Mathematics in India: From Vedic Period to Modern Times
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Lecture-16
Mahaviras Ganitasarasangraha 2

Okay, so this second lecture on (FL) of Mahavira the outline is something like this.

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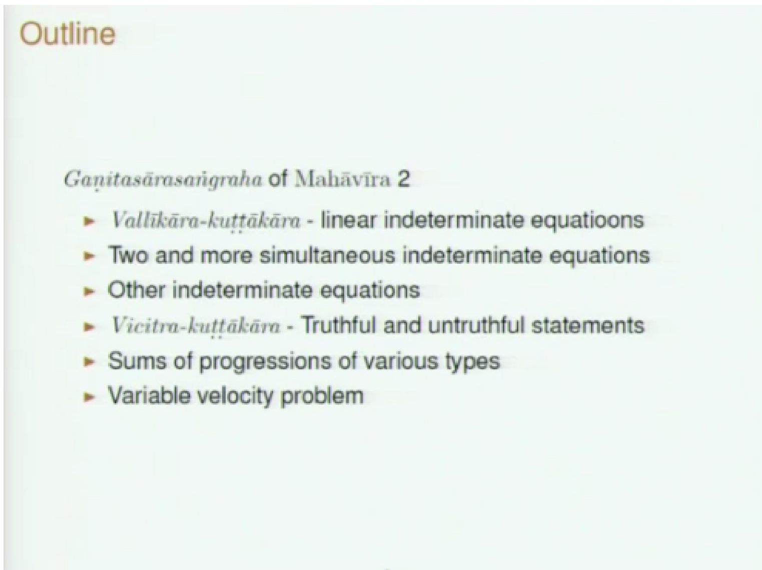

First I will be discussing this linear indeterminate equations are (FL) as it is called here then 2 and more simultaneous indeterminate equations and other kinds of indeterminate equations then verses (FL) truthful and untruthful statements then sums of progressions of various types, variable velocity these broad variable velocity problem we broadly the outline of this lecture.

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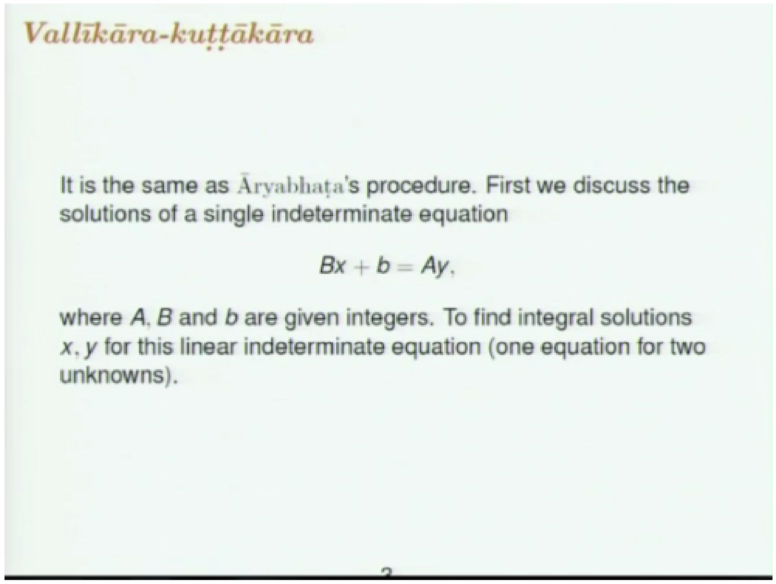

So, this is a linear indeterminate equation is of the following form, suppose you have given is equal to y y sorry.

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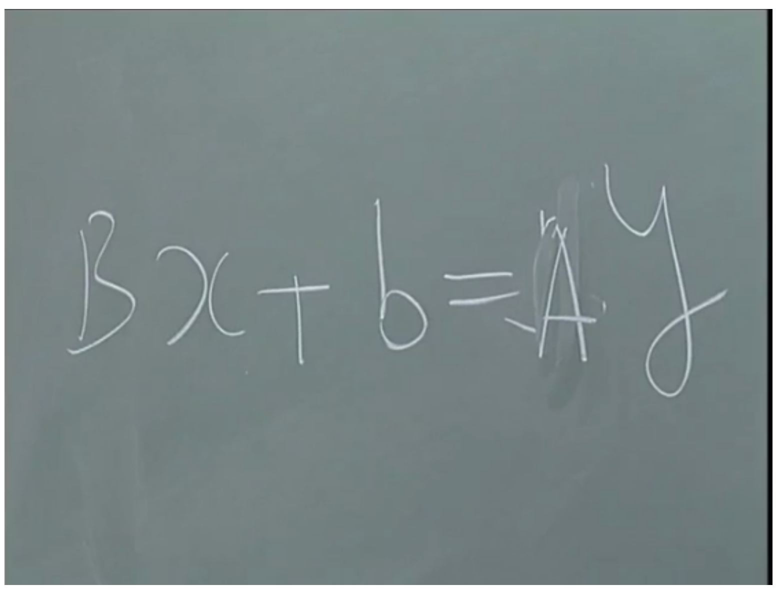

To use for consistency, so given this one equation in 2 unknowns x and y, so A and B are integers and you have to find integral solution for this right if you if it is you know if x is any real number then it can always be solved and y can be correspondingly you know will have a certain value but here it is specified that x and y are integers. So, only certain kind of certain integers will satisfy this equation.

It will not be unique, the solution will not be unique but there will be set of solutions for this equations with integral values for x and y so that is the linear indeterminate equation, so it is called (FL) in Indian mathematics and astronomy text, it has been discussed by Aryabhatta incur past in Aryabhatiya then Brahmagupta discusses it in his Brahmasphutasiddhanta and now mahavir also he has discuss this.

So, I will give how do you know mahavira has handled this and the examples he has given okay it by that interest. So, now (FL) so he considers so important in Indian mathematical tradition in fact algebra was called (FL) the chapter of an algebra in Brahmagupta's Brahmasphutasiddhanta is called (FL), so that is the important that was given to this and it has various applications the first we will discuss how it is solved.

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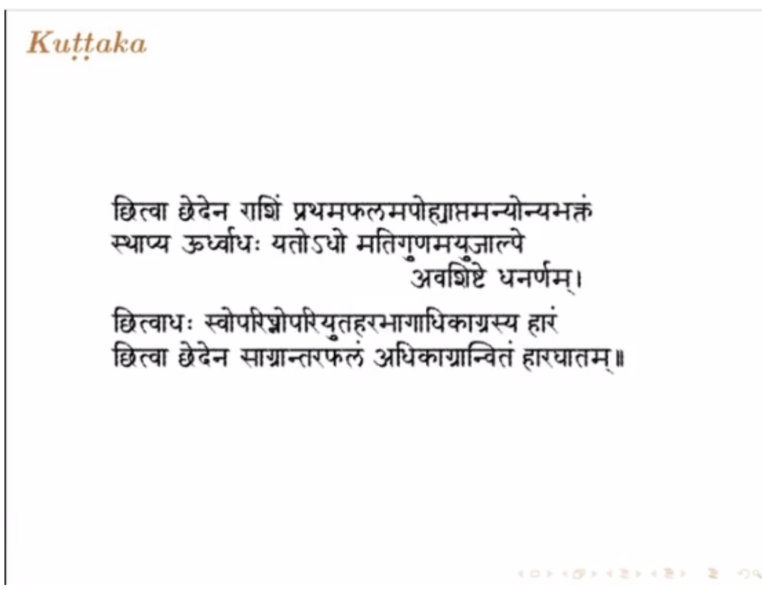

As I told you he is not the first solve Aryabhatta and Brahmagupta also have given the way to solve it what Mahavira how he has describe the solution is (FL).

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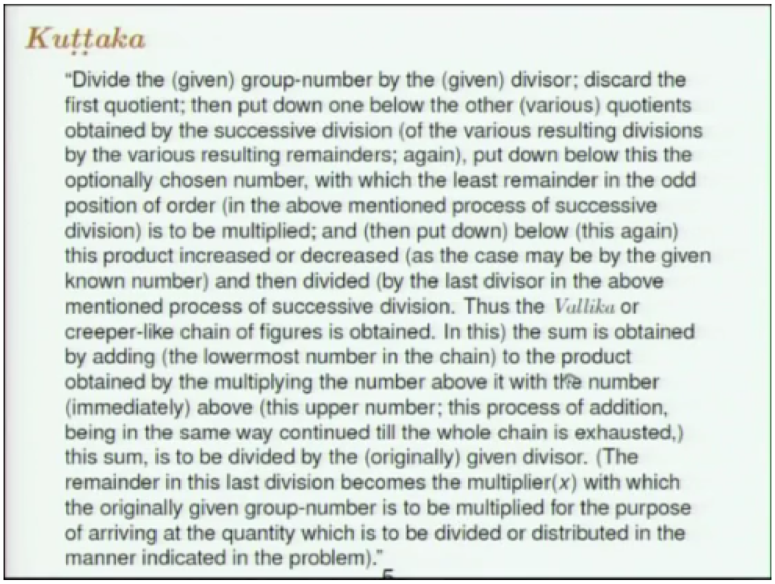

Kuṭṭaka

"Divide the (given) group-number by the (given) divisor; discard the first quotient; then put down one below the other (various) quotients obtained by the successive division (of the various resulting divisions by the various resulting remainders; again), put down below this the optionally chosen number, with which the least remainder in the odd position of order (in the above mentioned process of successive division) is to be multiplied; and (then put down) below (this again) this product increased or decreased (as the case may be by the given known number) and then divided (by the last divisor in the above mentioned process of successive division. Thus the *Vallika* or creeper-like chain of figures is obtained. In this) the sum is obtained by adding (the lowermost number in the chain) to the product obtained by the multiplying the number above it with the number (immediately) above (this upper number; this process of addition, being in the same way continued till the whole chain is exhausted,) this sum, is to be divided by the (originally) given divisor. (The remainder in this last division becomes the multiplier(x) with which the originally given group-number is to be multiplied for the purpose of arriving at the quantity which is to be divided or distributed in the manner indicated in the problem)."

So, divide the given group number I will read it and of course one will understand only when it describe it in the with using the notations. So, divide the given group number by the given divisor discard the first quotient then put down one below the other various quotients to obtained by the successive division of the various resulting divisions by the various resulting remainders again put down below this the optionally chosen number with which the least remainder in the odd position of order.

In the above mentioned process of successive region is to be multiplied and then put down below this again this product increase the decrease as the case maybe by the given known number and divided by the last divisor in the above mentioned process of successive division. Thus the (FL) or creeper-like chain a figures is obtained that is why it is called (FL) in the sum is obtained by adding the lower most number in the chain to the product obtained by the multiplying the number above it with a number immediately above this upper number.

This process of addition being in the same way continued till the whole is exhausted this sum is to be divided by the originally given divisor. The remainder in this large division becomes the multiplier x this applause this is a solution he describing with which the originally given group number is to be multiplied for the purpose of arriving at the quantity which is to be divided or distributed in the manner indicated in the problem.

I mean whatever is given in the brackets you know that is not there in the work itself but for completion of you know many things would have been implied in this works. So, the translation is including that.

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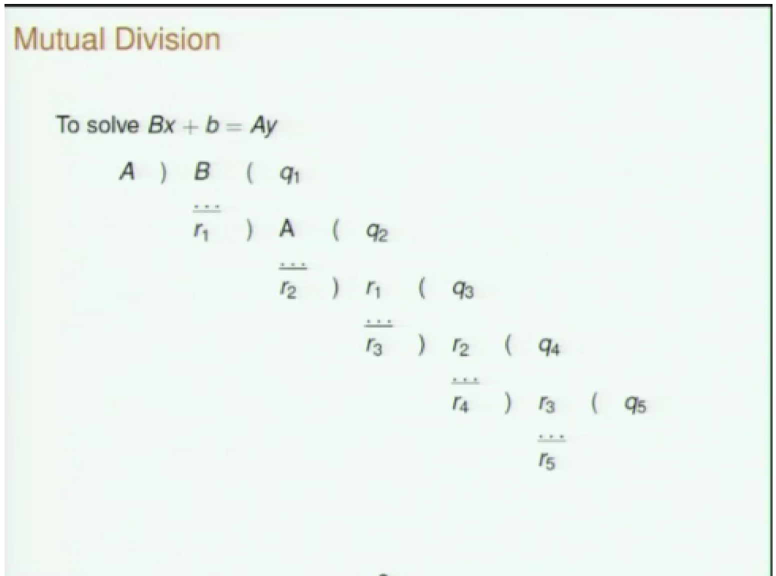

So, what is done is the following it is much easier dine than said, so he has solving the Bx+b is equal to Ay okay, now you are using essentially to use a similar procedure, procedure which is similar to what you do for finding the highest common factor between A and B (FL). So, B is simply B is equal to Ay, so you do this you know mutual division or a so called (FL) algorithm. So, then A, B this thing you know quotient is q1 then the remainder is r1.

So, now divide A by this remainder r1 then he get the quotient q2 and the remainder r2. Then again r1 you take here and divide r1/r2 quotient is q3 and remainder is r3 and so on till you get a small number here okay. The important thing is you know that this (FL) inderminate equation is much more in (FL) algorithm something more is involved here it is stop there you know but much more is required to get to solution of the problem which was post.

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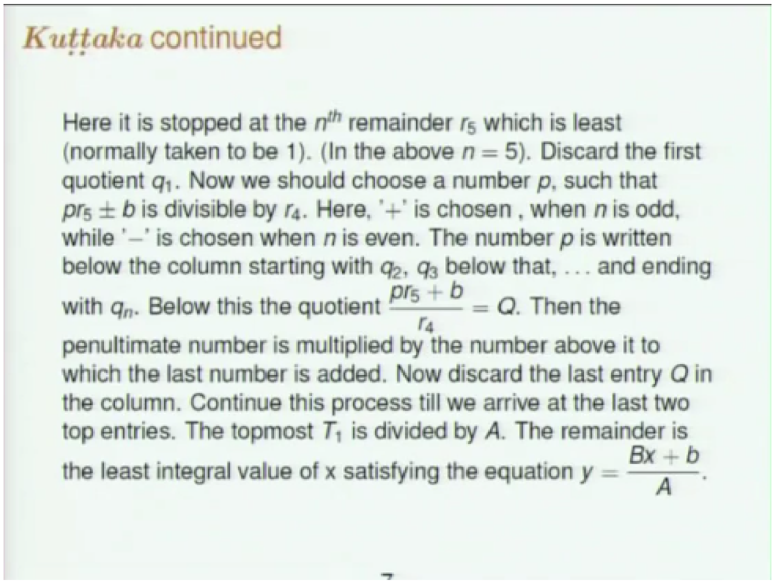

Here, so here it is stopped at the nth remainder r5 which is least it does not verse does not very it can be it not it can go on till you get 1, so in the above case n is equal to 5 okay as you were put it there like that now discard the first quotient q1 okay first quotient q1 discard the other quotient q2, q3, q4, q5 so that united of only that is a column one below the other and we should choose the number P such that Pr5 +/-b is divisible by r4.

Here + is chosen when n is odd and – is chosen when is even. The number p is written below the column starting with q2 etc., and ending with q1 and below this the quotient pr5+b/r4 is equal to Q okay.

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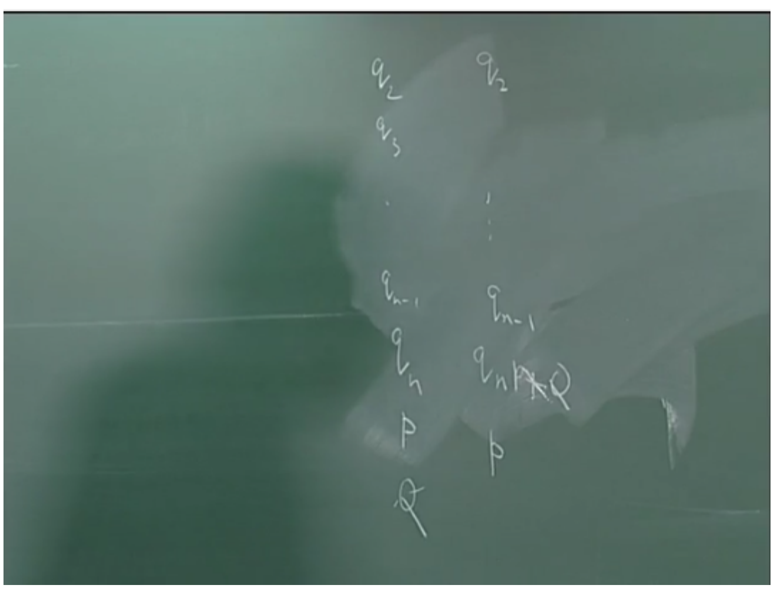

There going on right with the sequence of coefficients we write it like this q2, q3 etc., qn upper the first quotient q1 is not considered, so now after q1 you write the number p said that p multiplied by the last remainder and added to B is divisible by the previous remainder that is in what we have shown pr5+b is divisible by r4 and below that to write the quotient Q, so now in the next step you delete this, so then we do what we do is we take this into this +this qn*p+Q.

So, we write it here below this we write p and other things are the same above it, so in the next step what we do is this is multiplied by this and then this is added, so that will be the this qn p+Q so, this deleted and the upper entities are the same. So, you go on like this till only 2 quantities remain, so that you write as T1 T2 yeah.

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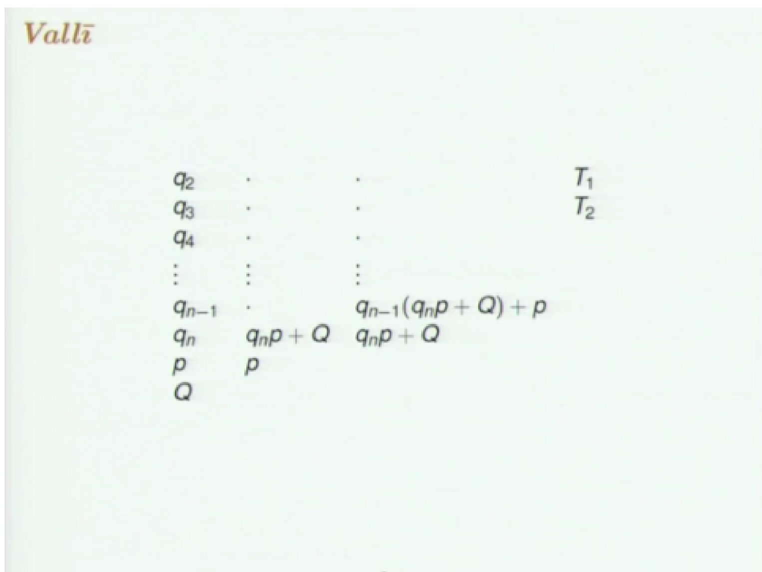

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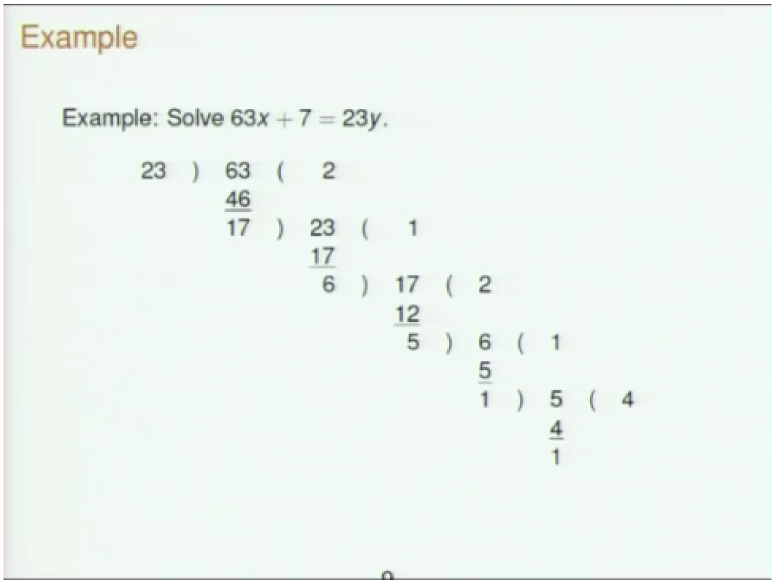

So, I will some examples then it will be clear okay it actually pretty simple, so we are given in a and the course we are given you know this procedure and almost everybody would be able to solve it. So, just have to understand a procedure you just write it down one then you will know. So, solve the equation 63x+y is equal to 23y, so you have to solve this equation for integral x and y right, so you do this mutual division 63 you know divided by 23 quotient is 2 remainder 17, then 23 is the divided by 17, so 1,6,6,17, 2.

So like this 1 in fact you could have stopped here but I want is a number of you know the total number of quotients should be odd is then only you will add be adding you know could not bother about adding and subtracting you can always you know you could have stopped here but I am just nothing prevents you doing this also, so such that you know the number of total number of quotient is odd that is why doing okay. Then there is r5 is 1 here r5 is 1 and r4 is 1 okay.

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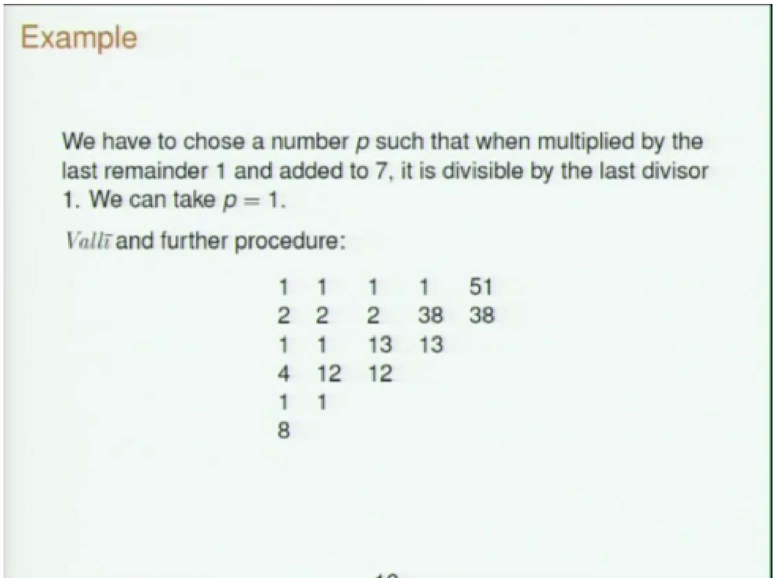

So, then you see we have to choose the number p such that we had got that 1, 2, 1, 4 right 1214 you had got because the first quotient 2 that you have to leave the all the other quotient you have to take 1, 2, 1,4 you have got then we have to choose a number p such that when multiplied by the last remainder 1 and added to 7 it is divisible by the last divisor 1 okay. So, here both the last remainder is 1, the last devisor 1 it is also 1 right.

So, we have to choose such that you know that is r5 p +B is divisible by r4 here both r4 and r5 are equal to 1 B is 7, so we can take p is equal to 1 okay. So, that is what you are written below this quotients you know from the list of first is removed as I told you, so 1214 then this p then quotient you see r5 p +B divided by r4 so that is 8, so that quotient you write, so now you know this into this+this 12, this is 1 again, so this is removed.

So, then next this into this+this, so 1, 2, 13, 12 this is removed, so now next this into this+this, so 38, 1, 13 this thing and then last is 51 you know this into this+this, so 51, 38, so you have got 2 entries, so now what you have to do is this itself actually is a solution.

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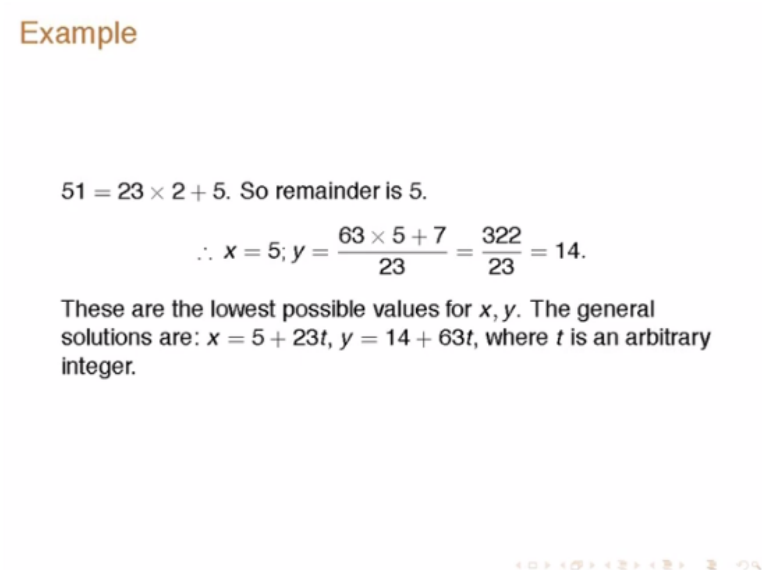

But you get the lease solution you note that 51 is 23 that is you divide 51/this 23 that is this Bx+sorry B is equal to Ay , so divide by this, so then 2 is the quotient 5 is the remainder, remainder is 5, so that remainder is the lowest value for x which that 5 this equation x is equal to 5 and y is 63*5+7/23 this 32/23 is 14, so this are the lowest solutions and the general solutions are clearly 5+23t and y is equal to 14+63t where t is an arbitrary integer know.

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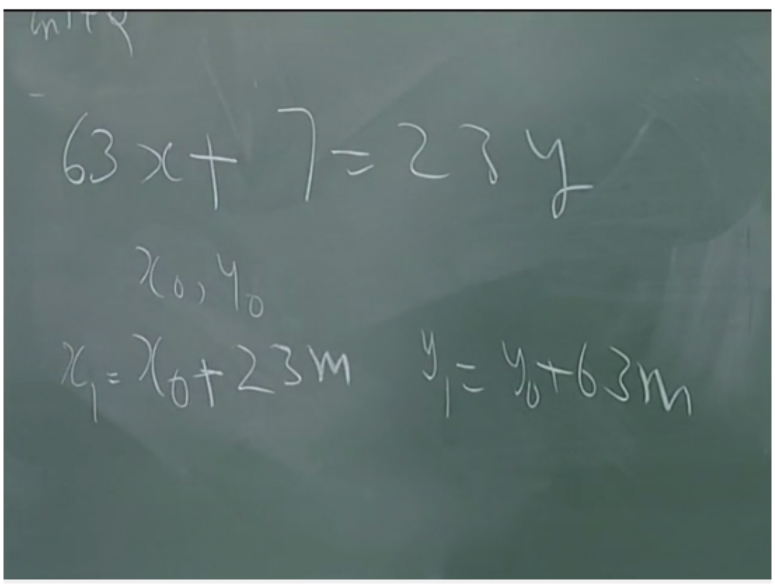

Because what we are solving is this 63x+7 is equal to 23y suppose x 0 is a solution, xo, yo then if you take x0+you know integral multiple of 23, 23m and y1 is equal to y0+63m then clearly that also will satisfy you know because this 63*23*m will come here and it will come they will cancel and if this satisfies this equation this also will satisfy the equation. So, you will get a into in fact your your infinite number of solutions.

Because m can go from in fact if you can have negative numbers are show, then it will be really it will go from –infinity to +infinity various integrals not all the integers but you know this it will free of this form okay .

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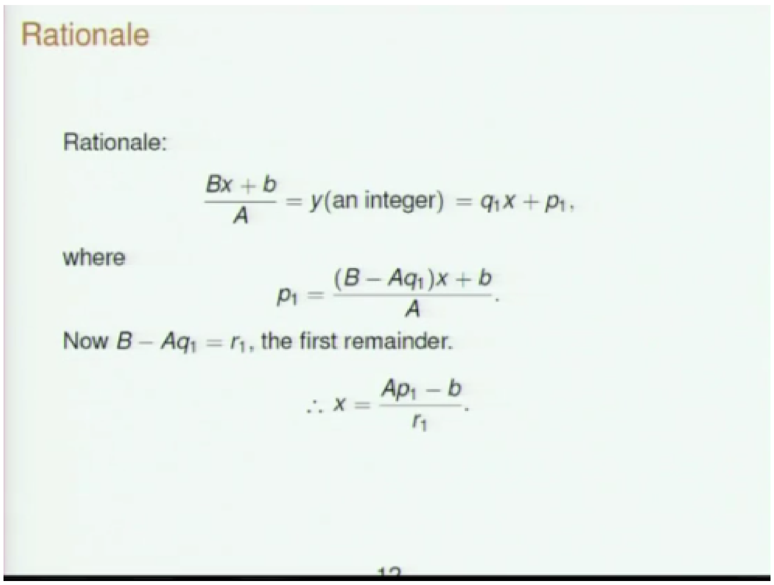

In fact rationale, so one can I will just give a brief why it does it work like this, so what you are doing is you know Bx+b/A, so that is the what you have to solve right Bx+b capital Bx+b/capital A that is why an integer that is what you have to get, so (FL) q1 is +p1 okay, so from this one can see that p1 is B-Aq1 x+b/A you know B-Aq1 is the first remainder okay, when B is divided by capital B divided by A, r1 is the first remainder say x is equal to Ap1-b/r1.

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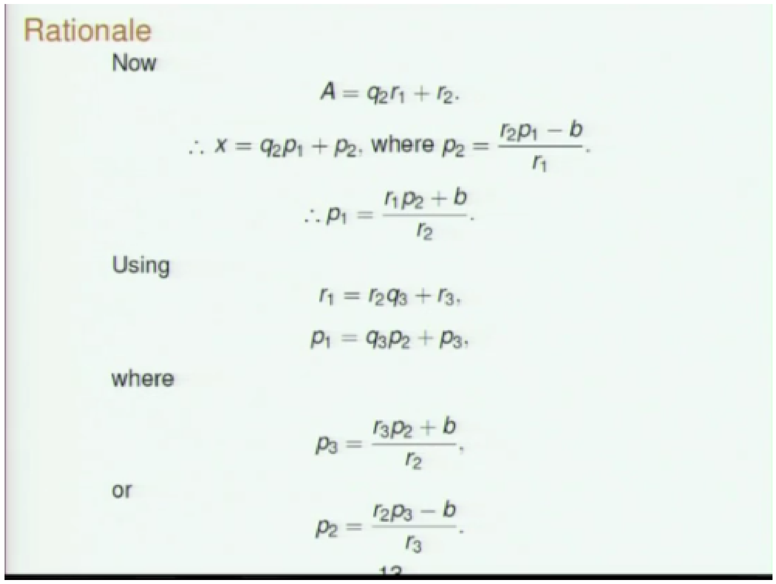

So, now A is actually again when A is divided by r1, the remainder is r2 and the quotient is q2, so A can be written like this, so x is q2p1+p2 and p2 is this, so from this one can write of p1 is equal to r1p2+b/r2 and again r1 you know when it is divided by r2 the quotient is q3 and the remainder is r3, so p1 you can write like this and so on you see.

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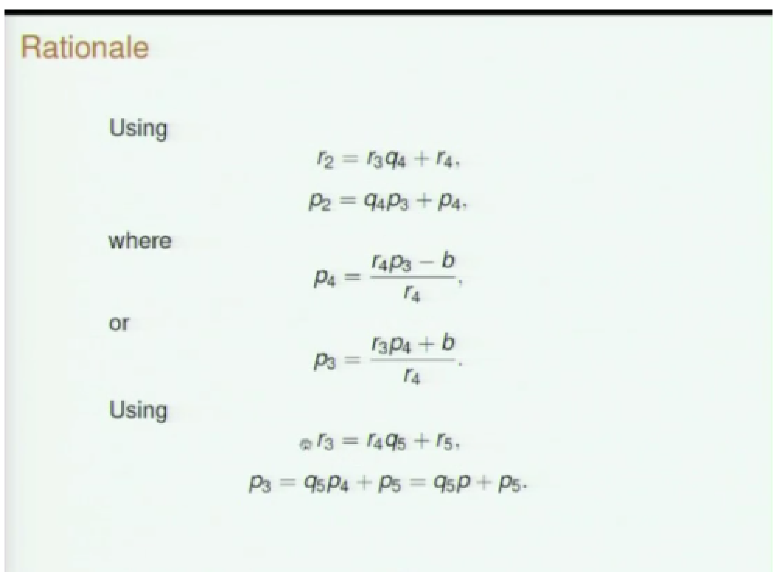

So, you get this kinds of equations you can have a closer look later, now you have to or some stage you are you know seeing back.

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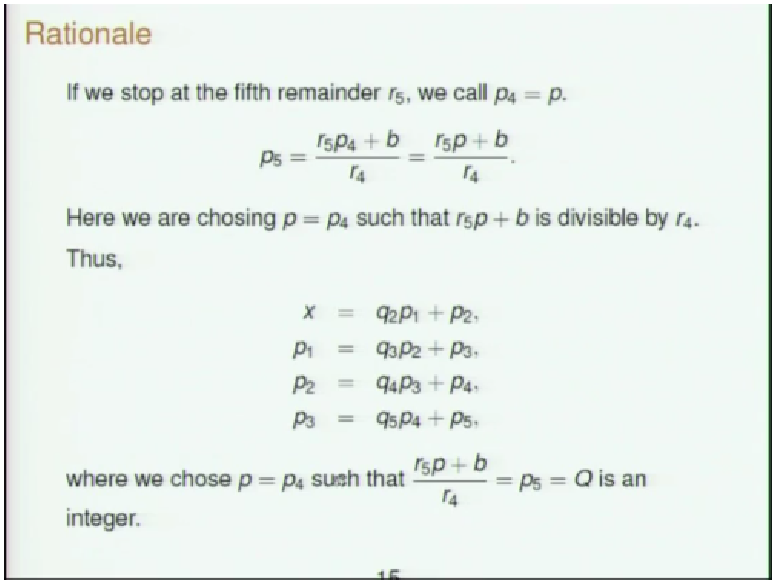

You are you know using a number let us say the suppose we stop at 5th remainder r5 and we call p4 as p, so the procedure which was you know given of that you know this r5p4+b/r4 that must be integer you call it p5, so it is equal to this. So, here we are choosing p is equal to p4 such that this is divisible by r4, so you get x is equal to q2p1+p2, p1 is equal to q2p3 etc., like this where we have chosen p such that this is equal to integer.

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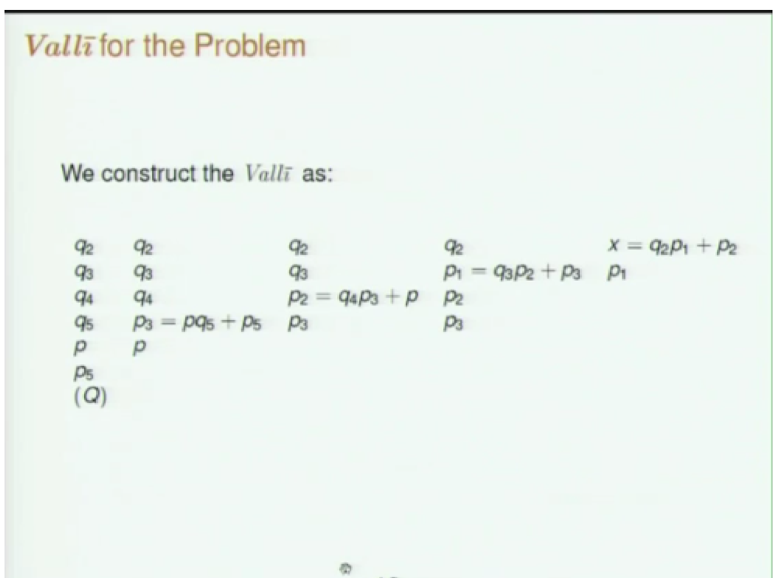

So, that is how you get the this thing you know, so you have to analyse it a little bit you know but nothing more than you know in a linear equation, so this will the justification of the procedure okay, so this is this.

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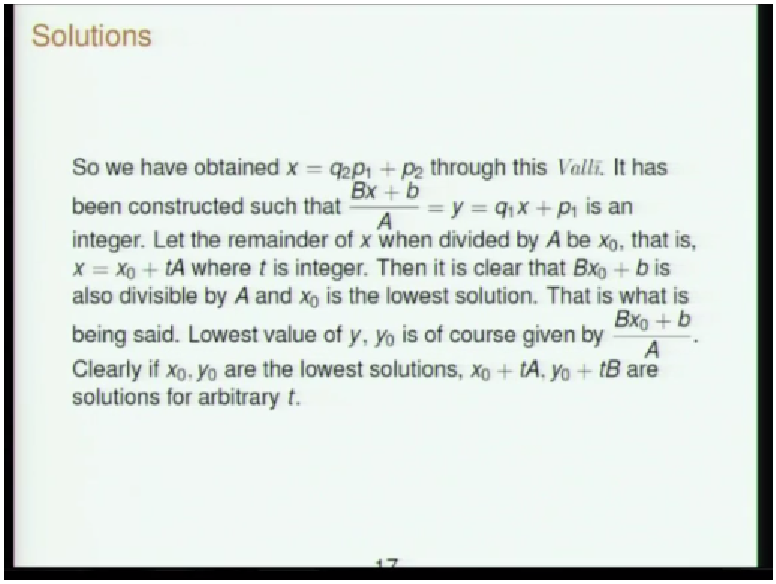

So, what we have obtained is x is equal to q2p1+p2,p through this valli, it has been constructed such that you know this is automatically integer Bx+b capital A by A is equal to y is equal to q1x+p1 is an integer. Now let the remainder of x when divided by A be x0, so then x is equal to x0+tA, so clearly one can sees that x0 is the lowest solution and the general solution is x0+tA for x and similarly y0 whatever you get Bx0+b/capital A is y0 and y0+tB is the general solution for y okay.

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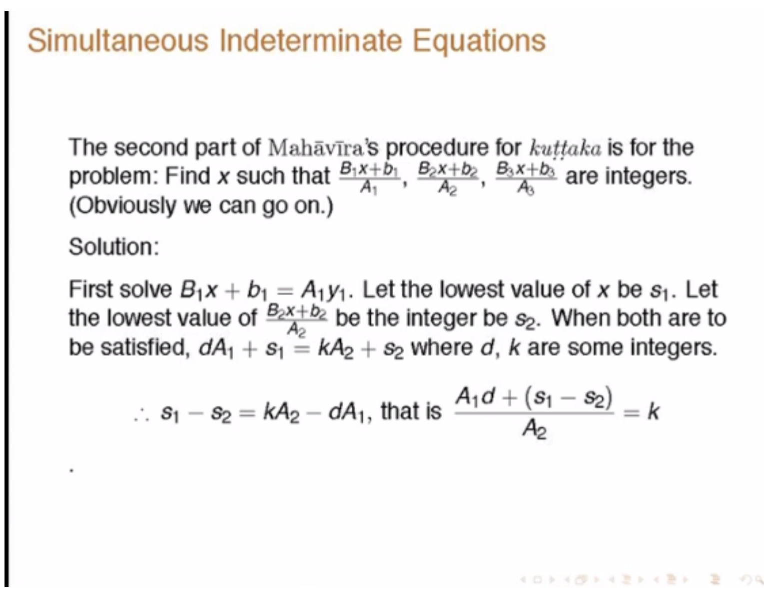

So, now this one equation is, so you get the point right I mean because see some equations maybe able to solve by inspection okay

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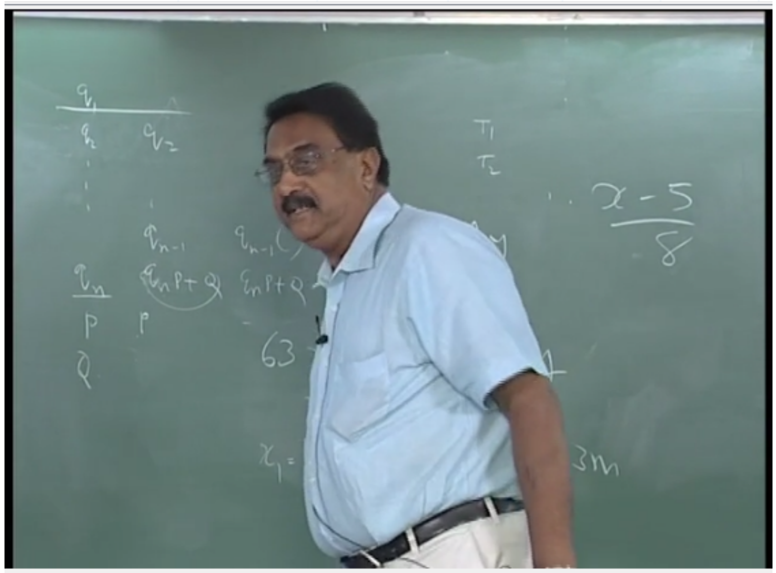

Suppose you want to solve this x-5/8 is an integer by inspection one can see that any integer see 5+8 we solve this or 5+some integral multiple of 8 will solve this you know by inspection when can see but all the problems can you solve this no, by inspection it is not easy, so that is what is given in the procedure which is work for all integers A and capital A and capital B. So, now the second part of Mahavira's procedure for (FL) is for the problem such that find x such that.

Let us say B1x+small b1/A1 is an integer, B2x+ b2/A2 is an integer like B3x+b3/A3 is an integer so on you see you can go on like that. So simultaneously I to solve this okay, so B1x capital B1x+small b1 is equal to capital A1 into y1, capital B2x+b2 is equal to A2 like that you know. So, how do we solve this in fact this also has been discussed by Mahavira, solve this first the equation he have already given the procedure.

Let the lowest value of s1 and solve this also the lowest solution for this is at stage s2 when both of them are to be satisfied see because the lowest value of this is s1, the general value is this dA1+s1 where d is an integer, similarly the general solution of this kA2+s2 where k is an integer. So, from this when you get s1-s2 is equal to kA2-dA1 that is A1d+s1-s2/A2 is equal to k, so now it is the one more (FL) you have to solve for a d you are finding you know d and k such that both the equations are satisfied right.

So, you have solve for d and k such that is you know integral value such that is this equation is satisfied okay.

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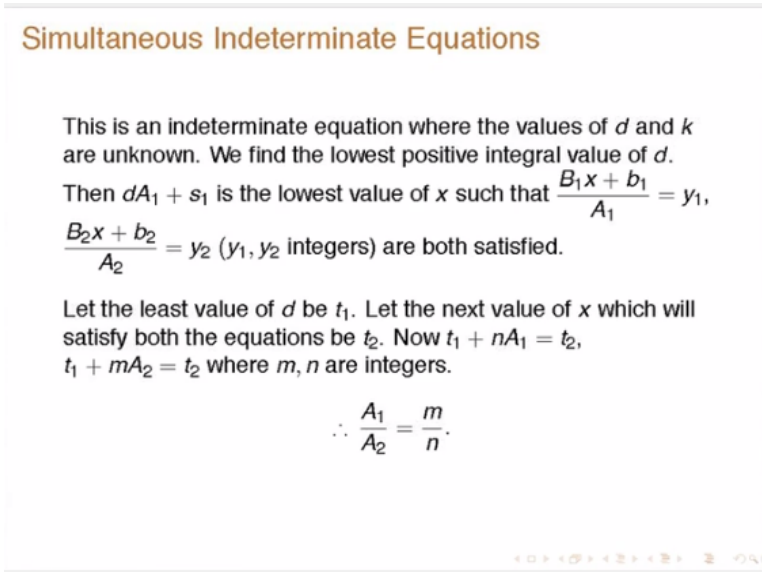

Indeterminate equation where the value of d and k are unknown, so we find a lowest possible integral value of d then dA1+s1 id the lowest value of x such that this is an integer, similarly this is an integer let the least value be t1, let the next value of x which satisfy this t2 then t2 must be t1+nA1 because the first equation if you take the lowest then if you multiply multiple of A1 that will be a solution.

Similarly for the second equation the lowest is t1 the common solution we have got a multiple of A2 must be a solution and both the equations had to be satisfied when you have to solve both the equation, so you get you know from this 2 equation you get A1/A2 is equal to m/n.

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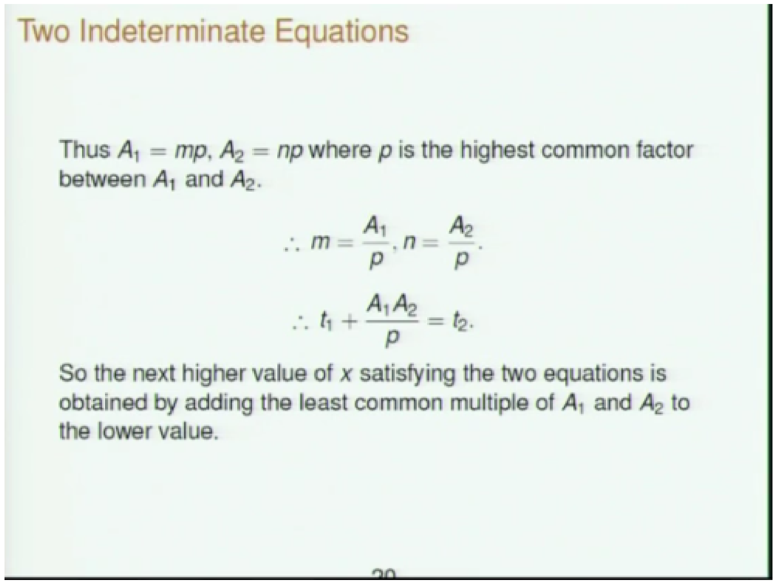

So, now let A1,A2 capital A1, capital A2 where p be the highest common factor than A1 is equal to mp, A2 is equal to np, so we can write it like this. So, p1+A1A/p is equal to t2 in this, so that will satisfy both the equation, so you got the point t1 is the lowest solution which satisfies both the equations and the next value is you know see both A1 and A2 are involved and actually the next integral value is p1+A1A2/p that is the next higher value.

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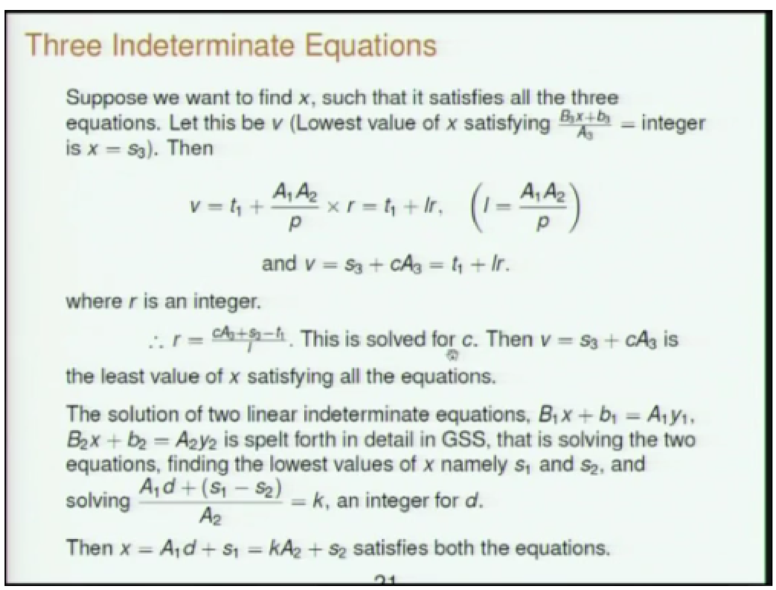

And if you want to go solve the third equation, so then you have to solve this you have to this also has to be satisfied this also has to be an integer, so the solution of this v is s3+cA3 let us say. Now the solution of the previous 2 equations will be like this, this order that is the lowest is this and a general is this okay where r is an integer, so we have to essentially again we have to solve this (FL) cA3+s3-p1/l is equal to r, so that is what you have to do.

But of course third one is a 3 equations (FL) does not given detail but the 2 equations he does discuss in detail it inverse and I wanted to describe what it is.

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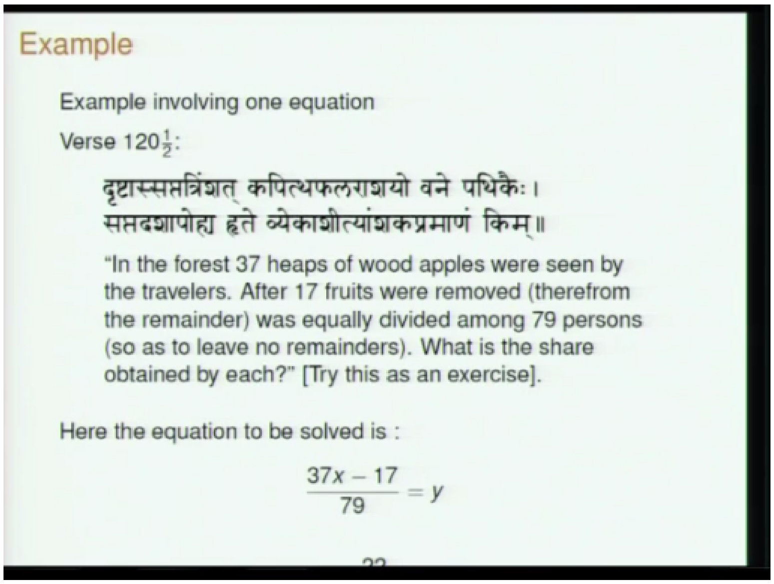

So, only if you work out some problem will be understand it is fairly simple but because so much is said you know among us maybe rattled. So, the problem is in the forest 37 heaps of wood apples were seen by the travellers, after 17 fruits were removed therefore the remainder was equally divided among 79 persons, so as to leave no remainders what is the share obtained by each okay.

So, 37 heaps were there okay, so then 17 were removed from that 37x, so suppose the number in each heap is x, so 37x, 17 were removed from that, so that could be divided among 79 people 37x-17/79, so that is an integer, so this is a (FL) equation. Of course this is negative okay, so this b could be negative also.

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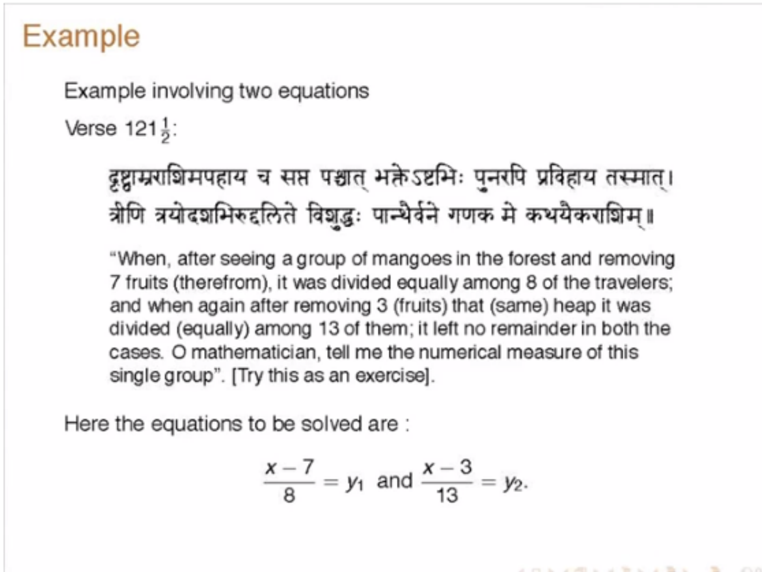

So, you have to use this procedure which I said, so the next also is a (FL) when after seeing a group of mangoes in the forest and removing 7 fruits therefore it was divided equally among 8 of the travellers and when again after removing 3 fruits that heap was divided equally among 13 of them, it left no remainder in both the cases, o mathematician tell me the numerical measure of this group. So, when you measure if when you from that heap if you remove 7.

So, then it is divisible by 8 because it is distributed among 8 people, so x-y x-7/8 is y1 then x-3/13 is similarly when you remove 3 from them it could be distributed among 13, so these are the 2 equations here we have to follow.

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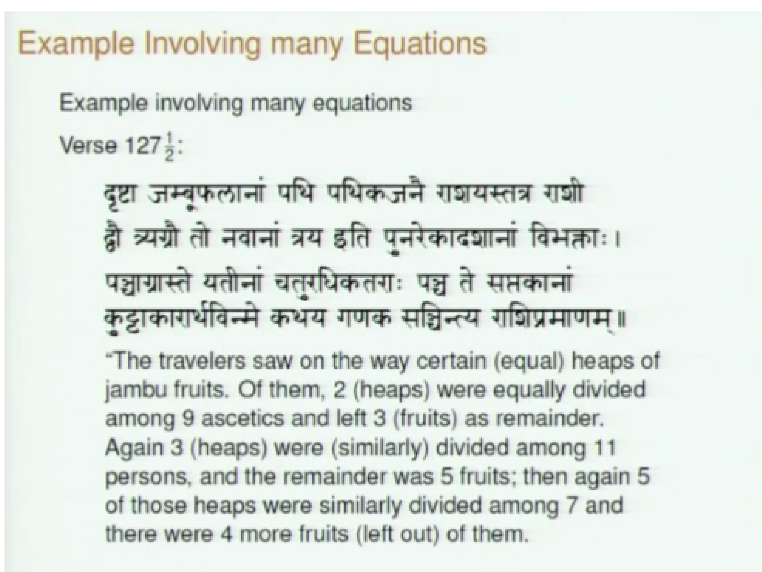

So, one can do that, so I have told you always Mahavira will have lot of problems for each kind of this is you know a topic that he discusses, so he will examples many also many equations also use the involving many more equations for instance 3 equations involving 3 linear inderminate equations here given this following problem (FL). The travellers saw on the way certain equal heaps of jambu fruits.

So, 2 heaps were equally divided among 9 ascetics and left 3 fruits okay. So, after distributing them 3 was the remainder again 3 heaps were similarly divided among the 11 persons, and the remainder was 5, 5 again were heaps similarly divided among 7 and there were 4 more fruits left.

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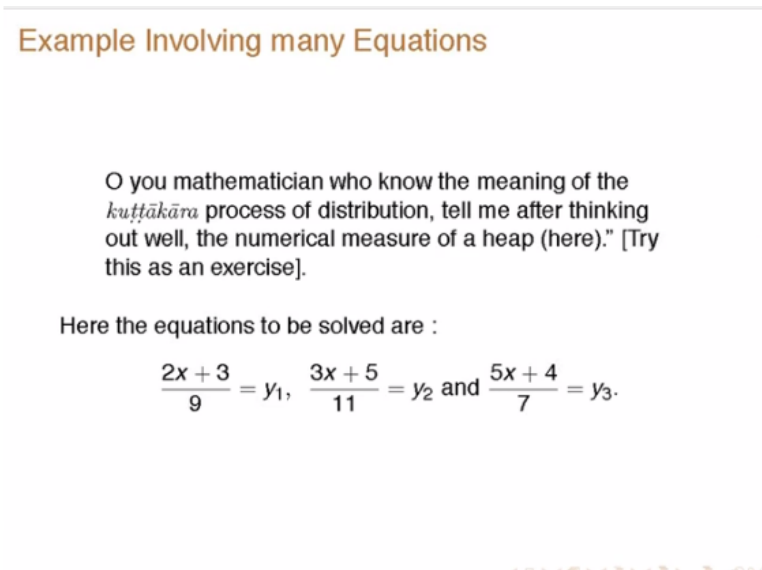

So, clearly essentially what you have to do 2x+3 among the 2 heaps you know (()) (26:39) was divided among 9, 3 was remainder 2x+3/9 is equal to y1, 3x+5/11 is equal to y2, 5x+4/7 is equal to y3, so this indeterminate equations you have to follow.

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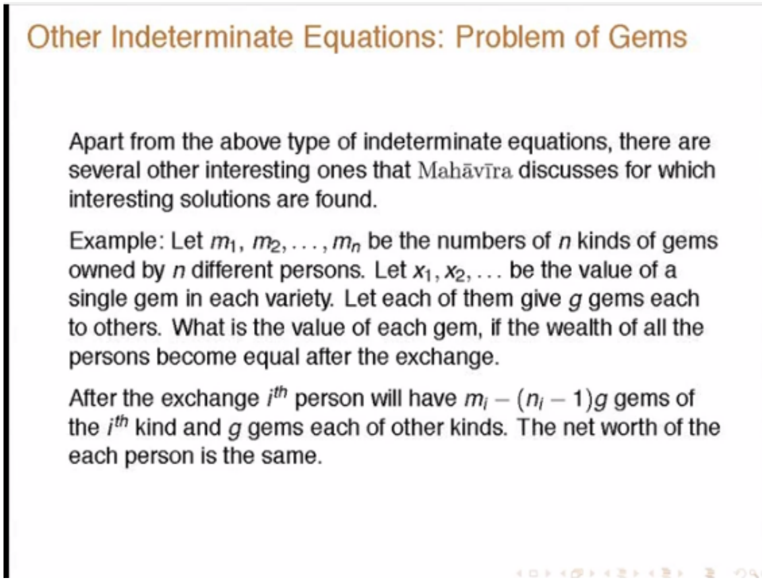

And you can solve it you know you just have to look at the procedure a little more in detail and it will be able to, this is one kind of linear indeterminate equations are full of the one of the most important but Mahavira considers several kinds of indeterminate equations whichever of interest. So, one of them another of them apart from this (FL) is only (FL) is the following thing following problem he considers.

For instance suppose let m1, m2, mn be the numbers of n kinds of gems, so n persons each of them has some different kind of gem some are diamond some are emerald, sees like that and x1, x2 be the value of the single gem in each variety. Let each of them give g gems each to others, what is the value of each gem, if the wealth of all the persons become after the exchange okay. So, that is each of them will have some gems particular kinds of gem with a particular value okay.

So, now are there will be some exchange program okay, so the for instance the ith person will have mi gems, so each of the other persons ni- 1 person he will give g gems of his own variety okay, so other he will receive also from each of the other he will receive g gems okay. So, finally he will have mi-ni-1 g gems of kind he is own kind and g gems have each of the other kinds. So, the net worth of each person is stated to be same okay.

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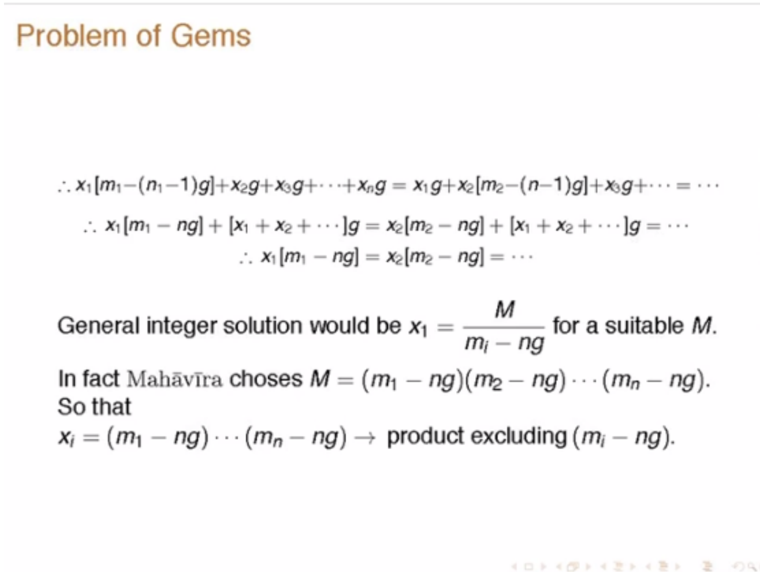

So, what you get his work from whatever the number gem he has the value of that will be clearly he received g gem from each of the other. So, x2g+x3g+etc., xng and his own what you are he has left with his own gems you know that is m1-n1-1 into g. So, this and there is other person also will have similarly will have this kinds of a the value of the gems will be like this. So, you will have x1 into m1-ng is equal to x2 is equal to m2-ng is equal to so on.

So, general integral solutions will be x1 is equal to capital M/m1, obviously is x1 x2 set of there are inversely proportional to this things. So, you through them you know and divided by mi-ng that will be xi, so in fact Mahavira chooses M is equal to this mn-ng, so that x1 is m1-ng etc., mn-ng in which this is excluded okay. In fact you can take a multiple of that all, so you know multiply all of them by the some number that also will be a solution okay.

Because what is said is that the wealth of each person will be the same okay, so that maybe any value, so there is some arbitrary and this in this but the in that the ratios are what you can actually determine from this.

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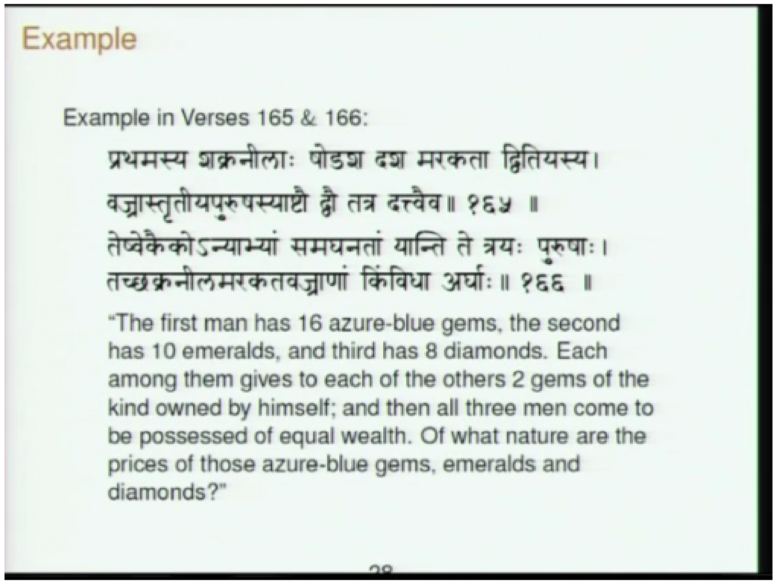

So, he has given the example (FL) okay, so the first man has this 16 azure-blue gems (FL) okay, the second has (FL) then emeralds okay, the third has 8 diamonds each among them gives to each of the others 2 gems of the kind owned by himself and then all the three men come to be possessed of equal wealth, so what are the values of this.

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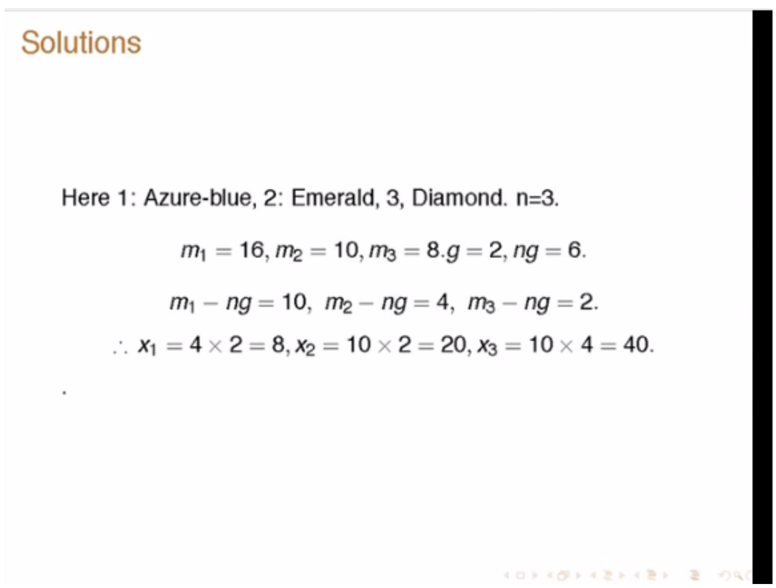

So, you can find that, so here 1 is Azure-blue, 2 is emerald, 3 is diamond, so the number of persons is n is equal to 3 and m1 earlier the first person had 16, second has 10, third is 8. So, g is equal to 2 here he they exchange each of them giving 2 of his own kind to the others, e is equal to 2, so ng is equal to 6, so you find this m1-ng and the way he has saved the solution m1-ng is equal to you can find these things.

So, then x1 is you know this into this so that is 8 x2 is this into this, so that is 20 and x3 is (()) (31:49). So, if the algorithm is given here you see for solving this, so similarly you consider what is the (FL) were this was the all this things now see everybody Bhaskara will talk about it in a then Naraayapunita will talk about this, these all (()) (32:07) straightening more and more slightly difficult problems in that category, so that is how Indian mathematics move.

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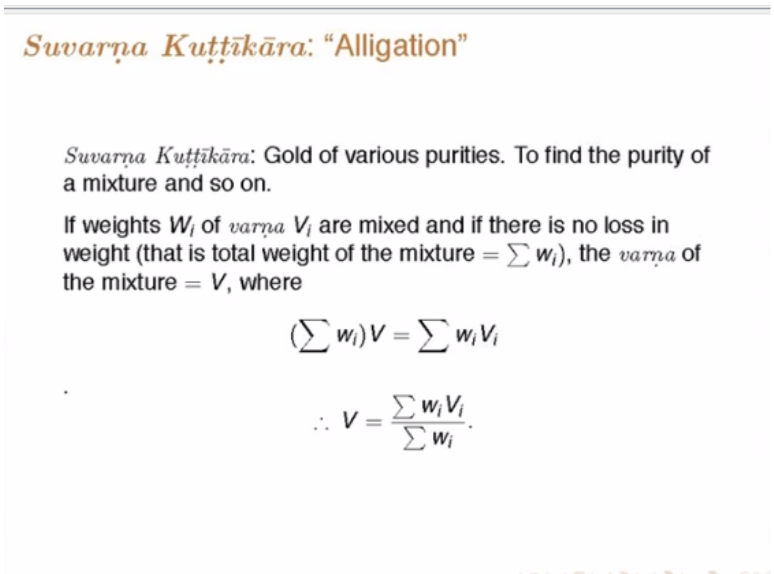

Similarly (FL) you know, so you have gold of various purities you know you can say because of a now call it is now 24 carrot, 22 carrot like that. so, that the measure of array purity is called (FL), so twice the blue eye of (FL) Vi are mixed and if there is no loss in weight, so then total weight of the mixture is this and suppose the (FL) the mixture is this okay, the various (FL) are mixed, so what is the varna of the mixture.

So, clearly so that will be the amount of the you know pure gold will be same in both if there is no loss, so then this is the equation, so varna of the mixture will be this and of course they will also consider some problems where in some amount will be lost and all that the weight may not be you know the some of the earlier weights, so that will be slightly different.

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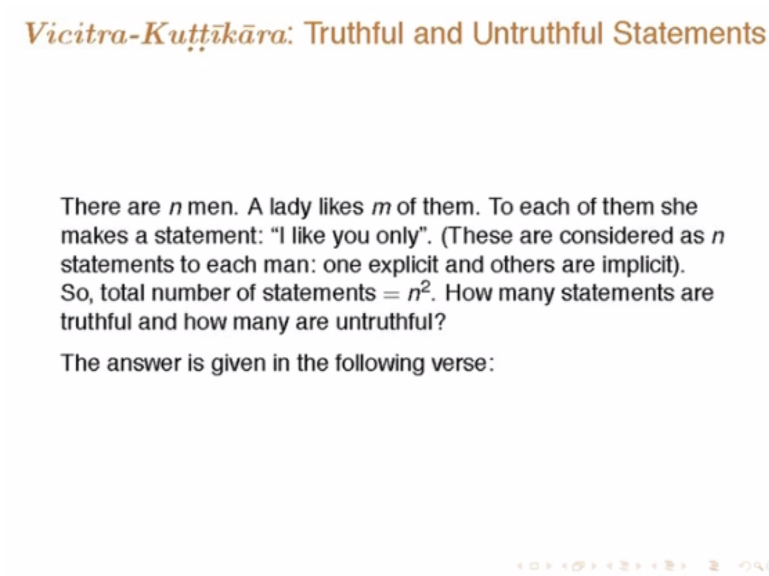

And then there are some interesting problem (FL) truthful and untruthful statement that also is you know various people after Mahavira onward they talk about that. So, the n men, a lady like m of them to each of them she makes a statement "I like you only" but that statement is considered as you know n statements I mean she is making 1 statement but the implicitly is assume that she is making n-1 statements you know I do not like that person I do not like he each of them she is making it.

Suppose he actually likes n of them then how many truthful statements she has made and how many untruthful statement, so total number of statement is n square, so each of them she is making n statements you know I like you I do not like this of course she might not say it is simplicity similarly second person also she will say I like you and it. and what actually she likes she like m of them okay, so then how do fall this.

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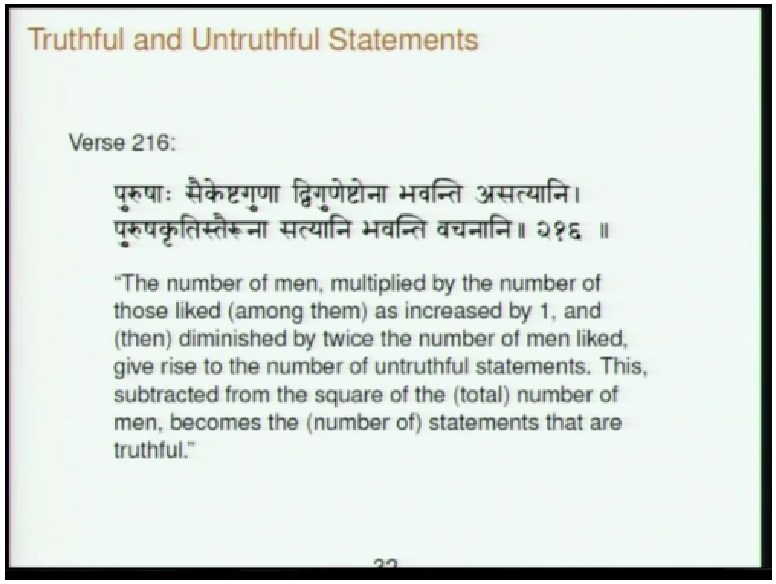

So, here you are given the solution (FL) the number of men, multiplied by the number of those liked among them as increased by 1, and then diminished by twice the number of men liked give rise to the number of untruthful statements, this subtracted from the square of the total number of men, becomes the number of statements that are truthful okay.

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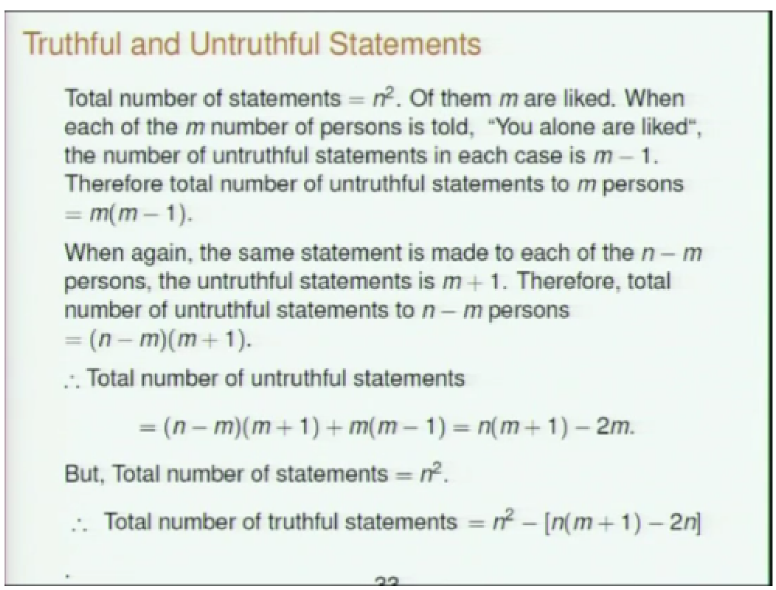

So, total number of statements is n square of them m are liked see, so m suppose you consider this subset of m people whom the lady actually likes when each of the m number of persons is told you alone are liked the number of untruthful statements in each case m-1 okay. Because she says I like you which is true then I do not like this you know m-1 others which is not true and he also saying I do not like those others the other subset which is true okay.

So, the untruthful statement is m into m-1 similarly same statement is made to each of te n-m persons, this subset of unliked entities is they for them the untruthful statement the each of them is m+1 because to that individual she is saying I like you but actually she does not like you, so that is one untruthful statement then she is saying you know I do not like this subset of this others which is not rue she likes them the m each of them.

So, that is untruthful and then she says in others in this other subset you know the disliked entities she say I do not like that which is true , so you have to carefully understand. So, the total of untruthful statements n-m persons is n-m into m+1, so the total number of untruthful statements is, so finally so you have to add this things m into m-1+this so untruthful statement (FL) n into m+1-2m, so total number of statements is equal to n square.

So, n square-okay, so this is the kind of thing I am sure you know in the present context also maybe we have to revise some algorithms you know so many into truthful and untruthful statements are made in the publics there must be a analysing it.

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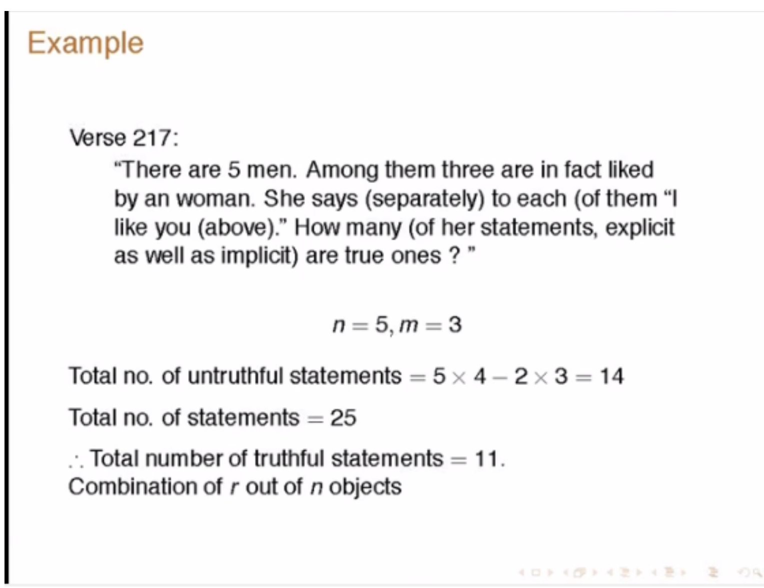

So, she has given a problem there are 5 men among them 3 are like in fact liked by an woman she says separately to each of them I like you alone then how many of her statements are true one n, so here n is equal to 5, m is equal to 3, so the total number of untruthful statement is 14, the total number of statements is 25 because this thing is there, so total number of truthful statement is 11and and the total number of untruthful is 14, 14 then.

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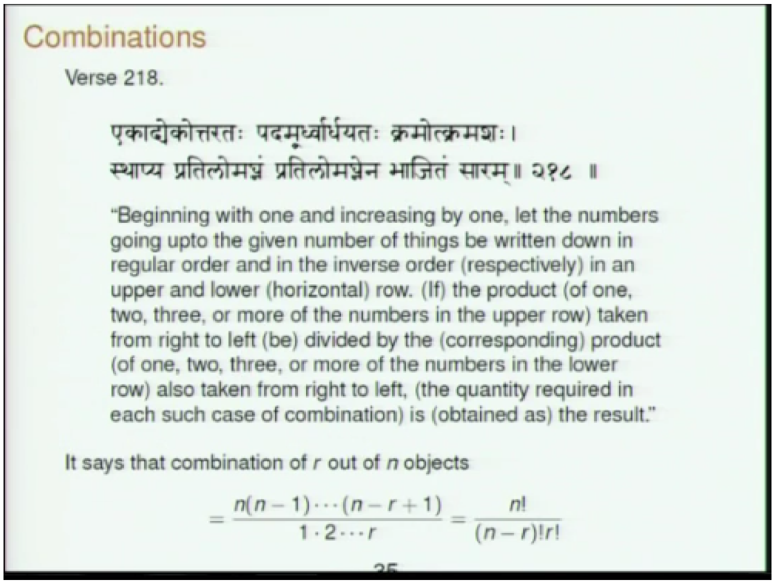

So, now comes a very important verse in Mahavira's (FL), so he now gives the expression for the combination of r out of n object the n objects are there then what is the combination of r out of n object (FL), so beginning with one and increasing by one, let the numbers going up to the given number of things be written down in regular order and in the inverse order respectively in the upper and lower horizontal row.

If the product of 1, 2, 3 more etc., the numbers in the upper row taken from fight to left we divided by the corresponding product of 1, 2, 3 etc., in the lower order also taken from right to left, so quantity required in each case is a result okay. So, essentially he is I mean of course he is taking 1 to n , so in this starting from n, so it is equivalent to this upper row is n into n-1 into this divided by 1*2*r, so factorial n by factorial n-r into factorial r.

So, this is the I mean what is given is this essentially you take the upper role like this and lower role like this he already combinatory this see you know this combination word discuss by professor Srinivas in this context of (FL) and makeable tasks and so on. He but here it is it has not this solution are not given in this form okay. So, in difference form, so if the particular of this particular form it is given for the first time for the here.

So, there is the very but strain it discuss too many problems and in exercises related to that he only gives some 3 problems in order of another problems the case is again the 6 rust you know taste, so 6 so then by mixing them then how many this thing you will get it taste kind of a thing, so that is as you know that is the totally 2 to the power of 6-1, then he talks about you know of course there are some flowers okay (FL) and (FL) okay 4 kinds of flowers.

So, how many kinds of gardens you can make, so that is simple thing, so let us say some 2,3 only you will discuss. But it is a you know really relate the foundation for this combinatory in a of course apart from what was actually done earlier in (FL) books and late row course Bhaskaracharya 2 carries forward this quite a bit and then Narayanapunita carries forward to a very great extend, so that will be discuss later, so this is an important thing is the in the text.

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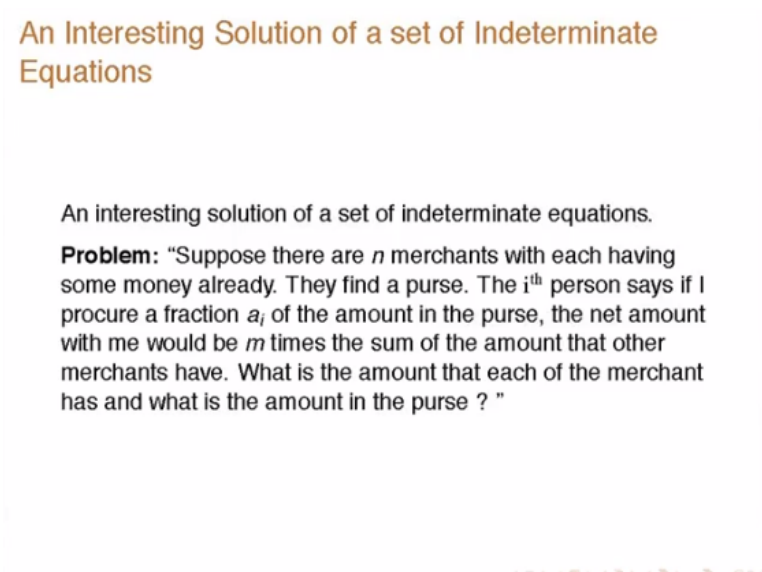

So, then he is as I told you that various kinds of linear indeterminate equations are there and (FL) was one of them and some other things also we discuss, so that one more interesting problem which he discusses. Suppose n merchant each having some money already they find a purse the ith person says if a procure a fraction ai of the amount in the purse the net amount of with he would be m time the sum of the amount that other merchants have what is the amount that each of the merchant has and what is the amount in the purse.

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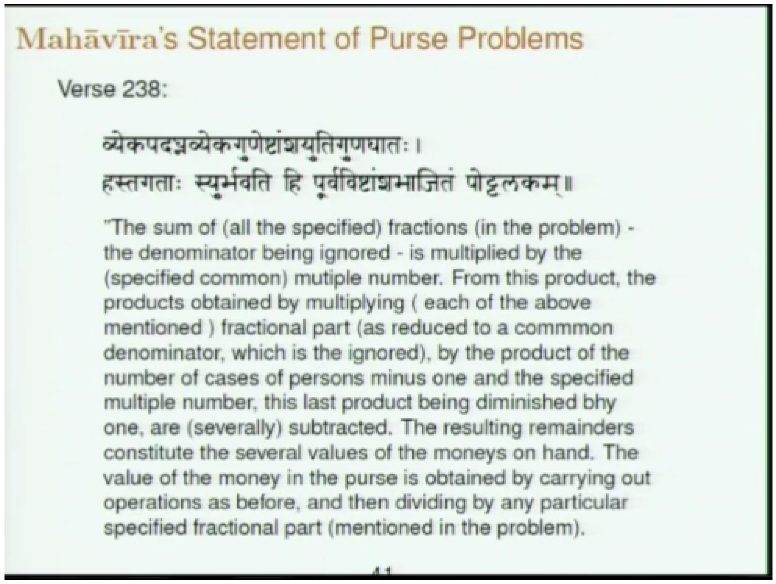

So, so essentially, so that is called you know (FL) the purse, so he has given this thing solution here, so I will keep this as do not too much time, so such a problem is the following you see

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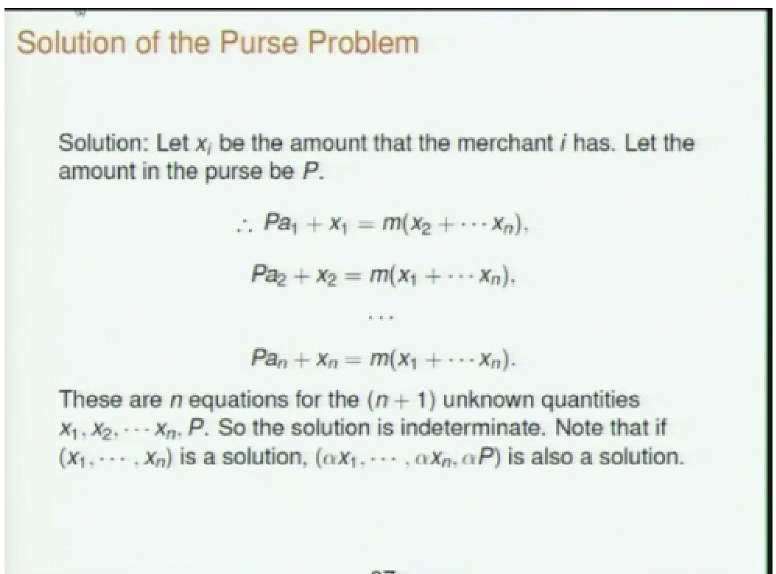

That is suppose some amount of money is found capital P this is the first person say that (FL) this amount x1 suppose I get a fraction a1 of this purse amount okay. So, then my total amount will be m times the whatever many that others have just now so this is this similarly Pa2+x2 is this etc., so n equations in n+1 unknown quantities. So, the solution will be given like this.

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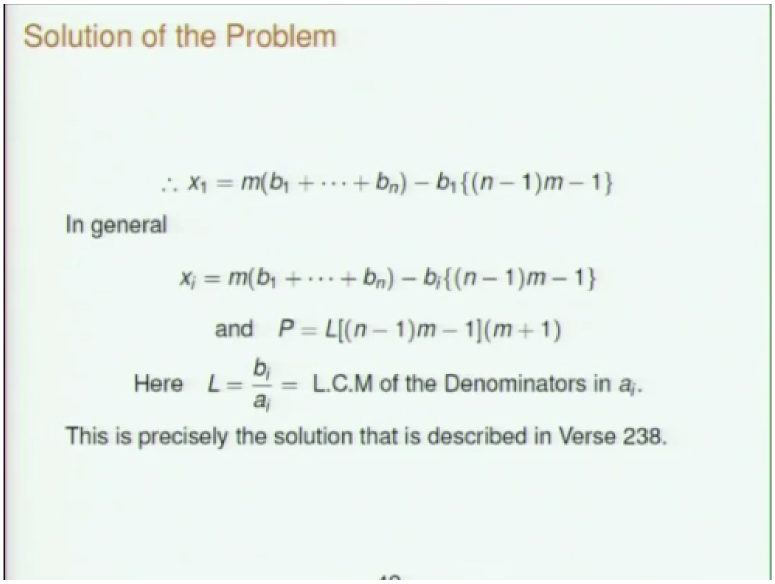

The solution is given in this from x is equal to m into b1+b2 etc., - this one. So, and the purse is this you can check this it is the matter of small details only but you should realize that is indeterminate equation because essentially see these are the equations right these are the n equations in n+1 unknown x1, x2 etc., xn they are the unknown and the amount of money in the purse p that is also unknown.

So, the n equations for n+1 quantities so and you can quickly see that you know if the solution is you know suppose 1 solution is x1, x2, xn and p then another solution if you multiply this by any quantity any number that also will be a solution because if you multiply this equation you see suppose instead of p you put alpha p instead of x1 you put alpha x1 if you instead x2 you put alpha x2 then same will be satisfied right.

I just multiplying all the equations by alpha and both left hand side and right hand side, so the actually solution is up to some arbitrary constant you can say the ratios are given, so that is what is interesting.

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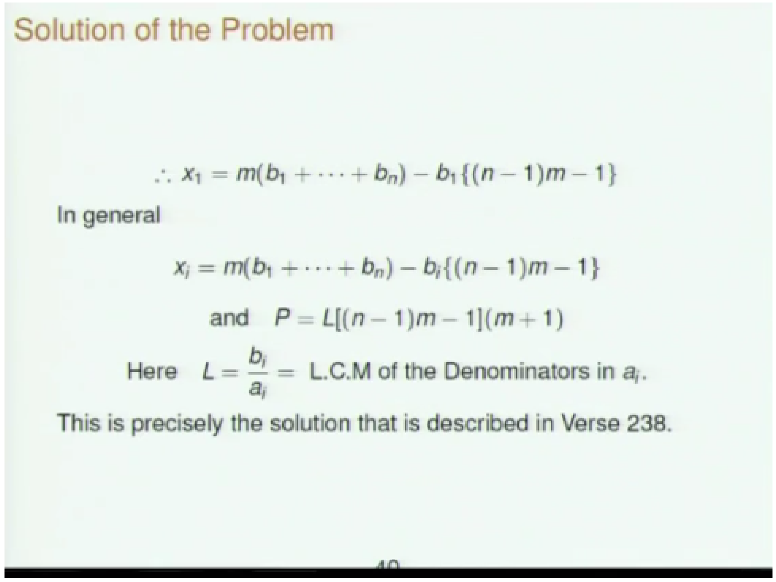

You have one (()) (43:44) try to solve this problems you know to get them integral solutions then you will know it is a very nicely here chosen. So, he has stated this.

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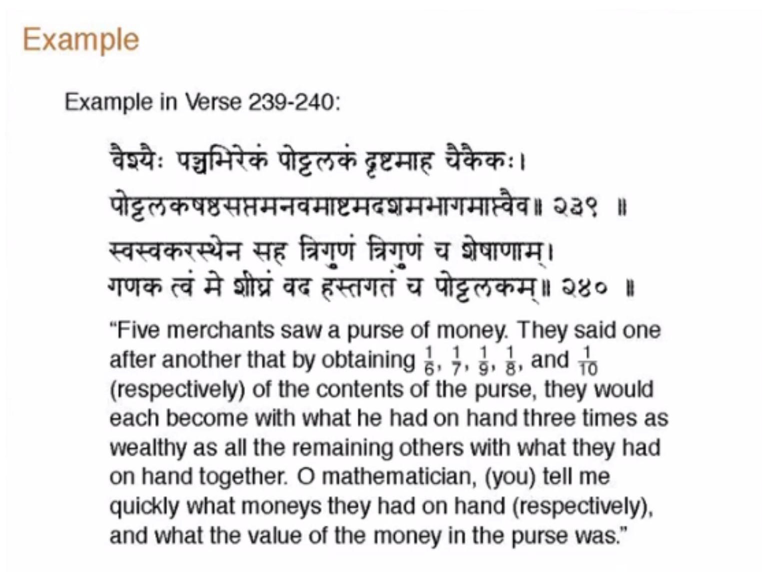

So, I given some this things also some example also 5 merchants saw a purse of money they said one after the another that by obtaining 1/6, 1/7, 1/9, 1/8 and 1/10 respectively of the contents of the purse, they would each become with what he had on hand 3 times as wealthy as all the remaining others with what they had on hand together O mathematician tell me quickly what moneys they had on hand at what the value of the money in the purse was.

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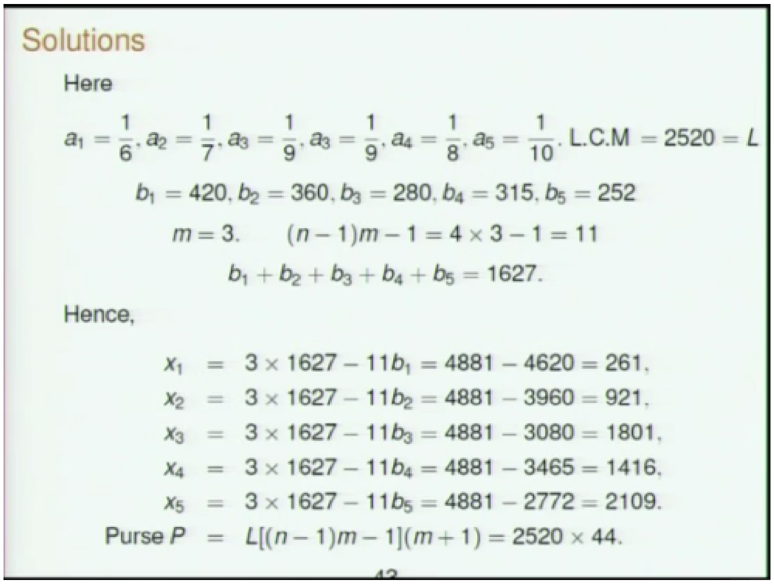

So, here the solution the thing is the fractions of this and you can take LCM will be this, so solution will be given here and the total number of the (()) (44:42).

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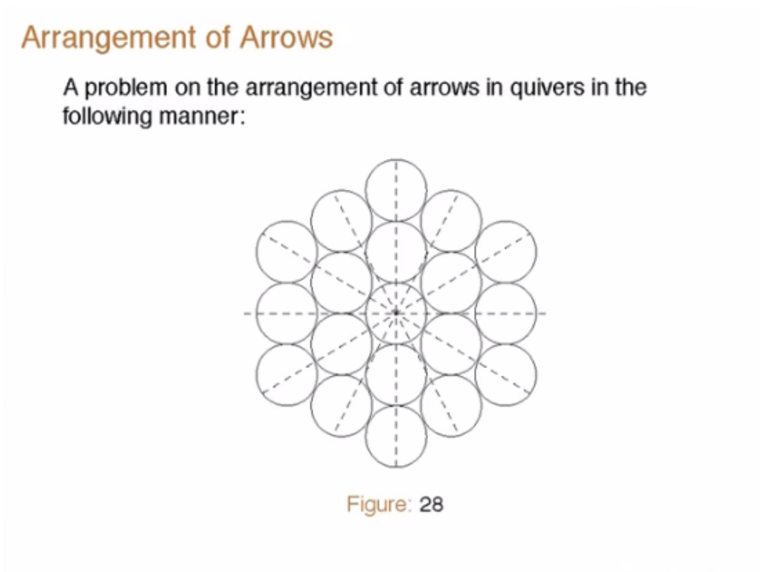

So, then he talks about some other problems problem of arrows okay I will keep this yeah.

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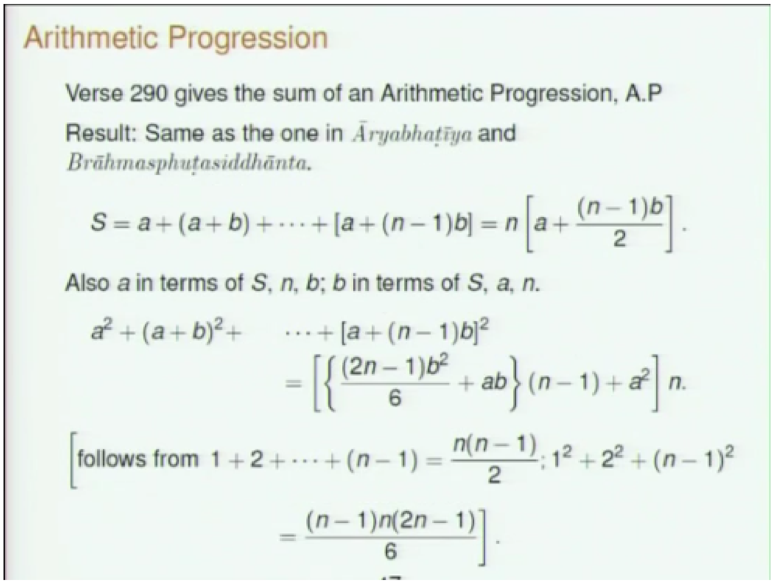

Arithmetic progression I told you again some advances are made you know a little more complicated things are done you see, see this is the arithmetic progression you know I will know already it has been hammered that this is the you know some by Aryabhatta, Brahmagupta etc., so this is a solution, so now you take this sums of the squares each number is a arithmetic progression you will take the sums of the terms in arithmetic progression.

So, then he gives this as the result you can see that one as to you this things some of the first n integer some of the square to the first n integer then you get the result.

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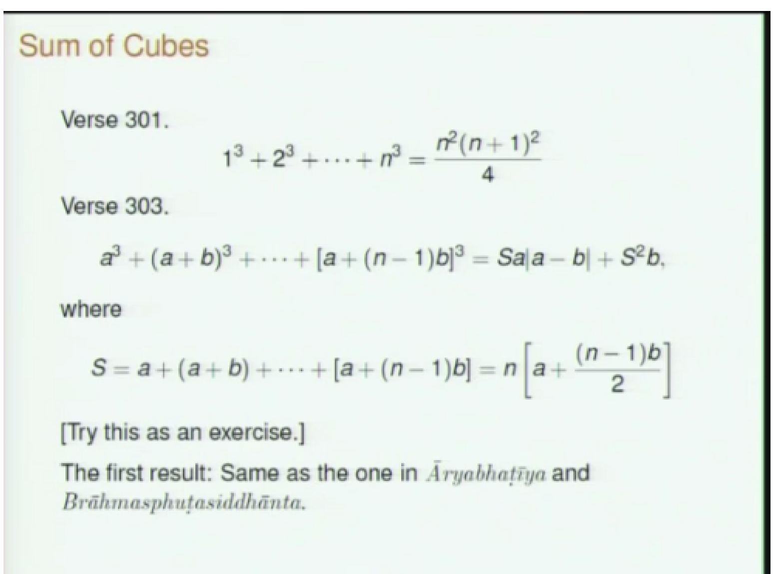

A cube also I will I said this already was tell Aryabhatta, Brahmagupta a cube +a+b whole cube etc., so then so that is you take arithmetic series. So, then take the cube of each of them the sum of that will be equal to this you can check that I think you should try this as an exercise.

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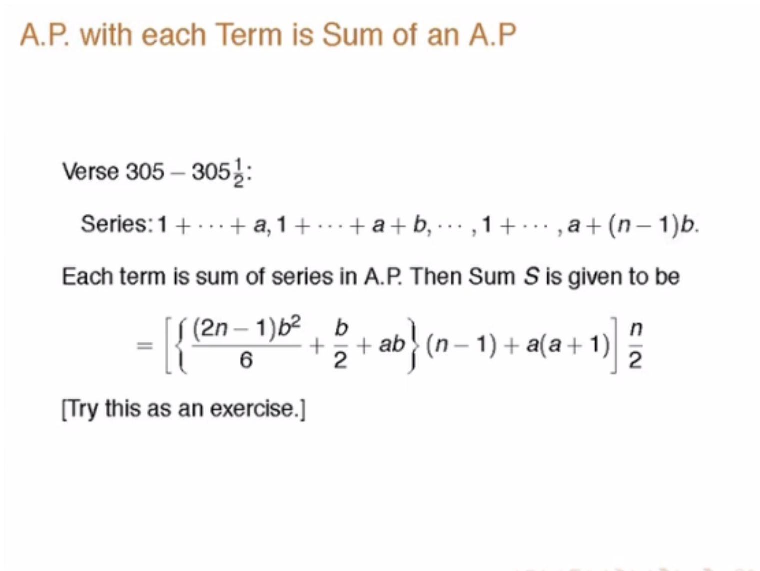

Then he also now goes to a arithmetic progression in which each term is again a sum of an arithmetic series okay. So, the series is 1+ etc., up to a and next term is 1+ etc., up to b and the term is 1+ etc., n-1b a+n-1b. So, this is the thing so each term of the sum of series each term is sum of the series in arithmetic progression the sum can be in fact it is also a good exercise from whatever you know you do not need anything expect that what have been done earlier you know on this you know sum of sum and sum of sums and all that. So, whatever results you got earlier that is good.

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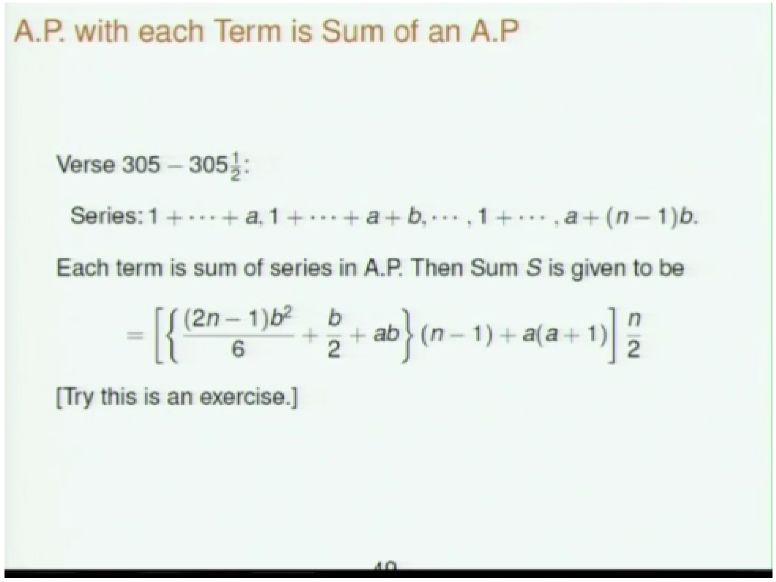

One can go up to a and then this is go up to a+b were amny more terms here in a second this thing term yeah suppose a is 20, b is 30, so the 10 more terms are there in which is second in this thing you know each is the sum actually each term in the each series is a sum by itself yeah.

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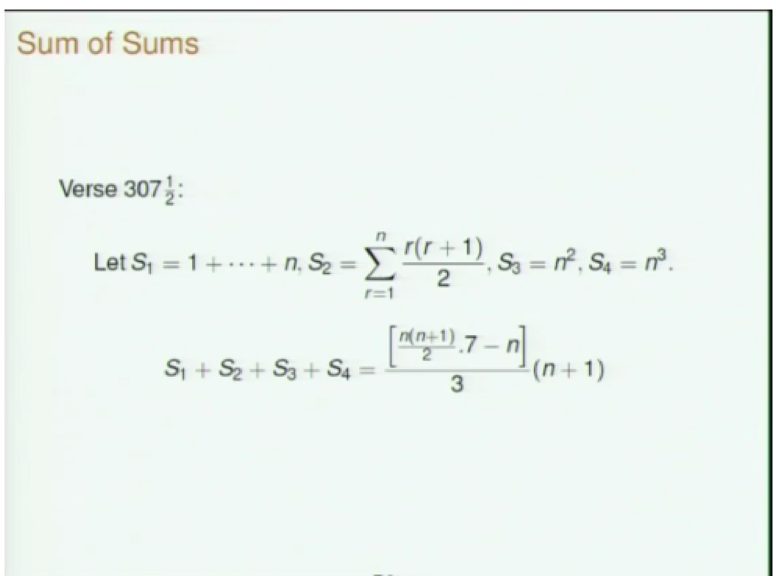

So, similarly this sum of sums and all that some combinations is given.

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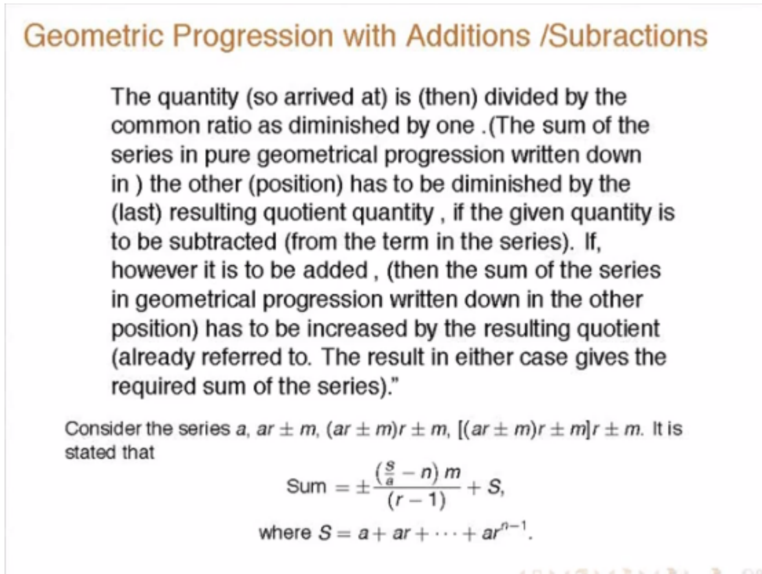

Then he has given some more problem you know various things you know suppose you had a, ar etc., so then suppose you have a series geometric some series like this a first terms is a, second term ar +/-m it is not just neither geometric or arithmetic so it is your multiplying by some quantity and then adding that subtracting sum series. Then this into r +/-m so those kinds things are also consider.

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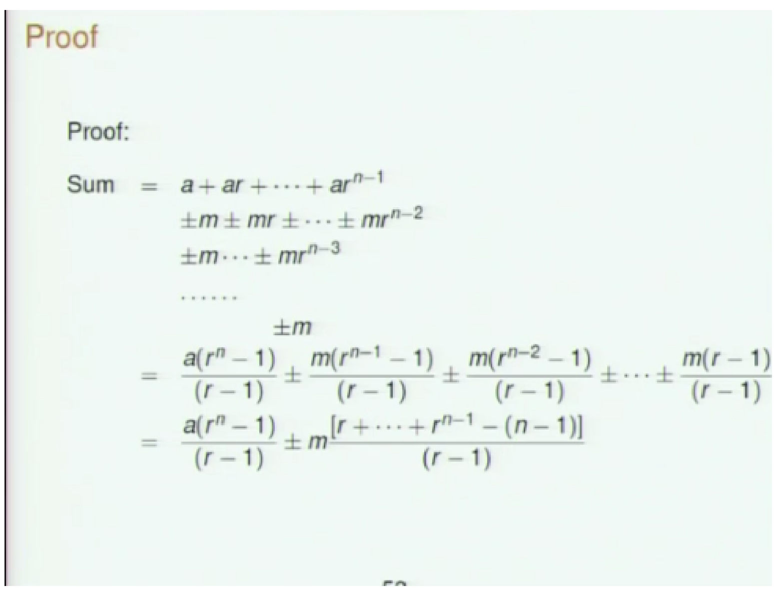

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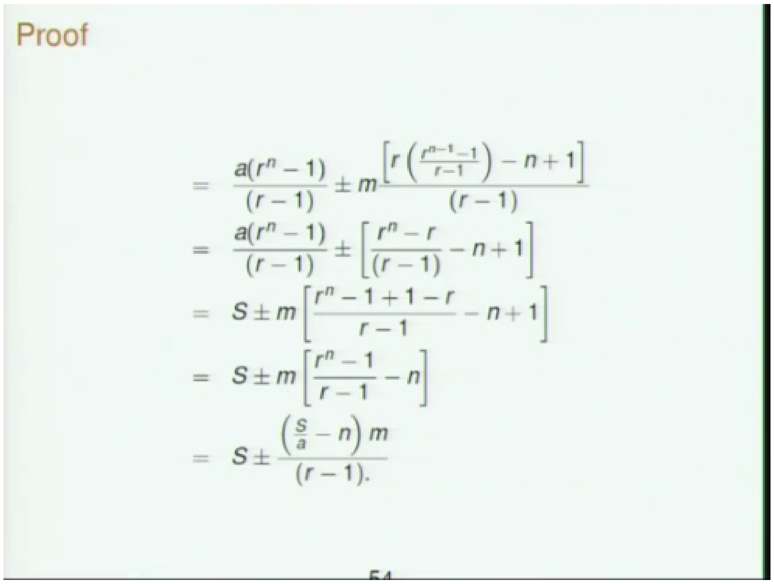

And one can sees that it will be one can you can find the sum okay you just have to see this slides at some leisure then you will get it.

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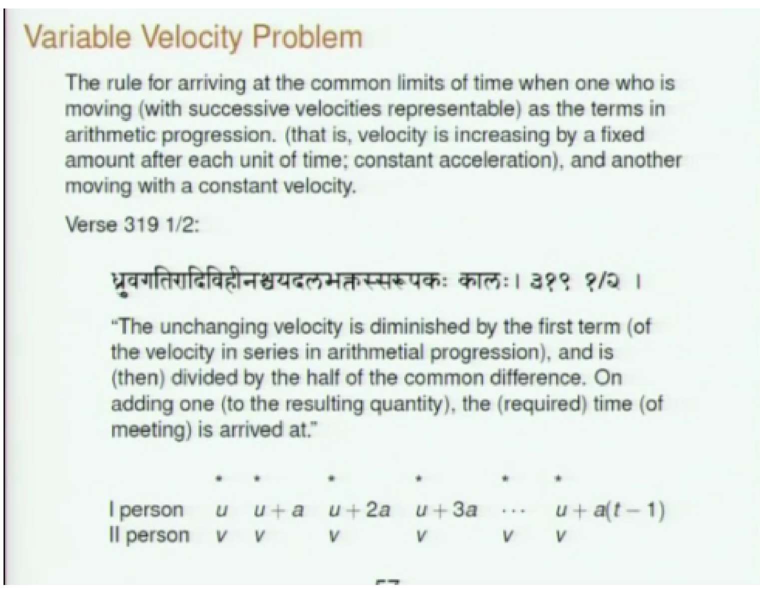

And then lastly I will discuss the variable velocity problem somewhat you know relate to a physics kind of problem, so the rule for arriving at the common limits of time when one moving with successive velocity represent as a terms in the arithmetic progression. So, that is velocity is increasing by a fixed amount after each unit of time, so that is some kind of a constant acceleration and another moving with a constant velocity.

So, it is (FL) the unchanging velocity is diminished by the first term of the velocity in series in arithmetical progression and is then divided by half of the common difference an adding one to the resulting quantity the required time this thing is arrival. So, what is the problem, the problem is the following you see, see this are you know 2 persons, 1 is moving with a constant velocity V, constants P, the second is accelerating.

So, when he go some here to here the velocity is u then here u+a okay, then here u+2a etc., see acquires some more velocity at each step, so that is called acceleration as constant acceleration right.

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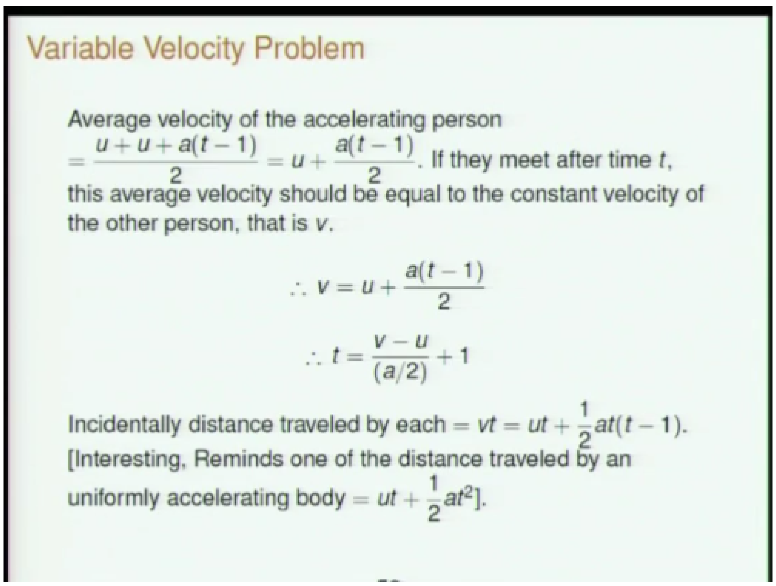

So, then clearly what is the time you know average velocity must be the same you see if they have to meet the average velocity when they meet must be the same see. They should have travelled a same distance okay. So, time is the same, so the average velocity should have been the same. So, then the average velocity is the first person is u+u this is the arithmetic series u+a into t-1/2 the second person of course is the constant velocity average is the his own constant velocity, so v.

So, t is v-u divided by a/2 +1 so that is what he saying, incidentally distance travel by each is V into t you see, so v into t and this is equal to ut+ half a into t into t-1 the distance you know I must sure it remains you know this formula constant acceleration a body is constant acceleration in school what you learn is that you know the distance travel is ut is state that is essentially given here.

Of course he discreetly accelerating you know at this speed steps, so that is why you half t into you will not get exact t, t into t-1 you are getting but in the limits when t is large okay and you know such that is product is finite, so then you will get actually that formula, so these a kind of a problem that you know and what is interesting is this algorithm which is given you see this is the formula is there okay and it is given in.

So, that is a constant question everybody asking how did they you know did they write equations like this. In the sloga itself he gives (FL) is add 1 kind of a thing okay add 1. So, then you take the difference between them and divide by the this (FL) is the difference you know in the arithmetic series, so divide by the **the** difference by 2 then there is a time. so, that is how express it.

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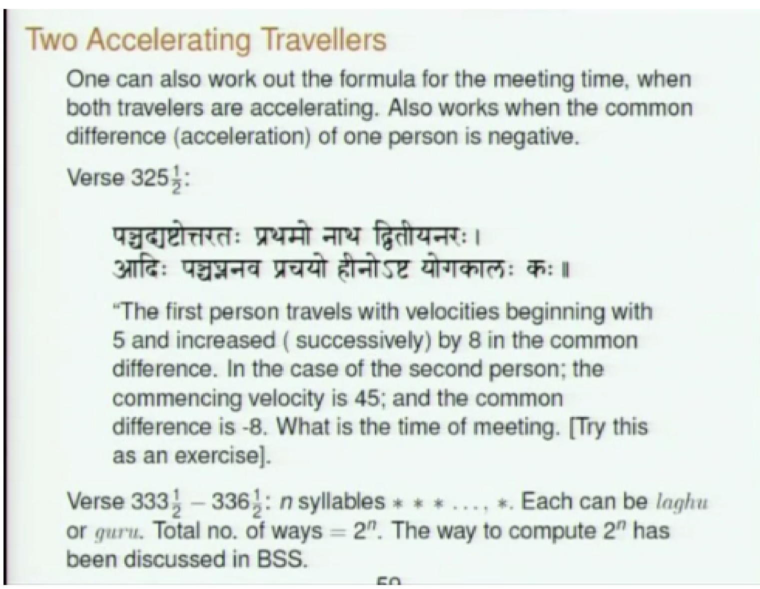

So, then he also considering 2 accelerating travellers and all that and he considers negative acceleration also you know positive negative sum one can be positive one can be negative because he is somewhat comfortable with positive and negative things right having formulated it in the beginning, so you ought to be refer that one. So, he gives some problems related to negative acceleration also finally he will give some.

I mean not finally I am saying another important thing n syllable how many (FL) and all that you know what is the total number of maze is that been you know discussed so many times I do not have to say that, to that answer is 2 to the power of n and the way to compute 2 to the power of n that also have been hammered, so that is also put here, so I think I will stop here, so these are the kinds of things that let discuss why discuss so many things.

But something I am you know just projecting you know for what about his most important, so some more things I will discuss in the last lecture or.

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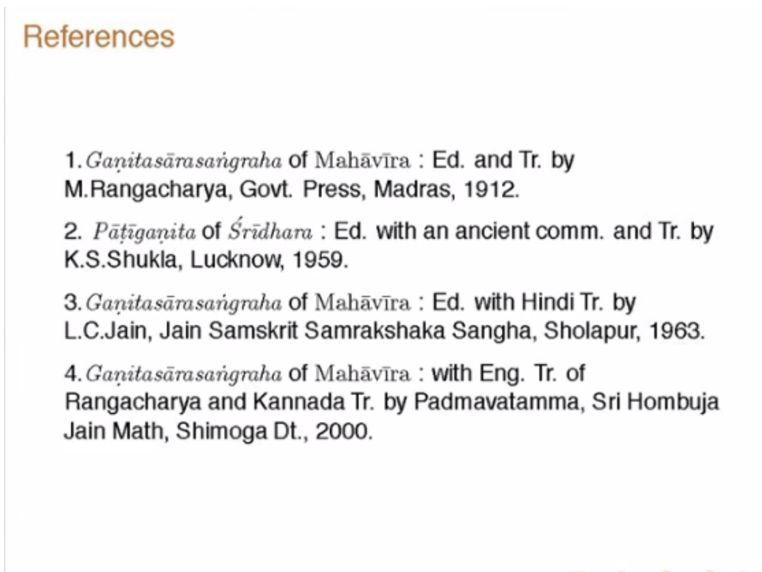

The references are given here.