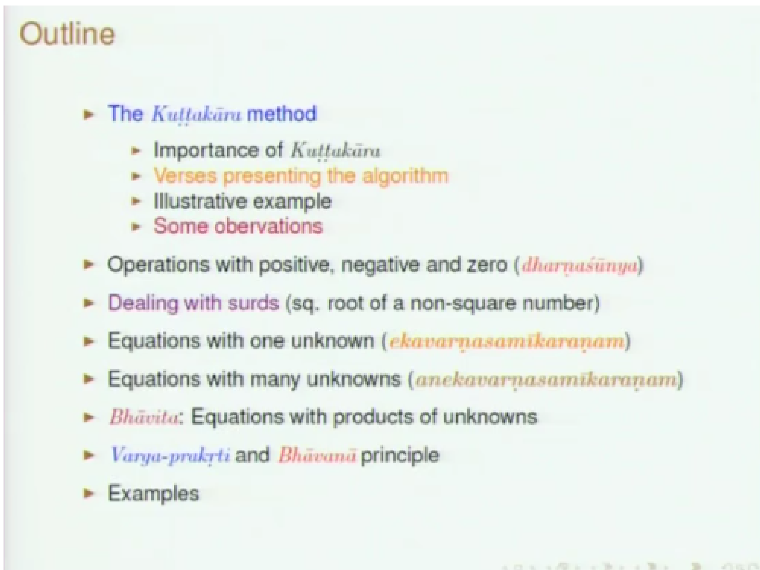
Mathematics in India: From Vedic Period To Modern Times
Prof. K. Ramasubramanian
Indian Institute of Technology-Bombay, Mumbai

Lecture-13
Brahmasphutasiddhanta of Brahmagupta-Part 3

So, earlier you had 2 lectures on Brahmasphutasiddhanta of Brahmagupta, so now is the third part which primarily deals with algebra. So, in brahmagupta's Brahmasphutasiddhanta which is a (FL) book in 18 chapter is called (FL).

(Refer Slide Time: 00:33)



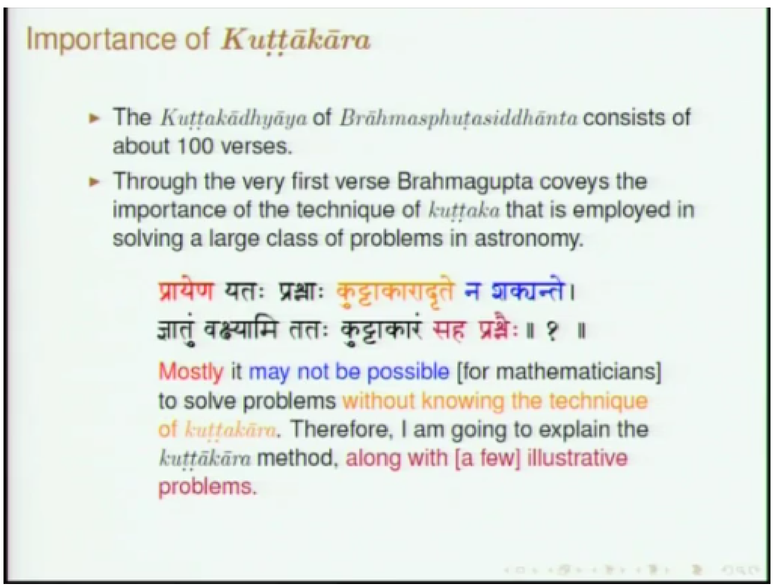
So, these saw about (FL) also discussed by Aryabhatta, so Brahmagupta starts this chapter which contain almost 100+ verses in the (FL) algorithm we briefly present the (FL) algorithm and then after making some of some observations we will proceed with what is known as (FL). So (FL) means a positive quantity (FL) means negative quantity (FL) is 0, so operations with positive, negative and 0, so this has been discussed in great detail in by Brahmagupta.

And then we will also discuss Brahmagupta's method for dealing with surds, so surd as you know is square root of a non square number and we will discuss (FL) and (FL) so (FL) here refers to an algebraic quantity say x so (FL) so you have equations on both sides, so with one unknown, so how do we equate them and solve them, so it is general approach and then discusses a few example.

So, (FL) so is many unknowns so how do we handle equations with many unknowns and (FL) refers to the product of 2 unknowns and he also discusses the approach by which we will able to solve see what is more interesting is to note that Brahmagupta gives general procedure see algebra was developed so we are talking about 6, 7 century and of course this is a very important problem, so (FL) and Bhavana principle.

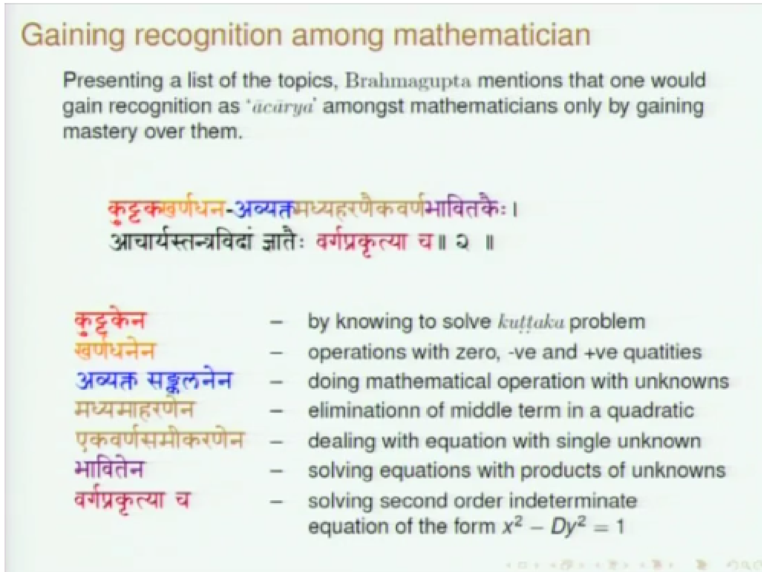
(FL) is basically a second order indeterminate equation and Brahmagupta gives a very important principle called Bhavana principle and this has various applications it is also will dealt with by professor Srinivas and then we will have a couple of examples.

(Refer Slide Time: 02:38)



So, he starts this chapter with this (FL) giving the importance of Kuttakara, so (FL) so in fact triggers the kuttakara algorithm and then about 20, 25 verses. So, giving various kinds of problem with various examples from drawn from different disciplines, so (FL) means without (FL) so it is almost impossible to handle equations with unknowns and therefore he says I describe this (FL) with various illustrations.

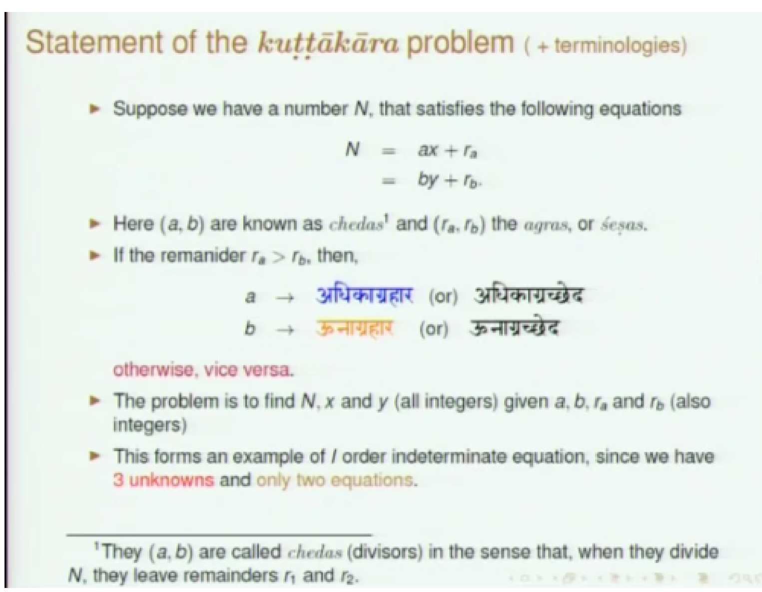
(Refer Slide Time: 03:19)



So, he says (FL) so this was basically summarises the various topic that he is going to discuss in this particular chapter which is called kuttakara (FL) so (FL) refers to the first order indeterminate equation and then as I was mentioning so (FL) so (FL) is 0 (FL) is negative (FL) is positive, so operation with all that (FL) so (FL) is basically an equation quadrilateral equation and how do we find (FL) is the middle term.

So, the coefficient of s and x both refers to as (FL) the context and we have to figure out the meaning (FL) and (FL) this of the various topics discussed in this chapter called kuttakara.

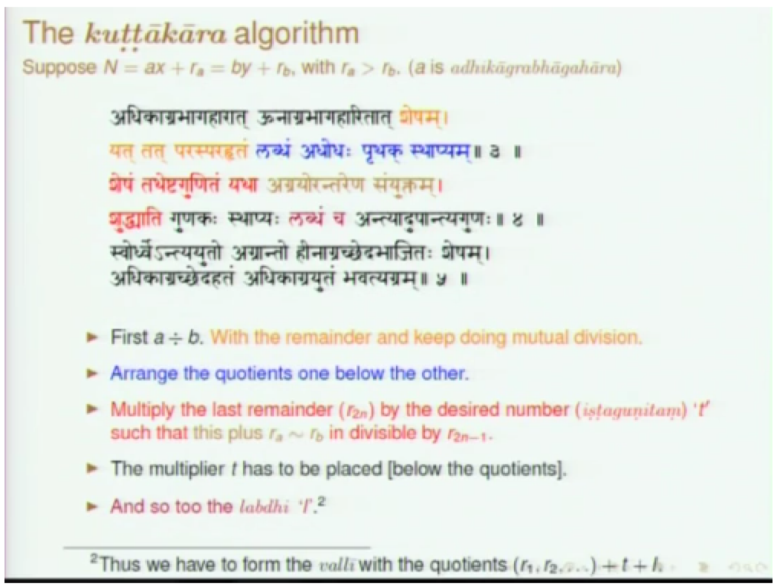
(Refer Slide Time: 04:21)



The very quickly recall so this is the kuttakara problem we have a number N and this number can be expressed in 2 forms ax+ra and b/+rb, so ra and rb are called as (FL), so if ra is going to an rb a is called (FL) and b is called (FL), (FL) in the sense, so if you want to find out x you have to divide N/a and of course ra and rb are the remainders. This is just to get the terms clear, so when we see the verse.

So, a and b are refer to as (FL) and (FL) respectively, so the problem is to find out N, x, y given A ra, B rb so this is the problem, so we have a 3 unknowns and we have 2 equations and therefore this forms an example of indeterminate equation of the first order.

(Refer Slide Time: 05:19)

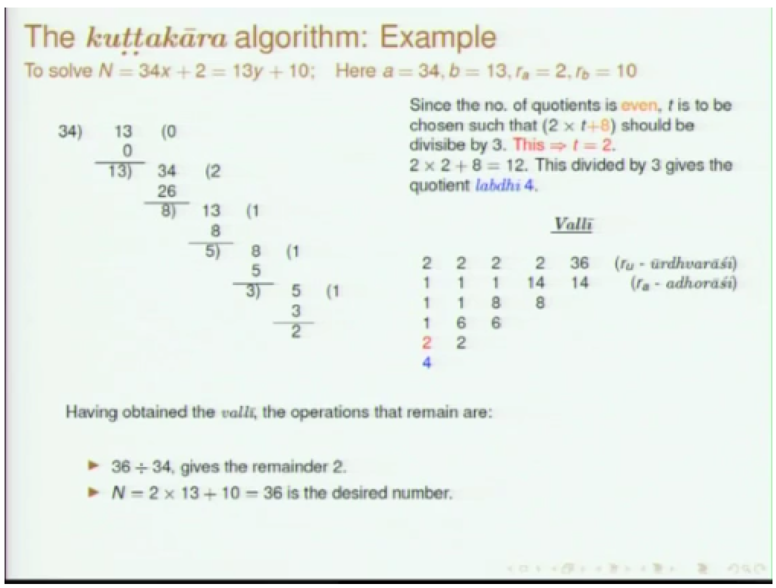


In fact the algorithm which has been presented by Brahmagupta in 3 versus or 4 more then what has been discussing, Aryabhatta also discuss in 2 verse but it was a big task and here a beautifully laced down the algorithm (FL) so, (FL) means one below the other we need to place, so what is to be placed, so (FL) quantities which are mutually divided so, they have to be placed one below the other.

But then where do we start with so his he says (FL), a is (FL) and (FL) so, you start with a/b and then you proceed, so with the remainders you keep on dividing and all the quotients that you get are to be placed one below the other. Then comes in fact the term (FL) was used by Aryabhatta to find out a quantity or some stage of the division you stop and the remainder has to be multiplied by some quantity and then the difference has to be added or subtracted.

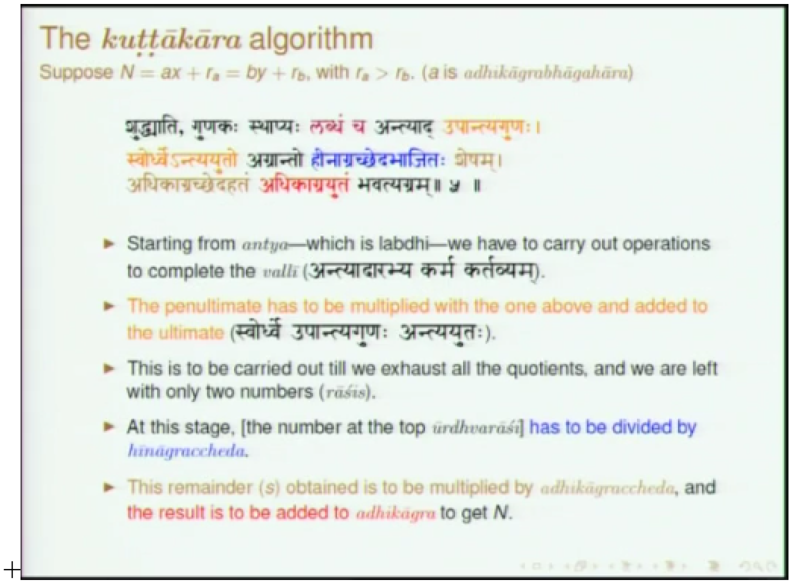
So, that this quantity is divisible by the previous remainder, so this has been very clearly stated here in Brahmagupta (FL) he calls it as (FL) so Aryabhatta call it as (FL) see (FL) as I was mentioning refers to the remainder (FL) is difference (FL), so when it is added or subtracted (FL) means divides without any remainder that is what is meant by the (FL). Then he says that (FL) so whichever is a multiplier (FL) has to be placed below (FL) means this when this is divided so, whatever be the quotient that is called it as (FL) that also has to be placed.

(Refer Slide Time: 07:39)



And then (FL) so, how do we so, having placed this only one below the other, so for instance if you see this problem, so 2, 1, 1, 1, 2, 4 so 2 is the (FL) and 4 is the (FL), so 2, 1, 1,1 so these are the quotients and they are placed one below the other what is to be done, so that operation is discussed in Brahmagupta here very clearly.

(Refer Slide Time: 07:58)



And this operation must missing in Aryabhatta's verses and it had to be supplied by the commentator Bhaskara, so here so he says (FL) see so, 2 into 1+4, so that has to be placed up, so that is 6 so this is the kind of description that Brahmagupta very clearly gives (FL) so, you go on and (FL), so this algorithm is more or less same which was explained while this Aryabhattiya was discussed.

So, 1 or 2 things which we are not very clearly stated then so I will just supply those details and then I will move on to other topics. So, here so let us choose the example 34x+2 is equal to N and 13y+10 is also equal to N, so here of the 2 remainders so, this 10 is larger and therefore 13 will be called as (FL), so 2 is (FL) and therefore it is called (FL), so the prescription was so (FL) had to be divided by (FL).

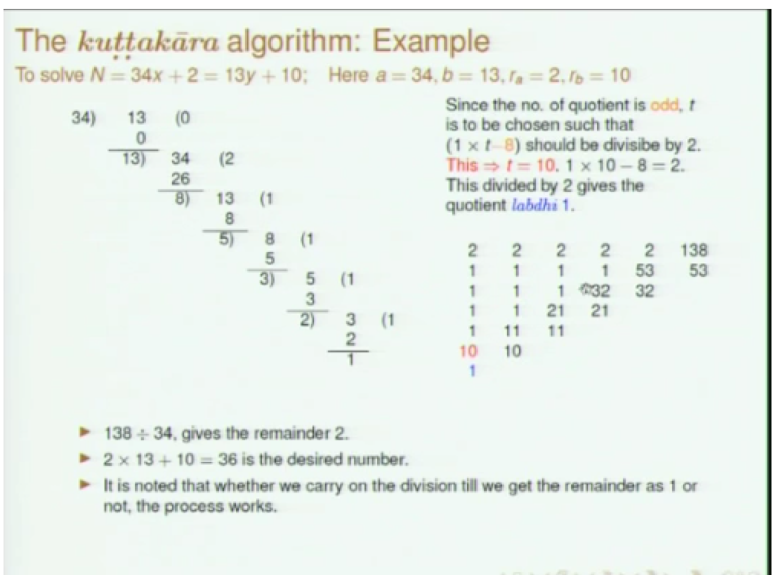
So, you divide this the quotient is 0 keep that so, you get 13 so and then you divide by 34 here after it is a mutual division, so this is dividing this you get 2 and 8 is the remainder, so this divides you get 1, so you get all the quotients, so you arrange them in this order one below the other. So, after the quotient see the last remainder is 2 here, and at this state what you need to do is so, you have to take a decision whether you have to add or subtract the difference of the remainders.

So, the statement was it has to be added if it is even and it has to be subtracted if it is add, so the number of quotients here is e1 and therefore so, you have to find out this T is refers to as (FL) here he calls it as (FL). So, the last remainder into t+distance f the remainder, so this should be such that t is chosen such that, so this divisible will be the previous remainder, so this is the prescription and obviously t is equal to 2 satisfies.

And what we get is 12 once you get this 12, so you have to divide by 3 and the (FL) is 4 that is placed here then it is **is** this so (FL) add to this you get 6 you complete the valli you read here. So, having reasons there so the 2 numbers one is called (FL) is number (FL) so all that that place to be done is the (FL) has to be taken and that should be divided by this (FL) fine. So, this 36 has to be divided by 34 the remainder is 2.

And this basically gives the see y so this into 2+10 gives you 36, so that is the desired numbers. So, you just happen that is 36.

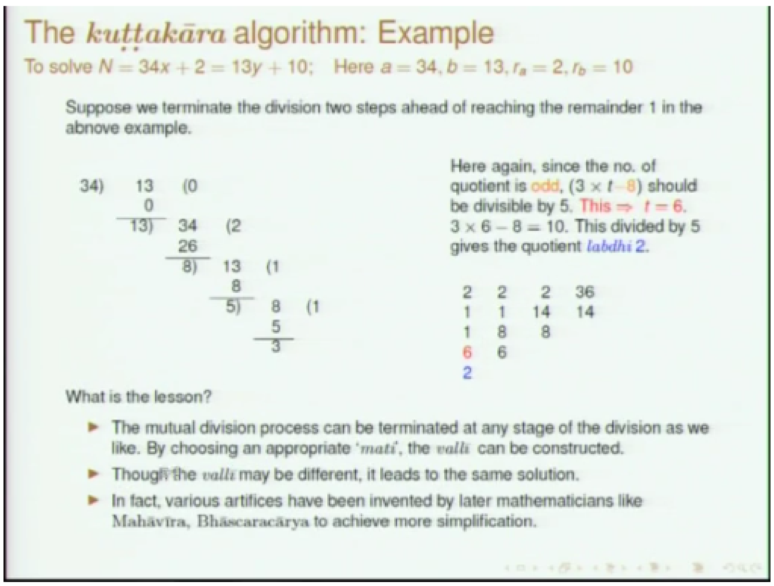
(Refer Slide Time: 11:25)



So, okay one interesting thing that you need to do is to observe that this process can be terminated at whichever stage we like. So, in fact the rational will be explained professor Sriram and here the point that I want to make is so, this in this I have done the problem and I have at this stage we stop in the previous thing. So, now I am just continuing one more division and what you get is remainder 1, in fact Bhaskara prescribe that you can continue till 1.

But this can be terminated at any stage. So, if you continue this till 1, so the number of quotient that you obtain is add. So, therefore here we have to subtract the difference in the remainders 10 and 2 is 8 and t first we chosen again with the same principle and t is 10 and the (FL) is 1, so you place this and what you get here is, so the numbers of different here but again the same principle. So, you will be able to get the number 36.

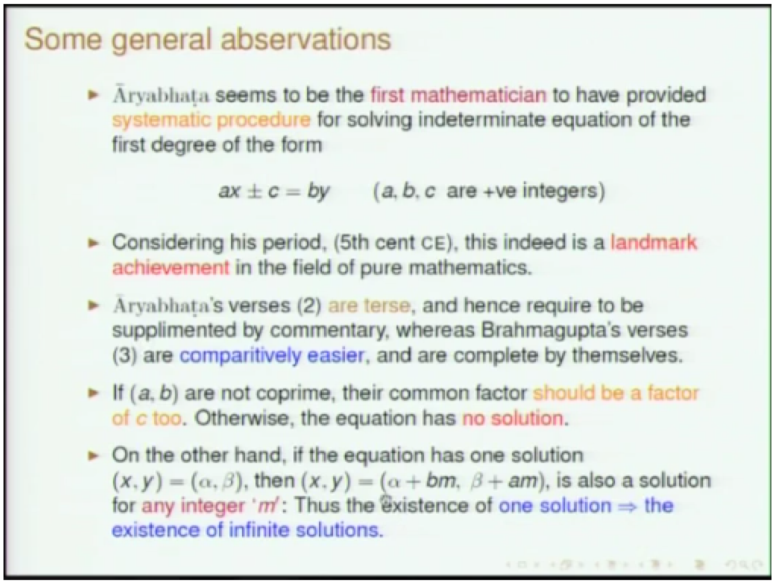
(Refer Slide Time: 12:34)



So, I do the other operation, so you stop before see in the previous thing so first I took with 2 then I added 1 more here I just stop at 3 itself, so you terminate at 3 and this become much simpler and the number of steps is less, so you will 36. So, the point is the mutual division can be terminate at any stage of the division and we choosing up that is that in fact explains why it is called (FL), so you can make it an intelligent choice at any point.

So, the basic point is you have to reduce the magnitude of the numbers and or some stage if you are able to see that you will be able to get an appropriate (FL) you can terminate the process and you can finish the problem okay. And as I was mentioning the algorithm has been described by later people so Mahaveera Bhaskara and so on and the basic thing which has been given by Aryabhatta has been slightly modified by later people and various artifices have been invented by the later mathematicians, so which will be discussed in other lectures.

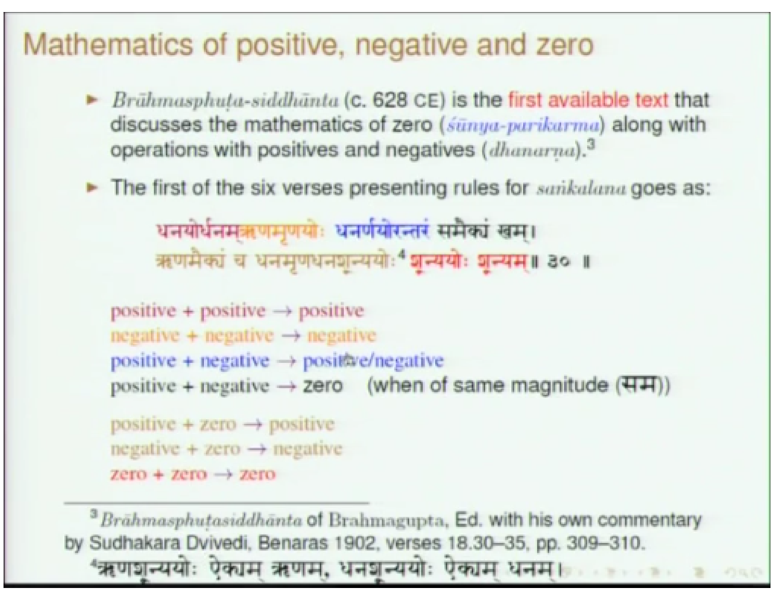
(Refer Slide Time: 13:47)



So, with this with only one more observation so I just proceed further, so this is something which is consider to be a landmark achievement in the field of pure mathematics solving an indeterminate equation and clearly laying out an algorithm for doing so. So, this is one important thing to know. The other is so if 1 solution is obtained some for instance if you look at this last point.

So, if you are able to find out xy is equal to alpha, beta is one solution, then x is alpha +bm and y is beta+am is also a solution for any integer m and therefore if you have obtain one solution you have obtain infinite number of solutions fine.

(Refer Slide Time: 14:37)

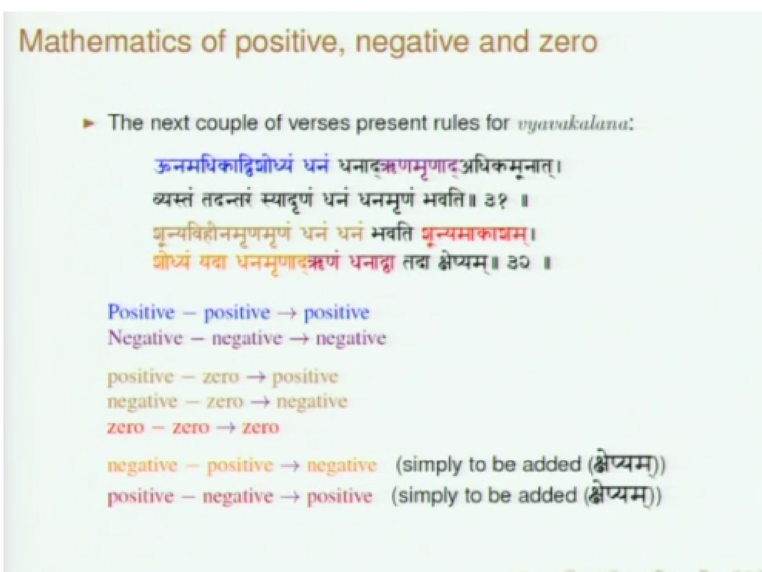


Now I move on to the second section of this (FL) wherein we discusses the (FL), so (FL), so these are the 4 fundamental operations (FL) is addition (FL) is subtraction (FL) is multiplication and (FL) is division. So, when we have quantities which are positive negative and 0. So, how do we conduct this 4 basic operations. So, how do we understand that, in fact here very clearly lay down.

So, this has to be understood and appreciated so thinking of the times in which this texts has been come composed. So, very very clearly he gives out all the operations. So, let me give the one or two examples taking 1 or 2 verses and explain, so the rest I will leave it because we have to cover a few more things. So, for instance for the first verse of this section starts with the operation (FL) Brahmagupta says (FL).

So, (FL) is positive quantity (FL) is negative quantity (FL) so (FL) so when you have 2 positive quantities we add them you get positive, negative+negative is negative (FL) when you deal with 2 (FL) so the resultant will be (FL), so when you have positive and negative it can be positive or negative then he says so consider 0, so (FL) is 0, so when you deal with 0 so positive+0 is positive, negative+0 is negative and finally he says (FL),so this will looks so trivial to us.

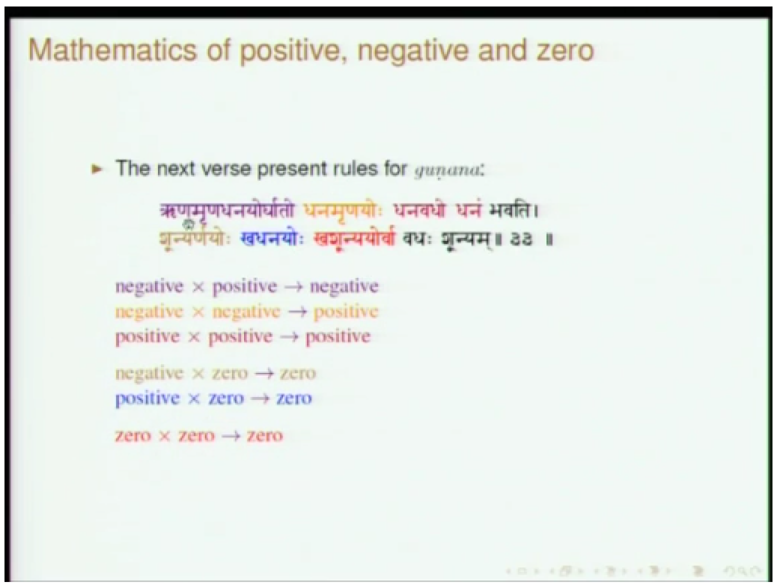
(Refer Slide Time: 16:37)



But laying it down so when things were not formulated well, so is something which is interesting, so (FL) so, he devotes 2 verses (FL) is smaller quantity, so (FL) so, is larger (FL) when you have a max larger quantity and you subtract the smaller one what you will get is positive. So, negative-negative will be negative and then so he goes on (FL), so (FL) is 0 (FL) a negative quantity so, (FL) so, what we will have is (FL) and (FL), so you will have (FL) positive.

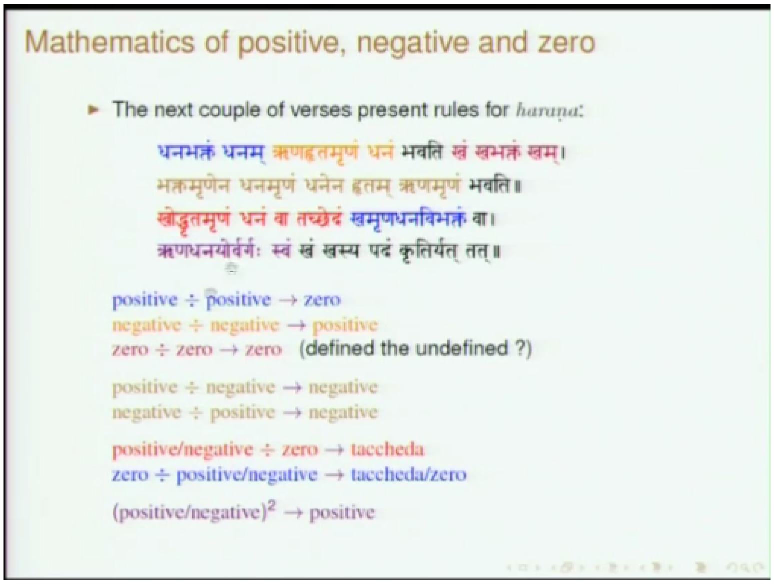
And (FL) see (FL) as I was telling in (FL) refers to space and all synonyms will be employed to refer to 0. So, (FL) all will be employed, so (FL)-(FL) will be (FL) so, then (FL) is positive quantity (FL) if you want is subtract what you will get is (FL), so and (FL) so all that has been all verses have been dealt with in great detail.

(Refer Slide Time: 18:07)



Then the (FL) so, (FL) so, (FL) is negative (FL) see is if you have negative into negative, so that will be positive see (FL) will be the resultant if you deal with 2 negatives. So, where it is (FL) so all those things have been stated and so, if you look at this last part of this verse (FL) see (FL) is 0 and (FL) is also 0, so he says (FL) 0 into 0 is 0.

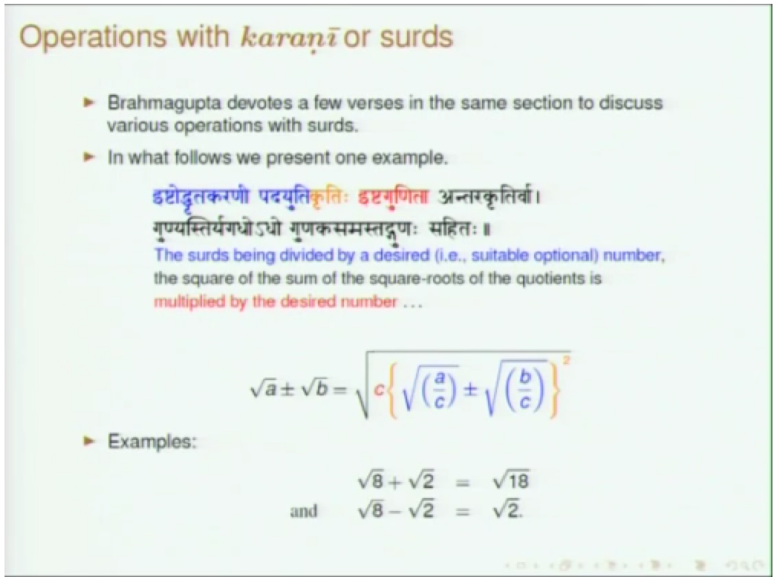
(Refer Slide Time: 19:01)



Then the last part so, (FL) so, (FL) see (FL) so, (FL) see so, (FL) is positive, so (FL) positive I made a mistake here positive should may positive should positive here okay. So, **so** then negative by negative, so that is positive again, so 0 by 0, so this also has defined as (FL), so this we know today is undefined but does mean defined as 0. So, (FL) so, positive by negative, negative by positive, so all that has been stated, so then (FL) this is the very important thing.

So, he introduces a term called (FL), so (FL) is division, so when 0 becomes the deviser see (FL) means it refers to a quantity wherein we find the deviser to be 0 (FL) refers to 0, so it should be a (FL) compound (FL), so for which 0 is the deviser, so they are introduce the term called (FL) and so, the other part is 0 is the numerator and any other quantity is the denominator that also has been stated to be (FL).

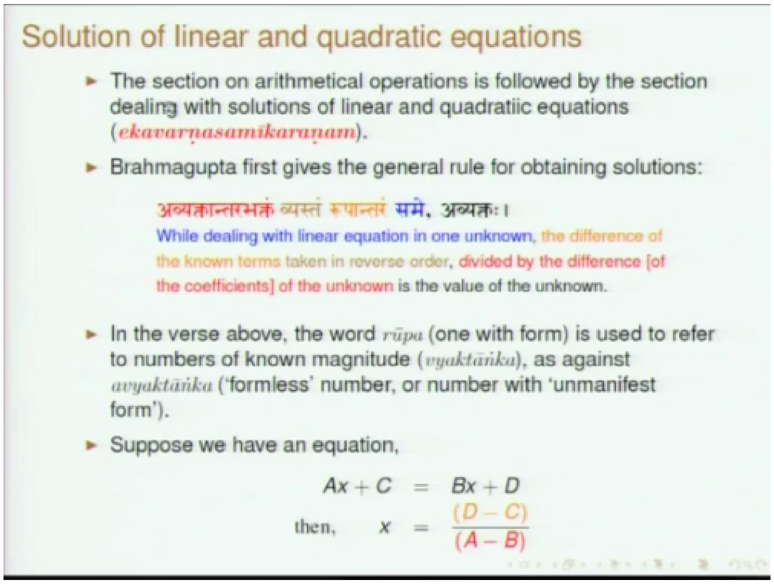
(Refer Slide Time: 20:45)



So, **so** with this he basically describe all the operations which can be done with positive negative and 0. Then this (FL) refers to surd, so, operations with surds this also we discusses in great detail I will give you only one example here (FL) see this suppose you want to add or subtract 2 surds. So, he gives a interesting method he says (FL) means you can divide it by some choice so which will make the operation convenient to you.

So, (FL) so divided by this (FL) is square root (FL) is add, so you have 2 numbers so AB o square root has to be added or subtracted. So, you divide by this and then he says take this square of that and then multiplied (FL) this quantity see whichever you chose you multiply and then take the whole square root. So, this is what I will be stated (FL), so and this will give you the difference or sum of the 2 surds with few more verses, so which deal with the square roots.

(Refer Slide Time: 22:10)

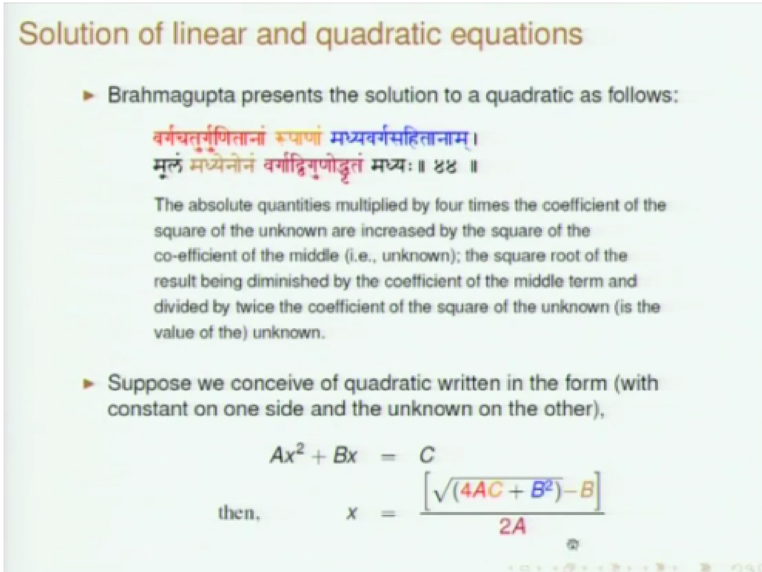


Then I move on to the (FL) and (FL), so the general principle is laid down by Brahmagupta in the very first verse of this section. He says (FL) suppose you have an equation of this forum, so Ax+C is equal to Bx+D. So, this coefficient say I said x is (FL), so (FL) something to form is known sometimes he uses the word rupa sometimes he uses the word (FL). So, rupa means actually a constant so whose form is known see, so in contrast with (FL), so whose value is un manifest rupa is manifest form which actually refers to the constant.

So, for instance D and c will be called as rupa, so that is why he says (FL) see the difference of the 2 rupas so D-C. So, (FL) in the sense so, this has to be taken the other order okay, so D-C it is in this sense use of the word (FL), so (FL) is A-B, so the coefficient of the unknown quantity is also refer to as (FL) ,so I have seen. So, this in the verse (FL) refers to the difference between the coefficients of the unknown quantities A-B (FL) is divided what is to be divided the (FL).

So, that gives the (FL) here refers to when you want to find equate this and then obtain the solution (FL) okay (FL) that is what it means.

(Refer Slide Time: 24:03)

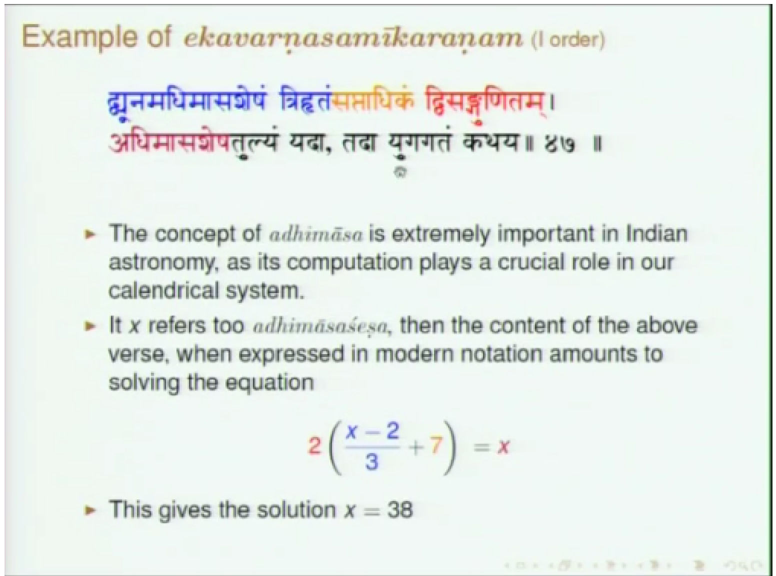


Then when you deal with quadratic Brahmagupta gives this (FL) before he moves on to the example see lays down the principles, so what is to be done that is very interesting here (FL) so suppose you consider an equation like this, so Ax square+Bx + if this is C, so he says he puts it on the other side. So, this is the general way in which the things have been handled. So, if you look at so the procedure that has been laid down.

So, you take rupa to one side and the unknowns on one side. So, and then handle this, so this C is refer to as rupa, so (FL) see so this (FL) here refers to the coefficient of x square oaky. So, (FL) is multiplied 4, rupa, so what you get is basically 4 AC, so (FL) see (FL) is this middle term, so (FL) is square (FL) is this, so 4AC+B square (FL), so you have to take this square root (FL) so, again –B, so (FL) see (FL) see (FL) here again refers to A (FL) okay.

It is basically the formula –B+- so, that is see away (()) (25:34). So, here so it has to be stated it in this particular forum and having laid down this here is a few examples.

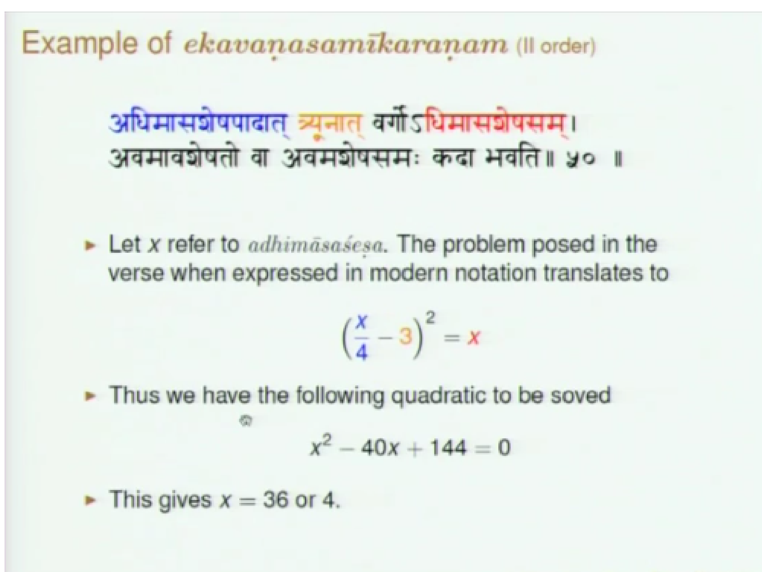
(Refer Slide Time: 25:41)



(FL) so, is very common problem which one encounters in astronomy, so the concept of (FL) so, which some of you would be familiar with, so we have to when we match this lunar and solar calendar, so the number of base which will constituted by 12 lunar months. So, will be falling sort about 11 days in a year and therefore, so every 2 and ¾ of the year we introduce a an average (FL), so it is has certain problem connected with that.

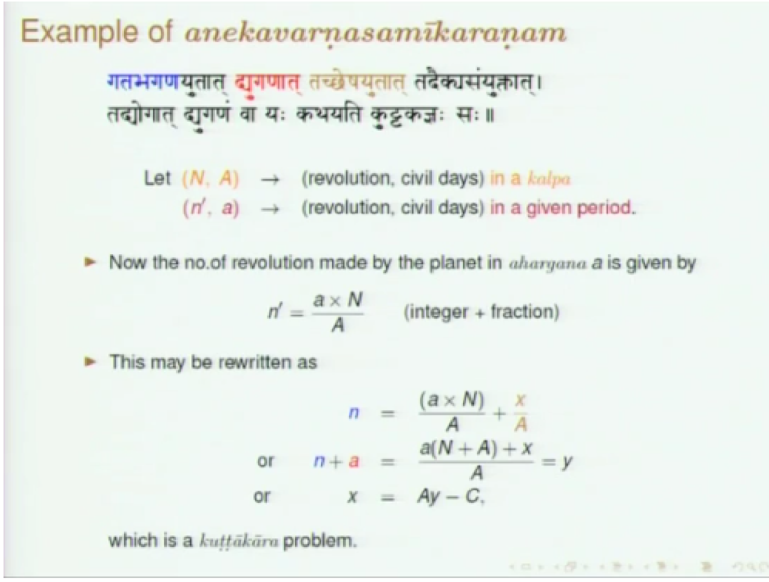
In that context he says (FL) means the remainder of a particular (FL) in a year (FL) so, he says (FL) means (FL) so, remove 2 x-2 (FL) divided by 3 (FL) so, +7 (FL) multiplied so (FL), so that itself is x, so what you get so is.

(Refer Slide Time: 27:01)



So, this is an example of (FL), so one more for (FL) see in fact this is second of (FL), so that is first order here again he says (FL), so (FL) is $1/4^{th}$ so, (FL) so (FL) -3, (FL) square (FL) it should be so, this is what it is (FL) so, how you will get, so you have a sort of quadratic and for which the solution is this, so one you choose 36.

(Refer Slide Time: 27:44)



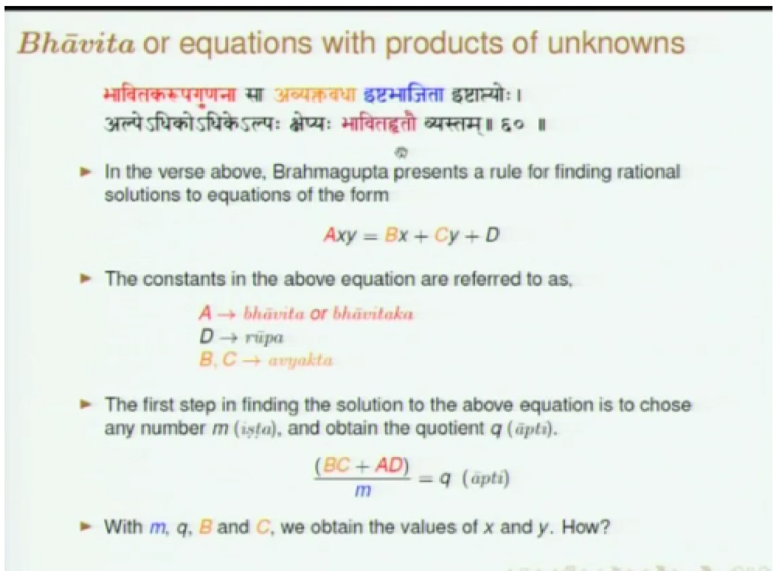
So, it is sums certain other considerations. Now I discuss this (FL), so you have a 2 variables, so we started with (FL) linear quadratic now (FL), so this again I just illustrate with 1 example (FL), so this problem will boil down to sort of (FL) but then so, presented in a context of finding the number of revolutions made by a planet in a given period and hence getting longitude.

So, here so, that is how it is post, suppose you have a very large period and this large period you call it civil days. So, in fact the Brahmagupta considers the period of (FL), so (FL) is 1000 times (FL) 1000 times 43 lakhs 20,000 years. So, he presents some huge numbers, so what is the number of revolution made by planet in this large period. So, that is how he goes about computing the planetary positions.

So, suppose N, A refer to the number of revolutions made by the planet in the total number of days in a kalpa. So, if n prime refers to the fraction of revolutions in a certain period called (FL), so (FL) is basically the count of days you start at a given period, so in this period this was the position from this, so in (FL), so how many number of revolutions have been made it to be fractional number.

So, obviously by rule of 3 we will get n prime is a times N/A, so this will be I mean an integer part under fraction. So, you try to express it as this where N refers to the integral number of revolutions okay. So, this is x/A and you add A on both sides and you get an equation of this forum. So, this is a (FL) problem, so if 1 is able to get, so then he says he is a (FL).

(Refer Slide Time: 29:59)

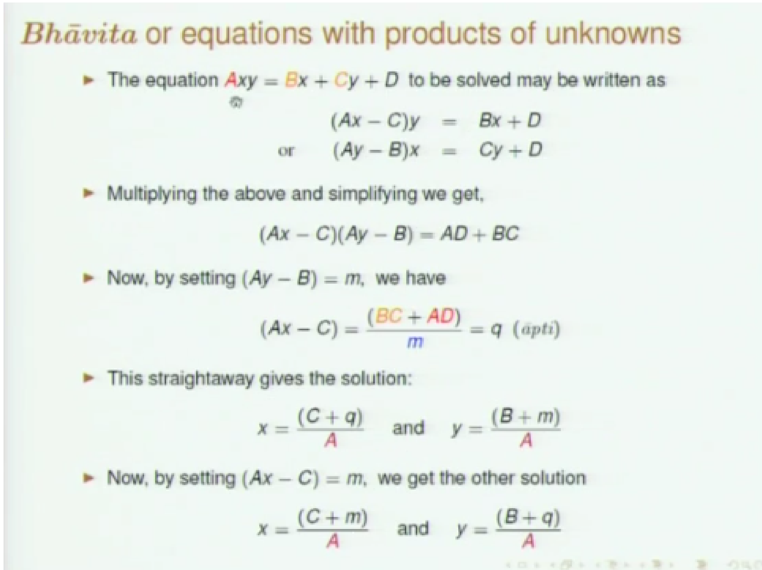


Then Bhavita problem, so Bhavita as I was mentioning refers to the product of 2 unknowns. Here he lads down the rule for first (FL) so, suppose we have an equation of this forum, so this coefficient of the product of xy is refer to as Bhavitha, so recall the term rupa refers to the constant. So, what he says is (FL) is multiplication take product of A and D. So, (FL) so, (FL) so, here x is (FL) y is also (FL) and the co efficient also refer to as (FL), so (FL) is product of (FL).

So, B and c so, then what is to be done, see if you look at the forst top of the verse (FL), so (FL) BC (FL) I mean divided by (FL) see the point is so, you want to find out solution, so the prescription is the following, so you take the coefficient which are known, so take the product and divide by some (FL) means whatever you get as the quotient, so (FL) of the numbers Q and m so, (FL) so where is what he saying so, you have to basically add this to this and this to this.

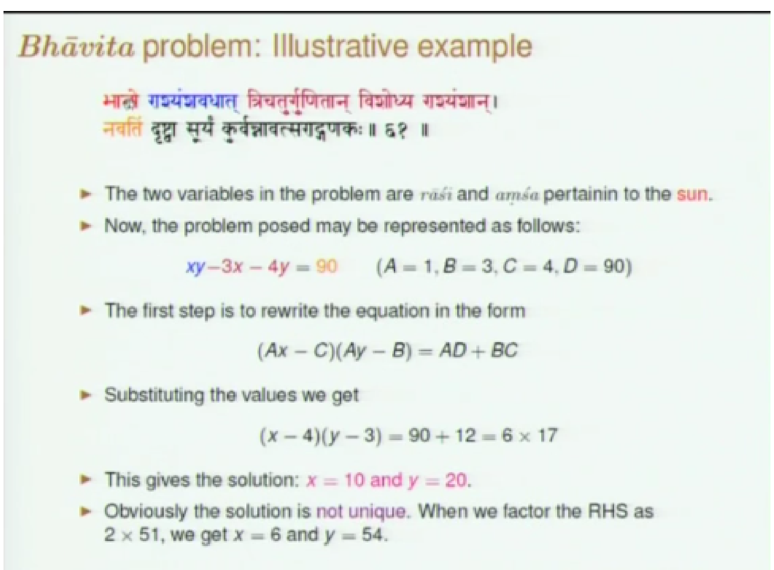
So, q and c have to be added to B and C, so depending upon which is smaller which is larger and (FL), so if you divide by A you will get the required x and y.

(Refer Slide Time: 32:13)



So, basically the principle that goes is the following see so, if you look at the C x is so, finally c+m/A and y is B+q/A or it is the other ways so, depending upon how you choose say basically this problem can be rewritten in this way and once you rewrite you get an equation of this forum and this he writes it as so, q times m, so this is all it does and then so, basically adding you will get your x and y.

(Refer Slide Time: 32:45)

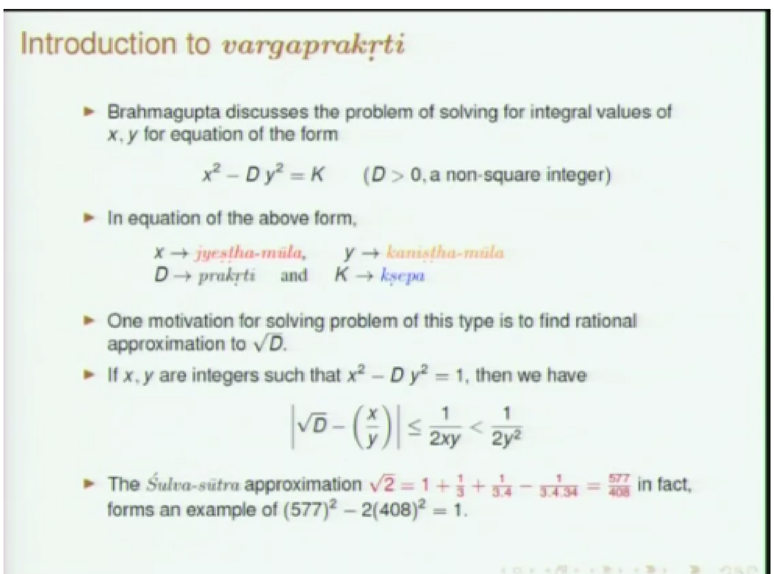


So, an example so he says (FL) so, (FL) say this phrase (FL) is interesting phrase (FL) means an mathematician, so (FL) you know what (FL) so, this is how the sutra goes if you have (FL), so (FL) so, even if a follower to do this problem for 1 year and if he completes, then I would call him mathematician, so it is in this sense I mean the kind of (FL) statement then algebra was in it is initial phase.

So, this is how in most of the examples he says (FL) anyway what he says is (FL), so (FL) refers to 30 degree segment (FL) is degrees, so 30 degrees called the (FL) is degrees, so (FL) related to sun (FL), so this of the 2 quantities x and y, so one is (FL) y is (FL) okay (FL) so, xy take the product (FL) see so, 3(FL), so 3 and 4 (FL) multiplied 1 is (FL) the other is (FL) having subtracted (FL) so, this is equal to 90.

Now from this you have to obtain what is the actual position okay, so that is what he says. (FL) one is able to tell what sun is, so then this is the problem and one can show that one gets x is 10 and y is 20, so this is an example of (FL) problem.

(Refer Slide Time: 34:42)



Now I come to the most important part of this chapter, so which is almost the last (FL) verses, the first (FL) verses (FL), so and then we had this meaning with surds and then operation and then (FL) bhavita then goes on to (FL), so (FL) refers to the second order indeterminate equation.

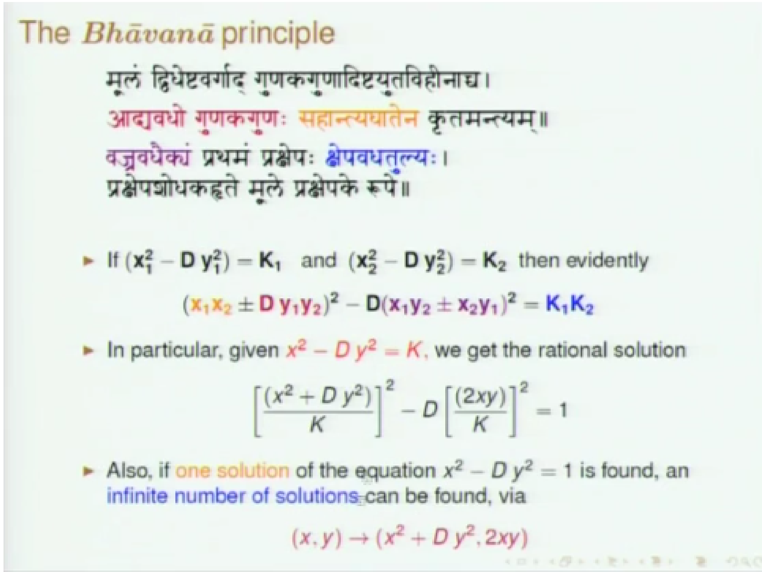
So, in equation of this forum is known as (FL), so x square-Dy square, so given D and K, so we need to find x and y, so which are integers.

So, later Bhaskara has very clearly stated the terminology so, (FL) but Brahmagupta does not call them as (FL) in the verses which prescribe this but we just take at this will be convenient when we see Bhaskara also later during the lectures later. So, here so this problem gives rise to this kind of unique quality, so the one can show that this one of the motivations are solving this problem is to obtain the value of rot D.

So, successive approximation of root D and we obtain using the Bhavana principle which has been given by Brahmagupta. So, we will demonstrate with the few examples but then so, just keep this in mind, so this is an any quality which will be satisfied root D see you can see that if x and y are large and suppose K is equal to 1, so then so, immediately you can get a first approximation, so root D is equal to, so y/x okay, so x/y root D is x/y.

So, this straight away you will get this, so root D –x/y is less than 1/2xy and that is less than 1/2y square. In fact we will show little later perhaps in the next lecture that the approximation which has been given for root 2 in the (FL) sutra text, so which we discuss much earlier, so this approximation which was given kindly straight away obtain using this, so this is not to mention that this was the approach taken (FL) that is the very interesting way of obtaining this approximation.

(Refer Slide Time: 37:19)



So, this set of verses basically describe 3 important things, so 1 is defining the (FL) equation and then so, defining the second, third line actually give the Bhavana principle and of course a special case in the third one K is equal to 1, the fourth line basically gives that. so, let us try to understand the terms employed in this verse, so in fact if we consider x1y1 and x2y2 which satisfy equations with this (FL) K1 or K2 are called as (FL) see.

So, D is called (FL) K is called (FL), suppose we have an equation which satisfies this, so given D and K1 you obtain x1, y1, so given D and K2 you obtain x2, y2 so, we have this 2 equations. Then to what is stated here is, so this equation will be satisfied and that is what has been stated in the second and third lines which you find here see (FL), so in fact Brahmagupta here uses the word (FL) and (FL) is multiplication (FL) so, this (FL) is the word which has been generally used.

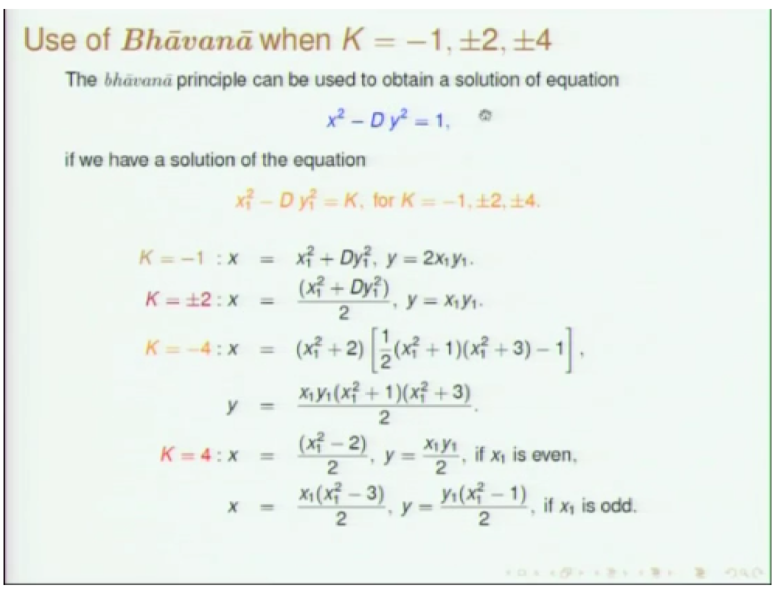
And here he is use the word (FL) to refer to D, so (FL) here refers to this y1y2 okay, so you have 2 equations. So, y1 is (FL) and y2 is also (FL) the product of y1 and y2, the product of y1 and y2 multiplied by (FL). So, this is the equation which has been stated the D times y1y2 (FL) so y is refer to as (FL) and x is refer to as (FL), so (FL) is product of (FL) say x1x2, so (FL) means added to that see, so x1x2+Dy1y2 then he says (FL) is (FL) basically refers to cross multiplication.

Suppose if have 2 equations x1y2, y2x1 this is refer to as (FL), so that is what he say (FL) see K1 and K2 (FL) is product of that, so this equation is satisfied. So, this is what see this is what is basically Bhavana principle fine. So, having stated the Bhavana principle you take a particular you have x1 is equal to x2, y1 is equal to y2. So which means you are Bhavana will get an equation of this forum and that is what is stated here.

And if one finds a solution to this, if suppose you find one solution with satisfies this equation x square- D y square is equal to 1, then by Bhavana principle see what is most important to understand here is by employing this Bhavana principle you can move from xy to x square+D y square and 2 xy. So, if one solution xy is known, so then all of Brahmagupta as approved. So, you have see if you do this also say this equation.

And as a particular case what one gets is 2xy and the x square+ D y D y square and you get infinite number of solutions. So, this is the most important aspect of Bhavana principle and using this principle one will be able to get successive values of (FL).

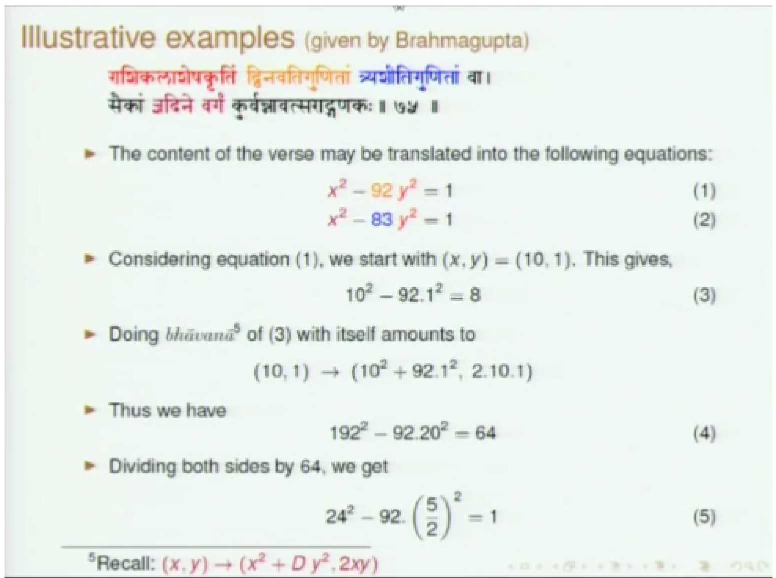
(Refer Slide Time: 41:43)



Brahmagupta also presents some other cases wherein K is -1+/- 2 +/-4, so all that has been stated by brahmagupta here so, this will be straight forward in certain examples, so you can simply guess x and y such that this equation is satisfied and once you know one solution then you keep on generating infinite number of solutions by using this Bhavana principle, so if you do not so, if solving a certain problem in fact which will be discussed in great detail by professor Srinivas later.

So, if at a certain case certain stage of the problem, so you get K is equal to -1, +/-2 +/-4 then also you will be immediately able to solve the problem and this particular have been stated down what will be the xy values, so have also been laid down by Brahmagupta.

(Refer Slide Time: 42:51)



I take a couple of more minutes so, to present an example of this Bhavana principle, so which has been given by Brahmagupta himself. So, after stating this (FL) so this is how the section on (FL) commences okay (FL) is the word which refers to D but Brahmagupta is the word (FL) here and this has been said as (FL) so Bhaskara's versus they use the word (FL), so it is (FL) is basically the second order equation.

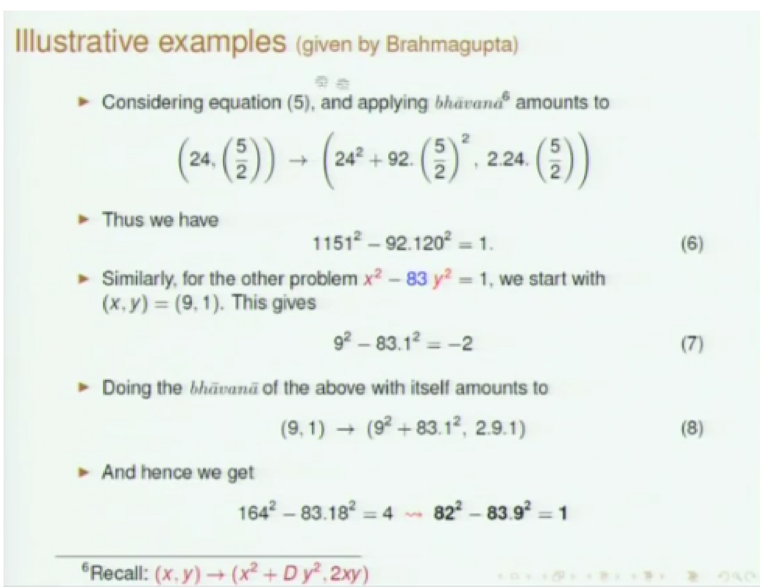
So, square and therefore it is refer to (FL) after presenting this what is (FL) equation and what is Bhavana principle and this particular case, so and then so how to find out, so even when K is not equal to 1 immediately jump into this solution, so after presenting what are the values of x1/1, so which will lead you to the solution, so he takes a example (FL) so (FL) refers to 30 degrees and (FL) is seconds so, (FL) refers to second sorry (FL) is minutes (FL) is seconds.

So, (FL) so, this is how it goes 30 degrees, degree, minutes and seconds, so (FL) see kriti is use to refer to square here, so krti so, we will have instance to discuss on this later we will discuss this Bhakshali manuscript also, so in colloquial language krti actually refers to once on work our (FL) krti when you say composition of (FL) okay. So, krti mostly it is used to refer to square in many of this mathematical text.

And (FL) so krti square of so 2 things one is (FL) the other is kala okay, (FL) is 92 okay (FL) or (FL) you can take D to be 83, so either 92 or 83 (FL), so that quantity K is 1 okay, so (FL) see, so let us take first equation and here, so you can straight away see that 10 and 1, so x is 10 and y is 1 you will get 8. So, now use the bhavana, so recall so according to Bhavana principle, so if x y is a solution then x square+D y square and 2 xy is also a solution.

So, this is the Bhavana principle straight you can find out and convince yourself that , so here so 10 square+92 into 1 square so this is the mew x and this is the mew y okay 2xy 2 times xy so, this moves to this also is a solution and if you do this and then you divide both these sides 64 you get an equation of this forum. Here D is 92 that is what the that is the example that we are doing. So, 24 square-92 into 5/2 square, so is equal to 1, so this is like x square-D y square is equal to 1.

(Refer Slide Time: 47:18)



So, once again do the Bhavana x those to x square+D times y square, so that is what it is and y goes to 2xy 2 times xy and you have this equation, so in the other problem so, this you have basically reduce this and you can further keep on doing and all this will basically give you better and better approximations to root D, so 1151/120 will be in approximate value of 92 root D root of 92 will be this ratio.

If you do Bhavana once more with itself, so you will get better and better approximations. Similarly so if you choose the D equals 83, so then you have straight away you can see that so it is not necessary that K has to be always positive, so you can choose anything and so we get 9 square-83 into 1 is -2 and doing Bhavana of this with itself, so gives you this, so which when you divide by 2.

So, **so** actually 2 square, so you can divide and you will get an equation of this forum, so this is x square-D times y square is 1 straight away. So, you can keep on doing Bhavana and you will be able to get better and better approximations for root D. So, with this we stop our discussion on Brahmagupta will continue with 1 or2 more examples and then we will proceed to Bhakshali manuscript in the next lecture thank you.