Mathematics in India: From Vedic Period to Modern Times
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Lecture-20
Leelavati of Bhaskaracarya 1

So, these of first of the 3 lectures on the famous work (FL).

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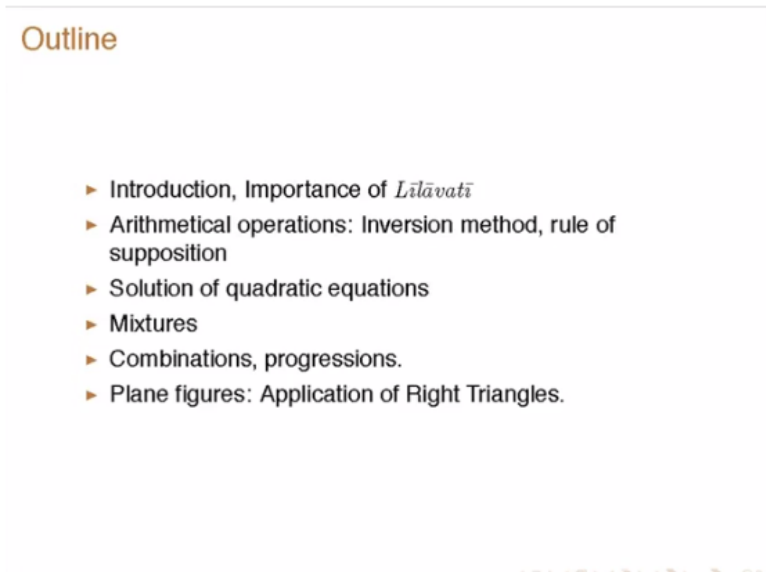

So, this give the outline, so introduction importance of (FL) then arithmetical operations which he considers and some particular methods which are he considers as a important and solution of quadratic equations. So, then in many other text earlier text, mixtures then combinations and progressions, and finally plane figures with application to right angles specifically.

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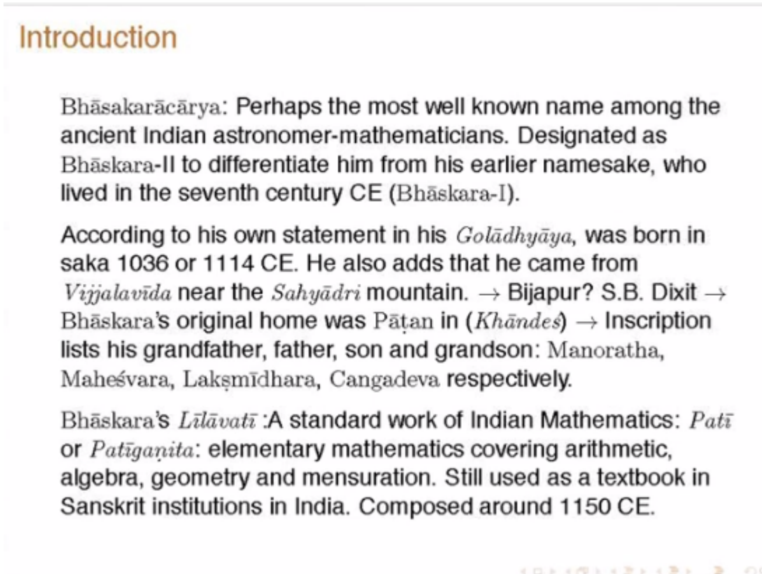

(FL) perhaps the most well known among the ancient Indian astronomer-mathematicians so, most people would have heard his name in India. And he is designated as (FL) to distinguish him from his predecessor is namesake (FL) 1, who lived in the seventh century. So, according to his own statement in the word (FL) which is the part of his famous work (FL) who will born in (FL) year 1036 or which corresponds to 11, 14 common era, so he also adds that he came from (FL) near (FL) mountain.

So, some people speculated that it was (FL) in northern Karnataka but many others do not think so, for instance the famous historian of mathematics Indian mathematics and astronomy (FL) who wrote famous work on Indian astronomy and mathematics in the 20th century beginning. He feels that the Bhaskara's original home was spotted in (FL) is the part of Maharashtra and the reason is inscription is here with list his grandfather, father, son and grandson whose names are Manoratha, Mahesvara, Laksmiidhara and Cangadeva respectively.

So, Bhaskara's (FL) is a standard work Indian mathematics it is a (FL), so elementary mathematics covering arithmetic, algebra, geometry and mesuration. In fact we will see that there is at least resultant trigonometry also in this work and what is important is that you know it is still used as a textbook in Sanskrit institutions in India like you know the (FL) courses and things like that, so they use (FL) and it is composed around 1150 Common Era.

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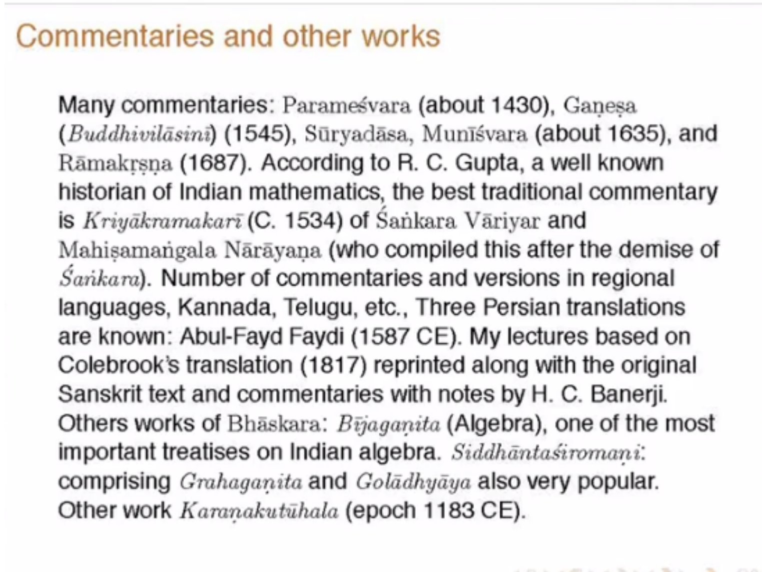

There are many commentaries on his famous work there is one (FL) around 1430 CE, so then one of the most famous ones which explains (FL) in considerable detail is (FL) of a (FL) composed around 1545, so in there is a commentary (FL) then one by (FL) around 1635, the one Ramakrishna commentary by Ramakrishna around 1687 and according to R.C. Gupta a well known historian of Indian mathematics the best additional commentaries (FL) composed around 1534 of (FL).

And his thought that he could not complete it and he is completed by 1 (FL) Nararyana who compile this after the demise of shankara. So, this is a very detailed commentary some of our colleagues are working on this detailed work the number of commentaries and versions in regional languages like Kanada, telugu etc., and now of course they in the lot of a literature available in (FL) in other languages also Hindi and so on and so forth yeah tree Persian translations are known one of the famous one is (FL) so it is around 1587 CE.

So, my lectures are based on (FL) books (FL) books translation which he did in the 19th century beginning base he which based of the original Sanskrit text and commentaries with notes by S.C Banarjee, now of course at many other accessible works in (FL) for instance there is work by name spelling, (FL) published by (FL). So, that is fairly easily accessible and easy to comprehend and there are many other work books on (FL) in the regional languages.

I mean which are modern you know which explains things in the modern notation or others I for instance no 1 work in Canada. So, there must be in other languages also now there are other works are Bhaskara (FL) which is also very famous work which will be discussed later in the lecture series. So, then (FL) which is comprising 2 parts called (FL) and (FL). Sometimes of course (FL) people talk as if that compost of 4 parts you know (FL) and (FL), so (FL) in the.

This is here I am different to the astronomy part which is (FL) and (FL) in the another work called (FL) who you know where he make some you know corrections to the parameters for the planets, the composed around 1183 common era.

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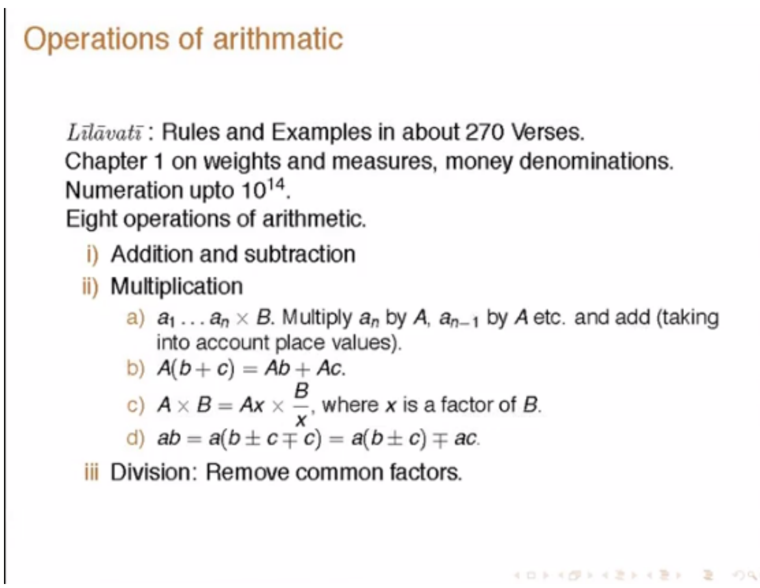

So, (FL) has rules and lot of examples in about 270verses the chapter 1 is an weights and measures, so which is there in many other works also you know they discuss the way it is you know and measures of time measures of length, measures of volume and so on and then money denominations and there is a numeration there is up to 10 to the power of 14, so then he discusses 8 operations of arithmetic addition and subtraction then multiplication, so suppose you have number A1 to An if you multiply by the B.

Then multiply an by B I am sorry this should replace capital A by capital B here yeah multiply An by B an-1 by B etc., and add all of them of course taking into account the place values because here it an corresponds to an itself. The an-1 you see here that will correspond to tenth place right, so you have to take this into account, then if you have a number a then multiply by b+c that is Ab+Ac all these things are stated you know see this is the modern algebra all this thing are stated you know it is called a distributive law.

So, that is interesting that they you know take they are careful about this things if we saw that in (FL) also. So, then sometimes it maybe simpler to write a into b as Ax into b by x where x is the factor of b and carry on the multiplication then if you have a into b then sometimes it may be easy to write it as A into B+/- c- or +c, so which is equal to a into b+/-c –or+ Ac, so then if you have division you remove common factors okay.

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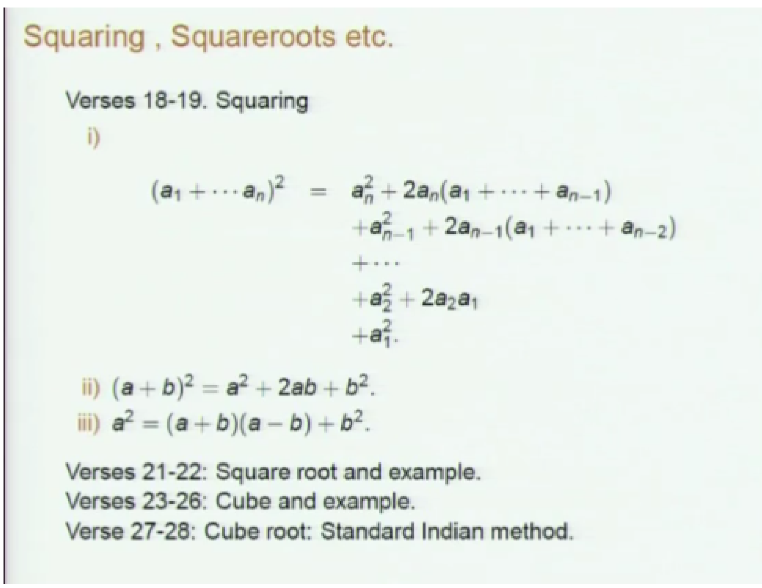

So, squaring, square roots etc., I will not spend much time upon those because earlier works work considered in great detail here in this lecture series, for instance if you have this an+etc., an whole square. So, then it is equal to an square you write first then 2 an into all are rest of the other digits okay and rather this is not this thing is just the a1+etc., I am not writing number as such the sum of these things, so it is 2 an into a1+etc., but of course decimal plays system is in the background you know the back of this everything.

And then similarly next is you take of an-1 square this and then multiply twice of that and multiply with the other numbers and so on. And specifically if you have 2 digits so then a+b whole square is equal to a square+2 ab+b square. Then a square sometimes it is convenient to write it as a+b into a-b+b square. So, that depends upon the particular situation where you know this maybe simpler for manipulations.

So, then square root he takes and you know explains a method of extracting the root, similarly in this 23, 26 he takes the cube and takes an example, so then he takes the cube root which is standard Indian method which we already discuss in the context of Aryabhattiya and (FL).

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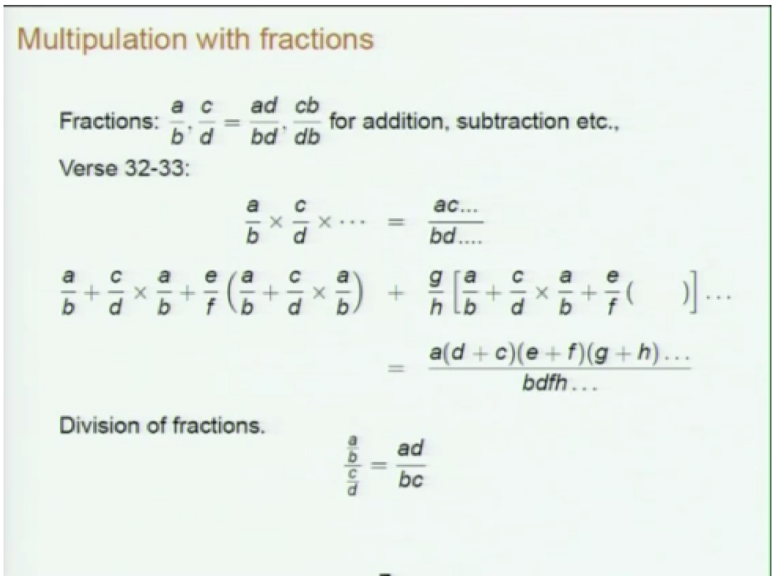

So, fractions, so sometimes it is you know convenient to do them to common denominator suppose you have a by b and c by d, so a by b is written as ad by bd and c by d is written as cb by db. So, the common you have multiply the denominators basically of course for many other numbers you carry on like this if it is specifically it is mentioned that a by b into c by d etc., etc., so multiply all are numerators and divide by product of all the denominators.

Then this kind of a thing you know suppose you have a by b then they are fraction c/d of a a/b then the fraction e/f of this resultant so, like that. So, this is considered in many this kind of a you know representation for many problems it is considered in the Indian works at similarly g/h into ths whole thing so, then that is equal to it is stated you know. He gives the simplified form a into d+c into e+, f into g+ etc. divided by the all the denominators basically product of all the denominators.

Then it consider division of fractions so, a/b divided by c/d is ad/bc okay.

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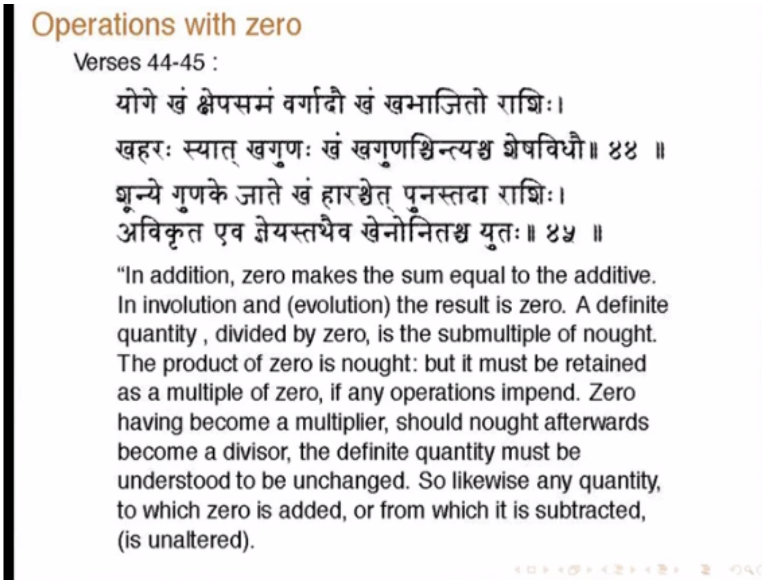

So, next he considered operations with 0 (FL) In addition zero makes the sum equal to the additive in involution and (evolution) the result is zero. A definite quantity, divided by zero, is the sub multiple of nought. The product of zero is nought but it must be retained as a multiple o of zero, if any operations is impend.

Zero having becoming a multiplier should nought afterwards become a divisor. The definite quantity must be understood to be unchanged. So, likewise any quantity to which zero is added or from which it is **sub** subtracted (is unaltered).

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Operations with zero

$a \pm 0 = a$, $\frac{a}{0} =$ *Kha-hara*: a fraction with zero for its denominator.

(In *Bījagaṇita*. Gaṇeśa: Infinity).

$$a \times 0 = 0$$

But it must retained as a multiple of zero, if further operations, impend. "Zero having become a multiplier, should nought afterwards become a divisor, the definite quantity must be understood to be unchanged".

$a \times \frac{0}{0} = a$? (But $\frac{0}{0}$ is indeterminate)

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So, what of course he is saying is that suppose you have a+/-0 then that is equal to a, so that a remains unchanged then a/0 is called it as (FL) a fraction with 0 denominator, it does not go explain that in great detail here because he consider things these things is (FL) you know, operation with 0 are considered in more detail in his work (FL) but he just say that he has (FL) and (FL) in the commentary mention that it is a infinity.

So, remember there would have a conclusion about that in (FL), now you know it is clear, so that a into 0 is 0, then what happens when a into 0 by 0 it must be retain in the multiple of 0 further operations impend, see suppose a into 0 then do not just put it is equal to 0 because afterwards some division by 0 maybe there. So, you retain that 0 having become a multiplier should not afterwards become a divisor the definite quantity must be understood be un change.

Perhaps he saying that a into suppose you have you know multiply by 0 on divide by 0, that is a itself. So, it is not very clear in this context this will be explained in detail later but now of course we know that you cannot just put it as a indeterminate you know 0/0 that depends upon the way the 0/0 comes but here the confusion is there it appears as if is writing it like this we do not know which is not correct. Here I do not have any this thing you know and he is not miss should that is a translation you know.

In fact it could be in not itself that is a peculiarity of his translation maybe his emphasizes it you know ought and knot kind of thing probably yeah.

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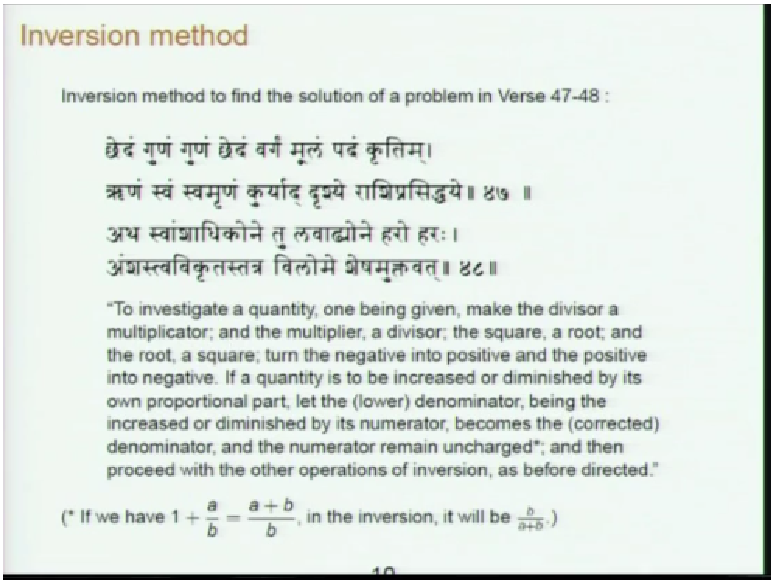

Then he considers what is known as inversion method the find a solution of a problem , for he is what he says (FL). So, to investigate a quantity 1 being given make the divisor a multiplicator and the multiplier, a divisor, the square, a root etc., etc., okay. So, what he is trying to say we should understand what he is if you see some example it will be clear.

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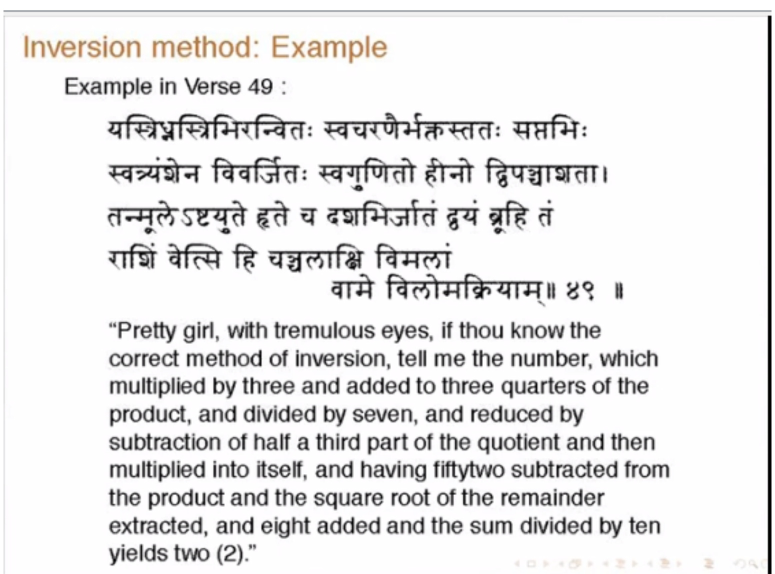

So, that is if you are saying a some operations are carried out and then you know the result is given okay and what is that original number. So, he is saying that you know you have to go in the inverse manner, so by, by when I deal with a deal with a example it will be clear. So, for instance he gives one example inverse 49 (FL) you know inversion you know do it the other way kind of a thing, so (FL) pretty girl with tremulous eyes, if thou know the correct method of inversion, tell me the which multiplied by 3 and added to three quarters of the product.

And divided by 7 and reduce by subtraction of half a third part of the quotient and then multiplied into itself and having 52 subtracted from the product and the square roots are remainder extracted and 8 added and the sum divided by 10 is 2, so this is a final result okay. So, you have done various operations on the number the final result is 2, then what is that number you see that is what is this thing okay.

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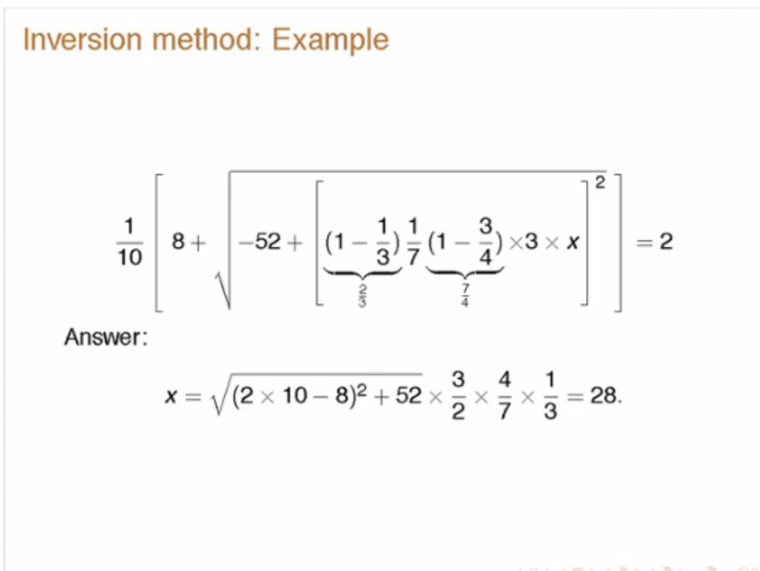

So, essentially he is saying you know if you translate that verse into our you know the an equation we know familiar with you know this writing equations in this form, so what he is saying you know the one tenth you know say x is there, so x into 3 okay. So, then you are 1-3/4 multiplied by that 1/7 is there 1-1/3 is there the whole thing square is that is what is he is saying right which multiply by 3 and added to 3 quarters of the product and divided by7 and reduce by subtraction of half.

The third part of the quotient and then multiplied into itself right, so that is what it is then you subtract -52 then take the square root of that then add 8 then divide by 10 that is 2, so now what he is saying to find the answer you do it the reverse way, so it is take 2, 2 is a answer you take that then see here 1/10 was given in the problem, so now you take it as 10, 2*10, so addition 8 was there here you -8 okay. So, then next square root is there then you have to take the square root then – is here, here you take +.

So, then here 2/3 is there you make it as 3/2 then here you see the whole thing is 7/4, so write it as 4/7 and a 3 is there 1/3, so this whole thing is equal to the answer which is 28. So, I mean this is inverted you know see 2 when you to in the modern you to taken into 10 write 10, then -8 right, square root of this number is 2*10-8 then you have to take the square of that it is, so you have to take the square, so like that, so that is the inverse method.

So, then the another method called rule of supposition or false position, I mean these are somewhat implied in the modern methods.

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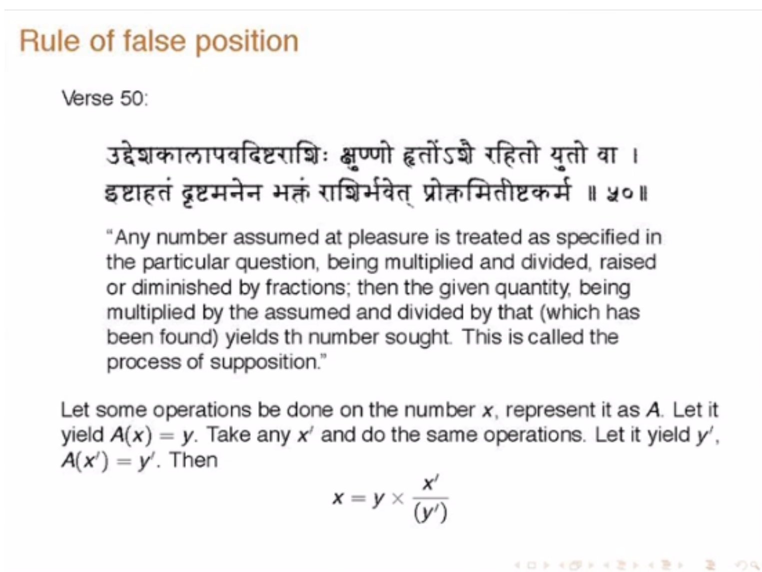

But in here explaining it you know in explicitly, so suppose this operates in the following (()) (18:54) what he is trying to say is the following. Suppose you have some operations you do on some number x okay we do not know that number x operations it do would on the number x and get the result y okay you know y see you have to find x, so what he is saying that take any number explain okay and do all this operations a on that okay and then suppose you get the result y prime.

So, then the result will be x prime/y prime into y, so that is by rule of proportions of course here you should be doing only squaring etc., should not be there it should be what is known as linear operation you know otherwise you will not get that. So, what he is trying to say is you know then you know if you take the number some example it will be clear.

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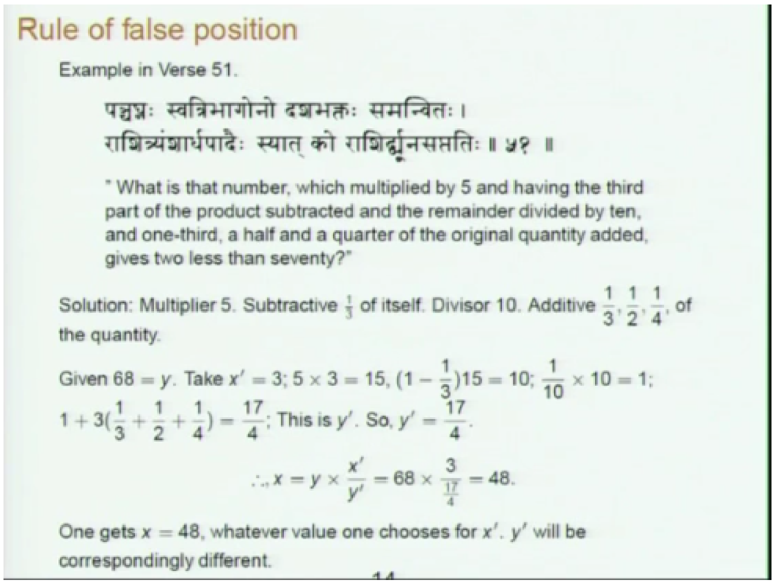

(FL) this is the example is giving (FL) what is that number which multiplied by 5 and having the third part of the product subtracted in the remainder divided by 10 and one third a half and a quarter at the original quantity added gives 2 less than 17 see here multiplied by 5. So, you write it as multiplier 5 third part of the product subtracted subtractive one third of itself.

So, now the finally what to get is 2 less than 70 so, 68 68 is y you have to find x. So, now take x prime is equal to 3 you do all these all operations and x prime okay. So, multiplied by 5 into 3 is 15. Then having the third part subtracted 1-1/3*15 is 10 then divide by10 1/10*10 is 1. Then add third part of the **1** 1/3 half and ¼ add all of them and multiplied by the number. So, the you have taken it as 3 and then add 1 okay added okay.

So, 17/4 this is y prime so, if you start from 3 you get y 17/4 so, but so, if the this is a quantity the original quantity is 3. Then the result is 17/4 but if the result is 68 what is the original quantity is just a rule of proportion. So, x is y into x prime/ y prime so, 68 into so, whatever you took so, suppose supposition 3 and got whatever result you got divide by this. And multiplied by the result so, this is 48 so, one gets 48 of course x prime you can take anything.

Instead of x prime is equal to 3 you can take 6 or 60 or whatever you can do that similarly correspondingly y prime will change. So, then x prime /y prime into x prime, so that will be the same quantity into result, with that will be the same quantity.

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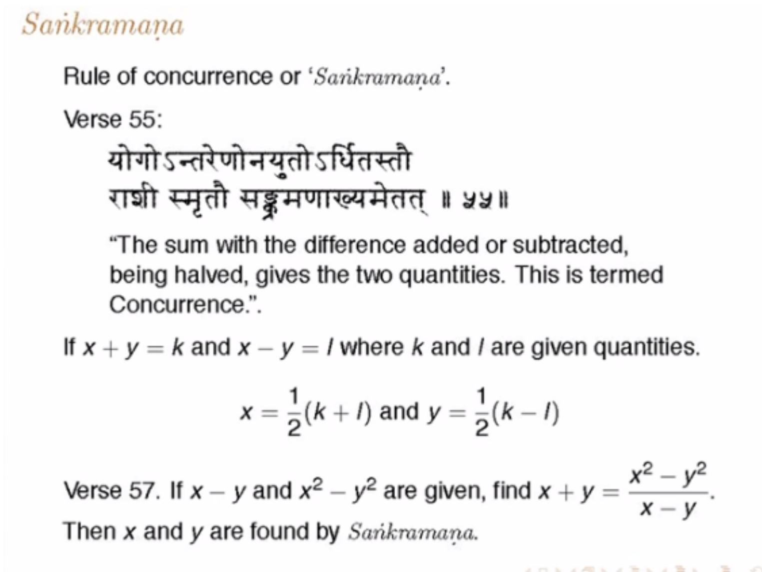

So, that is the so, now it is called the rule of false position so, now it is discusses what is known as sankramana. So, it is the very standard thing only you know this kind of things comes various places but various places so, that is what they stating is (FL). So, what he saying the sum with the difference added or subtracted being halved gives the two quantities this is termed Concurrence or sankramana, so what is the this is very simple thing. Suppose you are giving the sum of two numbers and difference of two numbers.

Then what are the two numbers that is the straight forward. So, if x+y is k and x-y is l then clearly x is half of k is l and y is equal to half k-l. Of course you can extend it into straightly different kind of problems based on this advanced things based on this. Suppose if x-y and x square –y square are given then you have to find x+y first x square-y square/x-y. Then x and y are found by sankramana in fact we will see that later.

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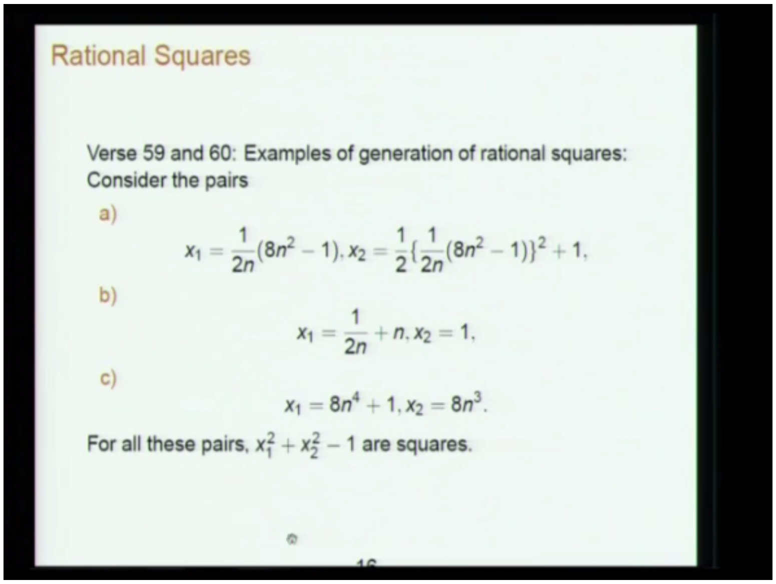

Narayana punita will discuss for more things you know some more difficult slightly the more difficult problems are given. Then you reduced to sankraman and then okay so, now generation of various rational squares etc.. the various rational triangles rationales, figures, and all that you know. You have to generate it okay so, as per you know pointed out you know we generate various words you see which are valued kind of a thing.

So, similarly you generate various the same kind of you know mind set seems to be there you know. It generate various kinds of figures or the kind of a thing miss some stipulation you know that all the sides must be rational or all the others segments must be rational and so on and so for. The for instance if you takes you know suppose he says that you consider two quantities x1 and x2 suppose you are stipulating that x1 square+x2 square-1 or squares.

So, this must be expressible as some quantity square square of that okay. So, then one particular kind of solution is x1, if you take this if you take any n integer. You take x1 is this then x2 is this so, then you get if you take this sum of the squares and reduced 1 then that will be a x square okay. So, similarly if x1 is 1/2n+n and x2 is 1. You will get a square for this and if x1 is 8 into the power of 4+1 and 8n cube again.

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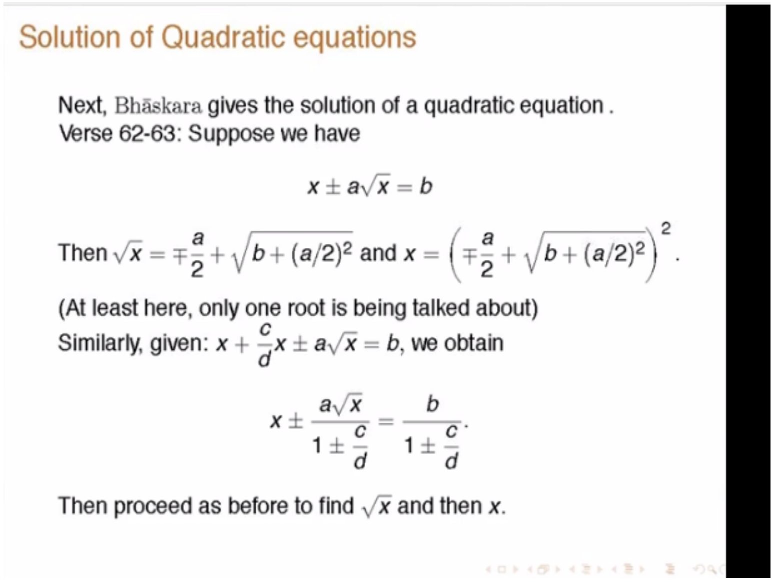

The sum of the squares-1 it will be a square you know that it will be some different quantity square you see I mean that quantity must not involve square root obviously you see. So, it must be some rational thing and then you square kind of a thing that is the what is then you cube the solution of a quadratic equation (FL) in the even earlier also we had seen that it is the expressed as the like this x+-a a root x is equal to b a solving for root x and then squaring it to get x.

So, then it is stated that root x is equal to if + or – a is there so, is – or + will be there it will come+ this. So, +a/2+ square root of b+a/2 whole square whole thing square that is the solution. So, here atleast you know only one root is being talked about. Because in many examples then negative root will not make sense so, he is actually talking one root only. So, it is standard thing you know that is ax square+bx square +c is equal to 0 you know.

You know the solution in the modern method at least the modern way of a expressing it. So, the same thing see here instead of b you got a and things like that you know. And here you know it is the right hand side so, you have to be careful you will get this at the solution so, then similarly if you have he considers you know suppose you have x+c/d x kind of everything you know. So, then this reduces to this then proceed has before you find root x and then x okay.

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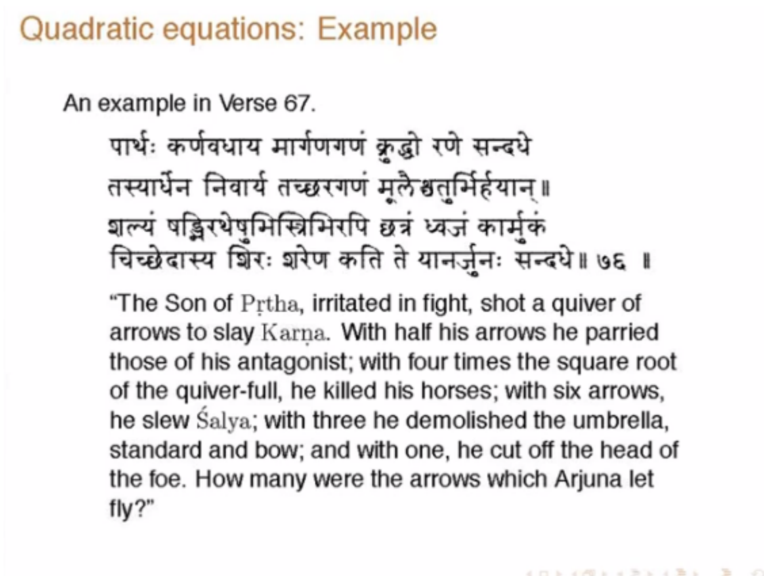

So, this famous example I will consider many of the you know famous examples which Baskara has you know by which are which are they are in that axis if we are illustrating various procedures so, (FL) the son of prtha irritated in fight (FL) okay shot a quiver of arrows to stay karna okay (FL) to slay karna so, shot a quiver so, a quiver of this of arrows (FL) with half his arrows he parried those of his antagonist (FL) is karna or this thing.

Then (FL) with four times the square root of the quiver-full, he killed his horses (FL) okay with six arrows he slew (FL) with three demolished umbrella (FL) umbrella standard and bow okay (FL). Then he cut off the head of the remaining this thing arrow cut off the head of the foe with the remaining I know. How many were the arrows which Arjuna let fly so, this is so, essentially son of prtha yeah (FL) okay that is the translation okay. You can come a tied as (FL) prtha irritated in this thing yeah sometimes prtha has son of (FL) you know kind of a thing is a Sanskrit.

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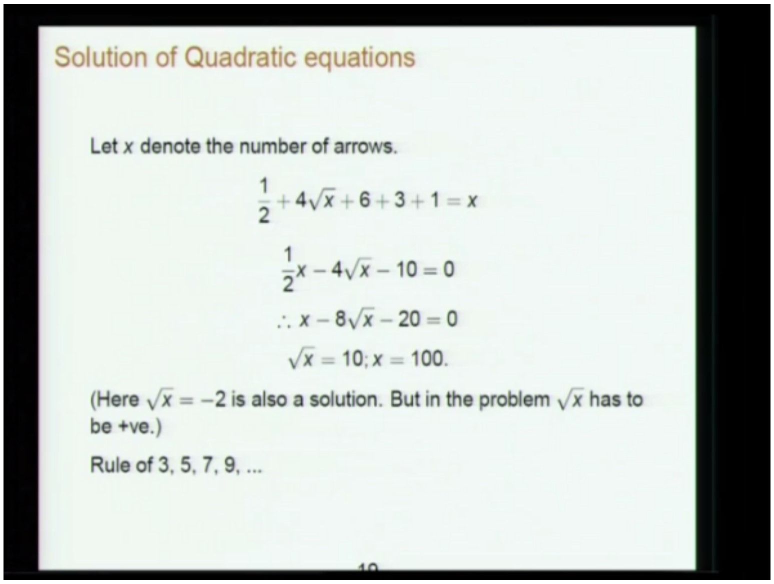

So, what is saying is half+4 root x+6+3+1 that is equal to x. So, in fact this must be half x half the arrows half x+4 root x+6+3+1 is equal to x, so these what to get x-8 root x -20 0. So, from the method that he has formula he has given for the solution root x is equal to 10 is x is 100. Of course root x-2 is also a solution in fact there is a valid solution for this.

But in the problem root x has to be negative because remember that he has taken you know four times, the square root of the with four times the square root the killed is arrows. So, the square root x also must be positive so, he is taking the positive root and the solution is this.

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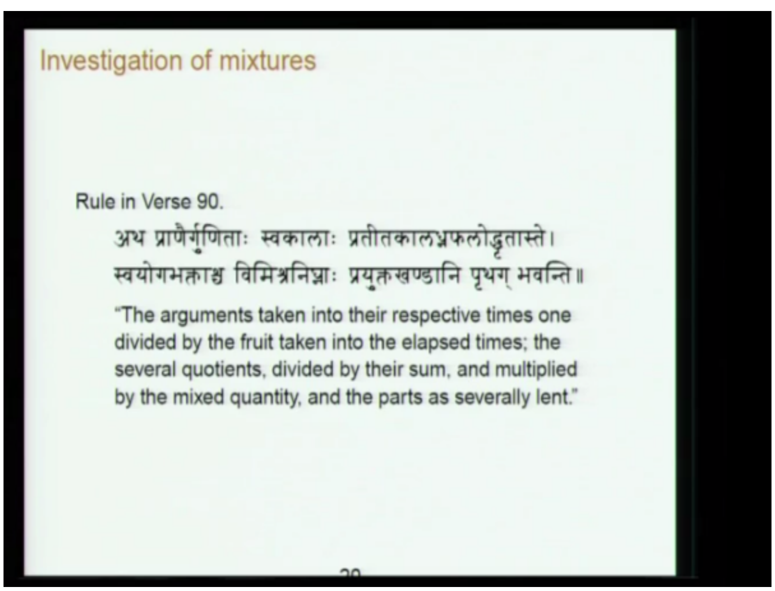

Then it talks about the rule of 3, 5, 7, 9 and all that similar to what he is but I forget to mention that apart from the 0 Brahmagupta and mahavira and he had there others also eminent (FL) and then sripati who wrote a very famous work was siddhantasekara which also the mathematics section. So, some of the results are taken from there also you know So, then he discusses investigation mixtures (FL) is a argument taken into the respective times.

One divided by the fruit taken into a elapsed times the several quotients divided by their sum and multiplied by the mix quantity and a parts as severally lent.

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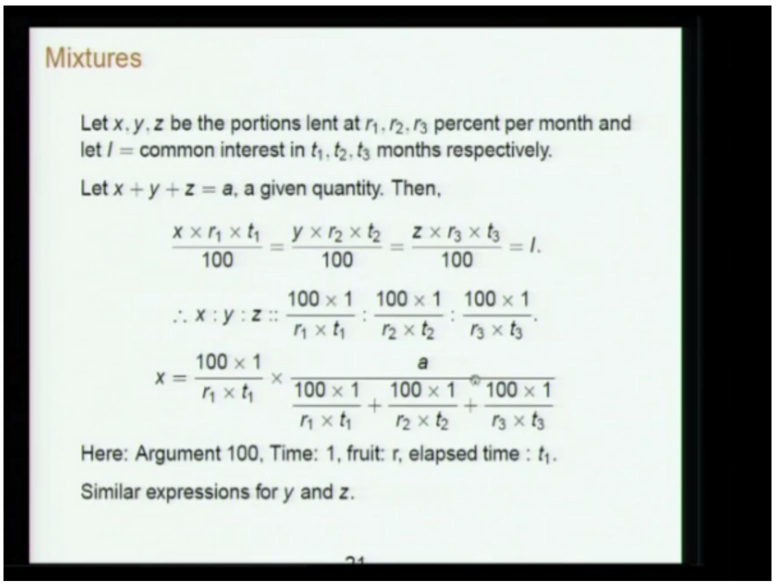

So, as usual write the things in the the form which is familiar to us. So, you will be able to understand suppose x, y, z are the portions lent the principle is lent at the interest rate r1, r2, r3 percent per month. And suppose common interest I is equal to common interest in t1, t2, t3 months okay. So, x being lent at rate r1 for t1 months, why being lent at rate r2 for t2 months z being lent at r3 rate r3 for 3 months.

So, the common interest is this so, x*r1*t1/100 etc… the these equal to common I and what is given is x+y+z so, then one can see that x:y:z is this thing. So, x itself is given by this what is saying 100*1/r1t1 into the mixed quantity divided by the sum of these. So, that is what is essentially saying a several quotients divided by their sum and multiplied by the mixture quantity. So, that is the several.

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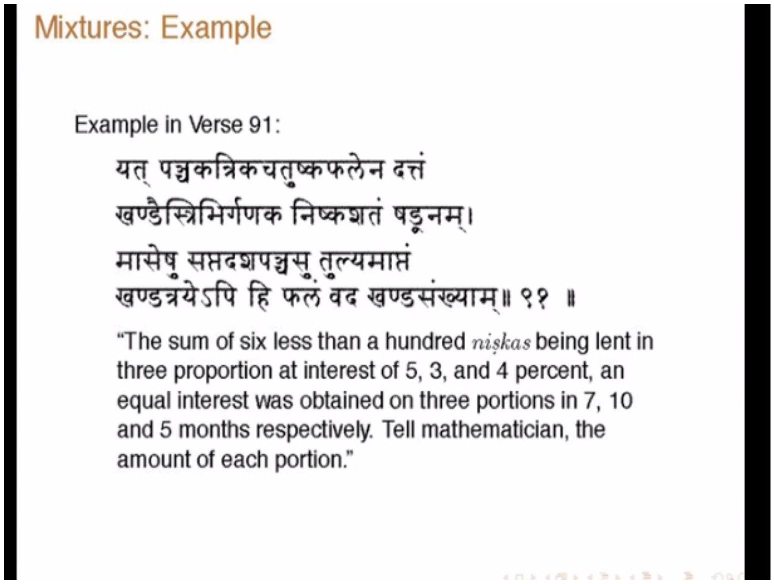

So, I gives an example the sum of six less than a hundred niskas being lent in the proportion at interest of 5, 3 and 4 percent at equal interest was obtained on three being lent in three proportions at interest of 5, 3 and 4 proportion is not known. An equal interest for the obtained and 3 portions in 7, 10 and 5 months respectively tell mathematician the amount of each portion (FL) okay (FL) that is the portion you see. Amount of each portion you have to tell.

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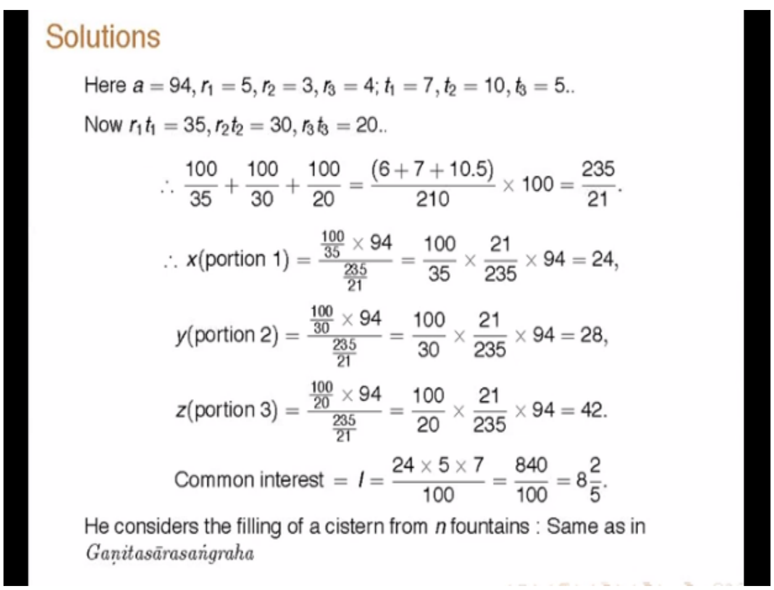

So, essentially here the rates of interest are given r1, r2, r3 5, 3 and 4 the number of months is this. The total interest is the same and r1t1, r2t2, r3t3 they are this. And then the amounts are different the total is given amount this thing so, by using is whatever the may procedure is given which just have to you see you see 100 divided by this. Then 100 divided by 30, 100 divided by 20 you take this had all of them you get this.

So, then this is the result you get so, then whatever the portion one lent so, that is the mixture quantity 94 comes here in the numerator this comes the denominator. And 100/35 comes in a that particular attaining to one that is this. So, these are the various quantities which are lent and a common interest is this. So, similarly so, next other mixture thing it talks about a filling the of a cistern from n fountains or n pipes. I discuss that in the context of (FL). So, that also he discusses here.

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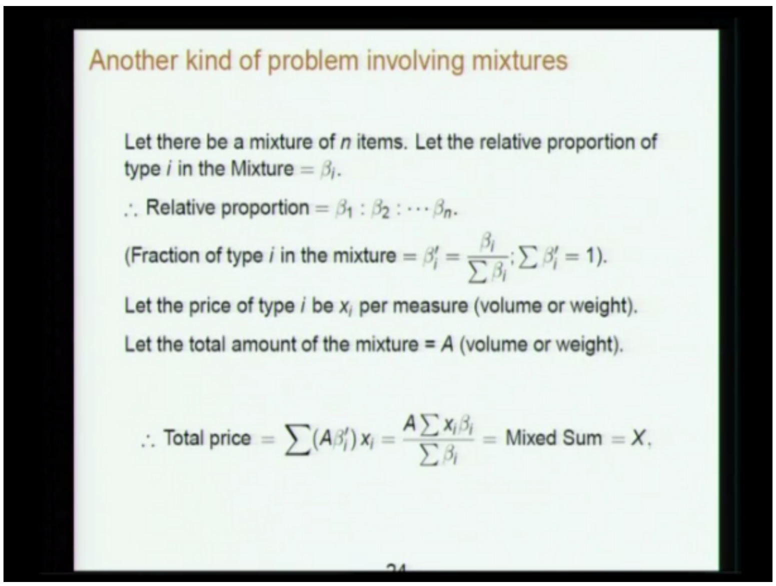

I do not want have to do it here then the other kinds of mixtured problems so, let the relative proportional of type I in the mixture is betai. You see **so, re** suppose the various n items are there and then the proportions are given not proportions. But the ratios the relative proportions are given beta1 to this thing so, they are not normally the total will not adapt to one. So, what you have to do is the relative fraction of type i in the mixture is you have to divide the particular quantity divided by the whole thing.

So, the sum of this fraction is equal to one but we do not need this see this will make the you know method of solution somewhat number we do not work with the slowly. So, suppose the total price is given that is x so, that is given by total amount of mixture is a. So, then total price will be this mixture sum is this.

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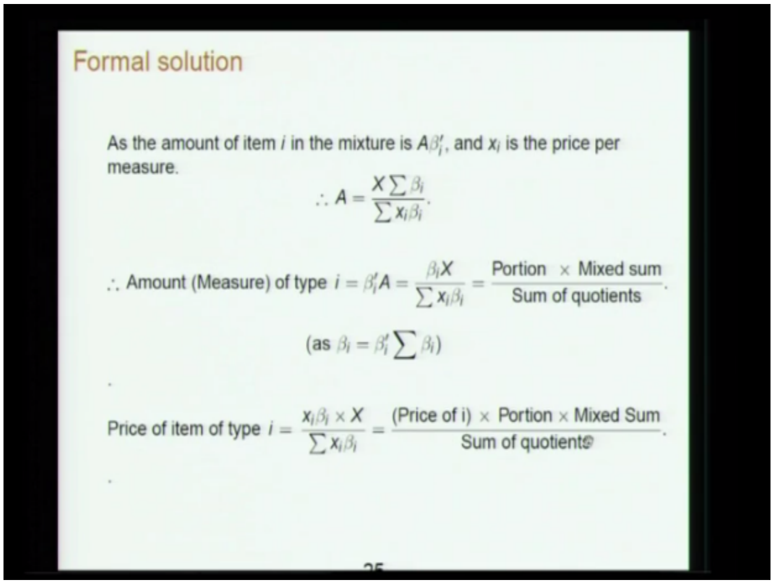

And the amount of item in the mixture is a beta prime i and xi is the price so, these the **sol** solution you know you have to multiplied by this proportional factor corresponding to i. Then divided by some of quotients then whatever you x is giving getting this is which is the stated result And that is the amount of measure of type I and price also of that you can give

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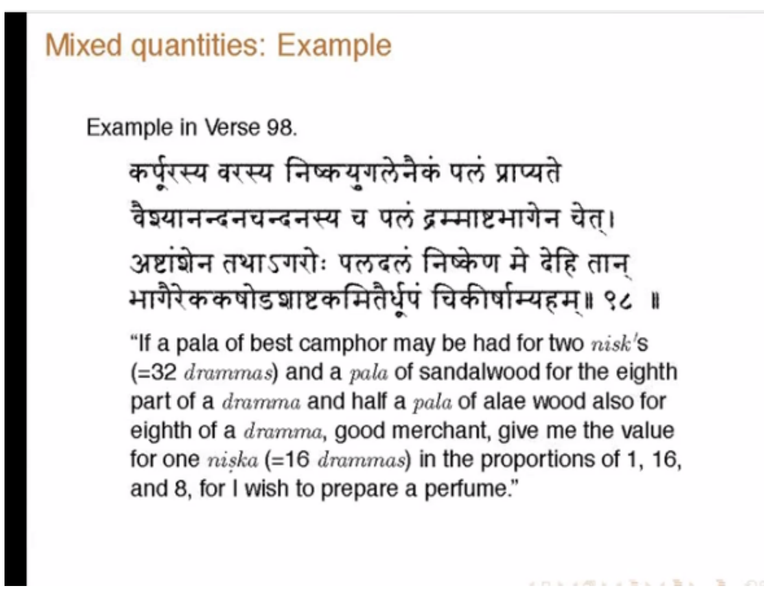

If you go through this that thing very difficult only you have to follow this and the understand the some small simplifications is making. So, if you tell an what an example it will be clearer to you (FL) if a pala some measure okay best camphor may be had for two niskas they are some units one niska is suppose to be some 16 drammas. So, and a palas sandal wood I sfor a 8 part of a drama and half a pala of from alae wood it calls it also for 8 of a dramas that is the transaltion niska (FL) and okay.

(FL) that also for a eighth of a drama, good merchant give the value of one nika a 16 drammas but they must be what is specify they must be what is specified is these must been the proportion 16 and 8 okay. So, the proportions of these things in the you know volume or weight that is given and then the relative prices are given. And what is the amount you are paid so, the total thing that is given then you have to give the quantities of those thing which are actually used to perform the .

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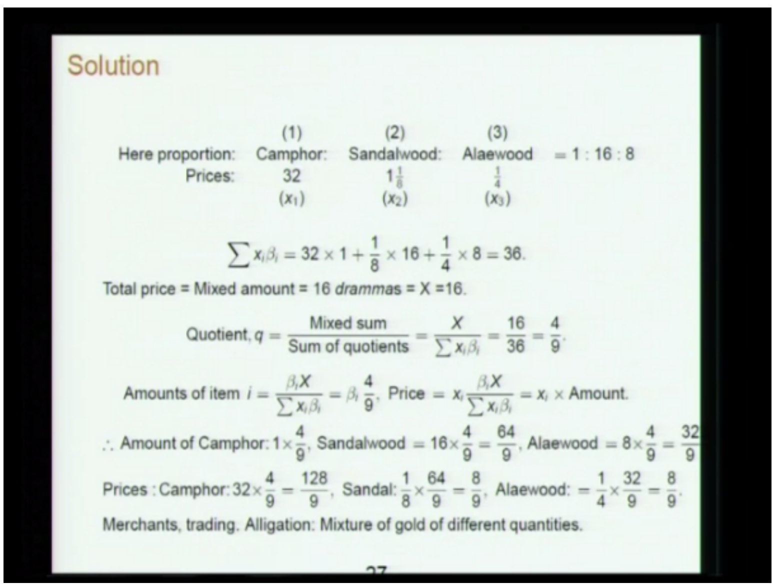

To prepare the perfume so, you saying camphor is you know suppose you called this 1, 2, 3 so, these are the relative prices and this on the proportions are 1:16:8 and the prices are 32 and 1 and 18 and ¼ I am sorry these not 1 1 should not be there it is only 1/8 so, the price of sandal wood is 1/8 only one should not be there 1/8 and ¼. So, the total you see see suppose your quantities are is so, 32 into 1 he amounts of prices are this 32 excess are the price this.

And these are the proportions so, you can keep it like that you do not have to normalize it I do not know make it convert into fractions. So, this is the total price so, the total price will be mixed amount I mean the if exercise the thing and the total price is given to be 16 sorry this is a quantity the total quantity is there whatever the intermediate quantity which come that is 36 here 36. The total price I s16 the quotient is x/this thing whatever is the mixed sum that is16 the mass.

So, this divided by this so, you get 4/9 so, amounts of item will be you know various proportions are given. So, 4/9 into this whatever is given here and then you compute this individually. So, the amounts are 1 into 4/9, 16 into 4/9, and 32/9 okay. So, that has the proportion 1:16:8 and a prices of course are you know you have to multiply by the prices for 1 unit kind of a thing 1 amount. So, you have to so, just a matter small details the nothing more significant, but it is useful simplification that is what it is important.

So, similarly he talks about the various stating kind of a situation then mixtures of gold gova different quantities.

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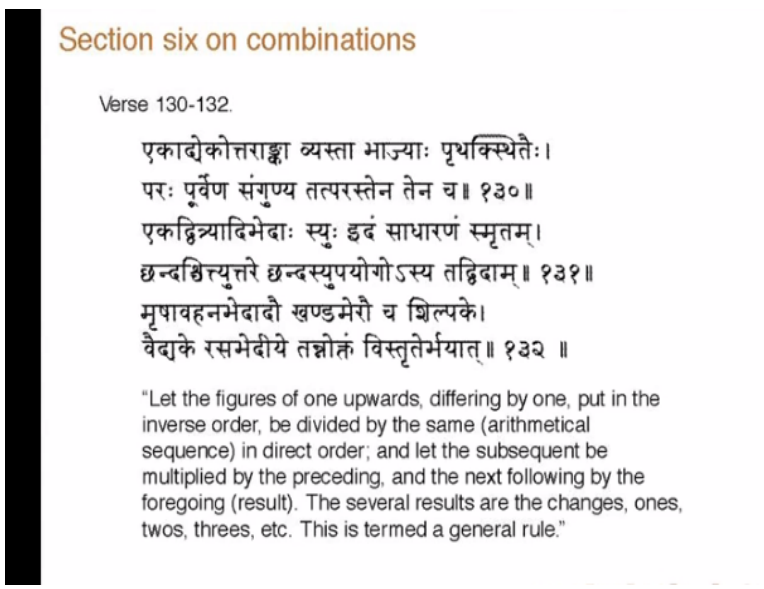

So, called eligation and so on so, next he comes to the combinations so, is the very very important topic but here of course you will not giving detail one one result only basically he is stating later on he will take it up in more detail over it is fermentations and combinas since in the later part of the text. But 4o I stelling here stating here is you know (FL) let the figures of one upwards differing by one putting the inverse order be divided by the same arithmeticals in direct order.

And let the subsequent be multiplied by the preceding and the next following by the foregoing result, the several results are changes, ones, twos, threes etc., this termed general rule and then you know u have to taught of the applications of this (FL) and so on it was in (FL) for those verse there in to find the variations of meter in the (FL) as in architecture.

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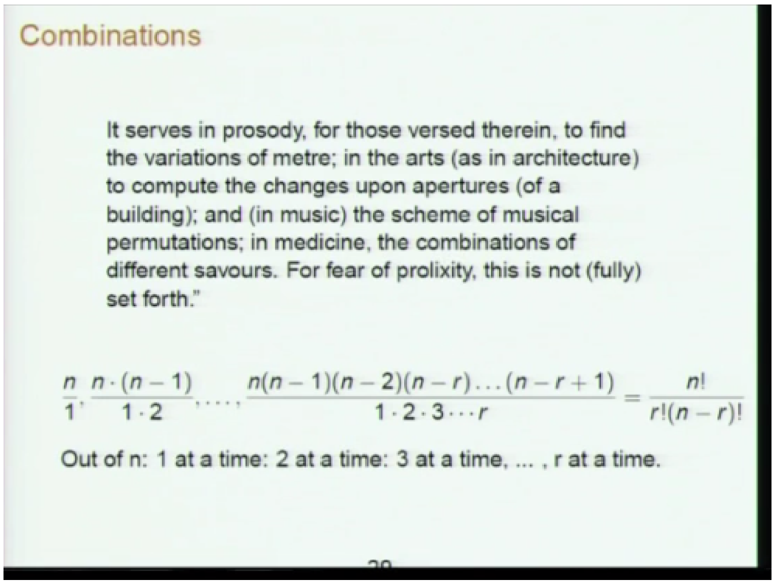

Then to compute the changes upon apertures of a building in music the scheme of musical permutations in medicine the combinations of different savours and for fear of prolixity this is not fulle set forth I mean for fear of you know talking too much about this you know. So, what is stating is the (FL) etc., so what he is saying is take you know to give some number 1 write n/1 and the next n into n-1/1 into 2 etc., etc., n into n-1 up to if there odd terms you go to n-r+1.

So, that in the numerator in the denominator 1 into 2 into 3 r okay. So, what these are the things you know, so out of n you can take 1 at a time 2 at a time and r at a time these are the quantities, these are the combinations right which we have already learnt, so in modern notation it is factorial n/factorial r into factorial n-r, so is state this result.

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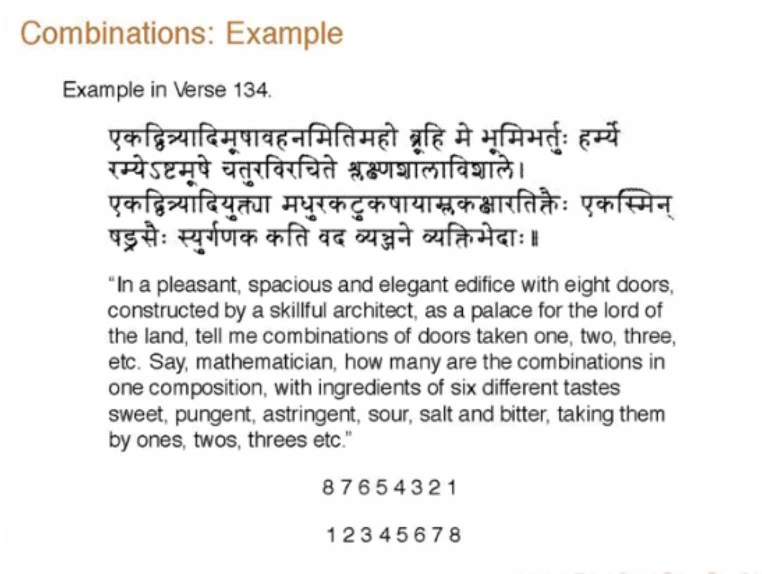

And immediately gives an example it was 2 examples contained in this verse (FL) so in a pleasant spacious and elegant edifice with 8 doors constructed by the skillful architect as a palace for the lord of the land tell me the combinations of appperture 1, 2, 3 etc… so, these one problem then say mathematician how many are the combinations in one composition with ingredients of six different tastes sweet, pungent astringent, sour, salt and bitter, taking them by ones, twos, threes etc…

So, the once the first problem is you know thus the building with 8 windows so, what are the ways number of ways of you know opening one window then two windows. I mean two out of the 8 then 3 out of the 8 and so on so, that is one problem. Then next is you know six kinds of tastes so, sweet, pungent, astringent etc… Madura (FL) amla ect. So, taking 1, 2, 3 etc… so, remember that is one of the very ancient example right so, this was the mentioned earlier you know regarding how many tastes are there in books and related to ayurveda.

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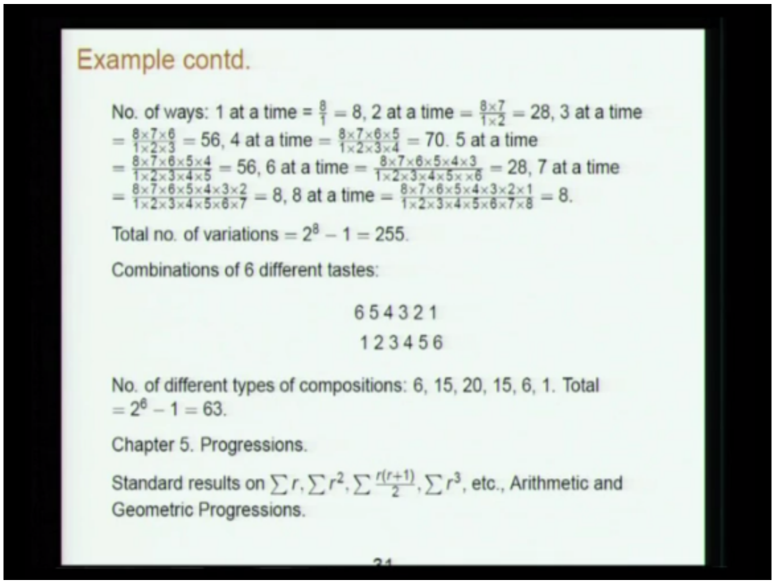

So, then number of ways if you take the first example so, 1 at a time is8/1, 2 at a time is 8*7/1*2 etc… 3 at a time is this so, these has been done earlier. So, I will not need not going to the detail but this are the simply put it is this and the total number of variations 2 to 8 to the power of 8-1 right. So, that is the total number of combinations so, adding all of them together 1 at a time, 2 at a time etc… 8 at a time for this particular example.

Similarly for this taste so, 1 at a time it is 6, 2 at a time is 15, 3 is 20 so like that total will be 2 to the power of 6 -1 that is 63. So, next the so, this is the and other I told you you will discuss about this combinations and all that and some more advanced results in later in the text. So, I will discuss that in my last lecture on Lilavati so, then progressions so, standard results and we already know how to sum of the first ten integer sum of the squares of the first ten integers.

Then some of sums then sum of cubes and all that right so, there been dealt with in great detail I do not have to repeat them.

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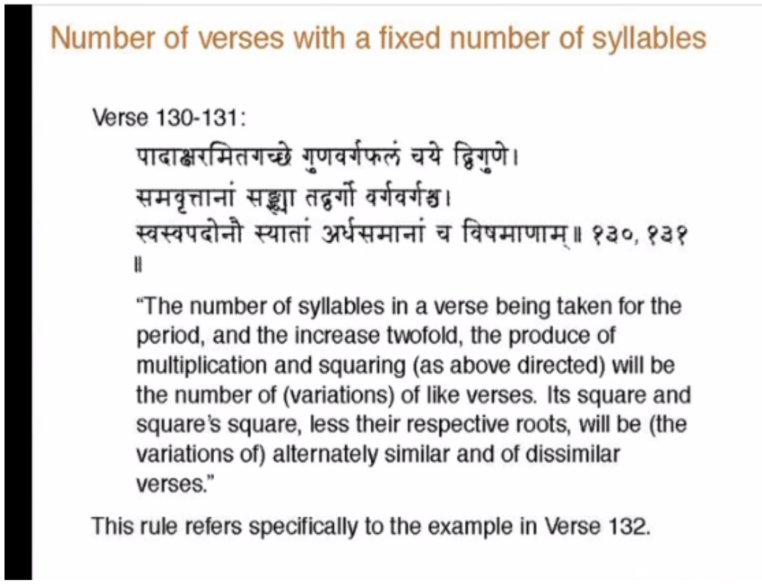

So, then chandas example number of verses with a fixed number of syllables so, this also has been told many times (FL) okay. The number of syllables in a verse being taken for the period and the increase twofold the produce of multiplication and squaring (as above directed) will be the number of (variations) of like verses. It is square and square is square, less their respective roots, will be (the variations of) alternately similar and of dissimilar verses.

So, in fact he is actually planning to explain the I mean explain the methods for the example which comes in the next verse.

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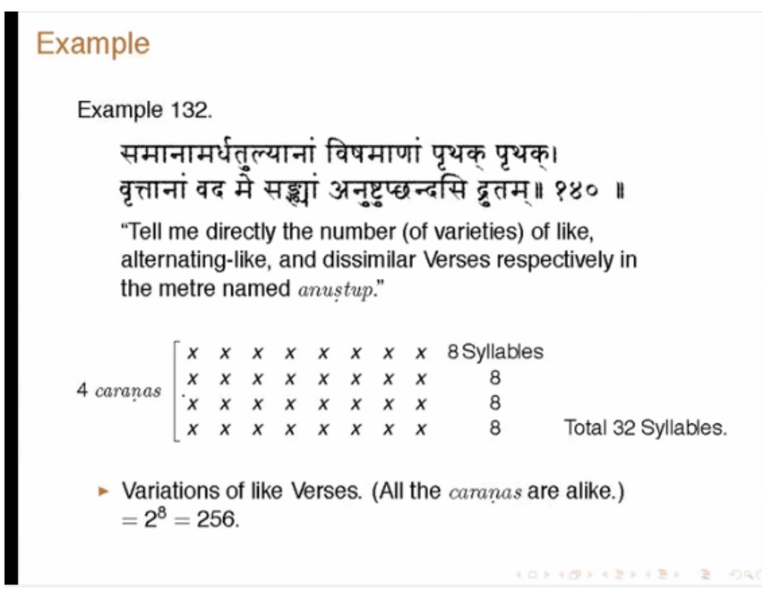

So, he saying tell me directly the number of varieties of like or alternative like and this similar verses respectively in the metre named anustup okay. So, there are 4 caranas as already has been this things so, you let it like this things at 8 syllables each. So, each of them can be (FL) right so, **so** which (FL) uniformly like by everybody so, I am sure you noted already detail and you know keeping it in the back of your mind.

So, these 8 syllables, 8distints so, total of 32 syllables all there okay suppose all the this thing caranas or (FL) they are identical. Then clearly of course each of them individually in each of them can be you know two factor of two right two kinds of things are (FL) for each of these syllable. So, 2 to the power of 8 but no more counting because all these things are repetitions are the same thing. So, 2 to the power of 8 is equal to 256.

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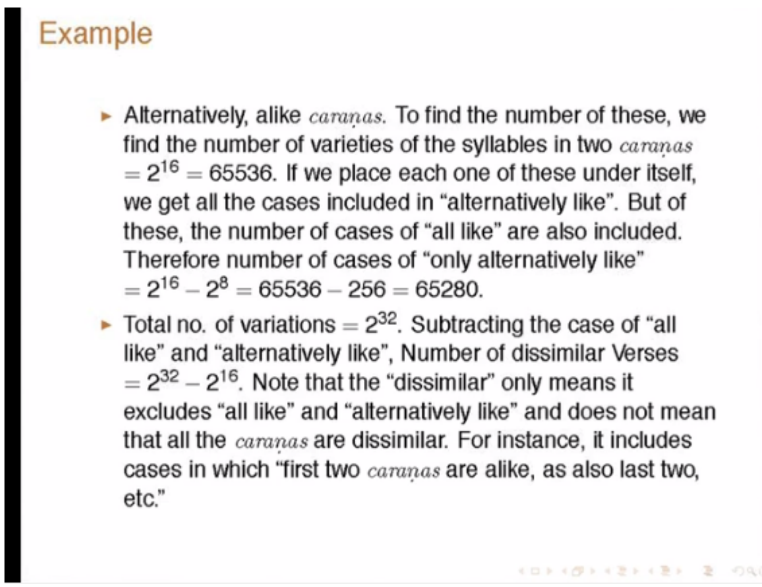

So, then alternatively alike caranas so, that is 2 to the power of 16-2 to the power of 8 so, this also has been discussed (FL) here, So, you will not showing to that so, that is the so, what you do is you know you take 2 caranas 2 caranas okay. So, one take this thing 2 caranas and then put the exactly the same thing below. So, then first and third will be alike and second and fourth will be alike okay.

So, then the number of this thing will be 2 to the power of 16 but it including the all alike also in this that also comes it have to subtract this. So, 2 to the power of 16-2 to the power of 8 and a total number of variations is 2 to the power of 32 obviously. If all of them are you know can be arbitrary each of them is logo or guru and that is not known which of them. So, then 2 to the power of 32 will be there because so, 32 is the total number of syllables.

So, then but these things these two cases the sum will be two to the power of 16. So, subtracting the case of all like and alternatively alike number of dissimilar verses is 2 to the power of 32-2 to the power of 16. So, here dissimilar only means it took all like alternatively like and does not mean that all caranas are dissimilar no. For instance it includes case such in which first two caranas are alike as also last two and so on okay only thing is excluding all identical and alternatively identical.

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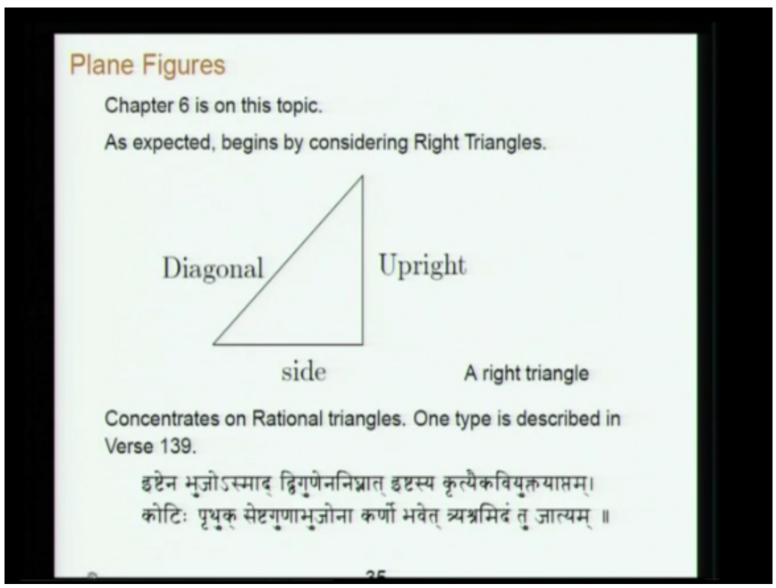

The rest of the things are that is how it is post oaky so, then (FL) right angle triangle so, he expectedly he begins been considering right triangles. So, side, upright and diagonal see concentrates and rational triangles so, one type is discussed in verse 139.

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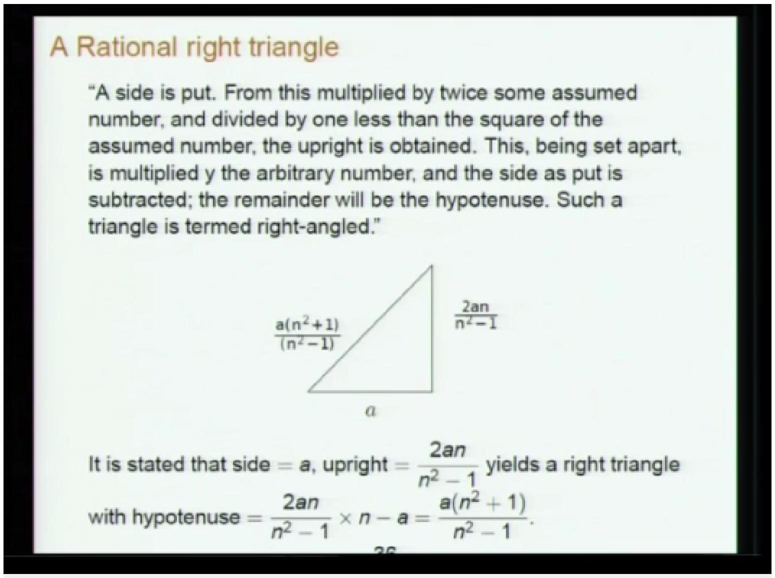

(FL) A side is put from this multiplied by that is a these arbitrary from this multiplied by twice some assumed number and divided by one less than the square of the assumed number, the upright is obtained. So, this is a so, then you are you know multiplying by twice some number 2a so, a into 2n and divide by the square of that number-1 n square-1. This being set apart is multiplied by the arbitrary number and the side as put is subtracted.

The remainder will be the hypotenuse so, the in fact hypotenuse will be this so, what is saying is this you take and then multiplied by arbitrary number. And then subtract the side that what he saying so, finally you get this so, a and then this so, these are right angle triangle. So, a is rational and n is an integer so, the rational right angle triangle.

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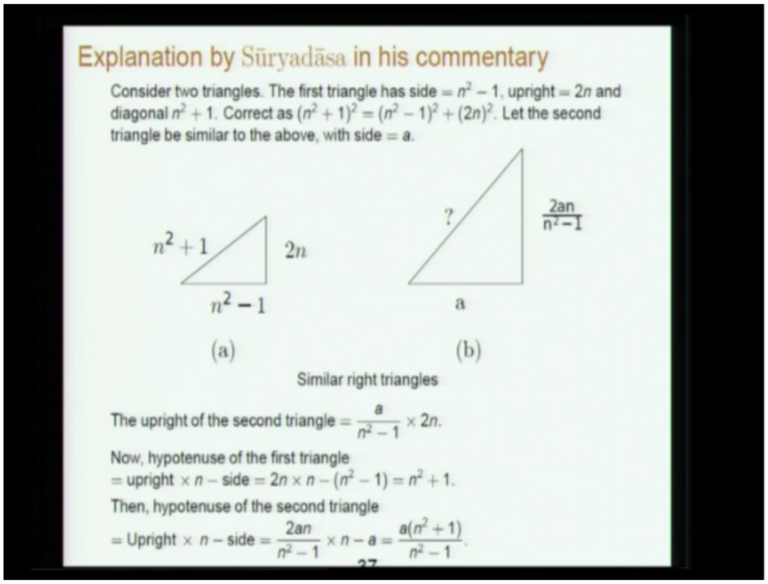

So, the explanation is interesting so, two triangle the first triangle as side n square-1 2n and n square+1 suppose is the standard thing which you are discussed already right. Because these the standard right angle triangle because these whole square n square- square, 1 square+2n square is equal to n square+1 square. So, now from this you have to get this a into this and then this is the upright and what is the diagonal of course one ways to just a proportion.

You can actually get it but his explanation is slightly interesting you know because it will be useful later on the upright is the second right triangle is this. Now hypotenuse the first rectangle you could take it as upright into n-side so, this into this-this you can view it like that. So, n square+1 and hypotenuse the second triangle also is construct like that upright into n-side. Because there proportional you know so, the way in which you construct the hypotenuse the first triangle.

You construct a hypotenuse of the second triangle also that is what he saying and you get this so, this will be a into n square+1/n square-1.

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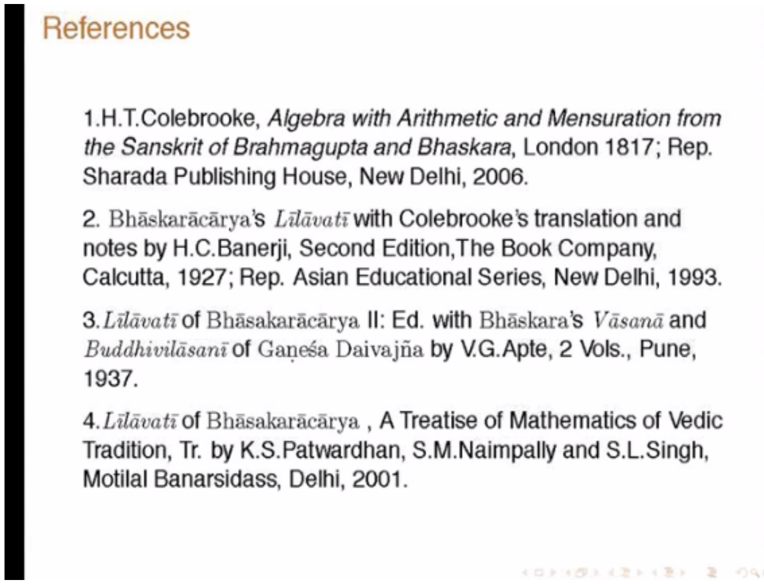

The references are given here okay I think that will consider this discuss more about this rational triangles in the next lecture thank you.