing on the shape of the fire-altar it is said that it should be akin to the shadow cast by the bird.

वयसां वा एष प्रतिमया चोयते यदग्निः। यन्न्यश्नुन्नयात्⁵

⁴*Taittirīya-saṃhitā* 5.6.8.
⁵वयसां वा एष प्रतिमया चोयते उत्पततां छायायेत्यर्थः (BSS.8.5)

(FL) desirous of having so large area (FL) designing current position today for various purposes, then as I was mentioning earlier so this height of the citi so they say (FL) perform it for the first time then it says (FL) third time. So this is how the prescription goes in Vedas, ok (FL). Then it also says (FL) sort of shape, image, ok usually suppose you have a certain ideal is called (FL) ok an image of something.

So (FL) means so you create the alter so this (FL) so does not refer to age, (FL) means bird which fly ok so you have to construct the alter in the form of a bird , so this is set of prescription which one finds in this (FL).

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Śyena-citi—Falcon-shaped fire-altars

- ▶ The origin of Śyena-citi can be traced back to *vedas*.
- ▶ For instance in *ṣaḍviṃśa brāhmaṇa* belonging to *sāmaveda*,
अथैष श्येन ... यथा श्येन आदधौत एवमेव एनमेतेन आदत्ते⁶
- ▶ Another version of the same statement perhaps on another *Brāhmaṇa* which is more popular goes as
यथा वै श्येनो निपत्य आदत्ते एवमेवायं द्विपन्तं भ्रातृव्यं निपत्य आदत्ते।
- ▶ These sentences are cited in the *Mīmāṃsā* text in connection with the discussion on deciding the meaning of the word *śyena* that appears in the *vidhi* (injunction)

श्येनेनाभिचरन् यजेत

⁶Ṣaḍviṃśa brāhmaṇa 4.2.3.

So what is this bird, so there are other statement which are found in various Brahmana. So I am just decide a couple of them and then proceed further. if (FL) comes down quickly something take up so which it wings spread ok. So it comes down quickly bounds on something graph and then proceed. So it is a sort of metaphorical description. So once you perform the sacrifice, so as it comes and takes it.

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Measurement units used in construction

अथाङ्गुलप्रमाणम्। चतुर्दशाणवः। चतुस्त्रिंशत्तिलाः पृथुसंक्षिष्टा हृत्यपरम्।
दशाङ्गुलं क्षुद्रपदम्। द्वादश प्रदेशः। पदं पञ्चदश। द्विपदः प्रक्रमः। द्वौ
प्रदेशावरनिः। पञ्चाराणिः पुरुषः। चतुररनिर्व्यायामः।⁷

anṅula	=	14 aṅgu	or	34 tila
kṣudrapada	=	10 anṅula		
prādeśa	=	12 anṅula		
pada	=	15 anṅula		
prakrama	=	30 anṅula		
aratni	=	2 prādeśa = 24 anṅula		
vyāyāma	=	4 aratni		
puruṣa	=	5 aratni		

⁷Baudhāyana-śulbasūtra, 1.3

So to all your wishes will be fulfilled and then that is kind of description (FL) so the one who hates you (FL) once you perform this, so this enemies will also be sort of finished something like that ok . In this connection so various measures have been specified, so from very small to purusa, the angula is one the small dimension and what is angula, so it says (FL) constituted angula and you can also specify in terms of tila.

So which is much smaller unit, so this is 34 tila and conclude angula, and then this goes on this table if 10 angula makes see all that has been very clearly stated (FL) so that will be in fact usually people say (FL) okay so that will be roughly angula, so this si how it goes and then it goes up to a purusa, so it starts with angula and then goes up to purusa, purusa measn a human being, the height of human being.

This measures, so if you just take this angula and then so if you see that so everything is with reference to 2 purusa ok. So finally the measurements will be given in terms of purusa larger measurements. So how much should be the width and breadth of Vedic the entire sacrificial place ground. So they will all be specified in terms of purusa, so if you are the performer then your height will be measured.

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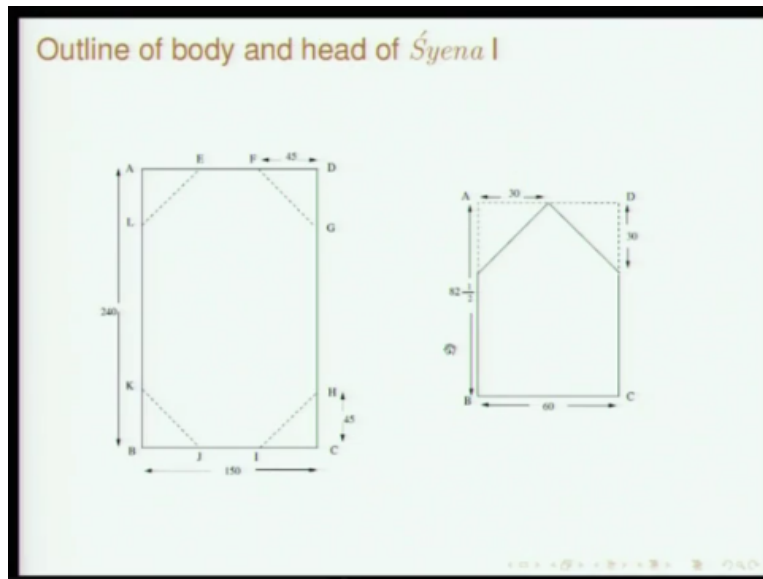
Construction of Śyenaçiti: I
Types of bricks: 1, 2 and 3

- Bricks of geometrical shapes other than rectilinear are needed.
- The five types of bricks used:
 1. B_1 —one-fourth brick (*caturthi*)— 30×30 *añgulas*; i.e., a square whose side is $\frac{1}{4}$ *pu*.
 2. B_2 —half brick (*ardha*)—obtained by cutting the one-fourth square brick diagonally; each of 2 sides equals *añgulas* and the hypotenuse $30\sqrt{2}$ *añgulas*
 3. B_3 —quarter brick (*pādya*)—obtained by cutting B_1 diagonally; each of 2 sides equals $15\sqrt{2}$ *añgulas* and hypotenuse 30 *añg.*

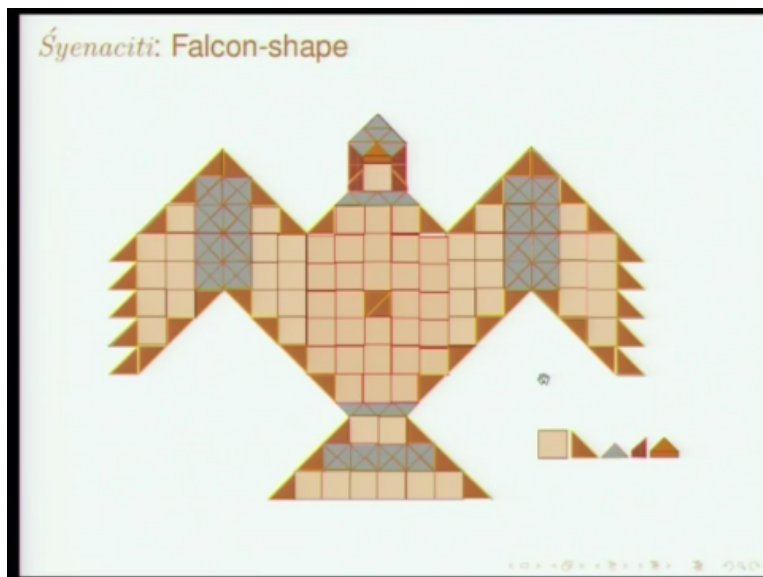
And the vedi will be constructed based on that it. Now I quickly ran through a few slides wherein the shapes of various bricks have been given in great detail, for all the measurements etc, I states her I will not spend much time here. This for instance if you want to construct (FL) itself of various types, so one particular (FL) have described here in this slide. So (FL) there are several types of bricks 1, 2, 3 bricks.

So we can see this one half of it and therefore you have this root 2 times and one-half again ok. So this is one set of bricks. If you put them together so you get another kind of a shape. So this will B5 is called hamsamukhi.

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So now slowly you can see so this is the body of the syena and this is the head of the syena.
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So you can see so this picture this is what shown earlier. So this is the head, so this is a body, this is a wing and this is called the (FL), we can see that so it is made up of basically 5 types of bricks.
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Number of bricks used

एवं षड्द्वारिंशदात्मनि। शिरसि चतुर्दश। द्वात्रिंशत्पृच्छे। पञ्चयोरष्टशतम्।
 अस्मिन् प्रस्तारे नवपट्टिशतुर्थ्यः। अर्धा द्वासप्ततिः पादा द्विपञ्चाशत्।
 पट्ट चतुरश्रपादा। एका हंसमुखी।

Parts of the <i>citi</i>	B_1	B_2	B_3	B_4	B_5	Total
Head	1		6	6	1	14
Body	30	6	10			46
Wings	30	62	16			108
Tail	8	4	20			32
Total	69	72	52	6	1	200

Total number of bricks as I was mentioning should be 200. So this is a constraint, so head will have 14 bricks, body will have 46 bricks, wings 108, tail 32, the total. So these are the 5 types of bricks, see all that has very very clearly stated in the sutra (FL) so 32 times will go in creating the tail of this (FL) 108 so all that has been stated.

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Śyenaciti: second layer
 Number of bricks used in the second layer

Parts	B_1	B_2	B_3	B_4	B_5	Total
Head		10				10
Body	12	28	4		4	48
Wings	48	28	34			110
Tail	8	4	18		2	32
Total	68	70	56		6	200

So this is the second layer the second layer will be such that no two tail will be exactly fitting, see they will be so start of interest is will be filled in second layer and this also has 200, but there are 5 difference thing, head has only 10 here and body has 48 in the previous if you see it has 14 and 46. So this is made up of only 4 types of bricks. So these four types they constitute the second layer.

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Fabrication of bricks

Ingredients to be added to the mixture of clay employed in manufacturing the bricks

- ▶ पर्णकषायनिष्पट्टा एता आपो भवन्ति। स्थेम्ने न्वेव।...
- ▶ Extracts of gum from certain trees (*pulāśā*)
- ▶ अथ अजलोमैः संसृजति। स्थेम्ने न्वेव।...
- ▶ Hair of the goat, of a bullock, horse, etc.
- ▶ शर्कराशमाहो रसः तेन संसृजति। स्थेम्ने न्वेव⁸
- ▶ Fine powder of burnt bricks ..
- ▶ उख्यभस्मना संसृज्य इट्टकाः कारयेदिति। संवत्सरभृतः एव एतदुपपद्यते। न गत्रिभृतः।⁹

The above process of strengthening is in practice till date.¹⁰

⁸ *Satapatha brāhmana*, 6.5.1.1–6.
⁹ *Baudhāyana Śulbasūtra*, 2.78–79
¹⁰ The addition of fly ash as well as pozzuolana is well known in the manufacture of cement.

The third layer will be again the first layer, fourth and fifth layer. So this is how this is constrain. So regarding this is fabrication of brick also there are some specifications in the sulvasutras. See these are all some interesting things, so it says (FL) extraction created on various, so in that sense they uses, so (FL) so in making this brick we have to add various extractions.

So it is primarily to add more strength to the bricks so then is he says (FL) something which is repeated in every sutra, so means to add more strength to that ok. So (FL) the hair of goat, so it is like today this fibre reinforce know there are they are the fibre ok. So (FL) various things which are added and it also specifies that once you fabricate the brick and it gets dry. So to the dimension of the brick will be reduced ok.

So it says (FL) one thirty of the size will be reduced some kind of prescription which has been given here and therefore in trying to create an altar you have to consider the factor also in talking, so this is was states and (FL) constructing so you have to consider how much gap you have to create so that I mean you fill those gap together, so that the dimension is order met with venue constructed the whole alter ok.

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Constructional Details

Specifications regarding the arrangement of bricks in different layers

- ▶ भेदान् वर्जयेत्।
 - ▶ Here the word “*bheda*” does not simply mean difference/distinction (in fact, this has to be maintained).
 - ▶ What is meant is a clear segregation between two rows across all the layers. This is to be avoided.
 - ▶ Joints should be disjoint! (not continuous)
- ▶ अधरोत्तरयोः पार्श्वसन्धानं भेदा इति उपदिशन्ति।¹⁴
 - ▶ The etymology could be: भेदहेतुभूतत्वात् भेदः ।
- ▶ अमृन्मयीभिः अनिटकाभिः न सङ्घां पूरयेत्।
 - ▶ (Arbitrary) foreign material should not be employed to fill the gaps.

The above-mentioned are very important principles from the view point of civil engineering.

¹⁴BSS. 2.22–23. (RPK's Book)

Then certain other subscriptions (FL) see when you construct so you do not give much gaps, otherwise the area of criteria which has been stated for will not be fulfilled, then (FL) so what is this veda that I am talking about between the two layers whatever be the joint. So then he says (FL) say this is interesting in the sense that see when you create certain structure so you should see to it that more or less the same kind of material used.

And used a completely different material then this will not stick with that and therefore he says (FL) is basically brick made out of clay. So (FL) is not made out of clay would not be using here. So all these prescription are been here.

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General observations

- ▶ The purpose for which the geometry got developed in the Indian context is **construction** and **transformation** of planar figures.
- ▶ We saw that Bodhāyana (prior to 800 BCE) not merely listed the so-called ‘Pythagorean’ triplets, **but also gave the theorem** in the form of an explicit statement.
- ▶ **Extensive applications** of the theorem in the context of scaling and transformation of geometrical figures was also discussed.
- ▶ Though *Sūlbasūtras* **did not explicitly give proofs**—which anyway was NOT a part of their “**oral**” tradition (of the antiquity)—it is evident from several applications discussed, that **the proof is implicit**.
- ▶ From the view point of history it may also be worth recalling:
 - Antiquity?** Though the Babylonians of 2nd millenium BCE had listed triplets in cuneiform tablets, there is **no general statement** in the form of a theorem.
 - Pythagorean?** Since there is hardly any evidence to show Pythagoras himself was the discoverer of the theorem, some of the careful historian call it **Pythagorean** theorem.

So with the few observations will end our task on veda and sulbasutras. As you would have seen the primary purpose for which the sulbasutra text came with existence is to see that you

have very clear rules which are stated which will facilitate as in constructing all those. Constructing is fire places on altars of different sizes and shapes. So and in this connection both construction as well as transformation ok.

So if you construct a certain (FL) and you will have another thing which will be of different shapes we should have the same and therefore this is equivalent to transforming one into the other. So this is the primary purpose, but this was not the purpose of geometry got developed civilization and I also demonstrated that how baudhayana would have arrived at the different triplets which has been mentioned by him.

And how the proof of these sutras are implicitly involved in the procedures which have been delineated for various construction or transformation of one figure to another figure. So this was demonstrated and regarding the antiquity of this baudhayana sulbasutra see this another thing you need to remember, so in this tradition we see that it has been an oral tradition and therefore any proof would have been combined by the (FL) not explicitly available in the sutra systems.

And regarding antiquity so we find various triplets in this Babylonian (FL) tablets but there is no general statement like the Baudhayana theorem in any of these other conditions. So regarding Pythagoras so Pythagorean theorem so many careful people they use Pythagorean the Pythagoras or directly involve or not we do not know and therefore they call it Pythagorean.

And this sulbasutra text are primarily meant for assisting the vedic please ok that is the purpose of this. So let me reiterate that and but looking at this we also are able to get a picture of the kind of the mathematics which was involved and how it got developed so in the antiquity at least 2500 years from now so much before that and we also see the use of fraction (FL) $\frac{1}{3} + \frac{1}{3} * 4$ so all that they are all very interesting things without found.

And the value of root 2 is remarkable accuracy and he also saw the use of algebra which is involve, for instance suppose you wanted to find out the construction in katyayana sulbasutra so it impossible for us to find out (FL) so without the algebra involve, so this prescription cannot be given (FL) also getting into, so we have various shape of alter (FL) and so on. As

regards this citis so it has been found that around 200 BCE, so we find (FL) construction, so in the excavations (FL) found. So these are the references, thank you.