Mathematics in India: From Vedic Period to Modern Times
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Lecture-25
Ganitakaumudi of Narayana Pandita 1

Ok this will be first of the 3 lectures in Ganitakaumudi so I will be giving, so the outline where we first talk about the importance of Ganitakaumudi .

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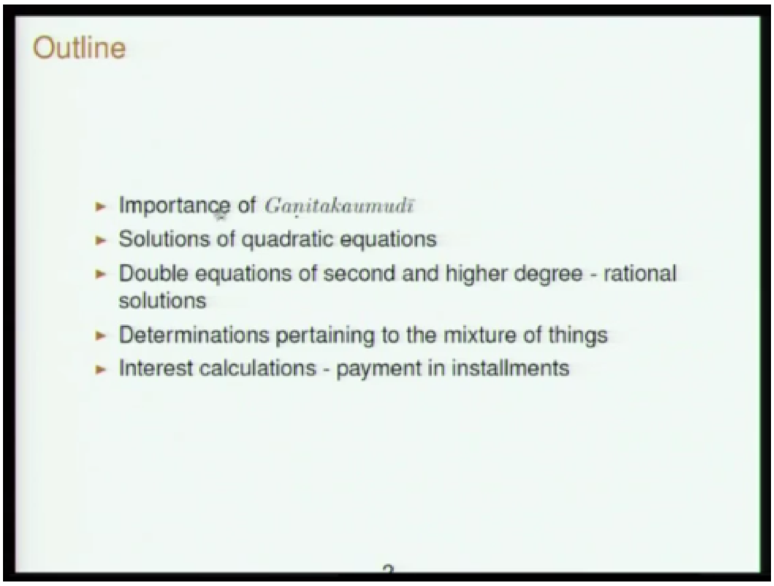

And some important topics here solutions of quadratic equations, double equations of second and higher and higher degree rational solutions, then determination pertaining to the mixture of things, the interest calculations, payment instalments have some interesting this is some methods which are given here which I have given today.

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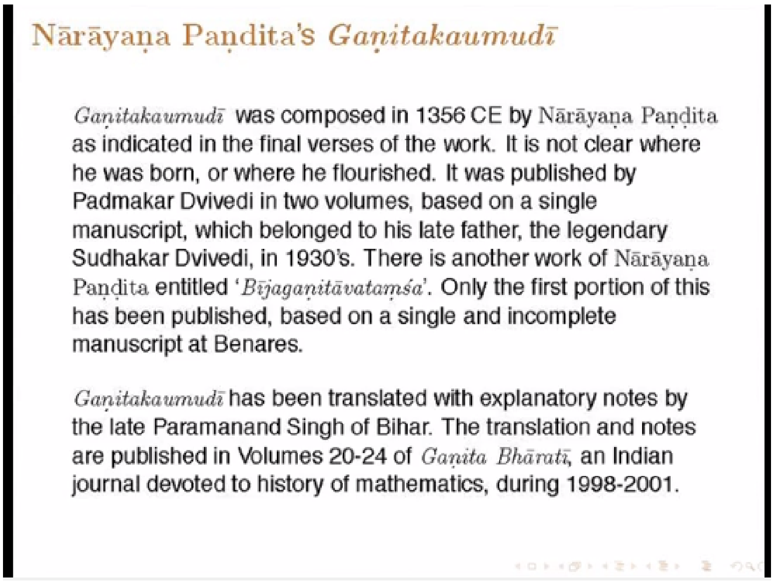

So Ganitakaumudi was composed in 1356 common era by Narayana Pandita as indicated in the final verses of the work. So it is not clear where he was born, or where he flourished. It has this work was edited in public by Padmakar Dvivedi in two volumes based on a single manuscript which belonged to his late father, the legendary Sudhakar Dvivedi in 1930s. Sudhakar Dvivedi famous person who edited and translated many of these important works.

In Indian astronomer in mathematics in 19th century onwards, he is the another persons who involved in the revival important part of revival of interest in in Indian mathematics in astronomy. So Padmakar Dvivedi his son. There is another work of Narayana Pandita called (FL) only the first portion of this have been published and based on a single and incomplete manuscript at Benares.

So Ganitakaumudi has been translated by Paramanand Singh when Parmanand Singh with explanatory notes it was published in these volumes of Ganitakaumudi general of Indian mathematics, history of mathematics mostly Indian, but others listing history of mathematician other cultures also is there in this should they completely devoted to the history and he published this between 1998 to 2001.

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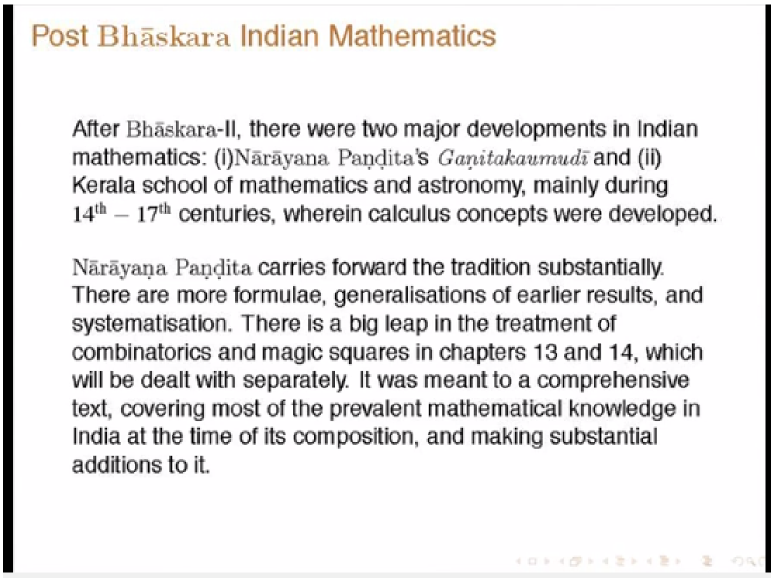

But this is a very big work after Bhaskara II there are 2 major development in Indian Mathematics, so 1 Narayana Pandita Ganitakaumudi and other the kerala school of mathematics and Astronomy mainly during between 14th and 17 centuries wherein calculus concepts for developer. They have already had a little bit about it, we will hear it in more detail in the lectures to follow.

So Narayana Pandita carried forward the tradition of mathematics in India substantially. There are more formulae, generalisations of earlier results and systematization, so entire we have seen it know from brahmagupta and others carried forward in putting up of Mahavira whose we discuss, so then carried forward more by Bhaskar Acharya in his 2 words (FL) and Lilavati. So we saw there are many advancements in that compared to the earlier hours.

So similar here also Narayana Pandita carries forward (FL) introduce more advances and he this is (FL) something more about it in a generalized decision that seems to be the way he seems to be working and that is the way it is written and especially with a big leap in the treatment of combinatorics and magic squares in chapter 13 and 14. So the other things are also there, there is a substantial improvement in cyclic quadrilateral rational figures and so on.

But one more very important result about the (FL) carry out some of integers (FL) already discuss in general result, so apart from those things the major results are contained in chapter 13 and 14, so which will be dealt with by others, should not be a comprehensive text covering mostly prevalent mathematical knowledge in India at the time of composition and making substantial edition to it ok.

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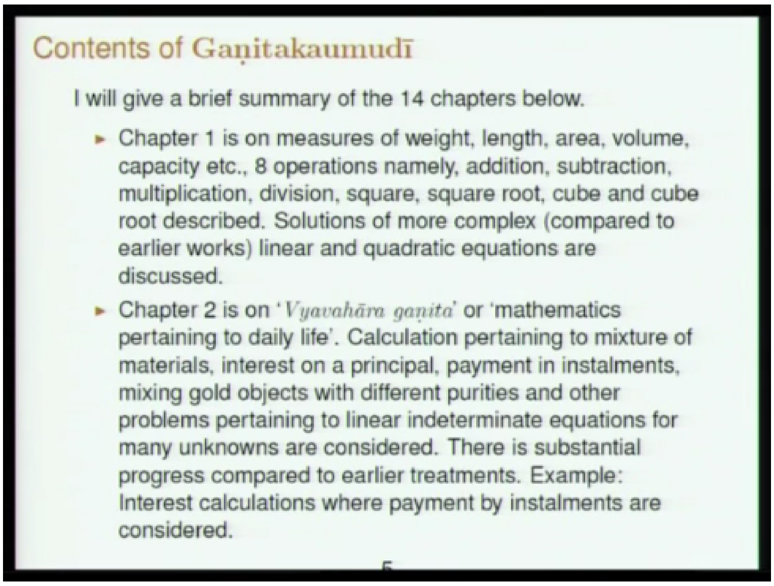

So I will give a brief summary of the 14 chapters in this verse, so chapter 1 is on measurers of weight, length, area, volume, capacity etc. as in other works put 8 operations we saw it in Lilavati also addition, subtraction, multiplication, division, square, square root, cube, cube root. So they are described and solutions of more complex linear and quadratic equations are discussed here because things are been discussed in earlier works.

And he is thinking to generalized the more and make it a little more complete, and chapter 2 is on Vyavahara Ganita or mathematics pertaining to daily life, so calculation pertaining to mixture of materials interest and principal, payment in instalments, mixing gold objects with different purities, and other problem pertaining to linear indeterminate equations for many unknown they Aryabhatta considered.

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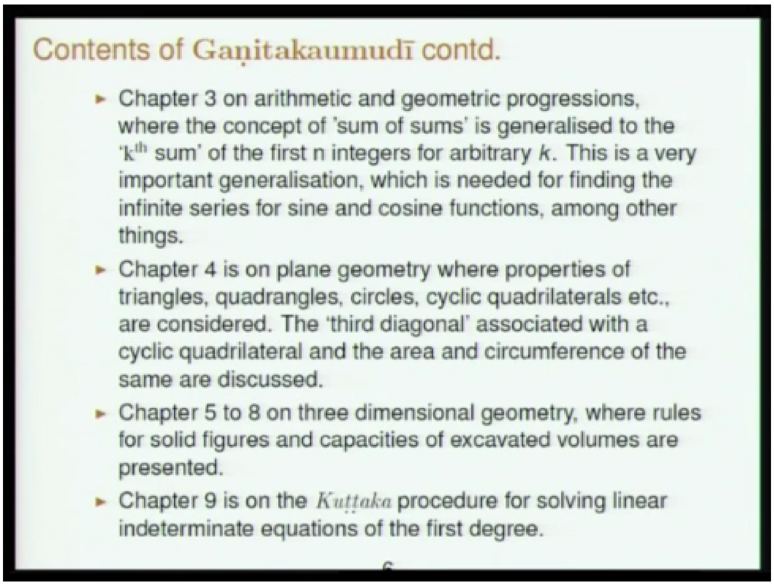

And there is what a progress compared to earlier treatment, example I will take this interest calculations where payment by instalments are considered I will deal with it a little in detail and chapter 3 is an arithmetic and geometric progression where the concept of sum of sum is generalised to the kth sum of the first n integers for arbitrary k. So where Aryabhatta himself has given the sum this si sigma (FL) then if you take r*r+1/2 it is sum 1-n.

So that is the second sum, so like that you can go on, so he has given the formula for the kth sum, it is a very very important generalisation which is needed for finding infinite series for sin and cosine functions among other things. So that will be discuss later how it is used and how exactly does it figure. Chapter 4 is on plan geometry where properties of triangles, quadrangles, or quadrilateral, circle cyclic quadrilaterals etc. are considered.

Now he introduced the new concept the third diagonal associated with a cyclic quadrilateral and then area and circumference of the cyclic quadrilateral are discussed. Then chapter 5-8 there are three dimensional geometry where rules of a solid figures and capacities of excavated volumes are presented and chapter 9 is on kuttaka procedure for solving linear indeterminate equations of first degree. So I will not deal with it already has been done in detail.

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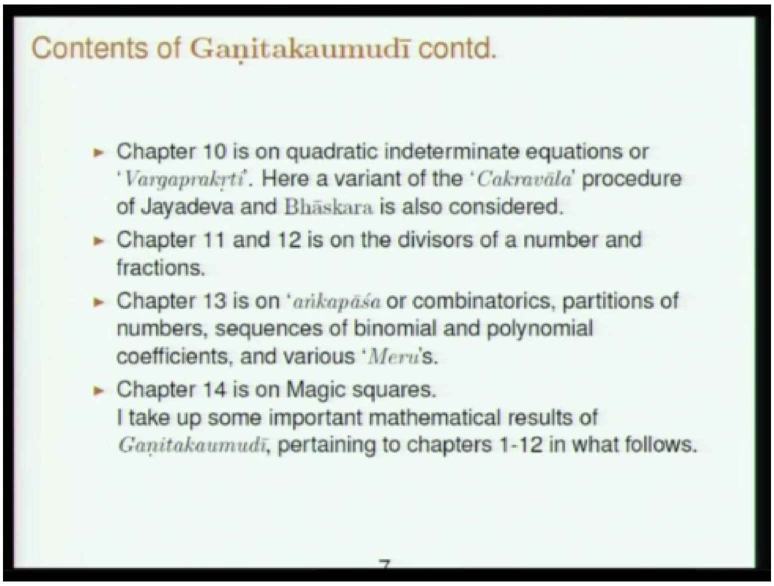

Then chapter 10 is on quadratic indeterminate equations or Vargaprakrti, so which was done earlier in yesterday here. Here a variant of the Chakravala procedure of Jayadeva and Bhaskara is also considered, yesterday it was mentioned that you know when you are going from one step to another this Pi will come (FL) quantity then you have to (FL) –d, so that is minimum. So he did a small change here that Naraya Pandita consider.

And you know that Chakravala procedure is due to Jayadeva and Bhaskara small change ok. So in chapter 11 and 12 is on divisors of a number and fractions and chapter 13 is on ankapasa or combinatorics, partitions number, sequences of binomial and polynomial coefficients and various Meru's. So will not be doing much but little bit I will do, little bit in this I will do, but it done by others.

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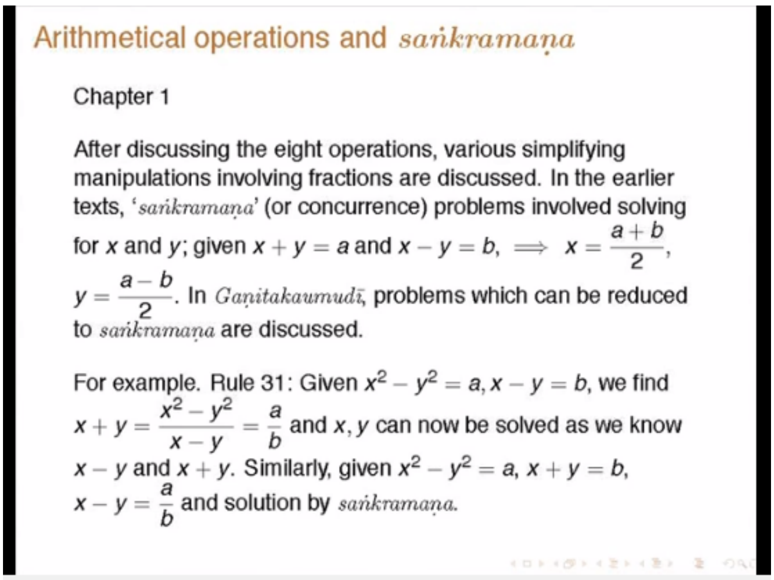

And chapter 14 is on magic squares and so basically I will do pertaining to chapter 1-12 in what follows 1 and 12 in what follows ok. So in chapter 1 I just discuss some important things only, got a comprehensive you know lecture giving me know the word content of Ganitakaumudi the content of Ganitakaumudi comprehensively, what were the important thing is there I will try to emphasize that.

Say for instance of discussing 8 operations various simplifying manipulations involving fractions are discussed and in earlier text sankramana already mentioned, so that is the sum and difference number of given you can find the number, so this is sankramana x+y+a x-y=b, then x is a+b/2 a-b/2, see in Ganitakaumudi problems which can be reduced to sankramana are discussed.

For instance suppose you are given x-y=b and xsquared-y square=b, so then you can find x+y=xsquare-y squared/x-y=a/b. So x and y cannot be solved as we know both x-y and x+y, x-y is given and x+y also found. Similarly suppose you have givne xsquare-ysquare nd x+y, x-y can be found out similarly and we can solve it by sankramana.

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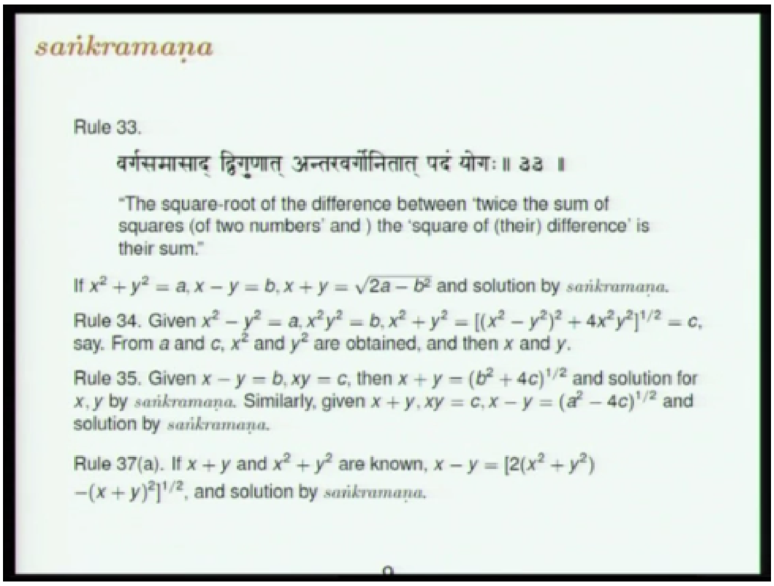

Then little more trivial (FL) the square root of the difference between twice the sum of the squares of two numbers and the square of the difference is their sum. So now suppose you are giving x-y=b, x squared+y squared=ab, so then x+y can see that will be 2a-bsquare and you can solve it by sankramana. Similarly suppose little one more these thing given that suppose xsquare-ysquare=Armstrong

And xsquareysquared=b, so then you can find out xsquared+ysquared as you know xsquared-ysquared whole squared 4xsquared ysquared whole to the power of 1/2. So you have got xsquared-ysquared and xsquared+ysquared. So from this xsquared+ysquared are obtained and then you can obtain x and y. So then similarly suppose x-y is given and xy is given, so x+y, b squared+4c whole to the power of 1/2 and again we can find out by x and y by sankramana.

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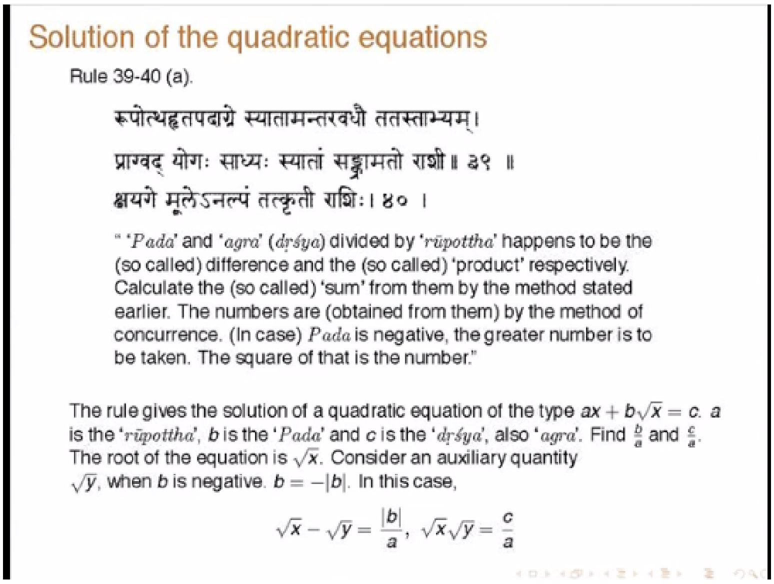

Similarly given x+y and xy also similarly we can do that and suppose x+y and xsquared are known then x-y=this quantity and you can solve it by sankramana. So then he discuss the solution of quadratic equations, so these all what he says (FL) divided by rupottha happened to be the so called difference and the so called product respectively. Calculate the sum from them by the method earlier.

The number should obtained from them by the method of sankramana in case pada is negative the greater the number greater the number to be taken, the square of that is a number. So whatever you doing this sankramana thing you know leading him onto the quadratic equation solution. So what you have to do is always had already written down equations like this in my previous lecture.

Ax+broot x=c, so a is the (FL) coefficient of x, b the pada, 2 portion of the root x and c is called (FL) now find b/a and c/a, suppose the root of the equation is root x, so consider auxiliary quantity root y when d is negative there is b=- not be in this case root x-root y+this and rootx*rooty=c/a. So then I mean these are the other root somewhat different, he is trying to use the things so you find this quantities.

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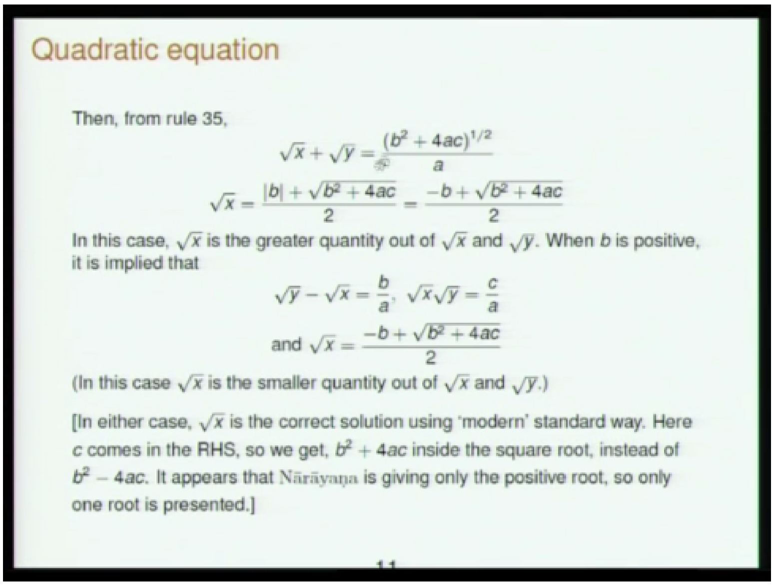

Similarly so then rootx+rooty is given, rootx-rooty is this, then rootx+rooty will be is giving this. So finally root x is not b here b negative so –b+square root or bsquare 4ac/2. So in this root x is the greater quantity out of root x and root y, when b is positive, it is implied that this is solution n. There are the same form, he say differently. I mean here we note that instead of -4ac normally +4ac comes.

That is because c is in the other side, you see he normally got ax at the root x and asquare b square+c=0, so here auxiliary becomes minus, so that is why that you know is coming like this ok. By at least here he is talking only one root but I will point out that later he does talk about 2 root, some problem he does talk about 2 roots, he wants this has been 10 to be positive, so root x is the correct solution during the modern standard way.

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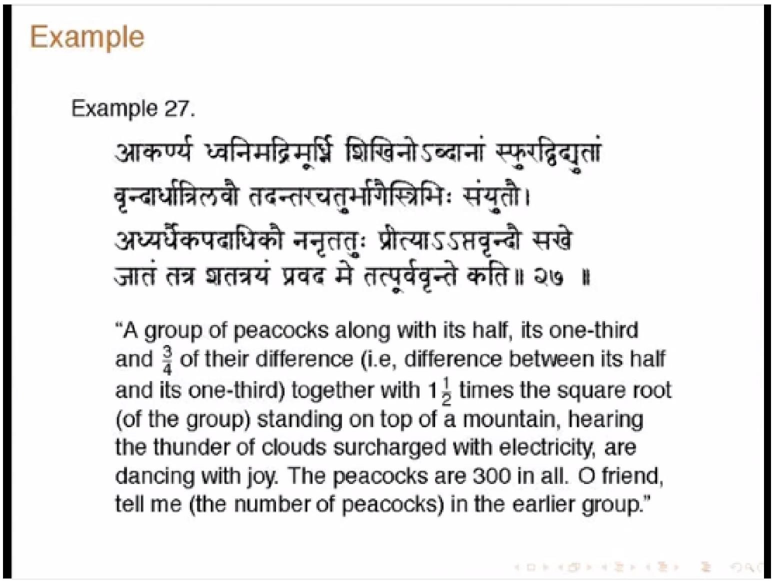

So various interesting problems are discussed (FL) so a group of peacocks along with its shalf, its one third and three fourth of the difference together with 1-1/2 times the square root of the group standing on top of a mountain, hearing the thunder of clouds surcharged with electricity, are dancing with joy. So the peacocks are 300 in all. O friend, tell macro environment the number of peacocks in the earlier group.

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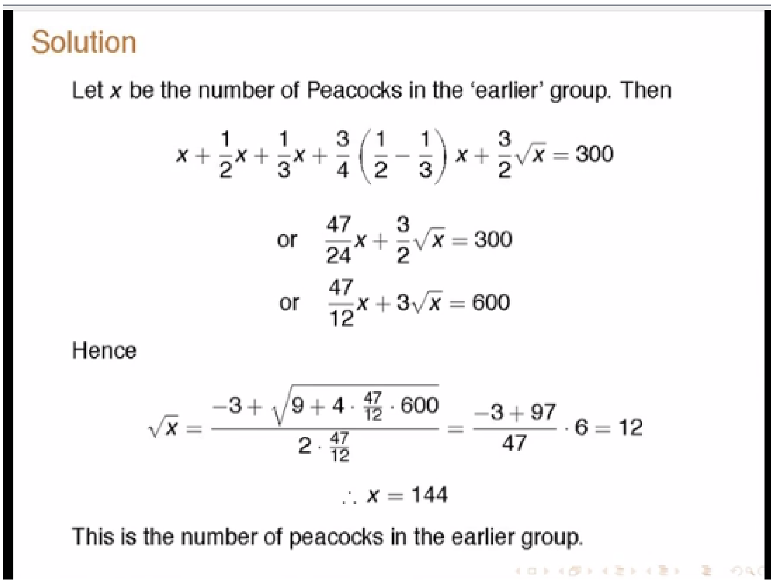

And whatever you left you know dancing around that is 300, so that is the final number, so it is always written in you know nice example you know it is amusing and it is present to have these things.

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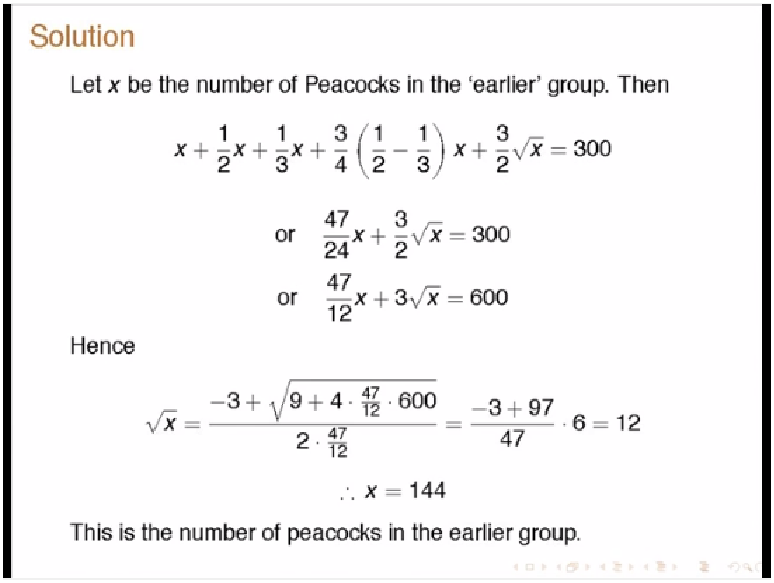

So suppose x is the number of peacock he says that right, what we say along which is half, right along which is half (FL) then you also take one third of the three fourth difference of this that also you take. All these things you multiply by this. So then 3/2 times root x you know he says together it is 1-1/2 times (FL) that is there is square root of the that is that, and finally 300, the total is 300 what is that after all that is 300.

So what is the totally 300 and what is left is you know what you have to find, sorry may be the peacock is 300 in all, so initial number you have to find, initial number only it, therefore the other, so this is the equation. So finally you get this, so fact rootx=12 and x=144. So x is 144. So if you do all this operation you know take this x and then multiply and half of it again you taken all the 650.

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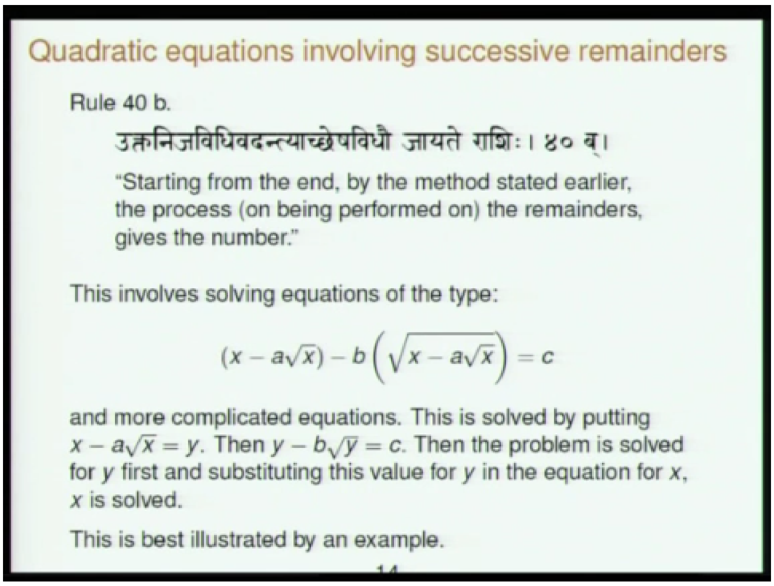

And what is initial number, so that is 144 which is less than 300 (()) (18:12) 47*12*x correct this is the final quadratic equation then you have to supply here. So then we talk about quadratic equation involves in successive remainder, for you also very interesting, so the rule itself is very simple in a very short (FL) by the method stated earlier, the process on being performed on the reminders give the number.

So this you know because just state this you will not be understand what you are saying to say the following you see suppose you have explained as a root x you just say, and then state the square root of that, so such things like (FL) some problems you know, so x-aroot x-b*square root of this, that is given to be something, so are more complicated things which I will consider soon enough.

So this is called putting see you just put this is equal to y, so then y-broot/=c, so you solve for a y and then you solve for x from this after solving from y from this, we are to plugging this y and solve for x.

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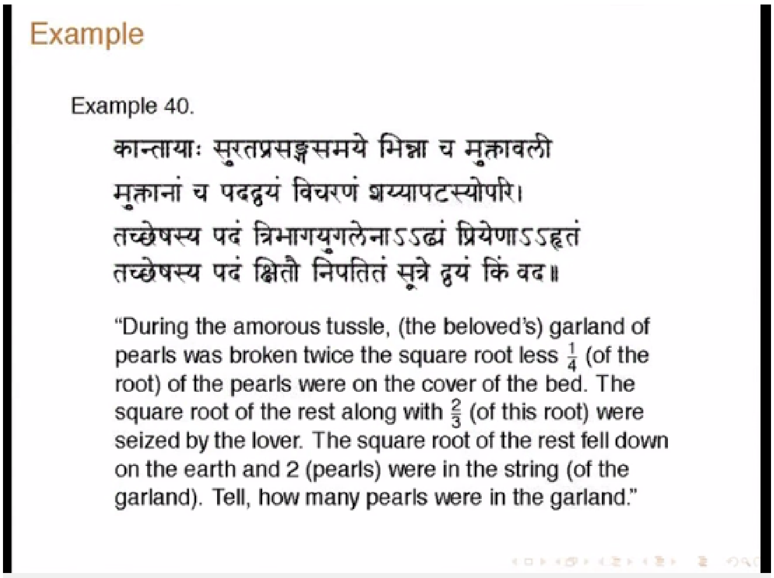

So this is giving an example (FL) ok during amorous tussle, playful for whatever so teh beloved garland was broken and simple for broken and find the square root less one fourth of the pears were on the cover of the bed ok. (FL) so that is the square root of a less than one before that was there and square roots along with two thirds of this route for these fell the level (FL) he took this thing.

And then the rest fell on the brown and 2 pearls remaining in the string you see he is wearing a garland of pearls so it is broken, so we have to find out how many pearls were there, so on the cover of the bed so it is 2-1/4*root x and that is make it as 7.

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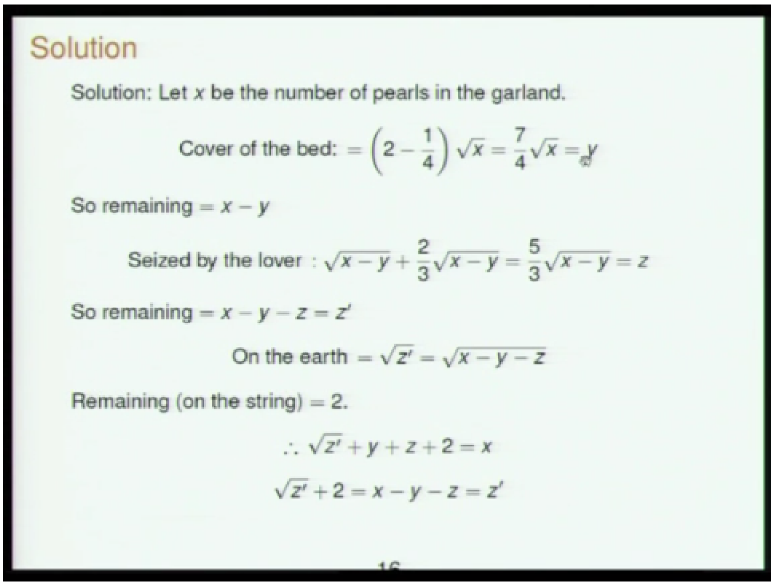

Sorry you make it as y so the remaining is x-y, initially it was x, you do not know that, then the lover x see this thing you know this one is taking a square root of this +2/3 times this ok. So that is that also we do not know how much he has seen, so that you call it is z and after these things so get for the initial number y is on the cover of the bed and z is caught by the lover, as far remaining is x-y-z=root zy ok.

And what is on the earth for the root you know on the root of zy, so that is they on the earth it is fallen on the earth is square root of the remaining whatever is remain (FL) and after all the reaming of the string is 2, so you have to find out the original number of is, so this whatever is remaining on the earth that is root of zy written+whatever was on the cover of the bed that is y, whatever was (FL) lover that is z, so root that time y+z.

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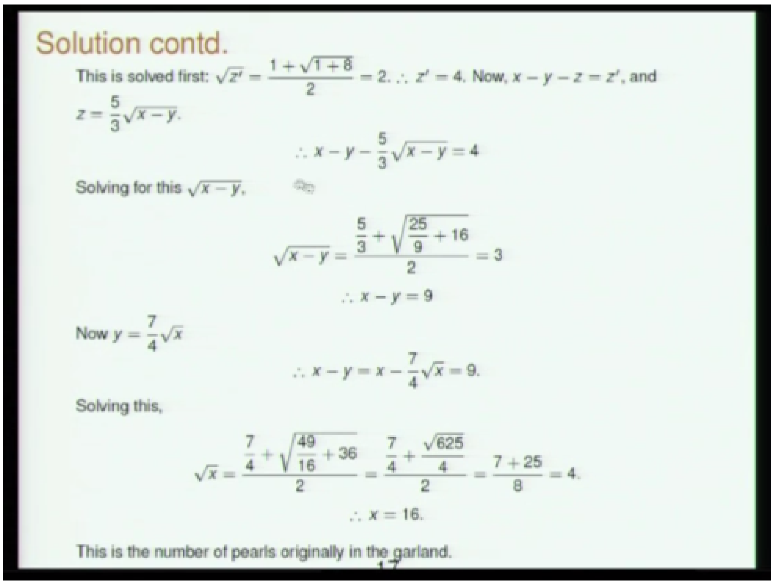

And 2 was remaining on the (FL) of the where yourself, so that is 2, the total of that is z, that the original number of us, so root zy=2=now, if you shift it to this side x-y-z=zy. So now solve for the so you will get the solution as 2 root z was 2, zy is 4, so now x-y-z=zy and z is we have already given 5/3 root x-y, so x-y-5/3 root x-y=4. So you solve for x-y, x-y will be 9, now y=7/4 root x, that is also given.

So x-y that is x-7/4 root x that is equal to 9, from this so solve for x root x=4 and x=16. So this square root are coming you know one more square root, so that is interesting. You can just carry on like this, so then again some remainder take equation.

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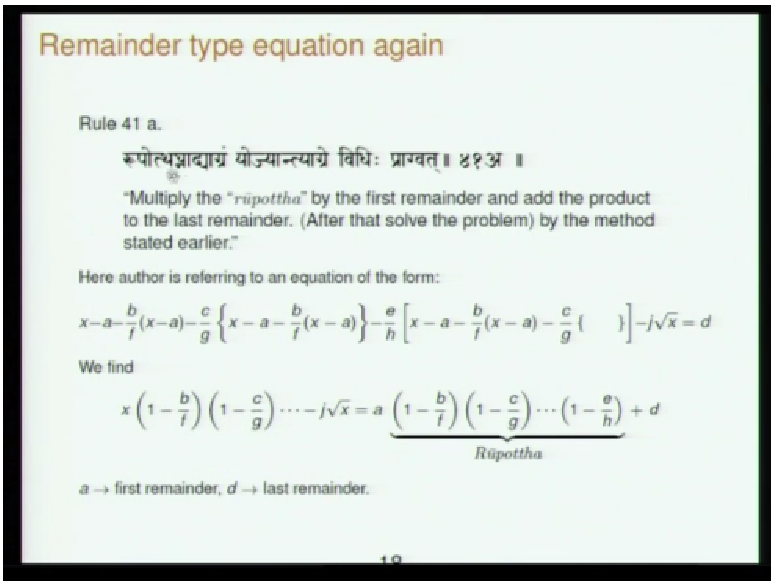

So again some reminder take equation (FL) multiply the rupottha by the first remainder and add the product to the last remainder like that. so essentially he is referring to an equation of this form x-a, so then fraction of this and subtracted to this, and fraction of this and subtracted to this like that you know, finally you are having this kind of an equation x*some quantity-j*root x a*some quantity+d, so he is giving the solution for this.

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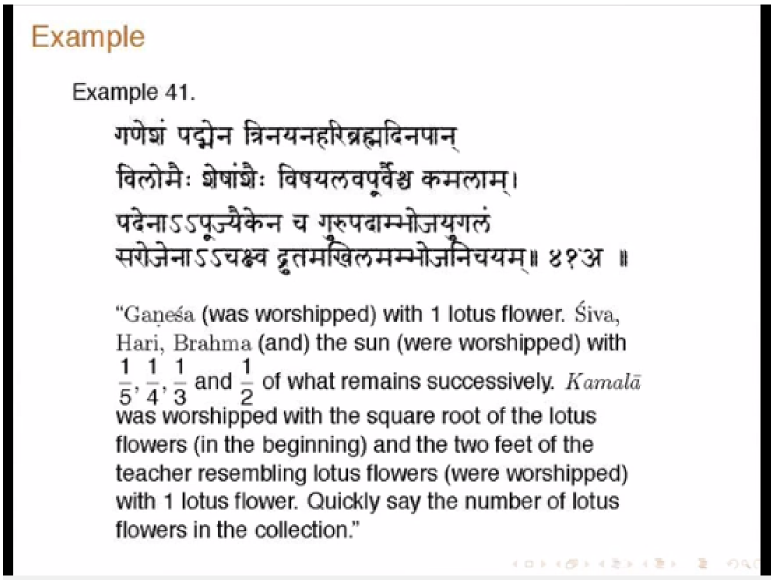

So if you give you take an example take up the example that will be clear (FL) Ganesa was worshiped with 1 lotus flower (FL) siva, hari, brahma (FL) the maker of a day the sun or worshiped with one fifth one fourth and 1 third and half of what remains successively. (FL) Kamala was worshipped with the square root of the lotus flowers in the beginning (FL) the 2 feet of the teacher resembling lotus flowers were worshiped with 1 lotus flower.

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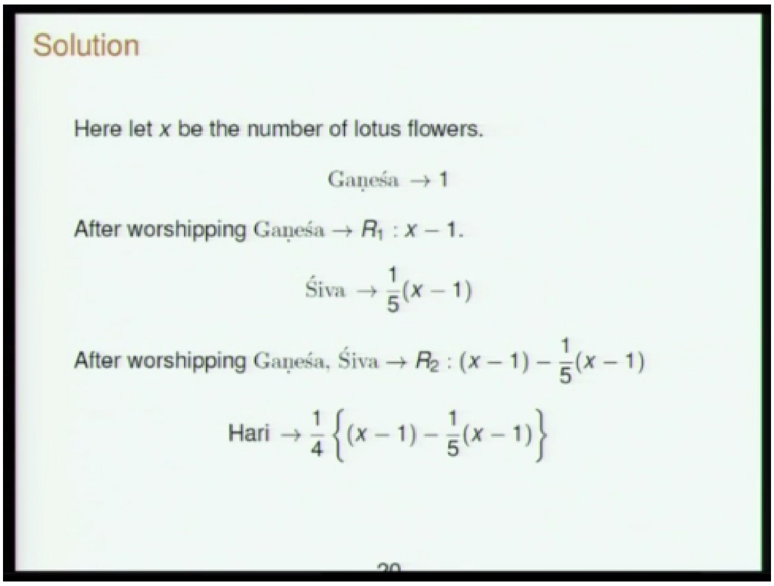

(FL) quickly say the number of lotus flowers in the collection. So let x be the number of lotus flowers, so Ganesa is worshiped 1 ok. So the remainder of this is x-1 and siva was worshiped one fifth of this, so one fifth x-1. So after worshipping Ganesa and Siva what we are just x-1-1/5*x-1 and Hari is worshiped with one fourth of this.

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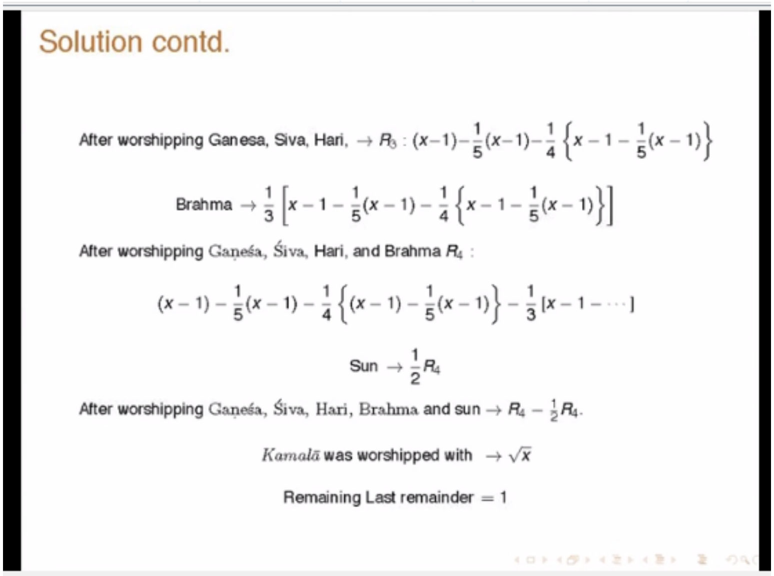

So after worshiping Ganesa, Siva and Hari what you get is x-1-1/5 and 1.4 these thing ok. So Brahma is worship with one third of what remains after worshiping Ganesa and Siva and Hari 1/3 of these 3. So after worshiping these 4 ok what is remaining is this and sun is worshiped with half of this. So after worshiping Ganesa, Siva, Hari, Brahma, and sun R4-1/2R4. And kamala was worshiped with root x and remaining is 1 ok.

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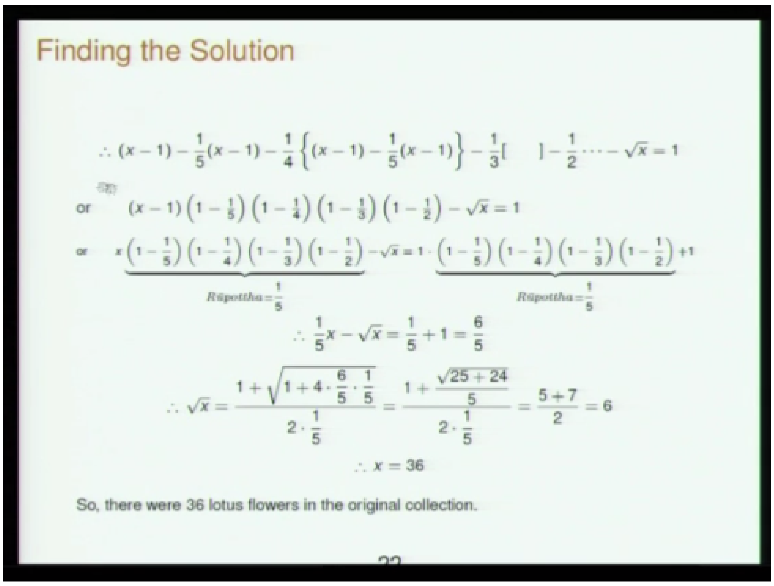

What you are getting is this kind of equation if my this you will get this equation, you just be little patient that nothing more is required in this. So you get this kind of an equation from in this is processes the form that he setting you know that the solution so you get x=6 by solving this quadratic equation finally you interpret that and root x is 6, so x is 36.

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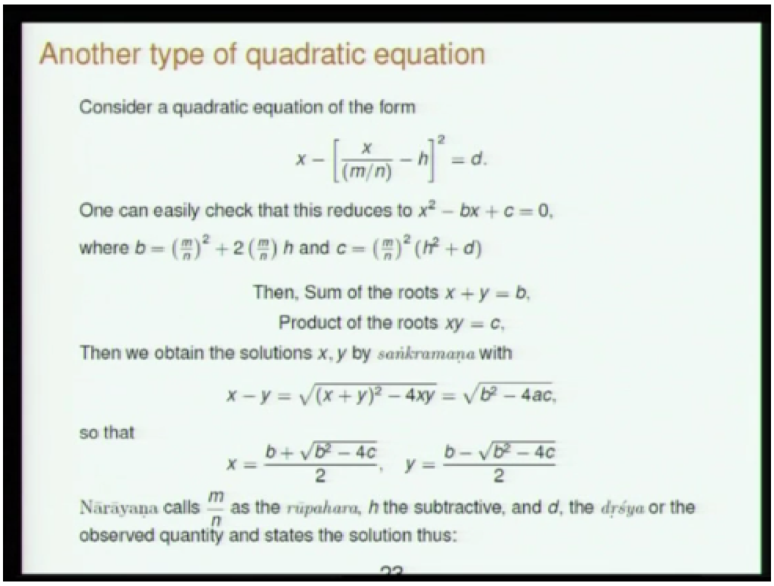

So another type of quadratic equation he discuss this kind of everything. So these are all very straight forward so gives some example of this ok.

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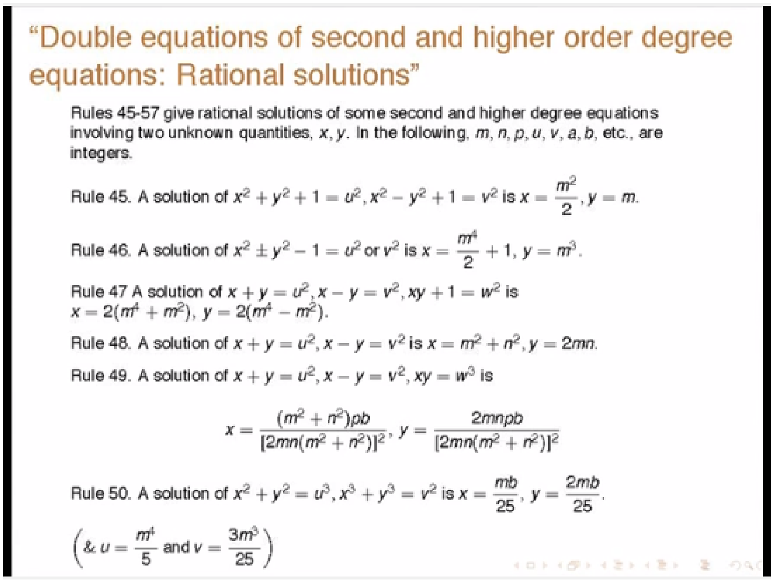

So then he talks various double equation of second and higher order degree equations, so some of them are discovered discussed earlier also, so x square+y square+1=you square nd x square-ysquare+1=bsquare, must also a square. That is equal to m square/2 and y=m that is solution and suppose this is equal to you square or v square this one then the solution will be this kind of everything.

And the solution of x=y=you square you know this is the square of a number, x-y is the leaner square of the integer and x+y+n so the solution will be this. So like this it is so on, it goes on particularly important is the various thing you are not trivial at all you have to go through this.

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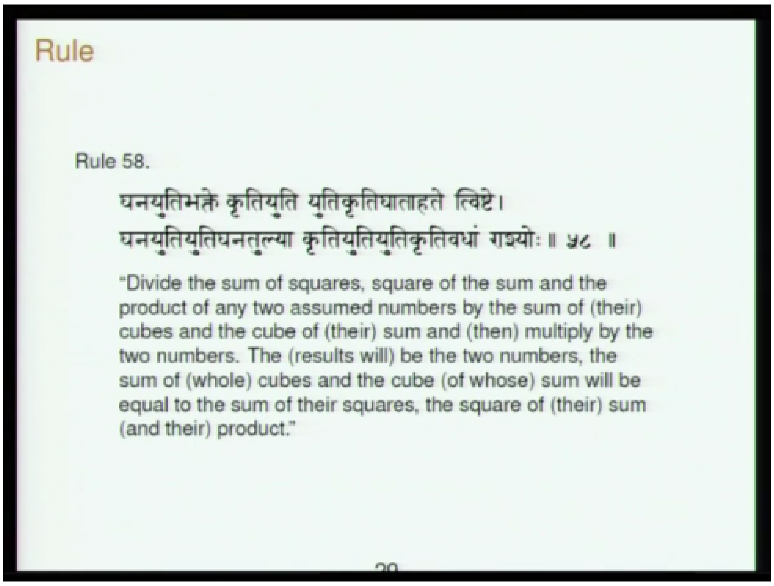

Next you goes you know in a 1 systematic set of you know equation which about this into the solutions (FL) very interesting. So the translation you know without going to this we can go through this is take the slides what is saying the following you know.

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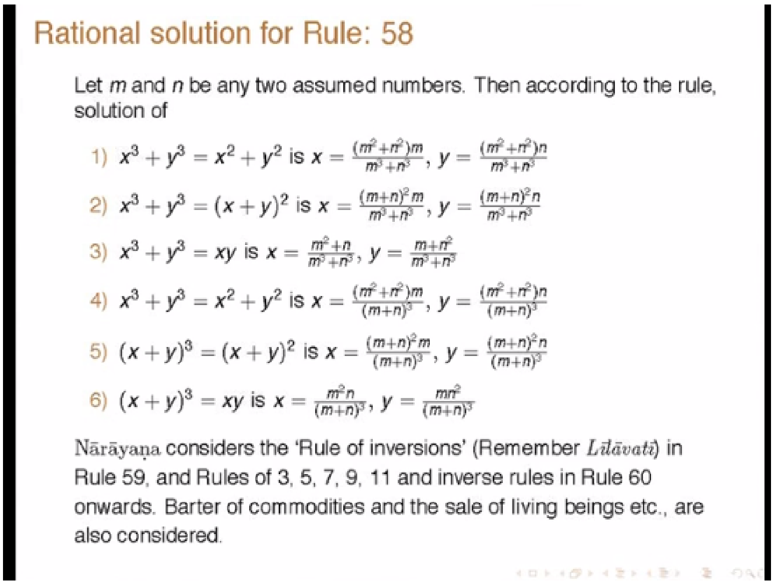

So what we are having is you have 2 numbers extend by have solution of xcube+y cube=x square I say ok , so are going to find rational solution for these, but trivial not at all trivial right so it is given if you take m and n to be any integers then this is the solution of this equation and x cube and y cube x+y whole square and xcube+ycube=xy, so then if you take any integer m and n and if you take this x and y like this then you get this.

So (FL) cube of the sum, so that is the kind of thing you know is doing. So similarly the solution of this is integral solution you will be having this kind of form and lastly x+y whole cube=xy, so this is the kind of solution you will have. (FL) discover the, consider the rule of inversion and all this tool proportion all that in great detail.

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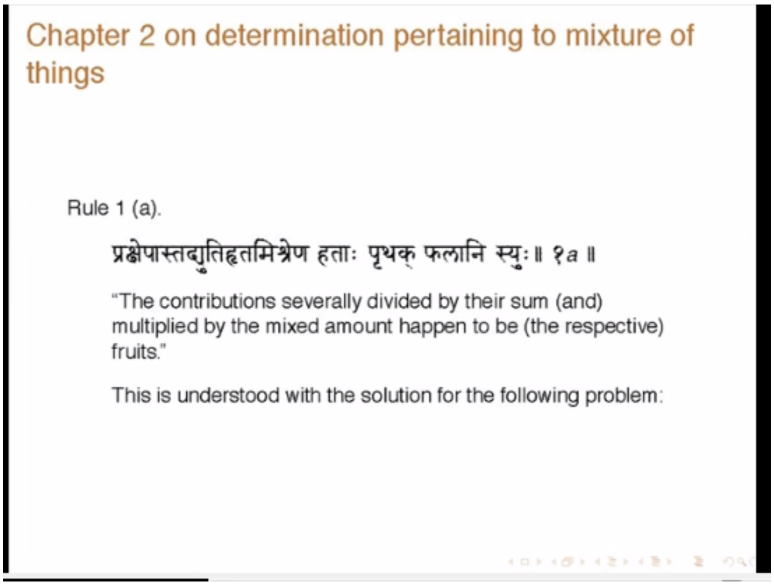

So I will not have time to do this, and (FL) is very interesting things.

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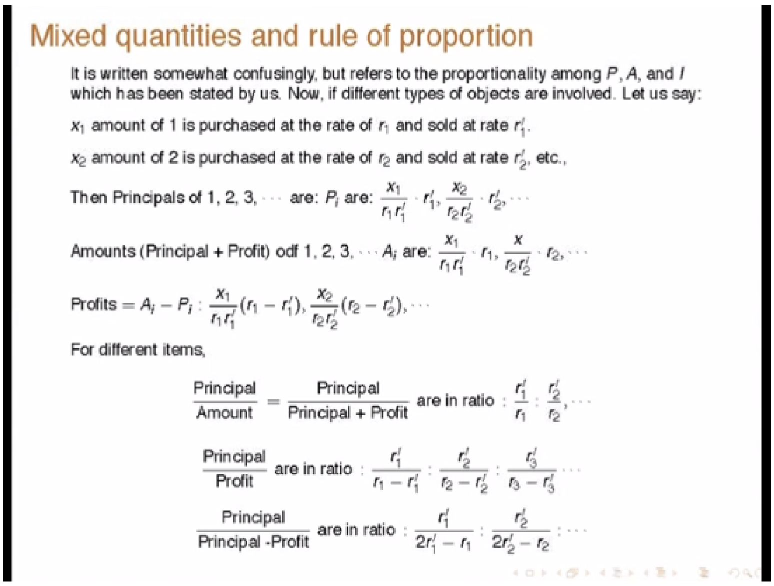

Mixed quantities and various proportions, we are rending money at various interest and you know suppose you are giving many people are giving suppose principle+profit=then what is the proportion, what is they know they showing which are lending money. So they are all very interesting problem, but do not have time, so I will one part may be I will pick up on a 1 or 2 basis.

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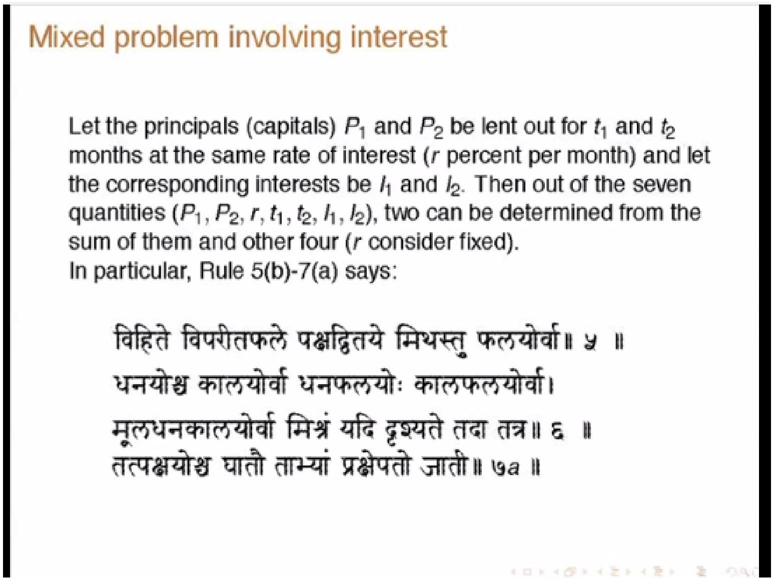

See suppose interest problem that principle capitals P1 and P2 be lent out for time t1 and t2 months at the same rate of interest and let the corresponding interests be i1 and i2. Then out of 7 quantities P1, P2, r, t1, t2, i1, i2, two can be determined from the sum of them and other four r consider fixed. So that is a very interesting way of which is generalization of you know direct interest kind of ah calculations.

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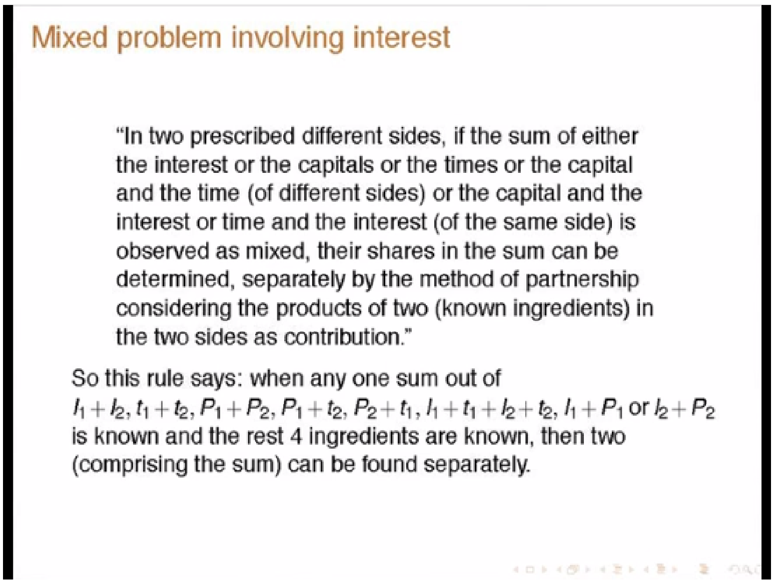

So what is happening is that so what the rule says is very anyone some out of you know, see suppose that this principles P1 P2, the time of lending is t1, t2, so interest out of them is i1, i2 ok. See suppose you are given any one sum of which is i1+i2 P1+P2, t1+t2, i1+i2 P1+P2, t1+t2, rest 4 ingredients are known, then 2 comprising the sum can be found separately.

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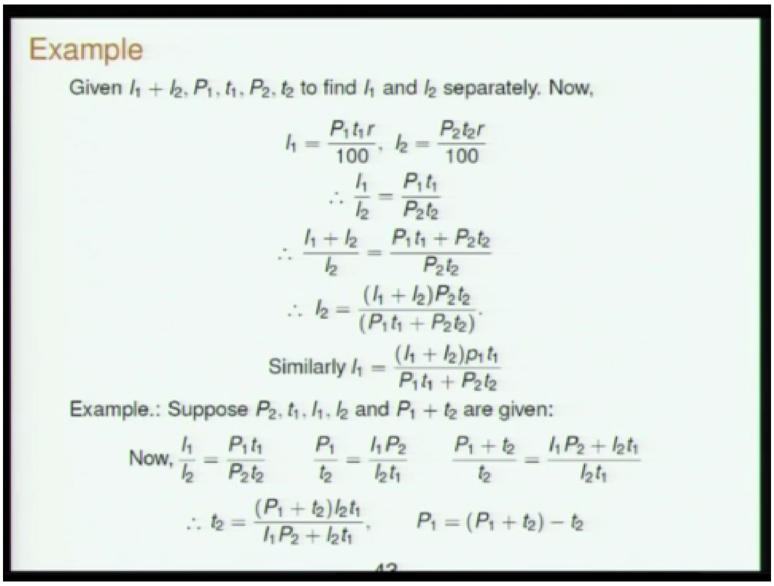

See these are all you know clearly this is of mathematics read the stage where they doing something from which are not directly relevant if nobody will give you know some of the principal and time, but to see you know that generally how one can solve this problem using mathematical probably we should have the mathematical problems all this kind of equation so that is why he is trying to solve this kind of equation.

He suppose you are given i1+i2, P1, t1, p2, t2, so you have to find i1 and i2 separately ok, so I giving the principal and the time length corresponding to this they are given and some of the interest is given. So then you have to find i1 and i2, so then they use the fact that they are rented at the same rate, rate is the same. So i1 is P1t1r/100, i2=P2t2r/100, so i1/i2 it is and finally so you get one equation, you know in terms of a i1 and i2.

I1+i2 to give the equation, so one give i1+i2 is already given you know one more equation involving linear equation involving i1 and i2. So we can find them separately. So that is what is done.

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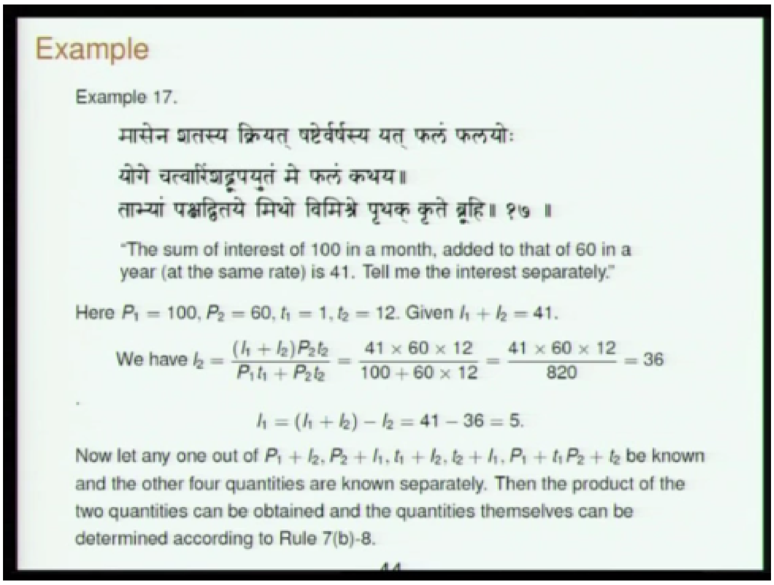

See for instance he says the sum of interest (FL) the sum of interest of 100 in a month, added to the 16 in a year is at the same rate is 41. So tell me the interest separately. You see so suppose P1 is given P2 is there, t1 is 1, t2=12 and i1+i2 is given ok, he suppose you are given post like this I am sure most people will give you (FL) and they all simple principles but still you have to use this logic and all that.

So you **you** find this equations, you get one more equation i2 you know, you will find that i1 will be i2 is 36 and i1 will be 5. So similarly if any of these other quantities you know P1+i2, P2+i1, t1+i2, t2+i1, P2+t2 be known for and the coma yet, so then also other things can be determined ok.

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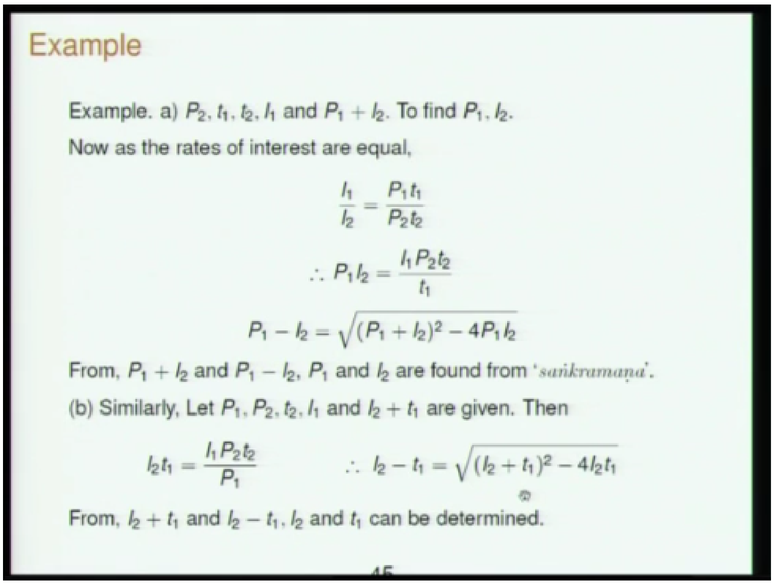

Some of them will involve some quadratic equations ok, see for instance you are given P2, t1, t2, i1 and P1+i2, we have to find out P1 and i2 you have to find, so i1/i2 is these remember the rates are equal, so P1i2=i1P2t2/t1 these things. So from this we can find out P1-i2 right, if P1i2 is this then P1-i2 is this, square root of P1+i2 whole square-4P1i2, this is the simple entity, you see and P1=i2 is already given.

So you have from P1+i2 and P1-i2 already knew P1+i2, so P1 and i2 can be found by (FL). So this is the way these are done, so intermediated various kind of you know, quadratic equation that have been come and as to square that have been come, so these are the kind of he gives some example of that.

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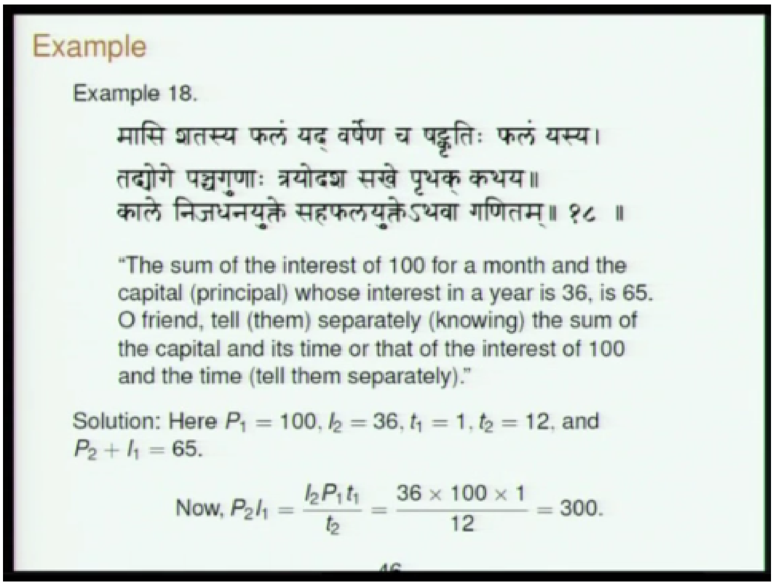

So now I think this is the of all interesting.

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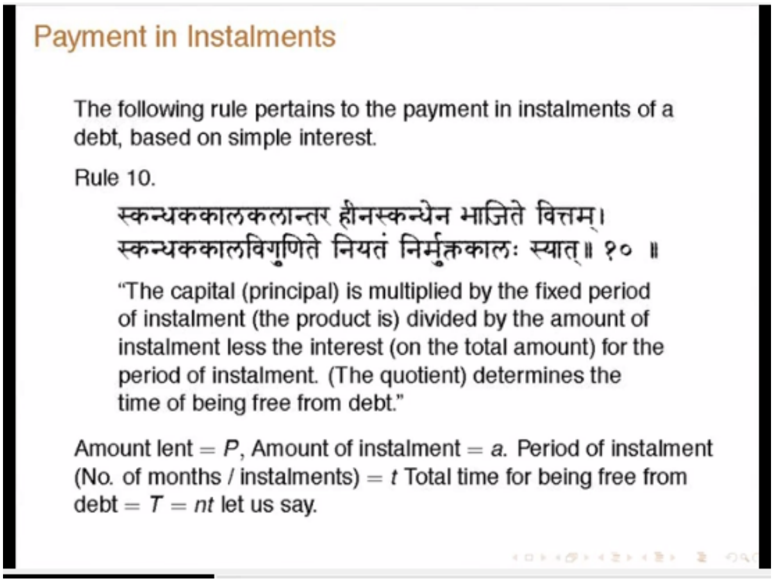

So I will discuss some payment in instalments which are discussed by Brahmagupta, so we all see this interest calculator even in Aryabhatta it is there, interest calculation is there, (FL) it is there, of course (FL) so here also I can find, but you know as I told you he advance it soon, so he talk of you know payment by instalments and what you know very close to these equation and the instalment we will come to that. so thus far want to discuss.

So he says the capital is multiplied by the fixed period of instalment is divided by the amount of instalment less the interest, the quotient determines the, time of being free from debt. So this is what is this verse says. So suppose the amount lent is P, the amount of instalment is Armstrong ok and period of instalment is number of months/instalments, so need to be given once in 2 months or once in 3 month whatever it is.

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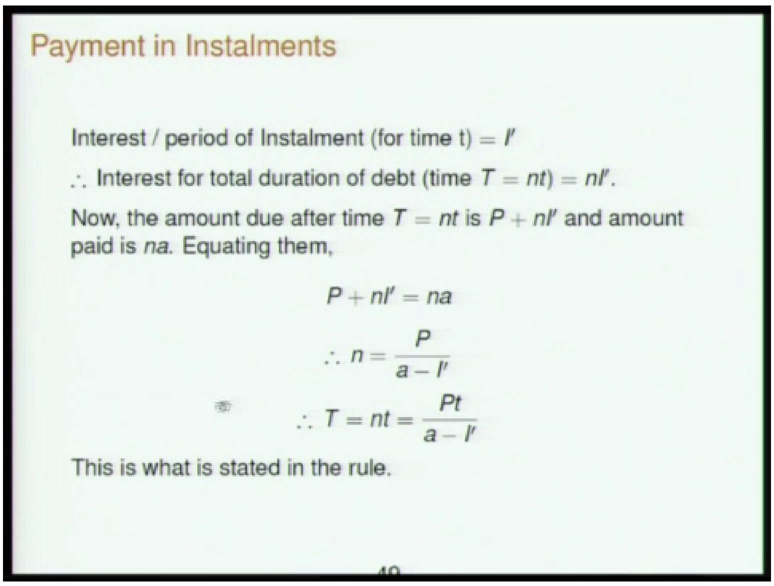

Let that be t and that total time for being free from debt that is T and nt let us say ok. So then in this case so interest/period of instalment id i prime and interest for total duration of debt is that is multiply by n ok n instalment period are there n*iprime. So the amount due after time T is P+niprime right n instalment and nprime is the interest for it which is calculated you know which the lender is specifying.

And the amount paid is ni because he is paying nm instalments of a each. So P+niprime=na, so n the number of instalment is given by nt, so pt/a-i, so this is what is stated in the rule. Ok so everything is simple interest ruling, compound interest is not coming, so these only calculate, these are simple enough, so one example you can write for this.

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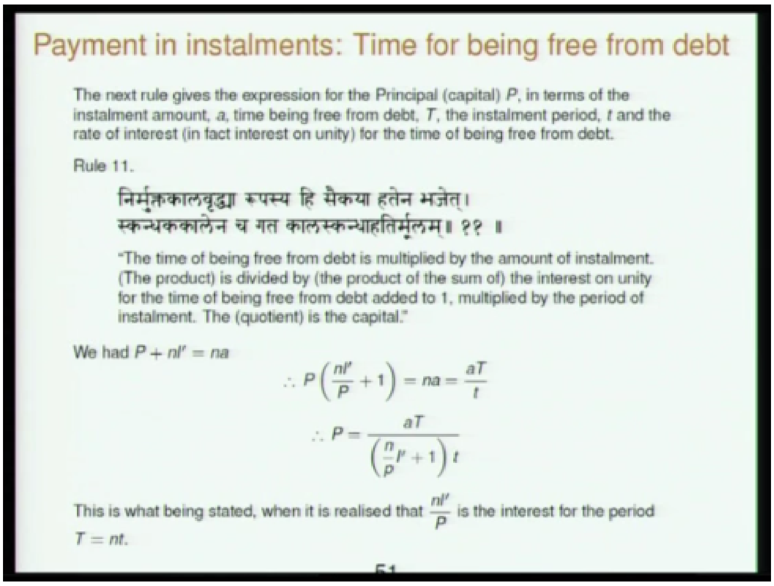

So again you can go on like this.

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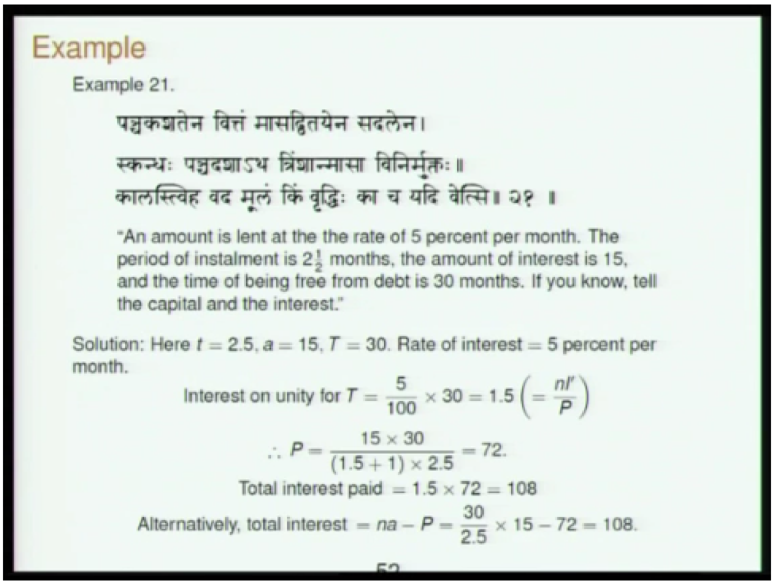

Now we will discuss a more realistic computation.

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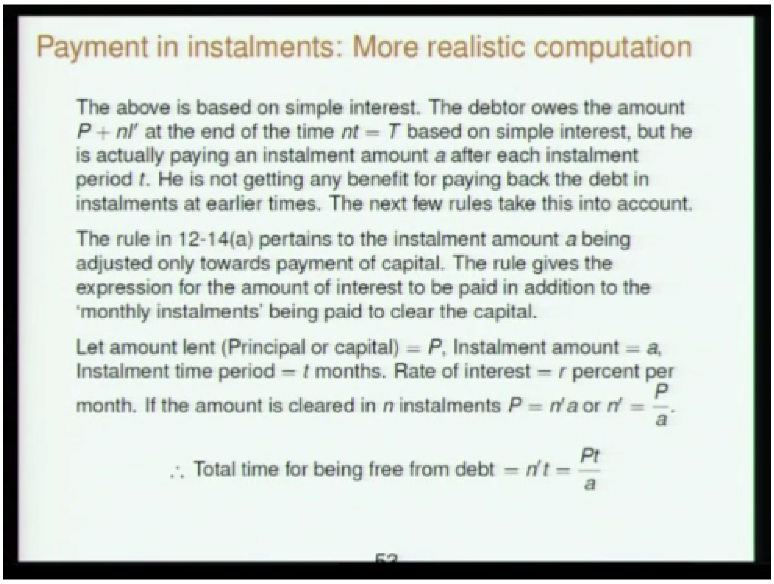

See here you are paying an instalment ok, see if your principle is P ok.

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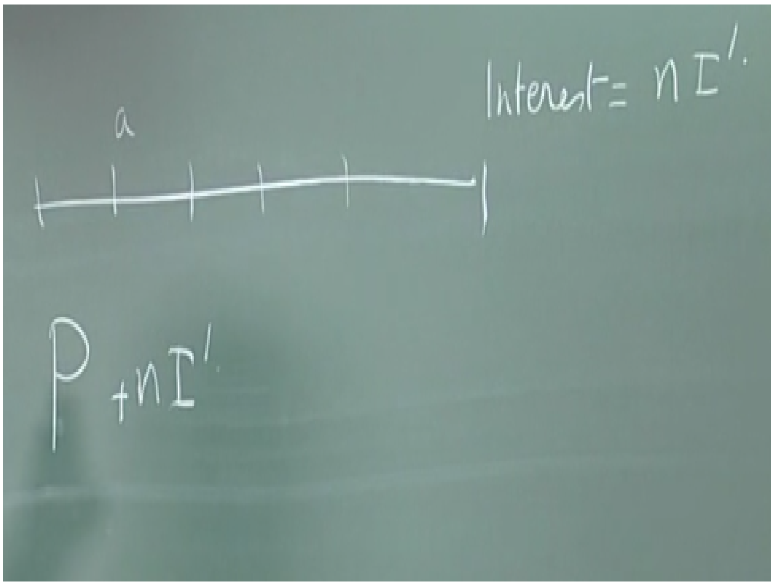

And you are paying between n instalment ok you write it like this of a each ok, os the total interest you have paid is equal to you know is n*amount for instalment niprime ok and finally you are giving P+niprime, so P+niprime you are giving principle+interest, but you are not paying at the end you know, you are paying in the instalment is beginning itself you are paying you see here you are paying 1/n of this and so on.

You know that is you are not getting any benefit for paying it earlier ok, suppose of 5000 rupees you have taken a loan and suppose 10% interest ok, so then after 1 year so you have to pay 5500, suppose you are paying 5500/12 you know every month see then you are loosing because you could have pay 5500 at the end you know and you could have done something with your money. So you are not getting any benefit from paying earlier in instalment.

So that is what is calculation now ok, so he says you know so this rule contains but in the instalment amount being adjusted only towards payment, he is considering only instalment amount adjust towards payment of capital, he has to pay interest also. The rule gives the expression for the amount of interest to be paid in addition to monthly instalment being paid to clear the capital. So let the amount lend will be P, so the instalment amount is a.

And instalment period is t month okay and rate of interest is t or interest r percent per month, so if the amount is cleared in n instalments, so P is nprimea or nprime is P/a. The rate of interest is is amount that should be n, so it cannot be integer ok, is the amount is cleared in n prime is instalment, P=nprimea ok. So emphasize it cannot be integer I am putting them as prime, so nprime is P/a ok, this is a is only for prime and the principle, interst is separately paid ok.

So this is the different problem today, so interest is separately paid, but a is paid only to clear the principle, so the total time of being free from debt is nprime*t, so that is The patient/a ok suppose P instalment is here, P is or n prime is number of instalment and instalment period is t month. So the total time of is nprimet, so n prime is P/a, it is Pt/a, so this is the total time, so n prime need not be the integer.

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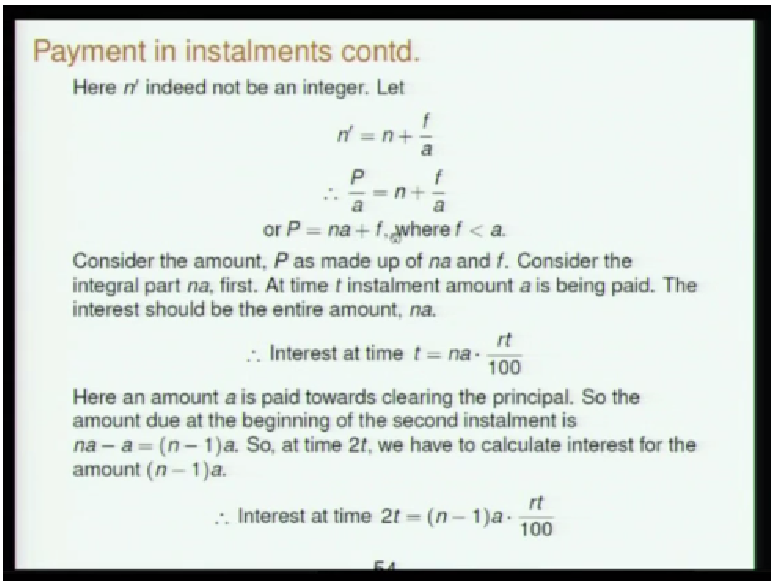

So that is why I wanted to say, n prime is n+f/a, that is integer (()) (42:40) n is integral ok and n prime is actual that is you know he may talk 2 and half month or 2 months and 1 of the month to clear, so that is allowed n prime is n+, so P/a is this, so P=na+f where f is greater than a,. The principle n* or there is nothing p n prime a n prime number of instalment is a nad so on, n i is n prime it is not a integer.

So which has na+f ok, so consider the amount p has made up of na and f, consider the integral part na, first if n is integer, so at time t instalment amount is a is being paid ok ok. So these are the instalment period. So here a is paid ok, so then the interest should be the entire amount na, the amount of interest now he is know 32 for this is n because the whole money is t, here whole money is here.

So you have to give the interest of the whole thing, so interest at time p that is at the first step is na*rt/100, because and now at this stage a amount a is already paid to clear the principle, the amount a is paid to clear the principle, so the amount due at the beginning of the second instalment is na-a or n-1a. So at the time the principle is n-na, so at the time of next instalment you have to calculate the interest for this.

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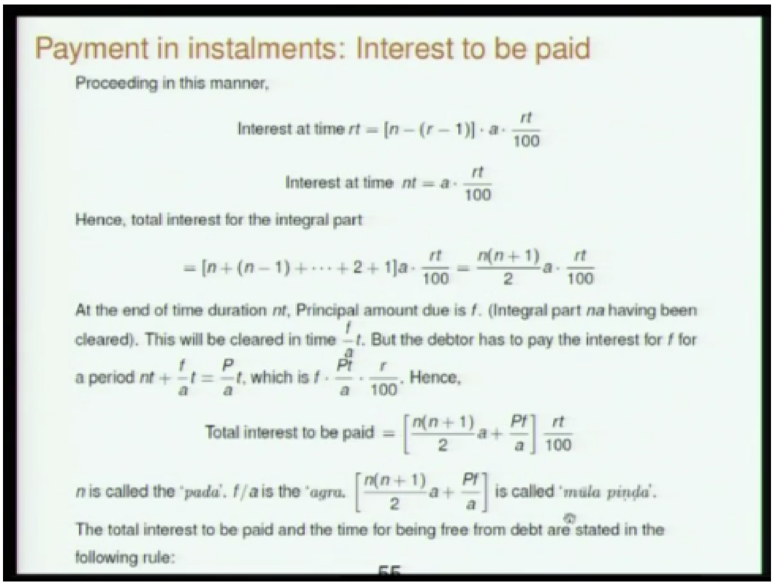

So time t2, t2 you have to calculate interest for the amount n-1a, you remember t is the amount period of instalment ok, t is the period of instalment ok. So interstate at time 2t is n-1a*rt/100 and similarly you can go on at interest at time r ok, of all period of instalment are this thing you will have, so them it will be n-r/a that is the amount of principle which is being which is own by the later right, for interest rtn-r-1 because this is the first full thing is, so r-1 you have to put, so this is the amount which is owned.

So that you have to calculate the interest for this ok, so n-r-1*rt/100 and interested time nt suppose after that you know he is it will just a at this point you see at end ok the only a will be the carried amount principle which he has these things, so that all will be clear there ok, so it is rt/100, so total interest for the integral part, I am going to take integral part see so p is you know remember P is nprime a=n+f whole square a, so this is how discuss.

So total interest for the integral part is n+n-1 etc. up to 1, so you are getting portion here say n*n+1/2*a*rt/100, and at the end of duration nt see now after the duration principle due is f, or it refers to na+f, so after this this is still f so that for you have to pay the interest also, so this will be clear in time f/at, but the debtor has to pay the interest for f for a period of nt+t/a*t it is calculation we have only consider integral part.

We have to have to pay the interest of the whole thing, the total interest to be paid n*n+1/2*a+Pf/a*rt/100. So this n is called the pada, f/a is called agra, and this called it is (FL) so this si the for this is the amount it has to be apart from the money he has given to create a principle you see at every instalment he is giving, he has to give the interest, suppose he when this interest will be create that every month to be paid you know instalment.

So these are thing, so here is the number of instalment is after this if a is specified so the fraction for the fractional month you know this is the fraction the whole thing is cleared in fraction number of integral+fraction number, so that is prosily what is so these are rule.

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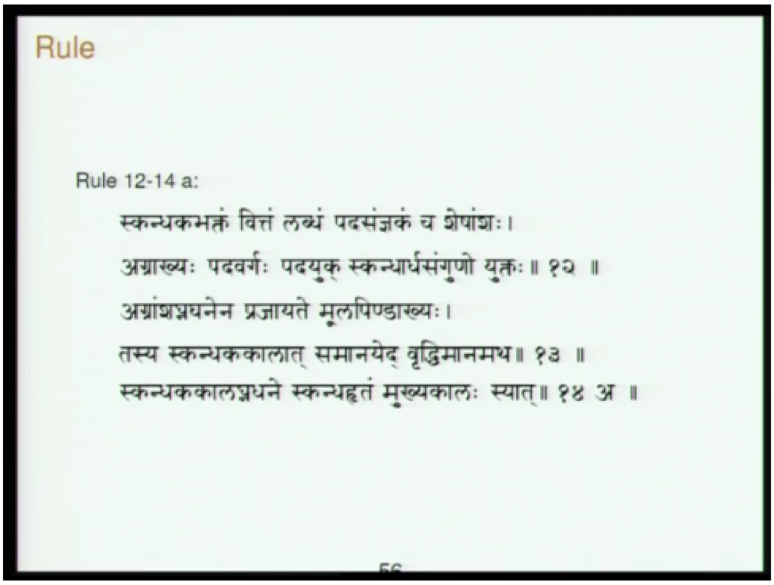

That is here (FL) etc.

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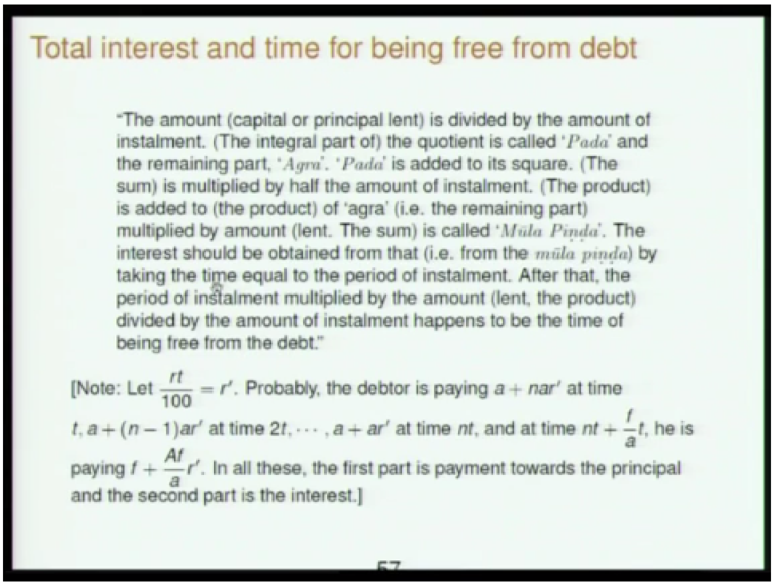

So what is saying the amount are capital and principle and divided by the amount of instalment and the integral part of the quotient is called (FL) the remaining part is called (FL) so on and so forth we just we have this what we have to what is this si the total interest this is what we have to do even this si the formula and that is expressed in verse (()) (49:10) so now we consider little more discuss of this, so here he is paying see the amount a is paying for instalment that is only for clearing the principle.

So the interest he is doing separately so that is why we did this thing you know whatever amount of principle he has to pay amount he has to be debt and consider as and interest he has calculate separately, so now he does get the thing all whatever amount is a he is paying, so that is partly towards principle and partly towards interest. So that is what he is considering next.

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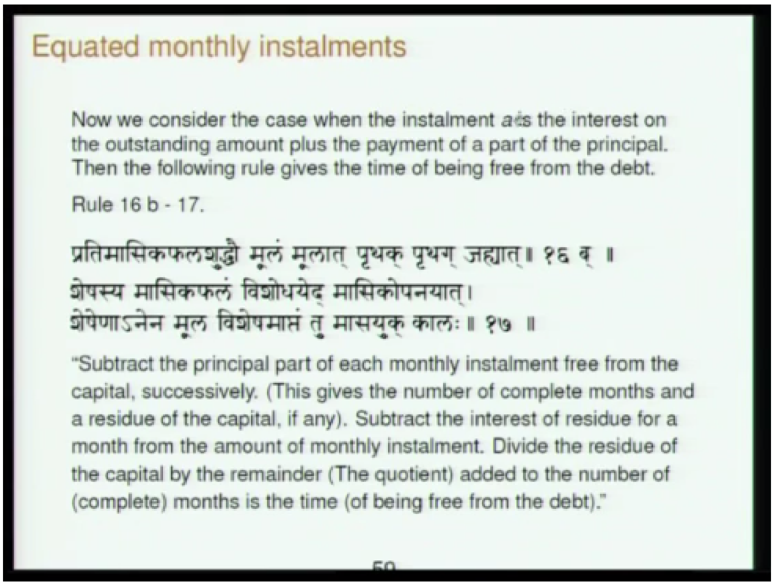

He consider the pay of instalment a is interest on outstanding amount plus the payment as a part of the principal. Then the following rule gives the time of being free from debt. (FL) subtract the principle part of each month instalment free from the capital, successively. THis gives the number of complete months and a residue of the capital if any and subtract the interest of residue of the capital by the remainder.

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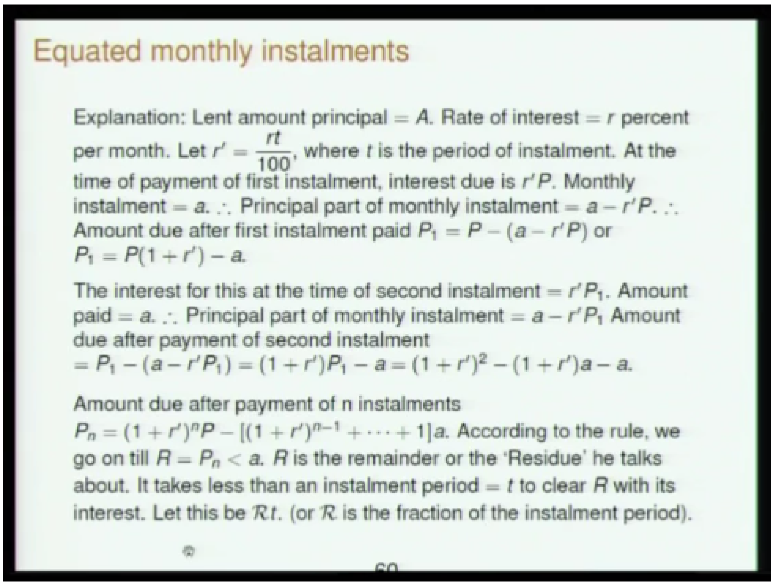

Divide the residue of the capital of the number, the quotient added to the number of complet months is the time of being free from the debt. What he is trying to say see that the amount of principle be a rate of interest is r+n, so I call rprime rt/100 t is the period of instalment at the time of first instalment interest due is r primeP right. P is everything and monthly instalment is a, and principle part of monthly instalment a-rprime.

Because interest for this at the is called this, so amount due of the first instalment pay is P-this, we have rate is P and then they calculate the principle part of how much have cleared, you know that is given clearly by this, so this P1 is this, the interest for this certain time of second instalment is r prime*P1 not r prime P, and amount paid is a, so then we have to find principal for whatever he says.

Now principle for the monthly instalment a-rprime P1, and amount due of the payment is second instalment is this, and amount pay die of the payment n instalment will be this kind of thing you know and this kind of geometric position will come, according to rule we go on till r=Pn greater than a, all the remainder of the residue he talks about it, so he takes less than an amount period to clear or which is interest ok, so is less than Armstrong

So amount of instalment you know or if it is full instalment period you know you have to pay a but only small part yes pay less than a, so that is good all the things can be calculated which P is the principal, and principle a actually I used it here amount of P as re-correct, a is the same as P, I use it here and I use it afterward here, P is the principle here, ok that fraction can be calculated or we say given example then it is very clear to u.

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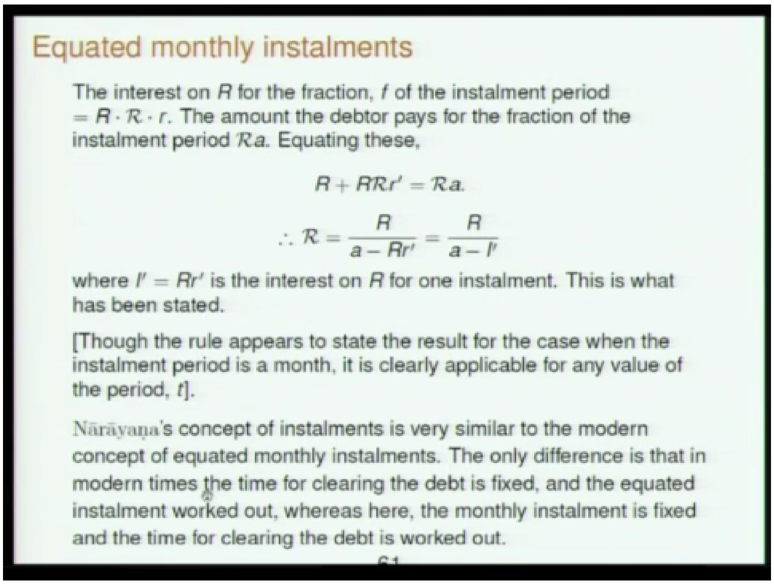

In (FL) instalment is very similar to the modern the time for clearing the dirty sticks, ok and of course compound interest is there, time for clearing the dirt is there and then you 5, 60 this is indicated month instalment, the best and compound interest, here the amount of instalment is fixed and the time is being everything calculated and taking into own fact that , so compound interest he did not talk about.

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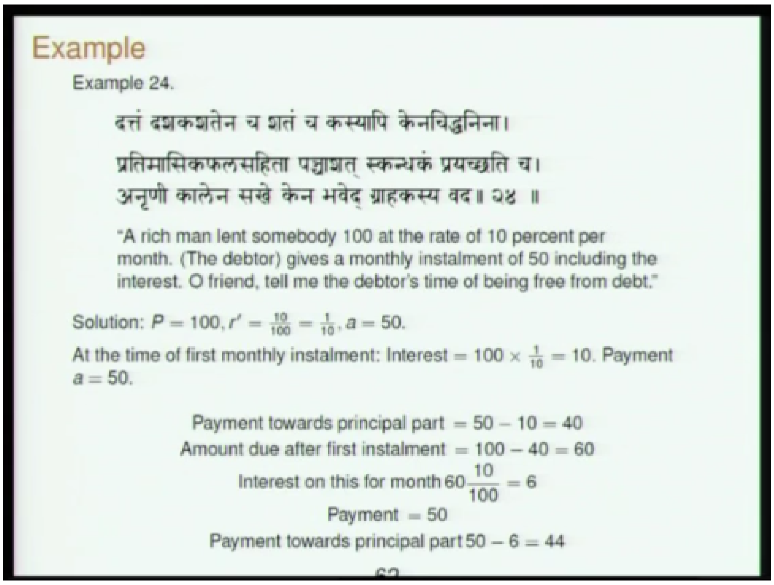

But still it is you know whatever amount of clearing that has been you know taken into account ok for giving relief for that, so realize that you know principle is you know being clear. So if I give an example sorry we ran out of time let me do it, a rich man lent somebody 100 at the rate of 10 percent per month, the debtor gives a monthly instalment of 50 including interest, O friend tell me the debtors time of being free from debt, so P is 100, the rate of interest is 10.

Ok, 10% from the high pertaining it, so a is 50 amount of instalment, so at the time of first monthly instalment in interest is 100*10 1/10 right and how see is, so 10, so payment is 60, so he is clearing the interest of 10, so payment was principle part, so rest is payment verse principle part 40, so amount due after first instalment is 100-40=60, ok to get a point after we saying 100 was taken, so one period of instalment one month 10, you know is the interest, so he is paying 50.

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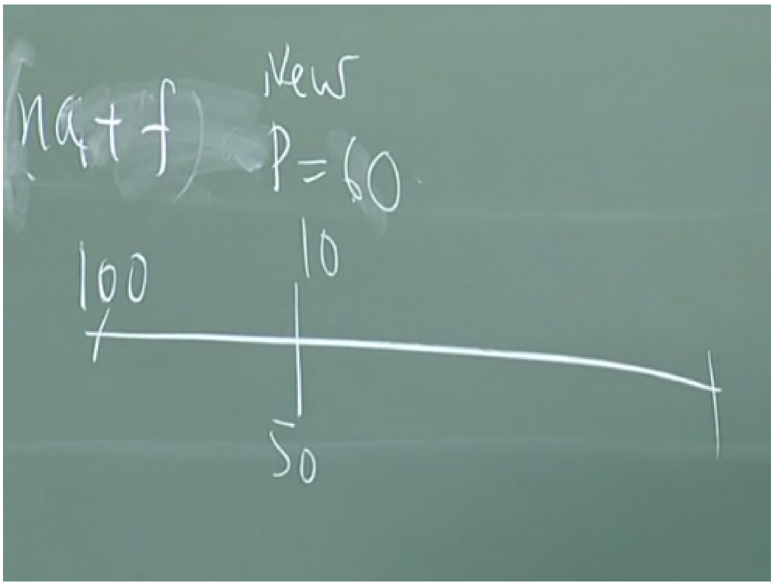

So if you subtract the 10 ok, so 40 is pay for specifically for clearing the principal, so principle now will be due principle, so that will be equal to 40, sorry 60, and interest for this month you know for month that is 36 ok, so after 1 month so the interest will be 6, ok so in that case the payment is 50, ok so the payment towards principle is 44, ok so that is 44, so how much is there, so it is (FL) 44.

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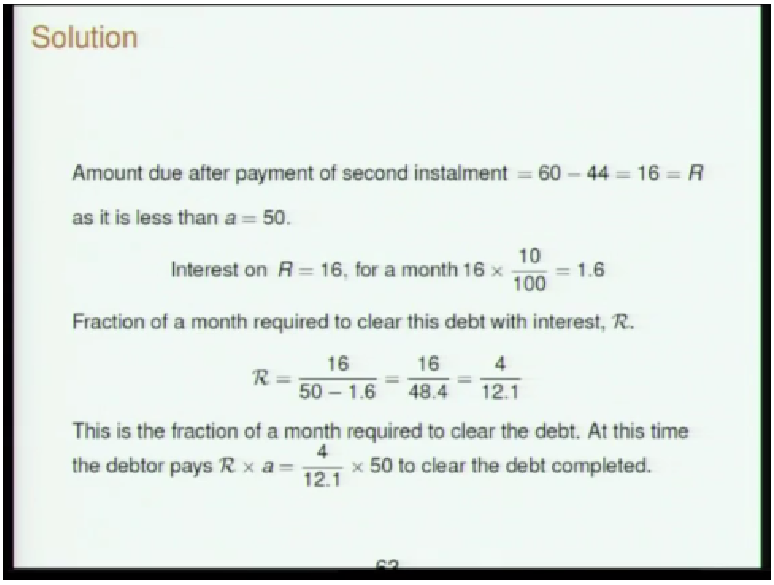

So the amount that you get is 60-44 is here you know so 44 is pay towards principle, 60 what was new or the principle, so 16 is there now the new principle is 16, so he has to pay the principle amount of 16 at the end of 2 months. Now 16 is less than 50, ok so interest and R=16 for a month is 1.6, so the fraction of the month is required to clear this date with interest are so the fraction updated one can find out for that is 4/12.1.

So after this, this is a fraction is a month required to clear the debt, see after this after this only this fraction 4/12/1 one month is clear, to clear the remaining principle amount 16 and the interest on that, so that is taken here of this denominator, so if you go through this it will be so that is how he has, so in everything I did was told you it is in some more advancement is there in some more generalisation relation and some new methods and new principle.

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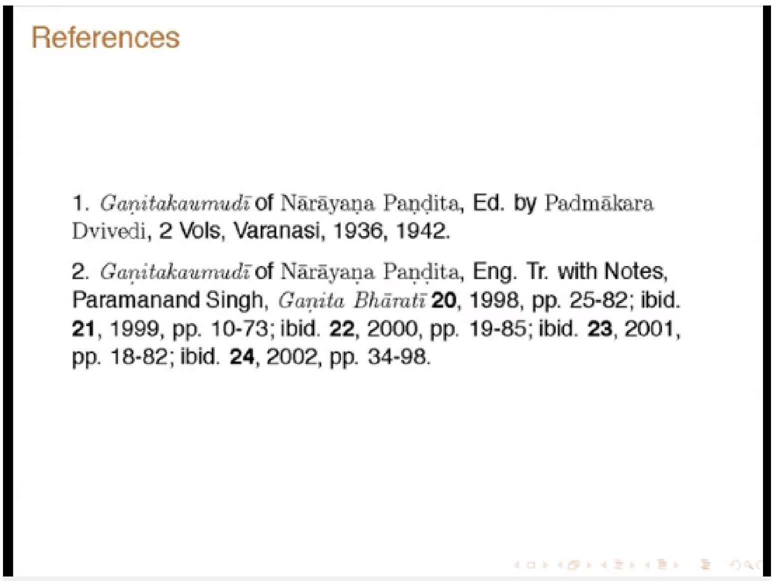

The references are given here, thank you.