Mathematics in India: From Vedic Period to Modern Times
Prof. M.S. Sriram
University of Madras, Chennai

Lecture-17
Mahaviras Ganitasarasangraha 3

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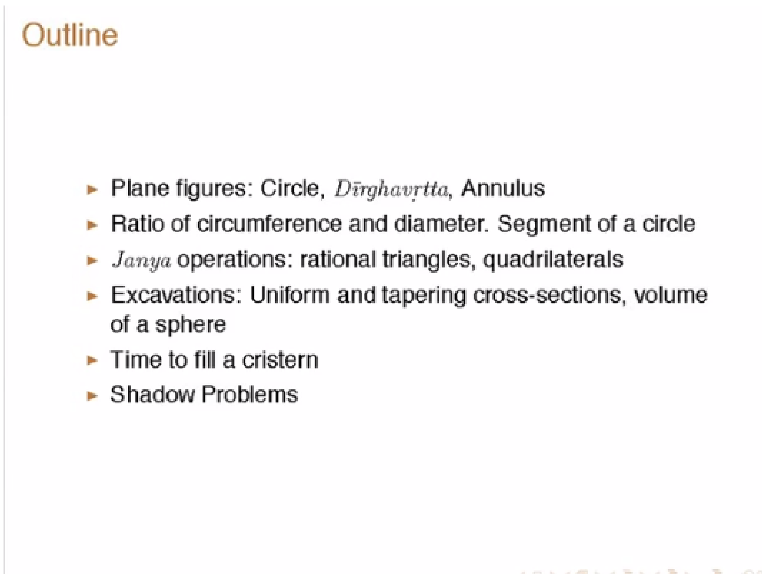

Okay. So, the third of this 3 lectures on Mahavira's (FL) will be on the areas and volumes, so this is the outline will first discuss the plane figures, regular figures, a circle (FL) annulus then the results of for this circumference and diameter for circle then segment of a circle then we talk about the so, called Janya operations which had introduce earlier rational triangles, quadrilaterals. Then regular volumes uniform volumes and tapering cross-sections.

Then volume of a sphere then time to fill a cistern which we discussed for a first time by mahavira and shadow problems okay.

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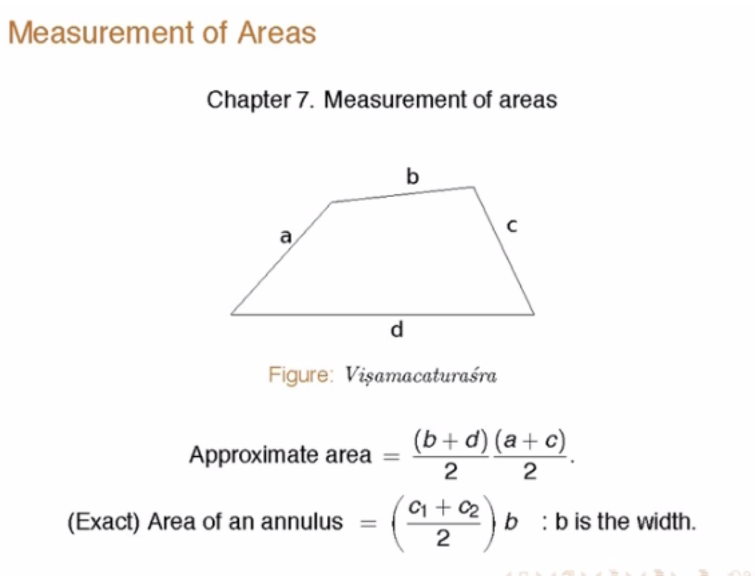

So, for instance if you have a (FL) any quadrilateral which is the though relation among the various sides abcd the (FL) approximated area (FL) Brahmasphutasiddhanta it is given as b+d/2 into a+c/2 approximate area. And but for an annulus he gives annulus means you know this is the region between the two circles okay between to the circumferences of the circle. So, c1 and c2 are circumference so, then the area is given as c1+c2/2 into b.

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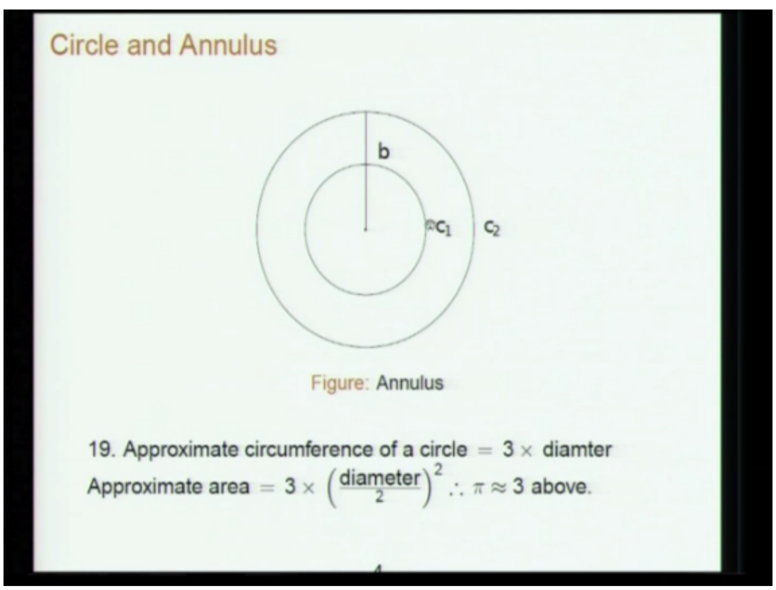

One can check that it is exact this exactly correct then the approximate circumference even according to mahavira it is approximate it is given as 3*diameter. So, essentially is taking pie is equal to 3 where as approximate area is 3*diameter/2 whole square okay. So, essentially pie is taken you know very crude value of pie z is 3 is taken.

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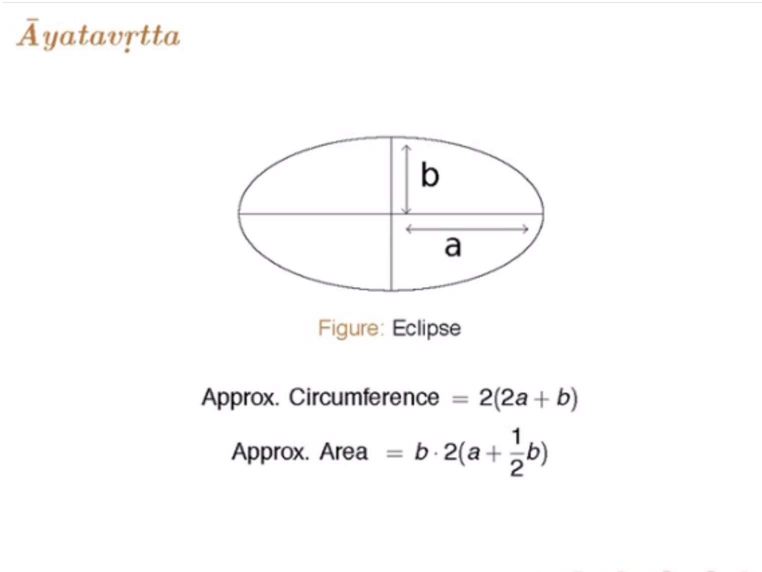

So, then for a first time is talking with (FL) is essentially an Eclipse we do not know whether he meant and a but it is some kind of a (FL) you know some one side is elongated with respect to (FL). The approximate circumference is given as 2 into is 2a+b this is shorter radii this longer radius 2 into 2a+b and approximate area is given as b into 2 into a+1/2b. So, this is the says.

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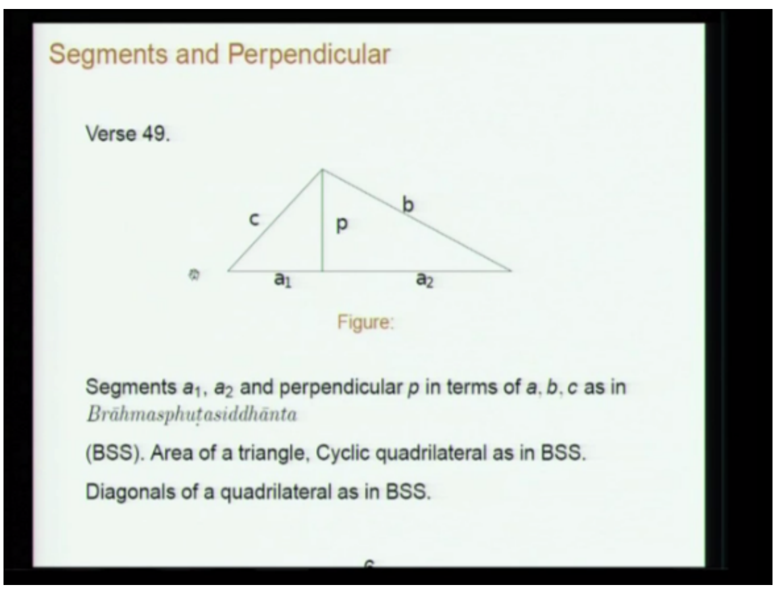

So, this and for the okay for a segments and a perpendicular he gives the usual results I will repeat it has been done several times right. So, if a triangle is there then is the base the segments are a1 and a2. And is the perpendicular so, the results for given in Brahmapurasiddhanta even earlier also ever given. So, same results mahavira has stating then area of a triangle and cyclic quadrilateral same as in Brahmasphutasiddhnata. And diagonals of a quadrilateral also a considers there is also same as in (FL).

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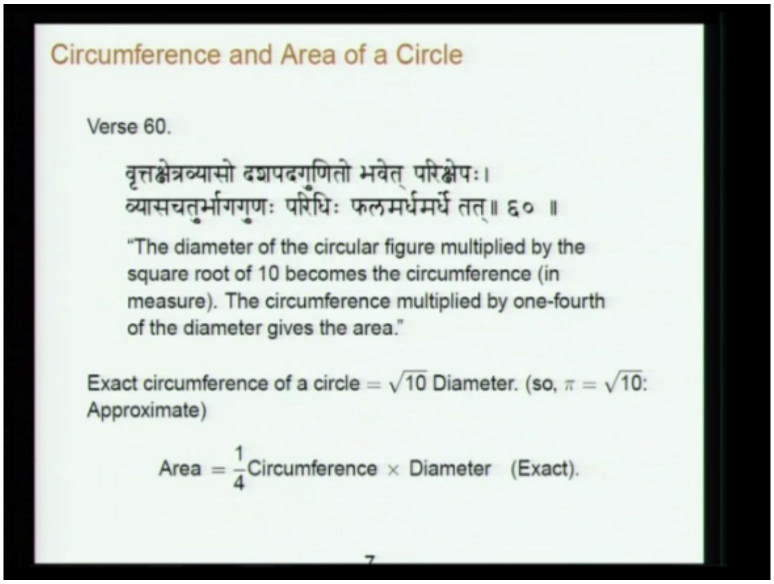

Now he goes back to the circumference and area of a circle and again so, in verse 60 says (FL) the diameter of the circular figure multiplied by the square root of 10 becomes the circumference(in measure). The circumference multiplied by one-fourth of the diameter give the area. So, he says that exact circumference of a circle is root 10 into diameter so. He is taking essentially pie is equal to root 10.

So, I told you earlier also discuss some kind of a Jyana value you will say because various (FL) x will give this number is an approximate and area is one four circumference into diameter. These of course exact whatever be the relation between circumference and diameter so, the area itself is exact. So, this is understandable probably in fact this is explain in commentaries to leelavati. So, what is being done is suppose you have a circle.

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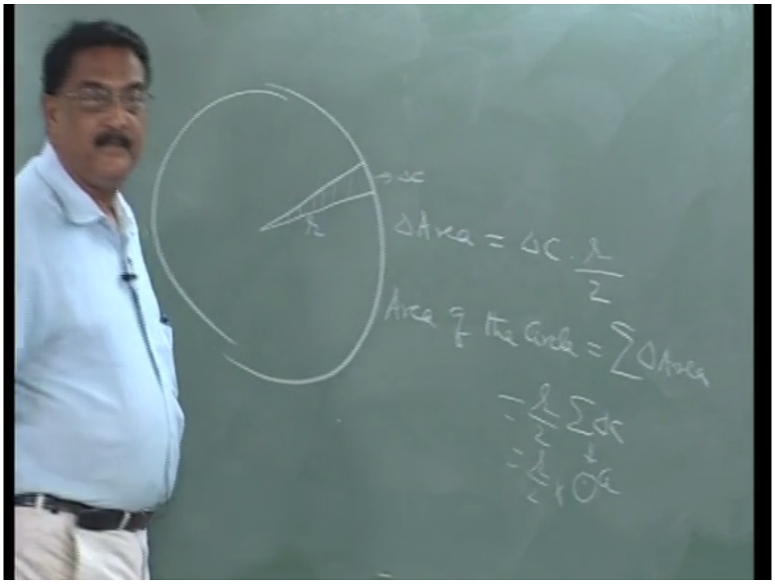

So, get divided into segments like this small segments so, this small segment this architecture can be approximated by the chord itself that it can be considered as state. So, then the area of this bit this segment is you know suppose this is some delta c. So, area will be delta c into the height, height is a radius itself into r/2. So, these the delta area that is the area of this segment and if you area of circle will be is equal to sum over all the segments.

So, when you sum over all the segments r/2 remain will constant the radius is a same that will come out. And sigma delta c that is the essentially circumference right r/2 into circumference is in it. So, because your going over the whole circle when you had measuring the whole area. So, with this preside the same thing instead of radius is using diameter. So, ¼ circumference into diameter.

So, this is independent of the relation between the circumference and diameter. So, probably that is how they got it and certainly that is how the y got it later you know this result by Baskara and others okay.

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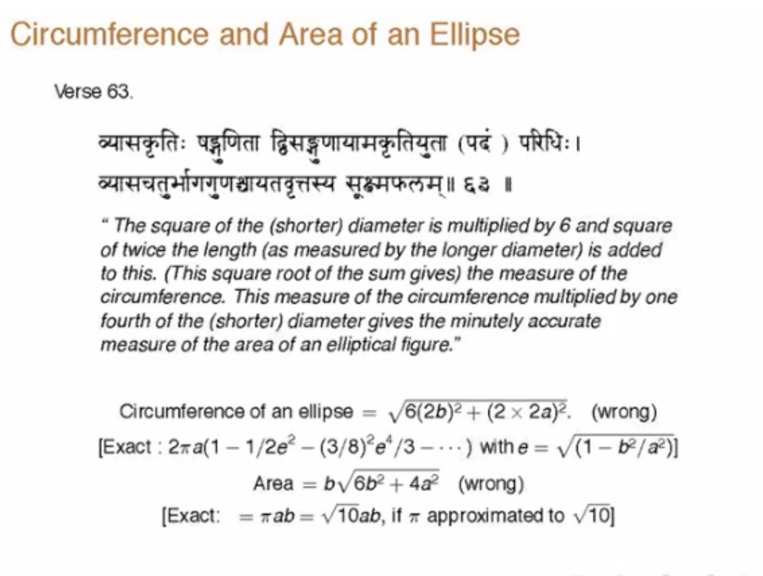

Then the circumference and area of an Eclipse the sphere of the (FL) the square of the (shorter) diameter is multiplied by 6 and square of twice the length (as measure by the longer diameter) (FL) is added to this. This square root of the sum gives the measure of the circumference and this measure of the circumference multiplied by one fourth of the (shorter) diameter gives the minutely accurate measure of the area of an elliptical figure.

So, what is saying it circumference of a so, we our notation is that you know so, this b is the or 2b is the shorter diameter, and 2a is the longer diameter. By in that case you have circumference is result is this and the area is b into square root of 6 into b square+4a square. Of course when you see that when you take b is equal to a he will get a value for the circle okay with pie is equal to root 10.

But for an ellipse this is not correct okay. But we do not know he might not meant ellipse so, it is not very clear one has to look at the commentaries little more in detail exact values of the circumference and the area are also showed in the slide for comparison.

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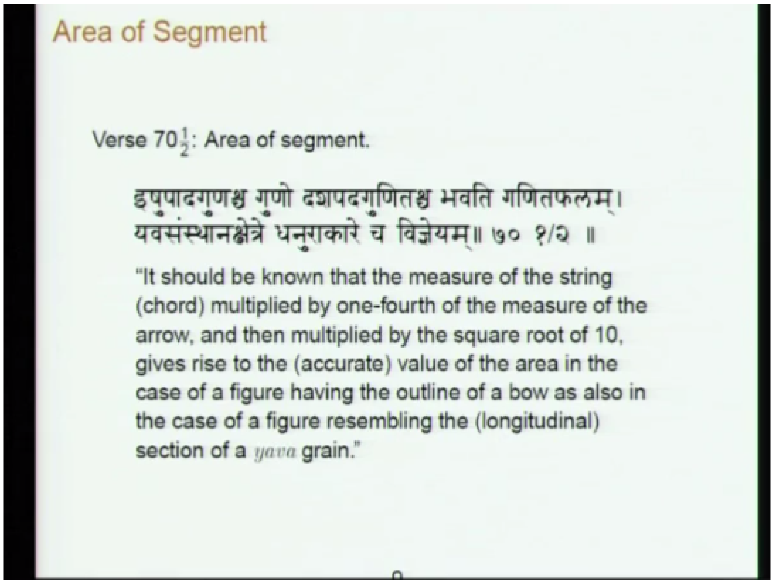

So, then next we will talk about area of a segment so, we will come to that.

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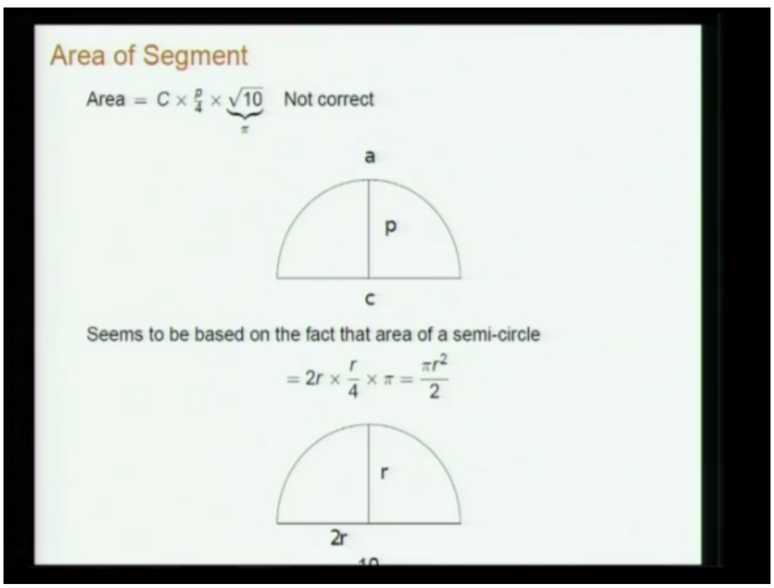

So, segment is you know that is if a circle is there so, this is the segment this segment that is what he is taking and this is the see what i stalking and this is the arrow okay. So, what is the yeah p oh so, these are segment is referring to.

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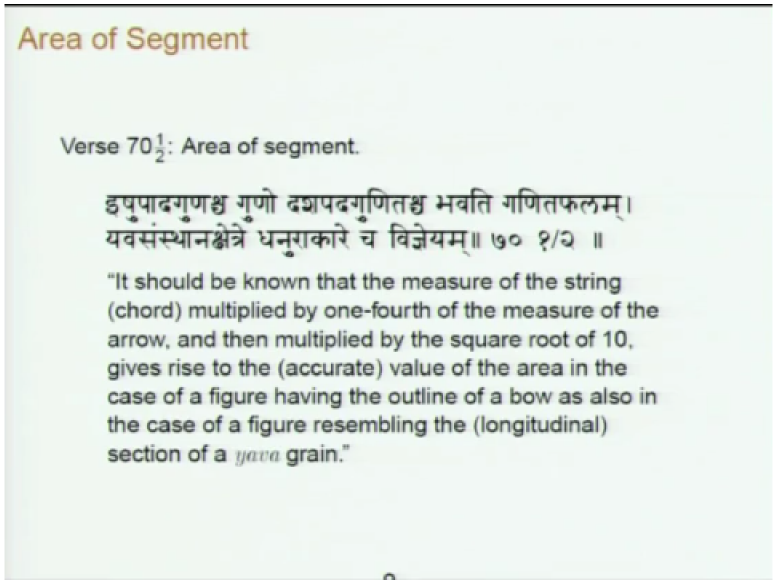

Then in that case he says (FL) so, (FL) is the essentially figure of a bow which should be known at the measure of the string chord you see multiplied by 1/4th of the measure of the arrow and then multiplied by the square root of 10 gives rise to the accurate value of the area in the case of a figure having the outline of a bow as also in the case of a figure resembling the longitudinal section of a (FL) grain I do not know that is what he says.

So, essentially what he saying is that if you have a segment like this the chord is C and the arrow is p then the area is C into p/4 into root10, so root 10 is basically what he says the value for pie is taking, so even if you replaced by pie it is not correct it seems to be based some scholars feel that the seems to be based on the fact that area of a semi-circle, see this kind of a thing will work, so here the the base is a 2r the diameter and this height, the arrow is r.

So, 2r into r/4 into pie, so pie r square/2 that is correct right that is the semi-circle area, so it seems to be base on that, so but it is a good attempt.

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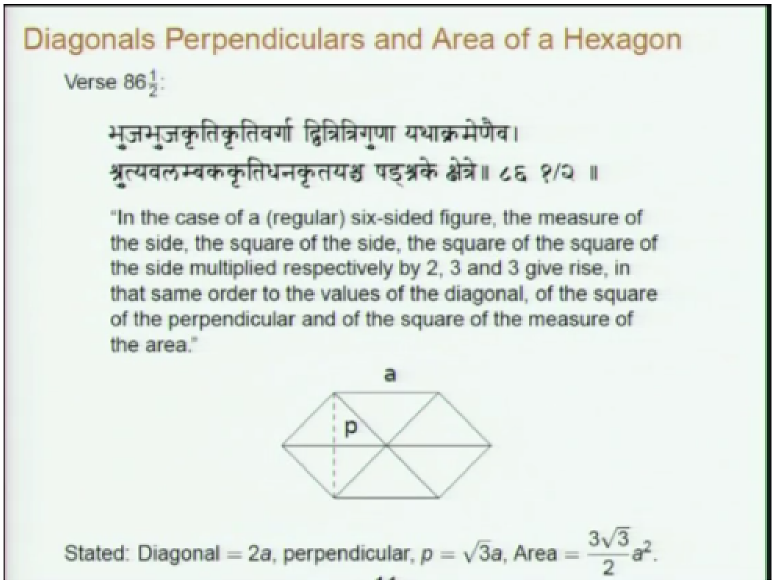

Then the diagonals perpendiculars and area of a hexagon, so he gives it in the next verse (FL). So, in the case of a regular six-sided figure the measure of the side, the square of the side, the square of the square of the side multiplied respectively by 2, 3 and 3 give rise in the same order to the values of the diagonal of the square of the perpendicular and the square of the measure of the area, so that is what he is saying.

So, essentially what he is saying is this is a hexagonal figure right, so a is the side and p is the perpendicular, so he is saying that the diagonal is 2a which is obvious the p is root 3 into a, so 1 can very easily check it and the area is 3 root 3/2into a square okay. I mean of course he is giving the square of the measure of the area, so square of this 9 into ¾ into a square see that what he is giving. But if you take the square root you get this, so this is the area of a hexagon.

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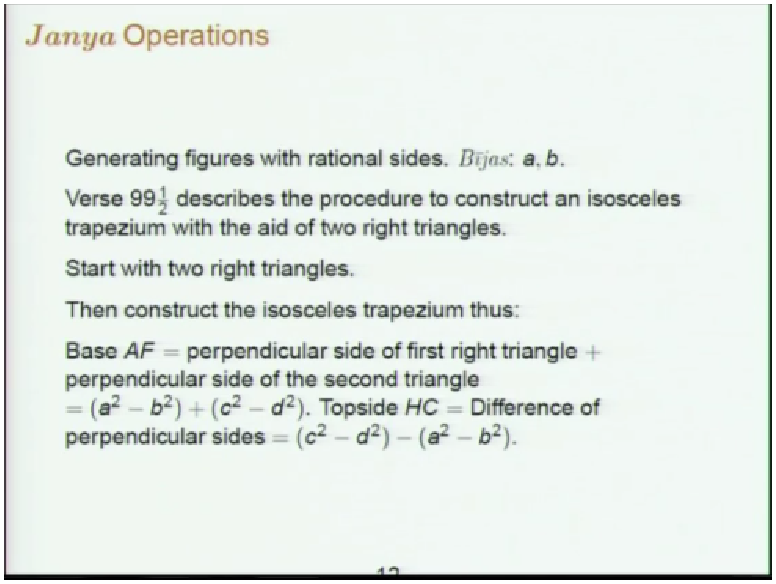

So, then he talks of the Janya operations okay generating triangles with rational sides, triangles, quadrilaterals and various figures which rational sides you see. So, as professor Srinivas pointed out of the generation of the verse, grammar I think some made same idea which will working you know generation of you know valid rational figures kind of a thing. So, see take the (FL) means the starting points you know the seeds it start with a, b.

That itself may not be the value of the side or of the operate of the triangle, so the verse one of the verses describes the procedure to construct an isosceles trapezium with the aid of 2 right triangles. So, start with 2 right triangles then the construct the isosceles trapezium in the following manner, so this is the perpendicular side of the first triangle+ perpendicular side of the second triangle.

So, **so** this is taken to be the perpendicular first angle is a square-b square in the perpendicular side and the second triangle is c square –d square and top side is the difference of the perpendicular sides.

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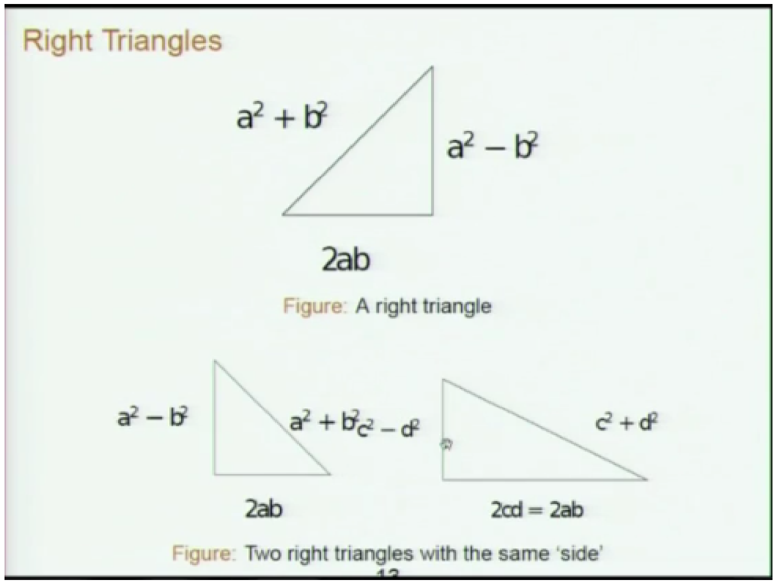

So, he is starting actually with this right triangles a square+b square, a square-b square is the diagonal 2a sorry upright, 2ab is the side and a square+ b square is the diagonal clearly I have already mention that this square equal to this square+this square right. So, similarly another triangle exist only thing is you take two cds is equal to 2ab c and d may not be equal to a and b. But this relation is there if that is true.

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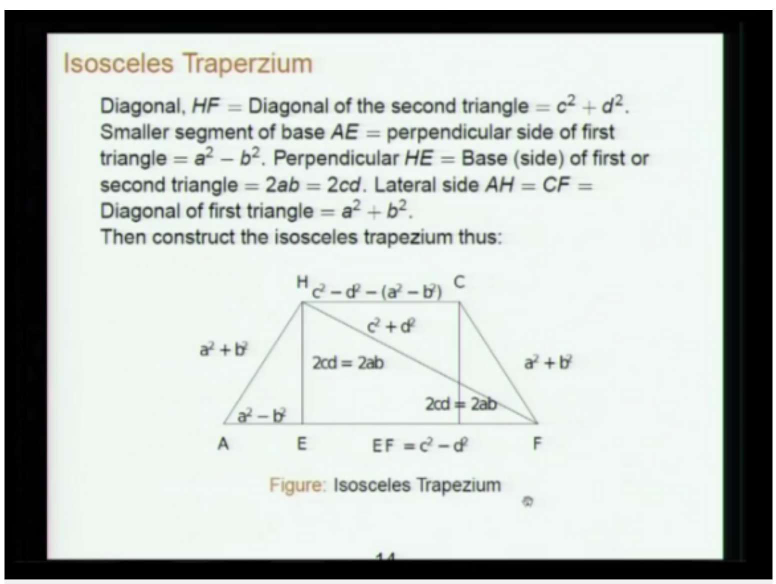

Then he will give a rational trapezium like this the perpendicular of a rational that what it is you know important. So, you take the diagonal to be second triangle diagonal c square +d square for smaller segment base ae is perpendicular side of first triangle so, a squared-b squared perpendicular h is base side of first or second triangle. So, these two cd is equal to 2ab and a lateral side ah other as sometimes they called.

So, is equal to diagonal of first triangle so, it a square+b square so, this is the finally you see Isoceles triangle what from this everything is you know rational I mean even this segments and all that right.

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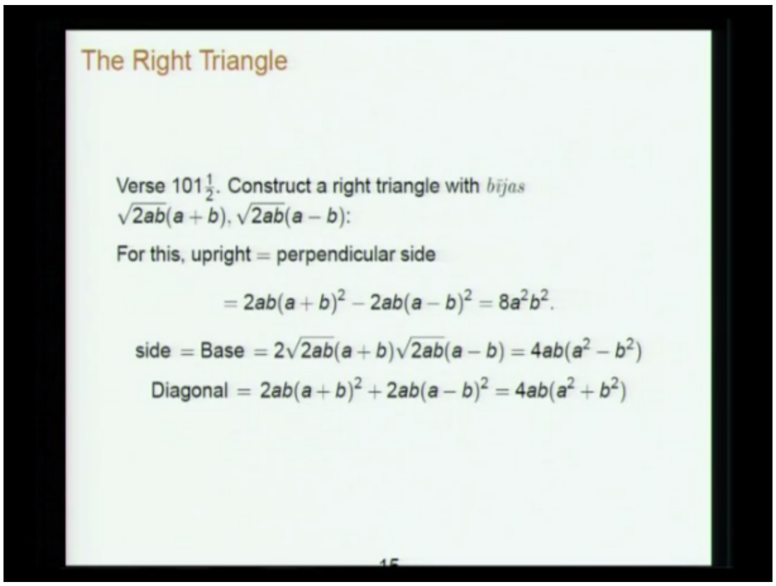

So, that is important so, next you construct a right several other right triangles are constructed. Suppose you have a right triangle with (FL) this square root of 2ab into a+b into square root of 2ab into a-b so, for this upright you take it to be this. So, this square-this square and base you take to be this into this. So, 4ab into a square-b square diagonal of course you take it to be square root of the sum of the squares. Squares of this so, 4ab into a square+b square.

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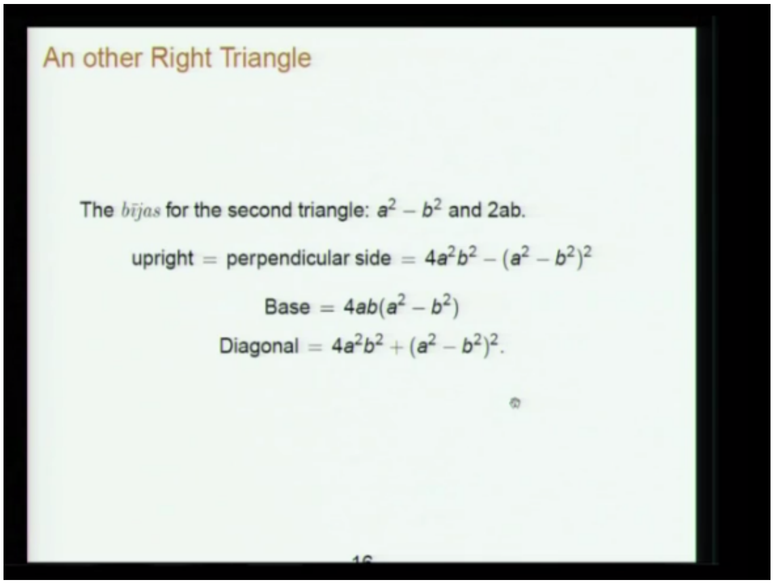

if you take this and a second triangle the bijas are taken to be this a square-b is to be that is the perpendicular is this base this and diagonal is this.

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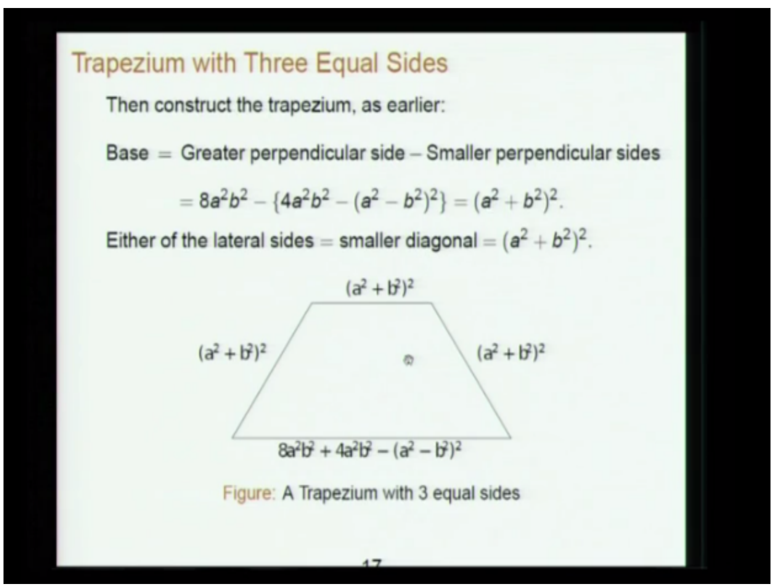

Then in that case you can get a nice trapezium with three equal sides okay base the summit and a are equal and base is. So, base is the academic great greater perpendicular side-smaller perpendicular sides so, it is given by this. And either of the lateral sides is a smaller diagonal this is a square+b square whole square so, like that you know. So, that is how it is constructing.

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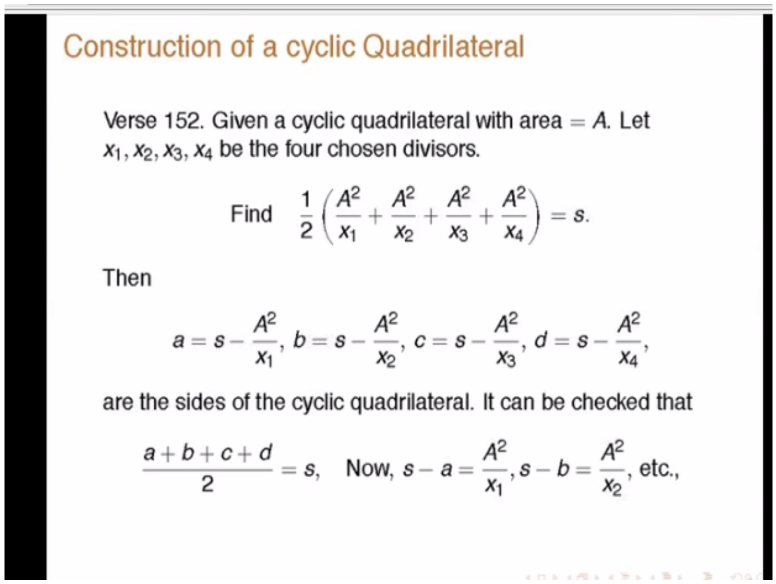

So, this will so, these are the various interesting things you know that will be certainly one of the aim of the book is not just scholarly work you know it is also to intersect students and mathematics. So, all these will give some good idea of you know how to manipulate with various things. So, then a construction of a cyclic quadrilateral it talks about suppose you construct a cyclic quadrilateral lateral with area A.

So, let x1, x2, x3, x4 be the four chosen some divisors okay so, arbitrarily you take of course everything is rational find this**.** So, then take the sides to be this a is equal to s-a square/x1 s-a square/x2 etc etc... So, then they are the sides of the cyclic quadrilateral and one can check that this square +this whole semi perimeter yes will get back that. And s-a is a square/x1 and so on and so for. But there is one tricky thing.

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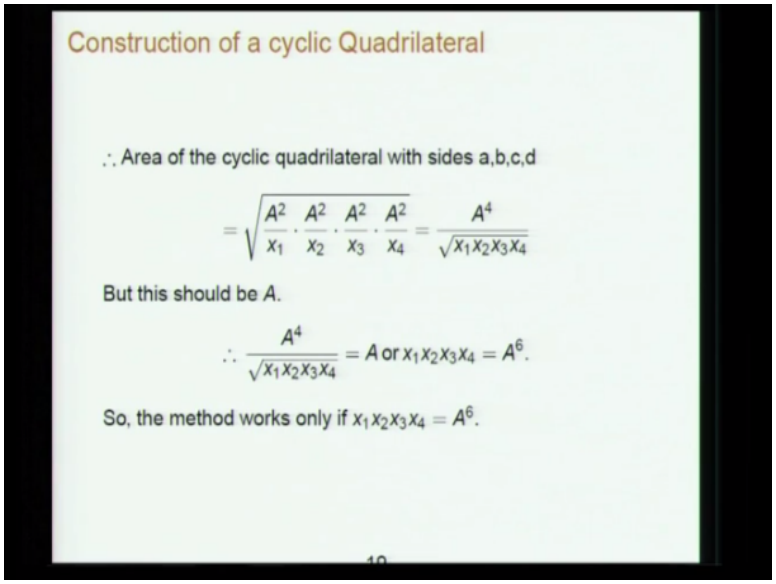

So, area of the cyclic quadril with sides abcd you know the square root of s-a into s-b into s-c. So, plugging that into you will get this is the area of this cyclic quadrilateral with these sides right with these sides. Because s-a will be a square/x1 so, area is this but originally you started with these area so, this must be equal to A or x1, x2, x3 is equal to it is a four of it. So, this method will work only if x1, x2, x3, x4 is equal to A to the power of 6.

So, that is not mention the problem he wanted the student to figure it out or it is a fit of no no so, any way this is certainly is not valid for arbitrary things x1, x2, x3 this has to satisfied that okay.

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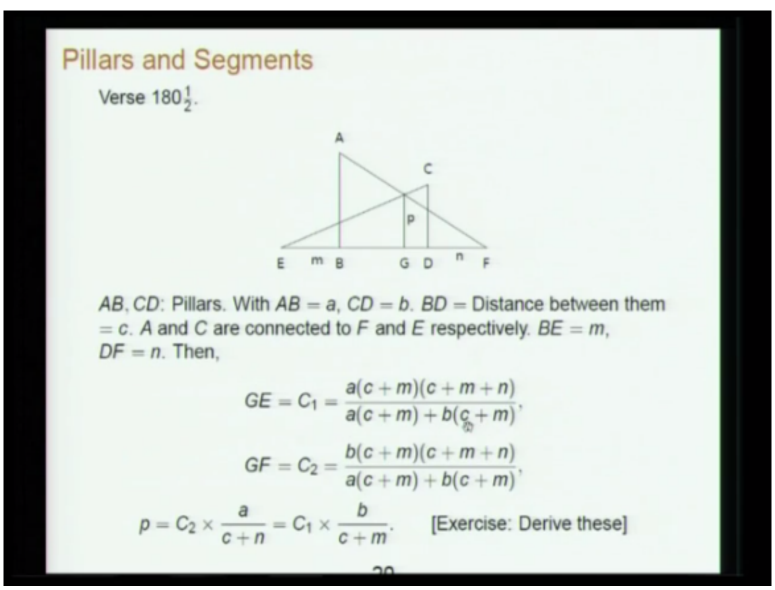

So, then pillars and segments so, this is also very even in Baskara later also you will see that you know lot of time is spent it you know constructing various kinds of you know similar triangles and things I get. So, here what is happening is so, ab and cd are pillars with ab is equal to some a height of one pillar cd is equal to b another pillar, bd is the distance between them. So, now this is stretch like this.

So, some point e with a string similarly this is from a you are stretching the string and (()) (18:49) so, a and c are okay corrected f and d. So, be suppose be is equal to m this segment is m so, then ge so, this ge is c1 is you get this the nice results. And gf is equal to c2 is b into this no b into c+mn into this and p is a perpendicular this perpendicular is c2 into a/c1+1 there some good cemeteries is this that is what is interesting you know.

There some good cemeteries in this so, this will be can be kindly derived okay using similar triangles okay.

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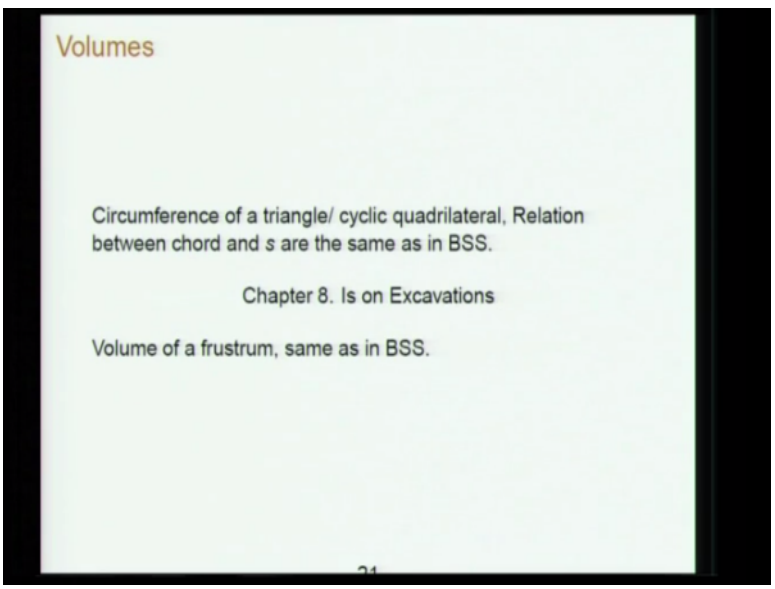

Then after the areas then it comes to volumes so, then sorry I am before that of course circumference of a triangle, cyclic quadrilateral and relation between a and is s are the same as in the earlier works. And the next chapter is an Excavations they are always called it excavations and it talks about the volume of a frustum same as in we spent some time yesterday explaining that right. So, how do we get the frustum this is a figure like this is in it

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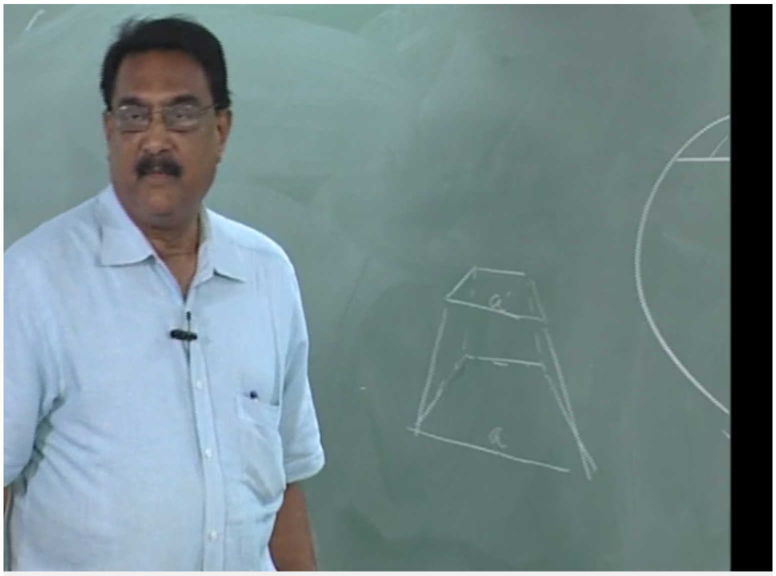

So, tapering kind of a thing is in it so, these the frustum kind of a thing so, here this bottom portion this taken to be square of side a is a prime. So, then Brahmagupta had given the approximate volume somewhat a better approximate volume then exact all that. And we saw that the exact volume is correct so, all these results are given by mahavir also.

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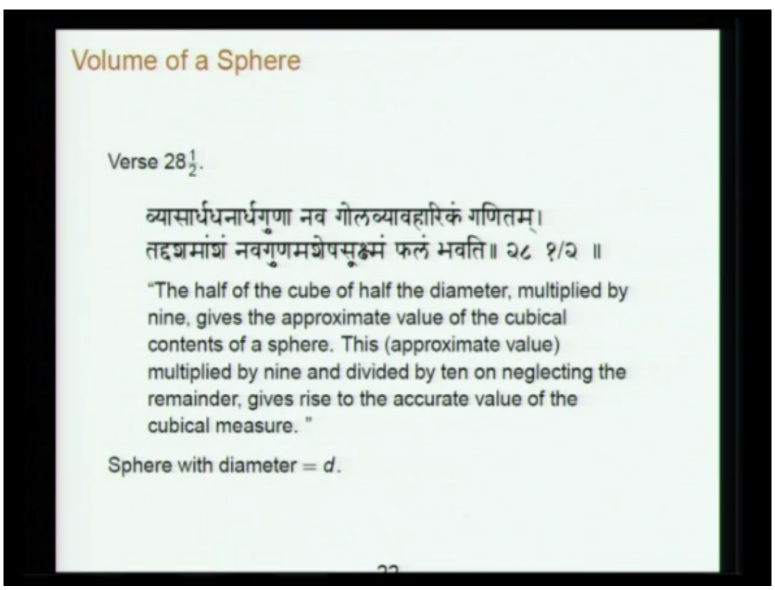

So, then he talks about volume of a sphere the half of the cube of half the diameter multiplied by 9 gives the approximate value of the cubical content of a sphere. This approximate value in multiplied by 9 and divided by 10 or neglecting the remainder gives rise to the accurate value of the cubical measure (FL) so, (FL) and all these multiplied by 9/10 so, those talked about.

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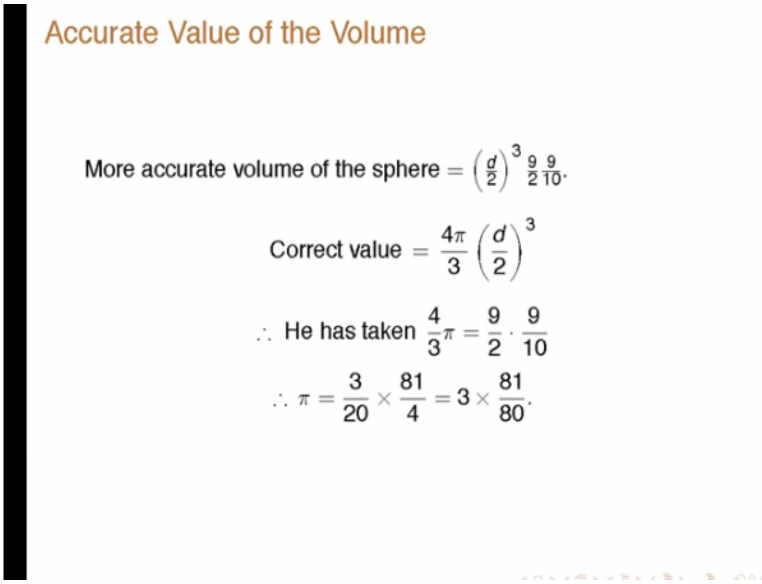

So, what is saying is that so, earlier also a the more accurate volume of the sphere is given to be this. So, definitely I this curious and d/2 whole cube into 9/2 into 9/10 the correct value is the of course 4 pie/3 into d/2 whole cube right this is what we know 4 pie/3 r cube so, if you equate this you will get 4 pie/3 is equal to 9/2 into 9/10. So, essentially is equivalent to so, the formula is okay.

If you take if you assume that here taken the circumference / diameter as the pie to be this 3 into 81/ 80. So, it take is better than 3 okay.

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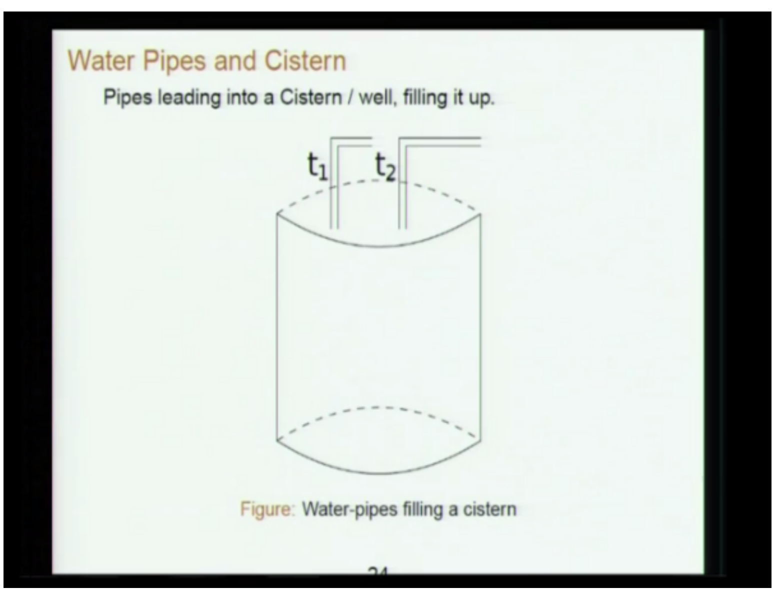

So, then it talks of some water pipes filling a system or some vessel okay so, pipes are there so, this is some you know ancient equivalent of a syntax tank. So, there are leading what I see these.

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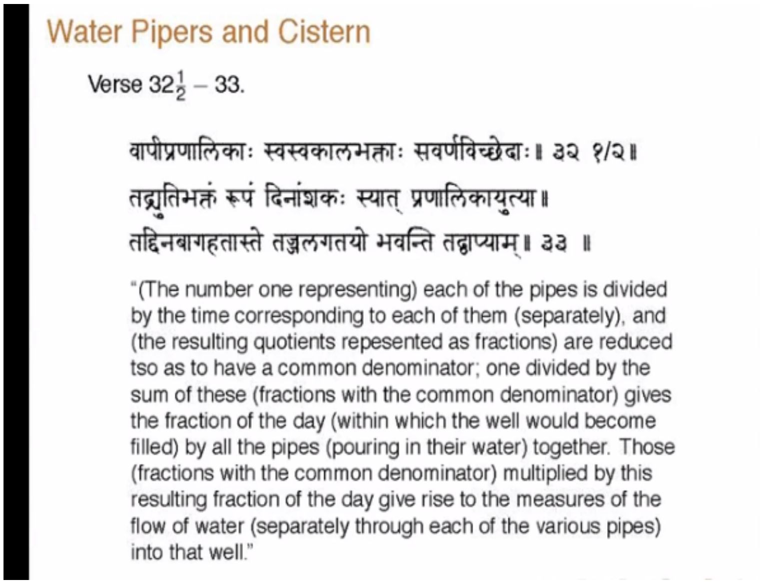

So, very relevant problems really for Chennai we should know this is things so, he gives the time is which is required to fill up the tank (FL). The number one representing each the bracket is as I told you sometimes where some things may be you know missing in the verse. But it has to be understood okay so, as no western scholars are objected. So, we can it that you know it is reasonably one can assume this.

(The number one representing) each of the pipes is divided by the time corresponding to each of them (separately) and (the resulting quotients represented as fractions) are reduce so, as to have a common denominator. One divided by the sum of these (fractions with the common denominator) give the fraction of the day (within which the well would become fill) by all the pipes(pouring in their water) together.

Those (fractions with the common denominator) multiplied by this resulting fraction of the day gives rise to the measures of the flow of water into that well.

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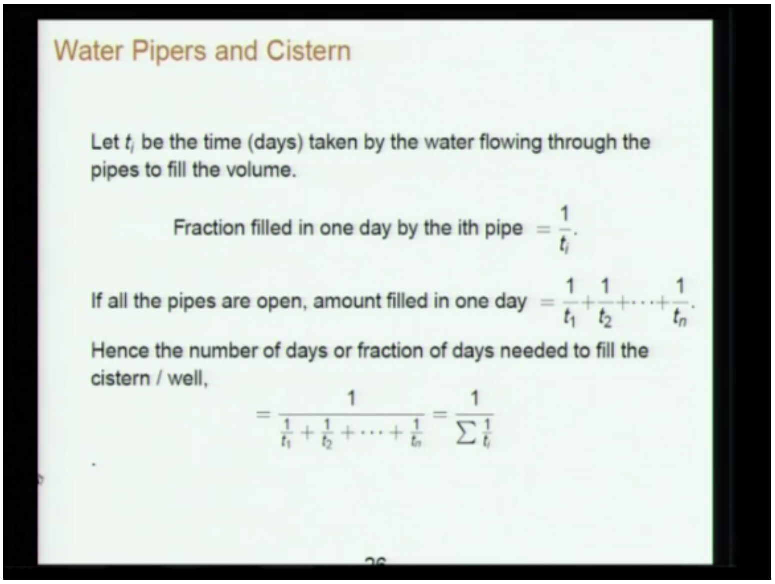

So, essentially one see let them the various pipes so, let ti be the time or the day time may be a units of day you can take so, it can be a fraction also taken by the water flowing to the pipe height to fill a volume okay right ti be this thing. So, fraction filled in one day/the height pipe you see so, to fill the whole volume it takes time ti. So, in Monday it is taking how much this is filling 1/ti right.

In if all the pipes are open amount filled in one day this okay that the sum over all these inverses. And hence the number of days or fraction of days needed to fill the cistern that is the whole thing that is equal to one over of these inverses so, one among this. So, that is the correct result exact results.

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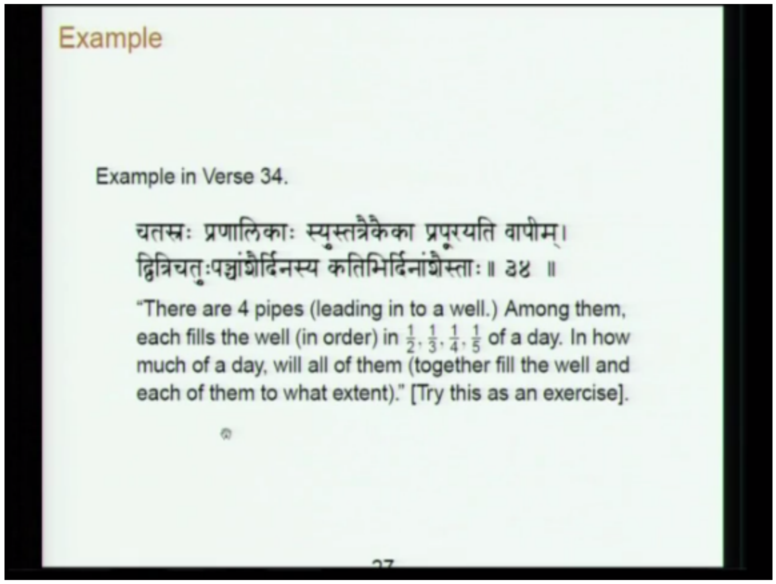

So, he gives an example (FL) the four pipes (leading in to a well) and among them each fills the well (in order) in ½, 1/3/ ¼, 1/5 of a day. Then in how much of a day will all of them (together fill the well and each of them to what extent) okay. So, how much the this thing and after the fill you know this thing you know what is the time taken. And what is the contribution from each pipe that a force can be calculated. So, that is what he saying okay.

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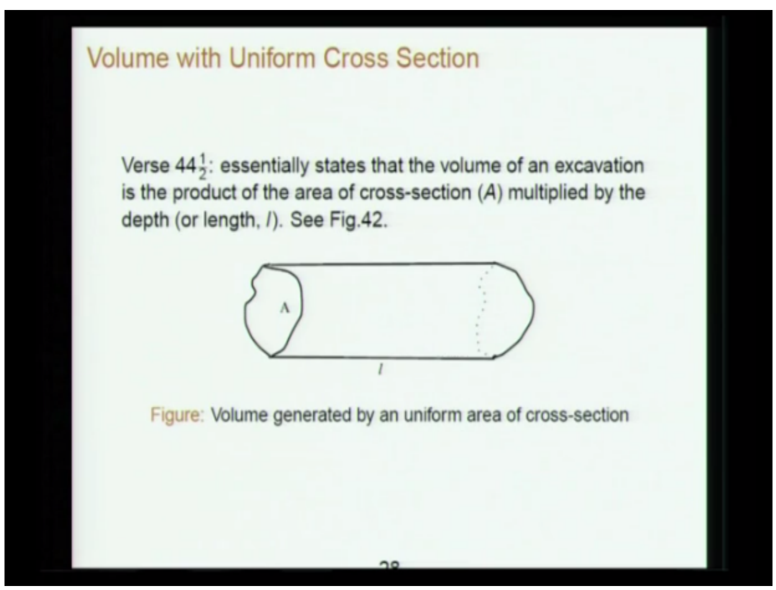

So, then again he comes to the volume it will you form cross section. So, the another verses will says that the volume of an excavation some of the use the excavation in all these things is the product of the area of cross-section multiplied by the depth or length. So, this is the see one some important topic we discuss it always discuss by the earlier later mathematicians with following you know.

So, this cistern this water pipe problem, and this excavations and all that they are discussed by Lilavati they are discussed by (FL) in (FL) would you like that. You know various people will there may be adding and little bit you know for modifying it to should some new things.

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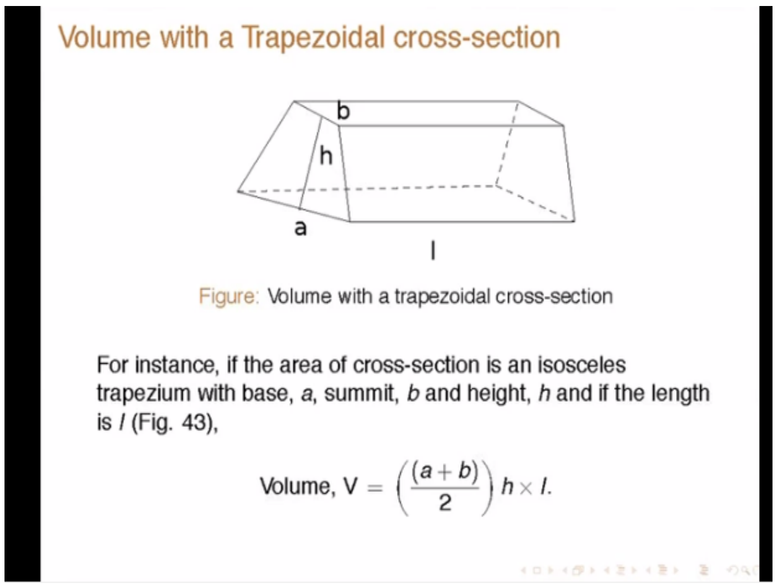

Similarly volume with a trapezoidal cross-section is given to be if the area of cross-section is you know that if a is the base and b is the summit and h is the height. So, area of cross-section is a+b/2 sorry a+b/2 into h right that is the area of cross-section. So, this multiplied by this l so, that is the volume with trapezoidal cross-section okay.

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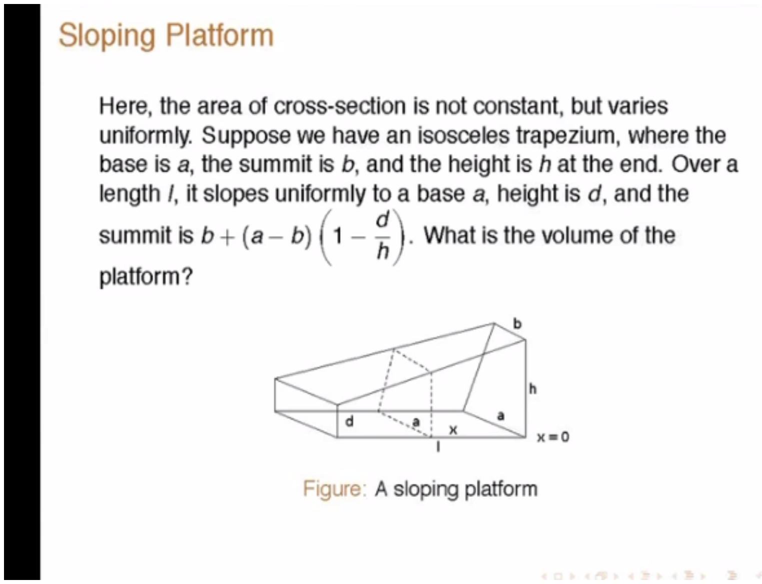

So, then it talks of a sloping platform so, here it is the area of cross-section is frustum etc. a talks about now, now he is talking about a new thing. So, area of cross-section is not constant but varies uniformly you suppose we have an isosceles trapezium, where the base is a, the summit is b, and the height h is at one end. And over a length l it slopes uniformly to a base a, height is d and the summit is b yeah b+a-b into 1-d/h so, what is the volume of the platform that where it is post is a following.

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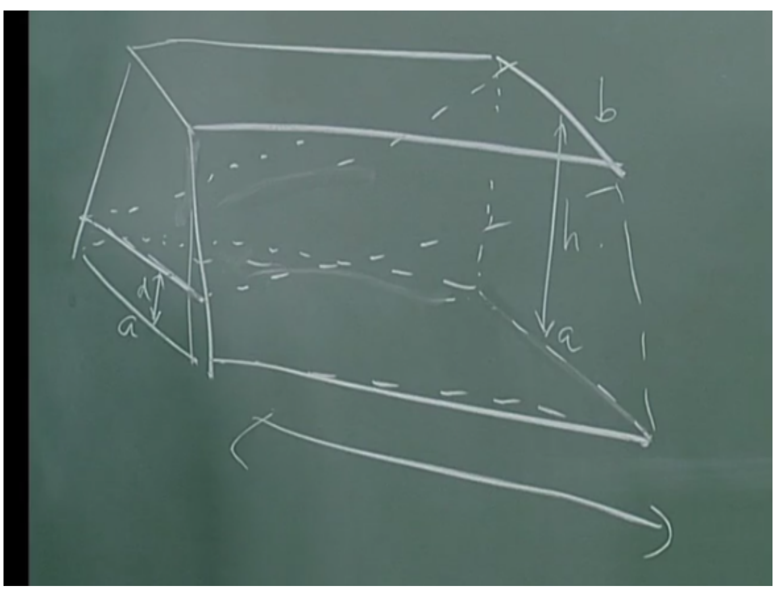

So, one starts with a uniform cross-section where the base is a, the summit is b, and height is h. So, the volume is here so, it is the area of cross-section into this length so, now suppose this structure is struck like this you know struck like this. So, what remains si this you know so, upto this so, only up to some height d from the bottom that is there. And then so, this is what the remaining so, this uniform this structure it is struck.

And so, that at this end the summit is a the base is a, the summit is b and height is h here at this end at the left end the base is a and the height is only d and these are decreased uniform corresponding. So, it is no longer uniform so, the area of cross-section is changing and also you see here not only is changing here over the cross-section it is changing in this direction also. And this much amount has fallen this per brick has fallen at this much structure has fallen down.

So, what is remaining so, that is what we are to find out the volume of that. So, in terms o f bricks for instance so, if the original number of bricks is there you know how much how many bricks are fallen and how many are remaining. So, that is how that is the thing we had to calculate so, the volume of the remaining structure so, that is what we had to calculated. The result he gives this start one end is the isosceles trapezium with sides you know.

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Sloping Platform

At one end, it is an isosceles trapezium, with base a, summit b and height h. At the other end, base is a, summit is $b + (a-b)\left(1 - \frac{d}{h}\right)$, height is d. At the distance x from the first end, base is a, summit is $b + \frac{(a-h)}{l}\left(1 - \frac{d}{h}\right)x$ and the height is $h - \frac{x}{l}(h-d)$, by the rule of proportion.

Then, the GSS result for the volume in Verse $54\frac{1}{2}$ is

$$V = \frac{lh}{6}(2a + b + d)$$

We find the volume to be (by using integration)

$$V' = \frac{lh}{6}\left[(2a+b) + (2a+b)\frac{d}{h} + (b-a)\frac{d^2}{h^2}\right] \quad ?\ \text{(Please Check.)}$$

Base a, summit b, height h and other side base a is a summit is b+a-b into 1-d/h you see it has proportionately you know this side become d. So, you can calculate at this base this one this summit will become b+a-b into 1-d/h. When d is equal to 0 top and bottom are the same at the left hand and width is a. When d is equal to h that is uniform cross-section the second term is 0 and with the top is b.

For an arbitrary d so, width the left hand is b+a-b into 1-d/h so, then at some intermediate this things you know so, the what is how much is this. The cross-section the base will be a only one can show that this is b+a- this is your end b+a-b into 1-d/h into x okay. So, this is sloping you see remember uniformly. So, this is the kind of a thing and height will be again the rule of proportion you see.

So, here d is the height d and here this is height h and what is the height here proportionately one could calculate and you will get this by a rule of proportion and (FL) result for this volume is lh/6 into 2a+b+d so, this is the what is saying is these the length l okay these the edge these b, a and this is the height at the other end by as fallen right. So, lh/6 into 2a+b+g and try to do it using integration exactly.

And this seems to be the result lh/ so, this first term is correct and this are the things which are there. So, 2a+b+d is getting but what should you one get in this. So you are may it, suppose these a more complicated thing you know simple arguments will not so, easily work it is like a frustum that cannot work here. So, probably that is why here does not got the correct result say anyway you should check this. What is a correct thing you can check it using.

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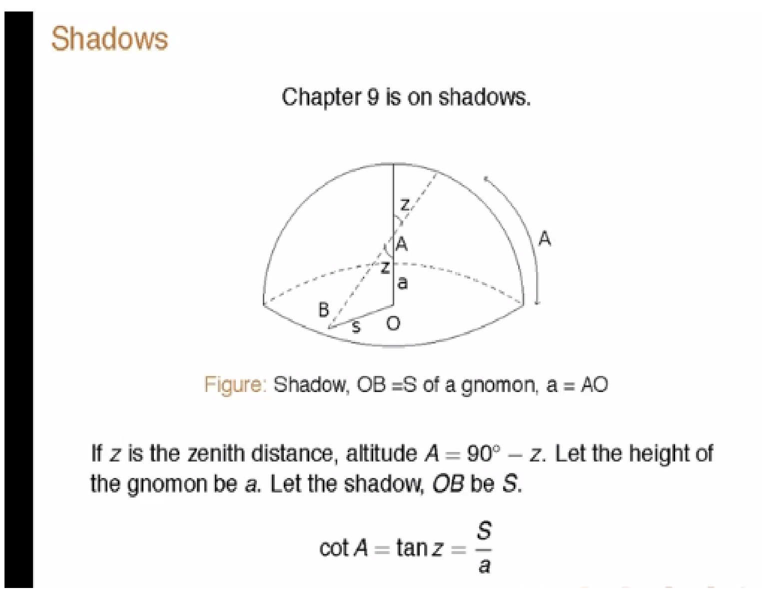

So, this as usual so, after this areas then volumes then he will talk about shadows. So, he is talking of the shadow of a gnomon so, this is the shadow is OB S is equal to S report it. So, gnomon is OA okay so, this is the sun let us say so, from that the shadow of gnomon is falling here. So, this is called a zenith distance okay is called zenith distance that is what is happening is that.

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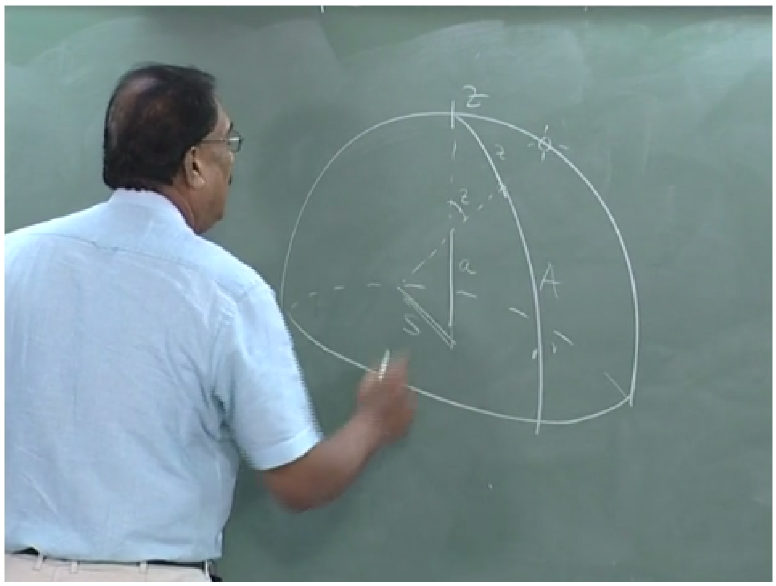

So, this is the is called a sphere okay so, these arise on so, these the zenith okay top most point of the sky okay. And then so, sun is here let us say so, this is your gnomon so, now the sun can be somewhere at some arbitrary time okay. And this is **is** falling like this so, you know so, these the shadow okay and this that is the shadow will depend clearly on this angle of course this is not to scale at all.

Actually this distance is you know much much more than this. So, just for clarity I have written like that so, this angle the angle at which is a sun shades are falling. So, that that is called a zenith distance okay and then 90-zenith distance is called the altitude, altitude of h. How many degrees above sun is on the sky see that is altitude and how much he is below from the zenith, so that is the zenith distance.

So, one can seize that, so this is the case if s the shadow and what is the taking for the yeah a is the noman height, so then this also there, so tan m is S/a, so that is the correct result tan z or cos a because a +that is 90 degrees, so that is s/a, so this correct.

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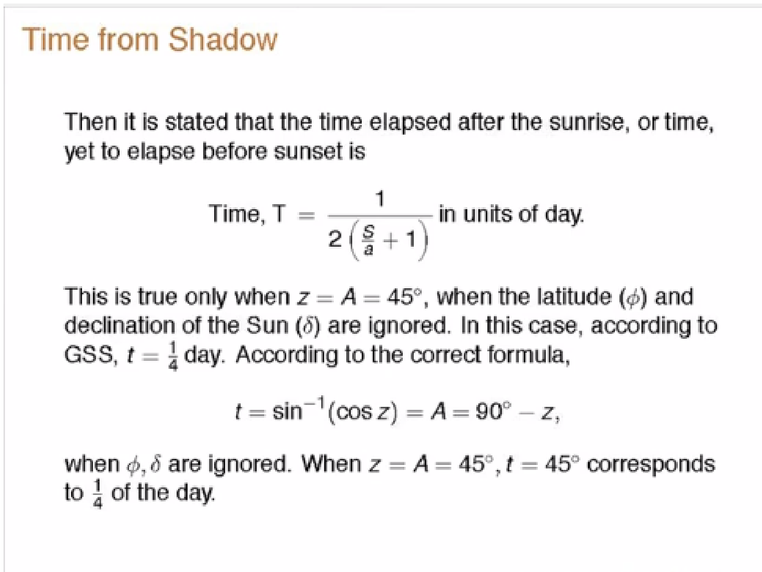

And he says Mahavira says that the time elapse after sunrise or time et to elapse before sunset is equal to 1 over 2 into S/a+1 units of a day okay. So, this is the result for he is giving into s/a+1 so, when does not know how he got this but he stated this result. Because time you know one certainly know this cannot be correct. You know because time is little more complicated it will not be given in terms of the shadow is so, easily.

In fact is correct when only for a very particular case when z is equal to a is equal to 45 degrees and the latitude and pie and the we call pie and the declination of the sun are ignored okay. So, that is a especially at the equator this is true especially that two only are marvh 21st and September 23rd. So, in that case and also when the z is 45 degrees then in this case according to the (FL) T is equal to ¼ of a day.

When this is 45 degrees and according to the formula also T will be so, in inverse car z and when z is equal to a is equal to 45 degrees, T is equal to 45 degrees corresponds to ¼ of the day okay. Because total 360 degrees is a day means from sun rise to sun set so, that is corresponds to 180 degrees okay. So, ¼ of the day will be 45 degrees so, you only very particular at only one instant that what a one particular place.

This is to but it may be approximate one should see may be for Chennai it cannot be two for certain days. Because the latitude is low okay so, one should invest to get the very truth formula.

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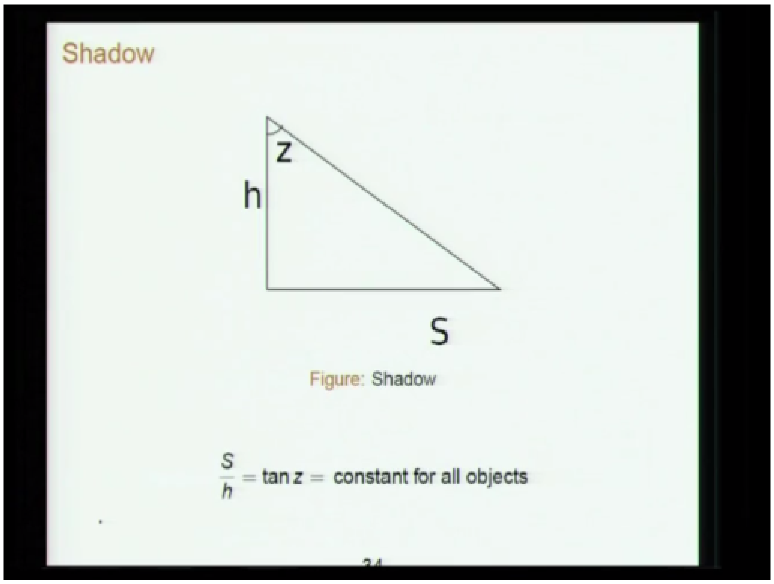

Then the shadow will continue so, he will note from is the a shadow divided by it is constant for all object. It naturally because you know if this depends only on this ratio only depend on the angle at we this sun rise is hitting right. So, it is constant for all objects.

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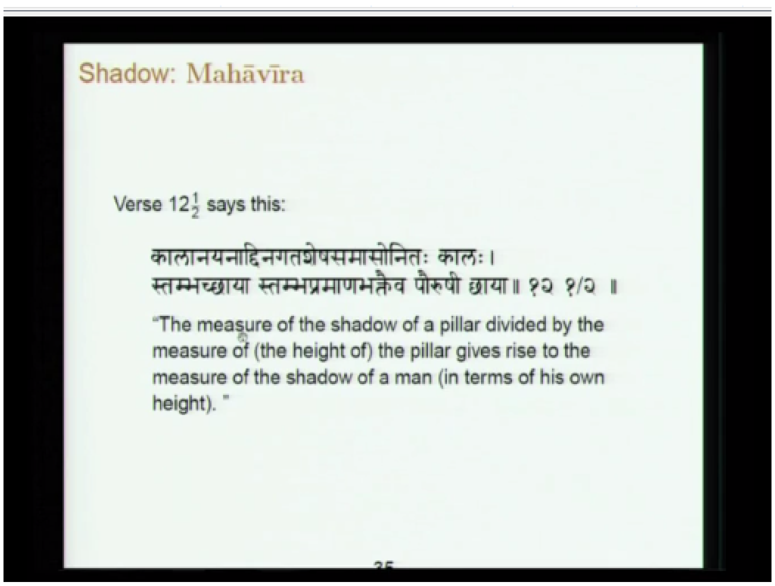

So, normally it is written in terms of the measure of the shadow of a man so, he says that the (FL) the measure 1of a shadow of a pillar divided by the measure of the height of the pillar gives rise to the measure of the shadow of a man (in terms of his height). So, where essentially saying is that the shadow of a pillar divided by height of the pillar is equal to shadow of the man and divided by height of the man. So, that is correct.

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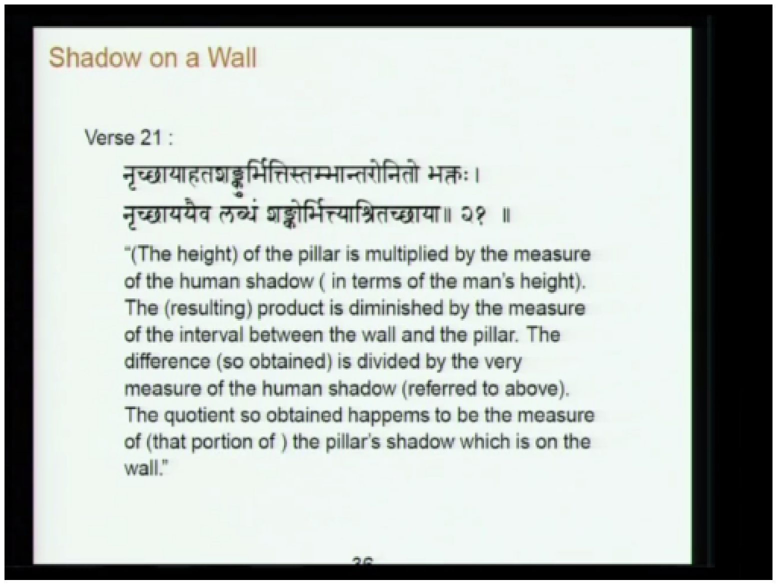

So, now is talking about shadow on a wall so, this is the some pillar is there.

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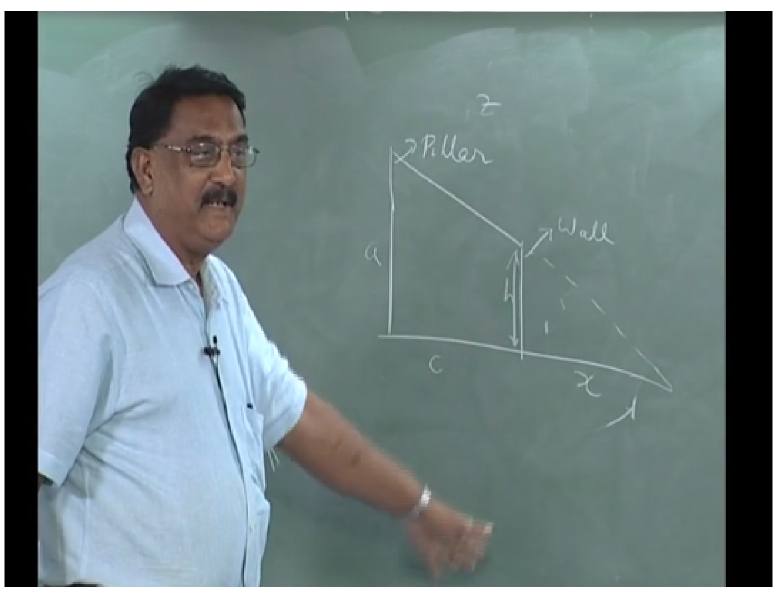

So, you are not observing the whole shadows that the wall inter meaning okay so, then what is the shadow what is this height what is the using that (()) (39:18) height. Suppose is a suppose the distance between the pillar it is the pillar. So, these the wall so, distance between c this thing is c and the shadow itself is x okay. So, then what is the height of this so, these also relevant.

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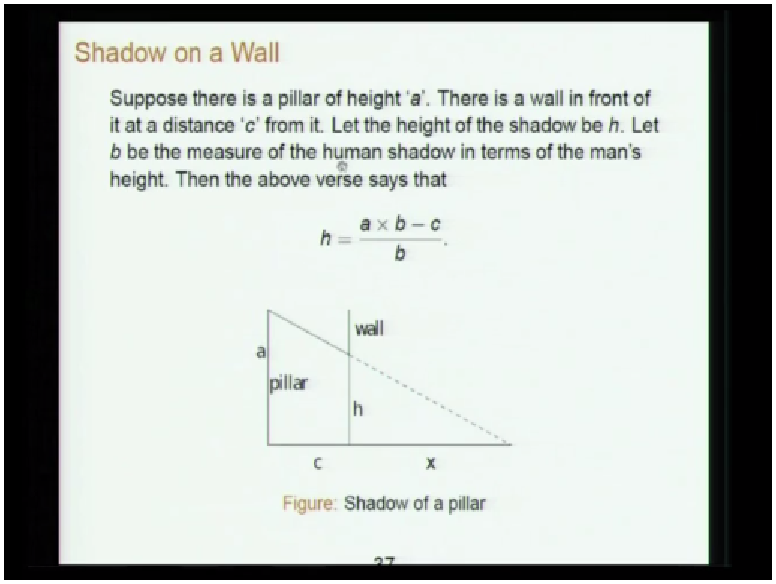

So, is he says that the essentially so, that the pillar of height of a that the wall in front of it at a distance c from it. So, let the height of the shadow be h sorry yeah height of the shadow on the wall that is what is this that is h. In a deep is the measure of the human shadow in terms of the man sight. So, then the above verse is it h is a*b-c/b okay. So, this is the shadow of a so, this is the important result.

And again is you know relevant for instance for lunar eclipse. Because so, this is the earth okay so, Sunday this is coming and suppose these the moons orbit. So, there is no wall but anyway so, this is where the moon is travelling so, then the shadow of a this thing that will be shadow regional be this okay. Then the moon is completely in this, this is then the full pull over the lunar eclipse and partially it will be partial this thing okay.

I come to that yeah b be the measure of the human shadow in terms of the mans height not shown in the figure yeah yeah yeah you have to draw another figure with the you know the man this thing yeah yeah yeah yeah yeah essentially you know the tanz is you know now human shadow divided by the man sight so. That tan quarter the angle that is that h is so, the these are result at given again it is comes from similar triangles.

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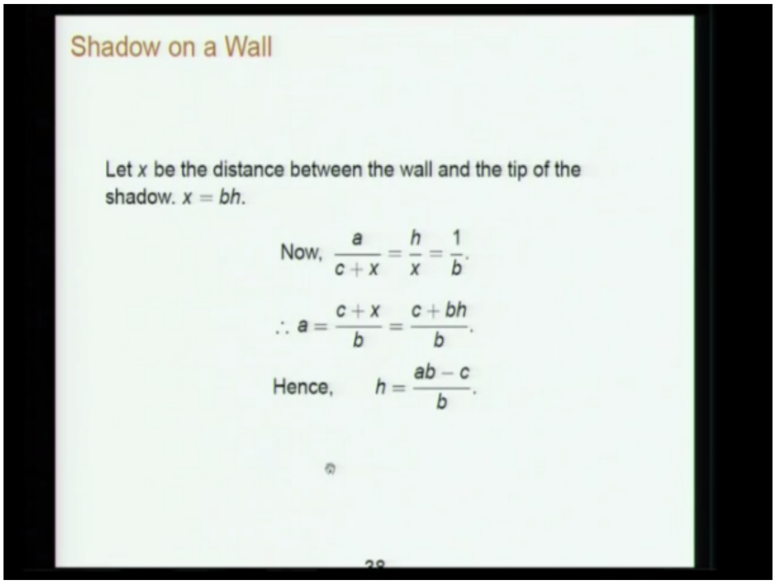

So, if a x is the distance between the wall and the tip of the shadow then x is bh. So, taht is because this is constant x is b into h because x/h is constant okay shadow divided by height is equal to shadow of a man that is get the b comes is a proportionality constant basically x/h is essentially equal to the human shadow divided by is height. So, that is what it have comes here that is the b here.

So, x is equal to bh so, if you take this triangle a/c+x i equal to h/x so, these are two similar triangles right so, a/c+x is equal to h/x. So, which is 1/b so, that is the definition so, a is c+x/b is c+bh/b so, h is ab-c/b. So, that is how it is in height on the shadow at any given time then the calculated like this.

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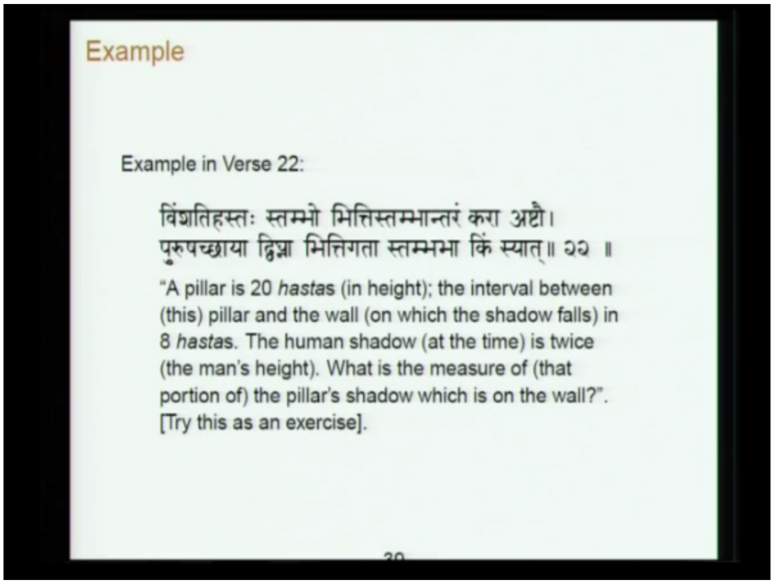

So, then again he gives some example (FL) A pillar is 20 hastas (in height) the interval between the pillar and the wall (on which the shadow falls) in 8 hastas. The human shadow (at the time) is twice (the mans height). So, that is get the b is given the human shadow at the time is twice the means height so, the b is given here essentially. What is the measure of (that portion of) the pillars shadow which is on the wall?.

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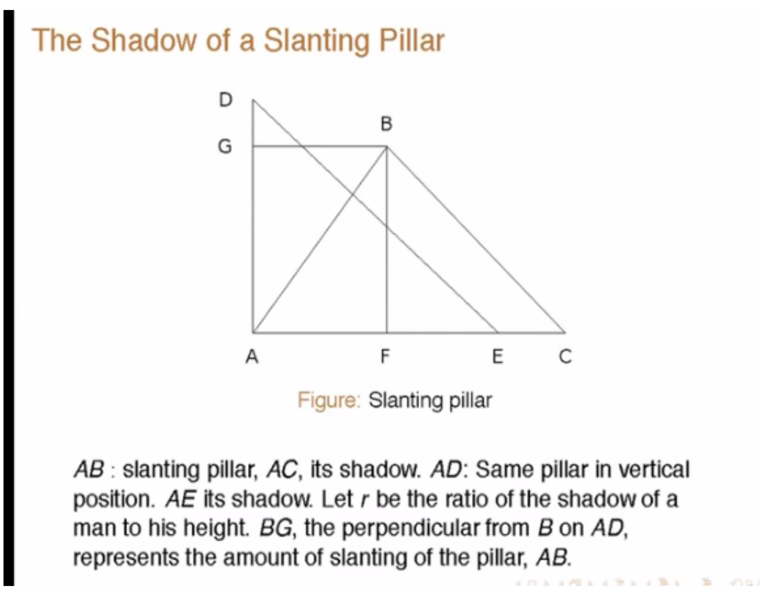

So, this can be try this as an exercise so, then one of the last problems which is discusses in th shadow is a a slanting pillar. So, is ab is a slanting pillar and ac is it shadow suppose ae ad is the same pillar in vertical position. Then if that is so, when in vertical position is shadow will be ae and let r be the ratio of a man to is man shadow to is I think it should be man shadow to is right and bg is the perpendicular from b and d, b on ad.

So, this is the distance between this and this so, this essentially gives the amount of slant. So, this you can say that the easy pillar and this slanting like this. Suppose that the perpendicular is bg so, it gives a measure of the slant of the pillar ab so, then in that case.

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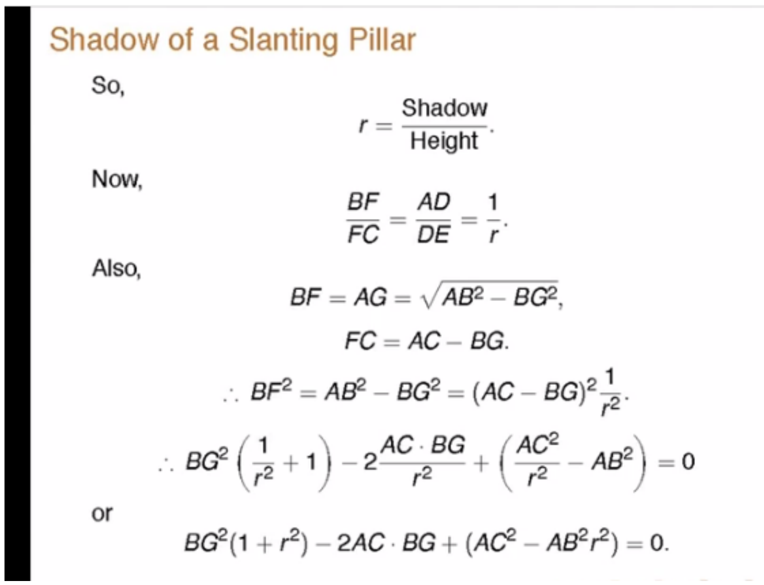

So, one can show that the essentially may be better too.

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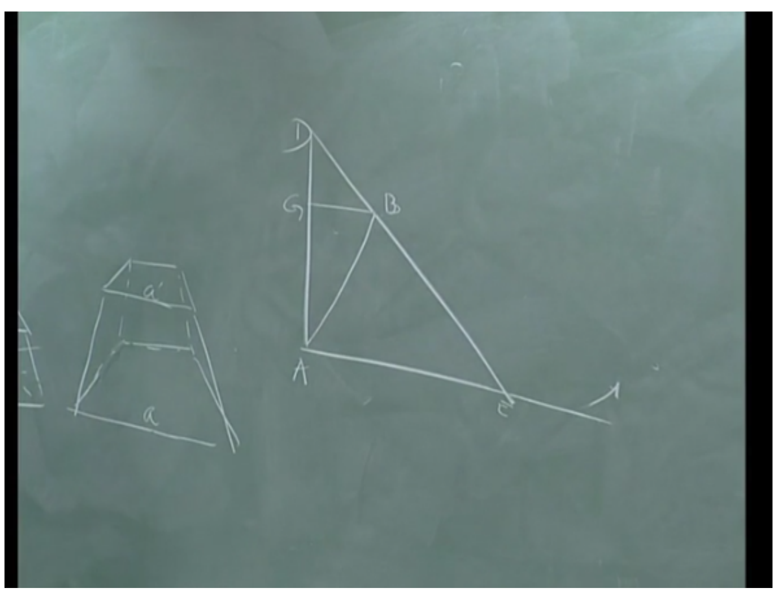

So, this is the this becoming one distance so, suppose it is slanting so, then in that case so, it is a so, this is your b, this is your g, this is your D. So, this is your e and ac when it is in ac so, this b this thing will be that will be the e one sorry go past that (()) (45:23) yeah

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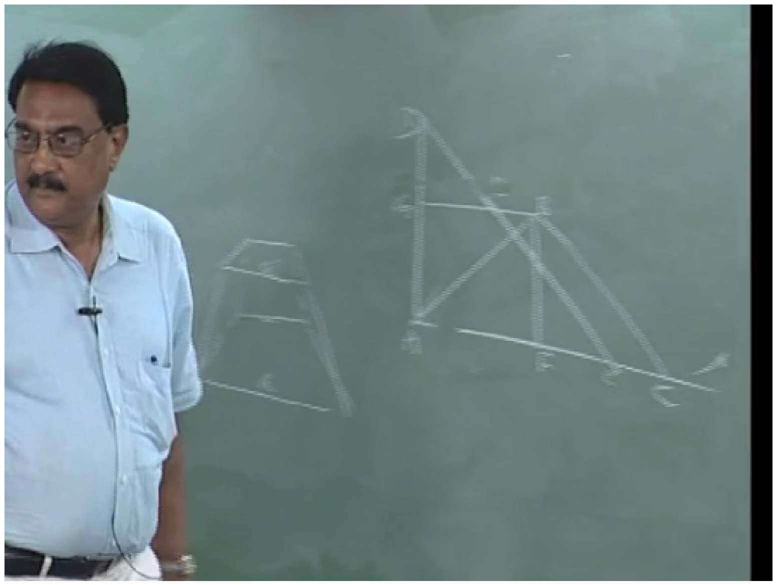

Slanting is this actually and suppose this is the slanting pillar here so, this would be like this okay. So, this is the right okay so, this is the figure and this is f oaky. So, the shadow of a in the vertical position that is ae shadow in the slanting position is ab shadow in the slanting position is a c ac yeah. So, then from the similar triangles Bf/FC so, that is equal to AD/AE okay (()) (46:16) similar triangles

Because the angle will be the same right that angle will be the same so, that is equal to the shadow by the objects effective height that is 1/r okay or rather the height divided by the shadow is 1/r. Because r is shadow/height so, then BF is equal to AG in the figure BFAG that is the perpendicular thing. So, that is square root of AB square-BG square and FC is equal to AC-BG. So, BF square is AB square-BG square.

You can see that AB AC-BG sorry AB square-BG square so, from this square root of AB square-BG square is equal to BF square and from this so, finally you get this whole square into 1/ r square. So, essentially what to get is a quadratic equation for BG so, quadratic equation was BG.

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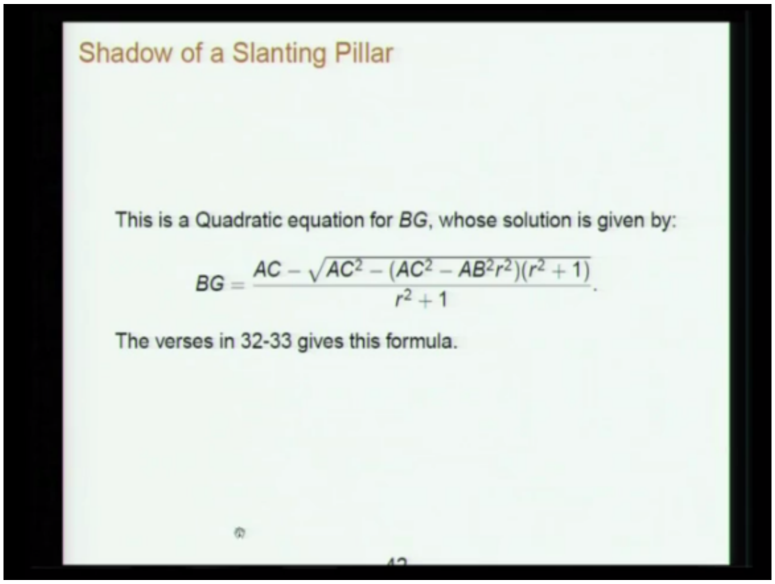

So, and solution is given by AC- this kind of a things so, you can find out this slant also suppose you are given the height of that thing or pillar and that is AB is the height of that pillar and which is slanting. And it is a shadow is AC okay so, then and r is the shadow /height so, then this will be the thing okay. Because if it is not slanting you see the shadow by the height will be given by r only right shadow/height is r.

If there is no slant it is vertical thing then the shadow varieties are built to slanting it is different and precisely this is the formula for that and satisfies the quadratic equation okay.

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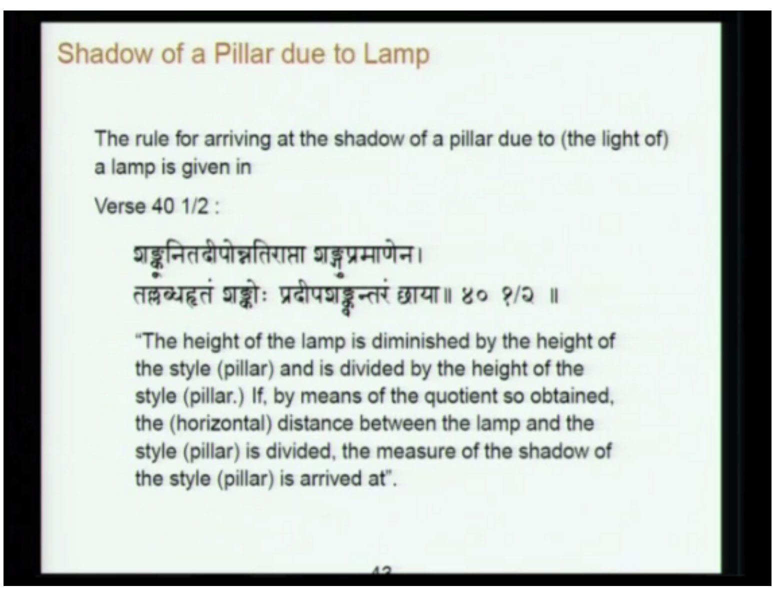

So, this is some kind of a new problem that here we discussed appear to the BrahmasputaSiddhaanta similarly it is a rule for arriving it a shadow of a pillar due to (the light of) the lamp. So, that is given the height of the lamp is diminished by the height of the pillar and is divided by the height of the style okay is the quotient so, obtained by mains of the quotient so, obtained. The horizontal distance between the lamp and the style pillar is divided. The measure of the shadow of the style is arrived at.

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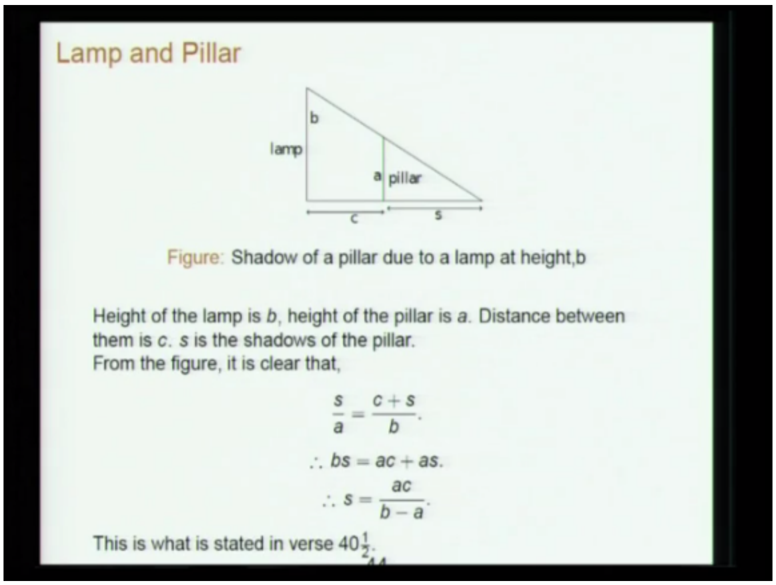

So, essentially what is the old result that you know that if this is the pillar and is aa lamp and these the height of the lamp. And so, then in that case see now it is a gnomon basically you know pillar means a gnomon. So, then s/a is equal to c+s/b so, the shadow s is equal to ac/b-a. This we are discussed you know earlier also so, is essentially giving the same problem. So, this is the some may essentially the last problem class of problem that we are discussed in this text in the shadow problem okay.

So, you seen the arithmetical operations arithmetic various operations and results connected with the arithmetic. So, and then you go to areas and then the excavations and then shadow. Then general typical way in which text book is organised so, this is essentially of covered some you know how were the basic things at the important things that (FL) okay. And also emphasize what is the new thing is doing compare to the earlier mathematicians.

And as I told is the very extensive work with lot of examples and given in nice examples in good verse so, is verse you know having the look at.

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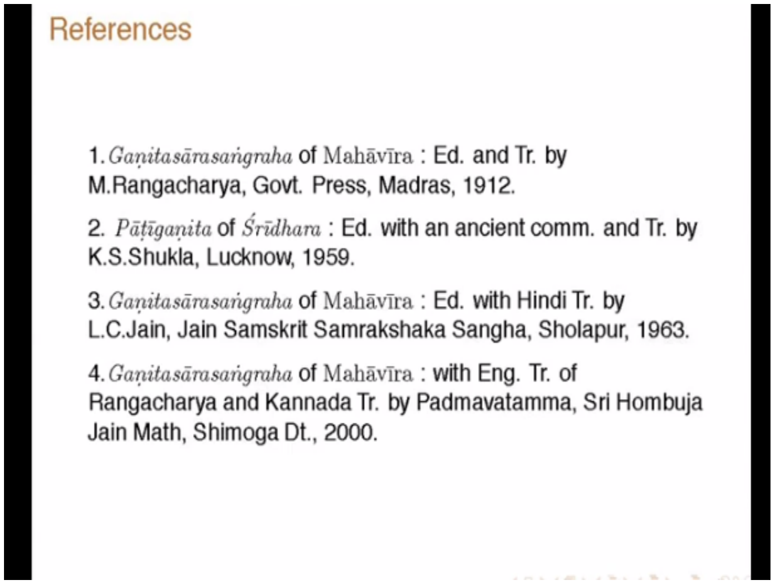

The references are given here okay thank you.