

**Mathematics in India: From Vedic Period to Modern Times**  
**Prof. K. Ramasubramanian**  
**Indian Institute of Technology-Bombay**

**Lecture-2**  
**Vedas and Sulbasutras-Part 1**

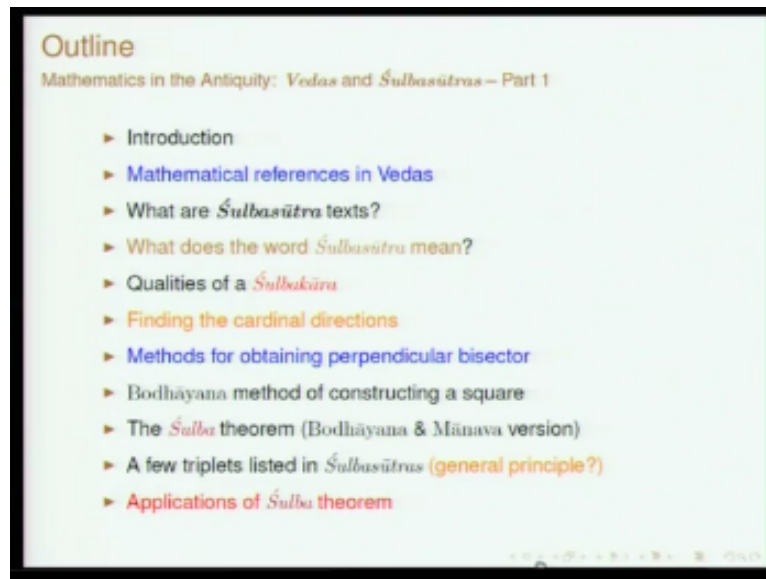
So you would have had a wonderful overview of Indian mathematics professor Srinivas. So in this talk basically I will be covering mathematics in Vedas and sulvasutras. So this will be run in two parts. Vedas as all of you know is the most ancient scripture that is available to us today and sulvasutras form a part of what are known as kalpasutras which itself is a part of Vedas. So what will be covering is basically the mathematical reference that can be found in Vedas.

And the geometry arithmetical algebra etc. so which can be found in the class of text called sulvasutra. The purpose of Vedas is obviously not to deal with mathematics, so it has to do something with the relation of human and nature and appreciation of nature in the form of divine been through humans etc. So this is primarily the content of Veda and in fact it has been subsequently states one of the most ancient text called vedanga jyotisha.

Wherein the purpose of Veda has been very clearly defined as Veda (FL) is primarily so propitiation of the divinity. So but you might have also heard perhaps in the overview by professor Srinivas wherein mahaviracharya, so while defining the scope of mathematics he says (FL) (()) (02:06) to (()) (02:14). So you will see mathematics in every aspect of life whether it is world later Vedic or religion.

So any way so what you will be trying to do is trace out some references which can be found in Vedas and then see what one can derived from those references with regard to the knowledge of mathematics which was available in those ancient period. So this is primarily the idea of catching up on the most ancient scripture, so vedic scripture and then will want to sulvasutras.

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So in this lecture we will start with the references in Vedas and then we move on discuss what are the sulvasutra text. So what do they deal with and what does the term sulvasutra mean and one of the most important things so as I was mentioning veda (FL) where are you have any how to perform sacrifices you had to create necessary platform (FL) and so and the construction so that is involved in creating such platforms etc. is the primary content of the text called Sulbasutras.

So the most important thing is to find out the coordinal directions. So that is one of the most interesting experiment it can be done with a very very simple device called (FL) I will be discussing that and this is a pre-requested for it. So for doing (FL) you have to create a such (FL) cannot be constructed without knowing the exact directions. So that is what any cardinal direction when will East, West, North, South.

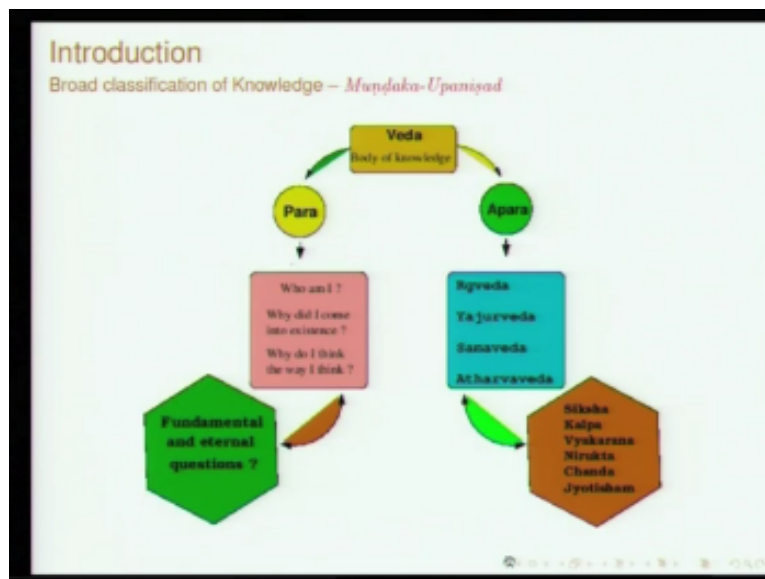
So everything that after all by looking at sunrise I will be able to determine the direction but that is not the case because of Uttarayan and dakshinayan those emotion. So we need to conduct a very simple experiment. But this experiment is one of the most full proof experiments. So we will be discussing all that. So then we also deal with the construction of the geometrical objects square, circle.

You see when I say both (FL) so this may be pretty simple so of tral if I need to construct a square I need to draw a line and then I can have a set square. I can have a T square and then I I can determine what is the great deal about it. So that is not thing, they did not have T square or set square but then (FL) has discuss the very interesting method by which all that you need

is a couple of nail and then a rope. So with that will be able to construct a perfect square. So that is what will be discussing there.

And then will be moving onto what is today called Pythagorean theorem or Pythagoras theorem. So we call it sulvasutra or sulba theorem and there are different versions of both (FL) so will be discussing that and then will also see how the triplets having listed in the this ancient text and then application of sulvasutra. So all that will be discussed in this lecture.

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Before proceeding further so we will try to look into this Upanishad for which is a part of the weather so finally 10 principle Upanishad and one of them is mundakopanishad, so where does this mathematics all within this Vedic purpose of knowledge. So here we have a passage mundakopanishad, so which price to classify the body of knowledge itself into two groups, one is called (FL) deals with most fundamental question.

So who am I, so what is the purpose of my existence, what is my connection with this all that, so this is the domain of para and apara to all the other things are put under para in fact even the Vedas, so the propaganda set in Vedas all under (FL). So we have 4 vedas as it is lisedet, so Rigveda, yajurveda, Samaveda, atharvaveda. Then we have 6 vedanga Shiksha Kalpana Naan irukka jyotisham.

It is under this jyotisham the mathematics also is plugged into, so ganitam is not listed, but the development of mathematics as we have seen is so is primarily in trying to solve problems in Astronomy and in fact has we will see later. So the connections between the

mathematical development and the application of astronomy will become quite evident when will look into the later part of a course as to why they had to solve first order intermediate equation and so on in the later part of the course.

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**Mathematical references in Vedas**  
Citations that unambiguously point to the decimal system being in vogue

- ▶ We find yet another passage presenting a list of powers of 10 starting from hundred ( $10^2$ ) to a trillion ( $10^{12}$ ).  
शताय स्वाहा सहस्राय स्वाहायुताय स्वाहा नियुताय स्वाहा  
प्रयुताय स्वाहावदाय स्वाहा न्यवदाय स्वाहा समुदाय स्वाहा,  
मध्याय स्वाहान्ताय स्वाहा ... परार्धाय स्वाहा<sup>2</sup>  
Hail to hundred, ... hail to hundred thousand ... hail to  
hundred million ... hail to trillion.
- ▶ We also find a list of odd numbers and multiples of four occurring in  
*Taittiriya-saṃhitā* (4.5.11):  
▶ एका च मे तिस्रह मे पञ्च च मे ... एकत्रिंशच्च मे  
त्रयस्त्रिंशच्च मे  
▶ चतस्रह मेऽष्टौ च मे द्वादश च मे ... चतुष्टवारिंशच्च  
मेऽष्टाचत्वारिंशच्च मे

<sup>2</sup> *Taittiriya-saṃhitā* 7.2.49.

So this is how one is able to relate mathematics with the most ancient scripture. So this jyothisham includes ganitham 2 weather has said I was mentioning that you find some references here and there and references will be only in the form of primarily numbers. So (FL) is primarily science of calculation calculating the number of days for calculating on which day what happened you perform and how many number of days in which the sacrifices to be done.

So there are various other things and how periodically how to be done and the numbers which we find in the vedic literature are mostly related to the propitiation which is done towards a certain particular date. For instance this manta which you can see here (FL) (()) (07:46) to (()) (07:53). So this basically list all the powers of 10 starting from 10 to the power of 2, to 10 to the power of 12. So (FL) refers to 10 to the power of 2 to 10 to the power of 12. So (FL) refers to 10 the power of 2 and towards the end of passage we find (FL) 10 to the power 12.

The list of all the words which refer to the power of 10 has been listed in this. Similarly in the next one so we find (FL) (()) (08:18) to (()) (08:22). So we find all the off numbers listed here and (FL) (()) (08:25) to (()) (08:28) the power multiples of 4. So these things are listed. Further there is an interesting passage in Rigveda, so here the passage (FL) (()) (08:38) to (()) (08:43). So all the multiples of 10.

So I am just coating this in order to convey a certain message that these people have been quite conversant with the decimal place value system. So in fact the one of the most fundamental discoveries. So we will deal with that exclusively in a lecture later but then you can see that so all the numbers which have been mentioned for instance look at this Mantra (FL) (09:10) to (09:15).

So this mantra (FL) (09:17) to (09:21), so nava refers to 9 and the rest 3, 3, 3, 9. So the very interesting number, so if you like at so this basically some of 33, 303, 3003 and you get this particular sum and just number you also very close to the period of 18 years, there have been articles written wherein they try to trace the origin of this number and what is so special about this number.

What is interesting here is to note that there (FL) evidence so by which will be able to see that vedic text has been very very conversion with the decimal place value system and we also have this mantra (FL) (10:05) to (10:10). So this shanthi mantra speaks of the notion of infinity (FL) (10:16) to (10:22). So you take Poorna Poorna and what is Poorna, so within very close to the notion of infinity.

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### Mathematics in the *Śulbasūtra* texts

What are these texts, and where do they fall in the Vedic corpus?

- ▶ One of the prime occupations of the vedic people seem to have been **performing sacrifices**, for which altars of prescribed shapes and sizes were needed.
- ▶ Recognizing that manuals would be greatly helpful in constructing such altars, the vedic priests have composed a class of texts called *Śulba-sūtras*.
- ▶ These texts (earliest of which is dated prior to 800 BCE), form a part of much larger corpus known as *Kalpasūtras* that include:
  - ▶ श्रौत – Employed in rituals associated with societal welfare.
  - ▶ गृह्य – Rituals related to household.
  - ▶ धर्म – Duties<sup>3</sup> and General code of conduct.
  - ▶ शूल्ब – Geometry of the construction of fire-altar.

<sup>3</sup>Ādi Śaṅkara in his commentary on *Upaniṣads* defines the term *dharmā* as *anuṣṭheyaṅāṃ sāmānyavacanam*.

So now I move on to the sulbasutra texts. So I was mention in earlier the sulbasutra text gone a part of what are know as kalpasutra. In fact kalpasutra includes (FL) (10:41) to (10:48) with certain virtual which are done for the society welfare. So (FL) so has to do with

the which was performed in the houseful. So certain text which will guide the performance of these things. So all refer to by these things (FL).

Then we have dharmasutra, so dharmasutra have to do with the general code of conduct it has to be followed or the well being of the pi. So that is what is and the duties which have to be done by the people and so on. So this is the subject matter of dharmasutra and when come to sulbasutra. Sulvasutras are texts which have been primarily composed to assist the sulvasara in the construction of various sacrificial altar.

The first sutra is some of this sulvasutras (FL) is basically a collection of bricks etc. so (FL) is putting together. So putting together to create a certain platform on which these rituals can be performed.

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**What does the word Śulba mean?**

- ▶ The word *śulba* stems from the root **शुल्ब-माने** (to measure).
- ▶ The etymological derivation of the word can be presented in more than one way:
  - भावव्युत्पत्ति - शुल्बनम् = शुल्बः ।  
Refers to the act of measuring.
  - कर्मव्युत्पत्ति - शुल्बयते इति शुल्बः ।  
Refers to the entity/result of measuring.
  - करणव्युत्पत्ति - शुल्बयत्यनेन इति शुल्बः ।  
Refers to the instrument of measuring.
- ▶ The complete derivation of the compound word *Śulbasūtras*, including the grammatical peculiarities is:  
**शुल्बनम् = शुल्बः (शुल्ब् + घञ्)<sup>4</sup>। तत्सम्बन्धि सूत्राणि।**

<sup>4</sup>This type of derivation based on *bhāṣavyutpatti* is governed by the *sūtra* 'bhāve ghañ'.

What does the term sulba mean, in fact sulbasutra is a compound word, sulba is one part and sutra is another part. Sutra is yet type of writing. In fact this has been the most ancient form of composing text. So if you look at Sanskrit that is available to the most ancient ones have been in the form of sutras. So we have this (FL) any sutras, you have Panini Sutra. So (FL) so we have Patanjali yoga sutras.

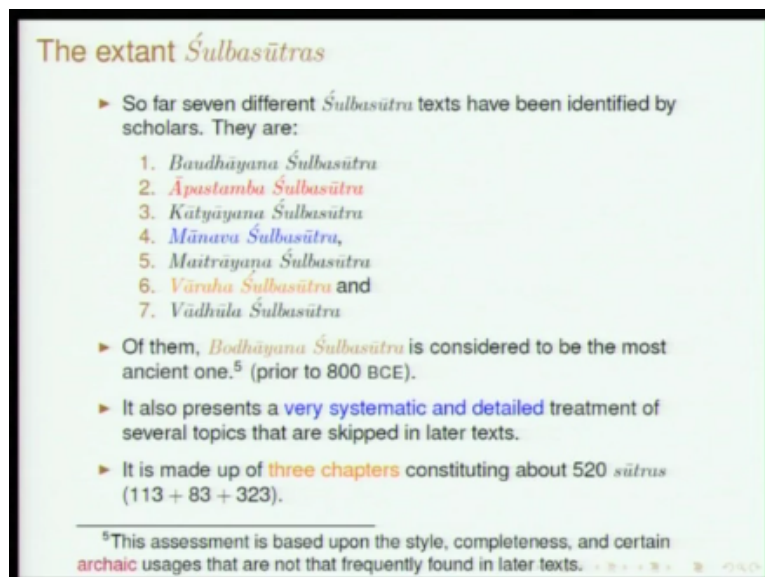
So all of them so have been composed in the form of what is referred to as sutra style, suppose the definition for sutras. So this sulbasutras is a compound word where in the sulba as a meaning of measuring. Sulba meaning in fact the Sanskrit the most of the word are derived from the roots and the root is sulba, sulbamanic. So mana is measuring. So

sulbamanae in fact this word can be derived by a different ways in fact what is called (FL) karmadhipati Karnavati.

And so on. So (FL) if you do this so sulba, so this is how the word can be derive and it refers to the act of sulab. (FL) sulba when you say you measure something and that entity is the object of measurement and that is the (FL) when you say so karana is an instrument, so by which you make measurement. So all of them are applicable when we refer to the content of the sulbasutra text.

So and therefore this compound sulbasutra actually means a text which discusses the act of measuring. So obviously the entity which is measure, the instrument which is used for measurement. So all of them will be referred to into this sulvasutra text.

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**The extant Śulbasūtras**

- ▶ So far seven different Śulbasūtra texts have been identified by scholars. They are:
  1. *Baudhāyana Śulbasūtra*
  2. *Āpastamba Śulbasūtra*
  3. *Kātyāyana Śulbasūtra*
  4. *Mānava Śulbasūtra*,
  5. *Maitrāyaṇa Śulbasūtra*
  6. *Vārha Śulbasūtra* and
  7. *Vādhula Śulbasūtra*
- ▶ Of them, *Baudhāyana Śulbasūtra* is considered to be the most ancient one.<sup>5</sup> (prior to 800 BCE).
- ▶ It also presents a very systematic and detailed treatment of several topics that are skipped in later texts.
- ▶ It is made up of three chapters constituting about 520 sūtras (113 + 83 + 323).

<sup>5</sup>This assessment is based upon the style, completeness, and certain archaic usages that are not that frequently found in later texts.

So the sutra itself is a very very very likely composed text, so you will not find so unnecessary words into the sutras. So that is the very definition of sutra (FL) (()) (14:27) to (()) (14:33) and so on and the sulvasutra was obviously cannot be understood on their own so without that the teacher or teacher plus commentator. So if teacher is not available we resort to commentary which have been composed by later period on the sulvasutras for understanding what does this sutra convey.

There are 7 sulbasultras, so which are available for us today. These are the text both Baudhayana, apastamba, katyayana, manava, varudha, and vadhula. So of the, 4 are more popular and more studies have gone into the first 4 and the later that have not been studied

that much. But anyway so according to scholar so baudhayana sulbasutra seems to be the most earliest of sulbasutra which are available to us today.

So this is based on certain analysis have been done in in the reference to the style, reference to completeness, reference to the party, who say the which is found in more text. So based on this felt that baudhayana must be the most earliest sulbasutra and one also points to certain sutras almost similar in later (FL) sutra so we will find the same sutra repeated with minor changes.

So and it is more extensive also with baudhayana sulbasutra and therefore they feel that it is the most ancient one. Baudhayana sulbasutra is made up of 3 chapter consisting of 520 sutra and composition is something like this.

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**Commentaries on Śulbasūtras**

The table below presents a list of some of the important commentaries on three 'earlier' Śulbasūtras:

Śulbasūtra	Name of the comm.	Author
Bodhāyana	Śulbadīpikā	Dvārakānātha Yajvā
	Śulba-mīmāṃsā	Venkaṭeśvara Dikṣita
Āpastamba	Śulbavyākhyā	Kapardisvāmin
	Śulbapradīpikā	Karavindasvāmin
	Śulbapradīpa	Sundararāja
	Śulbahāṣya	Gopāla
Kātyāyana	Śulbasūtravivṛtti	Rāma/Rāmacandra
	Śulbasūtravivarṇa	Mahīdhara
	Śulbasūtrabhāṣya	Karka

So they have commentaries as they was mentioning so for baudhayana we have 2 commentaries one by Dvarakanatha, the other by Venkatesvara Diksita, the apastamba we have 4 commentaries which are identified by Kapardisvamin, Karavindasvamin, Sundararaja, and Gopala and this Katyayana sulbasutra we have one very interesting commentary by Rama or Ramacandra.

So this Rama seems to have improved upon certain algorithms which have been stated in Katyana sulbasutra which I will be touching up on in my next lecture. So we have also very interesting commentary by Mahindra or this Katyana sulbasutra for wherein he has coated several verses some various other things is also another interesting commentary.



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**Qualities of a Śulbakāra**

► Mahādhara (c. 17th cent) in his *vyākṛti* on *Kātyāyanaśulbasūtra* succinctly describes the qualities of a *śulbakāra*.

सङ्ख्याज्ञः परिमाणज्ञः समसूत्रनिरञ्छकः ।  
समसूत्रो भवेद्विद्वान् शुल्बवित् परिपृच्छकः ॥  
शास्त्रबुद्धिविभागज्ञः परप्रास्त्रकृतुहलः ।  
शिल्पिभ्यः स्वपतिभ्यश्चाप्याददौत मतोः सदा ॥  
तिर्यङ्मान्याह सर्वार्थः पार्श्वमान्याह योगवित् ।  
करणीनां विभागज्ञः नित्योद्युक्तश्च सर्वदा ॥

A *śulbakāra* must be versed in arithmetic, versed in mensuration, ... must be an inquirer, quite knowledgeable in one's own discipline, must be enthusiastic in learning other disciplines, always willing to learn from [practising] sculptors and architects ... and one who is always industrious.

► The above anonymous citation clearly brings forth the point that a *śulbakāra*, is far more than a mere geometer.

Now I will decide a couple of verses so which is found in the commentary or Mahindhara. So wherein he defines what are the qualities that are expected of a Sulbakara. Sulbakara I mean the one who access so the vedic trees in the construction of the ah (FL) ok. So where sacrifice the perform. So he says (FL) (()) (17:25) to (()) (17:32) and so on. So (FL) means the one who is versed in Arithmetic science of calculation.

And (FL) so one who is verse with mensuration and (FL) could be of sort of enquirer and we also have the adjective (FL) certain enthusiasm, enthusiasm in learning other disciplines also. So today we have ended up into a certain situation where in, So civil engineering there will be some 20 disciplines. So we will know only true this even only read this so on so on. So in the name of getting deep into something.

So overall understand is all gone, anyway so here he describes the one who should be enthusiastic to know more and more things and so the point I want to drive through this versus is that primarily, so he is not the person who should be considered as the nearly geometry who is the only that kind of a thing, so this is quantities.

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Topics covered in the <i>Baudhāyana-śulbasūtra</i>	
Sanskrit name	Their English equivalent
रेखामानपरिभाषा	Units of linear measurement
चतुरस्रकरणौपायः	Construction of squares, rectangles, etc.
करणयानयनम्	Obtaining the surds/Theorem of the square of the diagonal
क्षेत्राकारपरिणामः	Transformation of geometrical figures
नानाविधवेदिविहरणम् <sup>6</sup>	Plan for different sacrificial grounds ( <i>dārśa</i> , <i>paśubandha</i> , <i>sutrāmāsi</i> , <i>agniśtoma</i> etc.)
अग्नीनां प्रमाणक्षेत्रमानम्	Areas of the sacrificial fires/altars
इटकसङ्ख्यापरिमाणादिकथनम्	Specifying the number of bricks used in the construction of altars including their sizes and shapes.
इटकोपधाने शैत्यादिनिर्णयः	Choosing clay, sand, etc. in making bricks
इटकोपधानप्रकारः	Process of manufacturing the bricks
द्वयेनचिदादाकारनिरूपणम्	Describing the shapes of <i>śyencati</i> , etc.

<sup>6</sup>In fact the text commences with the *sūtra*  
अथेमे ऽग्निचयाः । (Now we describe the fire altars).

And there are various topics which are discussed in sulbasutras to give you an idea as to what could be the content of a sulbasutra text. I have listed few of them, for instance we have (FL) say mana is measuring (FL) is measuring the length of the cord, so which means (FL) is a certain technical term. So they will define those technical term, but in some various unit of measurement are used.

So what are the units of measurements linear measurements, then (FL) 4 sided figure, so construction of 4 sided figure square, rectangle and so on. Then (FL) here refers to square root ok, so karani in fact karani have various conversation I will discuss that little bit later. So the (FL) means how do we find the square root of a non square number ok. So this things are discussed, there then (FL) so parinama is a transmission.

So if you have a geometrical of this particular shape you would like to transform that object into some other object and this is a area of preserving confirmation. So a square of a certain area should be transformed into a circle of the same area then obviously you need to know the value of pi. So in some sense. So all that so some approximations for that and how do you do geometrical.

So these are all discussed in this text, they are very interesting things and (FL) so vedi as I told you is a sort of sacrificial ground (FL) so for different sacrifices if it comes at the altar in different sizes and shapes and things are there are certain altar which have been constructed in the form of bird, so there is some constrain which is impose so it should have only one 20 square meters, so I am just saying in some units so 120 square meters, 100 square meter.

We should construct alter in the form of bird, so which means a lot of calculation has to go into has to go what kind of bricks you have to design and it has to be of certain height also. So different layers have to be arranged in a particular way. So that the structural stability is also taken care. So you have to just pile the bricks, so all that are discussed here and they also discuss about the kind of material which has to go into preparation of this.

This is another interesting aspects which point in, sulbasutras regarding the construction material and the process so which has to be done, to see that the bricks that you create are solid enough and so on and so forth , Then how to manufacture bricks, so these are all the contents of a typical sulbasutra text.

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**Expression for the surds given in Śulbasūtra texts**

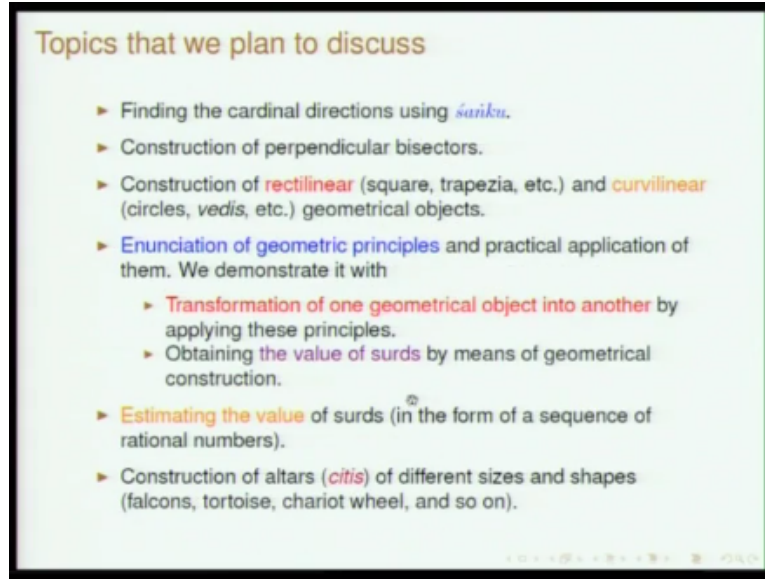
- ▶ Besides presenting the details related to the construction of altars—that generally possess a bilateral symmetry—the Śulba-sūtra texts also present different interesting approximations for surds.
- ▶ The motivation for presenting estimates of surds could be traced to the attempts of vedic priests
  - ▶ to solve the problem of “squaring a circle” and vice versa
  - ▶ to construct a square whose area is  $n$  times the area of a give square, and so on.
- ▶ The expressions for surds presented in the form

$$N = N_0 + \frac{1}{n_1} + \frac{f^2}{n_1 n_2} + \frac{1}{n_1 n_2 n_3} + \dots$$

can be understood in different ways, of which we will describe the Geometrical construction.

So not every text will discuss every aspect of it ok.

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The topics that would discuss here is roughly, so I will start with the determination of cardinal direction. So this is called (FL) direction on enormous finding. So that I will do as the first thing, then how to construct perpendicular bisector, suppose you have written in east west line then you have to construct a perpendicular bisector to get the what are the methods which are employed.

Then construction of figures like square trapezium, circle. So these are all very interesting things as I told you they are just done with nail and a rope. So emancipation of certain geometrical principles for instance the sulbasutra sulba theorem ok sulbasutra is the ancient text which has the Pythagorean theorem, so in the most general form not only templates. So this principle and how do we make use of these principles.

See with Pythagorean theorem which is called (FL) in Sanskrit, so this is so fundamental that it plays a major role in trying to even transform a particular geometrical object into another in shape and so on I will see little later then construction of *citis*. So these are the various topics that we will discuss ok.

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### Dertermining the east-west line

- ▶ Determining the exact east-west line at a given location, is a pre-requisite for all constructions, be it a residence, a temple, a sacrificial altar or a fire-place.
- ▶ The procedure for its determination is described thus:  
समे शङ्कुं निखाय 'शङ्कुसम्मितया रज्ज्वा' मण्डलं परिलिख्य यत्र लेखयोः शङ्कुग्रच्छाया निपतति तत्र शङ्कुं निहन्ति, सा प्राची। [Kt. Su. 12]

OA – forenoon shadow  
OB – afternoon shadow

Fixing a pin (or gnomon) on levelled ground and drawing a circle with a cord measured by the gnomon,<sup>7</sup> he fixes pins at points on the line (of the circumference) where the shadow of the tip of the gnomon falls. That gives the east-west line (*prācī*).

<sup>7</sup>This prescription implies  $r > 2OX$ , and has astronomical significance. ❧ 280

So determining the east west line, as you can see in the diagram so here OX refers to the (FL) a small stick which is well shaped and there is a tip at the corner ok. So it is sharpened at the top end which is OX. So then you draw a circle around there and this sutra so basically tells you that you have to create a certain horizontal plane, so that is what is referred to by the word (FL) means a horizontal plane.

So (FL) instead of fixing horizontal plane shanku is the stick ok, so this is the device, so nikaya is having place it well, then he says some (FL) so raju is rope okay the cord so with which you will draw circle and all that you will do all measurements. (FL) means so you have to choose a rope of appropriate dimension which can be measured with the Shank, so which means the rope has to be at least twice the shanku then only you can measure with that.

So it is in that sense and that has a certain astronomical significant which will not discuss now. So if a circle that you draw is too small then while doing a shadow measurements, so the shadow need not even enter the circle here so what you will do it so you create a platform you place the stick and then you see the shadow. So the shadow so the link keeps on decreasing and at a certain point it will enter into the circle.

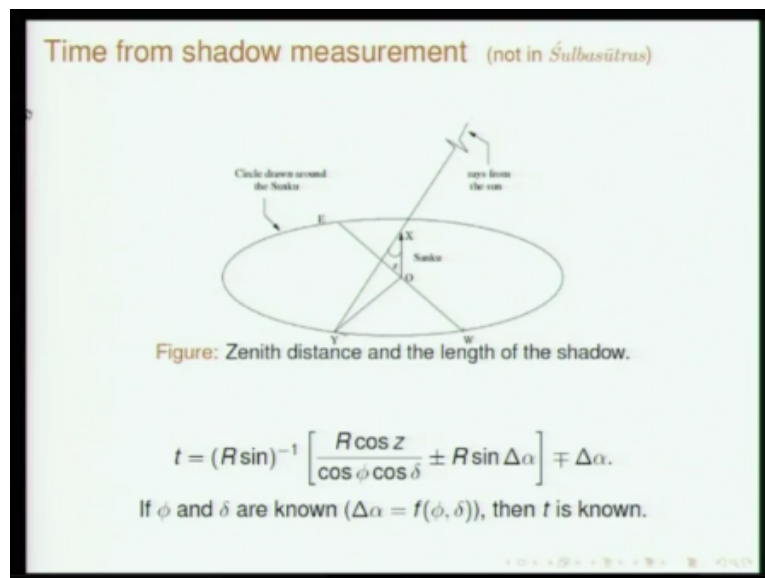
And then in the afternoon it will move out of the circle. So these two points to be primarily mark and they are A and B in this figure, points A and B so so finally define the east west and the east point ok. So once you draw a line A, B so A is marked in the forenoon as the shadow enters into the circle and B is marked as the shadow exist from the circle. So this experiment can be performed anywhere.

So the only thing that you have to ensure is a flat surface. In fact my friend professor Srinivas so once he was mentioning that his friend so consulted somebody regarding vasthu, so then the person came up and said your house is not properly oriented in the east west direction, he was married once, so this fellow said what I do and so on, so then apparently he asked how did that person who came for serving tell you that your house is not properly oriented.

Apparently he said he brought a small device where in the magnetic needle was there and then. So he said that so this is not properly oriented. So this magnetic needle so how big size it is when device can different magnetic your own card perhaps so can deflect the magnetic needle and that will not give you the exact is which direction. So in fact the card of full proof method is this method.

So it (FL) where in there are so many devices and this is not something, so this is so relation, so this is a very simple technique will be able to see and later when we saw that this is exactly oriented. Any way the point I am trying to convey here is the very very experiment and full proof experiment which can be done at any place on any given day. So which will give you the exact east west direction.

**(Refer Slide Time: 26:56)**



Ok, so this is a slide so which has been prepared based on certain work which is given in an astronomical work called (FL) so with myself and profession Sriram was studying at some point of time. So it is a very interesting of formula which is given here, so if you look at this

is the formula it is a lot of complicated (FL) complex formula which involved so lot of trigonometric functions and the inverse of trigonometry function.

In the left hand side what you find is t, so t basically refers to the time and in the right hand side what you find is pi and delta. So Pi refers to the latitude of place and delta the definition of the sun. So what I am trying to say here is so with this simple device Shanku you will be able to precisely tell what the time is provided you are able to see the where the sun is this any distance of the sun which is he said here once it is known will be able to get the time.

So this device is the very very profound device so once the latitude, the latitude itself can we of course found using this shanku at any given place, what is the latitude to the place, simple experiment will tell you so this formula is primarily convey so to give you a certain idea as to how even profound things can be done with this very simple device and this also tells you the kind of sophistication there gone in.

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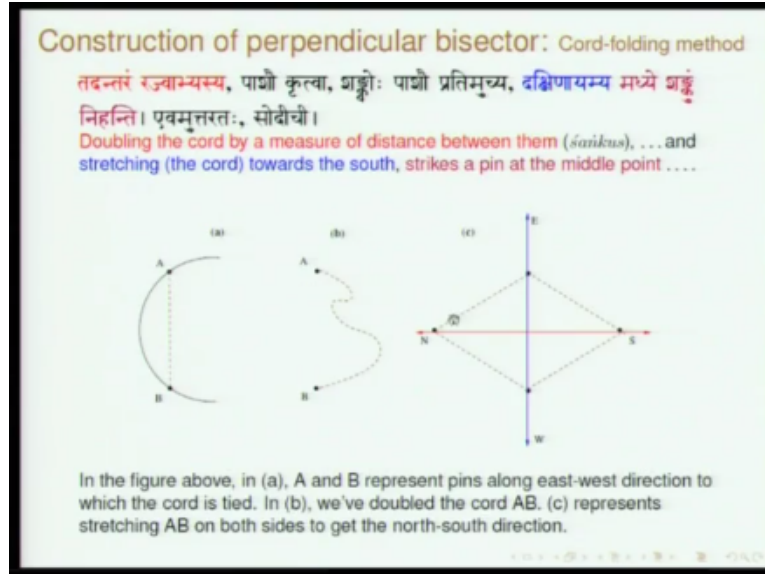
**Why perform experiment to determine the directions?**

- Posing the question – why not simply look at the sunrise or sunset, and be with it to find the east? – the commentator Mahidhara observes:  
...तस्य उदयस्थानानां बहत्वात् प्रतिदिनं भिन्नत्वात् अनियमेन प्राची ज्ञातुं न शक्या। तस्मात् शङ्कुस्थापनेन प्राचीसाधनमुक्तम्। दक्षिणायने चित्रपर्यन्तमर्कोऽभ्युदेति। मेषतुलासङ्क्रान्त्यहे प्राच्यां शङ्क्यामुदेति। ततोऽर्कात् प्राचीज्ञानं दुर्घटम्।  
Since the rising points are many, varying from day to day, the [cardinal] east point cannot be known [from the sunrise point]. Therefore it has been prescribed that the east be determined by fixing a śanku. ... Therefore, simply looking at the sun and determining the east is difficult.
- Having obtained the east-west direction, the next problem is to find out north-south. How to do that?

So as we was mentioning so one of the comment that was very clearly explain as to why you need to conduct this experiment to determine east west. So this Mahidhara says (FL) (()) (28:36) to (()) (28:45) rising point. So if you look at these eastern part of the risen so the rising part of the sun keep on 15 (FL) very very small amount every day and (FL) cannot be known by simply looking at the eastern part of risen.

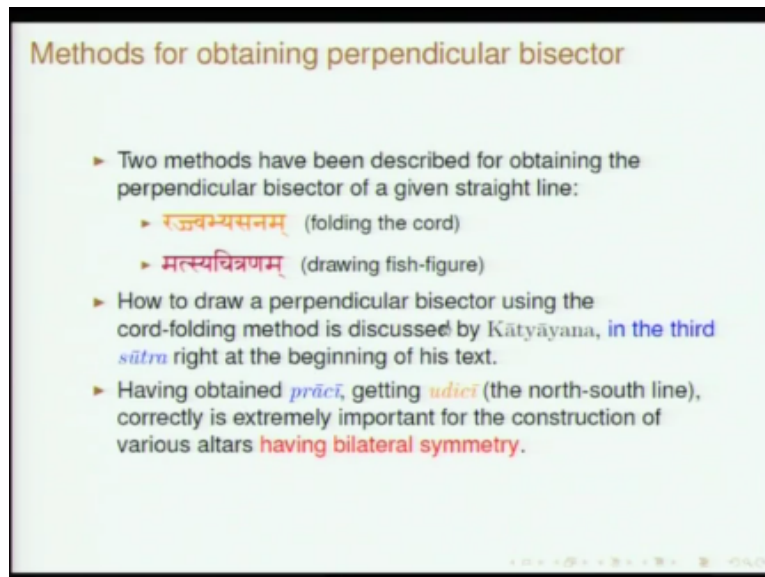
So finally he concludes (FL) (I) (29:09) to (I) (29:12) so it is difficult to simply determine by looking at the sun. So therefore you need to conduct this experiment. So having done this experiment so you have to find out the perpendicular direction that is the next problem.

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So this is actually done in a couple of ways.

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One is (FL) which is folding of the cord, see you have drawn a line now, you have determine the east west, now you have to draw a perpendicular. So all that you have is nail and rope. Nowadays what we do is primarily this latter method (FL) by drawing fish figure. So I will show this one by one. See the first by cord folding is as simple as that suppose you have determine point A and B.



All that this is you have to take care rope which is twice A B and then you have to mark the centre of this it by folding it you will be able to mark the centre of this rope. So once you that all that I need to do is you have to full. So tie at the end and then pull it on both the size. So once I did not give you not others will give you. So this is this will give you this perpendicular to the east west line.

(Refer Slide Time: 30:36)

**Construction of perpendicular bisector: Fish-figure method**

- ▶ In this method, as shown in the figure below, having obtained the east-west direction by the shadow of the *sanku*, we mark two points along the east-west line.
- ▶ With those points as centres, and choosing an appropriate radius, circular arcs are drawn.
- ▶ The line passing through the intersection points of these two arc gives the north-south direction.

So this is the first method. The other method is which is followed by even if you ask any school student all that you will do is will take a campus and then you will draw a semicircle and then put on other side draw the other circle. So this creates a figure, see so a figure like this so this is like a fish and therefore it is called (FL) means fish. So by drawing fish figure we will be able to get the perpendicular there.

(Refer Slide Time: 31:10)

**Bodhayana's method of constructing a square**

Systematic procedure that involves cord & nails, but NO OTHER MEASURING DEVICE

युतुरत्रं चिकीर्षन् यावदधिकीर्षत् तावतीं रङ्गं उभयतः पात्रं कृत्वा मध्ये लक्षणं करोति।  
 लेखांमालिख्य तस्य मध्ये शङ्कुं निहन्यात्। तस्मिन् पात्रौ प्रतिमुख्य लक्षणेन मण्डलं  
 परिलिखेत्। विष्कम्भान्तयोः शङ्कुं निहन्यात्। पूर्वस्मिन् पात्रं प्रतिमुख्य

Desirous of constructing a square, may you take a cord of that length, tie it at both the ends and mark its centre. Draw a line and fix a nail at its centre. Latching the ends...

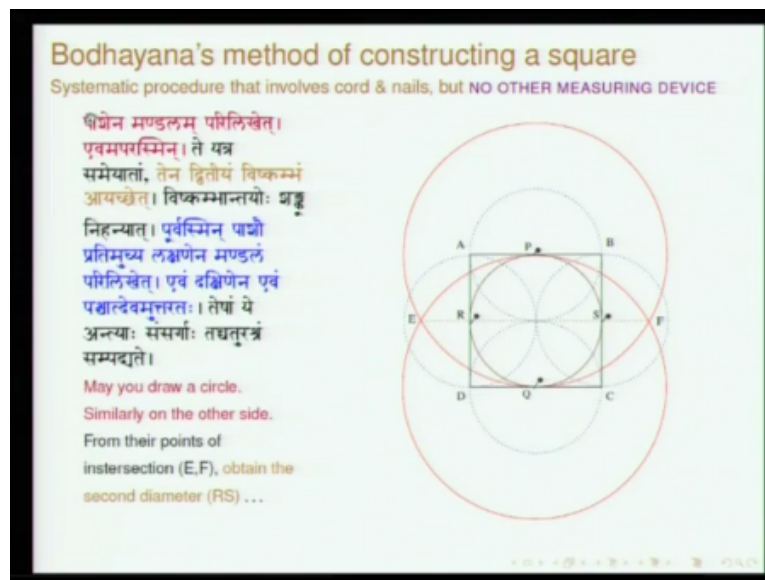
PQ — Cord of desired length.  
 O — Center of the cord (where nail is fixed)

Now I come to a very instructing construction which has been discuss in boudhayana sulbasutra for constructing a square. See (FL) so he says (FL) so whatever be the dimension the side of the square ok, so he says you take a rope of that link (FL) so he says (FL) so in this figure P and Q refers to the tip of the rope and P Q basically gives you the side of the square. So what is to be done is so mark the midpoint of it and you have to fix it.

Fix a nail then draw a circle with a so you just in a next figure so O is the nail and you draw circle. So having done this say all that we employee is a couple of nails and then rope that is all in trying to do draw a square so he says (FL) means you (FL) at the centre fix the nail. (FL) (O) (32:14) to (O) (32:20) to draw a circle. Then (FL) which will encounter refers to the diameter of the circle.

(FL) refers to the diameter P Q is the diameter here, (FL) (O) (32:31) to (O) (32:38) see you place a nail at t, you place a nail at Q then he says (FL) (O) (32:38) to (O) (32:48) so it is very special you say in fact there is one of the word which one can find in the ancient literature. So (FL) is normally sort of removing but pretty is a word when the suggest to go which is tying ok (FL) tying latching.

**(Refer Slide Time: 33:06)**



So then he says you tie it and then the same string is used and you have to draw a circle ok. Similarly you tie it and then draw another circle ok. So (FL) (O) (33:20) to (O) (33:30) wherever these 2 circle interface so which is E and F. So these are 2 points with them you draw a line. Then you have to mark points R and S ok. (FL) (O) (33:44) to (O) (33:52) again see we fix a nail here, we fix a nail here.

Then you say you fix 2 more nails here and then you call got the square how do you do. So then starting from P you have to draw a circle, it was done then you draw a circle, then you draw a circle, then you draw a circle (FL) (()) (34:05) to (()) (34:11) once this 4 circles are drawn, so this dotted circle. All that is done is where are those 2 circles interact intersect you have to mark the point and you will get the square A, B, C, D.

(Refer Slide Time: 34:28)

### The Śulva (Pythagorean?) theorem

► A clear enunciation of the so-called 'Pythagorean' theorem — called *bhujā-koti-karṣa-nyāya* in the later literature — is described in *Bodhāyana Śulvasūtra* (1.12) as follows:

दीर्घचतुरश्रस्य अक्षयारजुः<sup>१</sup> पार्श्वमानी तिर्यङ्मानी च यत्  
पृथग्भूते कुरुतः तदभयं करोति।

The rope corresponding to the **the diagonal** of a rectangle makes whatever is made by the **lateral** and the **vertical** sides individually.

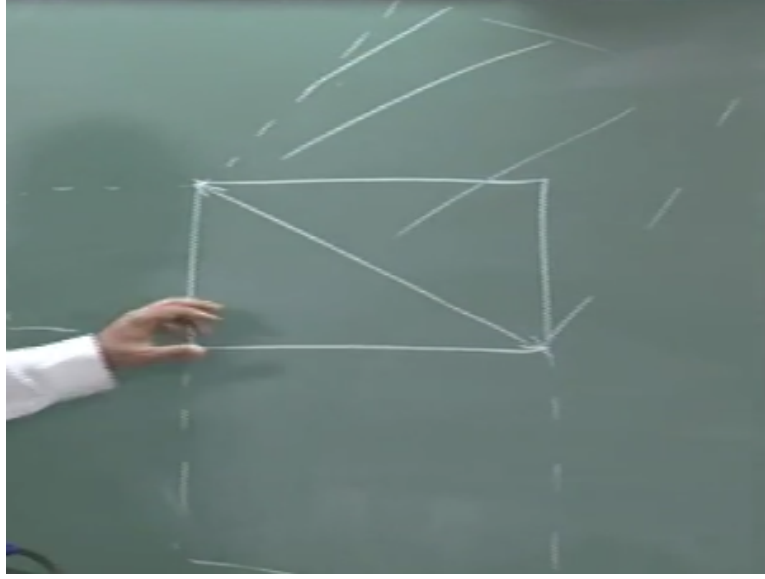
Terms	their meaning
दीर्घचतुरश्रम्	– Rectangle (lit. longish 4-sided figure)
अक्षयौ रजुः	– the diagonal rope
पार्श्वमानी	– the measure of the lateral side
तिर्यङ्मानी	– the measure of the perpendicular side

<sup>१</sup>The word *akṣayā* is archaic and hardly occurs in classical literature:  
अक्षयया व्याघारयति।... तस्मादक्षयया पञ्चवोऽङ्गानि प्रतिष्ठन्ति।

This is a very beautiful construction which has been discussed in bodhayana sulbasutra to draw a square, whatever be the dimension ok as long as the long as biggest model. Then we have this so called Pythagorean theorem discussed here and it is a split statement (FL) (()) (34:45) to (()) (34:54). This is the statement of Pythagorean theorem ok. So (FL) as I said refers to a 4 sided figure ok.

(FL) (()) (35:06) to (()) (35:11) refers to a rectangle (FL) raju is rope or line (FL) is not to be found in the classical literature, it can be found in the vedic literature and in fact there are various instances in which the word (FL) has been used. So this have been defined by the commentator of the sulbakara also in fact (FL) that today I incidentally saw in chandogya Upanishad the word (FL) has been used.

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And shankara in his commentary very beautifully define its (FL) (()) (35:51) to (()) (35:58) if you want to say for instance if you draw a rectangle for this is a corner, corner (FL) means this particular thing is what is referred to us (FL) refers to this (FL) (()) (36:20) to (()) (36:24) so this can be taken as (FL) and this can be taken as (FL). So all that is said here is if you think of drawing a square here and then drawing another square here if you draw a square and this will give you the area of these two.

So (FL) (()) (36:45) to (()) (36:53) this is something which is very well known (FL). So whatever be the area that is created by the 2 sides, so the hypotenuse also gives that. So this is basically the Pythagorean theorem.

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**Kātyāyana version of Śulva theorem (with comm.)**

- ▶ The Kātyāyana version of the theorem seems to be a redacted form of what appears in *Bodhāyana Śulvasūtra*.  
दीर्घचतुरश्रस्य अक्षयोरङ्गुः तिर्यङ्गानो पार्श्वमानौ च यत् पृथग्भूते कुरुतः तदभयं करोति इति क्षेत्रज्ञानम्। [KSS 2.7]
- ▶ But for swapping two words, there is only one difference; The phrase '*iti kṣetrajñānam*' has been added ⇒ that this is the most fundamental theorem in geometry to be known whose knowledge cannot be dispensed with.
- ▶ Commenting on this Mahidhara observes:  
दीर्घचतुरश्रस्य तिर्यङ्गानोपार्श्वमान्यौ रङ्गु पृथग्भूते सत्यौ यत्क्षेत्रं = यत्फलकं क्षेत्रं समचतुरश्रद्वयं कुरुतः, तदभयमपि मिलितं दीर्घचतुरश्रस्य अक्षयोरङ्गुः = कोणसूत्रभूतं रङ्गुः करोतीति इति क्षेत्रज्ञानम् = क्षेत्रज्ञानप्रकारो ज्ञातव्यः।

(FL) version as I was mentioning earlier the katyayana version is primarily the bodhayana sutra as it is with the only thing added (FL) can be understood in a 2 different ways. So I would say that this is the most fundamental theorem which is to be known in this plane or geometry, it cannot be dispense with and impact the application or many many many food. So even in very complex calculations like the period of eclipse etc.

So all that we use is primarily this is not a right angle triangle and so let see in fact the comment (FL) (( )) (37:48) to (( )) (37:53) it is a very very clear explanation (FL) area so it consider one side whatever be the area created by that side, whatever be the area created so that particular thing creates the area. So (FL) so this what is the meaning of the sutra ok.

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### Mānava version of the Śulva theorem

- ▶ The presentation of the theorem in *Mānava-śulvasūtra* differs from *Bodhayana Śulvasūtra* both in form and in style.
- ▶ Here it is given in the form of a verse as follows:
 

आयामं आयामगुणं विस्तारं विस्तरेण तु ।  
समस्य वर्गमूलं यत् तत् कर्णं तद्विदो विदुः ॥

Terms	their meaning
आयामं आयामगुणं	– the length multiplied by itself
विस्तारं विस्तरेण तु	– and indeed the breadth by itself
समस्य वर्गमूलं	– the square root of the sum
तत् कर्णम्	– that is hypotenuse
तद्विदो विदुः	– those versed in the discipline say so
- ▶ Using modern notation the result may be expressed as:
 

$$\sqrt{ayama^2 + vistara^2} = karna.$$

This manava sulbasutra also states this theorem (FL) so it says (FL) (( )) (38:20) to (( )) (38:30). This is a very very clear statement. So ayama and vistara they refer to the length and breadth of this rectangle. So (FL) we multiply ayama by ayama vistara by vistara. So which means so if you consider one side of A find A square and vistara find B square (FL) means adding them together.

**(Refer Slide Time: 39:13)**

Some 'Pythagorean' triplets listed in *Śulbasūtras*

► In the very next *sūtra* following the statement of the theorem, Bṛhadhāyana illustrates it with a few examples:

तासां त्रिकचतुष्कयोः, द्वादशिकपञ्चिकयोः, पञ्चदशिकाष्टिकयोः,  
सप्तिकचतुर्विंशिकयोः, द्वादशिकपटत्रिंशिकयोः,  
पञ्चदशिकपटत्रिंशिकयोः इत्येतासु उपलब्धिः । [BSS 1.13]

What is interesting to note is the use of the phrase  
इत्येतासु उपलब्धिः ।  
[the general rule stated above] is quite evident in these pairs.  
Is there a rationale behind the choice of these examples?

► A few triplets listed in the *Āśvalāyana-śulbasūtra* includes:  
(15, 20, 25) (16, 12, 20)

See vargamulam is square root of A square B square is C square that kind of a square root and vargamulam is square root, so that gives you so this is what essentially lead is ayama square+vistara square, square root is karna. So soon after giving the theorem (FL) (( )) (39:11) to (( )) (39:23) gives several triplets. So here is the sutra (FL) (( )) (39:28) to (( )) (39:32) so one triplets is 3,4,5 ok then (FL) is 12 12, 12, 5, then (FL) 15, 8 so 17 square.

Then (FL) 7 and 24, so it is 25 square, (FL) 12, 35 is 37, (FL) so that it says, (FL) he says we can go on constructing such picture, so uses a few triplet as examples of the theorem which he stated in the most general form in the previous sutra (FL) sutra gives couple of other triplets. So I am just listing them and from all the question as to why he chose only this triplet, so many triplets which can be constructed.

So one can think of the (FL) that this sulbasutras, sulbakara, thought of a certain general formula and plugging in certain values will be able to get those triplets. So that is what I show you in the next couple of minutes.

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**Rationale behind the choice of examples**  
 Conjecture put forth by Datta (pp. 133-136)

- ▶ One of the Kātyāyana-sūtras presents the relation
 
$$na^2 = \left(\frac{n+1}{2}\right)^2 a^2 - \left(\frac{n-1}{2}\right)^2 a^2$$
- ▶ Substituting  $n = m^2$ , and  $a = 1$ , we at once get
 
$$m^2 + \left(\frac{m^2-1}{2}\right)^2 = \left(\frac{m^2+1}{2}\right)^2 \quad (1)$$
- ▶ Here, putting  $m = 3, 5, 7$  immediately  $\rightsquigarrow (3,4,5), (5,12,13), (7,24,25)$ .
- ▶ Rewriting the above equation in the form
 
$$(2m)^2 + (m^2-1)^2 = (m^2+1)^2, \quad (2)$$
 and substituting  $m = 2, 4, 6 \rightsquigarrow (3,4,5), (5,12,13), (7,24,25)$ .
- ▶ How about the other example of Baudhāyana (15,36,39)?

For instance if we look at one of the relations which has been given in katyayana sulbasutra, so this is a algebraic relation na square so can we written in this particular form. So fine so katyayana sulbasutra does not presented in this particular form, but in a certain context, so in finding the value of root and you give a to find a certain description which can be written in this particular form ok.

Datta (FL) (()) (41:20) to (()) (23:26) stating the study of Indian mathematics contribution of India, so it is a very great scholar you have written a text called a since of sulba so wherein I mean he has Pi to analyse as to what would be the rational in trying to present these examples (FL) theorem was stated. So this if we make a substitution n=m square and we plug in a=1. So then this transforms into this particular form.

So if you plug in the value m=3, 5, 7, so immediately you get these triplets, ok. So 3, 4, 5, 5, 12, 13, 7, 4, and 25. So if you rewrite this equation in this particular form, and then you substitute the value 2, 4, 6, it immediately gives you 3 more triplets. So in this you can see that so this 2 triplets along the same, but other are different ok. The other example which has been given by baudhayana for instance 15, 26, 39.

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**Principle behind generating right-rational triangles**  
 Described by Āpastamba in the context *Saumiki-vedi*

त्रिकचतुष्कयोः पत्रिका अक्षयारजुः। तामिः त्रिरभ्यस्तामिः अंसौ।  
 चतुरभ्यस्तामिः श्रेणी। [ASS 5.3]  
 द्वादशिकपत्रिकयोः त्रयोदशिका अक्षयारजुः। तामिः अंसौ। द्विरभ्यस्तामिः  
 श्रेणी। [ASS 5.3]

$$3^2 + 4^2 = 5^2$$

$$(3 + 3.3)^2 + (4 + 3.4)^2 = (5 + 3.5)^2 \quad (A)$$

$$12^2 + 16^2 = 20^2$$

$$(3 + 4.3)^2 + (4 + 4.4)^2 = (5 + 4.5)^2 \quad (B)$$

$$15^2 + 20^2 = 25^2$$

$$5^2 + 12^2 = 13^2$$

$$(5 + 2.5)^2 + (12 + 2.12)^2 = (13 + 2.13)^2 \quad (C)$$

$$15^2 + 36^2 = 39^2$$

It seems Āpastamba has invoked the principle that if  $(a, b, c)$  satisfies the relation  $a^2 + b^2 = c^2$ , then  $(ma, mb, mc)$  also satisfies the same relation—where  $m$  is an arbitrary rational number.

How did baudhayana get, so here the principle behind generating the rational triangle, so this is another way of looking at it, so here there is a *Verdi* which is called *saumiki-vedi* and in the *saumiki-vedi* what should be the length, what should be the it is not actually a rectangle, but it is a quietly different kind of thing. So wherein it is a sort of trapezium kind of a thing. So wherein so the length of the size and so on are specified.

In that context so we kind these sutras (FL) (()) (43:16) to (()) (43:46) in fact so this can be sort of written like this. (FL) multiplication ok. So you take 3 and then do this process, so then you will get 12, 16 and 20. So if you take 4 and then do this process, then you will get 15, 20 and 25. Then he says (FL) (()) (44:07) to (()) (44:21) 15, 36 and 39. So this is the kind of triplet.

As I was mentioning so this has been given by baudhayana also. Any way the principle that seem to have employed here in arriving at this triplet one of this speculation is the that they had used general formula and then substituted different that you see 2, 3, 4, 5, 6. SO you get all these triplets that are found and given by baudhayana and later by other (FL) because you get various in triplets.

**(Refer Slide Time: 45:16)**



### Constructing a square that is sum of unequal squares

An application of the *Sulva*-theorem

**नानाचतुरश्रे समस्यन् कनौयसः करण्य वर्णयसो वृध्रमृद्विसेत् । वृध्रस्य  
 अक्षयारजुः समस्यतोः पार्श्वमानो भवति ।** (BSS 1.50)

Desirous of combining different squares, may you mark the rectangular  
 portion of the larger [square] with a side (*karanya*) of the smaller one  
 (*kaniyasah*). The diagonal of this rectangle (*vrdhra*) is the side of the  
 sum of the two [squares].

- ▶ The term *vrdhra* in the above *sūtra* refers to the rectangle ABEF.
- ▶ Asking us to mark this rectangle, all that the text says is the cord AE *akṣayārajūḥ* gives the side of the sum of the squares.
- ▶ In other words,

$$\begin{aligned}
 AE^2 &= ABCD + CGHI \\
 &= AB^2 + CG^2 \\
 &= AB^2 + BE^2.
 \end{aligned}$$

And all these measures can be taken to the measure of the various vedis so which they have used for various sacrifices. So there is an interesting application of this sulva theorem. So I will discuss one application now. So (FL) putting together which means you have 2 square of different size and you want to create a square whose area will be the sum of these squares ok. (FL) so it is geometrical construction.

All that you need to do is closely follow this figure, so A, B, C, D, is one square and C, G, H, I is another square. So these are the 2 squares and now you want to construct a square whose area is sum of these 2 squares (FL) (()) (46:06) to (()) (46:19) by the side of the smaller square ok. (FL) the one which is larger in dimension, so (FL) all that he says is you have to take the dimension C, G and then mark at in the larger square which is same as D, E ok.

(FL) here the term (FL) refers to a rectangle ok so it is not (FL) it is (FL), so (FL) so the diagonal for this (FL) which is A, E, E, (FL) of this square which you want to construct as a sum of the area of these 2 square (FL) will give the side, so basically what you will get is A, A, H, E, so this is the square ok. So with this so I will conclude my lecture today and will have the next part of Vedas and sulbasutra in the next lecture thank you.

Yes sir, yeah you see you told how to find out these correct correct yeah similarly to find out the time yeah, there is a smallest yoga yeah could you tell me that we also only calculation from the shadow using the shadow, yeah to find out the time, no no see what I said was this formula is encoded in the form of a word that we have decoded in the form of modern notation and that what it translates to is what I said.

This (FL) is a very very effective device to determine so many things. So in fact there is a factor called (FL) there almost always feels fine and the device which forms the basis of this chapter to conduct various experiments and determine various quantities of astronomical where is the Shanku. They will place it and then so based on the shadow will be able to determine the lattice of it.

In fact (FL) means 3 questions (FL) (( )) (48:43) to (( )) (48:50) you means you want to know where you are, so where you are essentially is given by latitude and longitude and in fact latitude is most important thing, latitude basically tells you how the time will fluctuate, see how much duration of daylight will be available for you to do the gate way transaction. So this slogas of course we can go back and then tell you.

But this is not a sloga where it is sloga is basically gives the description of the terms in the formula. So what is the multiply by what, what is to be divided by 1. My question is by the same argument that the shadow of the no one should fluctuate on day to day basis, see I will tell you so namely it will sounded to you that way, but the point A this variation in the raising point is due to change in declination of the sun.

The ecliptic and equator are inclined, so above 23 degrees, so from the east point sun will move to 23 degree towards the north return back and then south, so if you know the day on which will be going to be exactly saying the east point and then it may so this will happen on the (FL) LD, but suppose you have to do the construction on any given day at any given place. So this will be the most reliable method.

The point is on a given day so the path traced by the sun will be almost parallel to the equator, so leaving out the very minor change in the declination, so within a few hours in which you take the shadow measurement, so the path will be so this is this path is apparent path which is due to the rotation of the earth and therefore the path that is placed by the sun in the background.

So will be more or less parallel to the equator, and therefore when say does not matter how far it is moved from the equinoctial point where will be the rise, so there will be a corresponding shift in this side also. So and therefore the points that you will get by this **by**

**this** will be. So the line that you get will be parallel to the east west line. So by connecting these two points.

So but looking at that so if you say this is a rising point that rising point will be shifted from the equinoctial point it is depending upon the declination, the declination will be 0 to the exact these points, so that you know.