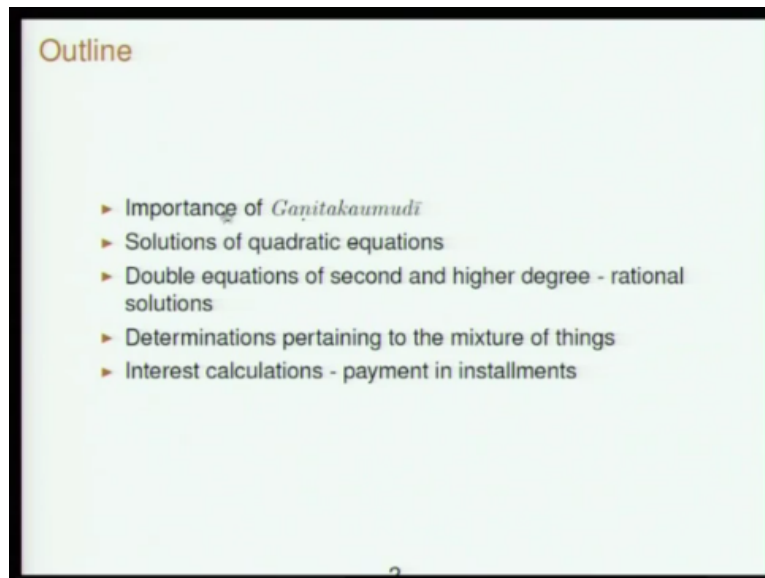


**Mathematics in India: From Vedic Period to Modern Times**  
**Prof. M.S. Sriram**  
**University of Madras, Chennai**

**Lecture-25**  
**Ganitakaumudi of Narayana Pandita 1**

Ok this will be first of the 3 lectures in Ganitakaumudi so I will be giving, so the outline where we first talk about the importance of Ganitakaumudi .

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And some important topics here solutions of quadratic equations, double equations of second and higher and higher degree rational solutions, then determination pertaining to the mixture of things, the interest calculations, payment instalments have some interesting this is some methods which are given here which I have given today.

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## Nārāyaṇa Paṇḍita's *Gaṇitakaumudī*

*Gaṇitakaumudī* was composed in 1356 CE by Nārāyaṇa Paṇḍita as indicated in the final verses of the work. It is not clear where he was born, or where he flourished. It was published by Padmakar Dvivedi in two volumes, based on a single manuscript, which belonged to his late father, the legendary Sudhakar Dvivedi, in 1930's. There is another work of Nārāyaṇa Paṇḍita entitled '*Bijagaṇitāvataṃśā*'. Only the first portion of this has been published, based on a single and incomplete manuscript at Benares.

*Gaṇitakaumudī* has been translated with explanatory notes by the late Paramanand Singh of Bihar. The translation and notes are published in Volumes 20-24 of *Gaṇita Bhāratī*, an Indian journal devoted to history of mathematics, during 1998-2001.

So Ganitakaumudi was composed in 1356 common era by Narayana Pandita as indicated in the final verses of the work. So it is not clear where he was born, or where he flourished. It has this work was edited in public by Padmakar Dvivedi in two volumes based on a single manuscript which belonged to his late father, the legendary Sudhakar Dvivedi in 1930s. Sudhakar Dvivedi famous person who edited and translated many of these important works.

In Indian astronomer in mathematics in 19th century onwards, he is the another persons who involved in the revival important part of revival of interest in in Indian mathematics in astronomy. So Padmakar Dvivedi his son. There is another work of Narayana Pandita called (FL) only the first portion of this have been published and based on a single and incomplete manuscript at Benares.

So Ganitakaumudi has been translated by Paramanand Singh when Parmanand Singh with explanatory notes it was published in these volumes of Ganitakaumudi general of Indian mathematics, history of mathematics mostly Indian, but others listing history of mathematician other cultures also is there in this should they completely devoted to the history and he published this between 1998 to 2001.

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## Post Bhāskara Indian Mathematics

After Bhāskara-II, there were two major developments in Indian mathematics: (i) Nārāyaṇa Paṇḍita's *Gaṇitakaumudī* and (ii) Kerala school of mathematics and astronomy, mainly during 14<sup>th</sup> – 17<sup>th</sup> centuries, wherein calculus concepts were developed.

Nārāyaṇa Paṇḍita carries forward the tradition substantially. There are more formulae, generalisations of earlier results, and systematisation. There is a big leap in the treatment of combinatorics and magic squares in chapters 13 and 14, which will be dealt with separately. It was meant to be a comprehensive text, covering most of the prevalent mathematical knowledge in India at the time of its composition, and making substantial additions to it.

But this is a very big work after Bhaskara II there are 2 major development in Indian Mathematics, so 1 Narayana Pandita Ganitakaumudi and other the kerala school of mathematics and Astronomy mainly during between 14th and 17 centuries wherein calculus concepts for developer. They have already had a little bit about it, we will hear it in more detail in the lectures to follow.

So Narayana Pandita carried forward the tradition of mathematics in India substantially. There are more formulae, generalisations of earlier results and systematization, so entire we have seen it know from brahmagupta and others carried forward in putting up of Mahavira whose we discuss, so then carried forward more by Bhaskar Acharya in his 2 words (FL) and Lilavati. So we saw there are many advancements in that compared to the earlier hours.

So similar here also Narayana Pandita carries forward (FL) introduce more advances and he this is (FL) something more about it in a generalized decision that seems to be the way he seems to be working and that is the way it is written and especially with a big leap in the treatment of combinatorics and magic squares in chapter 13 and 14. So the other things are also there, there is a substantial improvement in cyclic quadrilateral rational figures and so on.

But one more very important result about the (FL) carry out some of integers (FL) already discuss in general result, so apart from those things the major results are contained in chapter 13 and 14, so which will be dealt with by others, should not be a comprehensive text covering

mostly prevalent mathematical knowledge in India at the time of composition and making substantial edition to it ok.

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**Contents of Gaṇitakaumudī**

I will give a brief summary of the 14 chapters below.

- ▶ Chapter 1 is on measures of weight, length, area, volume, capacity etc., 8 operations namely, addition, subtraction, multiplication, division, square, square root, cube and cube root described. Solutions of more complex (compared to earlier works) linear and quadratic equations are discussed.
- ▶ Chapter 2 is on '*Vyavahāra gaṇita*' or 'mathematics pertaining to daily life'. Calculation pertaining to mixture of materials, interest on a principal, payment in instalments, mixing gold objects with different purities and other problems pertaining to linear indeterminate equations for many unknowns are considered. There is substantial progress compared to earlier treatments. Example: Interest calculations where payment by instalments are considered.

So I will give a brief summary of the 14 chapters in this verse, so chapter 1 is on measurers of weight, length, area, volume, capacity etc. as in other works put 8 operations we saw it in Lilavati also addition, subtraction, multiplication, division, square, square root, cube, cube root. So they are described and solutions of more complex linear and quadratic equations are discussed here because things are been discussed in earlier works.

And he is thinking to generalized the more and make it a little more complete, and chapter 2 is on Vyavahara Ganita or mathematics pertaining to daily life, so calculation pertaining to mixture of materials interest and principal, payment in instalments, mixing gold objects with different purities, and other problem pertaining to linear indeterminate equations for many unknown they Aryabhatta considered.

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### Contents of Gaṇitakaumudī contd.

- ▶ Chapter 3 on arithmetic and geometric progressions, where the concept of 'sum of sums' is generalised to the ' $k^{\text{th}}$  sum' of the first  $n$  integers for arbitrary  $k$ . This is a very important generalisation, which is needed for finding the infinite series for sine and cosine functions, among other things.
- ▶ Chapter 4 is on plane geometry where properties of triangles, quadrangles, circles, cyclic quadrilaterals etc., are considered. The 'third diagonal' associated with a cyclic quadrilateral and the area and circumference of the same are discussed.
- ▶ Chapter 5 to 8 on three dimensional geometry, where rules for solid figures and capacities of excavated volumes are presented.
- ▶ Chapter 9 is on the *Kuṭṭaka* procedure for solving linear indeterminate equations of the first degree.

And there is what a progress compared to earlier treatment, example I will take this interest calculations where payment by instalments are considered I will deal with it a little in detail and chapter 3 is an arithmetic and geometric progression where the concept of sum of sum is generalised to the  $k^{\text{th}}$  sum of the first  $n$  integers for arbitrary  $k$ . So where Aryabhata himself has given the sum this is  $\sigma(r)$  then if you take  $r^{k+1/2}$  it is  $\sigma(1-n)$ .

So that is the second sum, so like that you can go on, so he has given the formula for the  $k^{\text{th}}$  sum, it is a very very important generalisation which is needed for finding infinite series for sin and cosine functions among other things. So that will be discuss later how it is used and how exactly does it figure. Chapter 4 is on plan geometry where properties of triangles, quadrangles, or quadrilateral, circle cyclic quadrilaterals etc. are considered.

Now he introduced the new concept the third diagonal associated with a cyclic quadrilateral and then area and circumference of the cyclic quadrilateral are discussed. Then chapter 5-8 there are three dimensional geometry where rules of a solid figures and capacities of excavated volumes are presented and chapter 9 is on *kuttaka* procedure for solving linear indeterminate equations of first degree. So I will not deal with it already has been done in detail.

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**Contents of Gaṇitakaumudī contd.**

- ▶ Chapter 10 is on quadratic indeterminate equations or 'Vargaprakṛtī'. Here a variant of the 'Cakravāla' procedure of Jayadeva and Bhāskara is also considered.
- ▶ Chapter 11 and 12 is on the divisors of a number and fractions.
- ▶ Chapter 13 is on 'anikapāsa or combinatorics, partitions of numbers, sequences of binomial and polynomial coefficients, and various 'Meru's.
- ▶ Chapter 14 is on Magic squares.

I take up some important mathematical results of Gaṇitakaumudī, pertaining to chapters 1-12 in what follows.

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Then chapter 10 is on quadratic indeterminate equations or Vargaprakrti, so which was done earlier in yesterday here. Here a variant of the Chakravala procedure of Jayadeva and Bhaskara is also considered, yesterday it was mentioned that you know when you are going from one step to another this Pi will come (FL) quantity then you have to (FL) –d, so that is minimum. So he did a small change here that Naraya Pandita consider.

And you know that Chakravala procedure is due to Jayadeva and Bhaskara small change ok. So in chapter 11 and 12 is on divisors of a number and fractions and chapter 13 is on ankapasa or combinatorics, partitions number, sequences of binomial and polynomial coefficients and various Meru's. So will not be doing much but little bit I will do, little bit in this I will do, but it done by others.

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**Arithmetical operations and saṅkramaṇa**

Chapter 1

After discussing the eight operations, various simplifying manipulations involving fractions are discussed. In the earlier texts, 'saṅkramaṇa' (or concurrence) problems involved solving for  $x$  and  $y$ ; given  $x + y = a$  and  $x - y = b$ ,  $\implies x = \frac{a+b}{2}$ ,  $y = \frac{a-b}{2}$ . In Gaṇitakaumudī, problems which can be reduced to saṅkramaṇa are discussed.

For example. Rule 31: Given  $x^2 - y^2 = a$ ,  $x - y = b$ , we find  $x + y = \frac{x^2 - y^2}{x - y} = \frac{a}{b}$  and  $x, y$  can now be solved as we know  $x - y$  and  $x + y$ . Similarly, given  $x^2 - y^2 = a$ ,  $x + y = b$ ,  $x - y = \frac{a}{b}$  and solution by saṅkramaṇa.

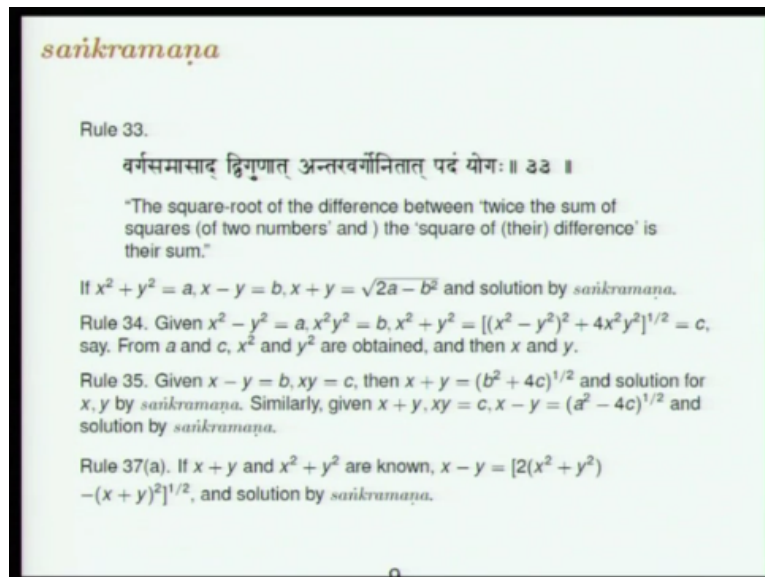
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And chapter 14 is on magic squares and so basically I will do pertaining to chapter 1-12 in what follows 1 and 12 in what follows ok. So in chapter 1 I just discuss some important things only, got a comprehensive you know lecture giving me know the word content of Ganitakaumudi the content of Ganitakaumudi comprehensively, what were the important thing is there I will try to emphasize that.

Say for instance of discussing 8 operations various simplifying manipulations involving fractions are discussed and in earlier text sankramana already mentioned, so that is the sum and difference number of given you can find the number, so this is sankramana  $x+y+a$   $x-y=b$ , then  $x$  is  $a+b/2$   $a-b/2$ , see in Ganitakaumudi problems which can be reduced to sankramana are discussed.

For instance suppose you are given  $x-y=b$  and  $x^2-y^2=b$ , so then you can find  $x+y=x^2-y^2/x-y=a/b$ . So  $x$  and  $y$  cannot be solved as we know both  $x-y$  and  $x+y$ ,  $x-y$  is given and  $x+y$  also found. Similarly suppose you have given  $x^2-y^2$  and  $x+y$ ,  $x-y$  can be found out similarly and we can solve it by sankramana.

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Then little more trivial (FL) the square root of the difference between twice the sum of the squares of two numbers and the square of the difference is their sum. So now suppose you are giving  $x-y=b$ ,  $x^2+y^2=ab$ , so then  $x+y$  can see that will be  $2a-b^2$  and you can solve it by sankramana. Similarly suppose little one more these thing given that suppose  $x^2-y^2=Armstrong$

And  $x^2 + y^2 = b$ , so then you can find out  $x^2 + y^2$  as you know  $x^2 - y^2$  and  $4x^2 y^2$  whole squared to the power of  $1/2$ . So you have got  $x^2 - y^2$  and  $x^2 + y^2$ . So from this  $x^2 + y^2$  are obtained and then you can obtain  $x$  and  $y$ . So then similarly suppose  $x - y$  is given and  $xy$  is given, so  $x + y$ ,  $b^2 + 4c$  whole to the power of  $1/2$  and again we can find out by  $x$  and  $y$  by *sankramana*.

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### Solution of the quadratic equations

Rule 39-40 (a).

रूपोत्थहृतपदाग्रे स्यातामन्तरवधौ ततस्ताभ्यम् ।  
 प्राग्बद् योगः साध्यः स्यातां सङ्गामतो राशी ॥ ३९ ॥  
 क्षयगे मूलेऽनल्पं तत्कृती राशिः । ४० ।

" 'Pada' and 'agra' (*drśya*) divided by 'rūpottha' happens to be the (so called) difference and the (so called) 'product' respectively. Calculate the (so called) 'sum' from them by the method stated earlier. The numbers are (obtained from them) by the method of concurrence. (In case) *Pada* is negative, the greater number is to be taken. The square of that is the number."

The rule gives the solution of a quadratic equation of the type  $ax + b\sqrt{x} = c$ .  $a$  is the '*rūpottha*',  $b$  is the '*Pada*' and  $c$  is the '*drśya*', also '*agra*'. Find  $\frac{b}{a}$  and  $\frac{c}{a}$ . The root of the equation is  $\sqrt{x}$ . Consider an auxiliary quantity  $\sqrt{y}$ , when  $b$  is negative.  $b = -|b|$ . In this case,

$$\sqrt{x} - \sqrt{y} = \frac{|b|}{a}, \quad \sqrt{x}\sqrt{y} = \frac{c}{a}$$

Similarly given  $x + y$  and  $xy$  also similarly we can do that and suppose  $x + y$  and  $x^2 + y^2$  are known then  $x - y =$  this quantity and you can solve it by *sankramana*. So then he discuss the solution of quadratic equations, so these all what he says (FL) divided by *rupottha* happened to be the so called difference and the so called product respectively. Calculate the sum from them by the method earlier.

The number should obtained from them by the method of *sankramana* in case *pada* is negative the greater the number greater the number to be taken, the square of that is a number. So whatever you doing this *sankramana* thing you know leading him onto the quadratic equation solution. So what you have to do is always had already written down equations like this in my previous lecture.

$Ax + \sqrt{x} = c$ , so  $a$  is the (FL) coefficient of  $x$ ,  $b$  the *pada*, 2 portion of the root  $x$  and  $c$  is called (FL) now find  $b/a$  and  $c/a$ , suppose the root of the equation is root  $x$ , so consider auxiliary quantity root  $y$  when  $d$  is negative there is  $b = -$  not be in this case root  $x -$  root  $y +$  this



and  $\text{root}x \cdot \text{root}y = c/a$ . So then I mean these are the other root somewhat different, he is trying to use the things so you find this quantities.

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### Quadratic equation

Then, from rule 35,

$$\sqrt{x} + \sqrt{y} = \frac{(b^2 + 4ac)^{1/2}}{a}$$

$$\sqrt{x} = \frac{|b| + \sqrt{b^2 + 4ac}}{2} = \frac{-b + \sqrt{b^2 + 4ac}}{2}$$

In this case,  $\sqrt{x}$  is the greater quantity out of  $\sqrt{x}$  and  $\sqrt{y}$ . When  $b$  is positive, it is implied that

$$\sqrt{y} - \sqrt{x} = \frac{b}{a}, \quad \sqrt{x}\sqrt{y} = \frac{c}{a}$$

and  $\sqrt{x} = \frac{-b + \sqrt{b^2 + 4ac}}{2}$

(In this case  $\sqrt{x}$  is the smaller quantity out of  $\sqrt{x}$  and  $\sqrt{y}$ .)

[In either case,  $\sqrt{x}$  is the correct solution using 'modern' standard way. Here  $c$  comes in the RHS, so we get,  $b^2 + 4ac$  inside the square root, instead of  $b^2 - 4ac$ . It appears that Nārāyaṇa is giving only the positive root, so only one root is presented.]

Similarly so then  $\text{root}x + \text{root}y$  is given,  $\text{root}x - \text{root}y$  is this, then  $\text{root}x + \text{root}y$  will be is giving this. So finally root  $x$  is not  $b$  here  $b$  negative so  $-b + \text{square root of } b^2 + 4ac/2$ . So in this root  $x$  is the greater quantity out of root  $x$  and root  $y$ , when  $b$  is positive, it is implied that this is solution  $n$ . There are the same form, he say differently. I mean here we note that instead of  $-4ac$  normally  $+4ac$  comes.

That is because  $c$  is in the other side, you see he normally got  $ax$  at the root  $x$  and  $a \text{ square } b \text{ square} + c = 0$ , so here auxiliary becomes minus, so that is why that you know is coming like this ok. By at least here he is talking only one root but I will point out that later he does talk about 2 root, some problem he does talk about 2 roots, he wants this has been 10 to be positive, so root  $x$  is the correct solution during the modern standard way.

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## Example

Example 27.

आकर्ण्य ध्वनिमद्रिमूर्ध्नि शिखिनोऽब्दानां स्फुरद्विद्युतां  
वृन्दार्धात्रिलवौ तदन्तरचतुर्भागैस्त्रिभिः संयुतौ।  
अध्यर्धकपदाधिकौ ननृततुः प्रीत्याऽऽसवृन्दौ सखे  
जातं तत्र शतत्रयं प्रवद मे तत्पूर्ववृन्ते कति ॥ २७ ॥

"A group of peacocks along with its half, its one-third and  $\frac{3}{4}$  of their difference (i.e, difference between its half and its one-third) together with  $1\frac{1}{2}$  times the square root (of the group) standing on top of a mountain, hearing the thunder of clouds surcharged with electricity, are dancing with joy. The peacocks are 300 in all. O friend, tell me (the number of peacocks) in the earlier group."

So various interesting problems are discussed (FL) so a group of peacocks along with its half, its one third and three fourth of the difference together with  $1\frac{1}{2}$  times the square root of the group standing on top of a mountain, hearing the thunder of clouds surcharged with electricity, are dancing with joy. So the peacocks are 300 in all. O friend, tell macro environment the number of peacocks in the earlier group.

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## Solution

Let  $x$  be the number of Peacocks in the 'earlier' group. Then

$$x + \frac{1}{2}x + \frac{1}{3}x + \frac{3}{4}\left(\frac{1}{2} - \frac{1}{3}\right)x + \frac{3}{2}\sqrt{x} = 300$$

$$\text{or } \frac{47}{24}x + \frac{3}{2}\sqrt{x} = 300$$

$$\text{or } \frac{47}{12}x + 3\sqrt{x} = 600$$

Hence

$$\sqrt{x} = \frac{-3 + \sqrt{9 + 4 \cdot \frac{47}{12} \cdot 600}}{2 \cdot \frac{47}{12}} = \frac{-3 + 97}{47} \cdot 6 = 12$$

$$\therefore x = 144$$

This is the number of peacocks in the earlier group.

And whatever you left you know dancing around that is 300, so that is the final number, so it is always written in you know nice example you know it is amusing and it is present to have these things.

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### Solution

Let  $x$  be the number of Peacocks in the 'earlier' group. Then

$$x + \frac{1}{2}x + \frac{1}{3}x + \frac{3}{4}\left(\frac{1}{2} - \frac{1}{3}\right)x + \frac{3}{2}\sqrt{x} = 300$$

$$\text{or } \frac{47}{24}x + \frac{3}{2}\sqrt{x} = 300$$

$$\text{or } \frac{47}{12}x + 3\sqrt{x} = 600$$

Hence

$$\sqrt{x} = \frac{-3 + \sqrt{9 + 4 \cdot \frac{47}{12} \cdot 600}}{2 \cdot \frac{47}{12}} = \frac{-3 + 97}{47} \cdot 6 = 12$$

$$\therefore x = 144$$

This is the number of peacocks in the earlier group.

So suppose  $x$  is the number of peacock he says that right, what we say along which is half, right along which is half (FL) then you also take one third of the three fourth difference of this that also you take. All these things you multiply by this. So then  $3/2$  times root  $x$  you know he says together it is  $1-1/2$  times (FL) that is there is square root of the that is that, and finally 300, the total is 300 what is that after all that is 300.

So what is the totally 300 and what is left is you know what you have to find, sorry may be the peacock is 300 in all, so initial number you have to find, initial number only it, therefore the other, so this is the equation. So finally you get this, so fact  $\text{root}x=12$  and  $x=144$ . So  $x$  is 144. So if you do all this operation you know take this  $x$  and then multiply and half of it again you taken all the 650.

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### Quadratic equations involving successive remainders

Rule 40 b.

उक्तनिजविधिवदन्त्याच्छेषविधौ जायते राशिः। ४० ब।

"Starting from the end, by the method stated earlier, the process (on being performed on) the remainders, gives the number."

This involves solving equations of the type:

$$(x - a\sqrt{x}) - b\left(\sqrt{x - a\sqrt{x}}\right) = c$$

and more complicated equations. This is solved by putting  $x - a\sqrt{x} = y$ . Then  $y - b\sqrt{y} = c$ . Then the problem is solved for  $y$  first and substituting this value for  $y$  in the equation for  $x$ ,  $x$  is solved.

This is best illustrated by an example.

And what is initial number, so that is 144 which is less than 300 (()) (18:12)  $47*12*x$  correct this is the final quadratic equation then you have to supply here. So then we talk about quadratic equation involves in successive remainder, for you also very interesting, so the rule itself is very simple in a very short (FL) by the method stated earlier, the process on being performed on the reminders give the number.

So this you know because just state this you will not be understand what you are saying to say the following you see suppose you have explained as a root  $x$  you just say, and then state the square root of that, so such things like (FL) some problems you know, so  $x$ -aroot  $x-b$ \*square root of this, that is given to be something, so are more complicated things which I will consider soon enough.

So this is called putting see you just put this is equal to  $y$ , so then  $y$ -broot/=c, so you solve for a  $y$  and then you solve for  $x$  from this after solving from  $y$  from this, we are to plugging this  $y$  and solve for  $x$ .

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Example

Example 40.

कान्तायाः सुरतप्रसङ्गसमये भिन्ना च मुक्तावली  
मुक्तानां च पदद्वयं विचरणं शय्यापटस्योपरि।  
तच्छेषस्य पदं त्रिभागयुगलेनाऽऽद्धं प्रियेणाऽऽहृतं  
तच्छेषस्य पदं क्षितौ निपतितं सूत्रे द्वयं किं वद ॥

“During the amorous tussle, (the beloved’s) garland of pearls was broken twice the square root less  $\frac{1}{4}$  (of the root) of the pearls were on the cover of the bed. The square root of the rest along with  $\frac{2}{3}$  (of this root) were seized by the lover. The square root of the rest fell down on the earth and 2 (pearls) were in the string (of the garland). Tell, how many pearls were in the garland.”

So this is giving an example (FL) ok during amorous tussle, playful for whatever so teh beloved garland was broken and simple for broken and find the square root less one fourth of the pears were on the cover of the bed ok. (FL) so that is the square root of a less than one before that was there and square roots along with two thirds of this route for these fell the level (FL) he took this thing.

And then the rest fell on the brown and 2 pearls remaining in the string you see he is wearing a garland of pearls so it is broken, so we have to find out how many pearls were there, so on the cover of the bed so it is  $2 - \frac{1}{4} \sqrt{x}$  and that is make it as 7.

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**Solution**

Solution: Let  $x$  be the number of pearls in the garland.

$$\text{Cover of the bed:} = \left(2 - \frac{1}{4}\right) \sqrt{x} = \frac{7}{4} \sqrt{x} = y$$

So remaining =  $x - y$

$$\text{Seized by the lover: } \sqrt{x - y} + \frac{2}{3} \sqrt{x - y} = \frac{5}{3} \sqrt{x - y} = z$$

So remaining =  $x - y - z = z'$

$$\text{On the earth} = \sqrt{z'} = \sqrt{x - y - z}$$

Remaining (on the string) = 2.

$$\therefore \sqrt{z'} + y + z + 2 = x$$

$$\sqrt{z'} + 2 = x - y - z = z'$$

Sorry you make it as  $y$  so the remaining is  $x - y$ , initially it was  $x$ , you do not know that, then the lover  $x$  see this thing you know this one is taking a square root of this  $+2/3$  times this ok. So that is that also we do not know how much he has seen, so that you call it is  $z$  and after these things so get for the initial number  $y$  is on the cover of the bed and  $z$  is caught by the lover, as far remaining is  $x - y - z = \text{root } zy$  ok.

And what is on the earth for the root you know on the root of  $zy$ , so that is they on the earth it is fallen on the earth is square root of the remaining whatever is remain (FL) and after all the reaming of the string is 2, so you have to find out the original number of is, so this whatever is remaining on the earth that is root of  $zy$  written+whatever was on the cover of the bed that is  $y$ , whatever was (FL) lover that is  $z$ , so root that time  $y + z$ .

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**Solution contd.**

This is solved first:  $\sqrt{z'} = \frac{1 + \sqrt{1+8}}{2} = 2$ .  $\therefore z' = 4$ . Now,  $x - y - z = z'$ , and  $z = \frac{5}{3}\sqrt{x-y}$ .

$$\therefore x - y - \frac{5}{3}\sqrt{x-y} = 4$$

Solving for this  $\sqrt{x-y}$ .

$$\sqrt{x-y} = \frac{\frac{5}{3} + \sqrt{\frac{25}{9} + 16}}{2} = 3$$

$$\therefore x - y = 9$$

Now  $y = \frac{7}{4}\sqrt{x}$

$$\therefore x - y = x - \frac{7}{4}\sqrt{x} = 9.$$

Solving this,

$$\sqrt{x} = \frac{\frac{7}{4} + \sqrt{\frac{49}{16} + 36}}{2} = \frac{\frac{7}{4} + \frac{\sqrt{625}}{4}}{2} = \frac{7 + 25}{8} = 4.$$

$$\therefore x = 16.$$

This is the number of pearls originally in the garland.

And 2 was remaining on the (FL) of the where yourself, so that is 2, the total of that is z, that the original number of us, so root  $zy=2$ =now, if you shift it to this side  $x-y-z=zy$ . So now solve for the so you will get the solution as 2 root z was 2, zy is 4, so now  $x-y-z=zy$  and z is we have already given  $\frac{5}{3}$  root x-y, so  $x-y-\frac{5}{3}$  root x-y=4. So you solve for x-y, x-y will be 9, now  $y=\frac{7}{4}$  root x, that is also given.

So x-y that is  $x-\frac{7}{4}$  root x that is equal to 9, from this so solve for x root  $x=4$  and  $x=16$ . So this square root are coming you know one more square root, so that is interesting. You can just carry on like this, so then again some remainder take equation.

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**Remainder type equation again**

Rule 41 a.

रूपोत्थञ्जादाद्यं योज्यान्त्याग्रे विधिः प्राग्वत् ॥ ४१अ ॥

"Multiply the "rupottha" by the first remainder and add the product to the last remainder. (After that solve the problem) by the method stated earlier."

Here author is referring to an equation of the form:

$$x - a - \frac{b}{f}(x-a) - \frac{c}{g} \left\{ x - a - \frac{b}{f}(x-a) \right\} - \frac{e}{h} \left[ x - a - \frac{b}{f}(x-a) - \frac{c}{g} \left\{ \quad \right\} \right] - j\sqrt{x} = d$$

We find

$$x \left( 1 - \frac{b}{f} \right) \left( 1 - \frac{c}{g} \right) \dots - j\sqrt{x} = a \underbrace{\left( 1 - \frac{b}{f} \right) \left( 1 - \frac{c}{g} \right) \dots \left( 1 - \frac{e}{h} \right)}_{\text{Rupottha}} + d$$

$a \rightarrow$  first remainder,  $d \rightarrow$  last remainder.

So again some reminder take equation (FL) multiply the rupottha by the first remainder and add the product to the last remainder like that. so essentially he is referring to an equation of

this form x-a, so then fraction of this and subtracted to this, and fraction of this and subtracted to this like that you know, finally you are having this kind of an equation  $x \cdot \text{some quantity} - j \cdot \text{root } x + a \cdot \text{some quantity} + d$ , so he is giving the solution for this.

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**Example**

Example 41.

गणेशं पद्मेन त्रिनयनहरिब्रह्मदिनपान्  
विलोमैः शेषांशैः विषयलवपूर्वैश्च कमलाम्।  
पदेनाऽऽपूज्यैकेन च गुरुपदाम्भोजयुगलं  
सरोजेनाऽऽचक्ष्व द्रुतमखिलमम्भोजनिचयम् ॥ ४१अ ॥

"Gaṇeśa (was worshipped) with 1 lotus flower. Śiva, Hari, Brahma (and) the sun (were worshipped) with  $\frac{1}{5}$ ,  $\frac{1}{4}$ ,  $\frac{1}{3}$  and  $\frac{1}{2}$  of what remains successively. Kamalā was worshipped with the square root of the lotus flowers (in the beginning) and the two feet of the teacher resembling lotus flowers (were worshipped) with 1 lotus flower. Quickly say the number of lotus flowers in the collection."

So if you give you take an example take up the example that will be clear (FL) Ganesa was worshiped with 1 lotus flower (FL) siva, hari, brahma (FL) the maker of a day the sun or worshiped with one fifth one fourth and 1 third and half of what remains successively. (FL) Kamala was worshipped with the square root of the lotus flowers in the beginning (FL) the 2 feet of the teacher resembling lotus flowers were worshiped with 1 lotus flower.

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**Solution**

Here let  $x$  be the number of lotus flowers.

Gaṇeśa  $\rightarrow 1$

After worshipping Gaṇeśa  $\rightarrow R_1 : x - 1$ .

Śiva  $\rightarrow \frac{1}{5}(x - 1)$

After worshipping Gaṇeśa, Śiva  $\rightarrow R_2 : (x - 1) - \frac{1}{5}(x - 1)$

Hari  $\rightarrow \frac{1}{4} \left\{ (x - 1) - \frac{1}{5}(x - 1) \right\}$

(FL) quickly say the number of lotus flowers in the collection. So let  $x$  be the number of lotus flowers, so Ganesa is worshiped 1 ok. So the remainder of this is  $x-1$  and siva was worshiped

one fifth of this, so one fifth  $x-1$ . So after worshipping Ganesa and Siva what we are just  $x-1-1/5*x-1$  and Hari is worshipped with one fourth of this.

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**Solution contd.**

After worshipping Ganesa, Siva, Hari,  $\rightarrow R_3 : (x-1) - \frac{1}{5}(x-1) - \frac{1}{4} \left\{ x-1 - \frac{1}{5}(x-1) \right\}$

Brahma  $\rightarrow \frac{1}{3} \left[ x-1 - \frac{1}{5}(x-1) - \frac{1}{4} \left\{ x-1 - \frac{1}{5}(x-1) \right\} \right]$

After worshipping Ganesa, Siva, Hari, and Brahma  $R_4 :$

$$(x-1) - \frac{1}{5}(x-1) - \frac{1}{4} \left\{ (x-1) - \frac{1}{5}(x-1) \right\} - \frac{1}{3} [x-1 - \dots]$$

Sun  $\rightarrow \frac{1}{2} R_4$

After worshipping Ganesa, Siva, Hari, Brahma and sun  $\rightarrow R_4 - \frac{1}{2} R_4.$

Kamalā was worshipped with  $\rightarrow \sqrt{x}$

Remaining Last remainder = 1

So after worshipping Ganesa, Siva and Hari what you get is  $x-1-1/5$  and 1.4 these thing ok. So Brahma is worship with one third of what remains after worshipping Ganesa and Siva and Hari  $1/3$  of these 3. So after worshipping these 4 ok what is remaining is this and sun is worshiped with half of this. So after worshipping Ganesa, Siva, Hari, Brahma, and sun  $R_4-1/2R_4$ . And kamala was worshiped with root  $x$  and remaining is 1 ok.

**(Refer Slide Time: 26:46)**

**Finding the Solution**

$$\therefore (x-1) - \frac{1}{5}(x-1) - \frac{1}{4} \left\{ (x-1) - \frac{1}{5}(x-1) \right\} - \frac{1}{3} \left[ \dots \right] - \frac{1}{2} \dots - \sqrt{x} = 1$$

or  $(x-1) \left(1 - \frac{1}{5}\right) \left(1 - \frac{1}{4}\right) \left(1 - \frac{1}{3}\right) \left(1 - \frac{1}{2}\right) - \sqrt{x} = 1$

or  $x \underbrace{\left(1 - \frac{1}{5}\right) \left(1 - \frac{1}{4}\right) \left(1 - \frac{1}{3}\right) \left(1 - \frac{1}{2}\right)}_{\text{Ripattha} = \frac{1}{5}} - \sqrt{x} = 1 + \underbrace{\left(1 - \frac{1}{5}\right) \left(1 - \frac{1}{4}\right) \left(1 - \frac{1}{3}\right) \left(1 - \frac{1}{2}\right)}_{\text{Ripattha} = \frac{1}{5}} + 1$

$$\therefore \frac{1}{5}x - \sqrt{x} = \frac{1}{5} + 1 = \frac{6}{5}$$

$$\therefore \sqrt{x} = \frac{1 + \sqrt{1 + 4 \cdot \frac{6}{5} \cdot \frac{1}{5}}}{2 \cdot \frac{1}{5}} = \frac{1 + \sqrt{25 + 24}}{2 \cdot \frac{1}{5}} = \frac{5 + 7}{2} = 6$$

$\therefore x = 36$

So, there were 36 lotus flowers in the original collection.

What you are getting is this kind of equation if my this you will get this equation, you just be little patient that nothing more is required in this. So you get this kind of an equation from in



this is processes the form that he setting you know that the solution so you get  $x=6$  by solving this quadratic equation finally you interpret that and root  $x$  is 6, so  $x$  is 36.

(Refer Slide Time: 27:16)

**Another type of quadratic equation**

Consider a quadratic equation of the form

$$x - \left[ \frac{x}{(m/n)} - h \right]^2 = d.$$

One can easily check that this reduces to  $x^2 - bx + c = 0$ ,  
 where  $b = \left(\frac{m}{n}\right)^2 + 2\left(\frac{m}{n}\right)h$  and  $c = \left(\frac{m}{n}\right)^2(h^2 + d)$

Then, Sum of the roots  $x + y = b$ ,  
 Product of the roots  $xy = c$ ,

Then we obtain the solutions  $x, y$  by *sankramaṇa* with

$$x - y = \sqrt{(x + y)^2 - 4xy} = \sqrt{b^2 - 4ac},$$

so that

$$x = \frac{b + \sqrt{b^2 - 4c}}{2}, \quad y = \frac{b - \sqrt{b^2 - 4c}}{2}$$

Nārāyaṇa calls  $\frac{m}{n}$  as the *ripahara*,  $h$  the subtractive, and  $d$ , the *drśya* or the observed quantity and states the solution thus:

So another type of quadratic equation he discuss this kind of everything. So these are all very straight forward so gives some example of this ok.

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**“Double equations of second and higher order degree equations: Rational solutions”**

Rules 45-57 give rational solutions of some second and higher degree equations involving two unknown quantities,  $x, y$ . In the following,  $m, n, p, u, v, a, b$ , etc., are integers.

Rule 45. A solution of  $x^2 + y^2 + 1 = u^2, x^2 - y^2 + 1 = v^2$  is  $x = \frac{m^2}{2}, y = m$ .

Rule 46. A solution of  $x^2 \pm y^2 - 1 = u^2$  or  $v^2$  is  $x = \frac{m^4}{2} + 1, y = m^3$ .

Rule 47 A solution of  $x + y = u^2, x - y = v^2, xy + 1 = w^2$  is  $x = 2(m^4 + m^2), y = 2(m^4 - m^2)$ .

Rule 48. A solution of  $x + y = u^2, x - y = v^2$  is  $x = m^2 + n^2, y = 2mn$ .

Rule 49. A solution of  $x + y = u^2, x - y = v^2, xy = w^3$  is

$$x = \frac{(m^2 + n^2)pb}{[2mn(m^2 + n^2)]^2}, \quad y = \frac{2mnpb}{[2mn(m^2 + n^2)]^2}$$

Rule 50. A solution of  $x^2 + y^2 = u^3, x^3 + y^3 = v^2$  is  $x = \frac{mb}{25}, y = \frac{2mb}{25}$ .

( $u = \frac{m^4}{5}$  and  $v = \frac{3m^3}{25}$ )

So then he talks various double equation of second and higher order degree equations, so some of them are discovered discussed earlier also, so  $x^2 + y^2 + 1 = u^2$  and  $x^2 - y^2 + 1 = v^2$ , must also a square. That is equal to  $m^2/2$  and  $y=m$  that is solution and suppose this is equal to  $u^2$  or  $v^2$  this one then the solution will be this kind of everything.

And the solution of  $x=y$ =you square you know this is the square of a number,  $x-y$  is the leaner square of the integer and  $x+y+n$  so the solution will be this. So like this it is so on, it goes on particularly important is the various thing you are not trivial at all you have to go through this.

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Rule

Rule 58.

घनयुतिभक्ते कृतियुति युतिकृतिघाताहते त्विट्टे ।  
घनयुतियुतिघनतुल्या कृतियुतियुतिकृतिवधां गश्योः ॥ ५८ ॥

"Divide the sum of squares, square of the sum and the product of any two assumed numbers by the sum of (their) cubes and the cube of (their) sum and (then) multiply by the two numbers. The (results will) be the two numbers, the sum of (whole) cubes and the cube (of whose) sum will be equal to the sum of their squares, the square of (their) sum (and their) product."

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Next you goes you know in a 1 systematic set of you know equation which about this into the solutions (FL) very interesting. So the translation you know without going to this we can go through this is take the slides what is saying the following you know.

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Rational solution for Rule: 58

Let  $m$  and  $n$  be any two assumed numbers. Then according to the rule, solution of

- 1)  $x^3 + y^3 = x^2 + y^2$  is  $x = \frac{(m^2+n^2)m}{m^2+n^2}$ ,  $y = \frac{(m^2+n^2)n}{m^2+n^2}$
- 2)  $x^3 + y^3 = (x+y)^2$  is  $x = \frac{(m+n)^2 m}{m^2+n^2}$ ,  $y = \frac{(m+n)^2 n}{m^2+n^2}$
- 3)  $x^3 + y^3 = xy$  is  $x = \frac{m^2+n}{m^2+n^2}$ ,  $y = \frac{m+n^2}{m^2+n^2}$
- 4)  $x^3 + y^3 = x^2 + y^2$  is  $x = \frac{(m^2+n^2)m}{(m+n)^2}$ ,  $y = \frac{(m^2+n^2)n}{(m+n)^2}$
- 5)  $(x+y)^3 = (x+y)^2$  is  $x = \frac{(m+n)^2 m}{(m+n)^2}$ ,  $y = \frac{(m+n)^2 n}{(m+n)^2}$
- 6)  $(x+y)^3 = xy$  is  $x = \frac{m^2 n}{(m+n)^2}$ ,  $y = \frac{m n^2}{(m+n)^2}$

Nārāyaṇa considers the 'Rule of inversions' (Remember *Līlāvati*) in Rule 59, and Rules of 3, 5, 7, 9, 11 and inverse rules in Rule 60 onwards. Barter of commodities and the sale of living beings etc., are also considered.

◀ ▶ 🔍 🔄 🏠

So what we are having is you have 2 numbers extend by have solution of  $x^3+y^3=x^2+y^2$  square I say ok , so are going to find rational solution for these, but trivial not at all trivial right so it is given if you take  $m$  and  $n$  to be any integers then this is the solution of this

equation and  $x^3 + y^3 = (x+y)^2(x-y)$ , so then if you take any integer  $m$  and  $n$  and if you take this  $x$  and  $y$  like this then you get this.

So (FL) cube of the sum, so that is the kind of thing you know is doing. So similarly the solution of this is integral solution you will be having this kind of form and lastly  $x^3 + y^3 = (x+y)^2(x-y)$ , so this is the kind of solution you will have. (FL) discover the, consider the rule of inversion and all this tool proportion all that in great detail.

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**Chapter 2 on determination pertaining to mixture of things**

Rule 1 (a).  
 प्रक्षेपास्तद्व्यतिहतमिश्रेण हताः पृथक् फलानि स्युः ॥ १a ॥

"The contributions severally divided by their sum (and) multiplied by the mixed amount happen to be (the respective) fruits."

This is understood with the solution for the following problem:

So I will not have time to do this, and (FL) is very interesting things.

**(Refer Slide Time: 30:36)**

**Mixed quantities and rule of proportion**

It is written somewhat confusingly, but refers to the proportionality among  $P$ ,  $A$ , and  $I$  which has been stated by us. Now, if different types of objects are involved. Let us say:

$x_1$  amount of 1 is purchased at the rate of  $r_1$  and sold at rate  $r'_1$ .

$x_2$  amount of 2 is purchased at the rate of  $r_2$  and sold at rate  $r'_2$ , etc.,

Then Principals of 1, 2, 3, ... are:  $P_i$  are:  $\frac{x_1}{r_1 r'_1} \cdot r'_1, \frac{x_2}{r_2 r'_2} \cdot r'_2, \dots$

Amounts (Principal + Profit) of 1, 2, 3, ...  $A_i$  are:  $\frac{x_1}{r_1 r'_1} \cdot r_1, \frac{x_2}{r_2 r'_2} \cdot r_2, \dots$

Profits =  $A_i - P_i$ :  $\frac{x_1}{r_1 r'_1} (r_1 - r'_1), \frac{x_2}{r_2 r'_2} (r_2 - r'_2), \dots$

For different items,

$$\frac{\text{Principal}}{\text{Amount}} = \frac{\text{Principal}}{\text{Principal} + \text{Profit}} \text{ are in ratio } : \frac{r'_1}{r_1} : \frac{r'_2}{r_2} : \dots$$

$$\frac{\text{Principal}}{\text{Profit}} \text{ are in ratio } : \frac{r'_1}{r_1 - r'_1} : \frac{r'_2}{r_2 - r'_2} : \frac{r'_3}{r_3 - r'_3} : \dots$$

$$\frac{\text{Principal}}{\text{Principal} - \text{Profit}} \text{ are in ratio } : \frac{r'_1}{2r'_1 - r_1} : \frac{r'_2}{2r'_2 - r_2} : \dots$$

Mixed quantities and various proportions, we are lending money at various interest and you know suppose you are giving many people are giving suppose principle+profit=then what is

the proportion, what is they know they showing which are lending money. So they are all very interesting problem, but do not have time, so I will one part may be I will pick up on a 1 or 2 basis.

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**Mixed problem involving interest**

Let the principals (capitals)  $P_1$  and  $P_2$  be lent out for  $t_1$  and  $t_2$  months at the same rate of interest ( $r$  percent per month) and let the corresponding interests be  $I_1$  and  $I_2$ . Then out of the seven quantities ( $P_1, P_2, r, t_1, t_2, I_1, I_2$ ), two can be determined from the sum of them and other four ( $r$  consider fixed).  
In particular, Rule 5(b)-7(a) says:

विहिते विपरीतफले पक्षद्वितये मिथस्तु फलयोर्वा ॥ ५ ॥  
 धनयोश्च कालयोर्वा धनफलयोः कालफलयोर्वा ।  
 मूलधनकालयोर्वा मिश्रं यदि दृश्यते तदा तत्र ॥ ६ ॥  
 तत्पक्षयोश्च यातौ ताभ्यां प्रक्षेपतो जातौ ॥ ७a ॥

See suppose interest problem that principle capitals  $P_1$  and  $P_2$  be lent out for time  $t_1$  and  $t_2$  months at the same rate of interest and let the corresponding interests be  $i_1$  and  $i_2$ . Then out of 7 quantities  $P_1, P_2, r, t_1, t_2, i_1, i_2$ , two can be determined from the sum of them and other four  $r$  consider fixed. So that is a very interesting way of which is generalization of you know direct interest kind of ah calculations.

(Refer Slide Time: 31:36)

**Mixed problem involving interest**

"In two prescribed different sides, if the sum of either the interest or the capitals or the times or the capital and the time (of different sides) or the capital and the interest or time and the interest (of the same side) is observed as mixed, their shares in the sum can be determined, separately by the method of partnership considering the products of two (known ingredients) in the two sides as contribution."

So this rule says: when any one sum out of  $I_1 + I_2, t_1 + t_2, P_1 + P_2, P_1 + t_2, P_2 + t_1, I_1 + t_1 + I_2 + t_2, I_1 + P_1$  or  $I_2 + P_2$  is known and the rest 4 ingredients are known, then two (comprising the sum) can be found separately.

So what is happening is that so what the rule says is very anyone some out of you know, see suppose that this principles  $P_1, P_2$ , the time of lending is  $t_1, t_2$ , so interest out of them is  $i_1, i_2$

ok. See suppose you are given any one sum of which is  $i_1+i_2$ ,  $P_1+P_2$ ,  $t_1+t_2$ ,  $i_1+i_2$ ,  $P_1+P_2$ ,  $t_1+t_2$ , rest 4 ingredients are known, then 2 comprising the sum can be found separately.

**(Refer Slide Time: 32:16)**

**Example**

Given  $i_1 + i_2, P_1, t_1, P_2, t_2$  to find  $i_1$  and  $i_2$  separately. Now,

$$i_1 = \frac{P_1 t_1 r}{100}, \quad i_2 = \frac{P_2 t_2 r}{100}$$

$$\therefore \frac{i_1}{i_2} = \frac{P_1 t_1}{P_2 t_2}$$

$$\therefore \frac{i_1 + i_2}{i_2} = \frac{P_1 t_1 + P_2 t_2}{P_2 t_2}$$

$$\therefore i_2 = \frac{(i_1 + i_2) P_2 t_2}{(P_1 t_1 + P_2 t_2)}$$

Similarly  $i_1 = \frac{(i_1 + i_2) P_1 t_1}{P_1 t_1 + P_2 t_2}$

Example.: Suppose  $P_2, t_1, i_1, i_2$  and  $P_1 + t_2$  are given:

$$\text{Now, } \frac{i_1}{i_2} = \frac{P_1 t_1}{P_2 t_2}, \quad \frac{P_1}{t_2} = \frac{i_1 P_2}{i_2 t_1}, \quad \frac{P_1 + t_2}{t_2} = \frac{i_1 P_2 + i_2 t_1}{i_2 t_1}$$

$$\therefore t_2 = \frac{(P_1 + t_2) i_2 t_1}{i_1 P_2 + i_2 t_1}, \quad P_1 = (P_1 + t_2) - t_2$$

See these are all you know clearly this is of mathematics read the stage where they doing something from which are not directly relevant if nobody will give you know some of the principal and time, but to see you know that generally how one can solve this problem using mathematical probably we should have the mathematical problems all this kind of equation so that is why he is trying to solve this kind of equation.

He suppose you are given  $i_1+i_2$ ,  $P_1$ ,  $t_1$ ,  $p_2$ ,  $t_2$ , so you have to find  $i_1$  and  $i_2$  separately ok, so I giving the principal and the time length corresponding to this they are given and some of the interest is given. So then you have to find  $i_1$  and  $i_2$ , so then they use the fact that they are rented at the same rate, rate is the same. So  $i_1$  is  $P_1 t_1 r / 100$ ,  $i_2 = P_2 t_2 r / 100$ , so  $i_1 / i_2$  it is and finally so you get one equation, you know in terms of a  $i_1$  and  $i_2$ .

$i_1+i_2$  to give the equation, so one give  $i_1+i_2$  is already given you know one more equation involving linear equation involving  $i_1$  and  $i_2$ . So we can find them separately. So that is what is done.

**(Refer Slide Time: 33:46)**

## Example

Example 17.

मासेन शतस्य क्रियत् षट्द्वयस्य यत् फलं फलयोः

योगे चत्वारिंशद्द्वययुतं मे फलं कथय ॥

ताभ्यां पञ्चदशितये मिथो विमिश्रे पृथक् कृते ब्रूहि ॥ १७ ॥

"The sum of interest of 100 in a month, added to that of 60 in a year (at the same rate) is 41. Tell me the interest separately."

Here  $P_1 = 100, P_2 = 60, t_1 = 1, t_2 = 12$ . Given  $i_1 + i_2 = 41$ .

$$\text{We have } i_2 = \frac{(i_1 + i_2)P_2 t_2}{P_1 t_1 + P_2 t_2} = \frac{41 \times 60 \times 12}{100 + 60 \times 12} = \frac{41 \times 60 \times 12}{820} = 36$$

$$i_1 = (i_1 + i_2) - i_2 = 41 - 36 = 5.$$

Now let any one out of  $P_1 + i_2, P_2 + i_1, t_1 + i_2, t_2 + i_1, P_1 + t_1 P_2 + t_2$  be known and the other four quantities are known separately. Then the product of the two quantities can be obtained and the quantities themselves can be determined according to Rule 7(b)-8.

See for instance he says the sum of interest (FL) the sum of interest of 100 in a month, added to the 16 in a year is at the same rate is 41. So tell me the interest separately. You see so suppose P1 is given P2 is there, t1 is 1, t2=12 and i1+i2 is given ok, he suppose you are given post like this I am sure most people will give you (FL) and they all simple principles but still you have to use this logic and all that.

So you **you** find this equations, you get one more equation i2 you know, you will find that i1 will be i2 is 36 and i1 will be 5. So similarly if any of these other quantities you know P1+i2, P2+i1, t1+i2, t2+i1, P2+t2 be known for and the coma yet, so then also other things can be determined ok.

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## Example

Example. a)  $P_2, t_1, t_2, i_1$  and  $P_1 + i_2$ . To find  $P_1, i_2$ .

Now as the rates of interest are equal,

$$\frac{i_1}{i_2} = \frac{P_1 t_1}{P_2 t_2}$$

$$\therefore P_1 i_2 = \frac{i_1 P_2 t_2}{t_1}$$

$$P_1 - i_2 = \sqrt{(P_1 + i_2)^2 - 4P_1 i_2}$$

From,  $P_1 + i_2$  and  $P_1 - i_2$ ,  $P_1$  and  $i_2$  are found from 'saiikramana'.

(b) Similarly, Let  $P_1, P_2, t_2, i_1$  and  $t_2 + i_1$  are given. Then

$$i_2 t_1 = \frac{i_1 P_2 t_2}{P_1} \quad \therefore t_2 - t_1 = \sqrt{(t_2 + t_1)^2 - 4t_2 t_1}$$

From,  $t_2 + t_1$  and  $t_2 - t_1$ ,  $t_2$  and  $t_1$  can be determined.

Some of them will involve some quadratic equations ok, see for instance you are given  $P_2$ ,  $t_1$ ,  $t_2$ ,  $i_1$  and  $P_1+i_2$ , we have to find out  $P_1$  and  $i_2$  you have to find, so  $i_1/i_2$  is these remember the rates are equal, so  $P_1 i_2 = i_1 P_2 t_2 / t_1$  these things. So from this we can find out  $P_1 - i_2$  right, if  $P_1 i_2$  is this then  $P_1 - i_2$  is this, square root of  $P_1 + i_2$  whole square  $- 4 P_1 i_2$ , this is the simple entity, you see and  $P_1 = i_2$  is already given.

So you have from  $P_1 + i_2$  and  $P_1 - i_2$  already knew  $P_1 + i_2$ , so  $P_1$  and  $i_2$  can be found by (FL). So this is the way these are done, so intermediated various kind of you know, quadratic equation that have been come and as to square that have been come, so these are the kind of he gives some example of that.

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**Example**

Example 18.

मासि शतस्य फलं यद् वर्षेण च षट्कृतिः फलं यस्य ।  
तद्दोगे पञ्चगुणाः त्रयोदश सखे पृथक् कथय ॥  
काले निजधनयुक्ते सहफलयुक्तेऽथवा गणितम् ॥ १८ ॥

"The sum of the interest of 100 for a month and the capital (principal) whose interest in a year is 36, is 65. O friend, tell (them) separately (knowing) the sum of the capital and its time or that of the interest of 100 and the time (tell them separately)."

Solution: Here  $P_1 = 100$ ,  $i_2 = 36$ ,  $t_1 = 1$ ,  $t_2 = 12$ , and  $P_2 + i_1 = 65$ .

$$\text{Now, } P_2 t_1 = \frac{i_2 P_1 t_1}{t_2} = \frac{36 \times 100 \times 1}{12} = 300.$$

So now I think this is the of all interesting.

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## Payment in Instalments

The following rule pertains to the payment in instalments of a debt, based on simple interest.

Rule 10.

स्कन्धककालकलान्तर हीनस्कन्धेन भाजिते वित्तम्।  
स्कन्धककालविगुणिते नियतं निर्मुक्तकालः स्यात् ॥ १० ॥

"The capital (principal) is multiplied by the fixed period of instalment (the product is) divided by the amount of instalment less the interest (on the total amount) for the period of instalment. (The quotient) determines the time of being free from debt."

Amount lent =  $P$ , Amount of instalment =  $a$ . Period of instalment (No. of months / instalments) =  $t$  Total time for being free from debt =  $T = nt$  let us say.

So I will discuss some payment in instalments which are discussed by Brahmagupta, so we all see this interest calculator even in Aryabhata it is there, interest calculation is there, (FL) it is there, of course (FL) so here also I can find, but you know as I told you he advance it soon, so he talk of you know payment by instalments and what you know very close to these equation and the instalment we will come to that. so thus far want to discuss.

So he says the capital is multiplied by the fixed period of instalment is divided by the amount of instalment less the interest, the quotient determines the, time of being free from debt. So this is what is this verse says. So suppose the amount lent is  $P$ , the amount of instalment is  $a$  and period of instalment is number of months/instalments, so need to be given once in 2 months or once in 3 month whatever it is.

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## Payment in Instalments

Interest / period of Instalment (for time  $t$ ) =  $I'$

$\therefore$  Interest for total duration of debt (time  $T = nt$ ) =  $nI'$ .

Now, the amount due after time  $T = nt$  is  $P + nI'$  and amount paid is  $na$ . Equating them,

$$P + nI' = na$$

$$\therefore n = \frac{P}{a - I'}$$

$$\therefore T = nt = \frac{Pt}{a - I'}$$

This is what is stated in the rule.



Let that be  $t$  and that total time for being free from debt that is  $T$  and  $nt$  let us say ok. So then in this case so interest/period of instalment is  $i$  and interest for total duration of debt is that is multiply by  $n$  ok  $n$  instalment period are there  $n \cdot i$ . So the amount due after time  $T$  is  $P + n \cdot i$  right  $n$  instalment and  $n \cdot i$  is the interest for it which is calculated you know which the lender is specifying.

And the amount paid is  $na$  because he is paying  $na$  instalments of  $a$  each. So  $P + n \cdot i = na$ , so  $n$  the number of instalment is given by  $nt$ , so  $pt/a - i$ , so this is what is stated in the rule. Ok so everything is simple interest ruling, compound interest is not coming, so these only calculate, these are simple enough, so one example you can write for this.

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**Payment in instalments: Time for being free from debt**

The next rule gives the expression for the Principal (capital)  $P$ , in terms of the instalment amount,  $a$ , time being free from debt,  $T$ , the instalment period,  $t$  and the rate of interest (in fact interest on unity) for the time of being free from debt.

Rule 11.

निर्मुक्तकालवृद्धा रूपस्य हि सैकया हतेन भजेत् ।  
स्कन्धकालेन च गत कालस्कन्धाहतिर्मूलम् ११ ॥

"The time of being free from debt is multiplied by the amount of instalment. (The product) is divided by (the product of the sum of) the interest on unity for the time of being free from debt added to 1, multiplied by the period of instalment. The (quotient) is the capital."

We had  $P + n \cdot i = na$

$$\therefore P \left( \frac{n \cdot i}{P} + 1 \right) = na = \frac{aT}{t}$$

$$\therefore P = \frac{aT}{\left( \frac{n \cdot i}{P} + 1 \right) t}$$

This is what being stated, when it is realised that  $\frac{n \cdot i}{P}$  is the interest for the period  $T = nt$ .

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So again you can go on like this.

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## Example

Example 21.

पञ्चकशतेन वित्तं मासद्वितयेन सदलेन।

स्कन्धः पञ्चदशाऽथ त्रिंशन्मासा विनिर्मुक्तः ॥

कालस्त्विह वद मूलं किं वृद्धिः का च यदि वेत्सि ॥ २१ ॥

"An amount is lent at the rate of 5 percent per month. The period of instalment is  $2\frac{1}{2}$  months, the amount of interest is 15, and the time of being free from debt is 30 months. If you know, tell the capital and the interest."

Solution: Here  $t = 2.5$ ,  $a = 15$ ,  $T = 30$ . Rate of interest = 5 percent per month.

$$\text{Interest on unity for } T = \frac{5}{100} \times 30 = 1.5 \left( = \frac{nI}{P} \right)$$

$$\therefore P = \frac{15 \times 30}{(1.5 + 1) \times 2.5} = 72.$$

$$\text{Total interest paid} = 1.5 \times 72 = 108$$

$$\text{Alternatively, total interest} = na - P = \frac{30}{2.5} \times 15 - 72 = 108.$$

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Now we will discuss a more realistic computation.

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## Payment in instalments: More realistic computation

The above is based on simple interest. The debtor owes the amount  $P + nI$  at the end of the time  $nt = T$  based on simple interest, but he is actually paying an instalment amount  $a$  after each instalment period  $t$ . He is not getting any benefit for paying back the debt in instalments at earlier times. The next few rules take this into account.

The rule in 12-14(a) pertains to the instalment amount  $a$  being adjusted only towards payment of capital. The rule gives the expression for the amount of interest to be paid in addition to the 'monthly instalments' being paid to clear the capital.

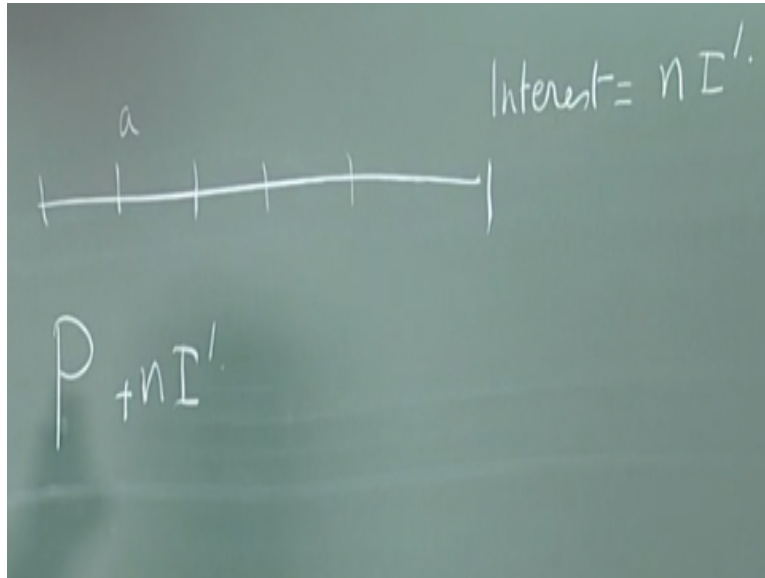
Let amount lent (Principal or capital) =  $P$ , Instalment amount =  $a$ , Instalment time period =  $t$  months. Rate of interest =  $r$  percent per month. If the amount is cleared in  $n$  instalments  $P = n'a$  or  $n' = \frac{P}{a}$ .

$$\therefore \text{Total time for being free from debt} = n't = \frac{Pt}{a}$$

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See here you are paying an instalment ok, see if your principle is P ok.

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And you are paying between n instalment ok you write it like this of a each ok, as the total interest you have paid is equal to you know is  $n \times \text{amount for instalment}$   $n \times \text{prime}$  ok and finally you are giving  $P + n \times \text{prime}$ , so  $P + n \times \text{prime}$  you are giving principle+interest, but you are not paying at the end you know, you are paying in the instalment is beginning itself you are paying you see here you are paying  $1/n$  of this and so on.

You know that is you are not getting any benefit for paying it earlier ok, suppose of 5000 rupees you have taken a loan and suppose 10% interest ok, so then after 1 year so you have to pay 5500, suppose you are paying  $5500/12$  you know every month see then you are loosing because you could have pay 5500 at the end you know and you could have done something with your money. So you are not getting any benefit from paying earlier in instalment.

So that is what is calculation now ok, so he says you know so this rule contains but in the instalment amount being adjusted only towards payment, he is considering only instalment amount adjust towards payment of capital, he has to pay interest also. The rule gives the expression for the amount of interest to be paid in addition to monthly instalment being paid to clear the capital. So let the amount lend will be  $P$ , so the instalment amount is  $a$ .

And instalment period is  $t$  month okay and rate of interest is  $t$  or interest  $r$  percent per month, so if the amount is cleared in  $n$  instalments, so  $P$  is  $n \times \text{prime}$  or  $n \times \text{prime}$  is  $P/a$ . The rate of interest is is amount that should be  $n$ , so it cannot be integer ok, is the amount is cleared in  $n$  prime is instalment,  $P = n \times \text{prime}$  ok. So emphasize it cannot be integer I am putting them as

prime, so  $n$  prime is  $P/a$  ok, this is  $a$  is only for prime and the principle, interest is separately paid ok.

So this is the different problem today, so interest is separately paid, but  $a$  is paid only to clear the principle, so the total time of being free from debt is  $n$  prime \*  $t$ , so that is The patient/ $a$  ok suppose  $P$  instalment is here,  $P$  is or  $n$  prime is number of instalment and instalment period is  $t$  month. So the total time of is  $n$  primet, so  $n$  prime is  $P/a$ , it is  $Pt/a$ , so this is the total time, so  $n$  prime need not be the integer.

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**Payment in instalments contd.**

Here  $n'$  indeed not be an integer. Let

$$n' = n + \frac{f}{a}$$

$$\therefore \frac{P}{a} = n + \frac{f}{a}$$

or  $P = na + f$ , where  $f < a$ .

Consider the amount,  $P$  as made up of  $na$  and  $f$ . Consider the integral part  $na$ , first. At time  $t$  instalment amount  $a$  is being paid. The interest should be the entire amount,  $na$ .

$$\therefore \text{Interest at time } t = na \cdot \frac{rt}{100}$$

Here an amount  $a$  is paid towards clearing the principal. So the amount due at the beginning of the second instalment is  $na - a = (n - 1)a$ . So, at time  $2t$ , we have to calculate interest for the amount  $(n - 1)a$ .

$$\therefore \text{Interest at time } 2t = (n - 1)a \cdot \frac{rt}{100}$$

So that is why I wanted to say,  $n$  prime is  $n+f/a$ , that is integer  $(n)$  (42:40)  $n$  is integral ok and  $n$  prime is actual that is you know he may talk 2 and half month or 2 months and 1 of the month to clear, so that is allowed  $n$  prime is  $n+$ , so  $P/a$  is this, so  $P=na+f$  where  $f$  is greater than  $a$ . The principle  $n*$  or there is nothing  $p$   $n$  prime  $a$   $n$  prime number of instalment is  $a$  and so on,  $n$  i is  $n$  prime it is not a integer.

So which has  $na+f$  ok, so consider the amount  $p$  has made up of  $na$  and  $f$ , consider the integral part  $na$ , first if  $n$  is integer, so at time  $t$  instalment amount  $a$  is being paid ok ok. So these are the instalment period. So here  $a$  is paid ok, so then the interest should be the entire amount  $na$ , the amount of interest now he is know 32 for this is  $n$  because the whole money is  $t$ , here whole money is here.

So you have to give the interest of the whole thing, so interest at time  $p$  that is at the first step is  $na*rt/100$ , because and now at this stage  $a$  amount  $a$  is already paid to clear the principle,

the amount  $a$  is paid to clear the principle, so the amount due at the beginning of the second instalment is  $na - a$  or  $n-1a$ . So at the time the principle is  $n-na$ , so at the time of next instalment you have to calculate the interest for this.

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**Payment in instalments: Interest to be paid**

Proceeding in this manner,

$$\text{Interest at time } rt = [n - (r - 1)] \cdot a \cdot \frac{rt}{100}$$

$$\text{Interest at time } nt = a \cdot \frac{rt}{100}$$

Hence, total interest for the integral part

$$= [n + (n - 1) + \dots + 2 + 1]a \cdot \frac{rt}{100} = \frac{n(n + 1)}{2} a \cdot \frac{rt}{100}$$

At the end of time duration  $nt$ , Principal amount due is  $f$ . (Integral part  $na$  having been cleared). This will be cleared in time  $\frac{f}{a}$ . But the debtor has to pay the interest for  $f$  for a period  $nt + \frac{f}{a} = \frac{P}{a}t$ , which is  $f \cdot \frac{P}{a} \cdot \frac{r}{100}$ . Hence,

$$\text{Total interest to be paid} = \left[ \frac{n(n + 1)}{2} a + \frac{Pf}{a} \right] \frac{rt}{100}$$

$n$  is called the 'pada'.  $f/a$  is the 'agra'.  $\left[ \frac{n(n + 1)}{2} a + \frac{Pf}{a} \right]$  is called 'mula pinda'.

The total interest to be paid and the time for being free from debt are stated in the following rule:

So time  $t_2$ ,  $t_2$  you have to calculate interest for the amount  $n-1a$ , you remember  $t$  is the amount period of instalment ok,  $t$  is the period of instalment ok. So interstate at time  $2t$  is  $n-1a \cdot rt/100$  and similarly you can go on at interest at time  $r$  ok, of all period of instalment are this thing you will have, so them it will be  $n-r/a$  that is the amount of principle which is being which is own by the later right, for interest  $rt - r-1$  because this is the first full thing is, so  $r-1$  you have to put, so this is the amount which is owned.

So that you have to calculate the interest for this ok, so  $n-r-1 \cdot rt/100$  and interested time  $nt$  suppose after that you know he is it will just  $a$  at this point you see at end ok the only  $a$  will be the carried amount principle which he has these things, so that all will be clear there ok, so it is  $rt/100$ , so total interest for the integral part, I am going to take integral part see so  $p$  is you know remember  $P$  is  $n$  prime  $a = n + f$  whole square  $a$ , so this is how discuss.

So total interest for the integral part is  $n + n-1$  etc. up to  $1$ , so you are getting portion here say  $n \cdot n + 1/2 \cdot a \cdot rt/100$ , and at the end of duration  $nt$  see now after the duration principle due is  $f$ , or it refers to  $na + f$ , so after this this is still  $f$  so that for you have to pay the interest also, so this will be clear in time  $f/a$ , but the debtor has to pay the interest for  $f$  for a period of  $nt + t/a \cdot t$  it is calculation we have only consider integral part.

We have to have to pay the interest of the whole thing, the total interest to be paid  $n \cdot n + 1/2 \cdot a + Pf/a \cdot rt/100$ . So this  $n$  is called the pada,  $f/a$  is called agra, and this called it is (FL) so this si the for this is the amount it has to be apart from the money he has given to create a principle you see at every instalment he is giving, he has to give the interest, suppose he when this interest will be create that every month to be paid you know instalment.

So these are thing, so here is the number of instalment is after this if  $a$  is specified so the fraction for the fractional month you know this is the fraction the whole thing is cleared in fraction number of integral+fraction number, so that is prosily what is so these are rule.

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Rule

Rule 12-14 a:

स्कन्धकभक्तं वित्तं लब्धं पदसंज्ञकं च शेषांशः ।  
 अग्राख्यः पदवर्गः पदयुक् स्कन्धार्धसंगुणो युक्तः ॥ १२ ॥  
 अग्रांशप्रघनेन प्रजायते मूलपिण्डाख्यः ।  
 तस्य स्कन्धककालात् समानयेद् वृद्धिमानमथ ॥ १३ ॥  
 स्कन्धककालप्रघने स्कन्धहृतं मुख्यकालः स्यात् ॥ १४ अ ॥

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That is here (FL) etc.

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Total interest and time for being free from debt

"The amount (capital or principal lent) is divided by the amount of instalment. (The integral part of) the quotient is called 'Pada' and the remaining part, 'Agra'. 'Pada' is added to its square. (The sum) is multiplied by half the amount of instalment. (The product) is added to (the product) of 'agra' (i.e. the remaining part) multiplied by amount (lent). The sum is called 'Mūla Piṇḍa'. The interest should be obtained from that (i.e. from the *mūla piṇḍa*) by taking the time equal to the period of instalment. After that, the period of instalment multiplied by the amount (lent, the product) divided by the amount of instalment happens to be the time of being free from the debt."

[Note: Let  $\frac{rt}{100} = r'$ . Probably, the debtor is paying  $a + nar'$  at time  $t$ ,  $a + (n-1)ar'$  at time  $2t$ , ...,  $a + ar'$  at time  $nt$ , and at time  $nt + \frac{f}{a}t$ , he is paying  $f + \frac{Af}{a}r'$ . In all these, the first part is payment towards the principal and the second part is the interest.]

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So what is saying the amount are capital and principle and divided by the amount of instalment and the integral part of the quotient is called (FL) the remaining part is called (FL) so on and so forth we just we have this what we have to what is this si the total interest this is what we have to do even this si the formula and that is expressed in verse (()) (49:10) so now we consider little more discuss of this, so here he is paying see the amount a is paying for instalment that is only for clearing the principle.

So the interest he is doing separately so that is why we did this thing you know whatever amount of principle he has to pay amount he has to be debt and consider as and interest he has calculate separately, so now he does get the thing all whatever amount is a he is paying, so that is partly towards principle and partly towards interest. So that is what he is considering next.

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**Equated monthly instalments**

Now we consider the case when the instalment  $a$  is the interest on the outstanding amount plus the payment of a part of the principal. Then the following rule gives the time of being free from the debt.

Rule 16 b - 17.

प्रतिमासिकफलशुद्धौ मूलं मूलात् पृथक् पृथक् जहात् ॥ १६ ब ॥  
 शेषस्य मासिकफलं विशेषयेद् मासिकोपनयात्।  
 शेषेणाऽनेन मूल विशेषमासं तु मासयुक् कालः ॥ १७ ॥

"Subtract the principal part of each monthly instalment free from the capital, successively. (This gives the number of complete months and a residue of the capital, if any). Subtract the interest of residue for a month from the amount of monthly instalment. Divide the residue of the capital by the remainder (The quotient) added to the number of (complete) months is the time (of being free from the debt)."

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He consider the pay of instalment  $a$  is interest on outstanding amount plus the payment as a part of the principal. Then the following rule gives the time of being free from debt. (FL) subtract the principle part of each month instalment free from the capital, successively. THIS gives the number of complete months and a residue of the capital if any and subtract the interest of residue of the capital by the remainder.

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## Equated monthly instalments

Explanation: Lent amount principal =  $A$ . Rate of interest =  $r$  percent per month. Let  $r' = \frac{rt}{100}$ , where  $t$  is the period of instalment. At the time of payment of first instalment, interest due is  $r'P$ . Monthly instalment =  $a$ .  $\therefore$  Principal part of monthly instalment =  $a - r'P$ .  $\therefore$  Amount due after first instalment paid  $P_1 = P - (a - r'P)$  or  $P_1 = P(1 + r') - a$ .

The interest for this at the time of second instalment =  $r'P_1$ . Amount paid =  $a$ .  $\therefore$  Principal part of monthly instalment =  $a - r'P_1$ . Amount due after payment of second instalment =  $P_1 - (a - r'P_1) = (1 + r')P_1 - a = (1 + r')^2 P - (1 + r')a - a$ .

Amount due after payment of  $n$  instalments  
 $P_n = (1 + r')^n P - [(1 + r')^{n-1} + \dots + 1]a$ . According to the rule, we go on till  $R = P_n < a$ .  $R$  is the remainder or the 'Residue' he talks about. It takes less than an instalment period =  $t$  to clear  $R$  with its interest. Let this be  $\mathcal{R}t$ . (or  $\mathcal{R}$  is the fraction of the instalment period).

Divide the residue of the capital of the number, the quotient added to the number of complete months is the time of being free from the debt. What he is trying to say see that the amount of principle be a rate of interest is  $r+n$ , so I call  $r$  prime  $rt/100$   $t$  is the period of instalment at the time of first instalment interest due is  $r$  prime  $P$  right.  $P$  is everything and monthly instalment is  $a$ , and principle part of monthly instalment  $a-r$  prime.

Because interest for this at the is called this, so amount due of the first instalment pay is  $P$ - this, we have rate is  $P$  and then they calculate the principle part of how much have cleared, you know that is given clearly by this, so this  $P_1$  is this, the interest for this certain time of second instalment is  $r$  prime  $\cdot P_1$  not  $r$  prime  $P$ , and amount paid is  $a$ , so then we have to find principal for whatever he says.

Now principle for the monthly instalment  $a-r$  prime  $P_1$ , and amount due of the payment is second instalment is this, and amount pay die of the payment  $n$  instalment will be this kind of thing you know and this kind of geometric position will come, according to rule we go on till  $r=P_n$  greater than  $a$ , all the remainder of the residue he talks about it, so he takes less than an amount period to clear or which is interest ok, so is less than Armstrong

So amount of instalment you know or if it is full instalment period you know you have to pay  $a$  but only small part yes pay less than  $a$ , so that is good all the things can be calculated which  $P$  is the principal, and principle  $a$  actually I used it here amount of  $P$  as re-correct,  $a$  is the same as  $P$ , I use it here and I use it afterward here,  $P$  is the principle here, ok that fraction can be calculated or we say given example then it is very clear to u.



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**Equated monthly instalments**

The interest on  $R$  for the fraction,  $f$  of the instalment period  
 $= R \cdot R \cdot r$ . The amount the debtor pays for the fraction of the  
instalment period  $\mathcal{R}a$ . Equating these,

$$R + RRr' = \mathcal{R}a.$$

$$\therefore \mathcal{R} = \frac{R}{a - RRr'} = \frac{R}{a - I'}$$

where  $I' = RRr'$  is the interest on  $R$  for one instalment. This is what  
has been stated.

[Though the rule appears to state the result for the case when the  
instalment period is a month, it is clearly applicable for any value of  
the period,  $t$ ].

Nārāyaṇa's concept of instalments is very similar to the modern  
concept of equated monthly instalments. The only difference is that in  
modern times the time for clearing the debt is fixed, and the equated  
instalment worked out, whereas here, the monthly instalment is fixed  
and the time for clearing the debt is worked out.

In (FL) instalment is very similar to the modern the time for clearing the dirty sticks, ok and of course compound interest is there, time for clearing the dirt is there and then you 5, 60 this is indicated month instalment, the best and compound interest, here the amount of instalment is fixed and the time is being everything calculated and taking into own fact that , so compound interest he did not talk about.

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**Example**

Example 24.

दत्तं दशकशतेन च शतं च कस्यापि केनचिद्दुनिना।  
प्रतिमासिकफलसहिता पञ्चाशत् स्कन्धकं प्रयच्छति च।  
अनृणी कालेन सखे केन भवेद् ग्राहकस्य वद ॥ २४ ॥

"A rich man lent somebody 100 at the rate of 10 percent per  
month. (The debtor) gives a monthly instalment of 50 including the  
interest. O friend, tell me the debtor's time of being free from debt."

Solution:  $P = 100, r' = \frac{10}{100} = \frac{1}{10}, a = 50$ .

At the time of first monthly instalment: Interest =  $100 \times \frac{1}{10} = 10$ . Payment  
 $a = 50$ .

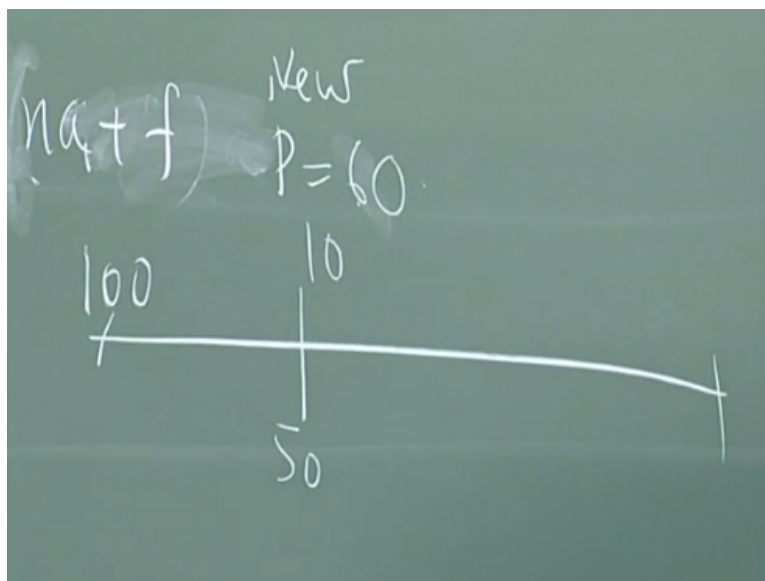
Payment towards principal part =  $50 - 10 = 40$   
Amount due after first instalment =  $100 - 40 = 60$   
Interest on this for month  $60 \times \frac{10}{100} = 6$   
Payment = 50  
Payment towards principal part  $50 - 6 = 44$

But still it is you know whatever amount of clearing that has been you know taken into account ok for giving relief for that, so realize that you know principle is you know being clear. So if I give an example sorry we ran out of time let me do it, a rich man lent somebody 100 at the rate of 10 percent per month, the debtor gives a monthly instalment of 50 including

interest, O friend tell me the debtors time of being free from debt, so P is 100, the rate of interest is 10.

Ok, 10% from the high pertaining it, so a is 50 amount of instalment, so at the time of first monthly instalment in interest is  $100 \times 10/100$  right and how see is, so 10, so payment is 60, so he is clearing the interest of 10, so payment was principle part, so rest is payment verse principle part 40, so amount due after first instalment is  $100 - 40 = 60$ , ok to get a point after we saying 100 was taken, so one period of instalment one month 10, you know is the interest, so he is paying 50.

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So if you subtract the 10 ok, so 40 is pay for specifically for clearing the principal, so principle now will be due principle, so that will be equal to 40, sorry 60, and interest for this month you know for month that is 36 ok, so after 1 month so the interest will be 6, ok so in that case the payment is 50, ok so the payment towards principle is 44, ok so that is 44, so how much is there, so it is (FL) 44.

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## Solution

Amount due after payment of second instalment =  $60 - 44 = 16 = R$   
as it is less than  $a = 50$ .

$$\text{Interest on } R = 16, \text{ for a month } 16 \times \frac{10}{100} = 1.6$$

Fraction of a month required to clear this debt with interest,  $\mathcal{R}$ .

$$\mathcal{R} = \frac{16}{50 - 1.6} = \frac{16}{48.4} = \frac{4}{12.1}$$

This is the fraction of a month required to clear the debt. At this time  
the debtor pays  $\mathcal{R} \times a = \frac{4}{12.1} \times 50$  to clear the debt completed.

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So the amount that you get is 60-44 is here you know so 44 is pay towards principle, 60 what was new or the principle, so 16 is there now the new principle is 16, so he has to pay the principle amount of 16 at the end of 2 months. Now 16 is less than 50, ok so interest and  $R=16$  for a month is 1.6, so the fraction of the month is required to clear this date with interest are so the fraction updated one can find out for that is 4/12.1.

So after this, this is a fraction is a month required to clear the debt, see after this after this only this fraction 4/12/1 one month is clear, to clear the remaining principle amount 16 and the interest on that, so that is taken here of this denominator, so if you go through this it will be so that is how he has, so in everything I did was told you it is in some more advancement is there in some more generalisation relation and some new methods and new principle.

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## References

1. *Gaṇitakaumudī* Of Nārāyaṇa Paṇḍita, Ed. by Padmākara Dvivedi, 2 Vols, Varanasi, 1936, 1942.
2. *Gaṇitakaumudī* Of Nārāyaṇa Paṇḍita, Eng. Tr. with Notes, Paramanand Singh, *Gaṇita Bhāratī* **20**, 1998, pp. 25-82; *ibid.* **21**, 1999, pp. 10-73; *ibid.* **22**, 2000, pp. 19-85; *ibid.* **23**, 2001, pp. 18-82; *ibid.* **24**, 2002, pp. 34-98.

The references are given here, thank you.