Mathematics in India: From Vedic Period to Modern Times
Prof. M.S. Sriram
University of Madras, Chennai

Lecture-21
Lilavati of Bhaskaracarya 2

Okay so, I will continue with Lilavati in this second lecture on the topic we continue with some applications of right triangles and what are known as suci problems.

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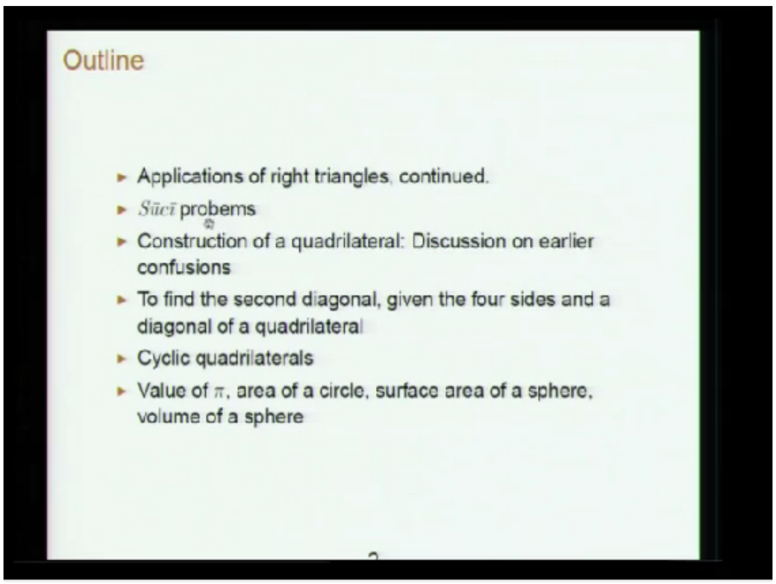

Then it is discusses a construction of a quadrilateral so, as a quadrilateral so, as a earlier confusions seem to be removed for the instance the area of a cyclic quadrilateral with given as remember square root of s-a into s-b into s-c into s-d so, earlier it is not stated specifically that you know it is for a cyclic quadrilateral. But Bhaskaracarya tells that it is only for a cyclic quadrilateral and it is not correct for any arbitrary quadrilateral.

So, then he will go on with a construction of a quadrilateral is essentially apart from only four sides it is not sufficient. You have to specify one more thing one more angle or a diagonal. So, then given the four sides and one diagonal go to construct find the second diagonal. Then it is continues with a cyclic quadrilaterals so, then the last this topic also is very important value of pie. I mean that is the ratio of the circumference and the diameter is circle.

Area of a circle, surface area of a sphere and volume of a sphere are discussed.

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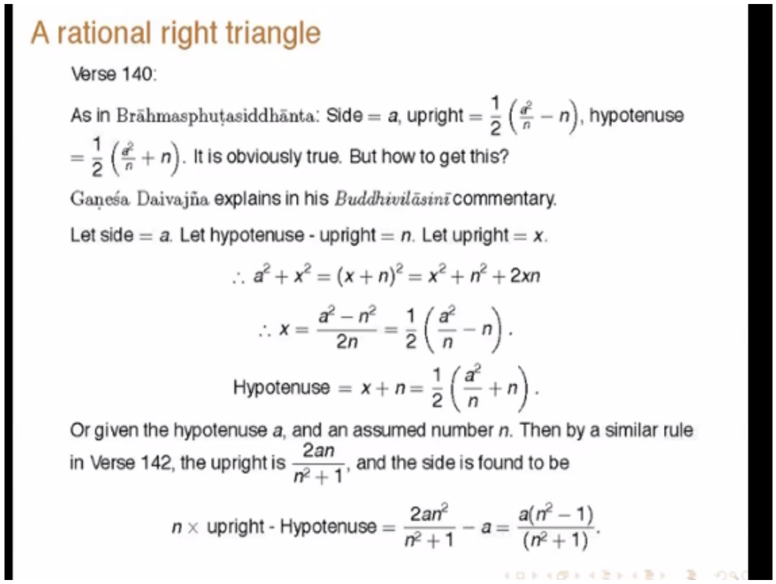

So, rational right triangle for instance suppose you have a side a upright or half of a square/n/-n then hypotenuse is half of a square/n+1. Of course is obviously true because this square+this square will be equal to this square. But how to get this so, I mean where do you start from so, (FL) explains in his famous (FL) any commentary. So, suppose the side is a so, then let hypotenuse-uprightb/n okay. You may takes side to be a and the difference between the hypotenuse and the upright is n.

And the let the upright be x so, then obviously a square+s square a is the side upright is x x square+x square is hypotenuse which is no upright+n so, that is x+n whole square. So, x square+n square+2xn so, immediately get x is equal to half of a square/n-n and hypotenuse is x+n is equal to half of a square/n+n. So, this ratio a square/n that is what is essentially you are starting with side a and hypotenuse s n+ the other perpendicular side.

You get this or given suppose you are given the hypotenuse an assumed number you see your side a and some number is given here hypotenuse is a and assumed number is given then by the you will find the upright to be 2an/n square+1 and a side is this a into n square-1/n square+1. So, these the some ways of construction okay.

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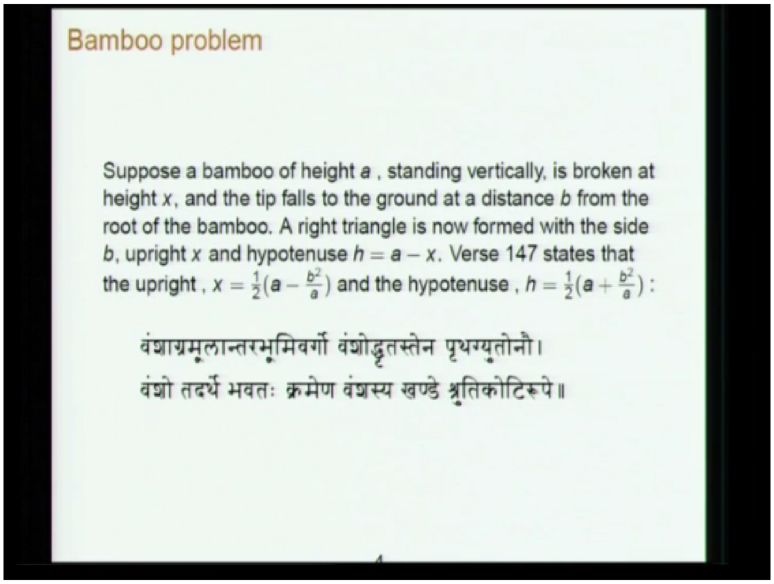

So, then the various problems he discusses some of them have become very famous most people who talk about Lilavati talk about this the so, called bamboo problem. Suppose the bamboo of height a is there standing vertically it is broken at height x and the tip falls to the ground at a distance b from the root of the bamboo. So, what is happening is that there is a.

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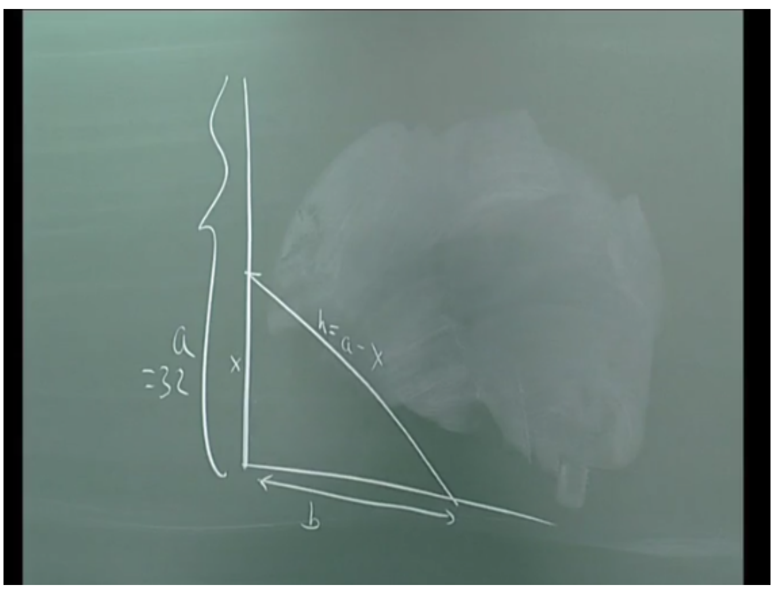

So, big bamboo is there so, then at some height you know it is broken okay and then this will be the thing. So, that this+this you see is equal to the original height right and this is yeah so, this is x, so, this is a-x. So, this is a and so, this is yeah d yeah so, (FL) okay of course you can enjoy the vertical beauty also of this (FL) coming various places.

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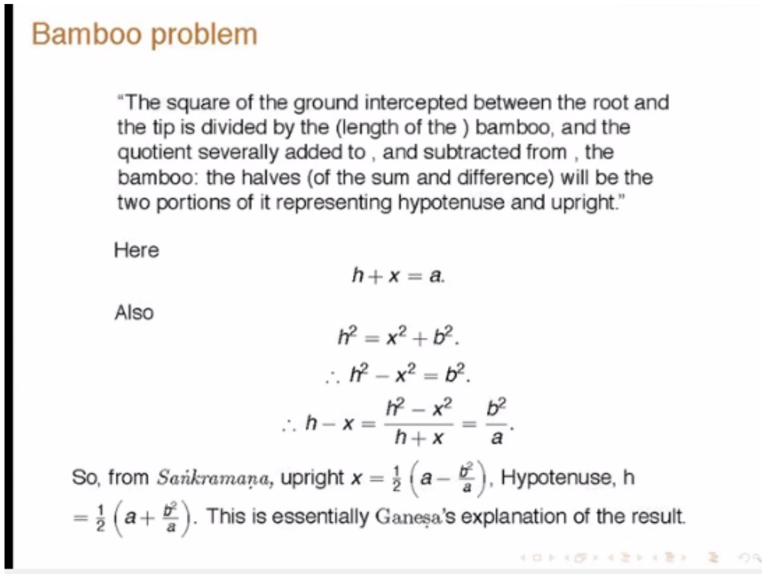

But what it means if the square of the ground into separate between the root and the tip is divided by the length of the bamboo. And a quotients severally added two and subtracted from the bamboo the hours of the sum and the difference will be the two portions of it representing the hypotenuse and the upright. So, essentially he is talking about situation where so, this is the hypotenuse so, h+x is equal to a right. So, h square is clearly h square is b square+x square.

So, h square-x square is b square so, h-x is h square-x square/h+x so, that is equal to b square/a. Because the h square-x square is b square. So, from (FL) so, h+x is a and h-x is b square/a so, from (FL) upright is x is equal to half of a-b square/a and hypotenuse h is half of a+b square/a. So, this essentially ganesa explanantion of the result.

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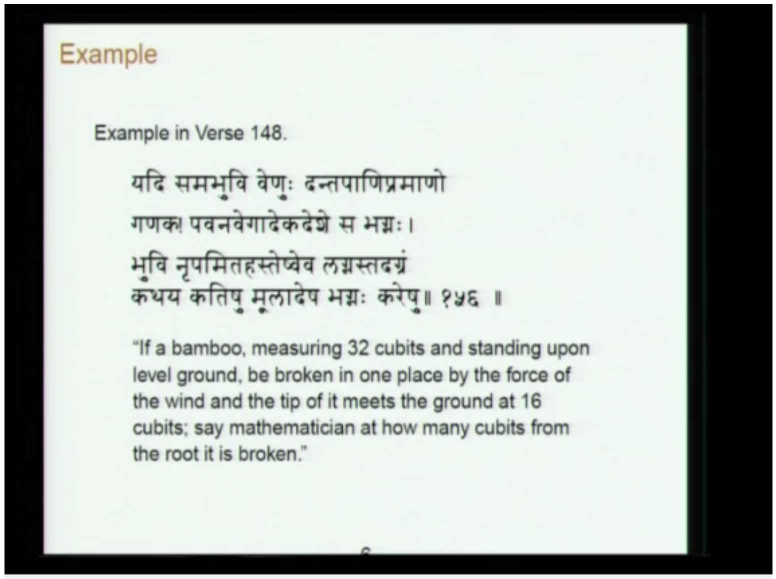

So, he gives an example also if a bamboo measuring 32 cubits and standing upon level ground be broken in one place by the force of wind and the tip o fit meets the ground at 16 cubits say mathematician at how many cubits from the root it is broken (FL) is 32 (FL) you know so, (FL) 32 cubits (FL) that is the measure (FL) so, because of this force of the wind it is broken at some this thing (FL) and a tip o fit meets the ground at 16 cubits (FL).

So, how is where it is broken okay so, here it is this is 32 the total height before it got broken that is 32 and this is 16 b is equal to 16. So, then you can find out x and h right (FL) is 32.

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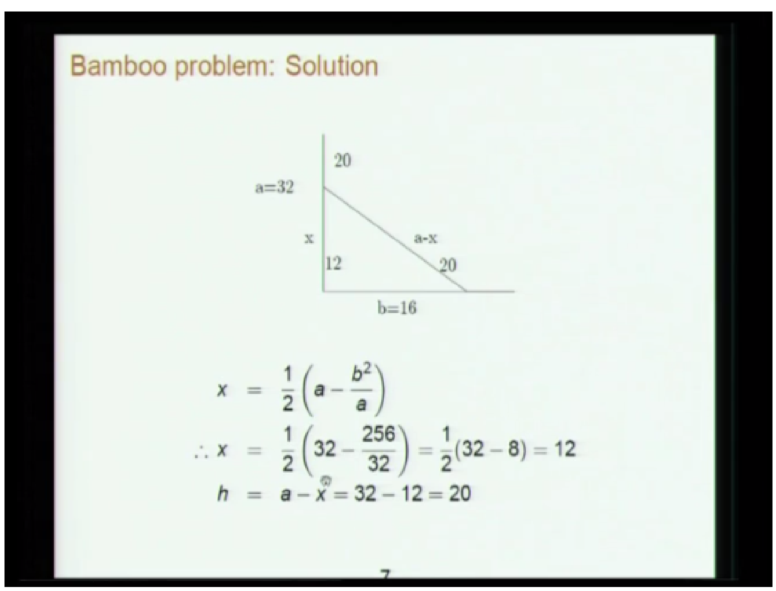

Is 32 teeth (FL) is (FL) you know so, then the cubit (FL) yeah so, that will of course satisfy the (()) (07:56) also you know so, and sounds nice so, x is finally the solution is x is equal to half of a-b square/a. So, half of a is 32, b is 16 so, b square is 256/32. So, this will be 12 and h is a-x is 32-12 is 20 okay so, this is the these the solution to the problem 12, 20 then this is 12. So, total so, this is 32 right 12+20 yeah.

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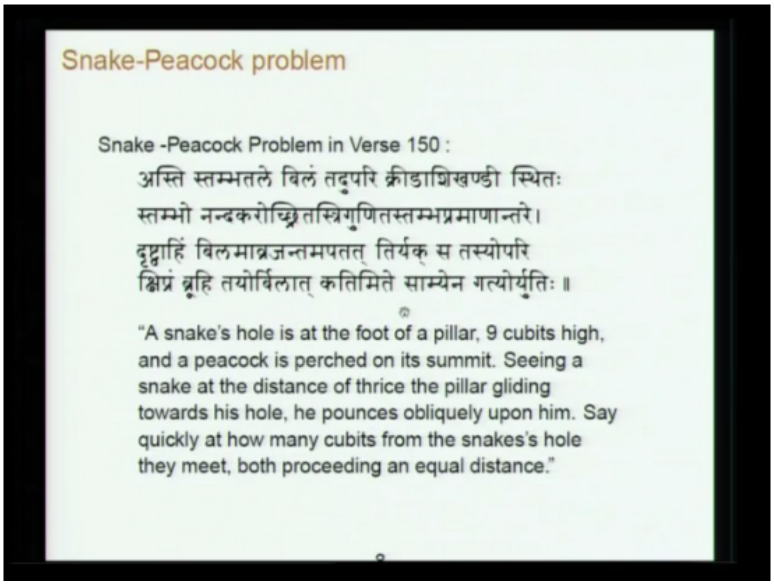

Then again you have this famous snake peacock problem verse 150 (FL) so, a snake is hole is at the foot of a pillar, 9 cubits high and a peacock is perched on its summit. Seeing a snake at the distance of thrice the pillar gliding towards his hole he pounces obliquely upon him say quickly at how many cubits from the snakes hole they meet both proceeding an equal distance okay.

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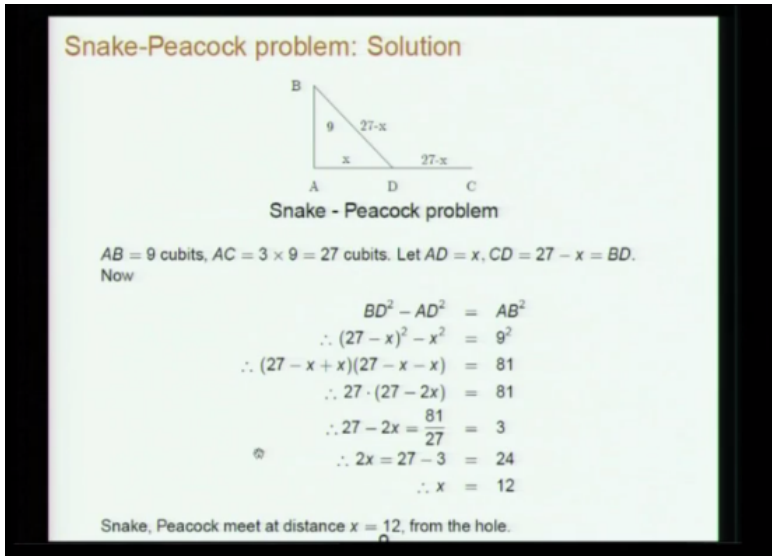

So, here is the problem see here this is some (FL) pillar so, upon that you know some peacock is there. So, this snake is here the (FL) that is the hole is here so, snake is proceeding towards this. So, and it is says that the peacock is proceeding to you know it is flying to moving to scared the snake. So, it is said that both of them are in a meet at the some points at that you know they the distance they are travelled is equal let is the move or the same speed okay.

It also given the this total AC that is the this is speed time this height so, 27. If you should know this x so, x 27x. So, AB is 9 cubits, AC is 27 so, this is x let us say so, CD is 27x-x so, this is equal to BD so, BD square-AD square is equal to AB square. So, 27-x whole square-x square is 9 square so, if you solve this you will get x is equal to 12 so, snake and peacock meet at distance 12 from the whole okay.

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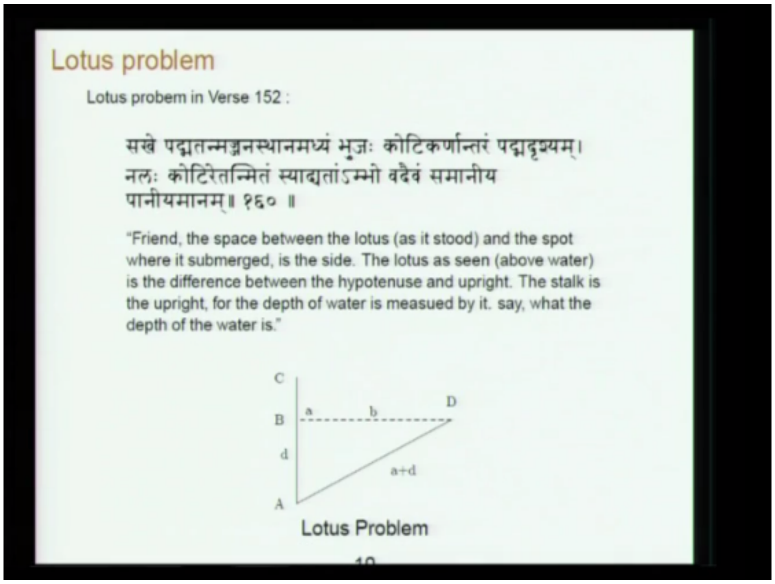

Of course this will lend itself good dramatraiation also rasimin some CD made on lilavati you know so, they are the demonstrate all these you know some of these problems are demonstrate it. So, snake is coming you know the music you know snake is coming and he (()) (11:02) everything so, nice in fact earlier chandralekha from Chennai also had done a play this thing on you know dance, drama based on the (()) (11:15) I am not seen that it has inspired so, many people in so, many ways.

Then again this is also a famous problem called the lotus problem (FL) so, earlier you see the lotus is there this is the root so, this is the base and because again the force of the wind so, let lotus will gets submerge okay. So, this will become the diagonal this will become the you know AC will become AD okay. So, then this is given this BD is given so, where does it getting where is it getting submerge okay b that is given.

So, then you have to find out the depth okay so, lotus has seen above water is the difference between the hypotenuse and the upright. The stack is the upright for the depth of water is measured by say what the depth of the water is okay.

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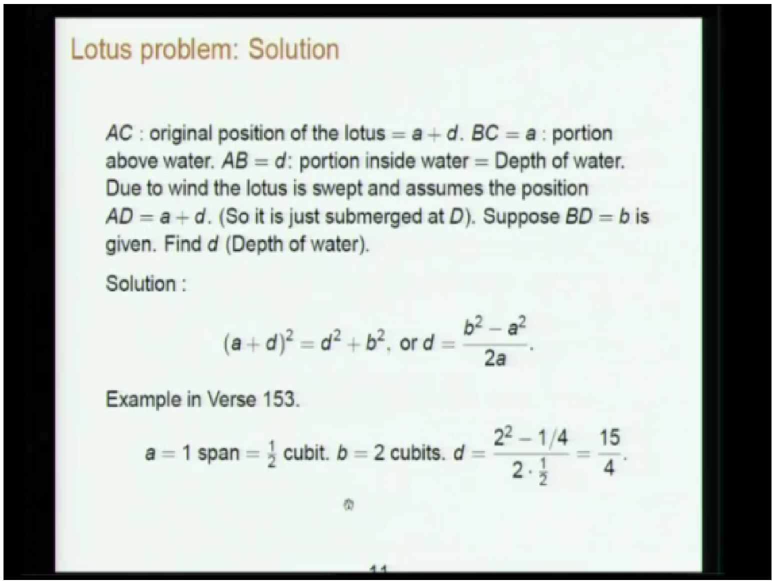

So, **so** this is the had already so, essentially you have is a+d whole square is equal to this thing d square+b square. So, d is equal to b square-a square/2a and a is 1 span suppose a I s1 span ½ cubit then b is 2 cubits given to be so, depth is 15/4 so, it is simple application of a right angle triangle clearly.

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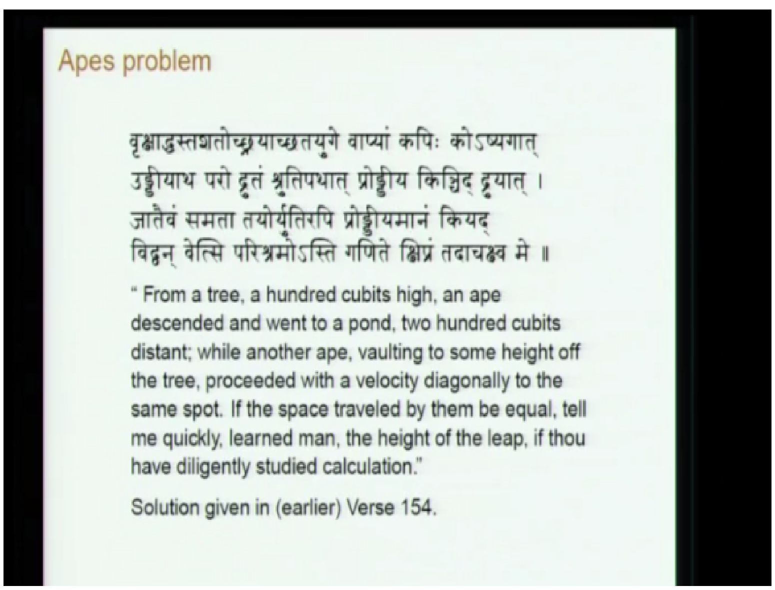

So, then again he comes to the Apes problem (FL) etc. so, from a tree a hundred cubits high an ape descended and went to a pond two hundred cubits distant while another ape vaulting to some height off the tree proceeded with a velocity diagonally to the same spot. If the space traveled by them be equal tell me quickly learned man the height of the leap if thou have diligently studied calculation okay. So, that is essentially you have this situation so, this is some tree.

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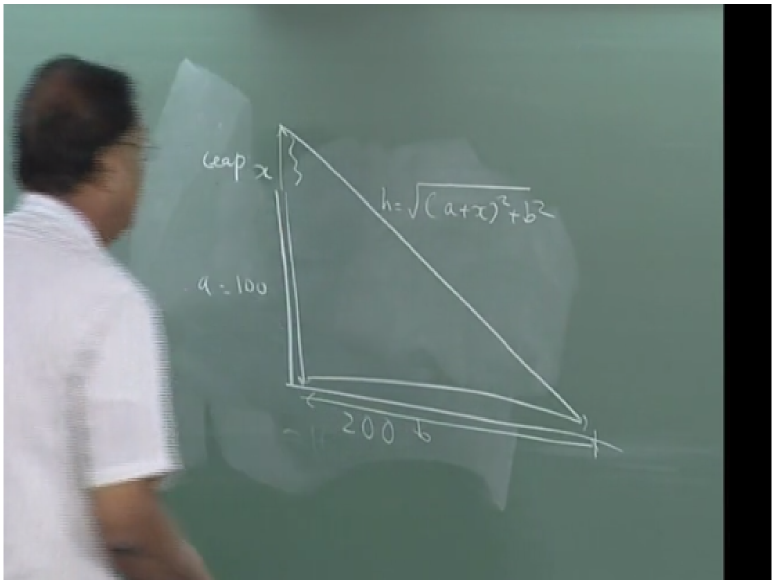

And then there is a pond so, tree this is 100 and this is pond is there some 200 okay so, then one (FL) you know descending and reaching this pond here. So another monkey is you know leaping okay and then goes diagonally okay. So, then you have to find this leap so, this is the leap okay so, again solution is given in again in earlier verse sometimes it happens.

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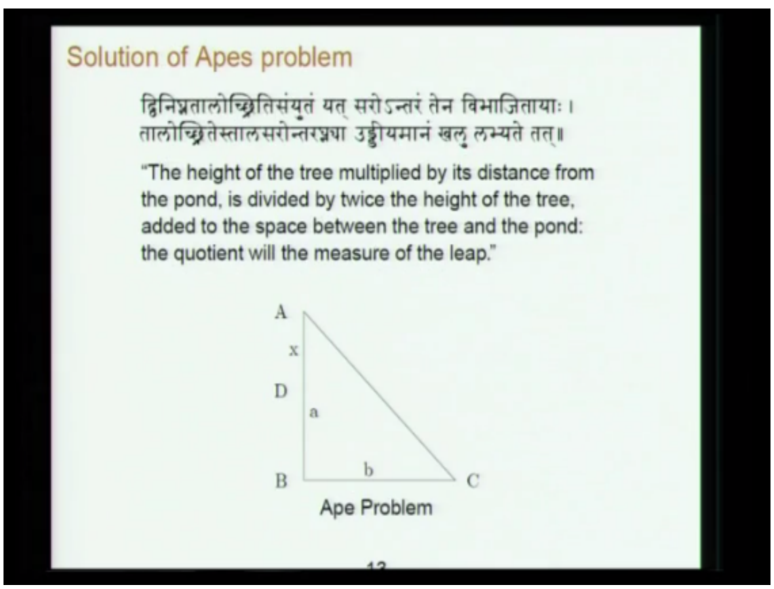

And then gives the method so, the height of the tree multiplied by it is distance from the pond is divided by twice the height of the tree added to this space between tree and pond the quotient will be the measure of the leap okay. So, (FL) leap so, essentially he is calculating this x is leap. So, this is a is equal to this height of the tree and b is a distance between the tree and the pond right. So, clearly and one see that actually x is equal to see the essentially same as a two ascetics problem in (FL) okay.

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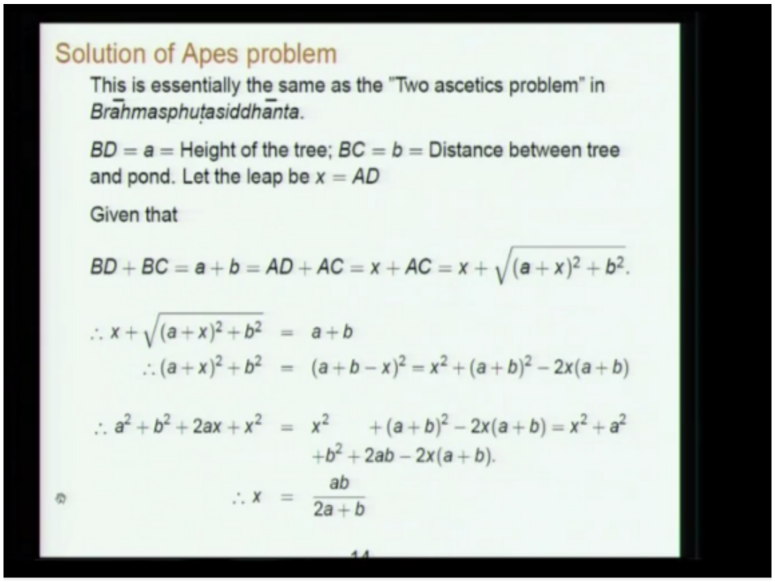

The same thing we are discussed this earlier so, only this some numbers have been change and the ascetics have been replaced by monkeys. You will get the solution as ab/2a+b okay where a is the this thing and height b is the distance between the is b so, these height is the leap. So, that is given as one can check that 2ab/a+b then in that case one can check that a+x whole square+b square is that you know the diagonal a+x whole square+b square okay.

So, that is the square root of the diagonal the hypotenuse okay. So, x+hypotunuse must be equal to a+b. So x+square rrot of a+x whole square+b square that is a+b right. So, this is h is so, x+h is equal to a+b. So, a+x whole square+b square so, if you take it to this side and square it you get essentially this.

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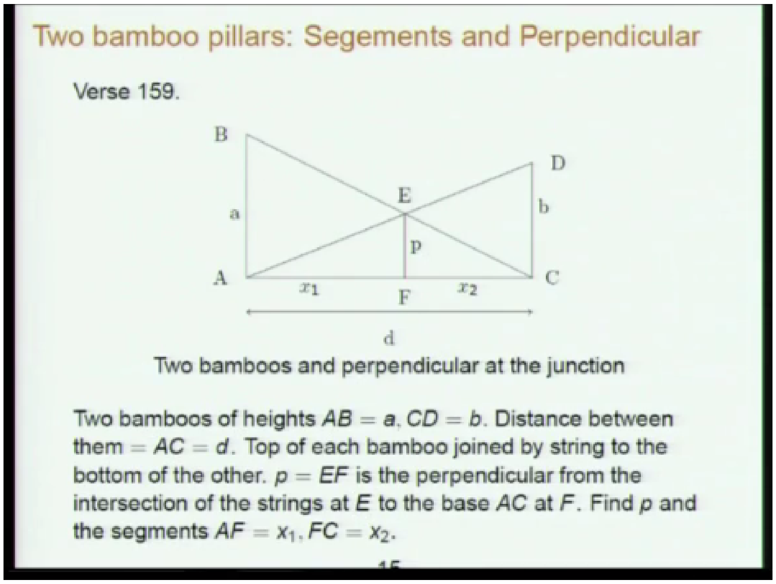

So, then he discusses the some segments and perpendicular here also consider the important suppose you have two bamboos ab and cd are the bamboos. And heights are ab is equal to A small a and CD is equal to b suppose the distance between them is d AC is d so, now top of each bamboo is joined by string to the bottom of the other. You see so, this situation and p is equal to EF is a perpendicular from the intersection of the string at E to the base AC at F.

And find you have to find p and the segments AF x is equal to x1 and FC is equal to x2 so, that is what you have to find.

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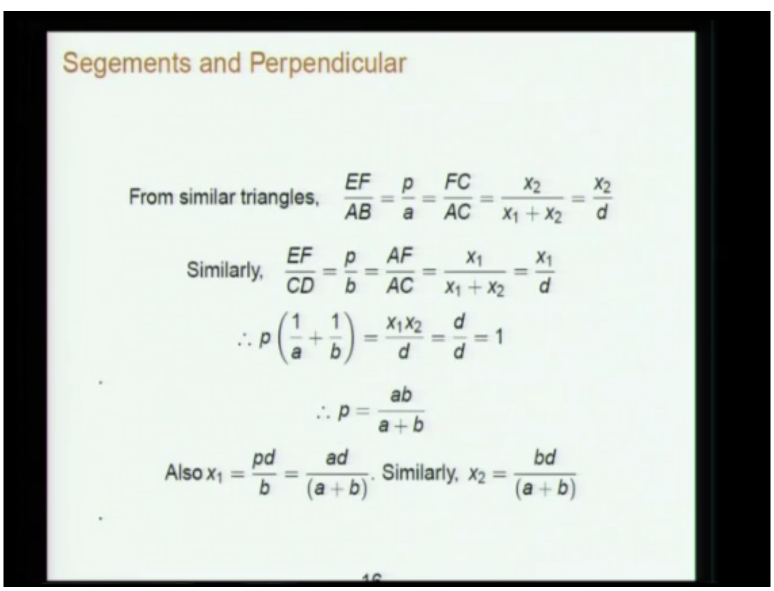

So, one can again we can you have to do the similar triangles EF/FC is equal to AB/AC so, like that various similar triangles you have to do finally so, you get p is equal to ab/a+b so, this perpendicular you see this is just given by ab/a+b so, interesting it does not depend on d is perpendicular does not depend upon d and of course once you know the perpendicular one can find this segments okay. So, x1 and x2 are given you know.

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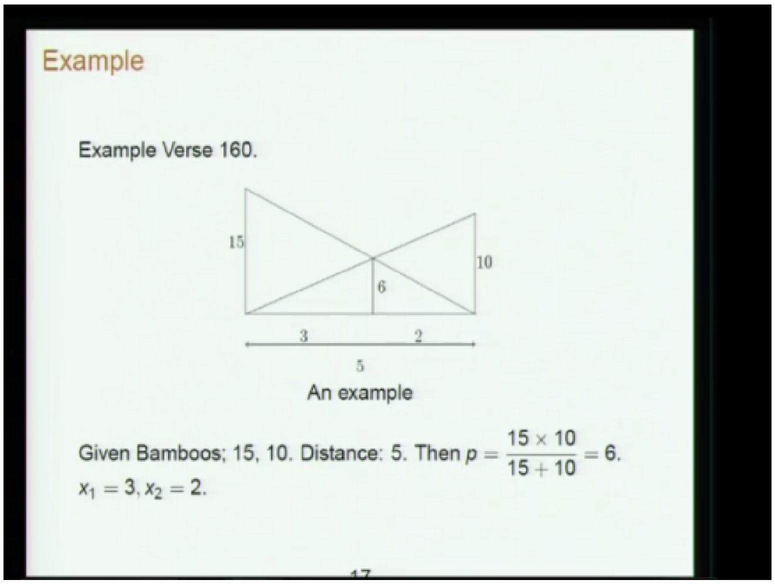

So, this kind of a so for example is given bamboos 15 and 10 suppose distance is 5 then what is the perpendicular, perpendicular will be 6 and this is 3 and 2.

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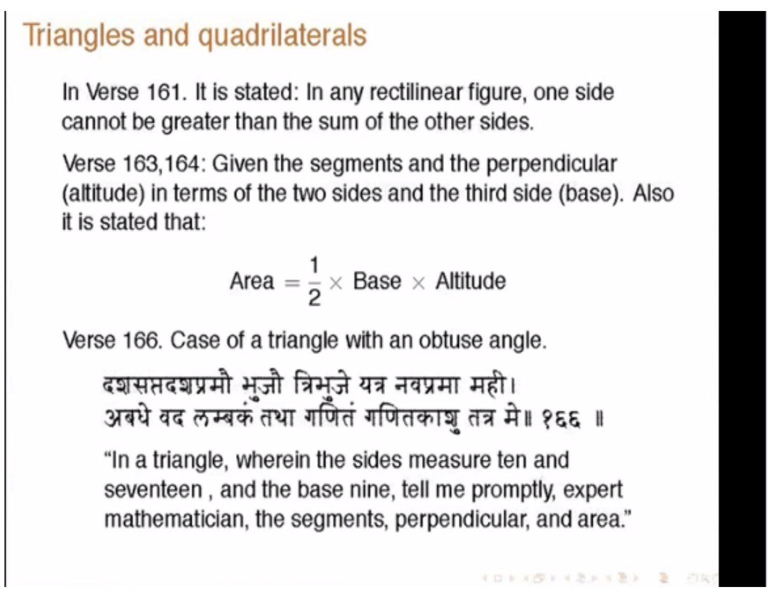

So, just similar triangles are used so, this is you know in fact this type of problems will come later in the quadrilaterals and all that you know various kinds of very complicated kind of a this segments and this perpendiculars will be there. So, it will be quite in stepped you even in (FL) it is there it is so, that is continuous and he gives a lot more you know detail of various segments is good to understand the things thoroughly and you know all the geometry configurations etc… will be clear.

That things to be the reason why they are do this so, then he continues with this triangles and quadrilaterals so, verse 161 it is stated in any rectilinear figure one side cannot be greater than the sum of the other sides. So, this is you know what kind of a theorem which had not been stated earlier I mean (FL) even Bhaskara one would realize being a very you know intelligent mathematician astronomer Bhaskara (FL) stated it.

So, he had stated it explicitly then given this segments and the perpendicular in terms of the two sides and the third side that is the usual standard thing (FL) right segments and perpendicular (FL). So, they are all stated and then area is half base into altitude so, that is also given then he considers a case of a triangle with an obtuse angle to illustrate a point. You know in a triangle varying the sides measure 10 and 17.

And a base 9 tell me promptly expert mathematician, the segments, perpendicular and area (FL) so, what is happening is that here is a triangle like this.

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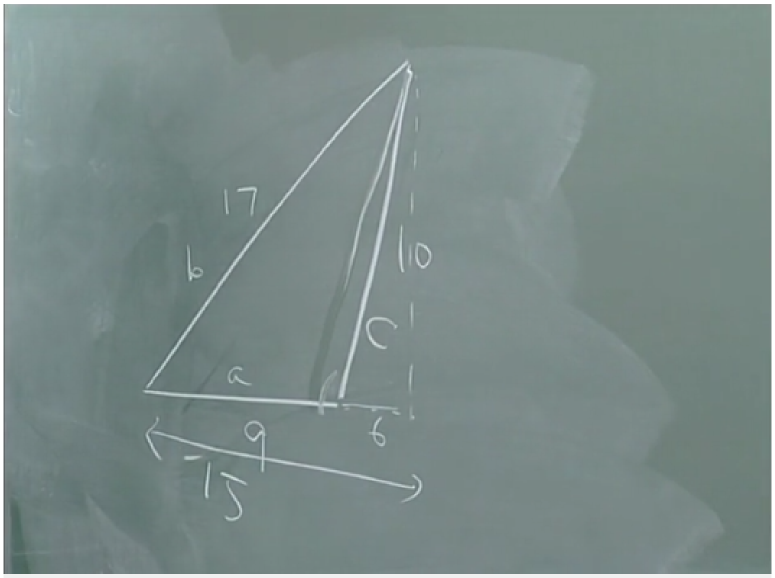

So, this is 17 and so, this is 10 okay and this is 9 and one can see that it will be an obtuse angle here you know. So, you have to find the segments and the perpendicular. So, one can do it.

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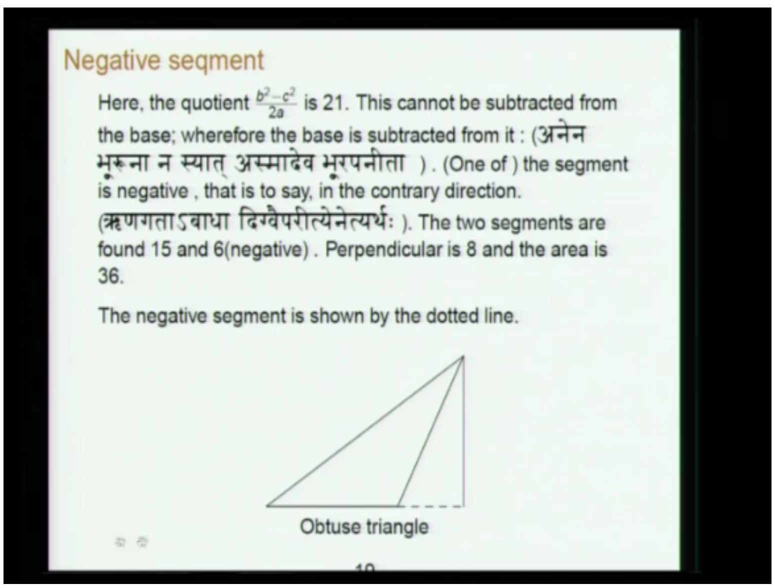

Here the quotient is see b square- c square by 2a you see a if you write it as is b and c and this is a, so remember in the segment calculation you get b square-c square by 2a it comes and that has to be added in subtracted from a, so b square-c square by 2a is 21. This cannot be subtracted from the base wherefore the base is subtracted from it (FL), so he says that therefore one of the segment is negative that is to say in the contrary direction (FL).

So, the 2 segments actually are found to be 15 and 6 and the perpendicular is 8 and so. So, this is 2 segments, so this will be 15 and this will be 6.

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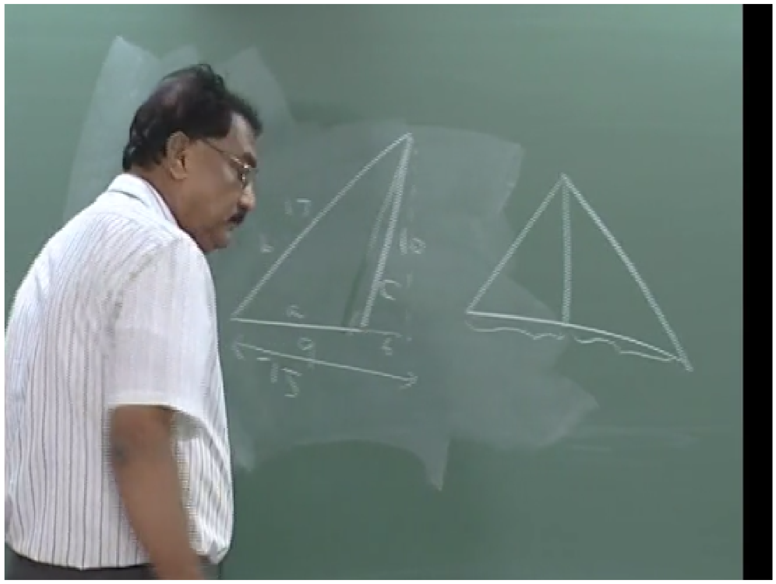

So, here you see normally earlier we are consider this kind of a thing in acute angle you see, so this are the segments and sum of the segments is the base so, here you have to take 15-6 you have to take you know. So, is explaining that you know (FL) you know it is going in opposite direction. So, see because here you know one segment is you know this tip to this and this tip to this here this into this.

So, other is actually this into this from here to here (FL) you see negative so, this is negative segment which is so, and by the dotted line.

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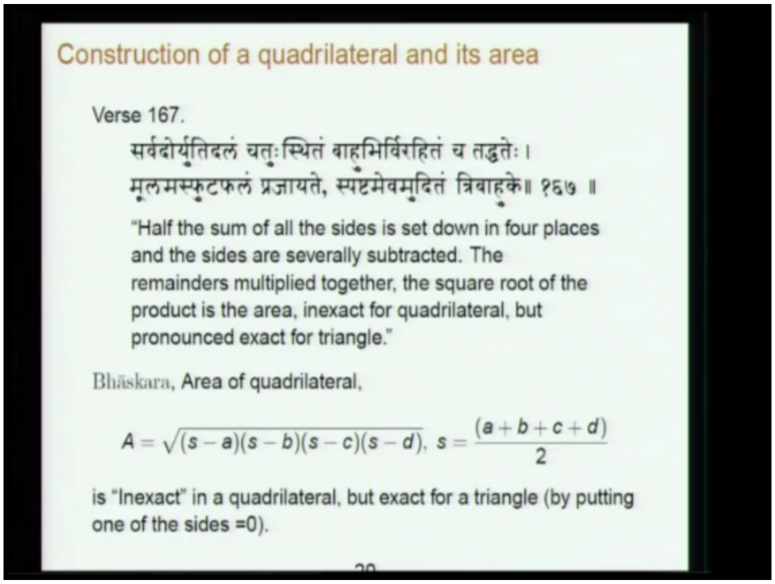

Then he discusses the construction of a quadrilateral and it is area (FL) is a important what he saying half the sum of all the sides is set down in four places and the sides are severally subtracted. That is sg the semi-parameter right semi-perimeter so, area is this the square root of the a product is area so, is very clearly saying in a exact for a quadrilateral moolam the moolam is the root (FL) is you know (FL) is clear or correct is given for a even longitude of a (FL) two longitude (FL) is inaccurate okay.

(FL) if the three thrice you know three side it is figure that is a triangle of course that is exact of course they you are have to put one of them it to be 0 right. The (()) (25:19) is 0 so, then he goes on to discuss this.

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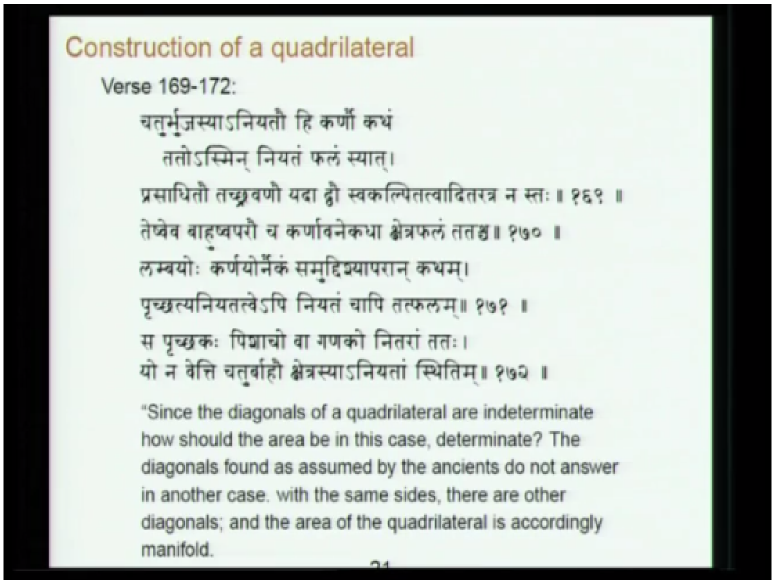

Because he says you know why is it indeterminate (FL) is the diagonals of a quadrilateral indeterminate how should the area being this case being determinate you know. So, that is what he saying (FL) etc… he saying so, that is the diagonals found assumed by the ancient do not answer in another case with the same there are same side other diagonals. And the area of a quadrilateral is accordingly manifold. So, that is what is what he saying is.

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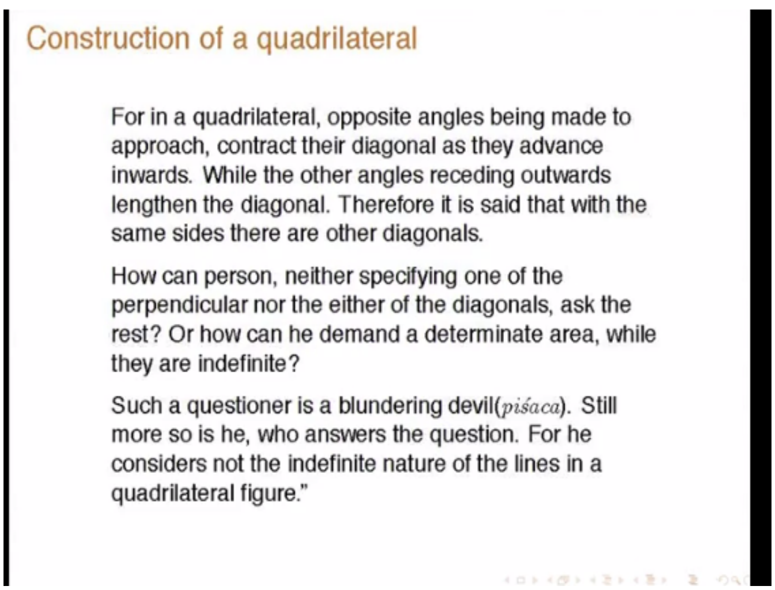

For in a quadrilateral opposite triangles is being made to approach contract their diagonal as they advance inwards while the other angles receding outwards lengthen the diagonal. Therefore it is said that with the same sides there are other diagonals okay. So, how can a person neither specifying one of the perpendicular nor either of the diagonals ask the rest? Or how can he demand a determinate area while they are indefinite? Okay.

Such a questioner is a blundering devil still more so, is he who answers the question for he considers not he indefinite nature of the lines in a quadrilateral is this thing is the so, in the of course (FL) stronger you know

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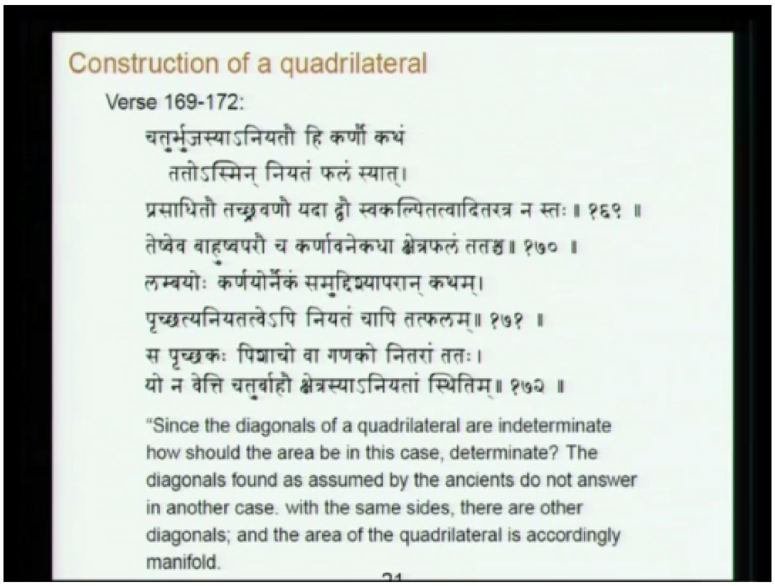

(FL) so, what is saying to say is the following okay see suppose is the quadrilateral suppose you just given four sides he is telling just by you know giving the four sides can you determine the area that is the question he is saying asking you know see suppose you see the quadrilateral okay.

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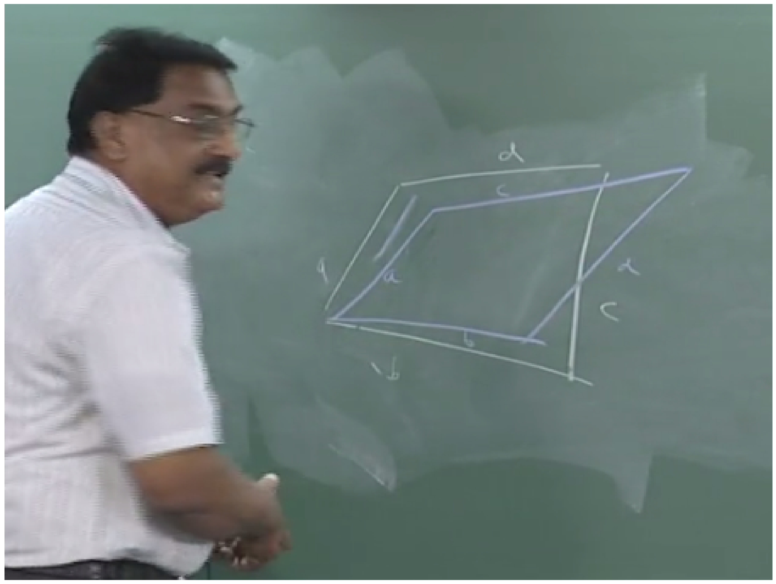

So, one quadrilateral you have got by you know abcd and everything what he saying that you know which is same side you construct the other quadrilaterals with the same sides. For instance these can come closer these can come closer so, let u say a okay and then d, d kind of a thing. So, this is also a, b, c, d so, here the diagonal is longer I do not know this diagonal is shorter and so, this angle is becoming increase is increasing.

And these angles are decreasing that is what he trying to say opposite triangles being made to approach are contracting the diagonals. So, this diagonal is shorter than the other diagonal. While the other angles receding outwards lengthen the diagonal okay. So, now this will be lengthening so, with the same sides you get different quadrilaterals various quadrilaterals with a same sides.

So, therefore you said with the same sides there the other diagonals oaky you given the sides the diagonals are not specified so, how can a person neither specifying one of the perpendicular nor the either of the diagonals ask the rest okay. So, or how can he demand a determinate area while they are indefinite. So, it is not determine okay because the several okay or several quadrilaterals are there.

So, how can you demand the determinate area did not specified completely okay so, he is you know the person who is if he states that result you know so, that is he is taking into task to say the least he is taking into task. And then of course says is a suppose some bodies ask that question you know what is the area this is the sides what is the area. So, that kind of a questioner is a blundering devil and somebody would tries to answer that.

He is one more of a blundering devil that is what he saying because (FL) without knowing you know without actually knowing the problem in the this thing question. So, he is trying to you know given a answer (FL).

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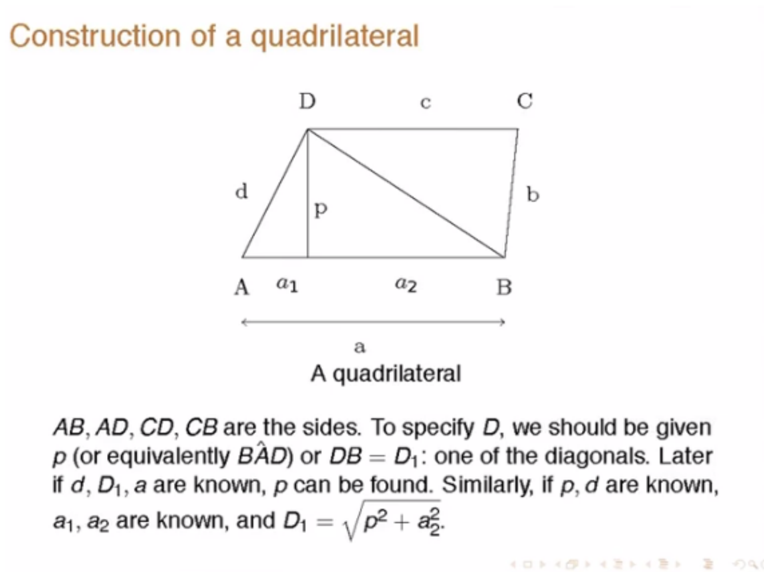

So, this is saying say so, essentially the AB these are the sides so, we have to specify you know to specify d for instance you know this diagonal or specify these d you see which has given the side which location of the point see it is not see in triangle that is not true. If you give the three sides that triangle is completely fixed but not in the quadrilateral.

So to specify this d either you should give sorry you should either gives ones the one the diagonal or you should give the perpendicular or gives the angle. So, apart from the sides you have to do one more thing so, that is.

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Finding the second diagonal

Determination of the second diagonal given the four sides and one diagonal in Verses 181-182 :

इष्टोऽत्र कर्णः प्रथमं प्रकल्प्यः त्र्यस्रे तु कर्णोभयतः स्थिते ये।
कर्णं तयोः क्ष्मामितरौ च बाहू प्रकल्प्य लम्बावबधे प्रसाध्ये ॥१८१॥
आबाधयोरेककुप्स्थयोः यत् स्यादन्तरं तत्कृतिसंयुतस्य।
लम्बैक्यवर्गस्य पदं द्वितीयः कर्णो भवेत् सर्वचतुर्भुजेषु॥ १८२ ॥

"In the figure, first a diagonal is assumed. In the two triangles situated on each side of the diagonal, tis diagonal is made the base of each; and the other sides are given; the perpendiculars and segments must be found. Then the square of the difference of two segments on the same side being added to the square of the sum of the perpendiculars, the square root of the sum of those squares will be the second diagonal in all quadrilaterals."

Then he goes on suppose they you are given four sides and a diagonal how to find out the second diagonal okay. So, that is what he is next problem (FL) or in all these quadrilaterals okay. So, this is the so, what you do first a diagonal is assume one diagonal in the diagonal situated in each side of the diagonal. The diagonal is made the base as each and the other sides are given so, the perpendiculars and segments must be found.

Then the square of the difference of the two segments and the same side being added to the square of the sum of the perpendiculars the square root of the sum of those squares will be second diagonal in all triangles are diagonals are quadrilaterals.

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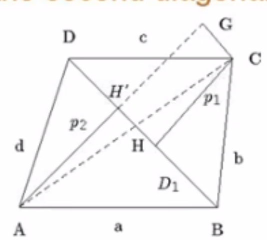
Determination of the second diagonal



Determination of the second diagonal

In the figure, BD is a diagonal. To find the other diagonal AC. p_1, p_2 are perpendicular to the first diagonal $BD = D_1$ can be determined. $DH = D_{1c}$ and $HB = D_{1b}$: segments associated with p_1. $DH' = D_{1d}$ and $H'B = D_{1a}$: segments associated with p_2. Extend AH' to G such that $H'G = CH = p_1$. Clearly, CG is perpendicular to $H'G$.

$$CG = H'H = DH - DH' = D_{1c} - D_{1d}$$

. Then the second diagonal $AC = D_2$ is given by

$$AC^2 = AG^2 + CG^2 = (p_1 + p_2)^2 + (D_{1c} - D_{1d})^2$$

So what i she trying to say is the following you see so, you are suppose you are given all the four sides and these diagonal. So, given the diagonal you see you draw perpendicular on this from a and c okay. So, then you see this DH frame and H frame B in this triangles so, DH frame and H frame is the segments are (FL) and p2 (FL) right (FL) similarly with the same base in this triangle you see BCD p1 is the perpendicular and now DH and HG they are the segments okay.

So, these are the so, you get segments and perpendicular different corresponding to which triangle you are considering. So, let us say DH prime is D1D I call it as that and DH prime B is D1A so, these the segments associated with p2 sorry D!C so, that is a DH, DH and HB HB D1B so, D1C and D1B they are the segments associated with this p1. And similarly DH prime = D1D and H prime B = D1A.

So, they are segments associated with p2 so, now you saying extend AH prime to G and he is not saying I mean I am getting the result. So, you would extend that you know subtract the side is p1 and you drop a perpendicular from here to here, So, then the your CG will be essentially D1c-D1d so, DH on the same side you know segment the segments are associated with two different triangles right D1c-D1d.

So, this will be this which is this and second diagonal will be you see this will be you see this is p2 this p2 and this sorry this p1. So, this AG will be p1+p2 sum of the perpendiculars you have to square that +you square the difference between the some segments you know associated with the different triangles. So, D1c so, which is this-this so, square this things add them and take the square root at the second diagonals. For instance you gives an example where D is 68 so, this D is 68.

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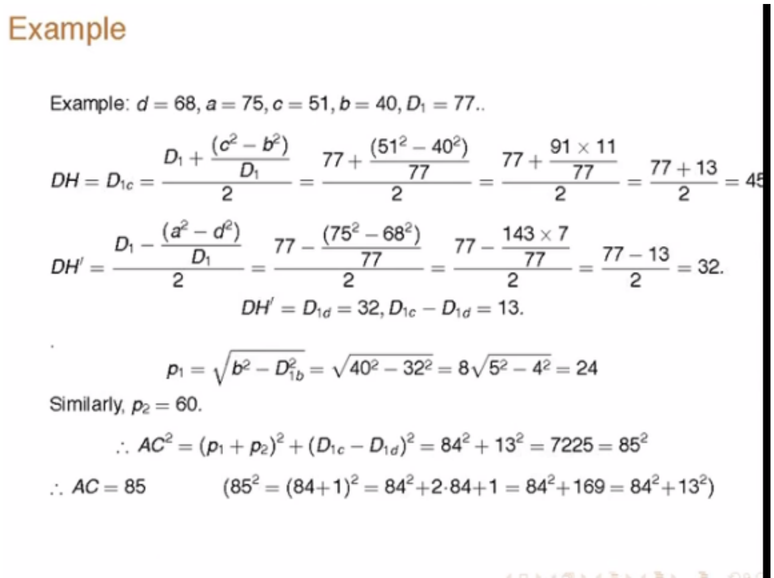

Then a is 75, b is 40 sorry these are the sides 68, 75, 51 and 40 and this diagonal is 77 suppose this diagonal is 77. So, then it only a matter of details you can find the segments and all that. And finally you get second diagonal to the 85 to get the get the point you see your given ABCD and this diagonal then you can get the diagonal AC using this method okay.

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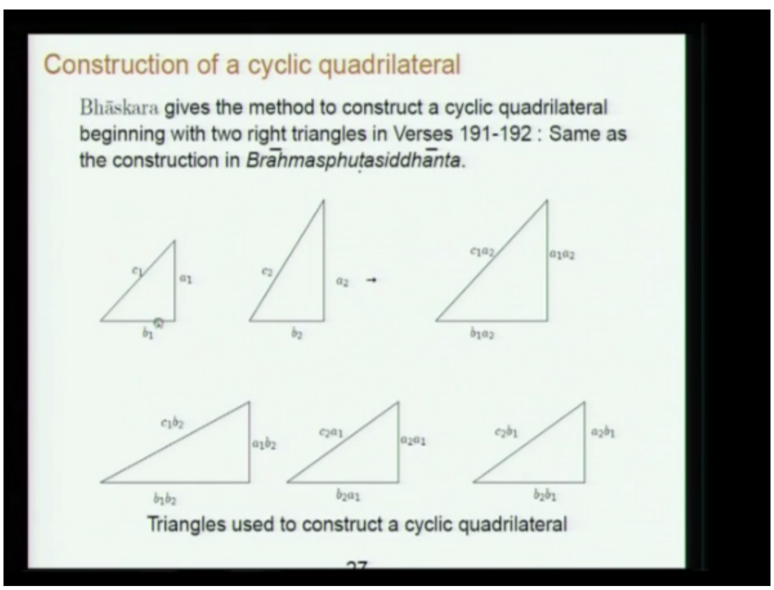

So, then you goes on to the construction of a cyclic quadrilateral so, same method has given in Brahmasputasiddhanta is given. So, you construct start with two right angle triangles b1, a1, c1, b2, a2, c2 now these are two right angle triangles. So, in construct four triangles from that okay so, this multiplied by a2 this triangle by a factor of a2 okay. And say increase by factor of b2 and similarly these increase by a factor of a1 these increase by a factor of b1.

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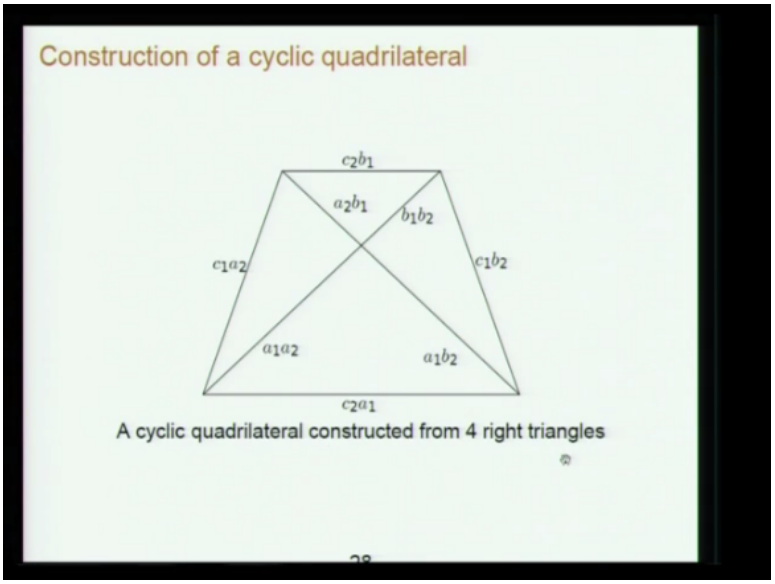

So, triangles and place them like this okay so, these cyclic quadrilateral which has been discussed earlier. You know very important thing now is that you know.

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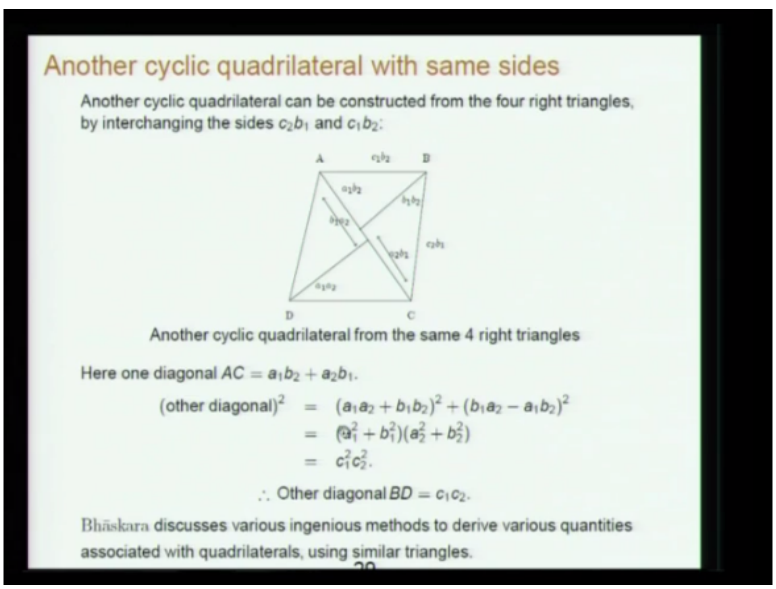

You can construct another cyclic quadrilateral that is what he saying that had not been discuss by Brahmagupta or mahavira so, what he saying is that essentially you exchange this two triangles okay. So, here these see in these construction that diagonals are intersecting perpendicularly they will be perpendicular to each other. So, now what I saying is that you know take this side to be this you know interchange.

So, essentially you get this kind of a thing and so, this will be the same a1a2, b1b2 earlier they proceed no they join together. And you know they constituted one the diagonal here there been separated right and this will be a2b1 and this will be a1b2. So, you can find the second diagonal and second diagonal will be c1c2 one can show that he is a second diagonal cd okay ac will continue to a1b2+a2b1.

And a second diagonal will be c1c2 we may also note that in this construction the diagonal BD is the diameter of the circle circum scribing the quadrilateral. So, Bhaskara discuss the various engineers method should derive various quantities associated quadrilateral and using similar triangles.

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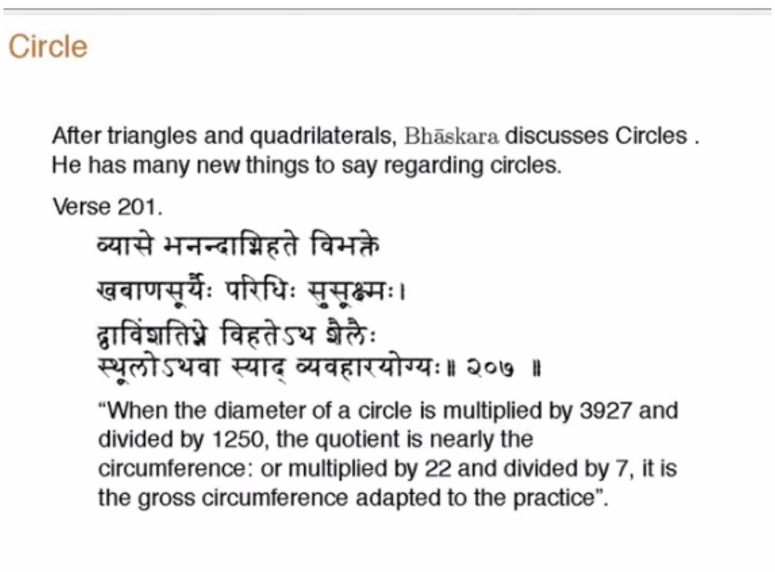

We do not have too much time so, now you will go to circle a discussion of a circle in Baskara so, his many new things to say regarding circles compare to what the earlier versions mathematicians Indian mathematicians right. He says that when the diameter of a circle is multiplied by 3927 and divided by 1250 the quotient is near the circumference or or multiplied by 22 and divided by 7 it is gross circumference.

So, (FL) 22 and (FL) is 7 (FL) mountain at this things so, that is 7 so, (FL) that is the approximate this thing. So, then (FL) 27 which stands for a 27 nakshatras or 24 27 fold division in this zodiac. So, nandha is 9, and agni is 3 so, (FL) is 3927.

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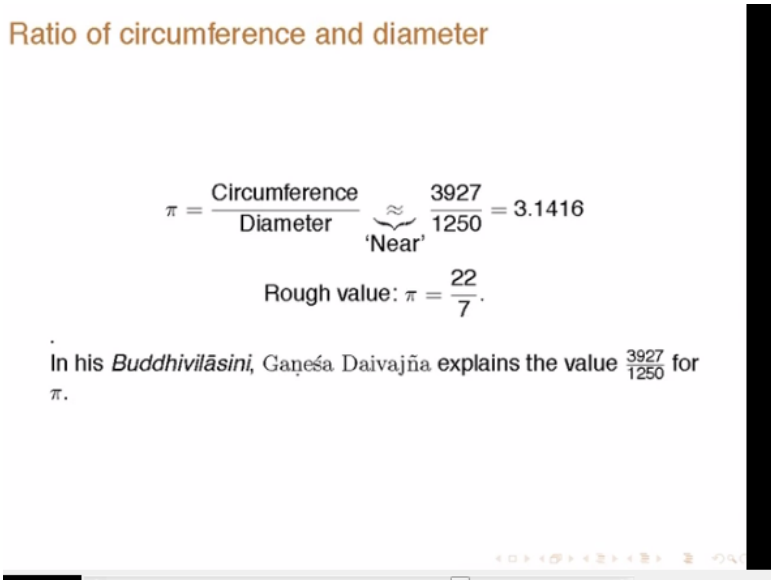

So, the circumference /diameter yeah 3927/1250 which is almost the same as the Aryabhatta and value 3.1416 and a rough value is 22/7 okay I sfl (FL) explains the value 3927/1250 for pie. So, it just Bhaskara stated this okay.

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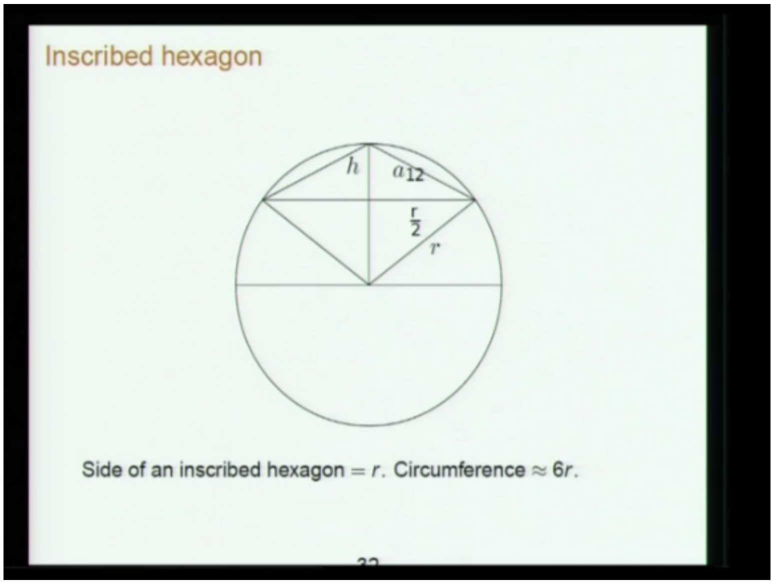

So, you start with a hexagon okay inscribe hexagon inside a circle so, you can take in the further the very approximate value you take the circumference of the hexagon to be the circumference is the circle that the very crude value because circumference as the hexagon will be 6 times the hexagon side which is r. So, circumference will be 6r so, which is or 3D 3 times the diameter.

So, now is a go to the (FL) explaining these things so, that is construct a polygan with sides 12 like this you know. So, this is a side of the (FL) okay and then suppose this is the h and this is side of that. So, these r/2 so, essentially so, **so** first you started with hexagon so, you do not good enough so, then you are doing the (FL).

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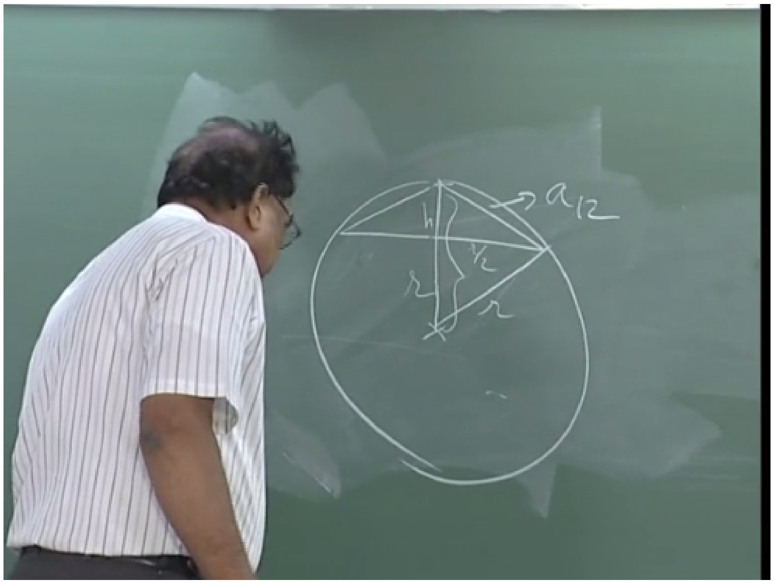

So, each this thing made into two kind of a thing so, these a centre of the circle. So, this is your h so, this is your r radius of the circle okay. And he is also r so, this is a side of the (FL) a 12 I write it like that and this clearly r/2 right at the hexagon side also is r. So, half of that is r/2 so, then clearly so, in this triangle r-h whole square+r/2 whole square = r square right.

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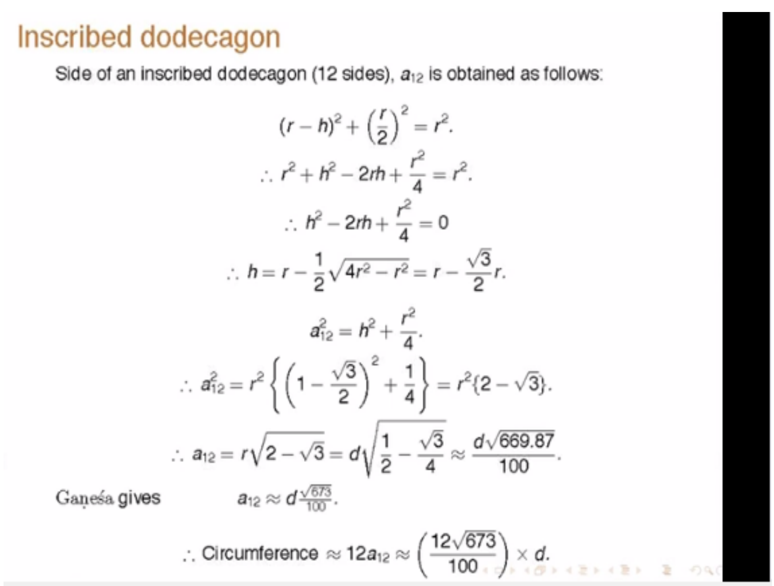

So, r-h whole square+r/2 whole square = r square so, you get h is equal to r-root 3/2*r so, that is that (FL) or (FL) or arrow right. So, that is r-root3/2r okay and so, a12 a12 square clearly this is equal to h square+this h we have found square root of that so, you get a12 is d= be fine you know using a calculator I get this. But Ganesha gives the value do d*square root of 673/100 so, circumference is approximately if you take use the dodecagon it will be 12*square root of 673/100.

This will be the value of pie if you start from s dodecagon somewhat better than this will be more than much better than 3.

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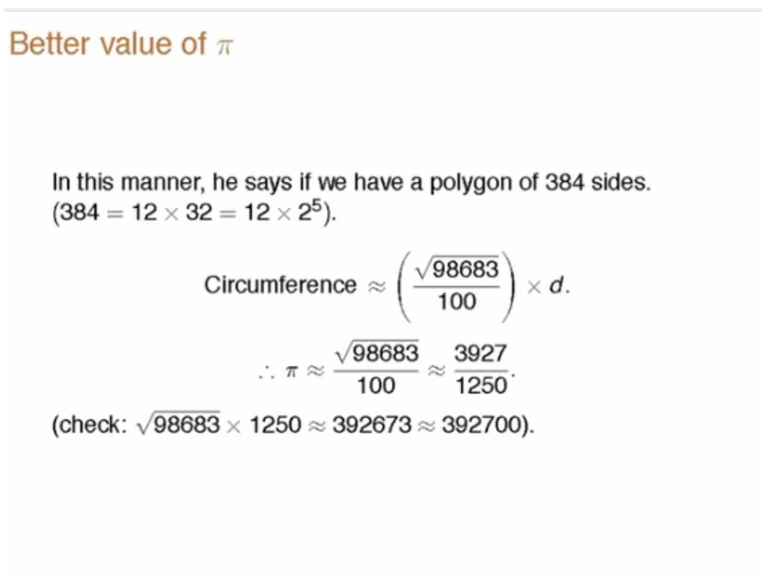

So, in this manner he says (FL) if you have a polygon of 384 sides so, 384 is 12*32 is 12*2 to the power of 5 okay. So, this you are to keep on dividing 5 times over you see and get a polygon of higher and higher number of sides. So, then the (FL) he says that it is square root of 98683/100 you know that is what it is this thing will be given. So, which is equal to 3927/1250 so, one can show that then this will be because this into this will be (()) (43:13) one can show that 392673 which is 392700. So, at least here the given procedure correctly so, and this is the better.

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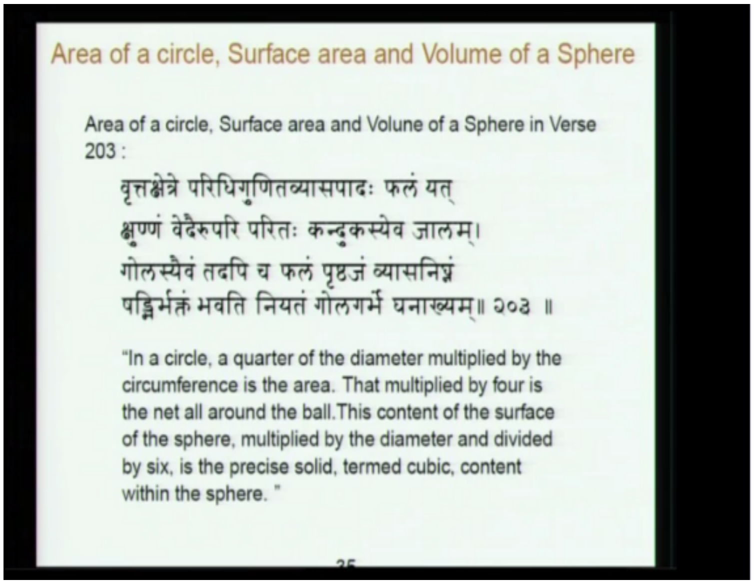

So, then the last topic in this second lecture and Lilavati they So, in a circle very important thing area of a circle surface area and volume of a sphere in verse 203 (FL). In a circle a quarter of the diameter multiplied by the circumference is the area that multiplied by four is the net all around the ball. This content (FL) this content of the surface of the so here multiplied by the diameter and divided by six is the precise solid termed cubic content within the sphere.

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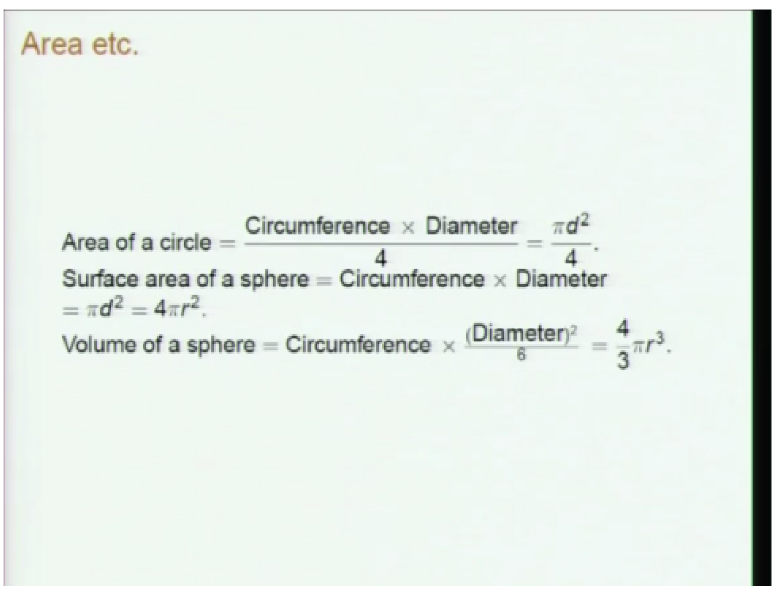

So, what is saying is a area of a circle he starts with that the circumference*diameter/4 so, is pie d square/4. I mean pie I have already mention for modern notation I am writing what they say is this and these correct surface area of a sphere is circumference*diameter*pie d square 4 pie r square which is this and volume of a sphere is circumference *diameter square/6 is comma to be 4/3.

So, there are exact results okay so, here he surface of a area of a sphere see what he saying is so, this is a 3 dimensional figure you know a 3 dimensional figure see says in the ball kind of a thing. So, in which you cover it with a thread (FL) you see so, that will be the area so, that is what he saying and volume of a sphere is this. Now in Lilavati none of these are derived are they are explain in detail ganesha derivation of a area of a circle in (FL).

So, that we can take up so, what he saying is which had indicated see make into two semi-circles and this semicircle you write it like this you know see this triangle you write it like this. Of course in a very large number means this will almost look like a straight line this arc. So, this is the straight line this one you know.

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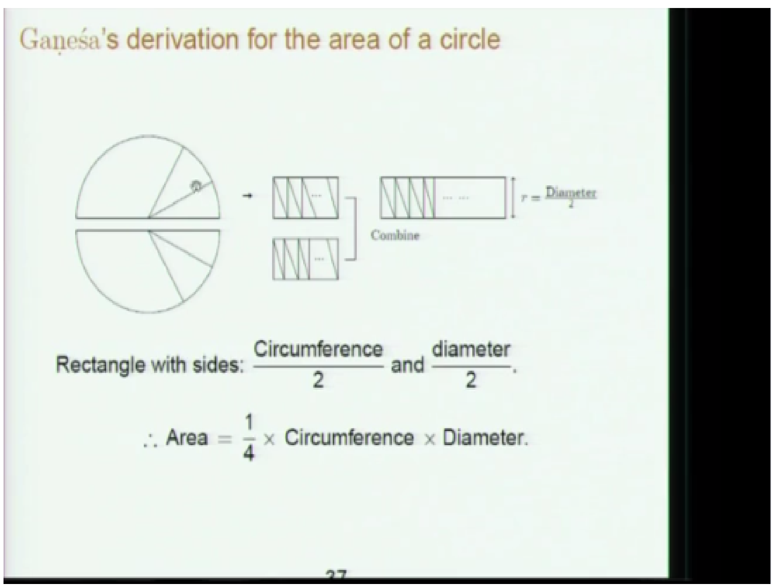

These all these triangles you put it like this one after the other second triangle is this not this, this all the lower things you know. And here also you take this you see this one is this and next one is this. You put it like that okay so, when you combine them you combine these two things. So this will be a you will get a rectangle okay. So, all these are all vacant here see in the first thing these are all vacant.

These are all triangles and these are all vacant upper things you know like this triangle with vertex feeling you know vertex is down much okay. Similarly here these are the vacant things triangle with vertex you know going upwards so, now you club them together so, they will exactly fit in and you will become a rectangle and this side is a rectangle is diameter. And the smaller side the bigger side will be the circumference naturally right.

Because you are adding all these things basically for adding these okay, and here and adding all these things here so, essentially you will get a rectangle and this will be the height of that rectangle is only a diameter/2 and this will be circumference okay. This will be half the circumference rectangle be the sides circumference/2 and diameter/2 radius right. So, area is really 1/4*circumference*diameter so, that is what we gets.

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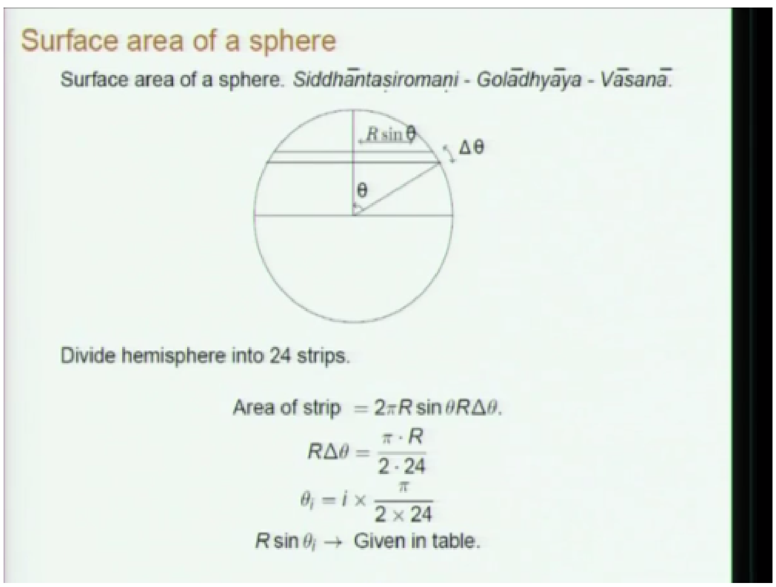

Then surface area of a sphere so, that is it will tricky so, he discusses this in (FL) that is the commentary so, here (()) (47:51) so, where there it discusses that so, what is does is divide the hemisphere*24 strips okay with this arc bit each of them is you know 90 degrees/24. And the area of the strip is into our language see this is the radius is the r sine theta. The radius will be 2pie r sine theta that will be the circumference into this side or delta theta you see.

So, area of the strip is this and all delta theta is r into either I told you delta the 90 degrees/24 in radius I am writing as pie/2*24. And thus the theta ice these are pie/2* pie/2*24*i I going from 1 to 24. Here dividing into 24 divisions that already written so, now you have to essentially some these area of the hemisphere it something things right. So, this is at a height level this will be rsine theta is the radius okay so, it some over all these strips.

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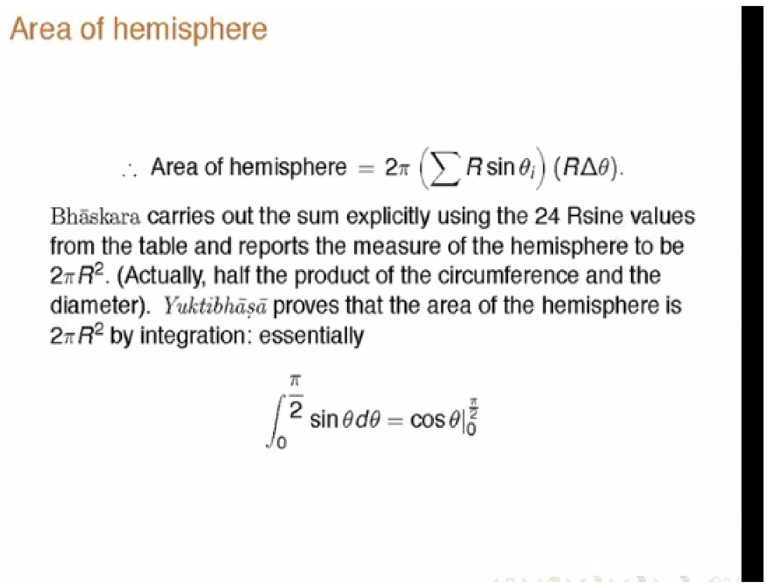

So, this is a thing and r delta to theta so, Bhaskara carries out the sum explicitly using the rsine values from the table and reports the measure of the hemisphere to be 2 pie r square. Then what is the say of course is actually this whole thing will be a half the product of the circumference in the diameter. And (FL) prove that the okay sorry so, what you gets will not be exactly this but when is dub this some you know using the values here taken.

It will be very close to that so ok, then he will you know take it to be it will be very close to what you get is you know 3, 4 some something you know, so the number will be close to 3 4, 3 8 square. So, that is R you know as we had you know discussed earlier many times (FL) or very correctly gets as you know that what you was getting you know if you are doing this things more accurately.

Then finally you should get you know something r square you should get so, he says that and finally he state it as half the product of the circumference and the diameter. But (FL) it will be done later proves that the area of the hemisphere is 2 pie R square it essentially does the integration sine theta d theta sine theta is there. So, they some they have they know how to infinite series were sine theta and all the they know that.

If you the differential of sine is cos and the differential of cos is sine so, they exactly get this result that is no summation you know. Summation is assume and finally you get what is recognize this very clearly that you know the derivative of sine theta is cos. That of course will be explain later so, you get the correct result Bhaskara but the method is slightly the slight divisions which will be covered up in (FL).

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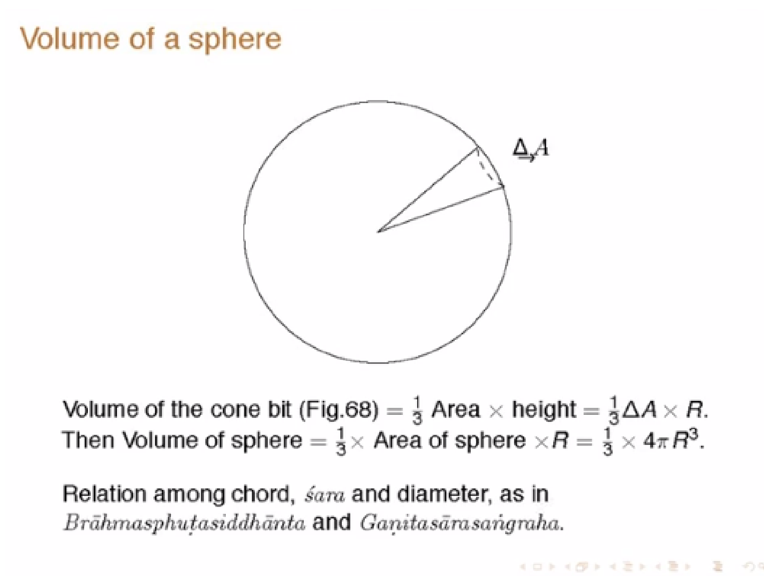

And volume of the cone also is very interesting volume of the sphere. So, you can take it as you know various cones. As if it is made up of various cones suppose they consider some cone with delta A as the area and volume of the cone bit is that is tapering you know to this thing. And we know that when there is a volume it is tapering. The volume is one third the base area into height right. This has been told and the is the base area is the delta A.

The height is r only radius that is all so, one third delta*A and the volume of the sphere is one third so, when you sum over all these things you have to take all these things into account. R will be the same you have to sum over the area elements so, no matter how you have divided it if you take the sum it will be the surface area of the sphere which has been derived earlier.

So, this will be area of a sphere one third of the area sphere nto all. So, which is one third*4 pie R cube so, it is thing. So, this is how he is you know they very important thing because I do not think they had discuss the area of a sphere and a volume of a sphere exactly they had area o fa circle was given earlier even mahavira had given that. Brahmagupta would have known that but area of a sphere and a volume so, this is the first time it I s explicitly spelt out and relation among chord sara and diameter.

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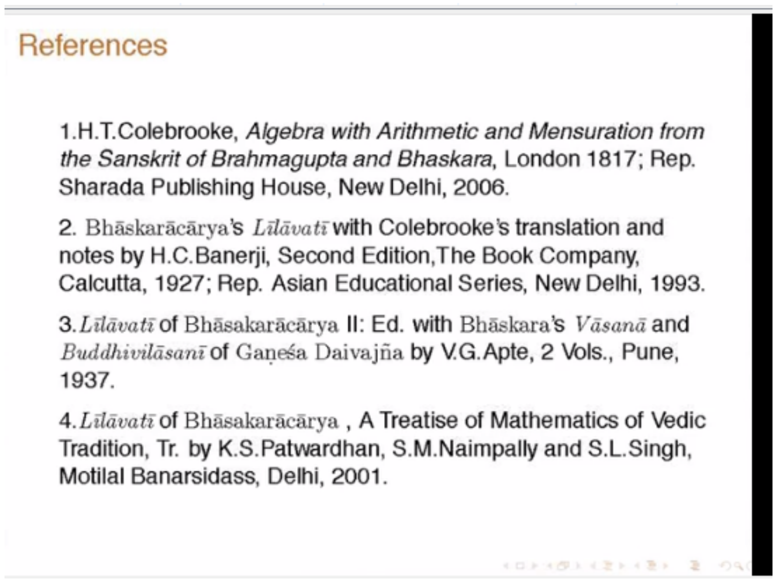

So, these things are standard things you have discussed here. The references are given here, thank you.