Mathematics in India: From Vedic Period to Modern Times

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Lecture - 28
Magic Squares - Part 1

So, this topic on magic squares will be dealt with in 2 lectures. So, today we know that there is a lot of research that is going on recreational mathematics. So, this actually forms a very interesting candidate so to be presented to students so in order that they cherish what they do particularly in the field of arithmetic. So, the topics that I would like to cover so in this part of the lecture is classification of magic squares.

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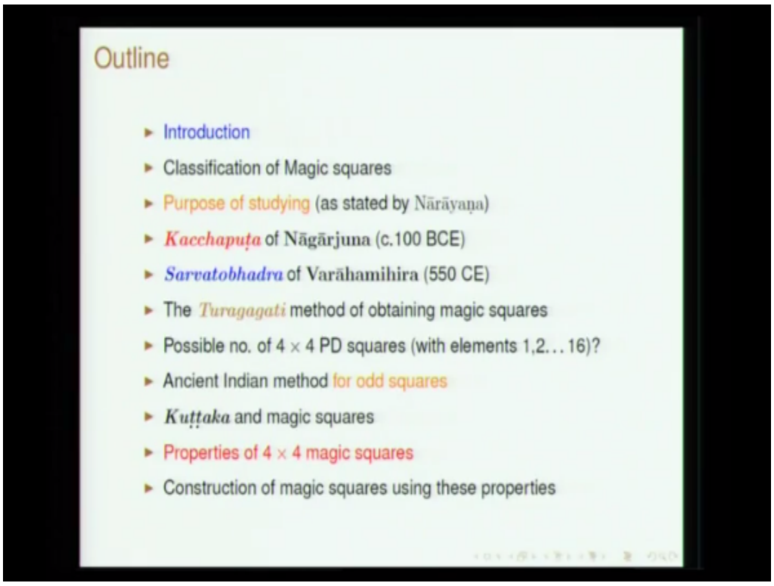

So, as it is done in the Indian texts, so and I will briefly discuss the purpose which has been stated by Narayana Panditha. So, as to why one needs to study the magic square and we will take a couple of examples of magic squares from very ancient texts for instance Nagarguna work. So, it is actually referred to as Kacchaputa. So, the term Kaccha in Samaskrit refers to tortoise. The shell of the tortoise will have a some kind of a hexagonal kind of a type.

So, may be so the magic square the numbers are represented in various cells, which resembles that and perhaps it is why it is called Kacchaputa. And we will also see an interesting magic square which has been presented by Varahamihira and it is Sarvatobhadra. So, bhadra is the name of magic squares. So, in fact the mathematics of magic square is called bhadrav ganitham okay. So, then we will discuss the horse motion method, horse move, turagagati.

So, turaga, turanga, turangama so all the three words refer to horse in Samaskrit. So, turagagati method of obtaining magic squares, in fact Narayana Panditha discusses this at great length at the beginning of his chapter on bhadraganitha and then he moves on to various other ways of constructing magic squares. So, we will start with 4 by 4 magic square, which is actually called samagarbha and PD here refers to pan diagonal.

So, we will quickly see what pan diagonal means and how many pan diagonal magic squares can be constructed. So, this is an interesting question which has been posed by Narayana Panditha in his ganitha gomathi and he also gives an answer so thus 384. We will see how it is and we will also discuss the ancient Indian method of constructing odd squares. So, then the relation between Kuttaka and magic squares, then we will see some of the properties of 4 by 4 magic squares.

And then we will see how from these property one can construct magic squares elegantly.

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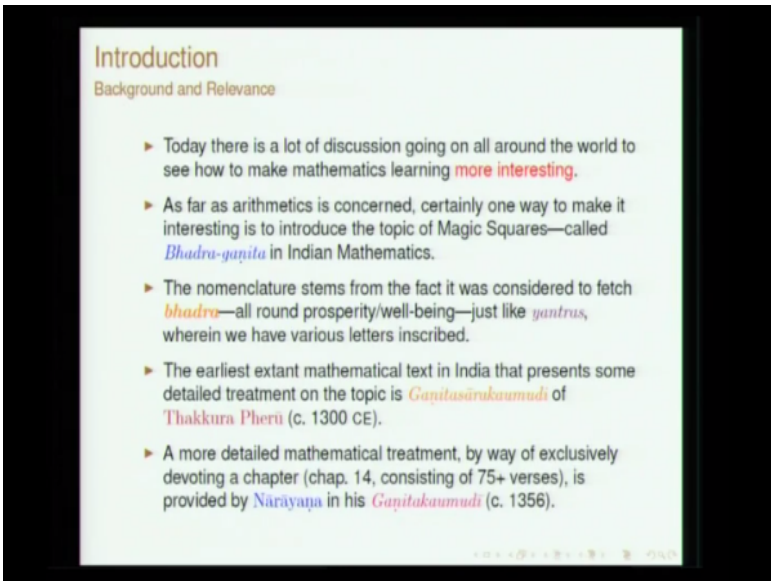

So, this is how we will progress. As I was mentioning magic squares is a very interesting topic so particularly children will definitely enjoy and unfortunately it is not something which is a part of the current curriculum. So, it seems lot of things can be done. So, from what one sees so at great length Narayana Panditha discuss in fact this chapter contains 75 + process.

So one complete chapter has been devoted by Narayana Panditha. So, the term bhadra in Samaskrit means an alround well being prosperity kind of a thing.

Magic square perhaps, I mean there are certain magic squares which one will find in temples and various inscriptions and so it is something like yanthra perhaps. Yanthra, you will have some aksharas inscribed in that. So, in magic squares you have some numbers inscribed in that. So, may be it had that kind of a significance one does not know and that could be the cause for calling it a bhadra ganitha. Or to be bhadra ganitha in order to protect oneself as Narayana says so among mathematicians you can post this question and it is not able to answer you save yourself.

So anyway so that could be the bhadra ganitha. So, a detailed exposition of mathematics of magic square or classification and so on method to construct is all found only in 13th, 14th century texts. So the earliest text is one of thakkura pheru, it is called (FL) and then Narayana Panditha's work is called (FL). In fact, somebody was asking me some time ago so what does this word kaumudi mean. Kaumudi normally refers to a text, which deals with grammar, but in fact the word kaumudi actually means moons light okay.

Ganitakaumudi, the text actually means it is a moon light so mathematics in the form of moon light perhaps. So, that is how the name ganita. (()) (05:16). Ganitakaumudi in fact the term kaumudi refers to moon light and this sisdhantha kaumudi, so these text came much later and this kaumudi is a general term, which is added to a certain text which is presented so lucidly kind of a thing okay. So, it is quite difficult to understand. So, the original text of Panini and therefore this kaumudi so the grammar presented in the form of a moon light.

So that is how it is. So, before proceeding further I would just like to introduce you, certain terminologies.

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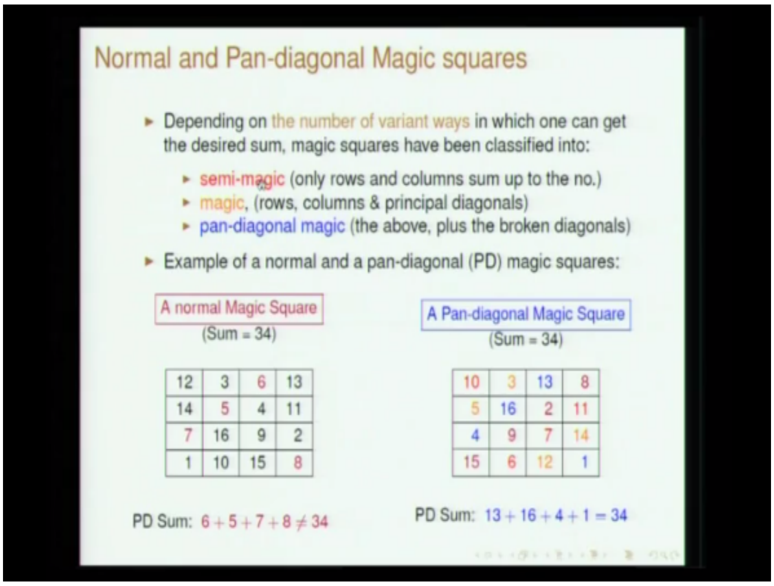

So, this magic square can be broadly divided into semi magic, magic and pan-diagonal. So, semi magic so where in only rows and columns sum up to a given number. So not even diagonal. So, if the diagonals also, the main diagonals okay also sum up it is a magic square and pan diagonal is something where in not only the main diagonals but even the other diagonals sum up to the given number.

So, for instance we just take this example so here, if you look at this so 6+5+7+8 so once you come across, you have to move here so that does not sum up to 34. So, this is basically a magic square where in all the rows and columns and the main diagonals they will sum up to 34 but these Pan-diagonals will not sum up to 34. But again the same elements actually fill up this magic square so this is a Pan-diagonal magic square where in you can see that 13+16+4+1 it is 34 or you can take 14+12+5+3 is 34 or 10+6+7+11 so all of them sum to 34, which is what is called Pan-diagonal magic square.

Indians are actually specialized in constructing Pan-diagonal magic squares and as will see Narayana Panditha presents various methods by which one can obtain Pan-diagonal magic squares.

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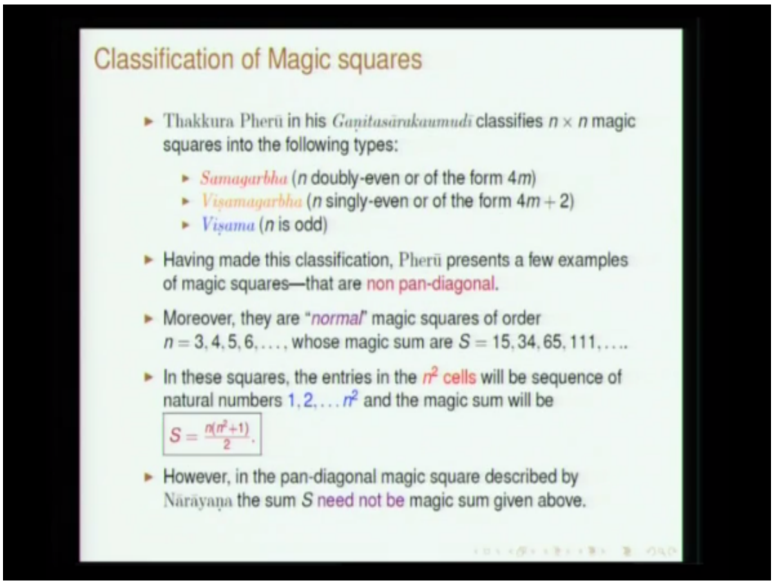

So, in the text of Thakkura Pheru as well as of Narayana Panditha, what one finds is the same kind of a classification. So, Samagarbha, Visamagarbha and Visama so these are the names which are given. So, Samagarbha actually refers to a n by n magic square, so it is of the type 4m so m is an integer so it can be 4,8 and so on divisible by 4 primarily and this is Visamagarbha, so this is also sum up but 4m+2 even and n is Visama any odd, it could be 4n+1, 4n+3.

So, the magic squares which have been presented by Thakkura Pheru in his ganitha sarakaumudi are of non pan diagonal nature, where as the one presented by Narayana is pan diagonal and he actually gives a examples of few magic squares so where in n = 3,4,5,6 all of them have been given by him and one will start with so 3 by 3 square one goes from 1 to n square so all the 1 to 9, they will fill up that and then we will get a magic square. So, there the sum will be 15 and if you have a 16, 1 to 16, then the sum will be 34 and then 65 and so on.

So the magic sum will be n*n square +1/2. So, Narayana actually gives a certain procedure where in one need not consider these elements. So 1 to 16 to construct a 4 by 4, you can give any number and you will be able to construct a magic square which is pan diagonal in nature. So, it is a kind of systematic procedure which have been presented by Narayana.

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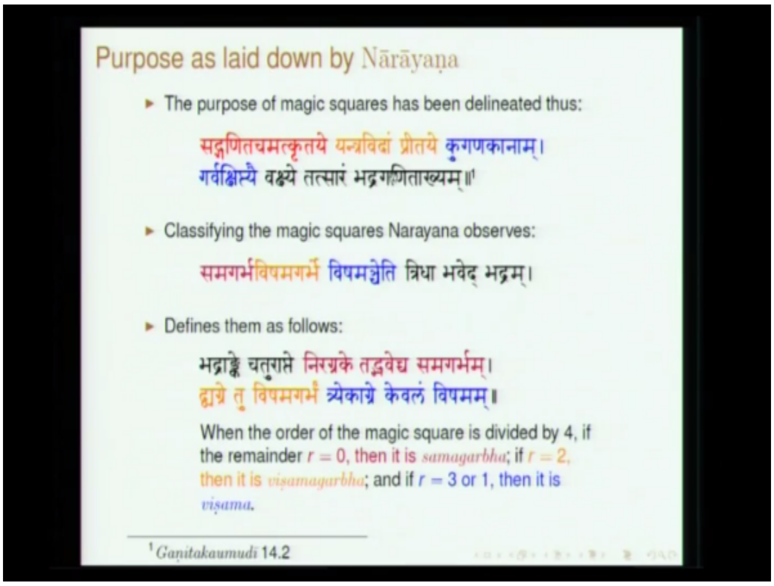

SO, before we proceed further so I wanted to quote a couple of verses, so which have been stated by Narayana right at the beginning of the chapter. So, he says so (FL). So, he has clearly stated out 3 reasons. One is (FL), if you are a good mathematician, so when you will enjoy so (FL) or you can excel in a certain forum by knowing this technique. So, (FL), so various kinds of permutations and combinations, the various ways of arranging things etc can be known.

And therefore preethi they will be pleased to know in this techniques and he also says (FL) shithyai, so, (FL) means so in order to knock of the arrogance of (FL), so, this is being useful (FL). So, (FL) then (FL), see one thing which is very interesting about Narayana is, he presents everything so systematically. So at the beginning of the chapter, so he presents the whole summary and whatever that is required in order to understand.

So he says (FL) so, this is the definition of samagarbha, chathurapthey means when it is divided by 4, so when it is divisible by 4, you call it as samagharbha, (FL) by this time most of you should be familiar with the word agra, agra is reminder, (FL) if 2 happens to be the remainder, when you divide by 4, so then it is called Visamagarbha. Here 3 or 1 (FL) you call it Vishama fine.

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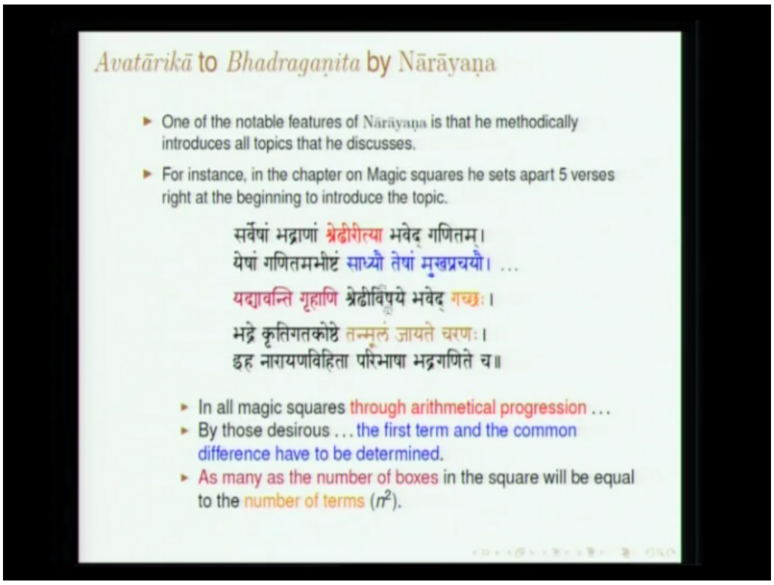

Then (FL: From 11:19 to 11:24), so this is an interesting statement and as you will see, he says, so all the magic squares, so that we will be discussing. So, the fundamental thing which will be you made use of is a certain arithmetic progression. So, this arithmetic progression is referred to as (FL) is a sort of step, so it increases by D, so that is why it is called (FL). So, (FL) you make use of a certain arithmetic progression and then you do the magic square.

So, therefore he says (FL), those who are interested, so in doing this magic squares, so, (FL), if you have to construct a magic square, you have to first of all find out a, see given a certain number s, so the magic sum has to be s, so for that, you have to find a certain appropriate arithmetic progression which into the square, you will be able to get the magic square. So, therefore he says (FL) is the first term, (FL) is the common difference okay by which it increases.

So, (FL), see the term gruha when it is used here refers to the cell in a magic square, when you draw that okay, so that particular cell is called gruha so in a 4 by 4 magic square, we have 16 gruhas okay. So, (FL) so that is termed as (FL) so if you recall in aryabhatiya also, it refers to the number of terms in a sequence, (FL) means the number of terms in a arithmetic sequence, which you have to deal with.

So, (FL) means so if you have 4 by 4 magic square, so the number of elements will be 16, so root of that will be the number of rows or number of columns. So, that is called (FL) is basically a row. So, then he says (FL) in fact so in kaumudi, you have the first (FL) is called

(FL) is all that is required terminologies have been clearly stated now. Now, let us get into the magic square.

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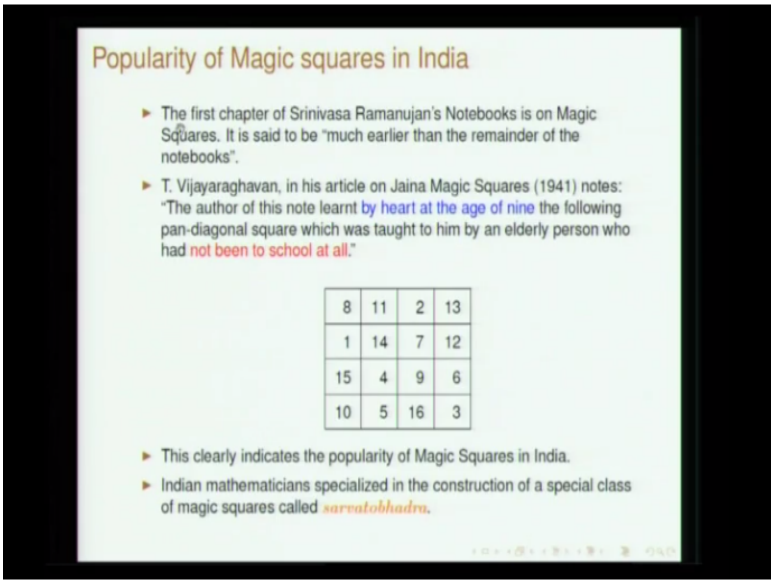

So, in order to understand the popularity of magic square, so in fact what one notes is, the famous mathematician Ramanujam, in his notebook, he starts with so magic squares okay and he will be referring to Vijayaraghavan, he has worked out some very interesting properties, so which we will be discussing little later.

So, he in his article on Jaina magic squares in 1941 he notes, the author of this note learned by heart at the age of 9, the following pan diagonal square, which was taught to him by an elderly person, who had not been to school at all. In fact, some of this interesting mathematics based on the Leelavathi and this kind of the problems, I remember when I was I think in primary school or so, a old man came, and then he was quoting some problems.

So, later now when I study this Indian mathematics, I realize so kind of problems that he was giving was sort of based on this Leelavathi. So, it has been sort of available so the construction of magic square is also, so many people know. So, he says by a person who had not been to school at all okay. So, it has been a part of tradition as a recreational tool so magic squares has been there.

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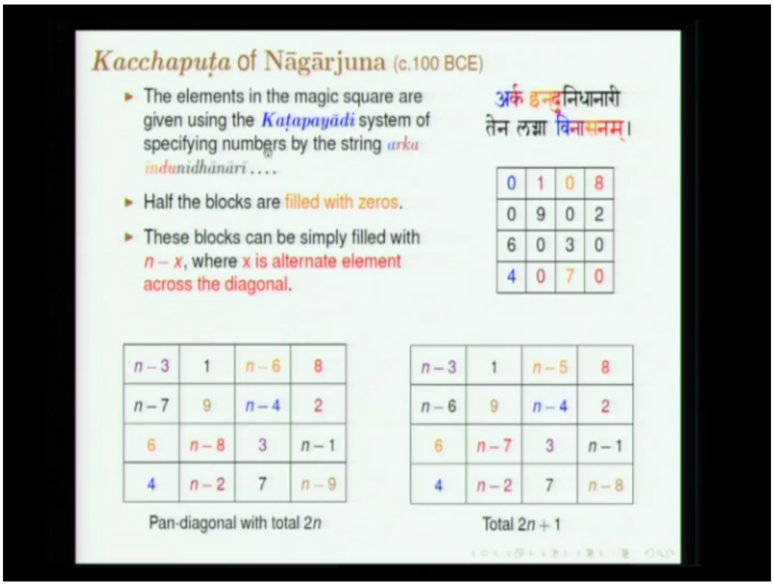

This Kacchaputa of Nagarjuna so there is a very interesting pneumonic so look at this (FL) there is typo here (FL). Now, I would like you to recall the (FL) system that we discussed so now, if you look at every syllable here and then try to translate it into this form of a magic square, you will see that he refers to 0, ka refers to 1, ee refers to 0. In katapayadi the system, all vowels refer to 0 and gna and na also refer to 0.

So, therefore if you see here, so na, hindhu, so dha refers to 8 ni dha na re ra is 2 and then (FL) the na lag na see vee naa san am. So, thi pneumonic I mean what is the significance of this pneumonic, see the point is see here, in this magic square, you see certain cells have been filed with 0 and certain cells are in fact, it is exactly 8 cells are 0, 8 cells are filled with numbers. And this cells which have been filled with 0, so can be filled with, for instance if you look at here, so, this cell is 0.

So 6 is here and I put a n-6 here and you find 0 here, I put n-2 here so since 2 is there, so if you sort of fill it in this way, you will see that so this actually turns out to be a pan diagonal magic square, whose sum is 2n. so, that way I have created very interesting pneumonic with whatever be the number that you want to get. So you will be able to construct a magic square. So, once you remember this pneumonic, that is all, that is one thing.

So, here if you want to get the sum as 2m+1, so then, all that you need to do is so, instead of n-8, we have to have n-7 so appropriately filling this. So, you will be able to construct a magic square provided you know this pneumonic in your mind that all. (()) (17:29) alternate element in the sense, if you see here, n-2 and 2, that is y I have put. So, 6 and n-6 so that is why I have put, so n-3 and 3, diagonally okay.

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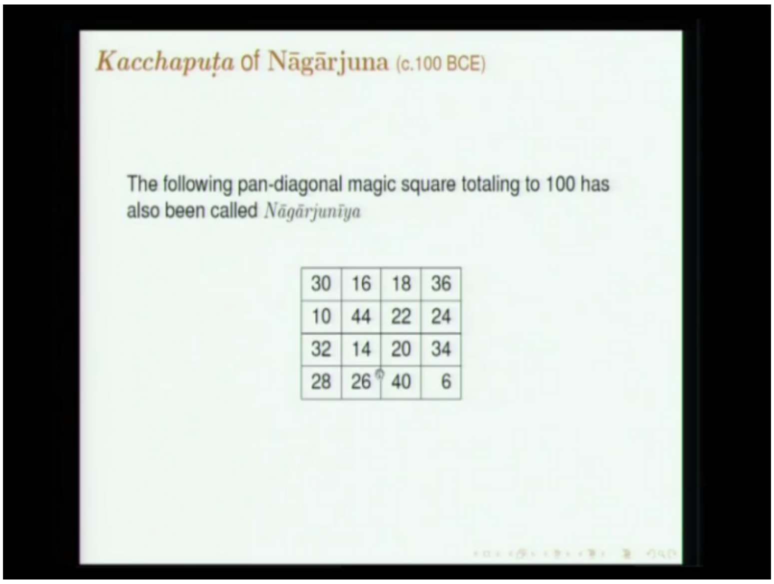

So, this magic square whose sum is 100 is also do just is convey you the kind of antiquity so we just have this, it is called Nagarjuniya associated with Nagarjuna. So, now I move on to the magic square, which has been presented by Varahamihira in a different context.

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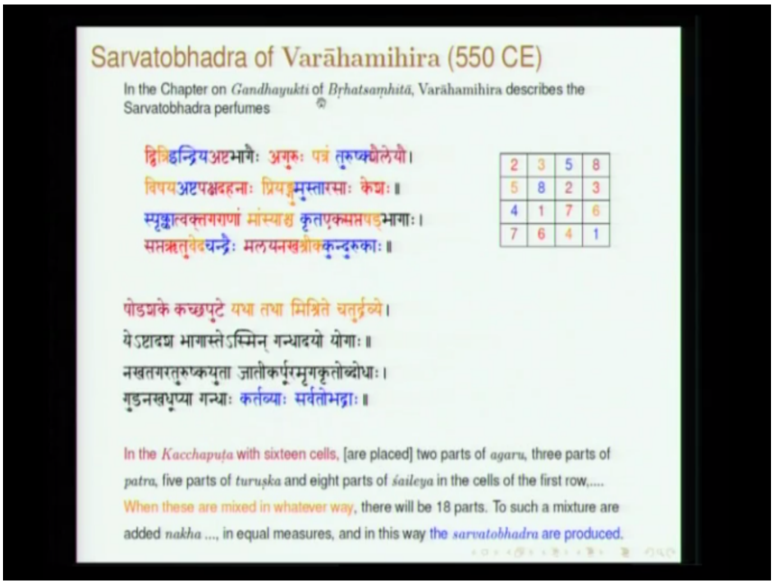

Varahamihira does not discuss bhadraganita per say but, in the context of (FL) is a huge compendium in which almost all topics will be found starting from geology to earthquake and various other rainfall and so on. So, here we have a chapter wherein he speaks of the mixing of various things and here we find this versus (FL) is sense organ, which is 5 in number ashta is 8, (FL) so the other half of this other quarter of the verse basically refers to the names of certain elements okay, so which will be used for producing say fragrances.

So, (FL) so they are supposed to be taken in this kind of proportions. So, then (FL) so dhahana is higher so which is actually 3 because of (FL) and these elements (FL) in one quarter of the verse is basically gives the number, other quarter he lists the elements. So, now in the set of two versus, so you have 8 quarters, half of them is number and half of them. So, 16 elements have been listed out here. And that is why he is calling it as (FL) so, it is a sort of a magic square, where in 16 cells are there, (FL) as you want, you can sort of mix them.

So, as you want in the sense, so this is a (FL) very interesting magic square, which is pan diagonal, if you look at this so, 5, 8, 4 and 1 they sum to 18. So, anyway you sum up so you take up any of these 4, so you take this 4, you take this 4, you take this 4 so all of them will sum up to 18, that is why he is saying (FL) so you can sort of. So, the point he is trying to say is so, if this final magnitude has to be 18, so you try to mix up various things, various elements, so that the total measure is 18.

And try to find out so which kind of a thing comes out to be I mean it is in that sense I mean he is trying to present it. So (FL) and in fact (FL) goes on

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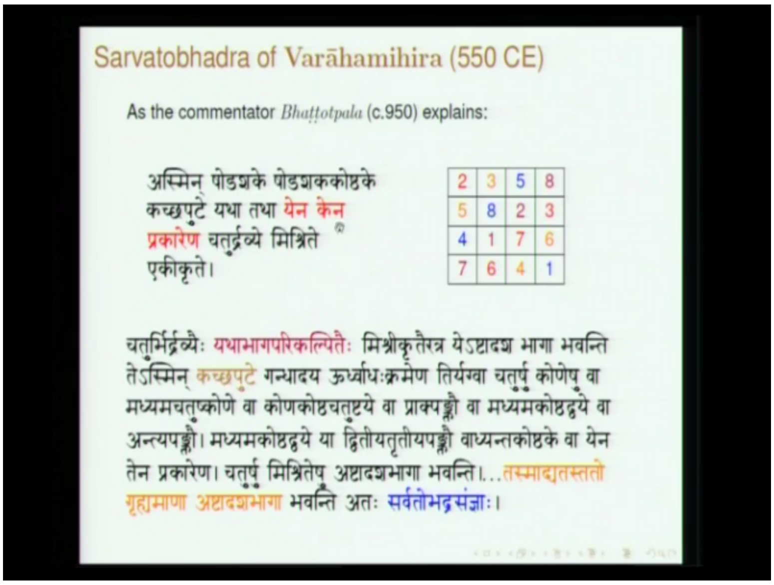

To comment on this, (FL) so, he is trying to explain so this verse. So, this is a translation which you can read at leisure.

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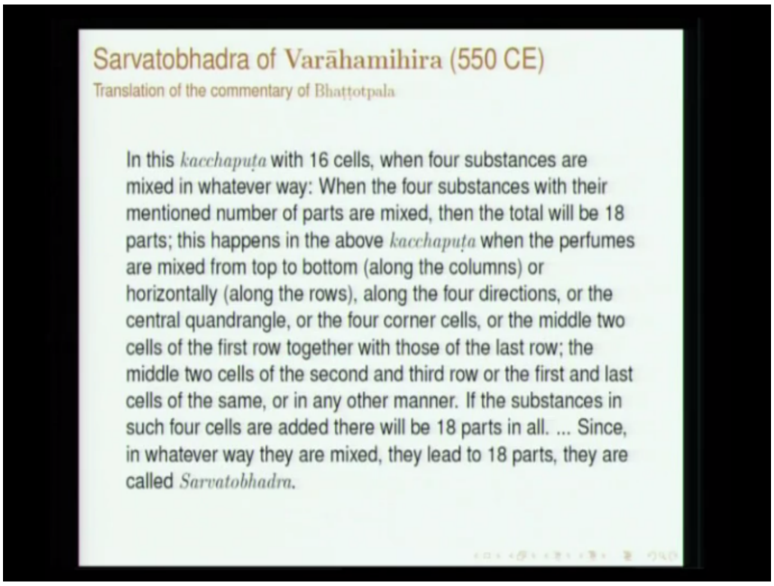

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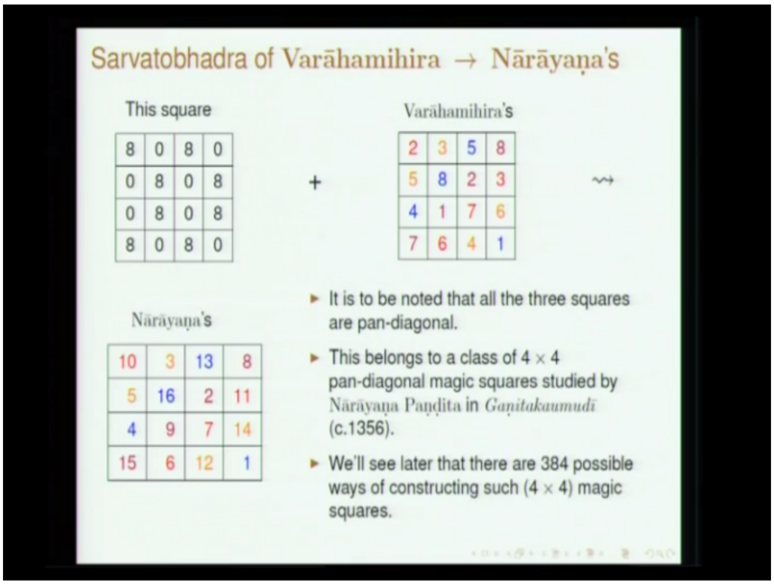

This magic square of Varahamihira, so, when it is sort of added with another square like this, so you will get this. So, in fact Narayana considers in great detail so, the elements is 1 to 16 and you fill this magic square and this 1 to 16 when it is sort of filled up, so what are the various ways in which it can be done. So we will show a little later as I was mentioning so it is only 384. So, the point I am trying to say is this Varahamihira's magic square added to this will give one of these 384.

So possibilities of the 4 by 4 magic square, which has been discussed at great lengths pan diagonal magic square which has been presented by Narayana.

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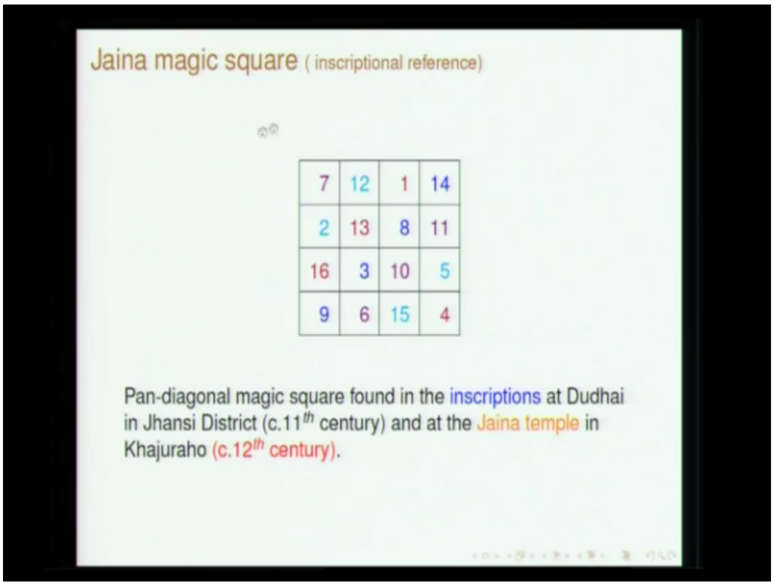

We also find the magic square in inscription. So, for instance in this Jhansi District in the Jaina temple you find this magic square and this magic square is again one of the 384. So, if you have a pan diagonal magic square, it has to be one of the 384, which Narayana discusses at great length.

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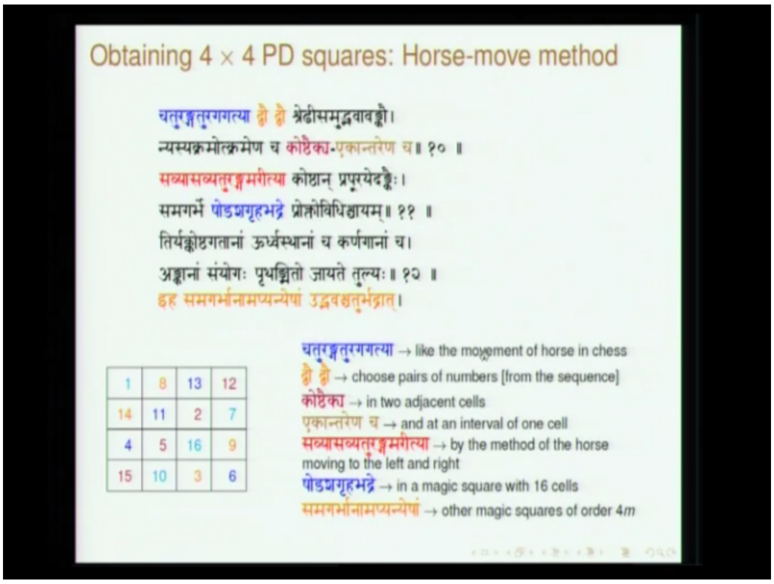

So, the first procedure, which has been discussed to obtain magic square is one of the (FL), so (FL) is a sort of chess kind of a game. So, he says (FL) that is all. So, these are the two versus 10 and 11, which actually present the way to construct a magic square by (FL). So, this is what he has stated. And then he says (FL). So, this basically explains that if you constructed this way, so then you will see that so, this will be a pan diagonal magic square.

So, how does it come out (FL) means, so suppose this is what is called (FL) you understand horizontal thing is called (FL) okay and the other thing is (FL) is up so vertical. So, whether you sum the cells horizontally or you sum the cells vertically (FL) then (FL). So, he does not say main diagonal, so all karnas okay. So, whether it is pan diagonal so that is how it turns out to be karnaganam, if you just add it like this so then also, so 9+3+14+8 or 1+10+16+7 or if you do it this way 8+2+9 and then 15 so all of them will sum to 34 okay.

That is what he is saying (FL) means all of them will sum to the magic sum. So, (FL) now let us try to understand so you closely try to follow this, this (FL) so you start with 1 in the left top most, then (FL) where do place 2, so you jump here, this will be 2 fine, this is a horse move, (FL) so read this verse carefully, (FL) the word (FL) has to be understood very carefully. You have 1 to 16, so 1 to 4 is one thing and this 4, you split into two, so, 1,2 and then 3,4.so, this (FL) is applicable for pair of them.

So in fact uses the word (FL) kind of a thing okay. So, 1,2 is (FL) 3,4 will be (FL) but not 2 and 3 fine. So, this will be true for every pair of 2's okay. So, of the 4, 2 will have (FL), the next 2 will have (FL). So, that is how it is. So, 1,2,3 and 4 fine, then (()) (25:44) very good. In fact, that is how this 384 comes into. I will explain that is very simple very interesting okay. 1,2,3 and 4. Now, let us start with 5 and then 5 you have to get 6, you get it this way, this is (FL), then 7 and 8.

If you closely follow this, you will see that 1,2,3 and 4 has been done in a sort of (FL) way okay. So, it is a sort of (FL) in the sense, you go in a sort of circular way. So, you will see we have 5,6,7 and 8. Then you consider 4 more, next 4. So you have to start with 9. So, start with 9 so and then so you have to get (FL) 10,11 and 12. So, this is again sort of cycle clockwise and then start 13,14,15,16 hogaya. So, this is how this (FL). (()) (26:56).

That is precisely why I am saying otherwise, you will not have this multiple choice. If it is sort of fixed, that is why we have this 384. I will explain why this 384. It is pretty simple now. Once you have understood this. So, pairs of them, you have to split into 4. So, this 4 has to again taken as pairs. So, pairs of them should have (FL) this is all. No more constrain on this fine. So, now so I will answer this question now.

So you start with (FL) 2. So, how many possible ways I can place in (FL) 2? So having placed 1 here, so I can have only 4 ways. So look at this, so this is one way, the second is to place at his place, thi is also (FL) then the third possibility is you can go up and then go there so that will be mapped here fine. So, it is a sort of torus. Other possibilities you move on this side, so you can go here and then you will reach here, this will be mapped there.

So, between 1 and 2, you have 4 possibilities. There are 4 ways in which 2 can be placed given this, you are placing the one at the top most left cell. So, having given this, so there is no constraint on where you place 3, that is why I said. So 3,4 has to be (FL) but where you place 3, I mean it is left to you. So, now having placed 2 in this place, so here Narayana has hinted it. See (FL) means 2 cells which are close by or (FL) is one difference. So, the 2 cells can be close by in choosing 3.

So suppose let us say we started with 2 here, now 3 can be placed here, here or here. There are 3 possibilities fine. So, with one there were 4 possibilities to place 2 having placed 2, there are 3 possibilities for placing 3 so 4*3 you get 12. So, now having placed 3, so 4 anyway is fixed by (FL) after placing 4, now you have to place 5. So, there is no constraint on where you place 5. So, look at this, so suppose I have placed 3 here, so 5 so 2 is filling here and 5 can be placed either here or here.

So, there are 2 ways. So 4*3*2, there are 24 ways in which. So, once these 4 things are fixed, so the rest of the square actually gets fixed. So, there is no way that you can play around and therefore see for 1, if this top most cell you have 24 possibilities and 1 can move anywhere of the 16 cells and therefore 16*24 you get 384 possibilities. So, this actually has been beautifully explained by Narayana. So, by drawing 24 and then he says so one can go anywhere of these 16 cells and therefore you have 384 possibilities.

So, right at the beginning, in fact he poses this as a question. So, and you can see that the greater part of the verse also (FL) is left (FL) is right okay. So, moving left and right kind of a thing. So, (FL) so left and right in the sense so having placed 2 here, we can place here to the left, to the right and so on. So, it is this kind of a thing. And even this (FL) can be taken as (FL). So if you play around, we will be able to see all that okay.

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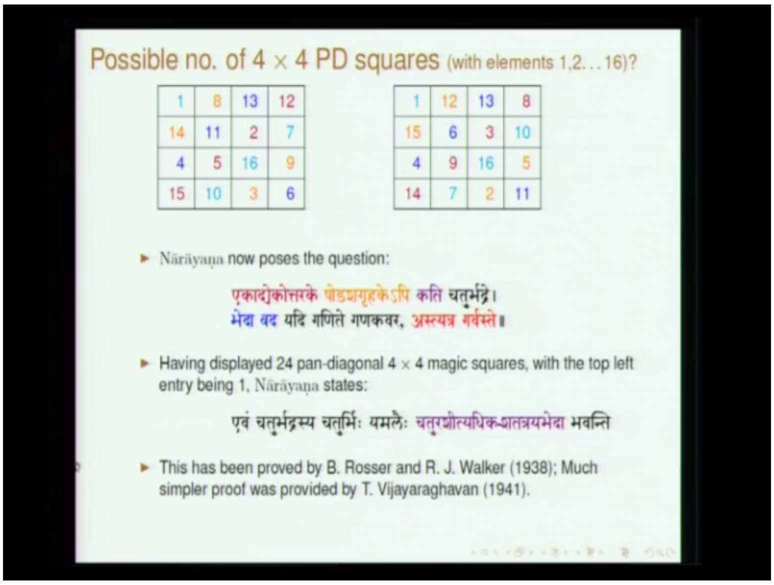

In fact, the verse which actually poses this question is as follows (FL) vadha means way you tell me. So, (FL) how many possibilities are there. So, (FL) means the first term is 1, (FL) common difference is 1. So, you have this 1 to 16 so in (FL), so in a 4 by 4 magic square, how many possibilities are there. So, (FL) if you feel that you are knowing enough mathematics. So may you tell me so, how many possibilities are there okay.

And then after discussing he finally concludes (FL) is 84, (FL) is +, (FL) 100. So, 384 possibilities are there. So, this is how. In fact, this has been proved by Rosser and Walker in 1938 and a much elegant proof has been provided by T. Vijayaraghavan. We will also discuss the properties which have been stated and based on the properties you can also construct. See, based on the properties, if you want to construct, this kind of a magic square, then you have to do addition, subtraction quickly mentally you can do that.

These are all small numbers you can do that. But, if you do by (FL) you can sort of mechanically quickly place it and then play around with that. This is a very interesting tool to play around. So, to construct odd squares.

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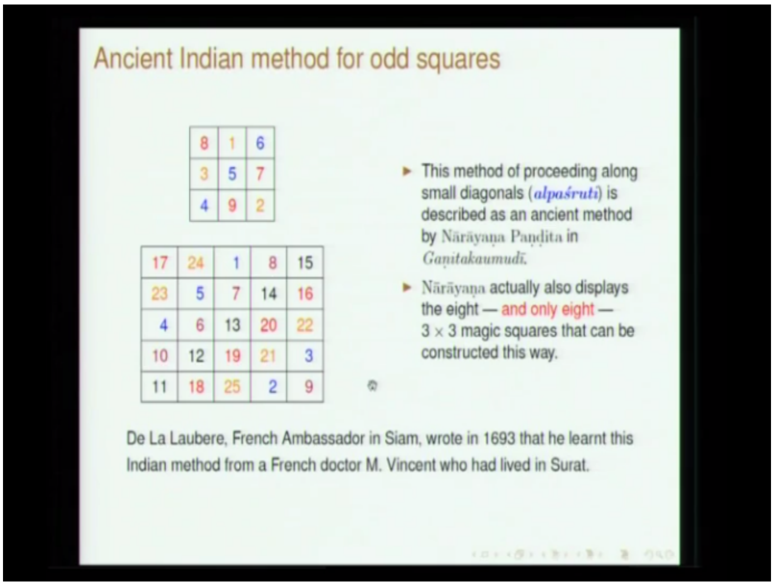

So, this is a even square in fact once we have 4 by 4, so to construct 8 by 8, so to move on to the higher order of some other bha, so you can start with this 4 by 4 and then once again play around like this (FL) you will be able to get that. So, we consider one example little later. But here another interesting method has been stated which has been discussed by Narayana himself. So, here I will not get into this versus but I will quickly explain so how one gets this kind of magic squares. So odd square. So, this is I think more popular 3 by 3.

So, we can have only 8 possible squares. May be I will just work out one example. See here you start with 1 and then the prescription is diagonally move up okay. If there is a cell to fill up, you fill so otherwise it is a sort of torus you move down. So where do i place 2? I place below, because there is no cell here. So, there is no gruha to occupy. So, this is like a torus, you map it so it comes here. Again you have to move diagonally up so you have to move here. So, since there is no cell here, it is like a torus so you have to place it here.

So, since the cell is already occupied with 1, so what you have to do is, you have to just move down okay. So, 4, you place 4 here, then diagonally keep filling 4,5,6 hogaya fine. So, at 6, so you have to move up here and this will be actually mapped to this point 4 here so in a torus but this is already occupied and therefore you have to place 7 below. So you go up so this will be mapped to this 8 and then so you go up so this will be mapped to this point 9.

So this is how you get this magic square and the same technique can be employed for any odd magic square. So, maybe I will work out one example in the board so that it becomes more evident. So, for instance, so let us take a 3 by 3. So, I started with 1 here so In the example which you find in the slide, so maybe we will start with 1 here. So, you have to move up one step so where will I place? So this will get folded here so I have to place 2 here.

So, then so I have to move up so this will get mapped here so 3. So, here I have to go up this way and I will tell you there is trick here. So, here in fact Narayana's verse very clearly tells so that is why I took this example. The prescription is the following may be I will just see if the verse is there. So that it becomes more or less like nail fixed on a green tree.

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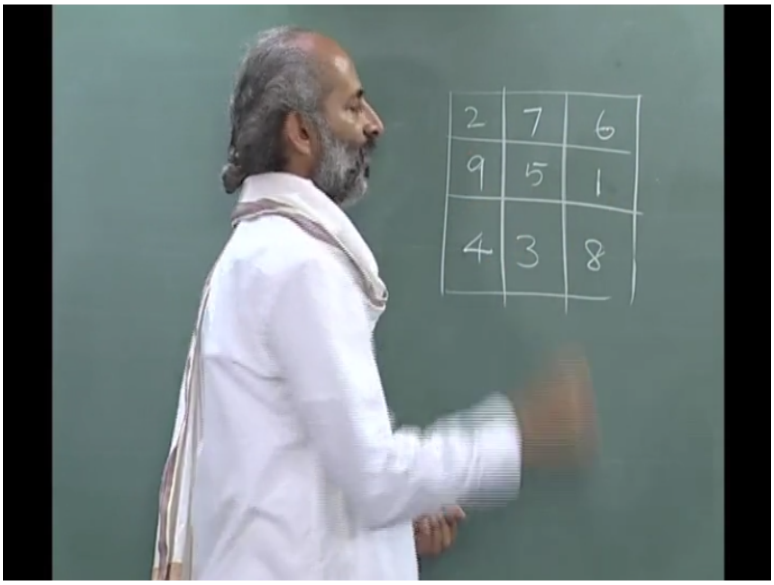

See, so (FL) aasha here means direction. So I will come back to this example. So, I will explain this couple of words here and how clearly Narayana prescribes. So, (FL) so, what I did was (FL) shredi is basically arithmetic progression, it need not necessarily be this number. Any arithmetic progression will do, but this is a simple example. So, (FL) I have fixed it, then what Narayana says is so, (FL).

So, (FL) prathyasha means in the opposite direction fine, so now, look at here so since I have hit it here, so, this is the direction in which I started so in fact (FL) is direction. So, in this example, which I have shown in the slide. So I started with this so and then we moved on so this was so I placed 2 here so I placed 3 here. So, when this was occupied, so what I did was I placed 4 here. Do you understand. So I started in this direction and therefore I had to move in the opposite direction, Prathyasha.

So, here I started in this direction and therefore I have to, when I hit here, so I have to place it here understand. So, then 4,5,6. So you go 7 here so this place actually maps to this place so it is a sort of torus which is filled so and then I have to place 7 here because it is the prathyasha direction. So, 7 you go here so this gets mapped 8 and you go here, this gets mapped here 9. So, the same technique is applicable to this higher order also. So you can perhaps take one more example.

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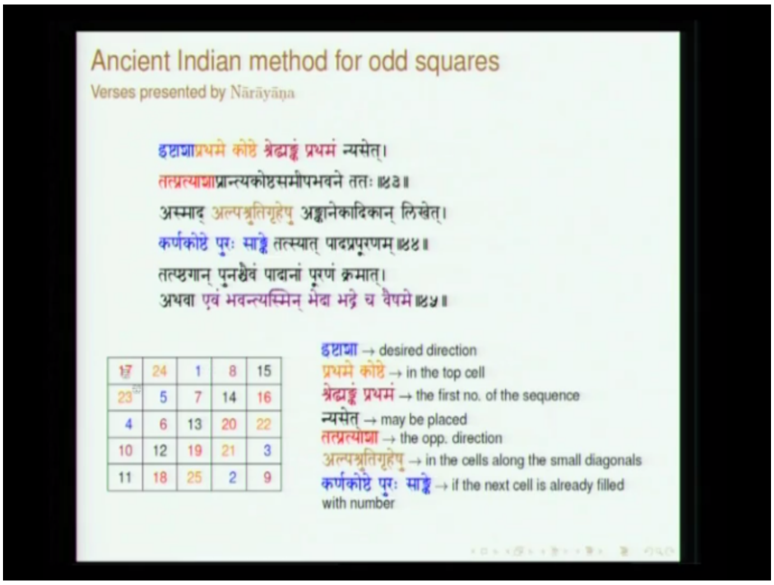

So, look at this where we have a 5 by 5 odd square here as an example. So, we can see this. You can see the diagram so we start with 1 and then we move up here so this gets mapped to this point. So, you move diagonally up you have 3 here. So, from 3, I have to go here so torus it gets mapped here. Then you go 5 so and then so hit with some other number which is already filled and since you have started in this direction, so you have to move down.

If you were to start in this direction, you have to move left. So, if you start with this direction, you have to move up. So that is the thing. So, here we move down 6 and then 7 and then 8 and then you move up here so this actually gets mapped here 9. So, it is a sort of torus and then you move up here so this gets mapped 10 and then so this is already filled so we move down 11 and the whole thing gets filled. So, 11,12,13,14,15 and then so at 15, so you have to move up here.

So, this will actually get mapped here right. So, which is already filled and so therefore we move down starts with 16 so you move up so this is 17 and then you move up you get 18,19,20. So this is already filled you move down 21. So on then 22, so 23 gets mapped and then 24 and you move up this gets mapped 25 hogaya. So this is all filled. So, this is the technique by which one can this is an ancient sort of technique in fact Narayana discusses this at a much later stage.

In fact, he presents his own new algorithm so by which one can construct not necessarily using this number. So, this actually start with one and then the common difference is one, you use these elements and the sum is fixed. Of course there is a lot of fun even in doing this. But, Narayana actually presents algorithm by which you can construct whatever number you want by a certain standard procedure by constructing appropriate arithmetic series.

That is why he already says start with (FL) and then you have to find out (FL) so and d you have to find out, then you can construct the magic square. In fact, (()) (40:20) no, no why? In fact, the (FL) refers to direction. In fact, if some of you know Vishnu Sahasranama, one of this slokas where we have this reference of this (FL) one of this dhyanaslokas (FL) so that is where this is named (FL) refers to direction. Here (FL) means you can choose east,west,north, south whatever you want. (FL) direction you choose.

Then you have to sort of move to (FL). (()) (40:52) why not? See you start with 1, so you go there, you do not have a cell, you come here (FL) opposite, you move down. You start in this direction, you move down. That is all, if it is not there it will have to move down. Down is basically you start with that direction. So it is in that sense, I mean they only give a sort of sutra from which we have to understand. So it is almost see if you want to explain everything in words, it is almost impossible.

So one has to guess. So, more or less, but the thing is, it is not like aryabhatia which is very tough. So, Narayana Panditha makes it as simple as possible in fact later mathematics works. So, as you might have seen, even (FL) is said much more elegant with various examples. Aryabhata does not do all that. So, Narayana Panditha does it. In fact, much more in fact, this seems to be the most voluminous work that we have on mathematics.

So composed around and lot of improvised techniques have been provided in almost every topic that he has discussed okay.

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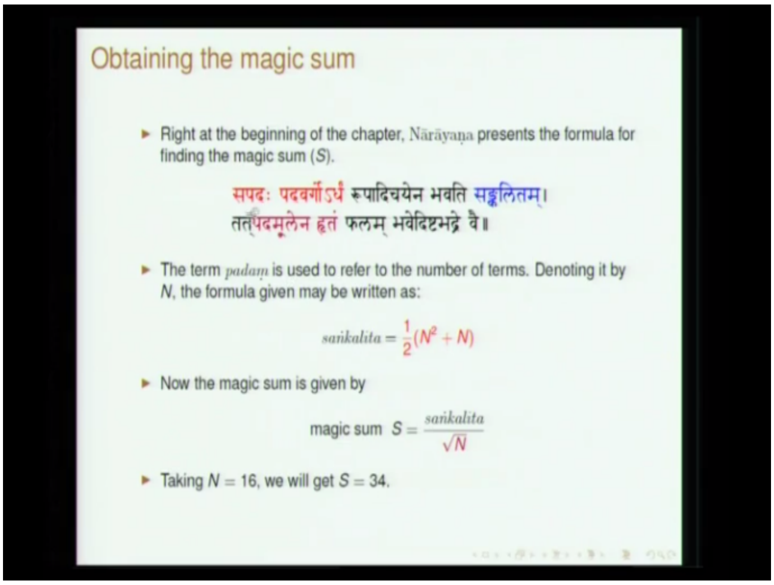

In fact, right at the beginning, he also gives certain verses, which actually present (FL). So, here so he basically says what the magic sum will be. The word padham so has to be understood as (FL). See number of terms. So, number elements which go in filling the square, that is what is called padham. See sometimes they use the word padha, sometimes they use the word (FL). To refer to the number of terms in the arithmetic sequence.

So, in this case, so (FL) refers to n square and ardham is half of that, (FL) means along with padha. So, basically he says n square+n divided by 2. So, you can easily see that basically what you have is an arithmetic sequence, so series and that is what it is. So, he is just saying that sum and (FL) so palam basically refers to the magic sum okay. So, that is what it is.

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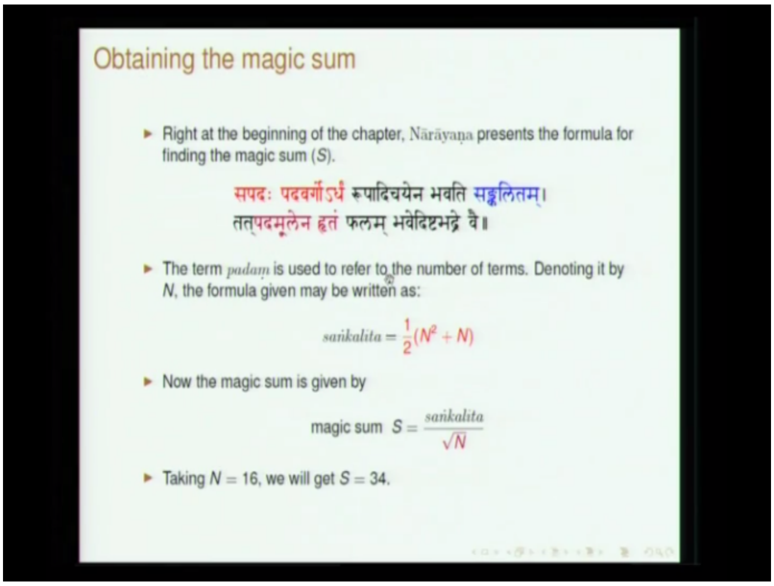

And in fact there is another verse I don't know if I have quoted the verse. I have not quoted, but I will just tell you. In the very next verse after the sapadham. So, Narayana gives how to find and d. to recall right at the beginning of the chapter so Narayana said so in dealing with magic squares, all that you have to do is see (FL) mukha refers to a, arithmetic sequence and prachaya refers to d. So, the problem is basically given sum, you have to find out a and d.

So, Narayana actually gives a verse by which the equation which has to be employed in order to get a and d has been clearly stated and we will just present this equation now and not the verse. So, given the magic sum s and the order of the magic square n, the first thing that is to be done is to construct the magic square is to obtain the sredhi. Sredhi is basically defined by a and d right. So, to obtain a, d basically n square elements have to be constructed.

So, Narayana makes use of this kuttakara. In fact, he uses the word kuttaka also. So, see in this equation 1, s is the sum, so by sum we mean either diagonal, pan diagonal, vertical, horizontal, that is the magic sum. So, what will be the sum of the all elements? So, n times s. So, that is the left hand side fine. And in the right hand side, so if you see that so a is the first term and this is the last term. So, first + last by half. So, that gives the mean and this has to be multiplied by n square.

Because there are n square terms here right. So, this is the equation which he has in his mind. So, all that he gives is this formula. So, therefore we get s as na n/2*n square-1*d. so, n is known, what is na? sum is also known that is given to you. The point is you give a sum, and then you say so what should be the dimension of the magic square 5 by 5, 4 by 4, 3 by 3 whatever. So, s and n are given and you have to find out appropriate a and d. so, this is the problem.

So, you have basically one equation and you have 2 unknowns, therefore it is a kuttaka problem fine. So a and d have to be determined. So, if you look at this equation, we have repeatedly said while we discuss this kuttakara problem that, this will have a solution only when if there is a factor okay. So, if there is a factor the gcd of n and n/2*n-1 so, that should be divisible. So, otherwise this will have no solution okay.

So this condition we have to just understand and anyway, the point is so you will be able to get several such possibilities in fact, right at the beginning of the chapter, Narayana says, so, if you have one solution, then you have infinite number of solutions. And therefore, given s and n, so, if you are able to get 1, then you will be able to get infinite number of magic squares in fact for that. So, that is the kind of implication of understanding this.

So, how many such magic squares can be constructed? The way Narayana has given. He proceeded a certain way of this algorithm, using this algorithm, so you will be able to construct infinite number right. So, that is the point I wanted to convey through this. (()) (46:50) you can get any number right. No, no, no. The point is you can also have fractions, that is also possible. So, nothing prevents us.

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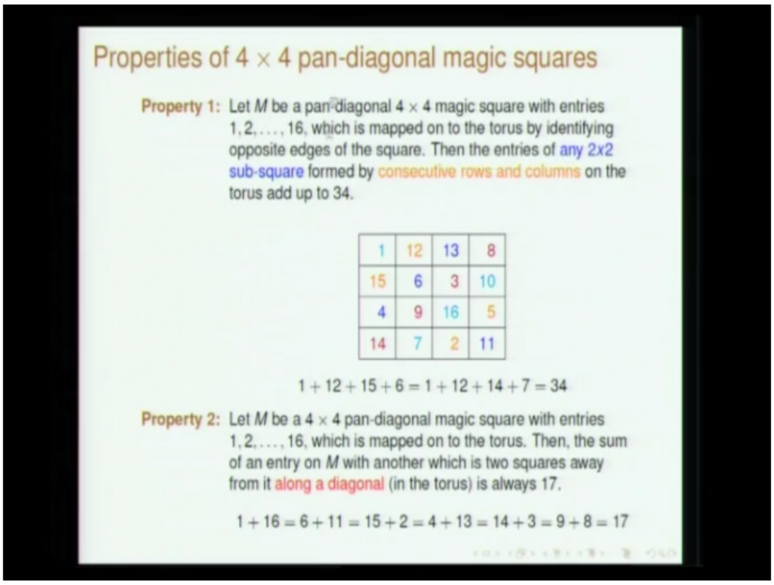

So, I will very quickly ouch upon these three properties. May be we will take it up once more in the next lecture and then see how we can construct magic squares using these properties. (()) (47:29) All are possible. In fact, you can do with negative, maybe we will see one or two examples. In fact, Narayana's steps itself so, if we see that in his is manuscripts, so present some examples where in some of the elements are negative.

So, this element which is negative will be denoted by a small dot above. So, that is the kind of thing. So, in the construction, which we find, so which has been a sort of transcript or what is there in the manuscript, so when they try to reproduce in printed version of the text. So, they have, see suppose this number is negative, we will put a small dot above this number, which actually means negative okay. So, this is also possible.

So, you can have a negative, you can have fractions all that we can have okay. So the property 1 is given a 4 by 4 magic square, and the entries are 1 to 16, which are sort of mapped into a torus, which we are trying to explain by identifying the opposite edges. So, this maps to this, this maps to this and so on. Then, the entries of any 2 by 2 sub square, so, how many such sub squares can be had now? So, 1,2,3,4,5,6,7,8 and then center 9 that is all, you can map this torus also.

This 1 and 14 and then 11 and 8. So, totally we will have 15 okay. So, any of these sub squares. So we will also add up to 34. So, this is one interesting observation which one has to make fine. (()) (49:22) that is a kind of a torus I would say okay. So, there are, this is the first property, the first property is so, any of the sub squares so 2 by 2 of consecutive rows and columns okay. So, not this and this. So, that is why when I said, so you have to just group this way.

And then so this 5,11 and 4,14 so, will sort of get mapped. So, that is the kind of torus. So, that way we can have 15 possibilities. So, this is how it is. So, you have the sum 34, this is first property. The second property is 4 by 4 pan-diagonal magic square. So, if you look at the sum of diagonal elements with 1 cell left in between. So, in another words, alternate diagonal elements. So, you look at this, 7+10 is 17.

You look at 4+13 is 17, you look at 12+5 is 17, so 6+11 is 17, you have anything so of this nature. So if you go here, so it will sort of get mapped to 4 here. So, this will be 17. So, all that is. So, if you look at here, you have to go here, so this will get mapped here. So 4+13 is 17. So, these alternate elements across the diagonal, so they will sum up to s/2. So, this is next property. I think I will stop here. So we will continue in our next lecture. Thank you.