Mathematics in India: From Vedic Period to Modern Times
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Lecture - 29
Magic Squares - Part 2

So this is the second part of our lecture on magic squares. In the last lecture, we discussed this turagagati method of obtaining magic square. So turaga refers to horse, so gati is jumping. Basically it is related to the movement of horse as we see in chess board, so how do we generate magic squares using that technique.

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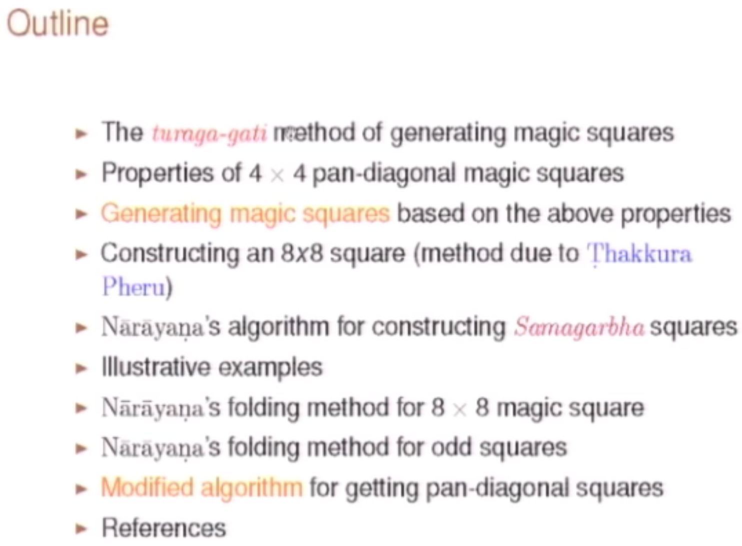

So I will quickly recapitulate that and then work out a couple of examples as if you approach me and then ask me so it would be convenient to see, how do we exactly use that method to generate. So we will work out a couple of examples, as we did not have much time in the last lecture. Then I will discuss the properties of the pandiagonal magic squares, so as has been neatly worked out by professor Vijayaraghavan and based on those properties, how is that that we can easily generate magic squares.

So these are the 2 things that we will do in the first, and then we will move on to the method which has been described by Narayana which is called samputa. So samputa basically means sort of putting together, so we may translate it as folding method, folding in the sense of 2 squares, folded like a hand and then added. So the elements of 2 squares each of them will have arithmetic progression written done in a particular form.

So then we sort of fold them together and add the sum, so we will get the magic square. So this is what is called samputa method of Narayana and we will do that for samagarba squares, and then we will also extend it for vishamagarba squares, and then we will work out a couple of examples for 8/8 magic squares, that will more or less bring the discussion on magic square to end.

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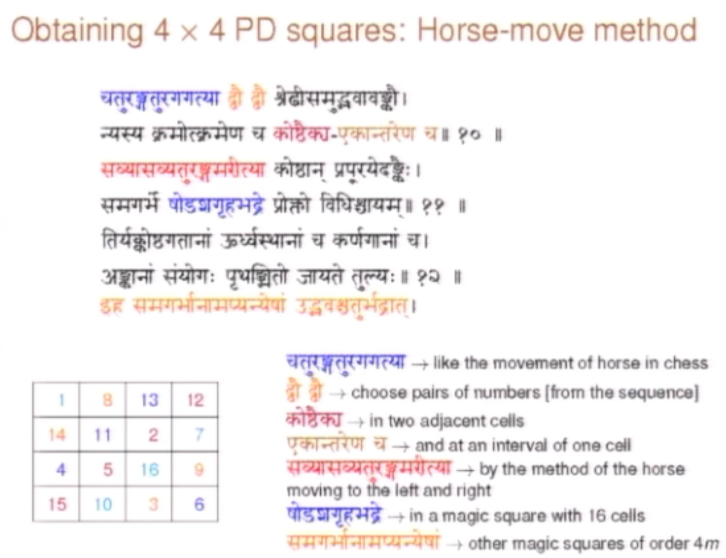

So this is the verse which prescribes this turaga approach to obtain magic squares. (FL) So this is the rule that has been prescribed, so prescribed to obtain (FL), so magic square having 16 cells, so 4/4 samagarba. And in fact the last line here he says (FL), once you have a 4/4 magic square, you can extend it to 8/8, you can extend it to 16/16. So more or less the process will be similar, so that is what Narayana points out.

(FL) So I will work out 1 or 2 examples, so that this turaga method becomes clear. So let us start with.

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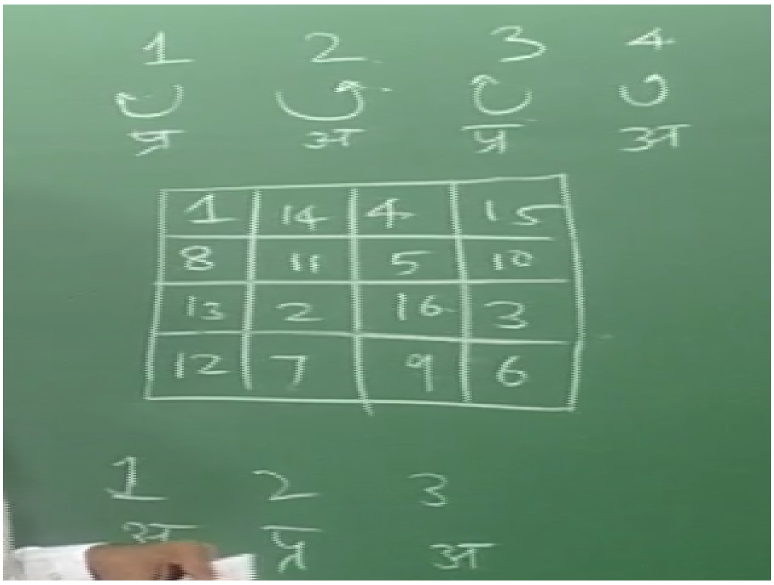

So we will start with 1 here. So this turaga, we have to place, it is a horse movement we have to place 2 here. So what is important note is (FL) that is what he says, pair of 2, suppose you consider 1, 2, 3, 4, you just split them into 2 pairs and then these 2, 1 and 2 will have a turaga, 3 and 4 will have a turaga, so not 2 and 3. Suppose I put 3 here, then turaga will lead 4 here, then I start with 5.

So 1 thing to note here is, it will be convenient to have a certain pattern recognition, so that we will be able very quickly fill this squares. So what we have done here is 1, 2, 3, 4. This could be sort of the first movement, sort of circular, you can call it pradhakshana. This will be very easy if you just have some kind of convention, so that it can be easily filled. Now I take 5, in this example which I have worked out 5, 6, this is turaga, then we move 7 and 8.

The 6 can be put in 3 different places, no that is why we have so many forms of 384 forms, so you chose 1 particular form. This turaga, you can have in many ways, you just chose 1 particular form, 1 jump. So that is why I am just saying this you can have, at the second state what I have done is these 4 elements, for all of them I have done this upradhakshana kind of thing, if you want to call.

In fact he says there are 2 ways, there are 2 terms which he says, kramayana, uthkramayana, sovyana, appasovyana. So in fact each 1 has its own way of generation, we will see 1 or 2 examples where it becomes evident, 1 has to play around with this. Then you can start with 9, you place 10 and then move on to 11 and then you place 12 here. So this is, the third movement which I has done is again sort of pradhakshana kind of a thing, see.

And the fourth so it should be sort of appradhakshana, so 13, 14, 15 and 16. So this you can see 13, 14, 15, 16 so it is sort of. Let me start with a different location for 2, so this turaga from 1, it can be jumped to 4 places it can either be here, it can be here, it can be here or it can be here, so this I tried to explain in the morning. Let me chose some other place for turaga. So I start with 1, and then I place 2 here, so we can place 3 here and then 4 will jump.

In fact, there are 2 things to note in the verse, 1 is look at this he says (FL) 1 difference, (FL) so possibility when you start with 3 here itself, we will do that. Suppose you take 2 here you can start 3 here you can put 3 here and then also you can work out. The turagagati will be for pairs. I write it this way so 3, 4, then we just take 5 turaga,so 6, then 7, then 8, so 5, 6, 7, 8. Once again you can see 1, 2, 3, 4.

I sort of in this particular thing the first movement was appradhakshana, 1, 2, 3, 4, 5, 6, 7, 8, the second movement is pradhakshana kind of a thing. Then I will start with 9, 10, 11 and then 12, this third movement 9, 10, 11, 12 is once again appradhakshana and want to do a certain pradhakshana, so this will get filled. So 9, 10, 11, 12, 13, 14, 15, 16. So this is what he refers to by saying (FL), there are 2 things.

1 is this kind of a circular movement, clockwise, anti-clockwise, I will just work out 1 more example. Instead of working out this example in the board, we will work out these examples using the slides themselves and I will explain them so that I will be able to save time. So in the previous examples you might have noted that the (FL) were placed in a sort of circular and anti-clockwise, anti-circular manner.

See 1, 2, 3, 4, 5, 6, 7, 8, so on and so forth. But in the couple of example that I am going to show you now, instead of having these (FL) placed in clockwise and anti-clockwise mode, we will have them placed diagonally itself.

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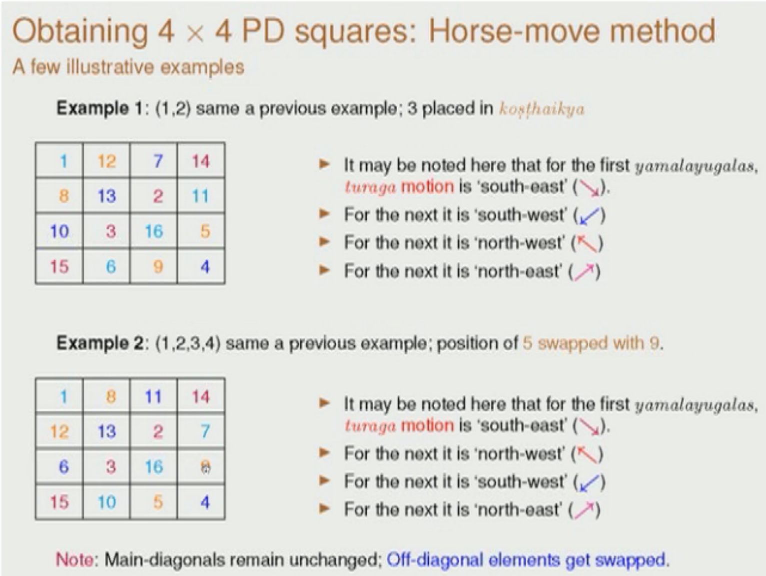

1	12	7	14
8	13	2	11
10	3	16	5
15	6	9	4

1	8	11	14
12	13	2	7
6	3	16	9
15	10	5	4

So for instance in this example, if you see 1 and 2, 3 and 4, then 5, 6, 7 and 8. Similarly, the rest of the things get filled by moving in south-east, south-west, north-west and north-east direction. For instance, if you look at the last 4 numbers, 13, 14, 15, 16 so they are all in the north-east directions, so assuming this to be the north. The next example that I am going to show you now here instead of placing 5 and 9 in this position we are just going to swap them.

So 1, 2, 3, remain in the same position and 4 also remain same. So instead of placing 5 above I am going to place 5 here to the left and then the whole thing will be placed. So if you note in this example 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15 and 16. In between these 2 examples you will see that the diagonal elements remain the same. See 1, 13, 16, 4 here 14, 3, 2, 15 and what you will notice is he off diagonal elements, so 7 and 11 they have got swapped here.

Similarly, 12 and 8 they have got swapped here, 6 and 10 got swapped, 9 and 5 got swapped. So this, if you look at this is the kind of motion that we have for the (FL), and the first and the last, they remain the same. These 2 have got swapped.

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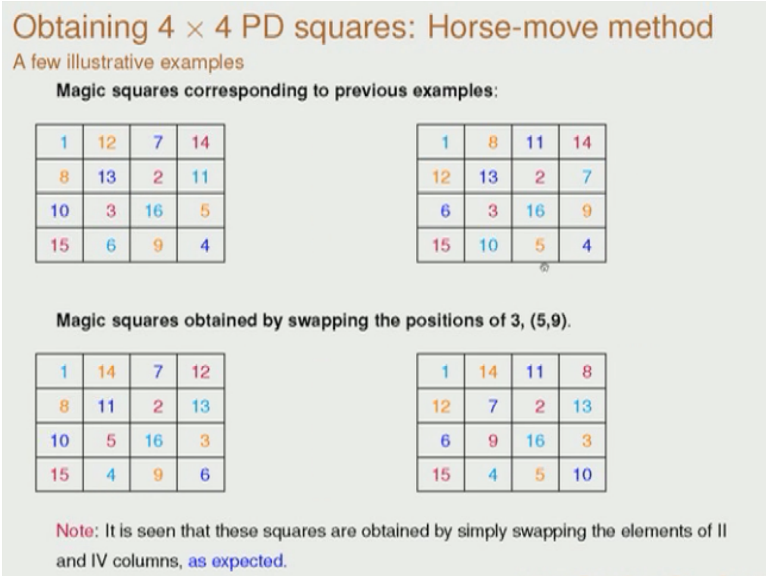

I will work out couple of more examples. These are the previous examples, and in the new example you will see that instead of placing 3 here, I am placing 3 here. So 1, 2, 3 and 4, as I had mentioned earlier see having placed 1 and 2, 3 can be actually placed in any of these 3 positions, 1, 2 or 3. So initially we started with 1, 2, and 3, so this is (FL) and in the other examples these are (FL).

So (FL) basically means 1 cell in between and (FL) 2 cells are close to each other, see 2 and 3 are placed in such a way that there is no cell in between them. So here again instead of placing 3 here, I have placed 3 here and 4 has to be placed here. So notice that all of them are turaga mode only. Any 2 pairs, 1 and 2 are in turaga 3 and 4 in turaga, so between 2 and 3 we do not see. Each pair you should start and they should have the turaga mode.

This much has to be ensured. And in this example, you notice 3, 4, then 5 and then 6 here, 7 and 8 and between, this example and this example, I am not going to explain all the details, you can see from this example. And 1 thing which is strikingly similar between this example and this example is the following. So here if you notice, 1, 8, 10 and 15, 7, 2, 16 and 9. See these have 1 and the same, 1, 8, 10, 15, 7, 2, 16, 9, between this example and this example.

By moving 3 from this side to this side, so you see that what has happened is, the elements 14, 11, 5 and 4, the elements of rows 1 and 3 are identical in, whereas 4 and 2 they have got swapped with each other. So this row has gone here and this row has moved here. So this is the difference between these 2 examples.

Similarly, in this example 1, 2, 3, 4 instead of placing 5 here, I am placing I am placing 5 here between 3 and 5 here, and here again you will notice that, between this and this column 2 and 4 have got swapped with each other and 1 and 3, they remain the same.

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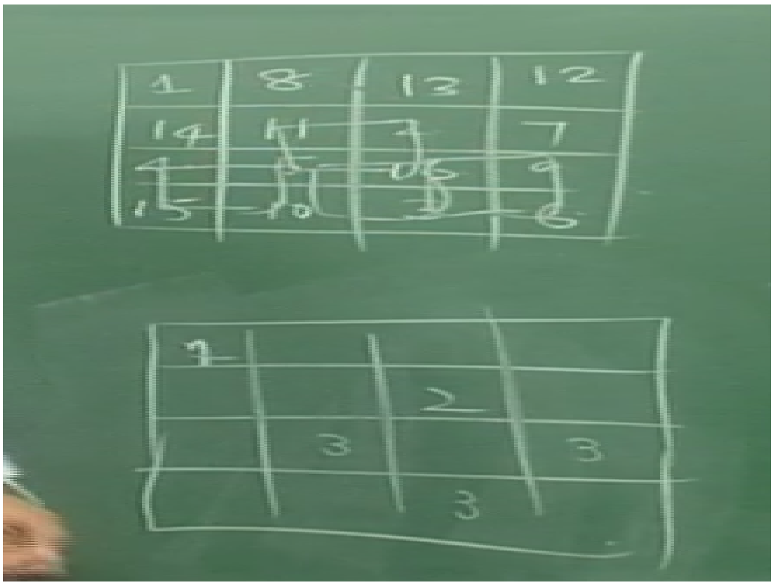

When you place the first element, you have 1 you have 2, so now you have placed your 3 here, 3 here or 3 here. So when you place 3 here, we say ekhandhara. (FL).

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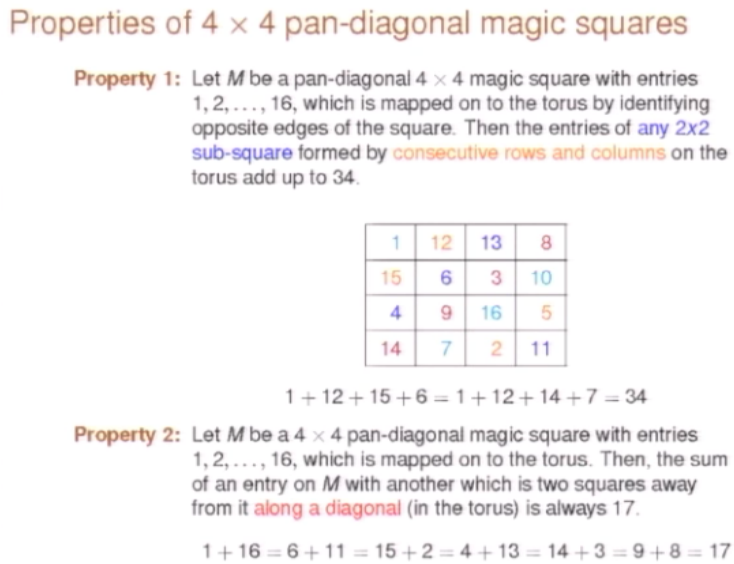

So we want to discuss this property, I just started discussion on this, property 1 essentially tells that in the magic squares you chose any 2, columns or rows, see that is what it says, consecutive rows and columns so any 2/2 sub-square the sum will be 34, so any 2/2 sub-square the sum will be 34, this is the first property that need to be. How many such things 1 can have, in fact morning I was mentioning that it is 15.

Apparently it should be 16, it was pointed out by Professor Kamala Keerthivasan was saying, that it should be 16, it looks like there are 16 such possibility 1 can have. Once you sort of fold it, see you have 2, 1, 2, 3, 4, 5, 6, 7, 8, and then the central will be 9. So then all the corners when we sort of fold, so we will have. So the second property will be extremely useful in constructing this magic square along with the third property.

So if you look at the square, you just add up elements which are alternate across the diagonal. They will be summing up to S/2, half the magic sum, 1+16 is 17, 6+11 is 17 and 12+5, 17 and 10+7, 17, so this is how it is, 13+4, 17, so it will be adding up to S/2, so this is the very important property.

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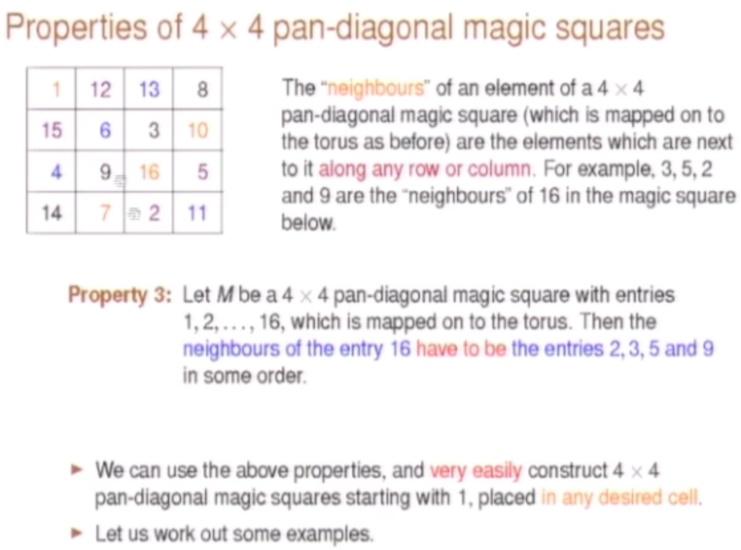

Then we move on to the third property, which is a very, very interesting thing which has been proved by professor Vijayaraghavan. Here he says that the neighbours of the elements are fixed, so neighbours in the sense of along any row and column. For instance, if you take number 16 which appears here, the neighbours are 2, 5, 3, and 9. So whatever be the configuration in which you arrive at, these will be the 4 neighbours of 16.

So this is true for any element that is in the matrix, in the magic square. So this 2, 5, 3, 9, once you remember this, then I think you will be able to construct the entire magic square. So along with the earlier property, I will just work out an example.

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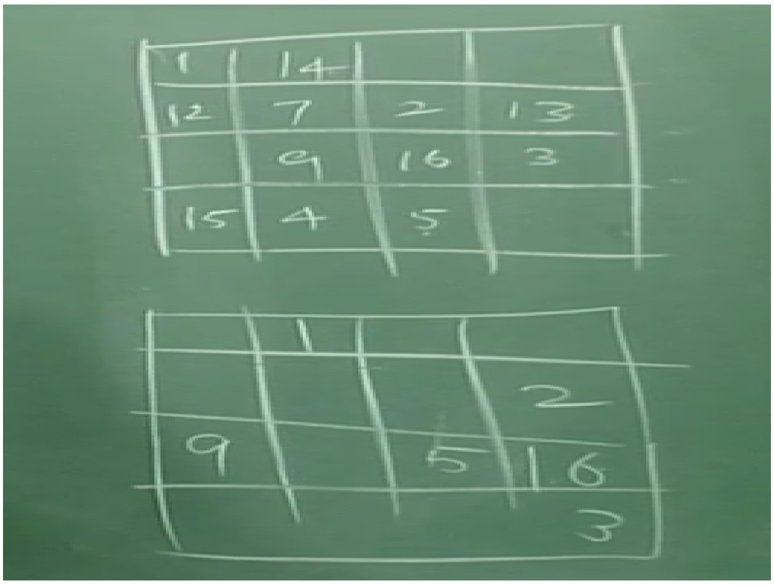

So you start 1, and then it says that, suppose you chose 16, if you remember the neighbourhood of 16, the neighbours of 16, 2, 3, 5, and 9. This can be placed in any way you want, these are the 4 neighbours. In fact, this immediately tells you that the number of possible combinations is 384. See this neighbours once it is fixed, so there are 4 neighbours and they can be placed in 4 factorial ways, which is 24.

And for this number this 1 can be placed anywhere, so there are 16 ways and 16*24 is 384. This is a very important property which has been worked out by professor Vijayaraghavan. So here, it tells you that the sum of any of these 4 blocks has to be 34. So you have 16+9, 25 and this should be 4, and if you look at, this and this, they have to sum up to 17, this has to been 13 and this and this has to sum up to 17, so you can place 14.

And all these has to sum up to 34, so this is 18 and 9, 27, this actually fix as 7, on this way we can easily fill the entire magic square. So this is 5, this has to be 12. So this way 1 can get all the elements fixed. How come we know which 2 boxes has to have the sum of such. Now this is true of all elements across the diagonal, any alternate thing if you chose, so that sum has to be 17, we have to choose alternately, that is how it is.

So this is 2, and this has to be 15, so 1 can, this is a very elegant way of working out magic squares, once you remember for 1 element, from 1 element if you remember the neighbourhood, then it automatically fixes all the elements of the magic square. That 16, you can place anywhere, right? See once you fix this 1 so it has to be here, alternate. So if you place your 1 there of course it will move.

If you place your 1 here so 16 would be at this place. So this neighbourhood see 2, 3, 5 and 9, you can place it anyway, so 2, 3, 5, and 9 it moves here it will be mapped here. This is valid for elements 1 to 16.

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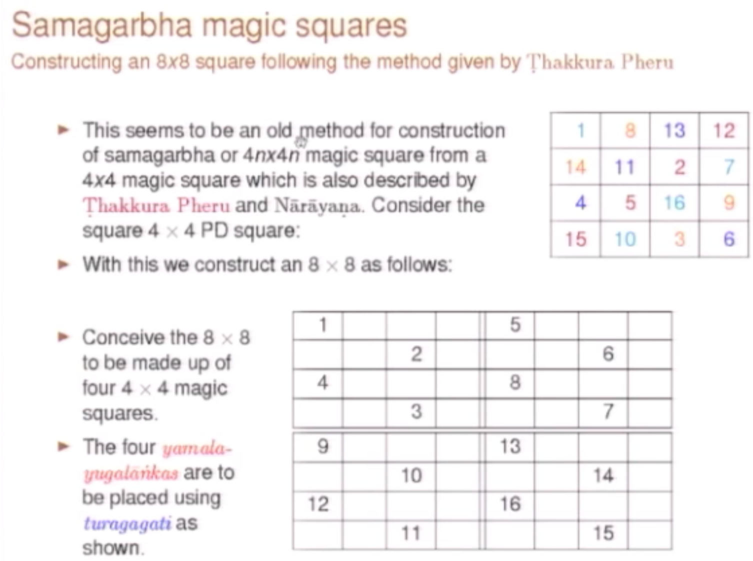

So samagarbha magic squares, suppose we have 8/8, this thakkura Pheru as I was mentioning earlier has discussed this magic square in his ganithasarakoudi, that is a text. It is composed around 1300 AD and he prescribes a method by which 1 can construct 8/8 from 4/4. So let us conceive of this magic square, initially we start with 1, 2, 3, 4, what to do is to do the same thing, you divide it into 4, 4X4. So 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16.

You just fill it, like a turagagati, you just fix it, these are actually referred to as (FL), so 1, 2, 3, 4, (FL) so this (FL) has to placed like this. Now the next step is, 1 thing which we need to notice, you moved from left to right, and then you moved from left to right, this has to be kept in mind. Then wherever you left, you left with this 16, you have to start here in the same 4/4. So 17, 18, 19, 20 that will be in the opposite way, so 17, 18, 19, 20, you move to here, so 21, 22, 23, 24.

Then you start here 25, 26, 27, 28, 29, 30 ,31, 32, this is the next, half of that is filled now. The other half is almost identical so you stopped here and therefore you have to start here. So 32, we came up to this, so then you start 33, 34, 35, 36. How do you select position for 33. That is a very interesting question, so why did I choose to place 33 here having placed 32 there.

In order to explain this, I have to go back then choose to explain how we chose to place the elements in a 4/4 magic square which is a much simpler case. So let us see this example here having placed 1, 2, 3 and 4 in sort of clockwise direction, I chose to place 5 towards the east of 4, having placed 4 here I just placed 5 here. And then it goes in turaga motion 5, 6, 7, 8, anti-clockwise direction and then having placed 8 here.

So once again I moved down and then 9, 10, 11, and 12 were placed in a sort of clockwise direction. I would like you to note the relative positions of 4, 5, and 8 and 9 in the case of a 4/4 magic square. Now we may want to the 8/8 magic square, in the case of 8/8 we just have to place 1, 2, 3, 4, 5, 6, 7, 8 all of them in the same clockwise manner and then in the next step, we place the 16 here and then immediately 17, so 17, 18, 19, 20 and this pattern will be followed in all the 4/4 sub magic squares which are there in the 8/8 magic square.

And having placed 32, we are placing 33 here and then we move on in the same pattern, 33, 34, 35, 36 and so on. 1 thing which you need to note here is whatever is the role played by 4 and 5 in the case of a 4/4 magic square will be played by 16 and 17, say for instance in the case of 4/4, so 4 and 5 are placed by moving towards the east. In the case of 8/8, this 4 and 5 are same as, are mapped to 16 and 17 in the case of 8/8.

So having placed 16 here we moved towards east and then placed 17. Similarly, this 8 and 9 in the case of 4/4 will be mapped to 32 and 33, so you can notice that. So we start placing them in almost an identical manner as 4/4. So that is why, so Narayana, in his verse, he very clearly said (FL). See once you understand the pattern in the case of 4/4, then all the multiples 4M type can be constructed by recognizing a certain pattern, which will be followed in all the higher dimensional magic squares.

This is something which one can play around with various things and then this is how I am getting in magic square. That does not follow the turagagati. Everything is placed in turagagati and where one should start is something which is to be filled in and then one has to understand. So in fact that initially people would have started with turagagati and then after this turagagati 1 actually finds out the explanation.

See how many possible combinations, that is what has been worked out by professor Vijayaraghavan later. So this is the only thing possible and it will be something derivable within the properties, which one has to study. Now I move on another interesting thing.

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Samagarbha magic squares

- In the previous step we moved horizontally to the right and filled the cells.
- Now, we start with the quarter where we left and move horizontally to the left.
- The four *yamala-yugalāṅkas* are to be placed using *turagagati* as earlier.

1	32			5	28		
		2	31			6	27
4	29			8	25		
		3	30			7	26
9	24			13	20		
		10	23			14	19
12	21			16	17		
		11	[illegible]			15	18

Finally we arrive at the pan-diagonal 8x8 magic square

- Again we start with the quarter where we left and move horizontally to the right.
- One of the properties of an 8x8 pan-diagonal magic square seems to be that the sum of four alternating cells along any diagonal adds to half the magic sum.

1	32	61	36	5	28	57	40
62	35	2	31	58	39	6	27
4	29	64	33	8	25	60	37
63	34	3	30	59	38	7	26
9	24	53	44	13	20	49	48
54	43	10	23	50	47	14	19
12	21	56	41	16	17	52	45
55	42	11	22	51	46	15	18

These are all just turagagati playing, so you just play around. So this is more interesting thing, which has been presented as an algorithm by Narayana is the following, which is the samputavidhi. Here he says (FL), that is what I was saying Samagarbha refers to magic square of dimension 4M. So we can work out examples with 4/4 and then 8/8, but Samagarbha refers to 4M, dhve karye to prepare 2 magic squares.

So you have to do that. 1 is called chadaka the other is called chadya. (FL) So chadya/chadaka is just placing one over the other, 1 is the coverer, the other is covered kind of a thing. So this is what he says. (FL) So morning I was referring to the kind of Kotaka equation that you will have once you have s and n. So A and D have to be determined. (FL) So this arithmetic sequence, which you choose, it is ishta. So it can be any number.

It can be 0, 1, 2, 3, it can be 2, 4, 8, 9 whatever. So it should a simple arithmetic progression, which you have to choose. (FL) Badhramitha means limited by badhra. So badhra if it is 4/4, you have to just choose a sequence of 4 elements. If it is 8, we have to choose 8. Badhramitha and it is referred to as moolapankthi. (FL) So this is all it says.

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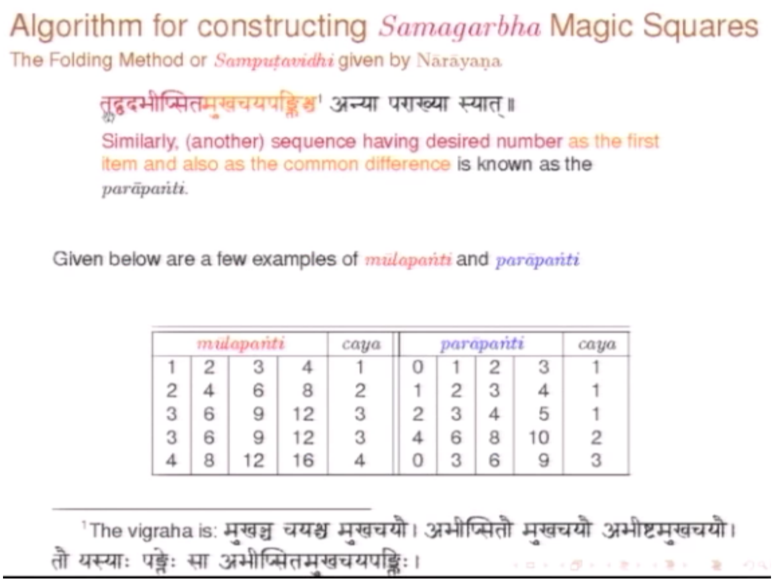

(FL) So parakya means it is called para. We call 1 as moola, the other is called para and abipshidha mukajeya, abipshidha is desired, muka is first number, jeya is difference. (FL). For example, we can have moolapnakthi 2, 4, 6, 8, we can have 3, 6, 9, 12. We can choose anything 4, 8, 12, 16, it can be even negative, so we can start with -2, 0, 2, 4, all that is fine. Parapankthi is also just anything that you like, 0, 3, 6, 9.

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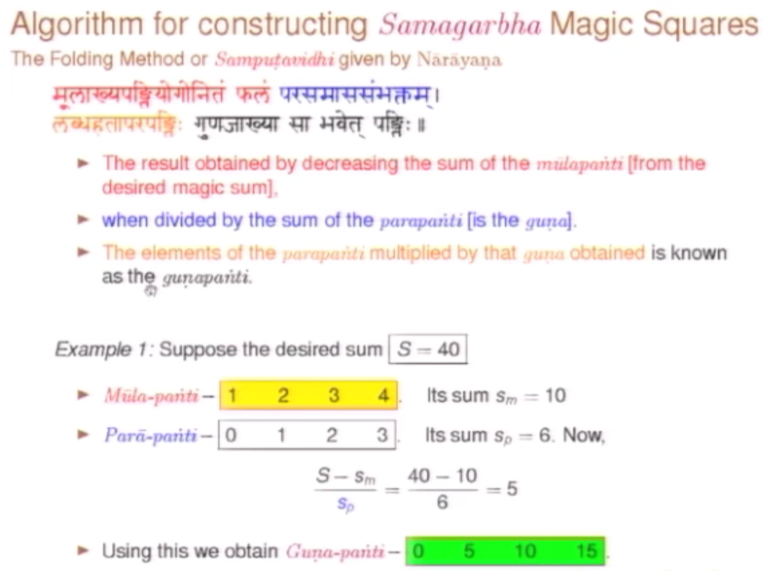

Then (FL) yoga is basically sum. So moolakthapankthi, you have a moola pankthi and you have an arithmetic sequence, yoga is sum, unitham is subtracted, (FL). It was mentioned earlier refers to the sum of this magic square. So sum is referred to as palam, so if you look at this equation, if suppose F is a sum, palam unitham, unitham means subtracted, so moolaktha pankthi unitham, suppose s, n 1, 2, 3, 4 is your moola pankthi, the sum is 10 and what we need to do is suppose you want to fine a magic square whose sum is 40, then s-sm is what is stated here.

And parasamasa sampaktham, Samasa also refers to summation, so parasamasa, the sum of the parapankthi, so thena smabaktham divided, so it has to be divided. So this is what it is and what we get 5 here in this particular case. Then, he says labdhahata parapankthihi. So this parapankthi has to be multiplied by the quotient that you obtained, (FL) That is referred to as gunapankthi. So since it is obtained from guna, so it is clled gunapnakthi.

Basically, we need to construct moolapankthi and then we need to have gunapankthi, so with them you will be creating your chadya and chadaka, then you do some puta, so you will get the desired magic square, which is pandiagonal. (FL) So having stated this, he states moola guna ke pankthi, this moola pankthi and parapankthi, so badhrardhathasthu parivruthe. Suppose we have a magic square, badharardha refers to half of it.

Parivruthe means they have to be placed in sort of circular form. So here let us take this example. So what we need to do is 1, 2, 3, 4 we have to place like 1, 2, 3, 4 badhrardhathasthu and then in the other 1, 2, 3, 4, 1, 2, 3, 4.

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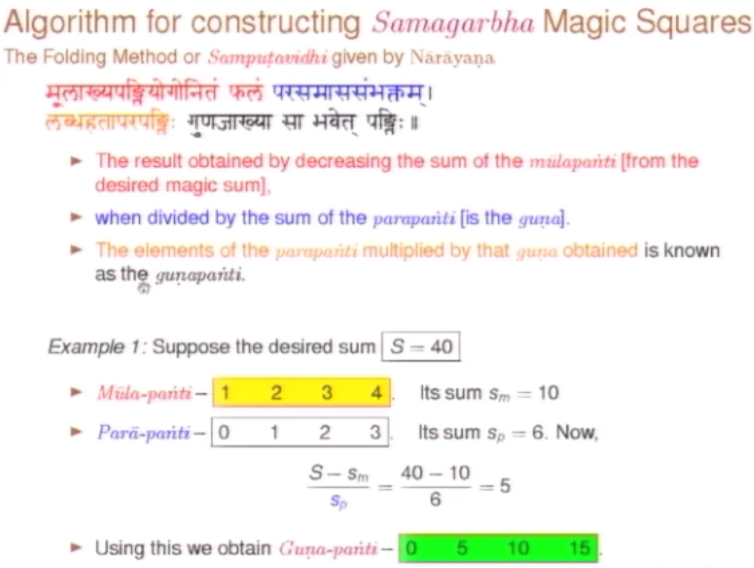

In fact, later he will say (FL). So adhye refers to this chadaka square and the other refers to chadya square. While arranging these numbers, he says we have to follow a certain pattern, so you just divide this horizontally and then place them clockwise here, so clockwise, anticlockwise, anticlockwise so in the case of parapankthi, you to have divide it this way. So when you deal with parapankthi and its guna, gunapankthi has to be.

Here we have to place it vertically divide and then 0, 5, 10, 15, 0, 5, 10, 15, 0, 5, 10, 15, so this is how they have to be arranged. So these are referred to in the verse by kramagaihi, uthkramagaihi. So krama is a certain pattern, uthkrama is the reverse pattern, so half of them has to be filled this way, half of it has to be filled that way. Here how do you know that you have to divide horizontally and vertically, so you can do this operation on this or that, does not matter.

So one of this has to be filled, broken horizontally. The other has to be broken vertically, so it will work out. So that is what he is saying thiriyakoshtani. Thiriyak means that which is horizontal and (FL) that which is vertical. That is why we have shown this way.

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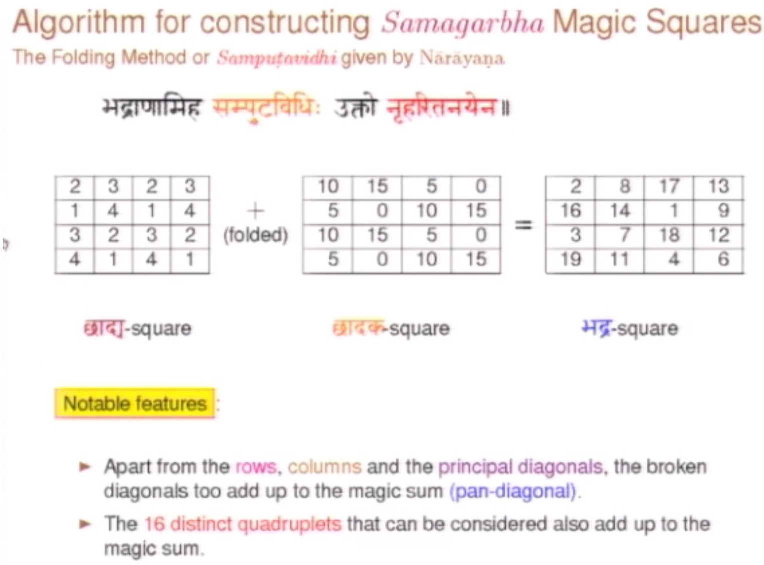

(FL) So this is how he describes Narayana as son of Narahari, so to Haridhanaya. So we had these elements placed here. So 1, 2, 3, 4, 1, 2, 3, 4, and then anticlockwise. So we have placed this. So now you have to do a samputa. This samputa amounts to 3 + 10, see that will be 13 and this 0 will overlap with the 2, so the first element will be 0 + 2 and the second element will be 5+3 8, third element will be 15+2 17 and the last will be 10+3 that will be 13.

Similarly, 5+1 so 16 and the last element will this 15+1, this actually goes as 16. So it is a folding and then at 10+4, we have 14 and then 0+1, you have 1, and the last 5+4 is 9. So this is how it is. This is a pandiagonal square. So this 16, 8, 12, and 4. This is 20, this 20 you have

40. So you start with 2, so if you take this pandiagonal 2, then you have to have 11, 18, 9. So this also gives you 40. So obviously all other rows and columns, they sum up to 40.

This is the prescription of samputa vidhi which has been given by Narayana. So suppose you choose 17, 9, and then 11, 3, so these 2 sum up to 20, these 2 sum up to 20, so we have 40. So this is pandiagonal. Suppose you choose all these 4, corner elements, see so 2, 19, 6 and 13. They form a torus. 2, 19, 6, and 13, they also sum up to 40. So this is the kind of prescription which has been given by Narayana to get a pandiagonal square of dimension 4/4.

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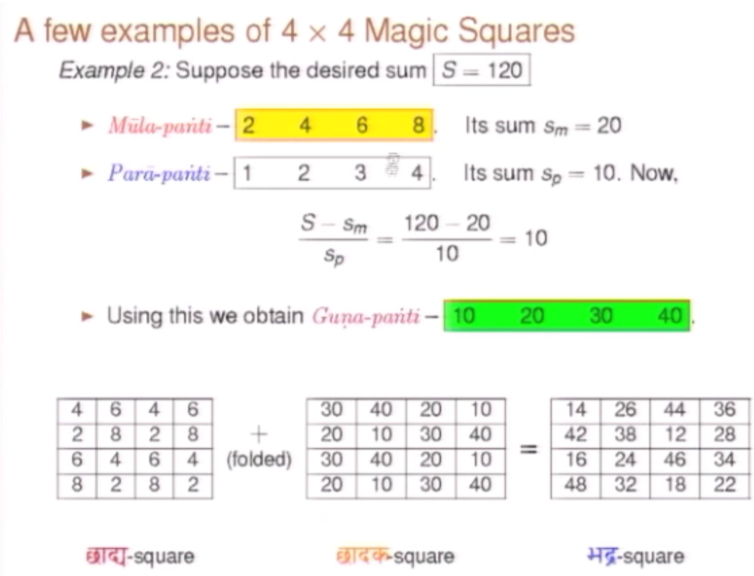

Let us just take one more example. Suppose you say sum is 120. So you choose moolapankthi to be, so it is all ista. So you can start with wherever you want 2, 4, 6, 8. Parapankthi 1, 2, 3, 4 so their sum is 20, this sum is 10 and what you have determined is guna. So the quantity to be multiplying this parapankthi, so s-sm/sp, you get 10 and gunapankthi is just product of this guna and the parapankthi, so 10, 20, 30 and 40.

So this has to be arranged as far as 1 square, chadya square is concerned. So you just arrange it, see 2, 4, 6, 8, so 2, 4, 6, 8. This top half of this you arrange in a clockwise manner, the bottom portion you arrange in anticlockwise 2, 4, 6, 8. So you arrange this way and here we have to divide it vertically 10, 20, 30, 40, 10, 20, 30, 40. So 10, 20, 30, 40 so here anticlockwise 10, 20, 30, 40, 10, 20, 30, 40. So you sum them up. So you have this square.

So this is samputa sum. So you have to remember that 10, 4 you get 14 and then 20, so 6, you have 26, and then you have 40+4 33 and the last column will be 30+6 36. Suppose you take this 22 and then 16, 38, 44 you can easily see that, this 120. So all the diagonals and pandiagonals, they will sum up to the same 120, you can choose a different. This is just to illustrate that. Say in 120, I chose 2, 4, 6, 8 and 1, 2, 3, 4 here, here 3, 6, 9, 12, 0, 3, 6, 9.

So it is all just any arithmetic progression will do and you will be able to get a different magic square which sum will be this. So you can take negative thing also, negative numbers. So any arithmetic sequence will do.

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A few examples of 4 × 4 Magic Squares

Example 4: Suppose the desired sum $S = 128$.

- *Mula-paṅti* – 3 6 9 12. Its sum $s_m = 30$
- *Parā-paṅti* – 2 3 4 5. Its sum $s_p = 14$. Now,

$$\frac{S - s_m}{s_p} = \frac{128 - 30}{14} = 7$$

- Using this we obtain *Guṇa-paṅti* – 14 21 28 35.

6	9	6	9
3	12	3	12
9	6	9	6
12	3	12	3

छाद्य-square

+ (folded)

28	35	21	14
21	14	28	35
28	35	21	14
21	14	28	35

छादक-square

=

20	30	41	37
38	40	17	33
23	27	44	34
47	31	26	24

भद्र-square

Now I move on to the next thing. So Narayana's folding method for 8/8 magic square. So when he describes this, he said samadharbe dve karve that is all. So samadharba when he mentions, it can refer to magic square of dimension 4M. So 8/8, the same thing will work out and he also said bhadhramitha. See the sequence that you have to choose should be same as the magic squares dimension, so here you have to choose 8 numbers, that is all.

So let us take 1 simple example. So this moolapankthi, suppose the sum is 260, so I want to construct magic square, sum is 260. So you just choose moolapankthi. So 1, 2, 3, 4, 5, 6, 7, 8. This parapankthi 0, 1, 2, 3, 4, 5, 6, 7. So these simple illustrations have been provided in the text itself, very simple. All that you need to do is, if you recall, there is sum, moolapankthi sum has to be subtracted from the sum of the magic square and then you have to divide it by the parapankthi sum, this is all it is.

So that prescription is same. So 260 is the sum, which is the desired sum, magic sum and the sum of this has to be subtracted from that, so divided by the sum of the parapankthi and what you get is 8 here. So this is guna. This guna has to be multiplied. See the advantage in choosing 0 will be addition, subtraction will become easier. So when you multiply, that is the idea of choosing 0 and choosing the small elements, so you can choose big number also, does not matter.

48, 96, I mean you can choose all that, but then that will not be convenient, so that is why choose small numbers. So then you get this gunapankthi. Again, the same principle, we have to divide this into half, place it in a particular way in 1 half of that and then in another way in the other half of the square. For instance, here what we have done is, you divide this in 2 halves. Then you place all this 1, 2, 3, 4, 5, 6, 7, 8, 1, 2, 3, 4, 5, 6, 7, 8, 1, 2, 3, 4, 5, 6, 7, 8, 1, 2, 3, 4, 5, 6, 7, 8.

So here it is reverse. So you have to do the anticlockwise in the other portion 1, 2, 3, 4, 5, 6, 7, 8, 1, 2, 3, 4, 5, 6, 7, 8. So this is how it is placed. So here you have to split it, vertically you have to divide it into half and then you have to place 0, 8, 16, 24, 32, 40, 48, 56. So you place it here, so you have to place it here the same way and then place it here the same way. So this half is filled. So you have to do the other way around. So you have to do it clockwise here.

So you add them, samputavidhi, so this 56+4 is 60, then 48+5 is 53 and then 40+4 is 44, so this colour will help you in identifying. So this is 32+5 is 37 and then 0+4 that is all. So you have got this magic square of 8/8 dimension.

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Nārāyaṇa's folding method for odd squares

पङ्क्ती मूलगुणाख्ये स्तः प्राग्वत्साध्ये तदादिमम्।
आदिमायामूर्ध्वपङ्क्तौ मध्यमे कोष्ठके लिखेत्॥
तदधः क्रमं पङ्क्त्याङ्कान् शिष्टाङ्कान् ऊर्ध्वतः क्रमात्।
द्वितीयाद्यास्तु तद्वच्च द्वितीयाद्यांश्च संलिखेत्॥
छाद्याच्छादकयोः प्राग्वद्विधिः संपुटने भवेत्।

Two squences referred to as the *mūlapaṅkti* and the *guṇapaṅkti* are to be determined as earlier. The first number should be written in the middle cell of the top row and below this the numbers of the sequence in order. The rest of the numbers are to be entered in order from above. The first number of the second sequence is to be written in the same way [in the middle cell of the top row]; the second etc. numbers are also to be written in the same way. The rule of combining the covered and the coverer is also the same as before.

1 more thing I just want to discuss. So Narayana's folding method for odd squares. So morning we had discussed this traditional method, so move along the diagonal and then so these verses were also discussed (FL). So now here he says (FL), almost identical. The prescription is pankthi mula guna ke, we have to derive 2 pankthi, pragvadhu sadhye. So like before you have to do this. (FL) it is only a question of arranging in a slightly different way.

So in the case of odd squares, this prescription is same. You have to obtain moolapankthi, you have to obtain this parapankthi and then with them, so he says (FL).

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Nārāyaṇa's folding method for odd squares

Example : 5 × 5 Square adding to 65

Mūlapaṅkti: 1, 2, 3, 4, 5; *Parapaṅkti*: 0, 1, 2, 3, 4

$$Guṇa = \frac{[65-(1+2+3+4+5)]}{[0+1+2+3+4]} = 5; \quad Guṇapaṅkti: 0, 5, 10, 15, 20$$

4	5	1	2	3
5	1	2	3	4
1	2	3	4	5
2	3	4	5	1
3	4	5	1	2

+

15	20	0	5	10
20	0	5	10	15
0	5	10	15	20
5	10	15	20	0
10	15	20	0	5

=

14	10	1	22	18
20	11	7	3	24
21	17	13	9	5
2	23	19	15	6
8	4	25	16	12

Nārāyaṇa's method happens to be an instance of combining two Mutually Orthogonal Latin Squares. However, it does not yield a pan-diagonal magic square as the diagonal elements of the squares are not all different.

I will quickly move on to the example. Let us choose a 5/5 magic square and let us say the sum we want is 65. Moolapankthi, parapankthi, so 65-sum of the moolapankthi/sum of the parapankthi, what we get is 5 and gunapankthi that you get is. So multiply this guna by parapankthi, you have this. Now how do you arrange them. So the arrangement is done this way. See if you look at the verse, (FL) the middle 1, you start with that.

So if you divide at some point, so all of them will be urdhuva, adhimaurdhuva means the top row. In the top row, choose the middle 1, and then start. How do you start writing? (FL) So you have to just place the sequence 1 below the other. So you had the sequence of moolapankthi. So you arrange them 1 below the other. So then you move on to the next 1, so there you have to arrange it 1 below. So you have to arrange it this way and then, here you have to arrange.

So once you come here, this will go to this side and then this will complete this. So this is the arrangement that he prescribes. The same will be for the chadaka square also. Start with this and then go, now you add this. This kind of method, which has been prescribed here does not yield a pandiagonal magic square and it can be converted to a pandiagonal by slightly rearranging this.

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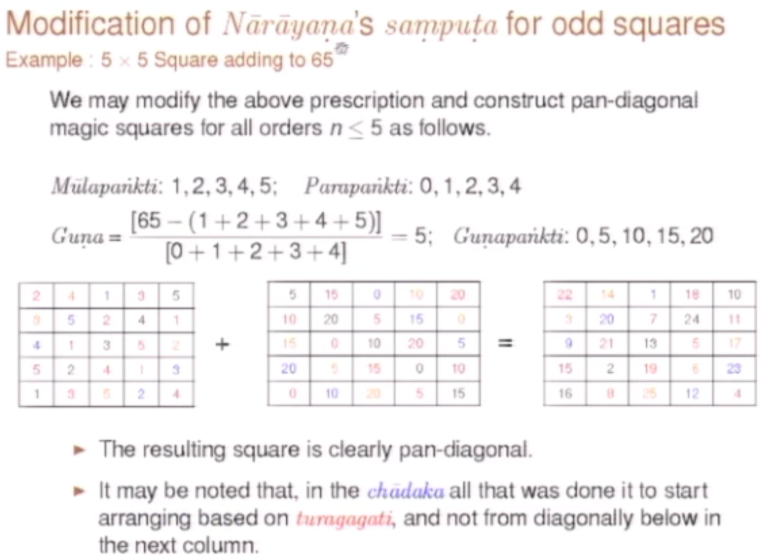

So modification of Narayana Samputa for odd squares. We choose the same example and so we arrange this in the same manner. See with moolapankthi, it has to be arranged in the same manner, but here when you arrange this chadaka to obtain this chadaka square, what to do is, you start with this, but then the reason that we have here is, see all the diagonal elements are the same and so all that has to be done is to slightly shift it.

It is a sort of turagagati, you start and then place here 0. Turaga, they are shifting, and then you add that, so you will be able to get this. 22+2, 20 here and then 2 22. So now let us see. We have this 14+3 25 and 6+17, so 17+3 is 20, 25 is 45 and this adds to 20, so you get the sum 65. So this is a pandiagonal square. So this chadya/chadaka generation is identical, but it is only the arrangement, so 1 can play around with magic square and then invent something new also.

This is all it is. So this is recreational mathematics, so with a couple of references, I will just conclude my talk on magic square.

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So this Ganithakaumudi of Narayana Pandita, as I was mentioning the earliest text that is available, so which has an elaborate discussion on magic square. So it has almost 75+ versus, discussing magic square. We saw a few versus right at the beginning as to how Narayana was. So clearly stating, what he wanted to do and the preliminaries were mentioned. This is the paper which I was referring to, professor Vijayaraghavan on Jaina Magic Square and this was referred to by Professor Srinivas also during his talk.

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So Kusuba, he has taken a couple of chapters of Narayana Pandita, so chapters 13 and 14, 13 deals with combinatorics and 14 deals with badhraganita. So this magic square and this was his Ph.D. dissertation submitted in Brown University and Paramandha Singh has also brought out English translation. So this book Ganithasarakaumudi also has a small section. So it does not deal with in great detail.

It does not discuss pandiagonal magic square, but it has a brief discussion on magic squares and Thakkura Pheru, this is the earliest text that we have and if you note, Sakhya refers to Sarma, Kusuba, Hayashi and Yano. So all 4 are friends. They have coined a nice term Sakya. Professor Srinivas and Raja Sridharan had also authored an article on this. So these are the references.