Mathematics in India: From Vedic Period to Modern Times
Prof. K. Ramasubramanian
Indian Institute of Technology-Bombay, Mumbai

Lecture-14
Brahmasphutasiddhanta of Brahmagupta-part-4 and The Bakshali Manuscript

So, in this lecture I will start with the portion which I want it to discuss in Brahmagupta's (FL) and then we will move on to discuss the content of a very important manuscript called the Bhakshali manuscript.

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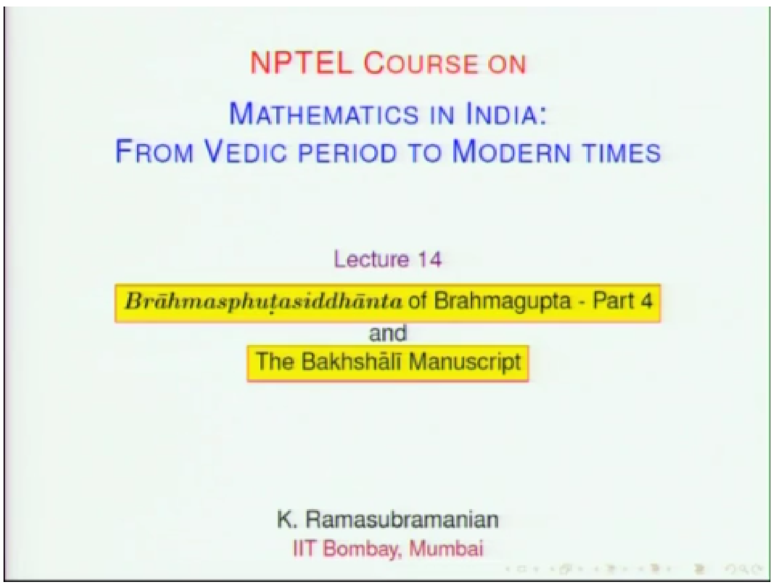

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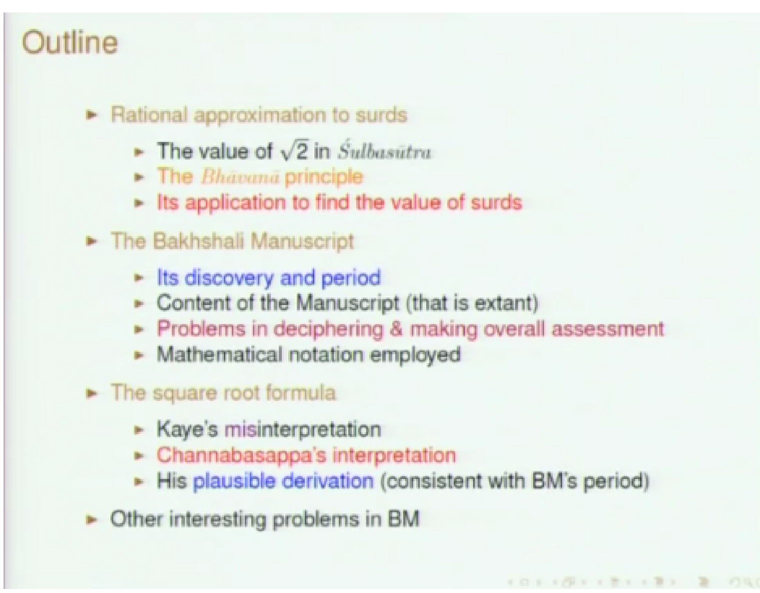

So, in the last lecture, so I discussed about this (FL) problem and the Bhavana principle. So, now I am going to start with that so I will quickly recapitulate this (FL) and Bhavana, so and then move on to illustrate this principle has to how this can be used to derive a certain expression which we found in (FL) (FL) give a very interesting expression for the value of group 2. So, it was expressed in the form of a sum of rational numbers.

And we will be able to see now how that same expression can be arrive by the application of Bhavana principle. So, that is differs part of our discussion and then we will illustrate with one more example and then move on to the Bhakshali manuscript. Bhakshali manuscript so is something which is very very unusual both in it is content as well as the way it was discovered. There are so many manuscript that will find for various works.

So, in the form of formally in the form of paper in the brisk bark all that, so this is the only manuscript that is very, very ancient to the period is something which is estimated that to be around third century, fourth century by some people and some others have expressed into be around seventh century but most scholar agrees that it can be beyond seventh century based on the content as well as the language.

So, we will discuss little more about it does it proceed and one important result which is found in Bhakshali manuscript is something which is quite remarkable in terms of the sophistication that it has in obtaining the value of surds. So, it is a very interesting formula so that one finds and this formula has been sort of misrepresented by Kaye and it has been thoroughly analysed by Channabasappa in his paper in 1975.

In fact Channabasappa has written some 3, 4 articles so primarily on Bhakshali manuscript all of them are scholarly articles and he actually analyses the way a particular verse, so which he calls as Bhakshali sutra a like Aryabhatta sutra, so it is also in the form of very small sutra just a small couplet and it gives a very interesting expression for obtaining approximate value of square root of a non-square number.

This we will discuss in great detail and obviously we would not have any kind of evidence thus to how the author of Bhakshali would have obtained this result. So, it is anybody's guess but we will think of a possible derivation. So, with that more or less our lecture will be over.

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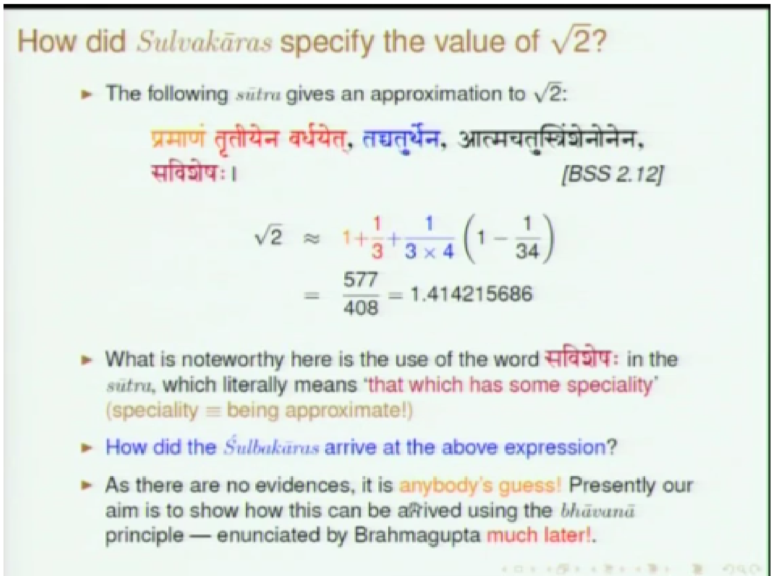

So, to recall the expression which I want to do derive using Bhavana principle, so which has been stated in (FL) is the following (FL) So, this is the (FL) which presents the expression for the value of root 2, so he says (FL) so increase by 1/3 (FL) ¼ of this means the previous value but this ¼ he says (FL) is subtraction (FL) is 34, so 134 has to be removed from the previous value which amounts to this expression.

So, what does it amount to so it is 577/408 and this value is almost corrects to 6 decimal places for root 2. So, this is the (FL) so you may recall this (FL) was written so around 800BC or prior to that, that is the estimate. Most interesting thing about this is so here it is said (FL) (FL) essentially translates to not putting an equal sign but approximate okay. So, (FL) so that is okay how did (FL) arrive so we showed a certain method.

So, in geometrical construction by which one could get this expression but now we are going to even this expression one does not know because (FL) in text form and therefore no explanations say in this sutras they text themselves but coming to this come much later and there to we do not find much of a explanation with reference to the approach which (FL) might have taken in arriving at this.

And therefore some 3,4 approaches have been suggested by scholars by which they would have arrived at this expression. Anyway so currently we will be trying to arrive at this expression using the Bhavana principle.

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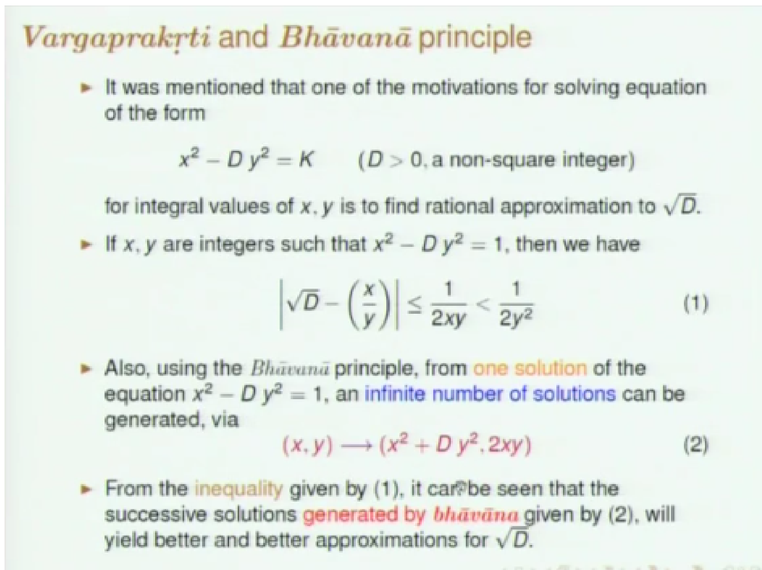

So, to recall the Vargaprakrti and Bhavana so we have an equation of this forum x square-D y square is equal to K where D is the non square number greater than 0. So, whose approximation we want to find out, so K is some integer, so the idea behind posing this problem was to find x and y which are integers and it turns out that the motivation could be in trying to find out the value of root D one of the motivations for solving this.

So, this inequality shows that the successive values which can be generated from xy which satisfies this equation will be better and better approximation for root OD. So, this can be shown some unique quality this unique can also be worked out which is not very difficult to see. So, all that you need to remember is we move from xy, xy is any guess so you given D given K you can always find some xy which will satisfy this equation.

So, and from that you move on to the other values by doing this Bhavana principle and we will see that we will get better and better approximation for OD, so this is the equation you start with so, to start with as you can see so what you can do is, so if x and y are sufficiently large you can easily see that D is going to be root D is going to be xy y, so this is how we start with in order to get better and better approximations.

So, even if it is not so after a few conversions you will see that it will be very close to root D okay.

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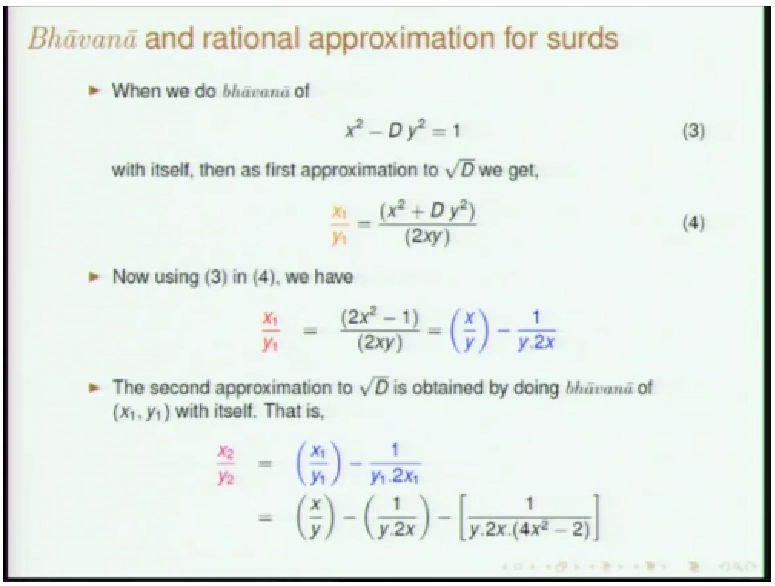

So, let us start with the equation so first approximation for root D he take xy, and then what to do is, so this is the that root of x/y can be taken as 0 third of approximation okay for this equation. The first approximation is by doing Bhavana with itself and x square+Dy square by 2xy so this is the first thing, so remember so this is what we do. So, this how we this is the equation which we are going to repeatedly do okay.

Now using so 3 in 4 see so you will get 2x square -1/2xy and this can be express in this particular formula fine. So, second approximation so all that we need to do is so we have to plug in x1y1 so in this and it turns out to be this, so x1/y1 is this so this equation so you just have it and then you express this. So, now you get this pattern you can see so this expression as such repeats so in the second approximation fine, so now let us go and see.

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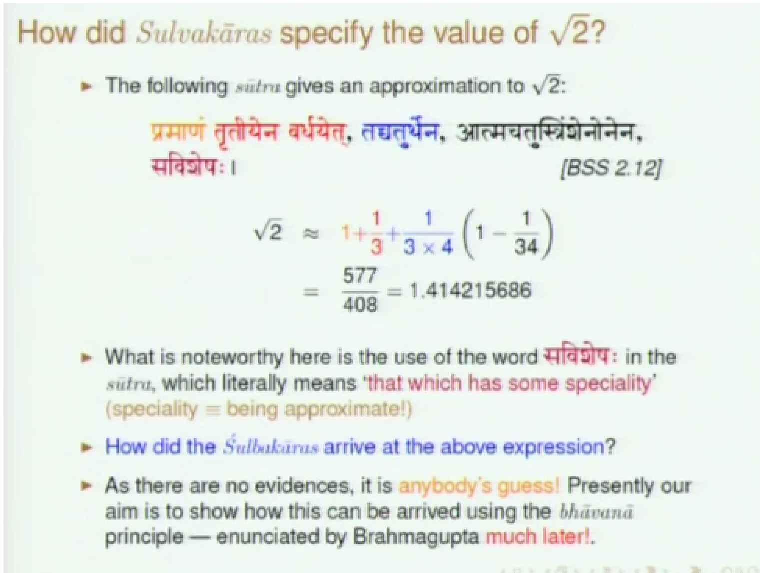

So, this approximation which was given in (FL) is 1+1/3+1/3*4 and the next term will be -1/3*4*34 in the denominator you will find the same terms appearing again. So, you will find see, so y then y*2x, then y into 2x into one more term so you can see that you get that.

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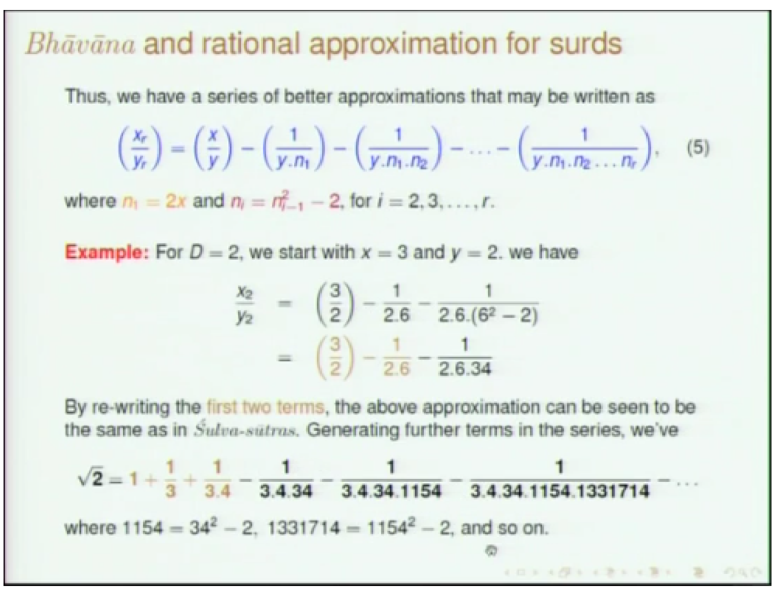

So, in general one can show that so the (FL) approximation will be of this forum. So, we will start with this equation see this equation x square-Dy square so suppose D is equal to 2, so this will be satisfied with x is equal to 3 and y is equal to 1. So, we get 3/2 and so by regrouping this, so we can show that, so this is the expression for root 2 and (FL) sutra stopped here. So, they gave the first 4 terms so (FL) and it said (FL.

So, that actually gives a value correct to 6 decimal places almost and they stopped there and one can show that the next term is going to this and the next term and so on. So, this is something which can be straight away obtained by using this.

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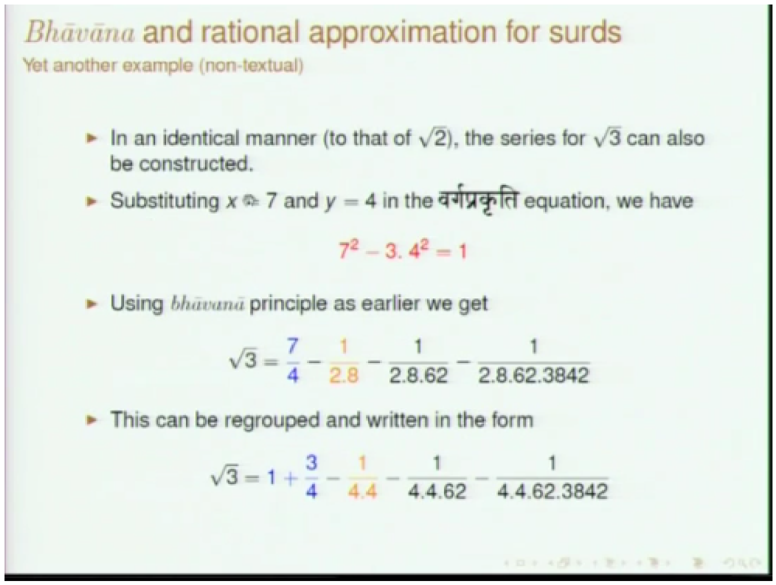

Now I will show one more example and then I will move onto so suppose you want to find out root 3 so you can start with this (FL) equation 7 square-3 into 4 square this satisfies this, so our initial choice is 7 and 4 xy. So, then so this successive application of Bhavana principle leads to this equation and this if one word to write so with 1 is the first thing so 1 can regroup this and then write it in this particular forum.

So that is why we see that as an application to (FL) and Bhavana principle one will be able to generate, so the expressions for surds in this particular forum and this is something which will go on. In fact here one should not have this follow equality it is not exactly equal because it is something which is going to done after each term we will see that the value increases the accuracy increases okay.

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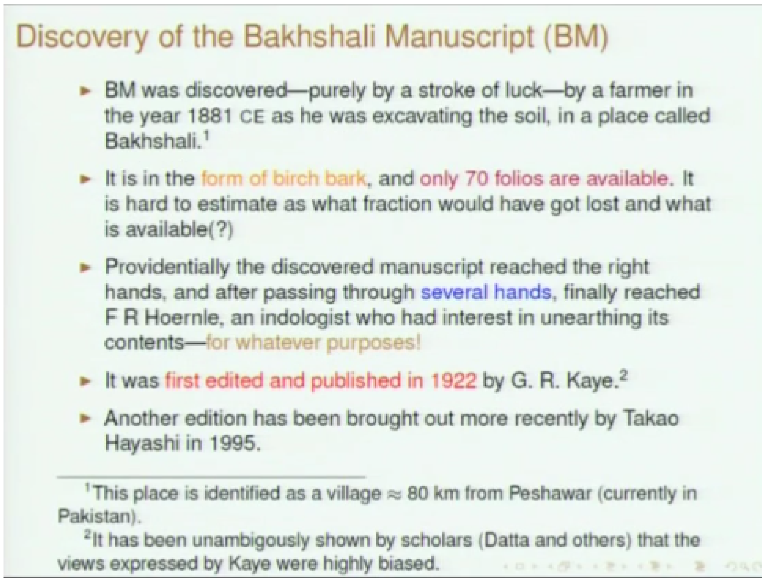

So, now I move on to the Bhakshali manuscript as I was mentioning this Bhakshali manuscript was something which was discovered by a stroke of luck apparently a former was digging somewhere and so he suddenly discovered a manuscript as he was excavating soil somewhere and this manuscript is in the form of bridge box. So, today it is very very hard to find manuscripts written in bridge bark but earlier that was one of the medium which was used for writing.

So, we can find manuscripts in palm leaves but bridge bark manuscripts are very very hard to see, so this is in bridge bark and they have found about 70 folios as you can easily understand, so if this has been lying under the soil for a quite long time, so it is something which is in a pretty damaged condition. So, anyway so that was discovered and this is in 1881 and this former so did not simply ignore it but took it meticulously to this person and the person took it to it.

They went into passed on into several hands and this was first published in 1922, so I was referring to kaye, so this kaye was the first person to publish it in the form of a text the manuscript is still there but it is there in London, so and recently it was edited once more and published by Hayashi, so Hayashi is a Japanese scholar and he has published quite a few works and one of them is Bhakshali manuscript.

And the place of discovery so they say it is close to Peshawar around the 80 kilometres and it is named after the village, so where it was discovered.

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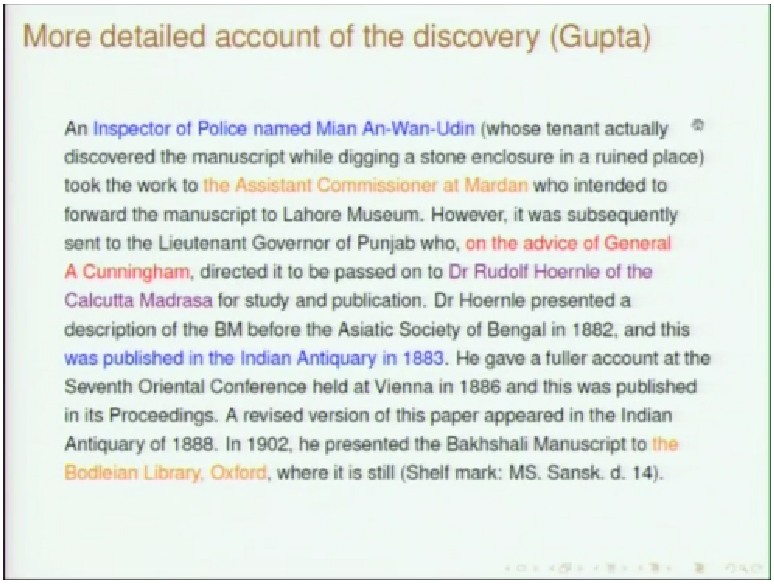

The story goes like this an inspector of police so discovered the manuscript while digging, so who was and it was handed over to assistant commissioner and it further went on to the general Cunningham and it was passed on to Horenile and then Horenile so in fact for some got interested in the manuscript and curiosity he studied it, so he made a few presentation I think some 3, 4 presentations so as this narration goes in the very next year, so it was discovered in 81 and 82, so in Bengali .

In Asiatic society of Bengal he made a presentation and then in 1883 and then once again in the oriental conference in Vienna in 1886 and revised version of the paper so were in 1889 in 1902 he presented this Bhakshali manuscript to Bodielian library and that where it is. So, the original manuscript is there in London fine. So, as regards the date of manuscript so as I was mentioning so there is a lot of uncertainty and kaye place it around 12 centuries CE.

Dhatta and others so place it around third century, so they placed between 200 to 400 Hayashi says it should be around 7 century, so all of them interestingly so say that we have arrived at this date based on the language which is that based on the content, which is that based on the script which is founded. The script so scholars says there it is Sarada and the language this is Gatha, so it is form of (FL) and of course the content of the manuscript.

So, certain problems which are being discussed have also been analyse to estimate the date of the manuscript.

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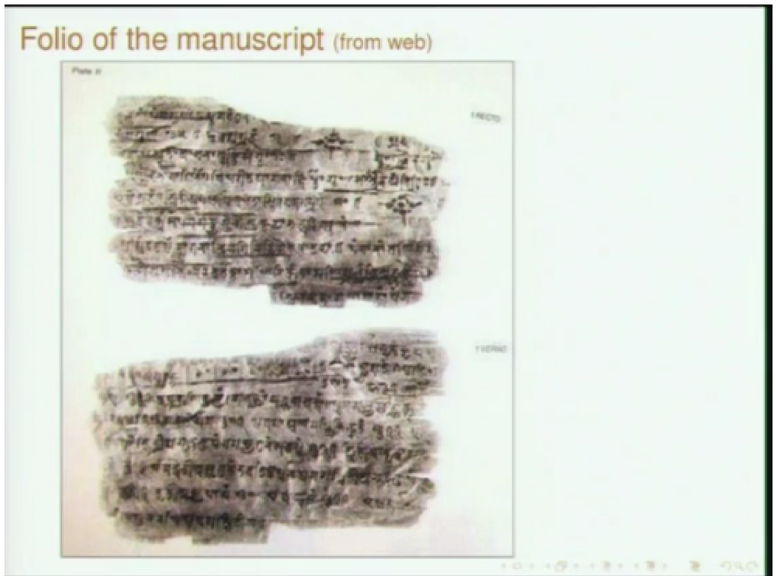

So, this is how it roughly like when this is very poor photograph.

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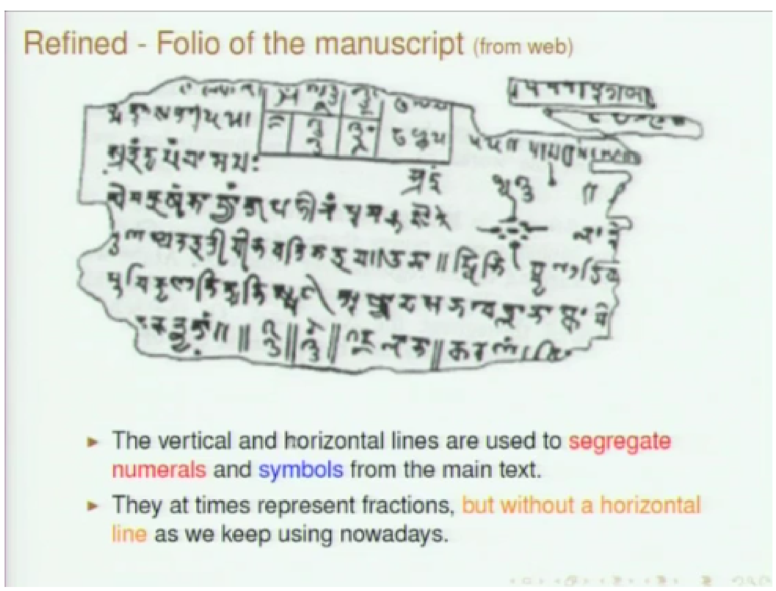

So, this is somewhat more refined, so this is how it a sort of a looks like a typical page of the manuscript. So, lot of things got damaged and whatever can be decipher in spite of this has been attempted by various scholars. One interesting thing about this Bhakshali manuscript is so, it sort of segregates this numerals and any kind of calculation will be put in the form of some tables some lines are drawn.

So, when things are arranged like this see suppose there are number one below the other so they represent some kind of a fraction, suppose it is suppose this is 7 this is 5, so this will be sort of 7/5 something like this, so this is the convention and if you have 3 numbers arranged the first will be so the suppose we have 5, 3, 8 so it will be 5+3/8 that kind of a thing. So, it is a kind of convention. So, we find certain notations employed and the numbers are segregated by bars.

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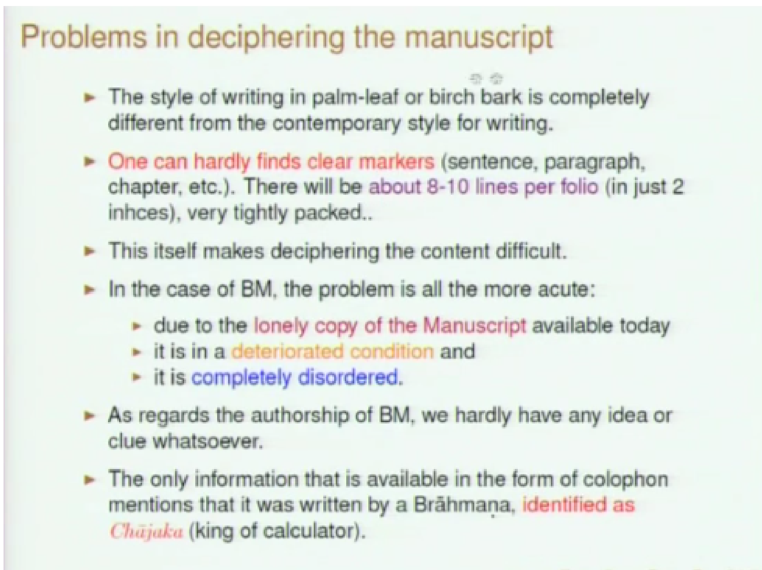

So, few remarks would be in place regarding the process of deciphering the contents of a manuscript. So, this style in which people have been writing in those days is far different from the style that is adopted by us today. So, if you look at any typical manuscript, so most of them are written in a very very very type manner, so what is written in something like 18 inches or 1 inches or 1 and half inches or 2 inches, so depending upon the palm leafs.

So, can we put in some 2 A4 pages, so that is the kind of very very tightly written and very very small, so it is a extremely difficult unless the scholar is trained to decipher the content of a manuscript the second thing is there would be no clear markers, so the chapter or full stop etc., so they are not found and only in a few rare manuscript do you find all this markers, so which actually help us deciphering the content.

And particularly in my case of Bhakshali manuscript the problem is all the more acute for the following reason, suppose you want to edit a text Aryabhattiya so you will find some 20 manuscripts in various places 200 manuscripts sometimes depending upon the popularity of the text but in the case of Bhakshali manuscript this is the only thing that is available and therefore collaborating or collating it some other thing is impossible and it is in a retreated condition.

And it is also completely sort of jumble, so disorder as it was sort of discovered there and how much is lost how much is there available so this is also not known. So, that is a sort of colafen (())) (19:41) so which has been identified by scholars that it is written by so this much is known by Chajaka whom they saw the cal the king of calculator.

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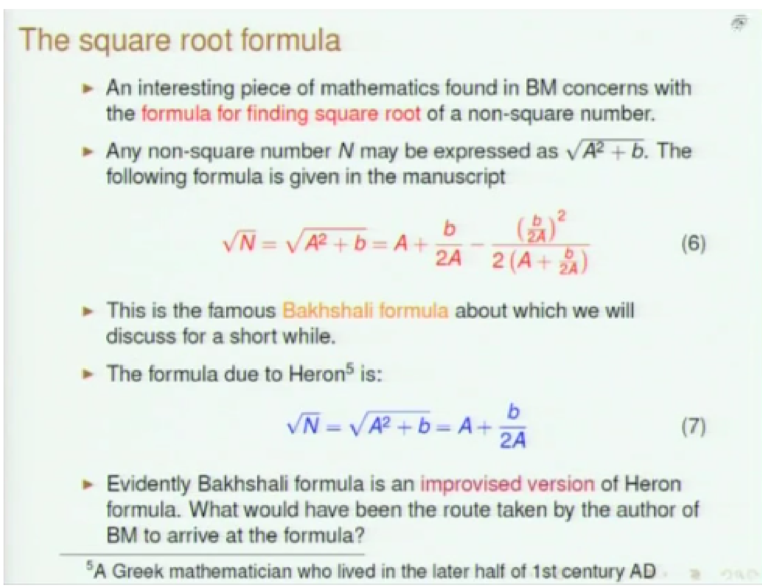

So, I was mentioning to you about an interesting formula presented in this Bhakshali manuscript, so the formula is to find out the square root of a non square number, so suppose we have a number n which can be expressed as A square+b, so let us assume A square is something which is close to n and less than n the formula which is given in the manuscript can be written in this form. So, root N is A+b/2A-b/2A square/ 2 times A+b/2A.

So, this quantity actually sort of repeats itself here, so this is the famous Bhakshali formula and this particular form A+b/2A is something which has been presented by Heron also you have one more term added to that, so naively one feels that this should be a much better approximation than A+b/2A how did this Bhakshali author arrive at this formula. So, this will be discussing towards the end of our lecture.

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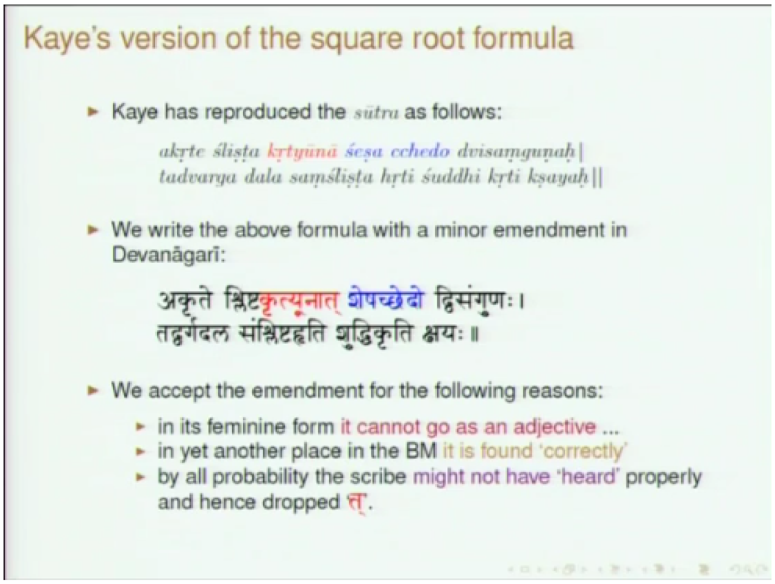

But before that I wanted to give you the verse, so which present this formula and Kaye in his first edition so he has presented the verse like this, so there is a lot of as I was mentioning it is too difficult unless the person is sort of quite conversion to the subject. So, wherein he knows the content, so it is almost impossible to split the words appropriately and there could be a lot of errors which occur in doing so.

And there seems have been committed by Kaye 2 anyway a sort of improvised version of this is this (FL) if you see in his edition he simply calls (FL) this is the formula which in modern notation translates to this expression 6.

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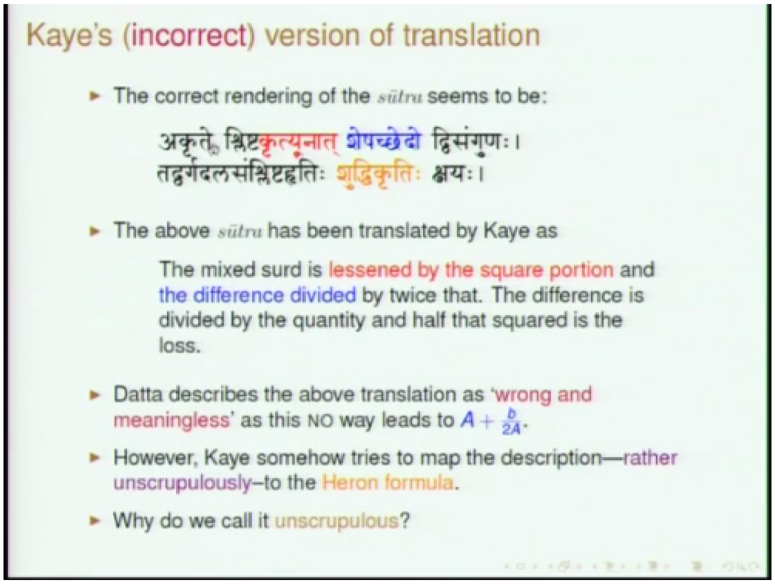

So, in fact after serious analysis so (FL) has arrived at the right formulation of the verse, so wherein this should be a sort of compound word (FL) so I will discuss in great detail has to how we can decipher this from this set of words which are found in this sutra. So, in fact it has been translated as the mixed surd is lessened by the square portion the difference divided by twice that and so on.

So, this is the kind of translation which has been presented by Kaye and Dhata who is a serious scholar, so to whom we owe a lot of our understanding to Indian mathematics he has said that this is all wrong and completely meaningless. So, to decipher the formula as simply A+b/2A and then say this is say must the Heron formula is something which is meaningless. It seems to be done rather unscrupulously that is what it amounts to because.

So there are various other evidences by which one will be able to clearly understand that this does not mean this. So, as has been clearly pointed out by Chanabhasapa why do we call it unscrupulous, so where a few reasons which I will quote 1 or 2.

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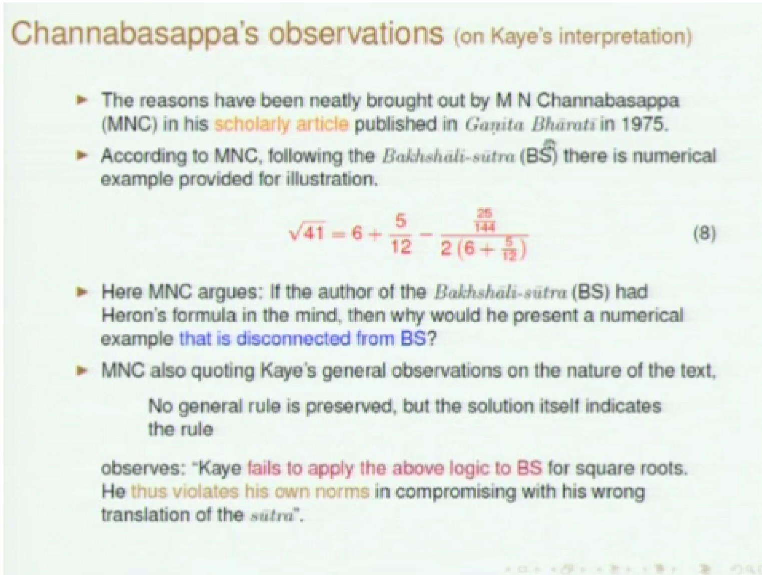

So, this has been brought out in an article a scholarly article in Ganitha Bharati, so it is there mathematics journal in 1975 by channabasappa. So, he says see as would have seen even other lectures which are given by professor Sriram on brahmaputasiddhanta or Aryabhattiya. Aryabhattiya of course does not give various examples whereas Bhaskara, Brahmagupta they always so giving a certain general formulation they immediately give illustrative examples.

So, channabassapa says so soon after the sutra is presented we have a numerical example and the numerical example simply amounts to this if this amounts to this, then why would one simply says that the formula is only confining to the first root on from the series. So, this is the first objection, so nobody is going to give a certain illustration which is completely disconnect from the sutra which is stated before.

And he also says that so no general rule is preserved but the solution itself indicates the rule, so this is an observation which has been made by kaye and Channabasappa says that this violates his own observation okay fine.

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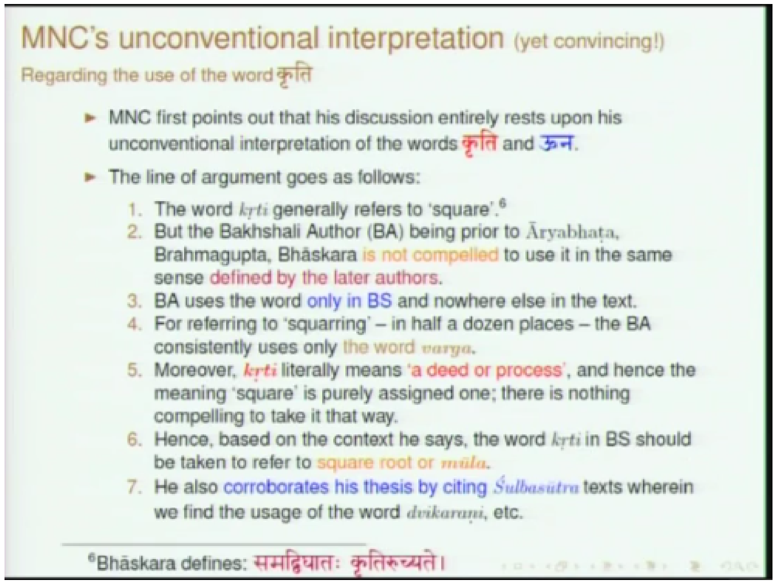

So, with this I move on to the explanation of this verse this is quiet instructive and therefore I will thought I spend some time in going to explain to you the kind of approach that has been taken by Channabasappa in arriving at this is the very scholarly article, see here we have the verse (FL so this is sutra. Here the word (FL) appears see (FL) you understand.

So, these are the 3 terms that appear 3 words and once again we have the word (FL) appearing here (FL) usually the word (FL) is understood to be square. So, this as acquire the meaning of square so much after the time of Aryabhatta, Bhaskara and so on wherein they very clearly says (FL) means square but (FL) as I was telling in the previous lecture the (FL) generally means a composition so (FL) right.

So, Kalidasa (FL), so (FL) is basically so it is derived from the root (FL) so (FL) means doing a certain process that is all it is, so and therefore it can mean the writing of a person it can mean the work done by a person, so in general in here in this context bassappa says that (FL) has to be not a square but a square root. So, (FL) therefore means fairly accurate square root fine, so (FL) so this is very interesting analysis which has been done.

So, that is nothing which prevents us from interpreting it as square root if we look at the derived meaning of the term because it has various other applications also in various other fields in various other senses, this is the first observation that he makes and the second observation that he makes is the word (FL) (FL) normally means minus, subtraction okay. So, this lessened from that, that is how it means.

So, here he says that this (FL) has to be taken as division and not as subtraction and in fact he quotes pictures to prove the point that he tries to make, so with these 2 observations we move on to look at the arguments presented by Channabasappa as I was mentioning the word (FL) generally refers to square at the Bhakshali author so being prior to Aryabhatta, so the first reason that he says is so this Bhakshali.

So, we do not very clearly know when it was composed but it might be following some tradition which has been much earlier also. So, according to bassappa this has been written even prior to Aryabhatta, so around that period the (FL) term was reserved to refer to square if it is occurs in a mathematical context. So, but prior to that the author is not compel to use it in that sense the words pickup various senses at the front points of time.

And therefore this is the first reason that he says and he also says that if you look at the entire Bhakshali manuscript the word krti does not appear anywhere else but for this sutra. So, wherever the process of square had to be referred the author uses (FL), so **so** he says reserved the word krti to refer to square root and not square, so this is the second observation. So, it occurs in a dozens of other places where he only uses the word (FL).

So, as I was mentioning krti is a generic term which can refer to a deed or a process and therefore there is no compelling evidence for us to take only to refer to the process of squaring and for this particular verse he says it has to be taken as square root or (FL) and he also corroborates this with other evidences from other earlier text. So, suppose we place the Bhakshali manuscript prior to Aryabhatiya. So, the only other extent that we have is the class of text which are called (FL) text.

And in the (FL) text, so when we discuss that we saw, so I repeatedly hammered the word, so this (FL) refers to square root and that also stands from (FL) so (FL) so and so the (FL) refers to square root and therefore this is also derived krti is also derived from the same rule and therefore it could refer to square root. So, in fact we have the word (FL) referring to square root of 2, square root of 3.

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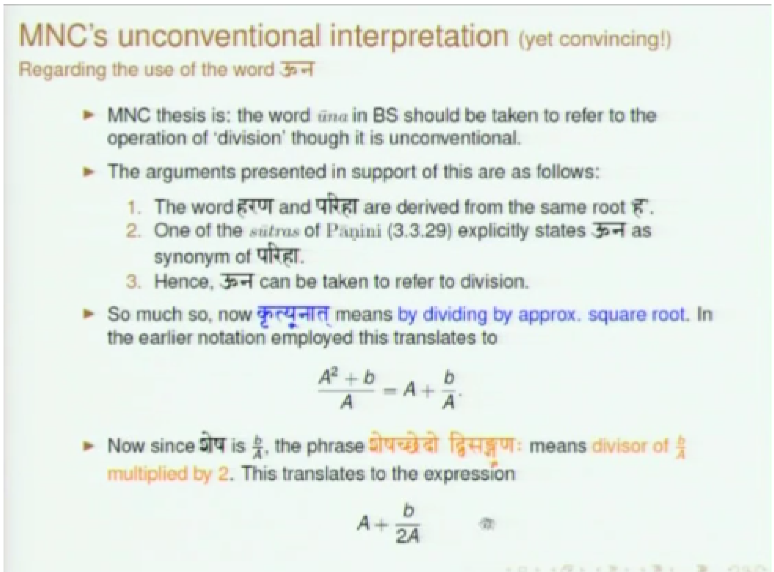

So, now we move on to the other word una, una so he says it refers to division, so how do we substantiate this. So, here he says the word (FL) and (FL) both are derived from the root(FL), so (FL) means division, so (FL) (FL) so they are also so it is used in the sense of close to this okay, so sort of eliminated so taken away, so it is in that sense. So, in fact he codes certain (FL) sutra wherein (FL) says (FL) so that result is stated.

And therefore it can be taken to refer to something which is removed and removal is something which is by division process and therefore una can be taken to refer to division in this sutra fine. So, now with this, so framework so we had the word (FL) so this is also sutra begins, so (FL) so here he says so is by dividing by approximate square root, so (FL) square root una is division so (FL) is this, A is taken as the approximate square root of root N.

See the number is A square+b, so (FL), so krti here (FL) (FL) means well known (FL) is well established so this square which is well established before this root N is A square and therefore (FL) so is a it is referring to division by A and it comes to A+b/A. Then we have (FL) so is the first half of the verse (FL) (FL) I leave it now (FL) so, (FL) is b/A (FL) is deviser (FL) multiplied by 2.

So, we have b/2A you understand (FL) denominator multiplied by 2, so A+b/2A you have obtain this term.

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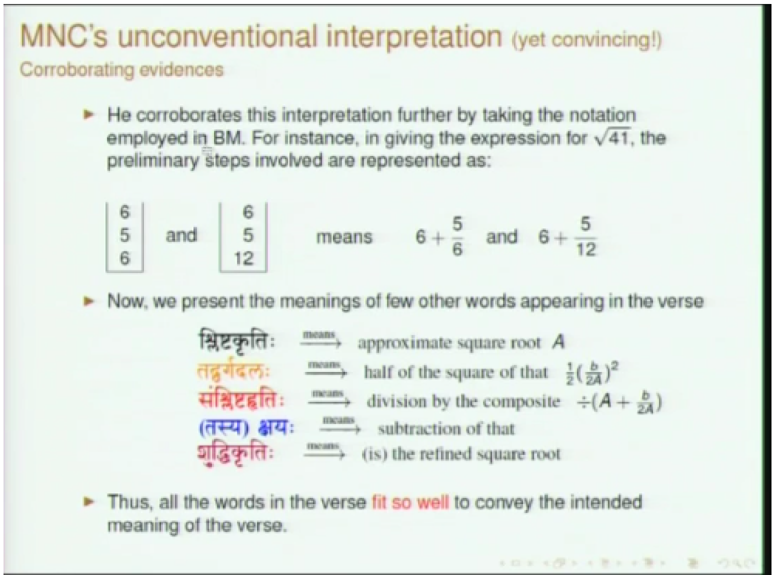

So, in fact as I was mentioning so you find this kind of markers in the manuscript, so suppose we find this number 6/6. So, this is to be understood as 6+5/6 and then 6512, so 6+5/12. In fact the illustration which has been provided in Bhakshali manuscript is 1 of root 41, so root 41 can be express as 36+5, so 6 square+5 so you have the same thing appearing see 6 is a and b is 5 and so in fact both of them are presented in this manuscript and he says 6+5/12.

So, this is the reference in the manuscript has to how this thought with. So, to complete the analysis of the sutra so (FL) is A, so the word (FL) see the forum okay N, so (FL) see in fact see approximate I say because this is (FL) is something sort of kwon kind of a thing. So, that is why it is referring to A, so we have A square+b is the number and the closest square root is A okay, so it is in this sense.

So, then we had see (FL) the second half of the verse starts with (FL) see so (FL) refers to the previous quantity so what was the obtain is b/2A see (FL) so this b/2A (FL) refers to this square (FL) is half of it, so (FL) refers half of this b/2A the whole square then we have the word (FL) so which appears in the verse see I will just quickly highlight the verse see (FL) okay.

So, (FL) is sort of put together okay so, (FL) is A+b/2A (FL) means division so, (FL) see so, you divide by this see so, that is how it is. So, what is to be divided is this factor (FL) refers to so, this divide by this quantity. So, then you says (FL) so, (FL) is shadow subtraction okay. So, this quantity this ratio has to be taken as negative and negative to this number which you already obtained.

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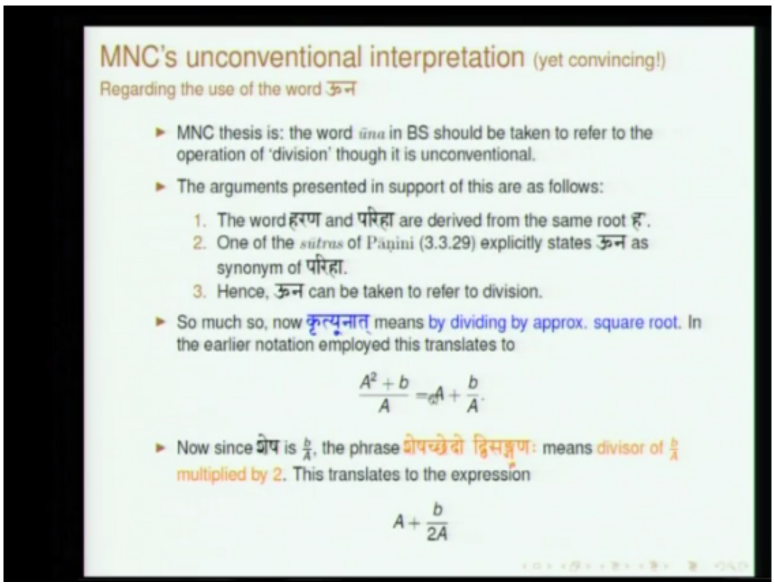

So, this is the quantity from which so, the ratio of this has to be remove and what to get is basically (FL) so, (FL) is (FL) square root so, a refined value of the square root ahh correct, that is how it is.

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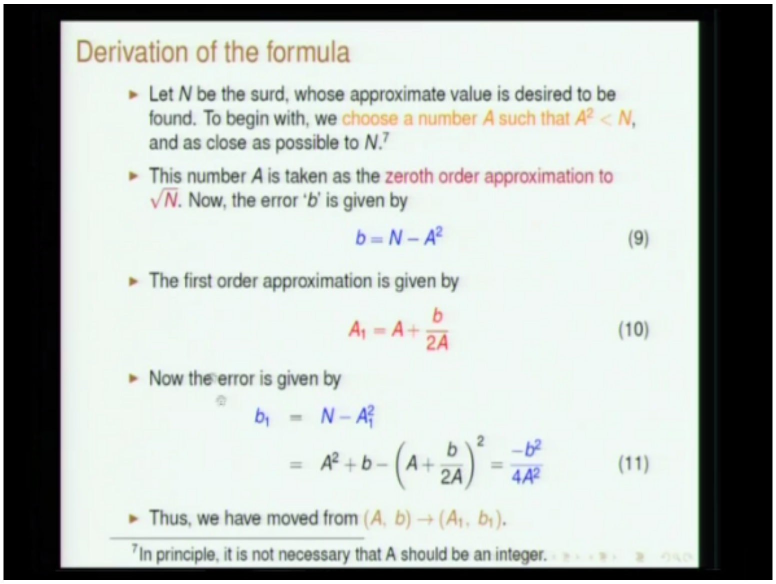

So, what is the way by which they might have arrival it so, a possible derivation is presented now suppose N is the surd and A is the nearest square which is less than N. So, we start with A and the error at this stage is N-A square that is why we write a error at this stage is this. So, as a first order approximation so, we write A+b/2A is A1 so, this is like this see suppose you have root of 1+x kind of a thing.

So, as a first order approximation 1+ so, 1/2x so, this is how we take so, that is how it is okay. The error so, at this stage is obtained by N-A1 square. So, 0th order it is and first order it is so, b1 is this. So, this A1 square see so, N is A square+b and A1 square we write. And what we get this –b/4A square. So, this is the error at this stage so, what we do is we start with the Ab and then we have moved to A1, B1.So, the first order approximation is this and this is the error.

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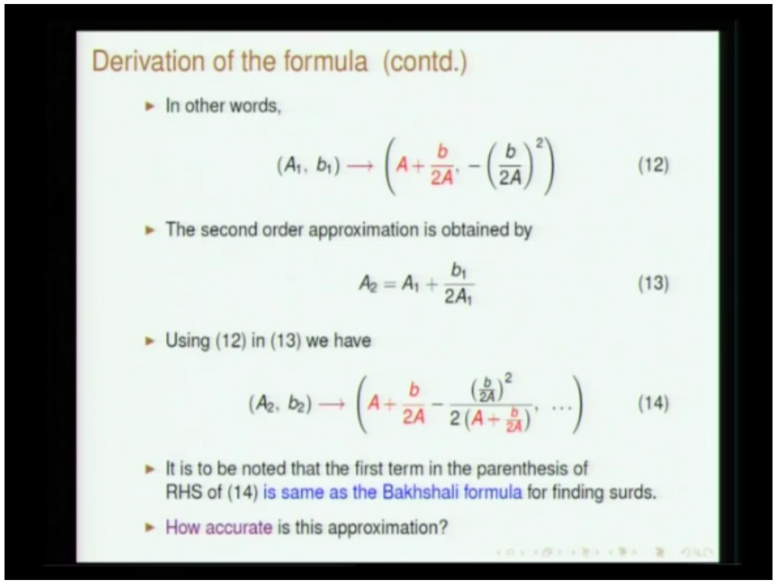

So, if you want to move to higher order. So, essentially so, this is what it is so, we have a started with this. So, this is A1b1 or this is the expression let we have A1b1. So, second order approximation so, we use A1 so, instead of A so, use A1 here in this expression. So, A1+b1/2A1 so, if plugging the value of A1 so, A+b/2A- so, b1 is this and so, this is how it is. So, this is basically the formula which has been stated in the Bakshali**.**

So, this is the shadow of approximation so, which one can keep on doing so, further and apparently stopped here whether the authors of Bakshali derived it this way is something which one cannot very clearly spell out. I mean it is only a possible derivation and given that so, Indians were sort of fond of this iterative process it is quite possible that this might have been the approach which they had taken to arrive at this expression.

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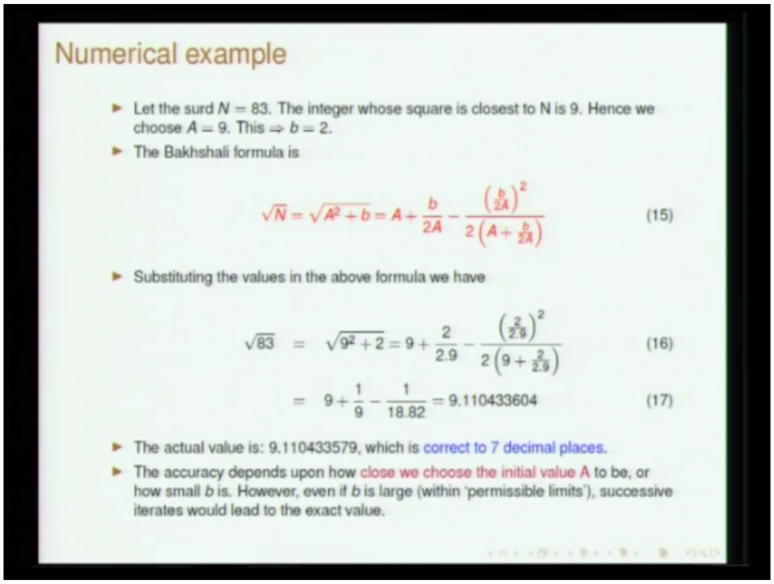

So, how accurate is this approximation so, we will take one numerical example so, however N is 83 so, the closest square i s9 square. So, A is 9 so, that implies b is 2 so, this is the formula so, we call A+b/2A so,- this square yeah and so, if you substitute this values. So, what one gets is so, 9.110433604 and you use modern calculators so, what we get is this expression. And you can straight away see that it is correct to 67 decimal places.

The suppose give where to choose so, 85, 87 so, the closer you take so, there is nothing which compulses to take A as even integers. So, this will work even otherwise the closer you take the higher the accuracy will be even if you take much so, it is basically a sort of zigzag function which will be close which will keep on converging to the actual value. I will quickly discuss one more problem. So, an interesting problem which has been presented in Bhakshali**.**

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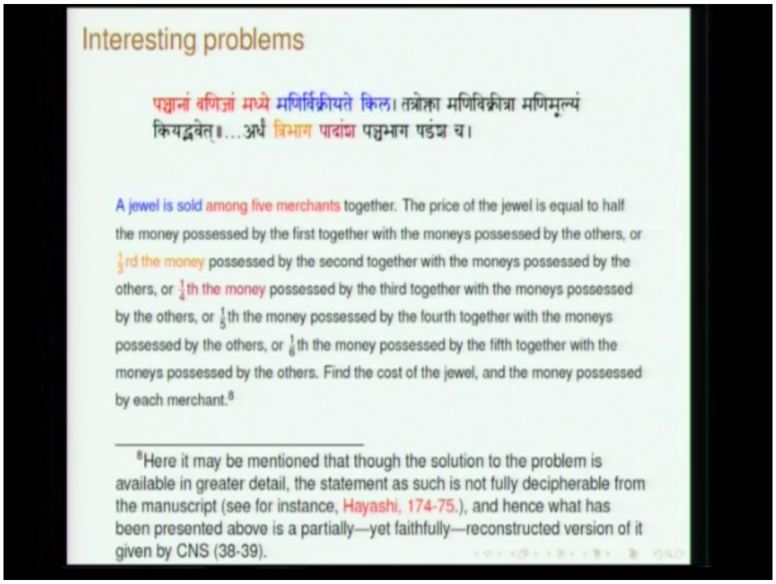

And then make a few observation before winding up this stack there are quite a few interesting example which have been presented so, what has been stated here is there are see I said so, he say only this formula has been stated here. So, A+b/2A is what is implied with the verse no one can get from this verse. So, whichever by you in fact the problem has been very beautifully analyse.

And this unconventional meaning has been assigned to the word (FL) after lot of thought by(FL) and (FL) so, also tried to interpret the verse but one or two places you will find the little difficulty in trying to make various terms. So, go one with the other and finally give the meaning so, (FL) I think was in first century yeah. So, (FL) see there are as I told you there are lot of problems**.**

So, the this sense of the problem has been understood with the words that have been presented there but not every word s available for us because this manuscript has been damaged. So, it is said (FL) so, there were five merchants so, where willing to purchase something some jewel. So, he says a jewel is sold among five merchants the price of the jewel so, he stated in a complicated manner.

So, if he says so, it is exactly equal to half the money which is possessed by the first fellow+the sum of the other. So, one third the money possessed by this second fellow and the sum of the others o, one third the money possessed by the other fellow and the sum of the others. So, one fourth and sum of the others so, what is the price of the jewel and what is the money possessed by this people. So, this is the problem fine so, (FL) one third (FL) so, one sixth so, this is how he takes the problem.

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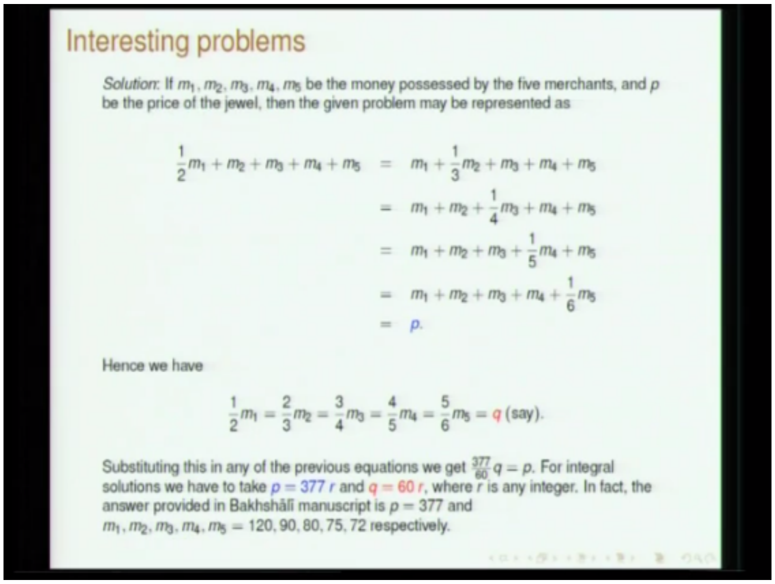

And so, if somewhere to write it so, using what a notation so, this is what it amounts to see half of the first merchant+the sum of the others. So, this will be equal because that is the price of the jewel so, one third the money of the second merchant+the sum of the others. So, one fourth the money of the third person+and so on so, this will be let us say this is equal to p. So, this translates to this so, let us say like this is equal to something q.

So, then so, what one gets is basically an equation of this forum so, 377/60q could be equal to p. So, this is what one gets and one can write it so, **so** this is 377r q is equal to where r is any integer. So, bakshali actually gives the value p is equal to 377 and q is equal to 60. So, of course any multiple is also valid and so, this is the one of the typical problem which can be found him Bhakshali manuscript.

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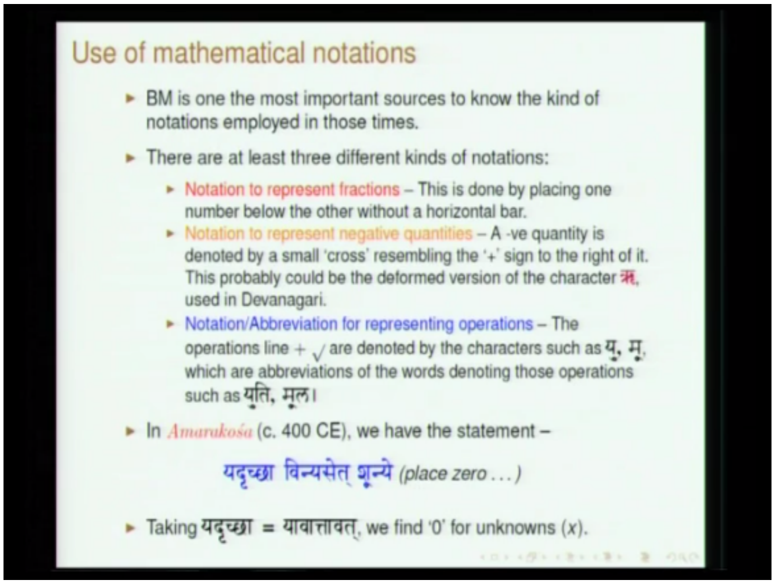

This manuscript is also important so, as I showed you one of the slides earlier so, in connection with the kind of Notations.

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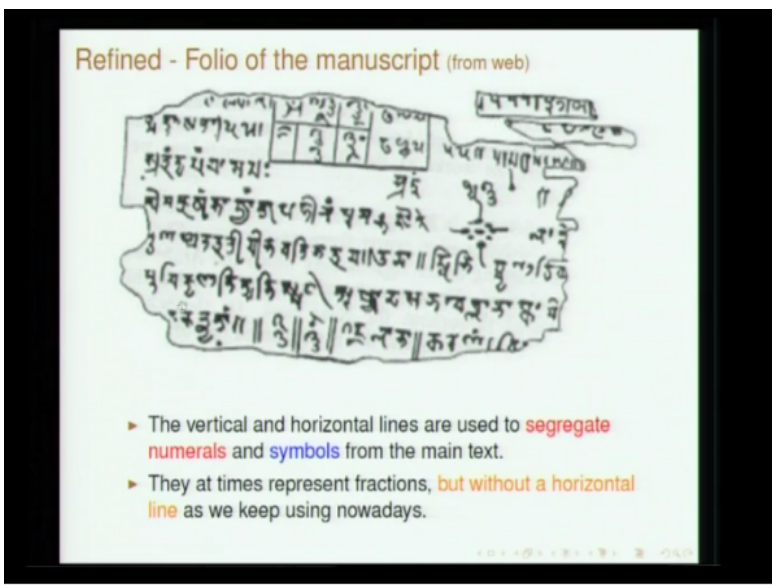

So, this is something which is frequently asked so, what was the kind of notations which might have been employed by those people in those days. So, this kind of Notation certain Notations are also available so, besides the kind of tabulation which one finds to segregate the numbers. So, how where the representing division how where the representing negative quantity how where the representing some unknown value.

So, m1, m2, m3 so, all of them are unknowns so, if one were to put in the form of notation, so what from the notation employed.

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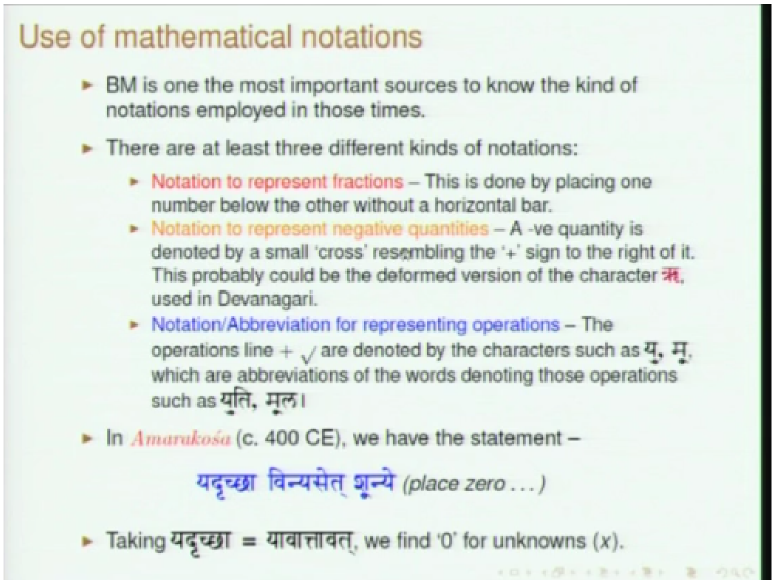

Sp, to a certain extent we get clue form what is available there and he make a few remarks this connection scholars were identified that there are at least 3 kinds of an notations which have been employed one is a notation to represent fraction as I was mentioning. So, you place one number below the other, so this is understood as ratio, so this is one notation, so which is sort of horizontal bar and notation to represent negative quantities, so this is something which is been noted.

So, kind of +kind of a symbol, so if a quantity is negative, so you write the and then put a small kind of a mark to the right above it and some scholars have opine that this + kind of a symbol has to do with the word which they been employing to refer to a negative quantity (FL), so this (FL) might have been so represented with this kind of a symbol, so that is one suggestion which has been given.

And so this represent certain operations, so this also can been seen suppose you have to add 2 quantity in between, so this+symbol is not used for the they use the word (FL) see so (FL) to refer to and then sometimes (FL) to refer to the (FL) square root kind of the thing. So, these are certain notations which one can find out in this text and it is in this sense also, so this manuscript is something which is quite unique and provides us lot of light on the kind of notations and the kinds of problems which have been attempting in those days.

So, in fact even (FL) so this is so (FL) so this is see that there are this quotation I have provided to the show that some kind of notational form has been employed to represent certain things even in those days though we do not have very clear evidences to what has been used.

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So, because this manuscript is handed from generation to generation and most of things done orally in this tradition so, with this few remarks, So, we conclude our discussion on Bhakshali manuscript thank you.