**Mathematics in India: From Vedic Period to Modern Times**
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**Lecture-22**
**Lilavati of Bhaskaracarya 3**

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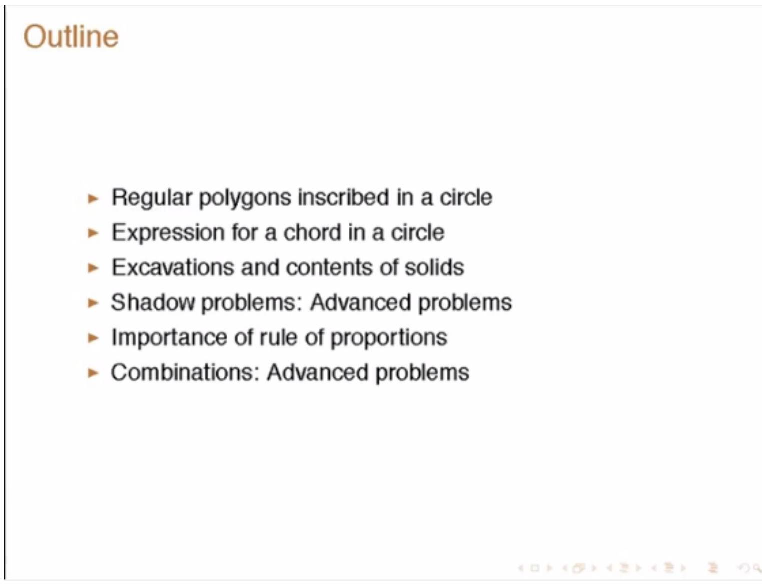


So, this is outline so, let me, so now he discusses the value of pie.

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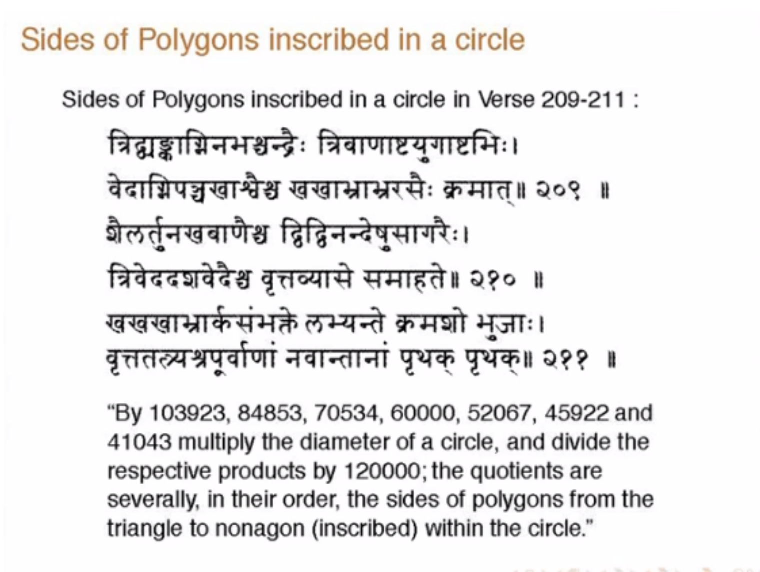


Determined by inscribing polygons in a solid, you know sorry sides of polygons inscribed in a circle, he. So, (FL), so he is giving the sides of triangle, squares, pentagon, hexagon etc., inscribed in a circle. So, for instance if you take the triangle inscribed in a circle, so you take the 103923 multiply by the diameter circle and divide by this and the quotient will be the side of the triangle, similarly for other things.

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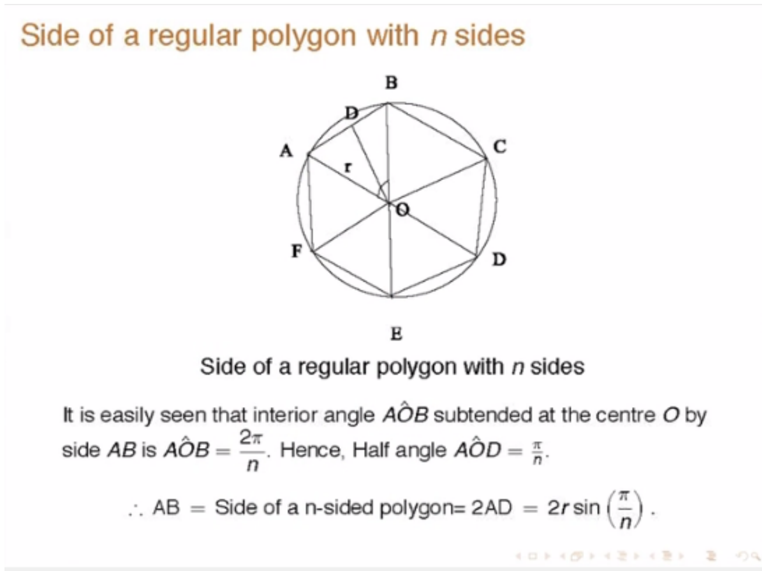


So, what actually the exact result is the following suppose the polygon of n sides is inscribed in a circle like this. So, then clearly it is interior angle AOB it is called interior angle you see. So then that is 2pie/n right because the 360 degrees 2pie, so divided by n each of this angles 2pie/n. so, this half angle will be pie/n, so the side of the polygon is twice AD and AD itself is r into side Pie/n the half angle is pie/n, so AB is 2r*side pie/n.

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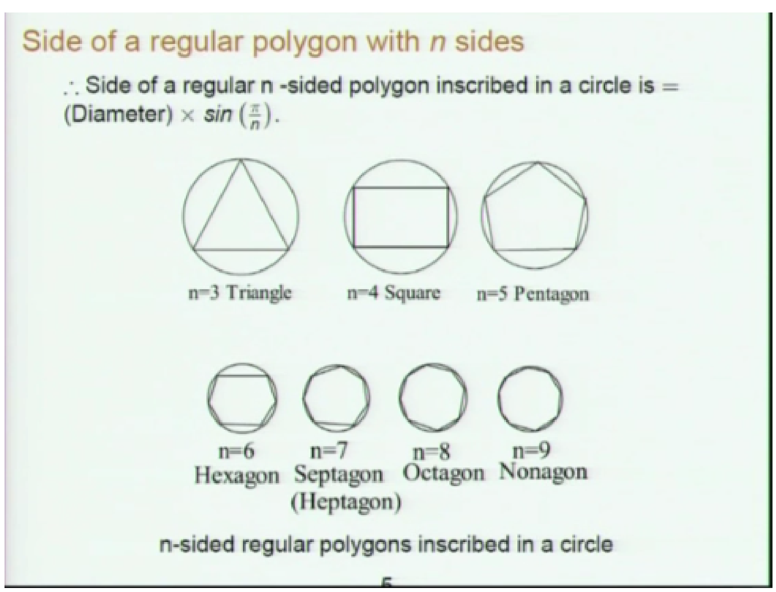

So, side of a regular n-sided polygon inscribed the circle is diameter*sin pie/n. so, if you have n =3 is a triangle, square, pentagon, hexagon, septagon or heptagon, octagon and nonagon.

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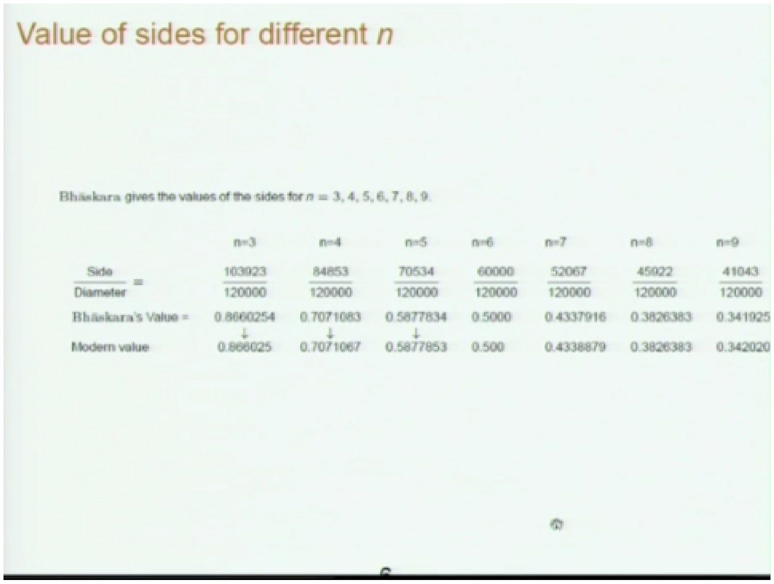


So, then value that Bhaskara gives is the following he said 103923/120000, 84853/120 etc., right so this is the things Bhaskara so, from this you can we can put in the decimal form like this we can do it. And a modern value is this modern value using you know I gave you the 2rsin pie/n, so it is very accurate kind of a thing. Of course the other way also as possible that there is what (FL) written right from the size of the polygon to get the value of the circumference by diameter okay.

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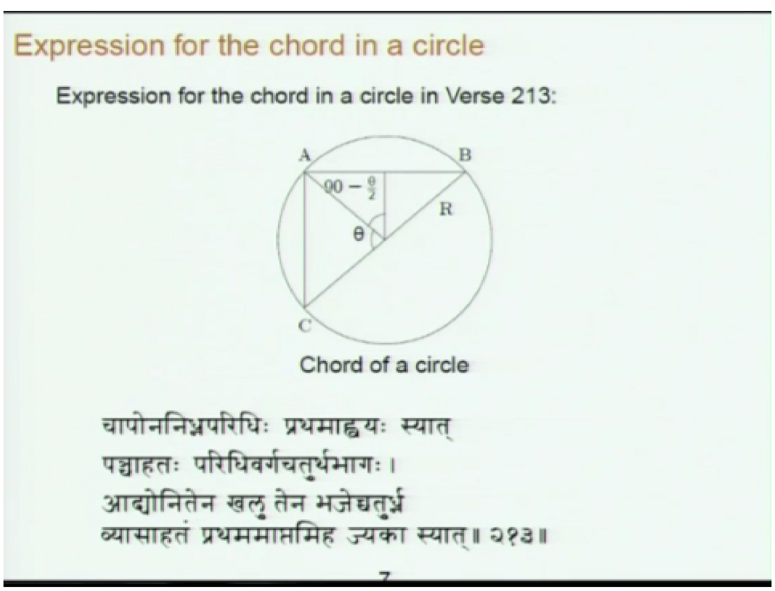

So, then he is one result in (FL) which is related to trigonometry, so chord of a circle, so this is in a circle chord in a circle. So, this is your arc kb, so corresponding to that this is chord AB. So, he gives a result for the chord in the following manner (FL), so what he is trying to say is that.

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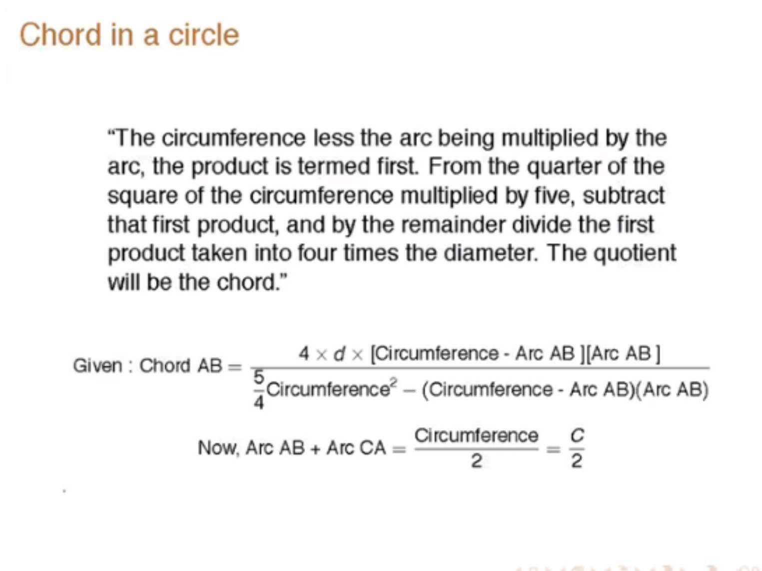


Circumference less the arc being multiplied by the arc, the product is termed first, from the quarter of the square of the circumference multiplied by 5 et., so what he saying this that this chord AB is 4* the diameter* * circumference-Arc AB* Arc AB/5/4*circumference square-circumference-Arc AB*Arc AB. So, now this is the result the very important results, so I will comment upon that in the following.

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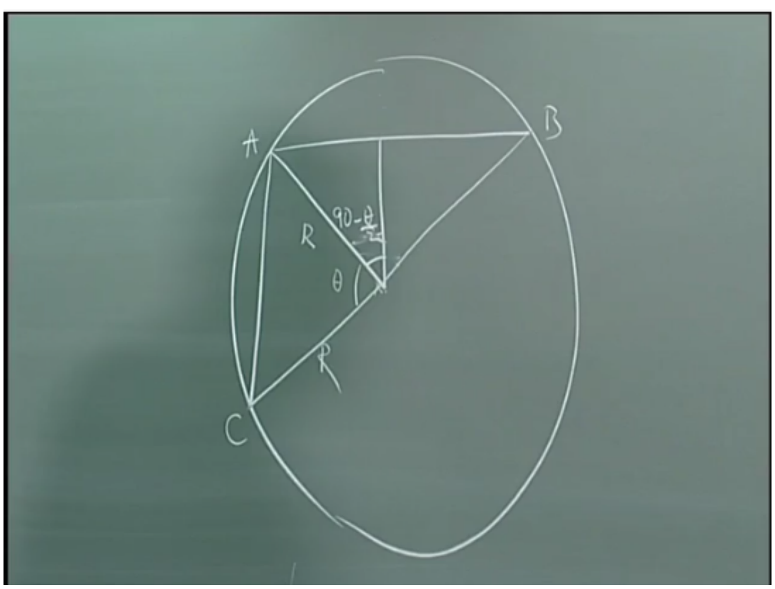

So, what are you having is better write the figure in the board also, so this is your AB, suppose so this is your diameter okay, so call this as C okay, so this need not be of course can be small or large you see but it be within this semi circumference, so let me put it like this. So, this angle is let us say so, theta, so then this angle sorry this angle is 90-theta/2 clearly right. So, I need this okay.

So, now Arc AB+Arc CA, so that is the semi circumference right C/2, so then Arc AB itself it is C/2-Arc CA your Arc AB is Arc semi circumference-Arc CA.

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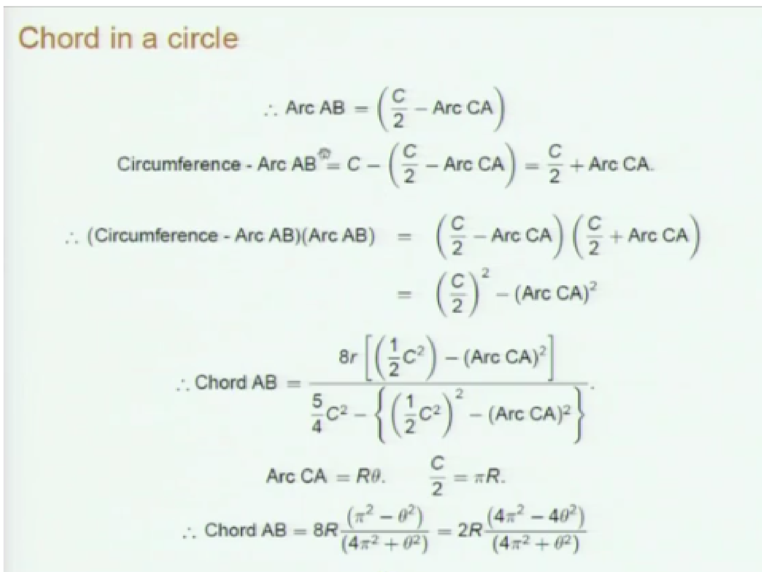


So, circumference-**A** Arc AB this C- C/2-Arc CA so C/2+Arc CA. so, circumference-Arc so you will get this things for finally called AB I mean according to if these a stated result it will be 8r*half C square-Arc CA whole square etc., so Arc CA is Arc theta you see Arc is theta this angle suppose you call it as theta Arc CA is clearly Arc this is R*theta, so that is the Arc CA and C/2 is pie*R, so chord AB is I mean in the modern notation 8R*pie square-theta square/4 pie square+theta square.

And so this I can write as 2R*, so this what is reduces to, so what he is stating of course is you know chord AB is like this whereas what we get is you know in the modern notation will be like this okay.

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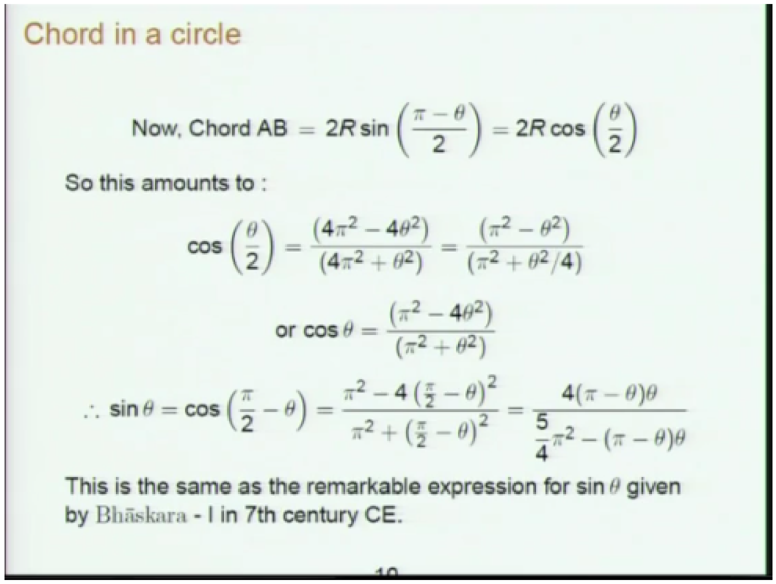


Then now chord AB, is clearly

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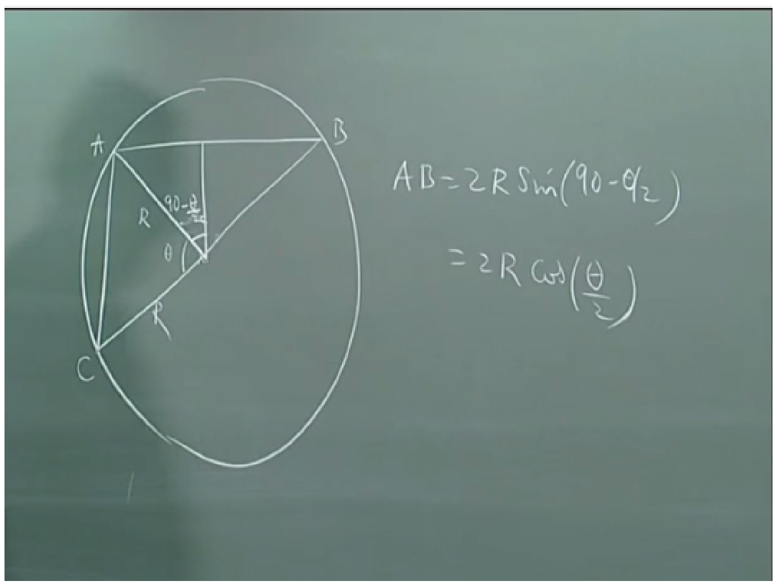


So, this is this is Ris clearly 2R sin of 90-theta/2 right, so AB=2R sin of 90-theta/2), so this is equal to 2R cos theta/2. I am working with Bhaskara Bhaskara-2's result to arrive at something. So, what it amounts to is that cos theta/2, so chord AB is the formula is given is 4 pie square-4 theta square/4 pie square+theta square, so that is cos theta/2 is pie square-theta square/pie square+theta square/4.

So, this valid for all theta, so if you write instead of theta/2 write theta, so cos theta will be pie square instead of theta I have to write 2theta. So, pie square-4 theta square/pie square+theta square. So, sin theta is cos of pie/2-theta, so you substitute that, so what to get is 4*pie-theta*theta/pie/4pie square-5-theta*theta see I mean of course the I am not derived this result I have only expressed whatever Bhaskara has said in this form okay, I have reduced into this why do I do is.

Because I have wanted into compare it with expression given by Bhaskara one way back in seventh century, Bhaskara-1, Bhaskaracarya 1 commentated every great commentator of Aryabhatta as well as he has done many other many works of his own composed of his work of his own (FL) and (FL) apart from his you know great Aryabhattiya (FL), so he has given this remarkable expression for sin theta by Bhaskara1 in seventh century.

And what is remarkable about this is that you know it is just you know it is given as a ratio of this to polynomials in theta and it is throughout the range 0 to 90 degrees this is correct it is you know it is accurate of up to about 1% of this. So, that is you know very because sin theta is very close to theta when theta is very small, so similarly for you know theta close to 90 we can get something you see.

But for it to be you know very good for throughout the range, so that is very very remarkable and he has just, so Bhaskara 2 has really restated the result due to Bhaskara1 in a slightly different language okay.

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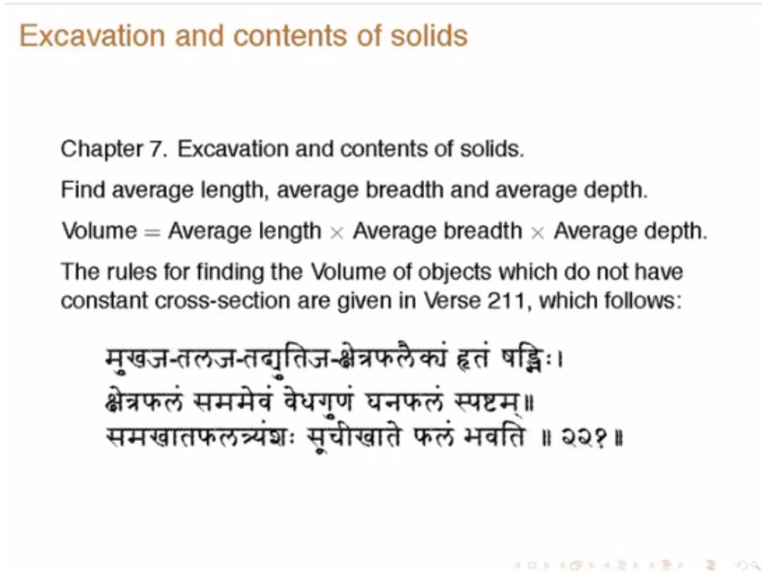


So, then again this is about the plane figures and now you goes to solve it is excavation and contents of solids. So, the get the volume the average volume is taken as a average length*average breath*average depth, so find average length, average breath average depth volume is product of this and a rule for finding volumes of objects we do not have cross-section are given verse 211 (FL).

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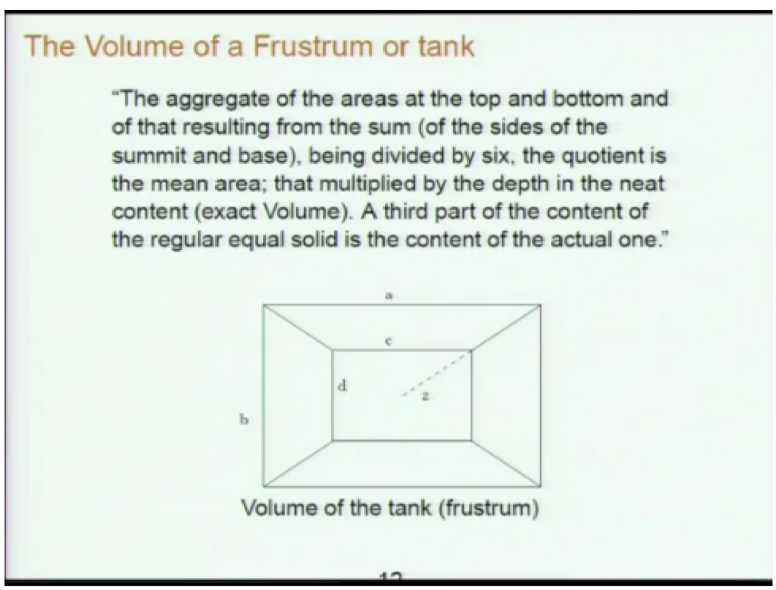


So, this is again the volume of a frustum or a tank whatever it is which is which as a tapering cross-section, so he is given that the aggregate of the areas at the top and bottom and of that resulting from the sum of the sides of the summit and base being divided by 6, the quotient is the mean area, that multiplied by the depth in the neat content not in is the neat content, a third part of the content of the regular equal solid is the content of the actual one.

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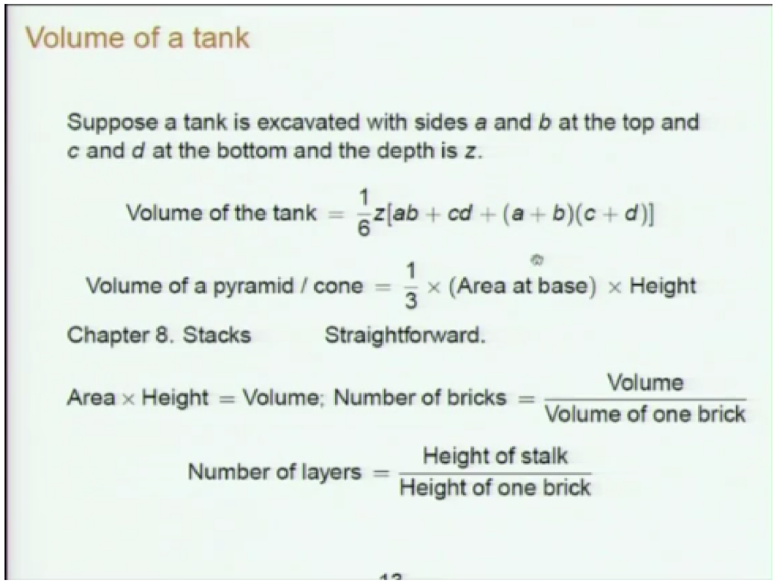


So, he is saying is amounts to the suppose you have a tank like this, so upper part is you know a square of you know rectangle of side A and B. the bottom part is a rectangle of side C and D, so it the depth is Z and you are assuming that it is tapering uniformly, so then in that case volume 1/6*z*ab+cd+a+b*c+d. similarly he state that the volume of the pyramid or cone is 1/3 area at base*height.

So, this is as a when you had mentioned it giving the top come from (FL) so it is written his result is given there also his owe he is just stating the well known result in Indian mathematics are that time. So, that chapter 8 is an stacks which is straight forward I mean know stacks of a layers of things, so the volume is area*height and if you are building it is bricks, so then the number of bricks is volume/volume of 1 brick.

And number of layers is height of stack by height of 1 brick because these are all important for wage calculation and so on.

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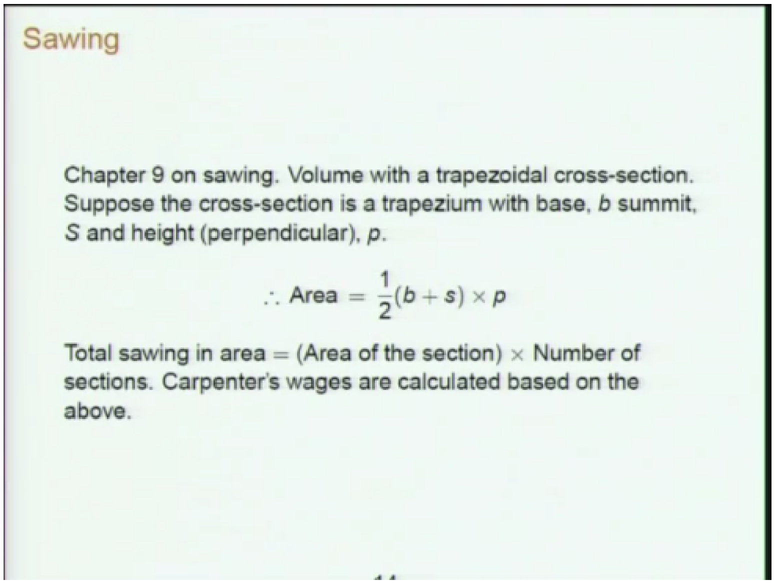


Similarly chapter 9 is an sawing, so volume with a trapezoidal cross-section, suppose the cross-section is a trapezium with base b  summit S and height p then area is half of b+s*p and we have seen that right.

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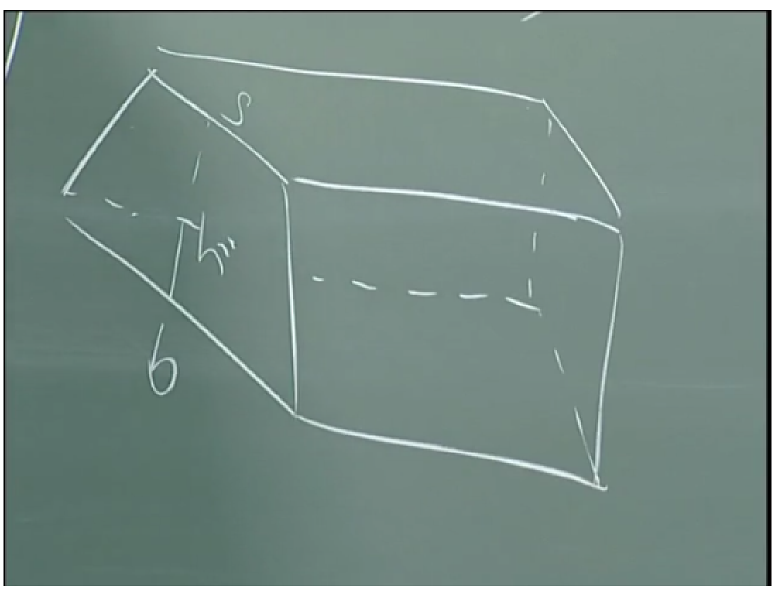


Again (FL) itself have this result suppose this b this thing this is S, so this is h and this is the volume okay. So, then, so this is the area of cross-section is b+s/2*h or p sorry p+h sorry there 1h area is there sorry this p cos theta is p here. So, the area is b+s/2*p and then so suppose you are you know having this kind of volume and in sawing it at various places you know cutting it.

So, total sawing in area is area of the section*number of sections, so because the carpenter's wages etc., are calculated based how much he has sawn right. So, how many cross-sections he has cut, so this is. So, these are all come in (FL) as I told it is you know in the earlier text also and these are standard topics you know volumes, stacks and things like that.

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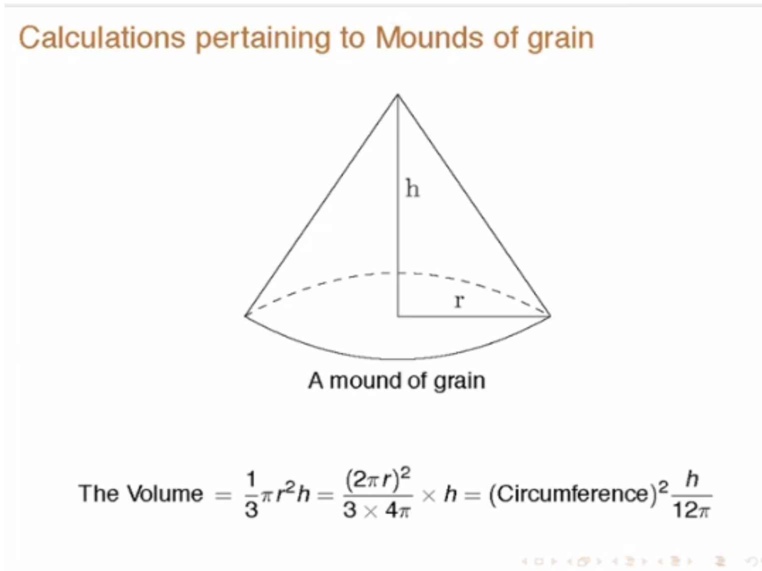


So, then he talks about calculations for pretending to **to** mounds of grain, suppose at mound of grain is in a conical figure is there, so in the volume of the this thing is we know that it is 1/3 the area of this base area*height, so this can be expressed as circumference whole square/h*/12pie.

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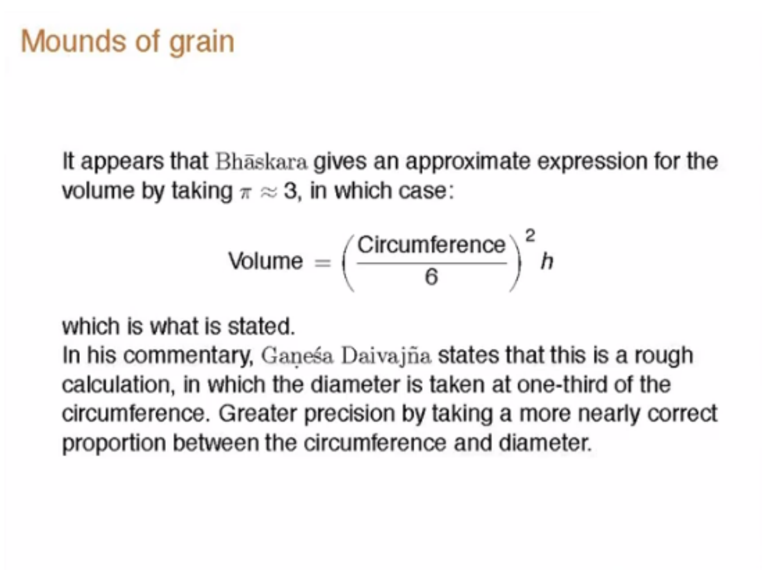

And it appears that Bhaskara gives an approximate expression for the volume by taking pie approximately =3 because volume in hew the expression for volume e gives in (FL) circumference/6 whole square*h, so that is what is stated. Of course in his commentary (FL) state that this rough calculation in which the diameter is taken at 1/3 of the circumference. So, greater position by taking a more nearly correct proportion between circumference and diameter.

So, this is you know for easier calculation you have just taking the value of pie to be 3.

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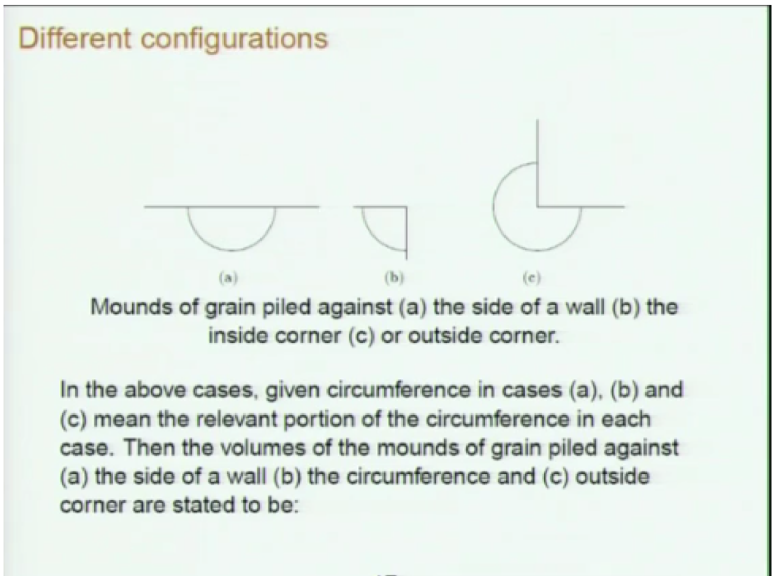


So, similarly you have different configurations you know the grains which are you know again the side of a wall. So, only half the circumference of comes into the picture, so then the grains stop you know in his this thing you know conical manner at the side you know at the corner only you are doing or outer corner you doing you know so, that is $3/4^{th}$ of this things circumference you are covering.

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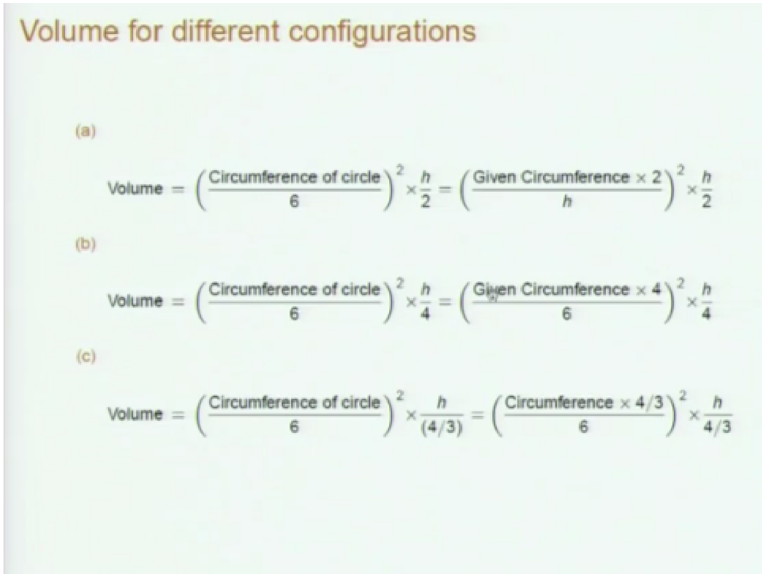


So, for that also volume is given this expressions are given, so again essentially this circumference/diameter approximate=3 is taken, so.

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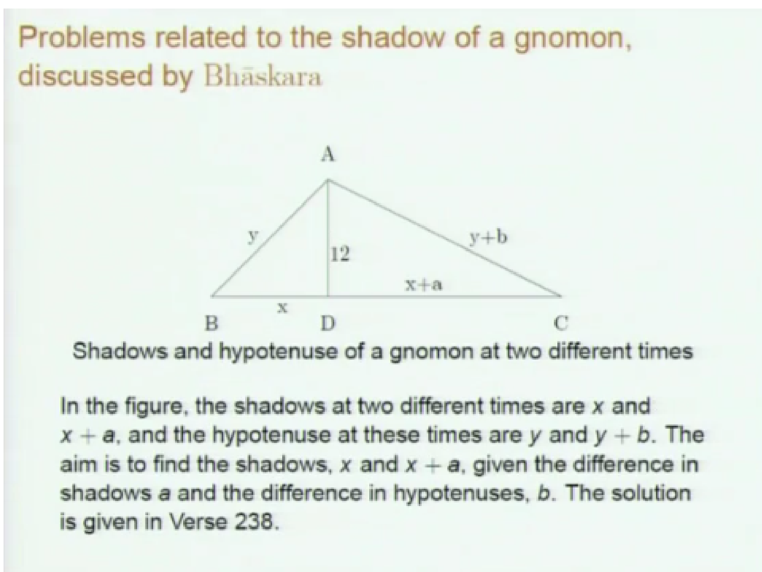


So, then the shadow of a gnomon that also we had seen earlier works invariably discuss this shadow problems, so here he is given a little more advance little advance problem. So, what he is saying is that there is a gnomon (FL) whose height is 12, so it is shadow is x or sometime and x+a at some other time. So, similarly then this (FL) a hypotenuse is y at the time when the shadow is x.

And y+b at the time when the shadow is x+a, so then you are given only a and b that is the difference in shadows and then the difference in hypotenuse, so then you have to find the shadow and a hypotenuse.

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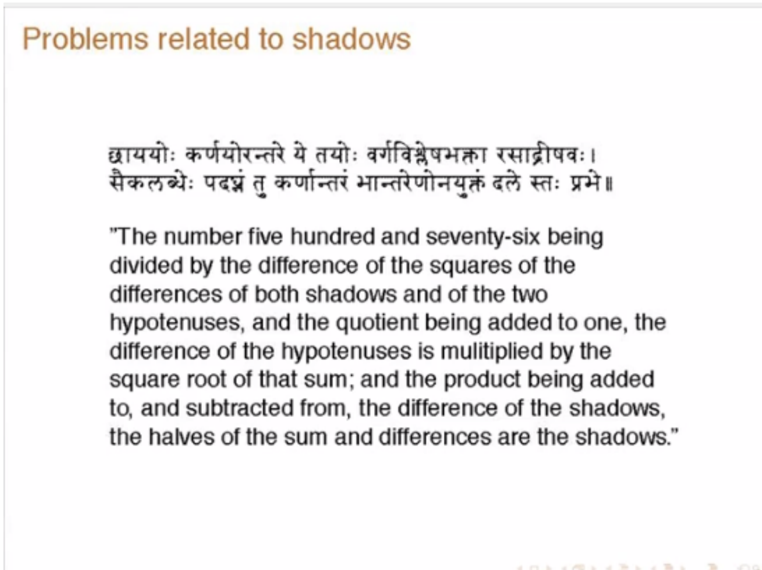


So, the solution is given here in this verse (FL) the number 526 is being divided by the difference of the squares of the differences of both shadows and of the 2 hypotenuses and the quotient being added to one, the difference of the hypotenuses is multiplied by the square root of that sum and the product being added to and subtracted from the difference of the shadows, so the halves of the sum and differences are the shadows. So, again if you write the expressions it will be clearer.

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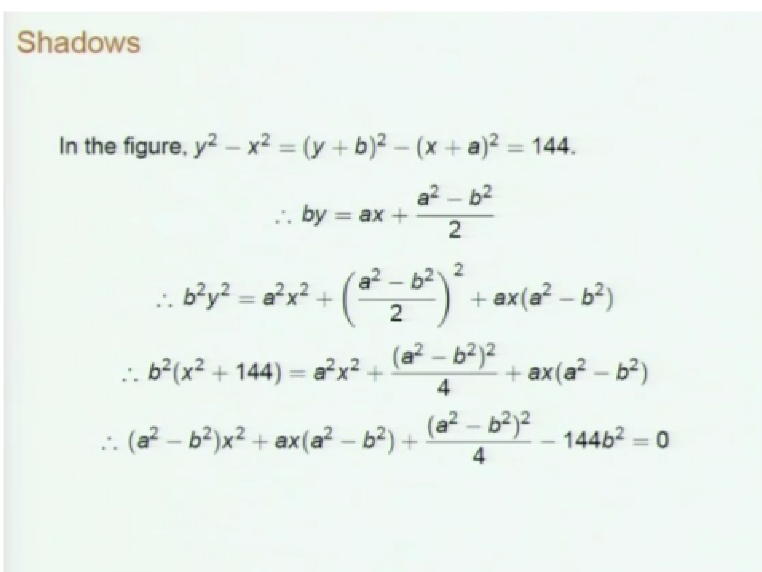

So, essentially you are having this right at 2 shadows at 2 different times is there.

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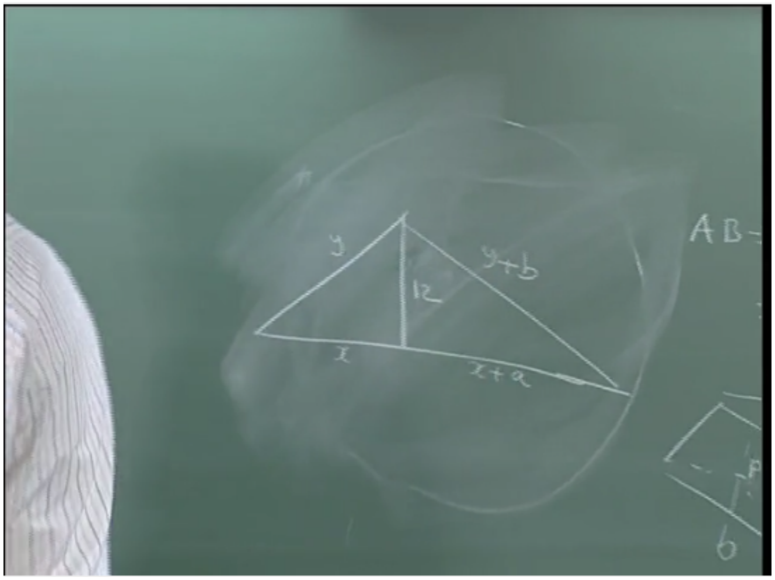


So, this is 12, so this x , this is x+a , so this is y, this is y+b. so, in that case clearly y square-x square is 12 square and also y+b whole square-x+a whole square is also there, so you get by=ax+a square-b square and so on. So, finally y, so y is given in terms of x basically by this equation okay. So, then he substitute back here, so finally you get an expressions a like this x the quadratic equation for x.

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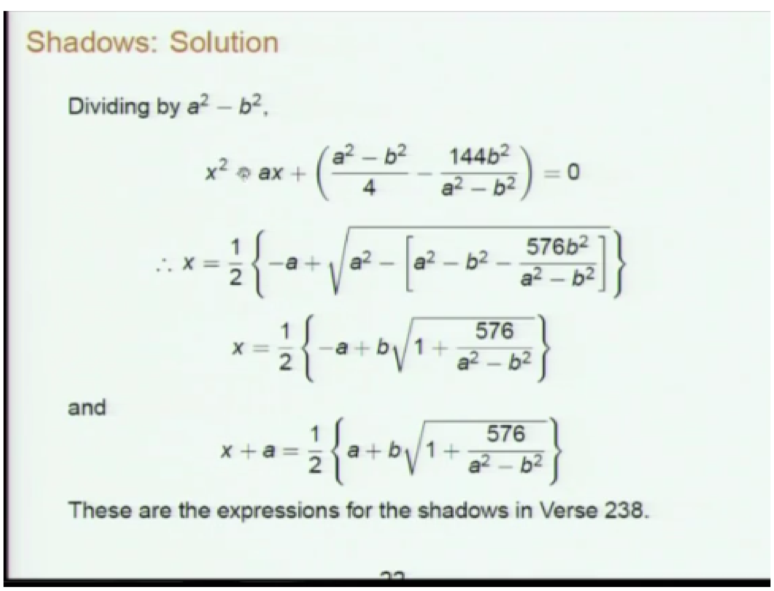


So, you have got x square+ax*this =0, so you can solve it and as it mention only the positive rule is root is taken, so x is the base and x+a is this okay.

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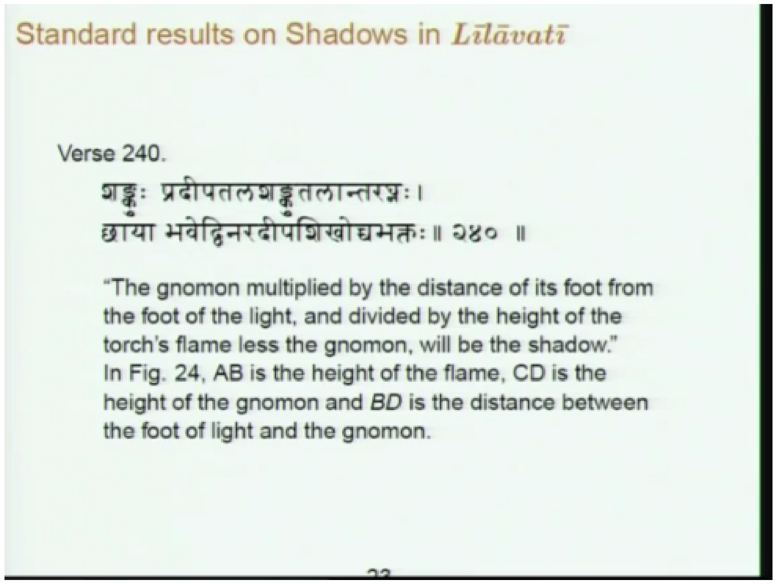


Now these standard results on shadows in (FL) again we know that if there is gnomon, so then the shadow is falls due to the lamp here okay then the shadow DE is CD*BD/AB-CD okay. So, these stated the expression for the shadow. So, we have seen you know it can be applied to eclipses and other things right. So, for instance in the lunar eclipse, so this will be the earth the radius this will be the sun's radius and this will be the place where the shadow is you know tapering to 0.

So, then when the shadow tip, so then the expression for that is there and you have to do a little more to find the shadow of the place where moon is moon's arbitrary places okay.

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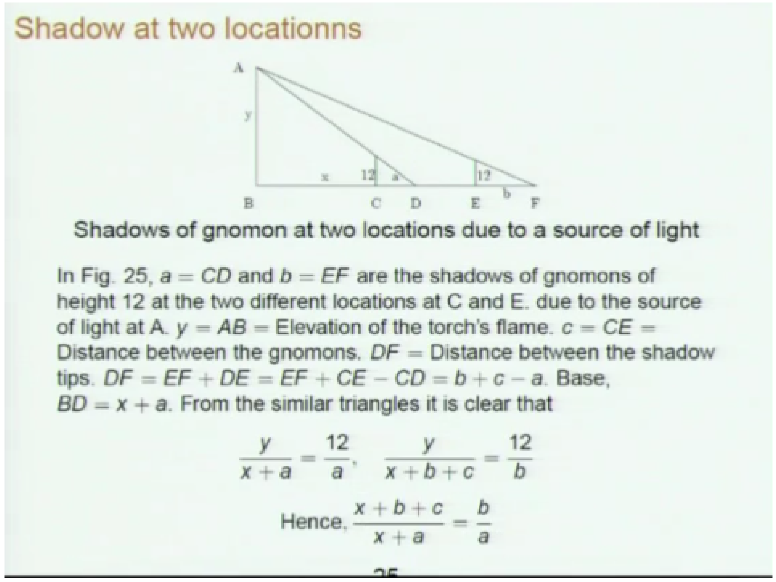


Then shadow 2 locations, so I do not have to go into too much into details of this.\

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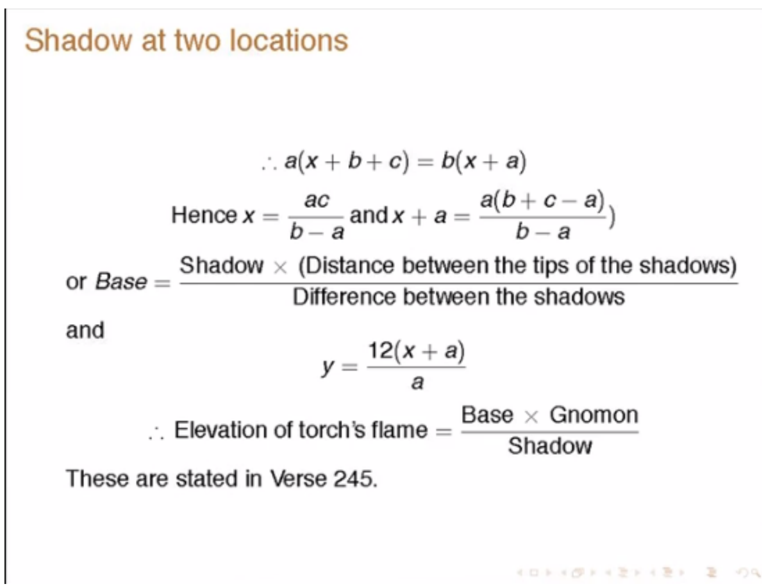


Because this also has been done earlier by in both in (FL) as well as in (FL) even earlier.

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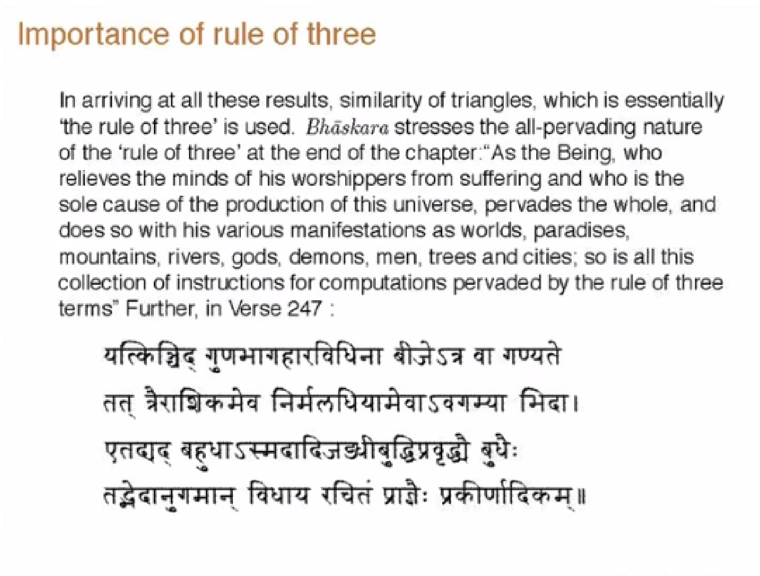


So, then he after discussing this things all these are of course are the crucial use is made of the similarity of triangles the all these results. So, then Bhaskara talks about the importance of rule of 3 he says that in arriving that all these results similarity of triangles which is essentially the rule of three is used. So, Bhaskara stresses the all-pervading nature of the rule of 3 at the end of the chapter.

So, he says that as the being, who relieves the minds of his worshipers from suffering and who is the sole cause of the production of the production of this universe pervades the whole and does so with his various manifestations as words, paradises, mountains, rivers gods, demons, men, trees and cities. So, is all this collection of instructions for computations pervaded by the rule of 3 terms oh is really you know very strongly get almost emotional about it (FL).

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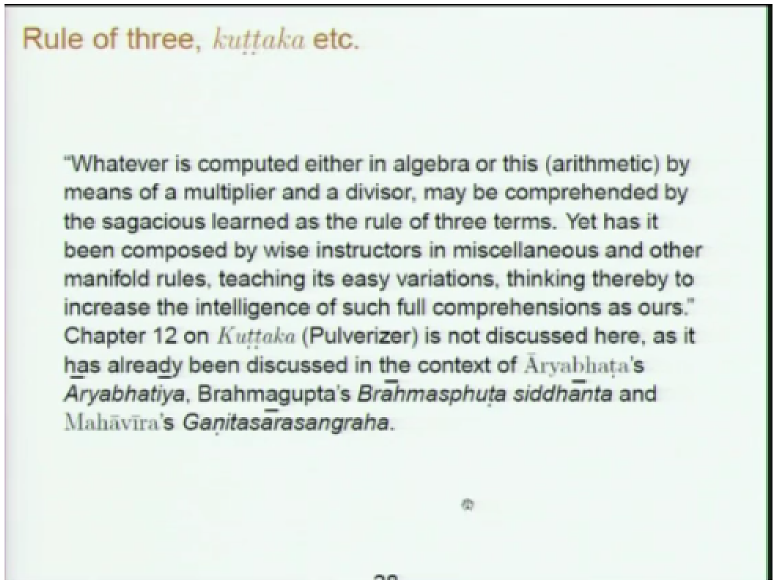


So, like that whatever is computed either in algebra or this arithmetic by means of a multiplier and a divisor, maybe comprehended by the sagacious learned as the rule of 3 terms, yet has it been composed by wise instructors in miscellaneous and other manifold rules, teaching it is easy variations thinking thereby to increase the intelligence of such full comprehensions as ours. Of course Bhaskara is very good that you know using verse you know he will use it to the maximum effect in all his works.

So, this is and in fact this rule of 3 is a actually when is astronomy you know that is where you know he has really Bhaskara in his (FL) shadow problem there is a normally the chapters and the shadows and you know measuring time and various things destination and longitude of sun etc., from the shadows. So, it is a normally there is a big chapter on that in Indian astronomy text.

So, really he is given very cleverly he used rule of proportions at various points you know and sometimes where it is not expected also you know one way one does not expect I will give you some example.

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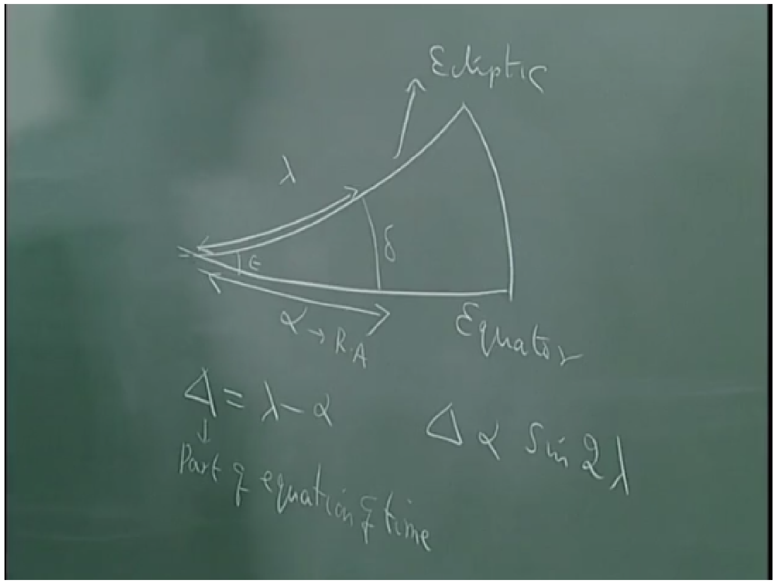


See consider for instance the sun is moving on the ecliptic which is inclined to the excavator. So, they are inclined by the circles and inclined by an amount equal to the oblique is ecliptic.

Now for instance suppose the sun is the longitude is lamda the declination is one of the find out actually declination is not proportional to lamda but sin of declination is proportional to sin of lamda. So, in astronomy at many places the sin's are proportional and not the original quantities themselves then now these apply to another problem. So, now this projection of these longitude on the equator, this is the equator and this is the ecliptic.

So, projection of this on the equator that is called as a right ascension it is denoted by the symbol alpha, so right ascension. So, now the time is essentially related to the right ascension, now there is what is known as a equation of time I do not want to technical, it is equation of time which comes in astronomy and that is essentially the equation of time one part of it is lamda-alpha, so this is the part of equation of time which determines when the sun is transiting the meridian.

So, now this is this thing now lamda=alpha at this point okay. So, lamda and alpha both are 0 and here at this point lamda and alpha are both 90 degrees, so now this equation of time is expected to be 0 here and o there of this part at least, so now in between what do we do. So, Bhaskara cleverly says that the delta is proportional to sin of 2 lamda, so that is it is maximum at lamda=45 degrees.

And then of course he will get the proportional the constant also in an engineer’s manner. So, this is the so you have to use this proportionality into the rule of proportion appropriately at sometime guess it also and in this case the guess is reasonably accurate.

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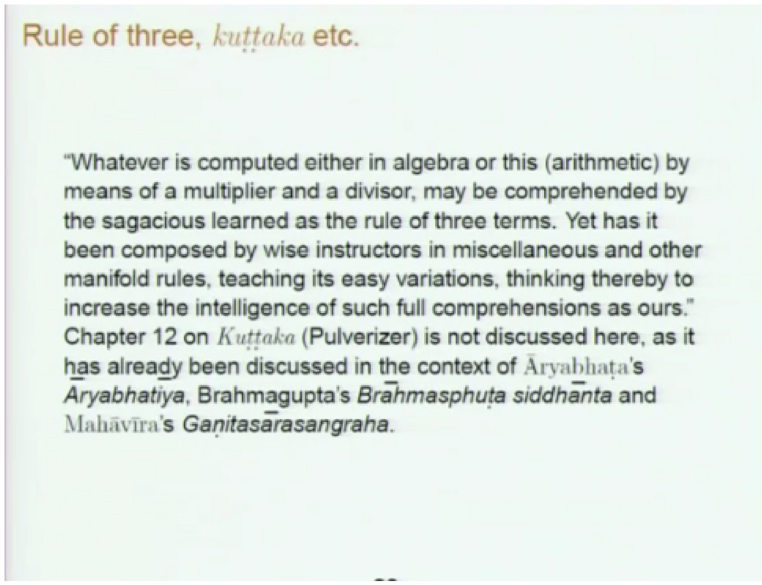


And rule of 3 is important in that you know, so these proportional to that kind of a thing , so one can says that he had a sense of proportion okay.

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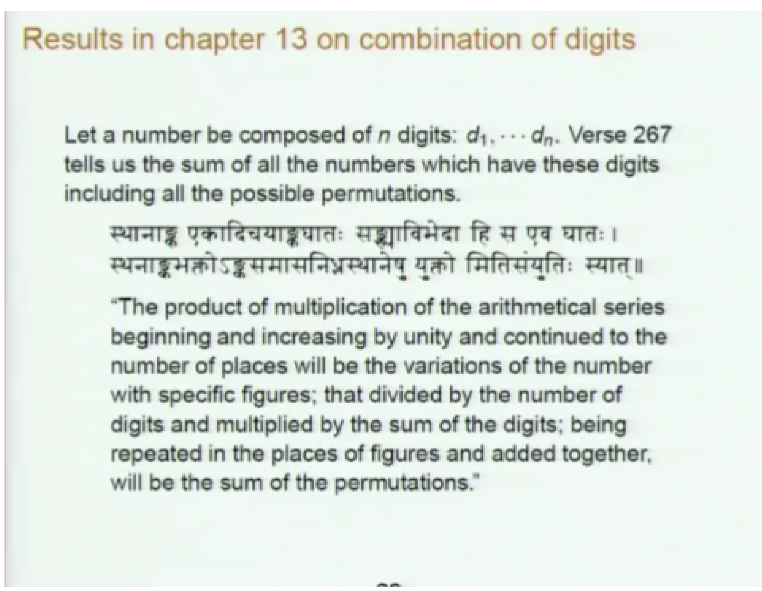


So, then the result are chapter 13 on combination of digits, so let a number be composed of n digits d1 to dn. So, verse 267 tells us the sum of all the numbers which have these digits including all the possible permutations. We see a we are seen that you know this combinatorics and all that are given even earlier (FL) but in the modern form n*n-1 etc., you have a 1*2, there are given by Mahavira and Bhaskara also stated this earlier in (FL), so now he is saying.

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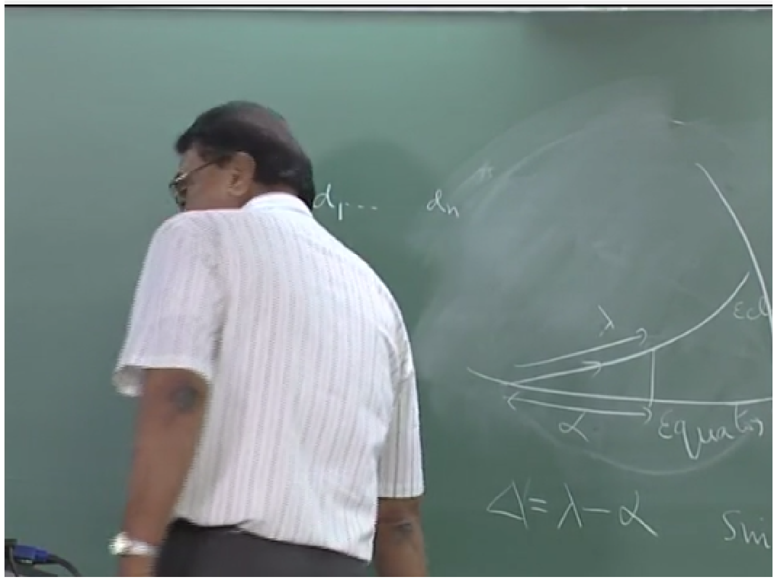


But sum of all the numbers suppose you have this dn, so you take all the permutation these all the digits okay to take all the permutations and then you get all those numbers and then sum of all of them what is that sum to that is all is what is talking about these are first time such a thing is discussed. So, the product of multiplication of the arithmetical series beginning an increasing by unity and continue to the number of places will be the variations of numbers which specific figures.

That divided by the number of digits and multiplied by the sum of digits being repeated in the places of figures and added together will be the sum of the permutations. So, sum of all the numbers inherited by this permutations, s that is what is given in this verse.

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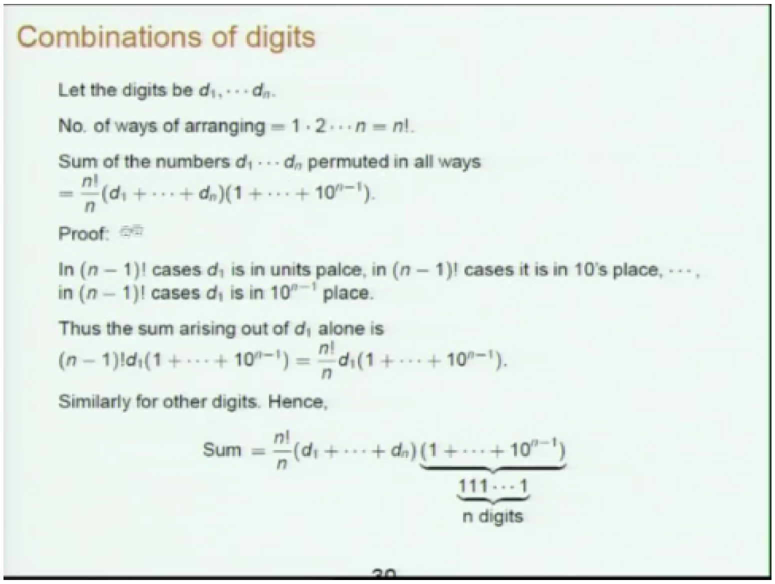


So, the let the digits to be d1 to dn, so number of ways of arranging by already seen that it is you know 1 to 2 etc., n right n things so the number of ways of permutation number of permutations you know that is factorial n and sum of the numbers d1 to dn permutation always he has given that all you know take this divide by n then this take the sum of all the digits and that you know in all the places 1+etc., 10 to the power -1.

So, that is essentially 1, 1, 1, 1 etc n digits, so that is the sum being repeated in the places and added together, so that is what is here okay. So, you understand it like this say d1 to dn suppose you take d1 alone it has taken in the sum while you take the sum arising out of d1 alone, so then d1 is there at some place and in factorial n-cases d1 is in units place okay, one work where is d1 you see, so one.

So, that is all the others can be interchange and you know permutated so factorial n-1. In factorial n-1 cases d1 is in units place. In factorial n-1 places it is 10th place etc., etc., in all the places you know it is factorial n-1, so thus the sum arising out of d1 alone is this I mean if you take the d1and then sum the effects you know whatever it is due to d1 in all the digits have all the permutations arising then we get this.

The factorial n/n*d1*this similarly of course it is 2 for all the other digits also, so factorial n/n*d1 etc., to dn*1+etc., 10 to the power of n-1, this of course just 1, 1 etc., n digits number with 1 at each digits.

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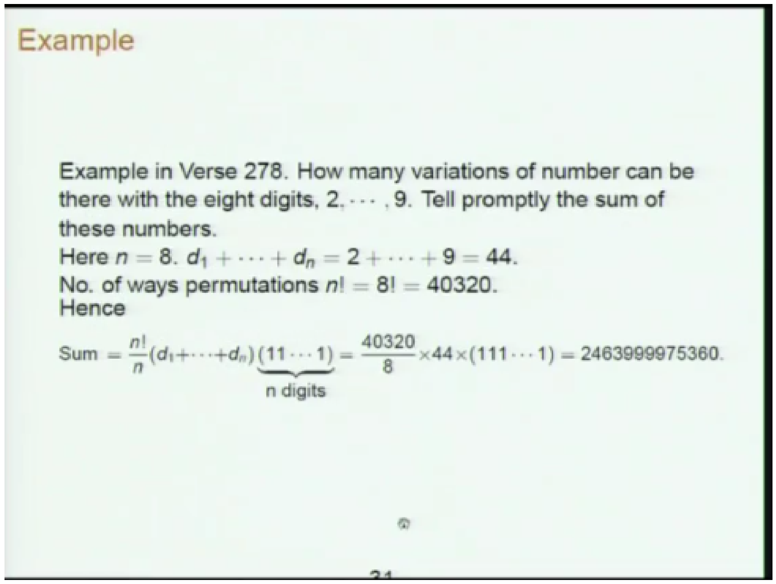


Here what is he saying, so example he gives in 278, how many variations of number can be 3 with the 8 digit 2 to 9, tell promptly the sum of these numbers. So, n=8, here d1+etc., dn is 2+etc 9=44 okay number of permutations is factorial n is factorial 8 it is 40320, so sum is given by this right factorial n/n*sum of digits*this n digit number with all ones, so this is the sum of these things.

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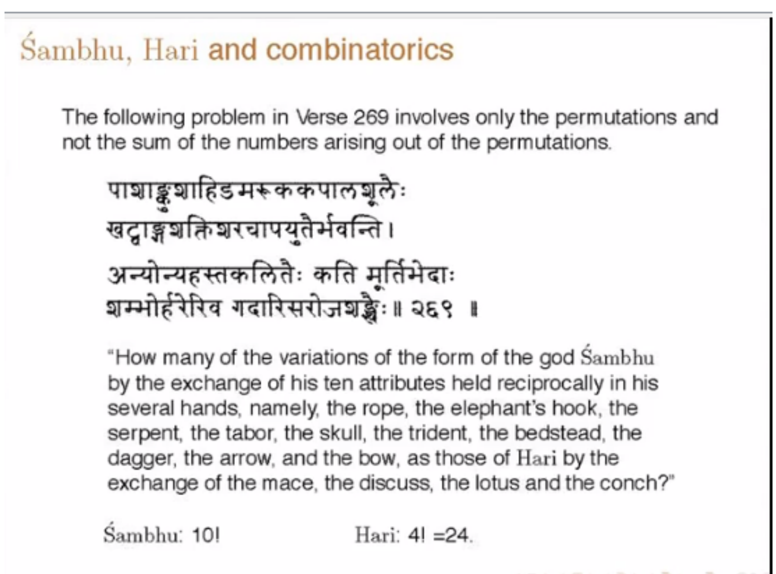

So, now the following problem in verse 269 involves only the permutations and not the sum of the numbers arising go to the permutations okay. It to it quite a comp earlier also but somehow he has taken this problem here (FL) how many of the variations of the form of the god sambu by the exchange of his 10 attributed held reciprocally in this hands several hands, so he has 10 hands here namely the you know (FL) rope, (FL) , the elephant's hook okay (FL) then the serpent (FL) the n (FL) then (FL) etc., you see dagger, arrow, bow okay also that .

And then as was the (FL) by the exchange of the maxe, the discuss the lotus and the conch hari is only 4 sambu is 10. So, factorial 10 and factorial 4 are the answer. So, this sambu, hari and combinatorics, so seeing this title some people will delight and some people maybe read with each okay.

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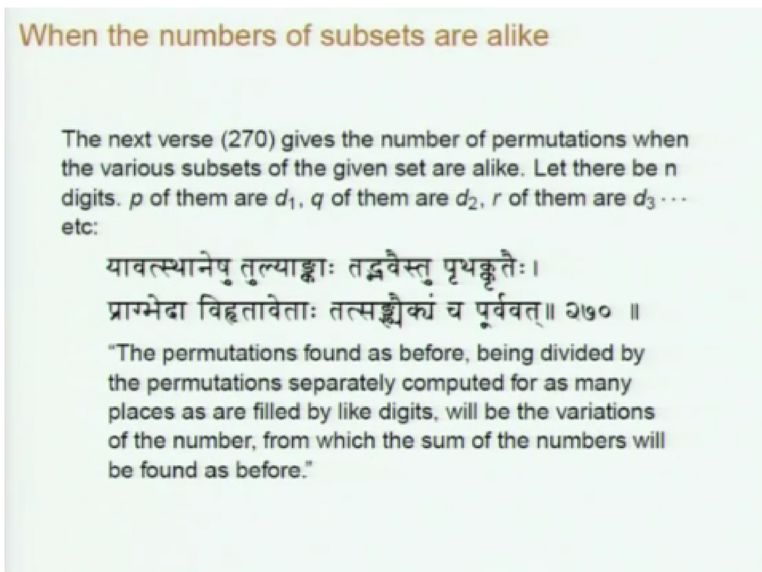


Now he talks about when the numbers of subsets are arise see there is a very important thing is doing this is the first time the next verse gives the numbers of permutations when the various subsets of the given set are arise. So, let there be n digits p of them are d1, q of them are d2, r of them are d3 etc., see when we did this arrangements okay. So, we almost assume that you know that if you took it for all of digits are distinct.

But suppose there 1 distinct are there obtain some 3 of them are the same, so if you are reeling them nothing new will come so we are over counting. So, we should avoid that, so that is what he is saying (FL) the permutations found as before being divided by the permutations separately computed for as many places as are filled like digits will be the variations of the number from which the sum of the numbers will be found as before.

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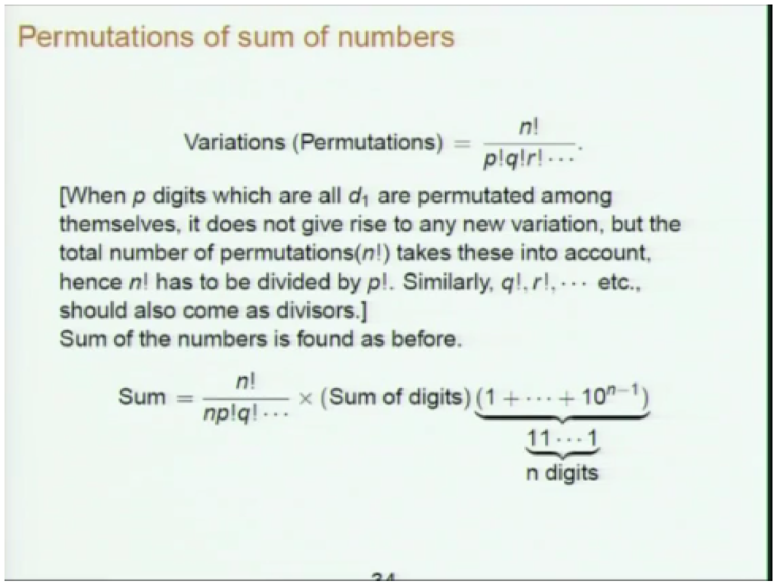


So, essentially he saying it variations of the permutations will be factorial n/factorial p, factorial q, etc., you have to divide by the you know is the subsets are there you know with p which are alike q which are you know alike and so on. Then you have to divide this factorial n/this thing because when you permute among themselves no new combination is going no permutation is going to occur. So, you have to for correct counting you have to divide by them.

This is the first time it is being stated in Indian mathematics, the sum of the numbers is found as before, so you have factorial n by n*factorial p etc., so sum of digits * this 1,1,1. So, that is this permutation with right objects.

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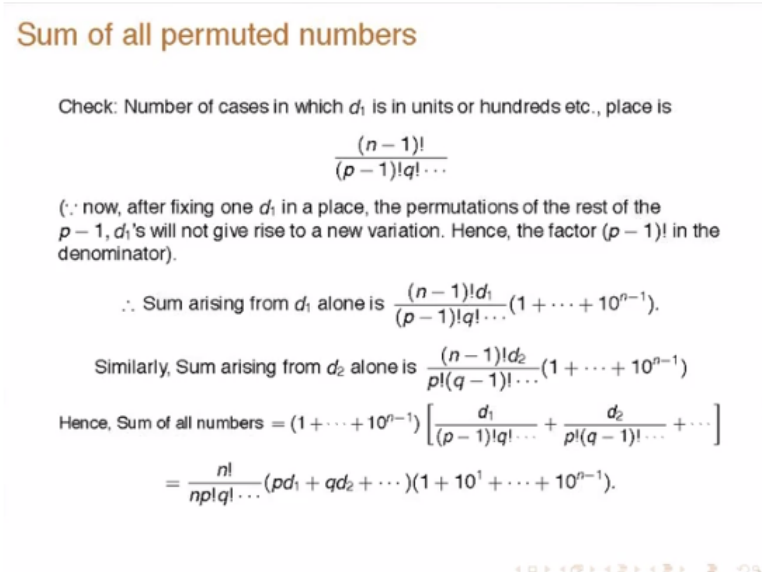


So, then the sum of all permuting numbers you can again check number of places in which d1 is in units or 100 place is d1 is assigned 1 place, so then the other places are n-1, so your n-1 factorial, in out of this few are alike etc., r are alike so you get this denominator factorial q etc., and p-1 are arise, 1 is already taken you know d1 is one of them, so p-1 factorial is are all alike. So, therefore because now after fixing 1 dn in a place the permutation for the rest of the p1 mid-1d1 will not give rise to new variation.

And the factor factorial p1 in the denominator, so some arising from d1 alone is factorial n-1*d1*all this/factorial 3-1* factorial q etc., so similarly some arising out of d2, so factorial n-1*d2 / this 1+this. So, sum of all the numbers you have to sum all this things. So, you have d1/p-1 factorial/q factorial q etc., so finally you get an neat result factorial n/n*factorial p*factorial p etc., pd1+qd2+etc., *this.

So, this is the sum of the digits, so this is new thing that he is doing you know that sum of the subsets are alike you know identical, so.

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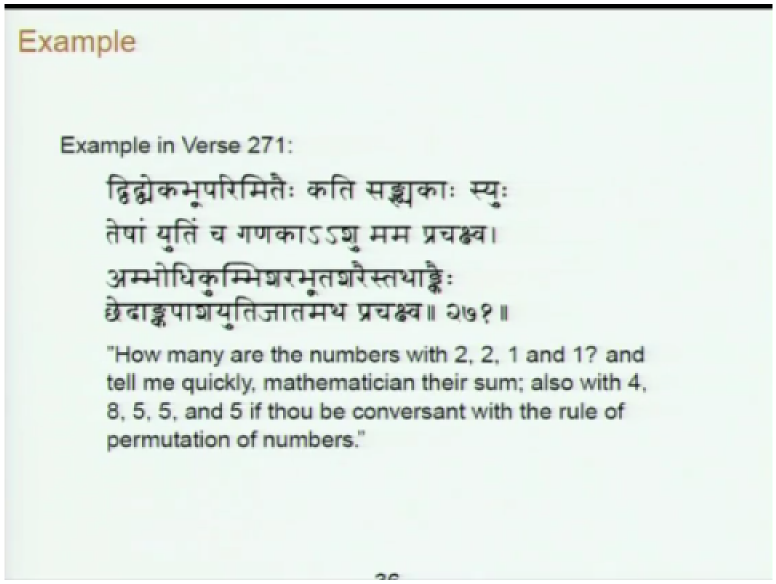


So, example for instance he takes the case how many of the numbers with 2, 2, 1 so here you know the 2 is apses 2 of them are alike and here 1 and 1 and tell me quickly mathematician their sum also with 4, 8, 5, 5if thou be conversant with the rule of permutation of numbers.

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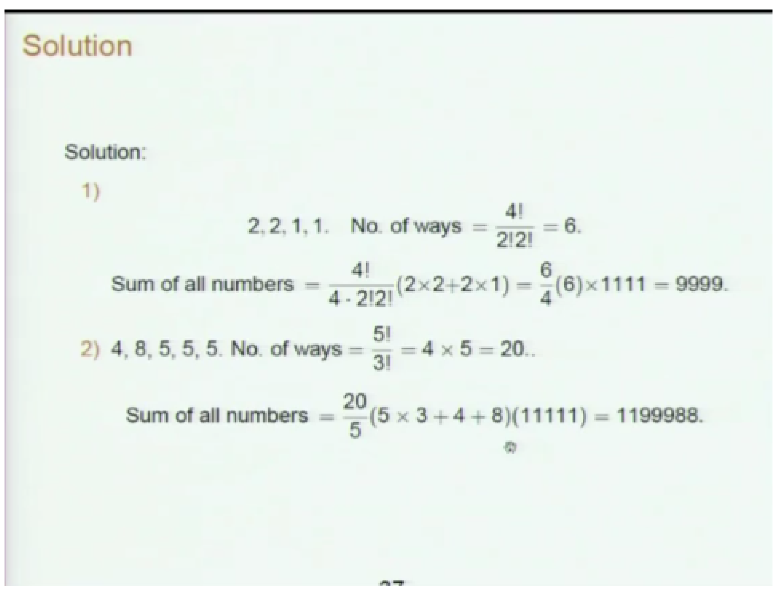


So, number only he is saying you know with we does not have distinct word number of cases you know how many see that is all (FL) he just tells the that is all there is no separate this thing for terminology for that. So, similarly I mean example for instance so here it is 4 factorial 4/the sets of to each so factorial 2/ factorial to 6, so then you have to divide by 4 and then you have to you know 2*2 the 2, twos and two 1.

So, you have to multiply by this, so you get 9999, so similarly if you have this, the number of ways is factorial 5/factorial 3 see these are all distinct and these are 3 of them alike so this is 4*5=20 and sum of all the numbers is this okay.

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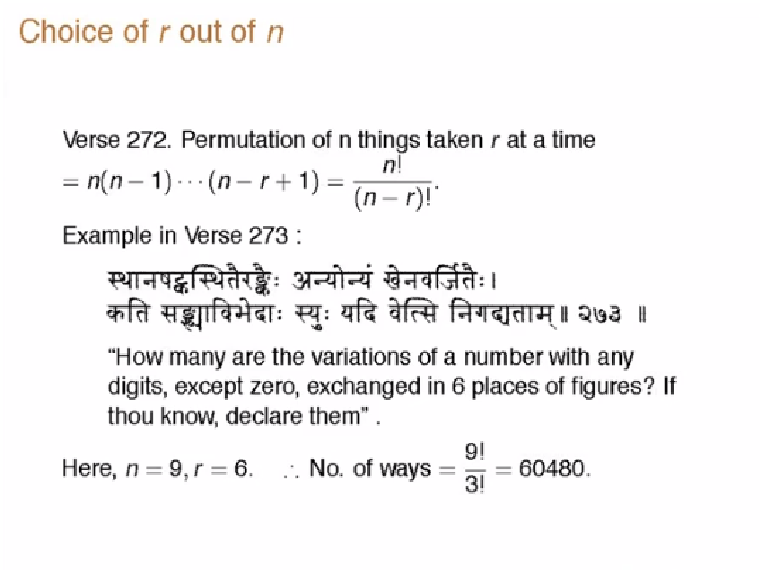


Then choice of r out of n again permutation have n things taken r at a time the again he is restating this things which had been discussed earlier also how many are the variations of number with any digits except 0, exchanged in 6 places of figures, so n =9, r=6, so the number of ways is this.

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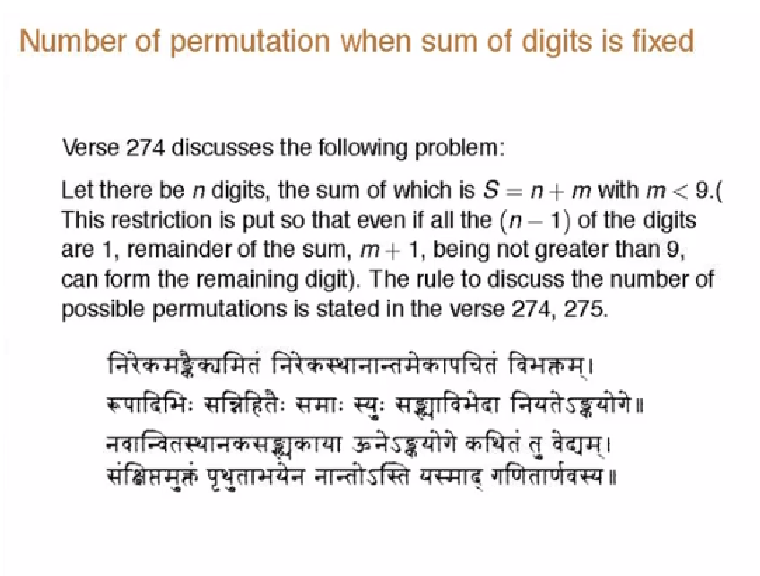

See earlier it was basically combinations only he had discuss (FL) this things, so now he is talking of permutation, so that is this (FL) ordered in important were not important here order is important you know. So, now it is a very important very interesting problem he discusses by for us one of the advance results in this work. So, let there be n digits the sum of which is S=n+m with m less than 9, so this restriction we will see is why it is restriction is forth okay.

Then rule is to discuss the number of possible permutations so that is stated in verse 274, 275 (FL).

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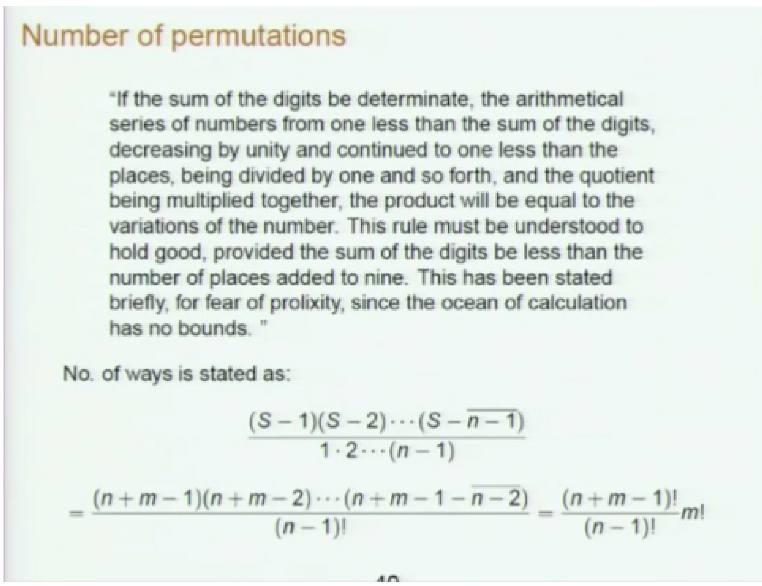


If the sum of the digits be determinate the arithmetical series of numbers from one less than the sum of the digits decreasing by unity and continued to one less than the places being divided by one and so forth okay and the quotient being multiplied together the product will be equal to the variations of the number, so the number of the digits is n, the sum of the digits is S, so you have to S-1 up to sum of the n this thing sorry n places here1, 2 sorry n-1 places okay between by the unity continue to 1 less than the places here right to that thing.

So, number of ways is stated this, so which S is n+m so I can write it as n+m-1 factorial/ factorial n* factorial m, so this is a very how do we understand this.

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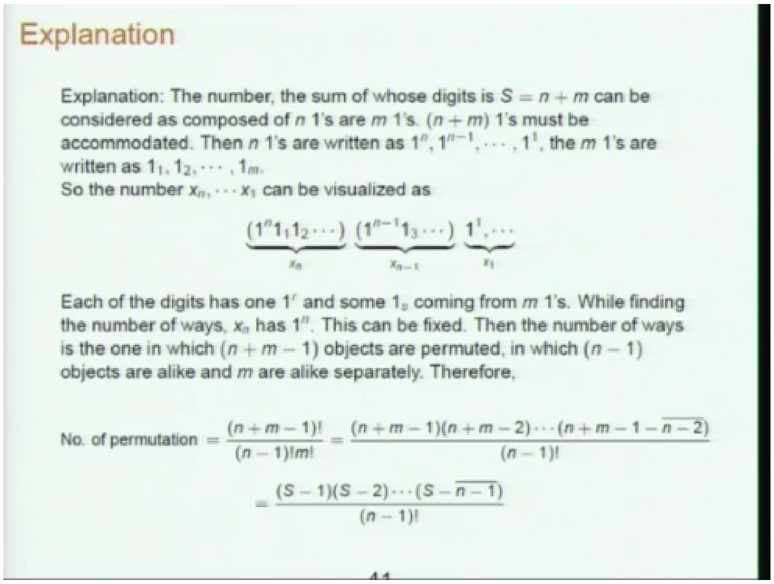


So, he does not give the proof but we can visualize it like this see the digits are n okay. So, sum of the digits is m.

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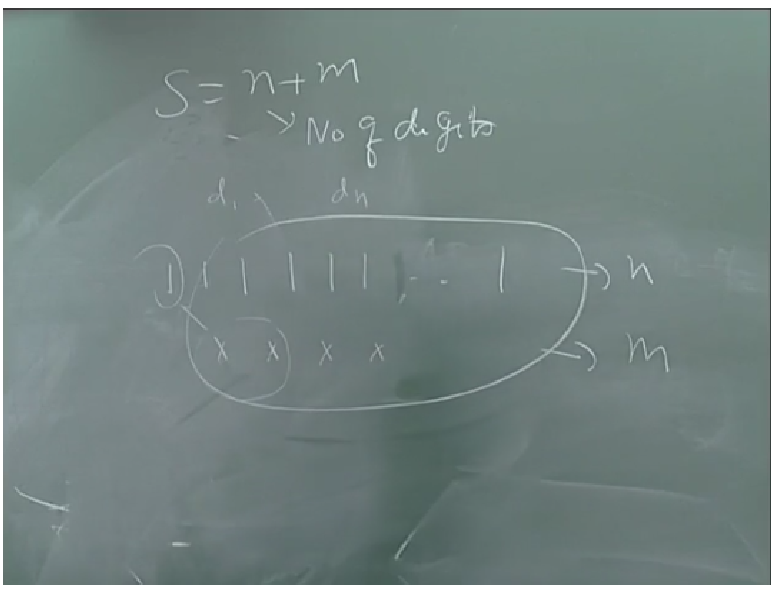


So, S is n+m, so these are number of digits, so you have to so various permutations we have to this thing okay, for now what you do is see each digit, so the d1 to dn you can visualize it as if it is compose of you know with write 1, n ones n of them and then you also write m of them okay. So, you can visualize this digital this number as you know each digit composed of 1 one you see each digit 1, 1 and then sum of these are attached to this kind of a thing okay.

And totally guarantee to be n+m, so this can be done in several ways, so that is what is being done did you get the point see n+m=number okay and you consider n+m ones, so each digit you know it is not 0, so 1., 2 8, 9 kind of a thing one to this. So, then total must come to be each of them cannot be 1, so sum of these things must be attached to each number. So, then total then come out to be n+m, so how many ways that is being done you see.

So, that is what I had written here number xn to xn would be n whatever you know can be written as this, so 1 this upper indent that is coming from this n ones and the row is can be coming from the lower ones, so 1n1, 1, 12 etc., the each of this digits is composed of this 1 which an upper index and several with lower index, so that this total of this is m okay. So, each of the digits has 1, 1r and sum 1 has coming from m ones.

So, while finding the number of ways xn has1n which is that by definition it is 1n this can be fixed okay this is at 1 plane and the number of ways is the one in which n+ these things see this you keep it fixed and all of these there can be permuted among themselves okay that is the number base, n+m-1is objects because this is n-1 will be left here and m is there, there can be permuted among themselves in which this n-1 objects are alike.

If you just permute themselves among there subset only in their subset then that will not give a new this thing and similar this also will not give a new thing if you permute among m only, so it has to be n+m-1 factorial/factorial n-1*factorial m, so this will be the result S-1-2, several ways of looking at it.

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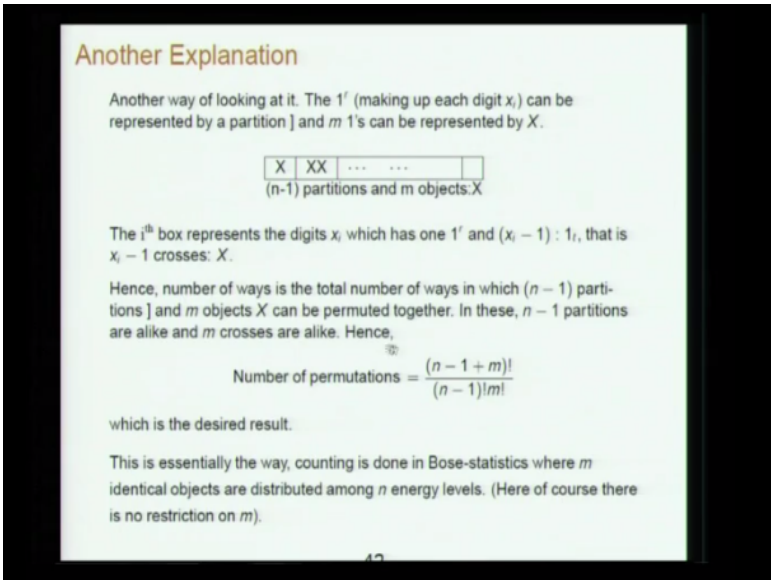


So, suppose you can also consider this you know that the each one of them can be represented by a partition, partition means I am consider physical partition not a mathematical partition physical partition and m ones can be represented by x okay.

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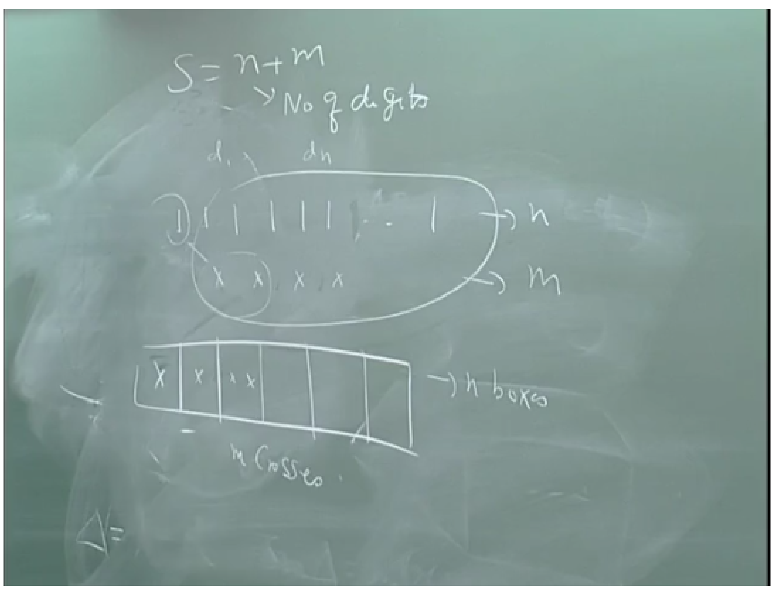


So, you can write it like this there are n boxes you know there are n-1 partitions okay, there are total n boxes and this crosses m crosses m crosses, so then you permute this you know, so the height box you know see suppose you have the box i, you have to pt 1, 1 is there and the number of x is here you see, so that will give the digit corresponding to the ith you know, the ith digit it will give so, that is what you do and then.

The number of permutations will be see you had taken permute all these things. You can permute the permutations for you see because this can be shifted you know see this can come here all the things can come here. And like that you know the both these vertical line and excess can be together they can be permuted. But if you permute this partitions themselves you will not get a new result new permutation. Similarly if you permute these alone you will not get a new permutation.

So, you get number of permutations will be n+1 n-1+m factorial/factorial n-1*factorial m is a little you know you have to think over it little bit is the result is the desire result. This essentially the way counting is then in both statistics by m identical objects are distributed among n energy levels okay. Suppose this n correspond to energy levels and m objects of there in how many ways are doing it you know identical objects okay.

Because m you know m is a identical objects are there okay. Then so, here the here of course I am not saying this everything they are same. Because here there is no restriction and m for both statistics there no restriction and n. It can be in fact for bosons it can be any in this thing a number for the Fermi it can be only one 0 or a 1 kind of a thing. So, that is the kind of thing you know which is done you know much more long back.

And here you can understand the distinction and m so, Baskaran himself is saying that mk should be less than 9 it can be 8 or lower. Because this digit you see can be 1 to 9 only and 1 is already there so, only (FL) add 8 you cannot add 9 so, because a 1 single digit cannot out two this things you know 10 cannot come. So, m has to be less than or equal to 8 or m is less than 8 so, this is the thing. So, this is the really a very interesting result which is there in Bhaskara Lilavati quite sophisticated is in it.

Considering it is you know twelfth century, so to be counting things like this and (FL) significant you know talked about this in the contexts of four so, that also is remarkable. Because himself in 1824 when you did this counting even quantum mechanics fully had not developed okay. And they had this plank law that derived it plank at given a law. He had derived it in some way and then instead of derived it using the principle of detail balance kind of thing.

But is is said that you know radiation and enclosure should be considered as the you know box of gas, a gas of photons you know and you have to do the counting write. So, he did the counting write and you know actually the statistics you know is really a very wonderful thing and significant he did it he did this comes some tradition you may of course he may not have we do not know the connection between him and the older Indian tradition.

But I am saying that the counting is and why it is consider in Lilavati also is because you know counting and this kind of manipulations are consider important.

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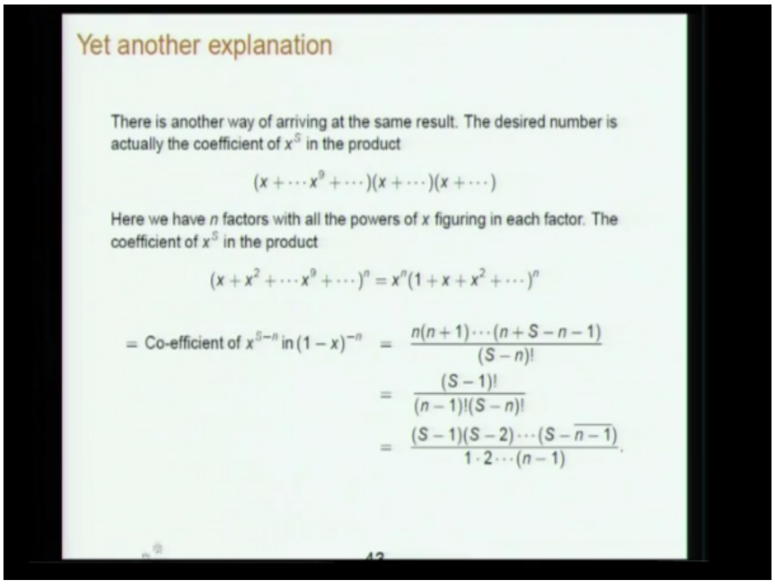


And these a particularly little difficult or little advanced topic in the counting for that time okay. So, there is another way of doing it also many other ways for this all them modern methods. You can say that you know suppose there are n, n is the number of digits right so, you write this number x+etc. up to any number of powers. So, n products like this n factors and with the powers of x figuring in each factor all powers of x figuring in each factors.

The coefficient of xs in the project sees this in can be consider as you know any digits n factors. So, essentially this what you are referring doing the number of ways in which you get the total number total sum as S which n factors. So, is x is a power of S in the product this you see x+x square etc… up to the power of n go to the power of n. So, what is the coefficient of x is the power of S so, that will be the actually the answer which can be written as x to the power of n into this.

1+x etc… whole square whole to the power of n so, which is x to the power of n you have already got. So, essentially coefficient of x to the power of S-n in 1-x whole to the power of –n. This see because this is 1-x to the power o f-n using the binominal theorem you get this like this. So, this is by for what you know Lilavati (FL) covered some important results covered in Lilavati.

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The references are given here thank you.