Mathematics in India: From Vedic Period To Modern Times
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Lecture-12
Brahmasphutasiddhanta of Brahmagupta-Part 2

Okay, so we will continue with the discussion of mathematics in Brahmasphutasiddhanta, so now it is a second part.

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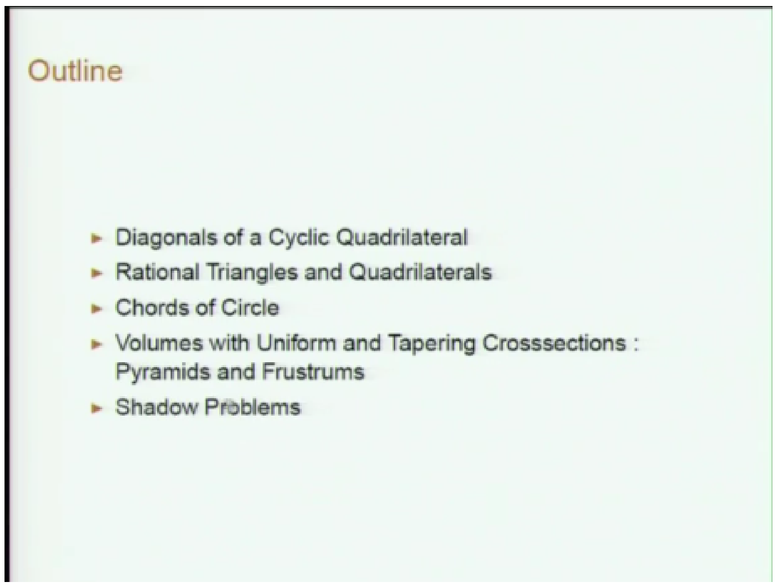

So, meanly we will be discussing the diagonals of a cyclic Quadrilateral, then rational triangles and quadrilaterals then chords of a circle, so that is a plain figures, so then we will go to the volumes with uniform and tapering cross sections: pyramids and frustums and shadow problem.

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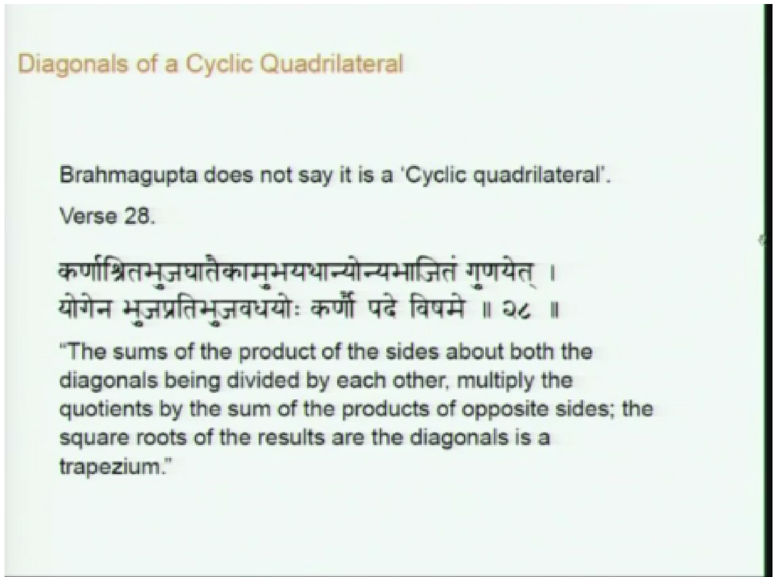

So, earlier we are discuss the circum radius of a triangle and cyclic Quadrilateral. So, we will now discuss the diagonals, so Brahmagupta does not specifically mentioned that it is a cyclic Quadrilateral but it is applicable only to that, so what is they say (FL) the sums of the product of the sides about both the diagonals being divided by each other, multiply the quotients by the sum of the products of opposite sides; the square roots of the results are the diagonals in a trapezium.

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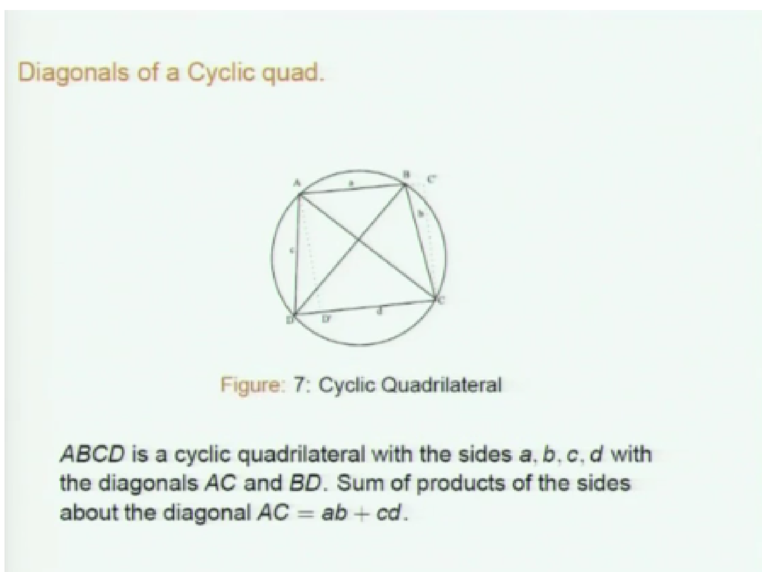

So, this is a cyclic Quadrilateral ABCD and the sides a,b,c and d and the diagonals of AC and BD and sum of the product of the sides about the diagonal AC okay. So, it is talked about the sum of the product products of the sides about the diagonal AC. So, that is AB see this is one side so, these are the two sides abating this diagonal and other if we take the other part it is CD.

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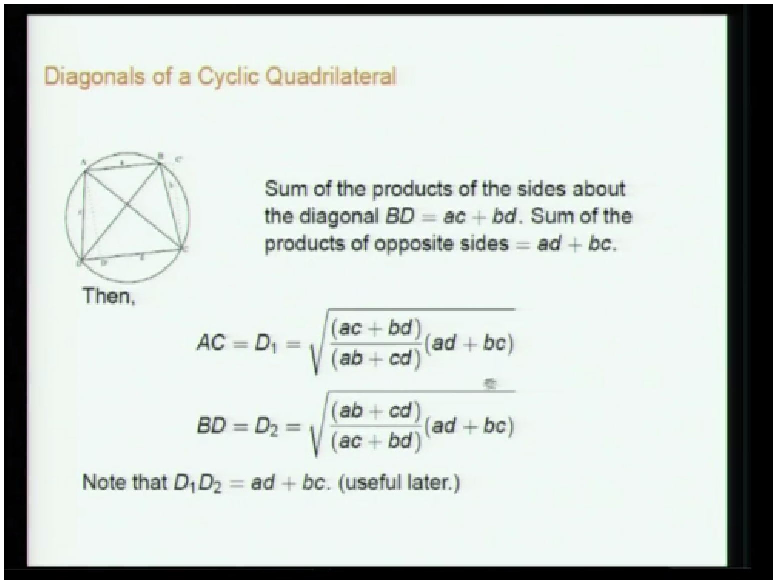

So, AB+CD so, similarly for the diagonal BD it is AC+BD and sum of these things you know sum of the products of opposite sides is AD+BC. So, these are three things you want then the result is AC is D1 is equal to square root of ac+bd/ab+cd into ad+bc and BD is the other diagonal is ab+cd. So, these the interchange you know so, ab+ a still go up and ac+bd this will come down is ad+bc that is the product of the sum of the products of opposite side that will be the same.

So, these are the results for the two diagonals and note the D1, D2, ad+bc which we will be using it will be useful later in many applications. So, this is the first time that the diagonals of a cylcis quadrilateral individually have been given and came to be discovered much later around 17 century years so in Europe. And earlier result had only that result and the product the diagonals not the individually diagonals.

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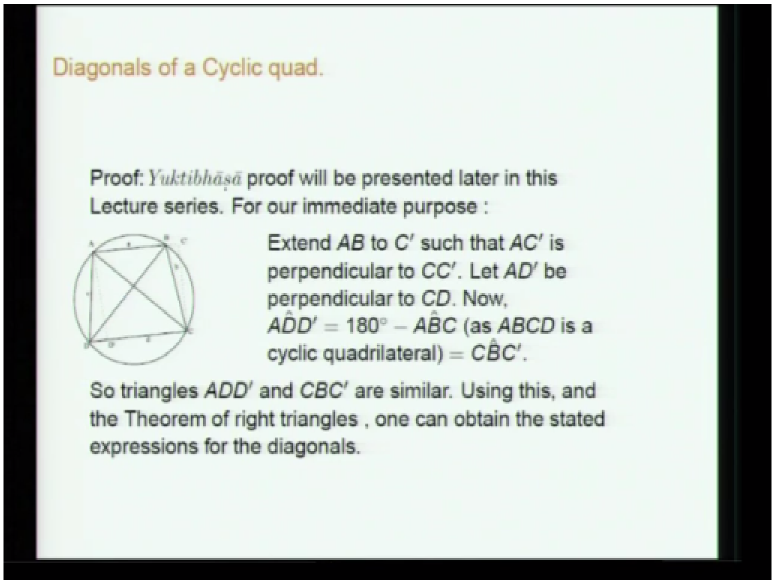

So, the individual diagonals are given by Brahmagupta for the first time. So, the proof hued have a proof I mean one cannot get a result like this without having a method to do it. But you are not given, it neither the commentary give it and later we will in this series of lectures Yuktibhasa proof will be presented in detail. But for our immediate purpose we will use you know what is in this so called modern we have doing things.

So, you have this cyclic quadrilateral ABCD. So, what do you do is extent AB to C prime and such that CC prime is perpendicular to AB ABC prime. Similarly you drop the perpendicular AD prime on the side CD okay so, then so, this AD D prime.

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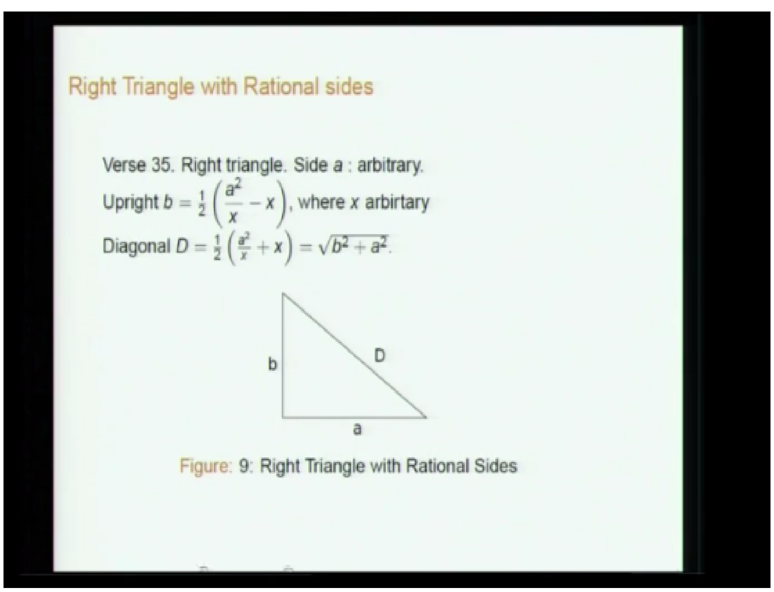

So, this angle this angle will be yeah so, this angle AD D prime is equal to 180 degrees-this angle. We know that in a cyclic quadrilateral right so, that for that happens okay so, and that is equal to CB CB prime. So, this angle is basically equal to this angle so, and one angle is a 90 degree new sub triangles. So, these two triangles AD D prime and CB C prime these two triangles are similar using these and theorem of right triangles.

One can obtained a stated expression for the diagonals, so what essentially do is AC square is AC square is CC prime square+AC prime square okay. And then CC prime square itself is equal to BC square-BC prime square. So, like that use these expressions and BC prime is equal to AC prime-AB so, using these results is a bit of algebra only nothing else, no great concept is involve. You can get this results and both here also it is a same thing is there.

You will get essentially two expressions for AC so, equating this two using the similar triangles using the theorem of right triangle one can get this.

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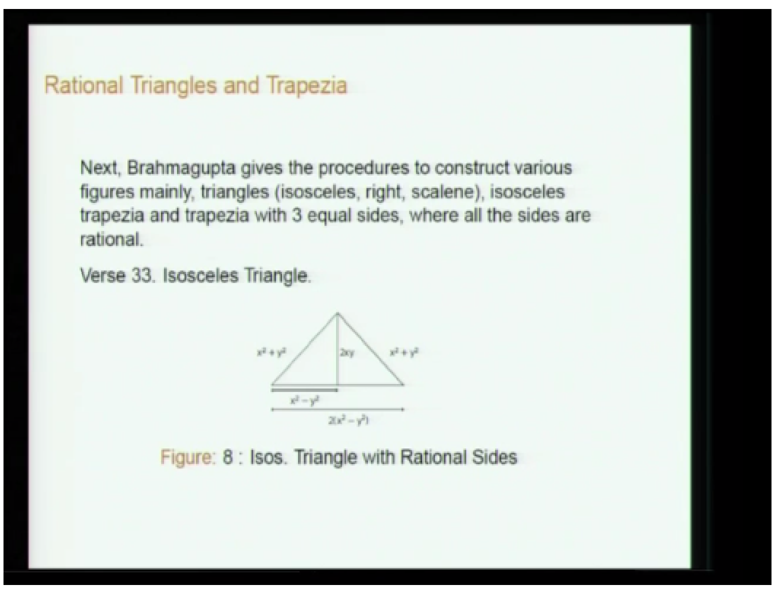

So, that the very important result which has been you know you to Brahamagupta so, now we will go to what are known as Rational Triangles and Trapezia. So, in Indian mathematics works so, there is a importance is given to construct you know triangles, quadrilaterals and various figures where all the sides are rational. So, that is not always easy to construct so, that is what is a being done.

He gives you the procedures to construct various figures mainly triangles with Isosceles Triangle, Right Triangle, Triangles, Scalene Triangles etc, Isosceles Trapezia and Trapezia with three equal sides where all the sides are rational for an Isoceles Triangle he gives the following construction, so you take X and Y to be rational okay. Then take the base to 2 into X square-Y square and two sides to be X square+Y square Isosceles Triangle then perpendicularly is 2xy.

So then one can seize that is rational only rational. Here is the very important thing, so if you take this straight angle triangle okay, so one side is x square-y square the other side is x square the diagonal is x square+y square and the perpendicular is 2xy. So, this is use to you know construct the phytoghorian triplets with Mr.Ramasubramaniam was telling okay. So, by using integral values for x and y you can get various things you know 3, 4, 5 and various other things you can get.

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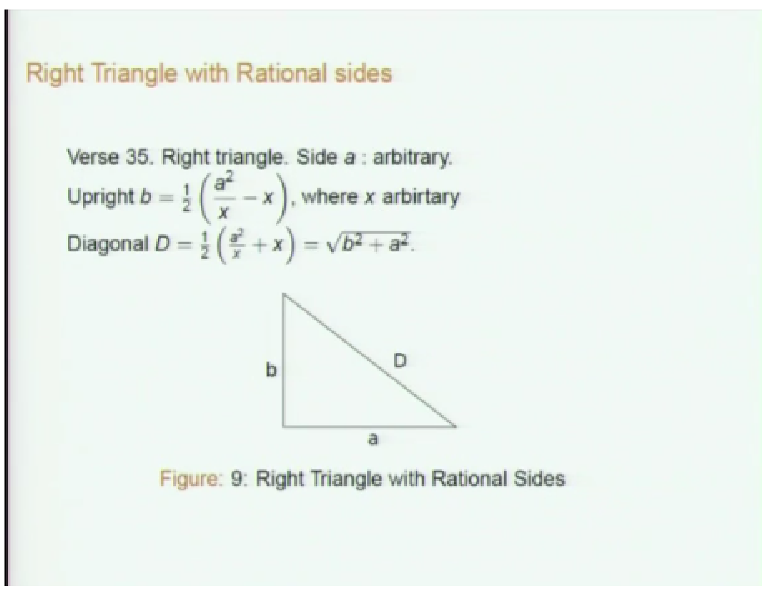

And underway to do it is essentially the same principle is involved. So, suppose you take the side A which is arbitrary and suppose you have some arbitrary x number means rational number again. So, when you take the upright to be half of a square/x-x then the diagonal will be half of a square/x+x and it is equal to square root of b square+a square, so it will be rational okay. So, that comes is slight non triviality please remember.

Because if you take you know 1 triangle right angle triangle which sides 1 and upright 1 the diagonal will be root 2 it is not be rational. So, you have to have some method of constructing the rational thing that is what is being then here.

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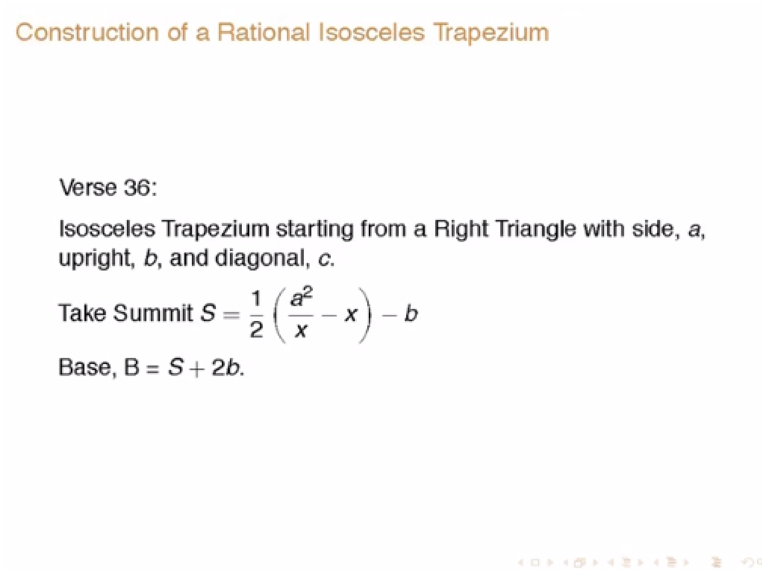

Now for an Isosceles Trapezium is essentially it uses this right triangles with the rational sides and for instance Isosceles Trapezium if you start with some right triangle with a side a upright b and diagonal c then you can take b to b some half of a square/y-y kind of thing and take the summit to be something like this and base to be S+2b okay where is x is another rational number.

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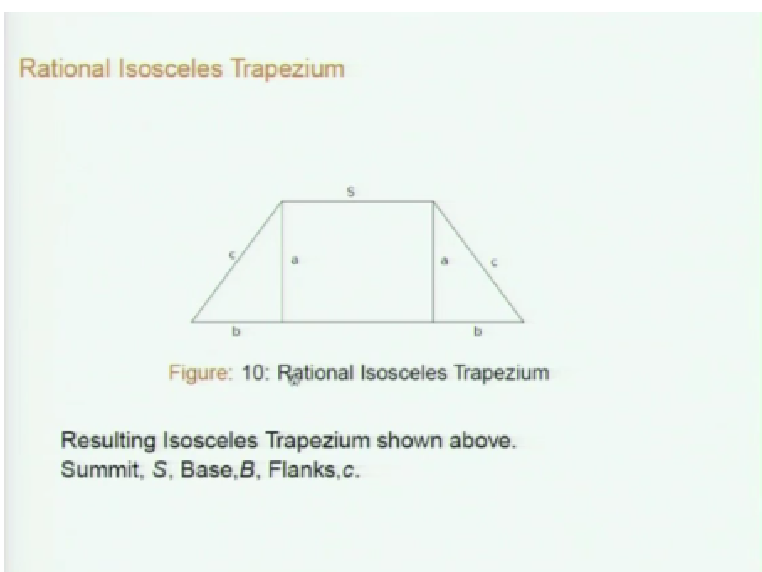

So, essentially what you have is this kind of rational Isosceles Trapezium these are the 2 abating triangles here a,b,c a is the upright these are all rational the summit S in fact you can take summit S is to be arbitrary rational number and you will get these kind of base will be of course 2b+S.

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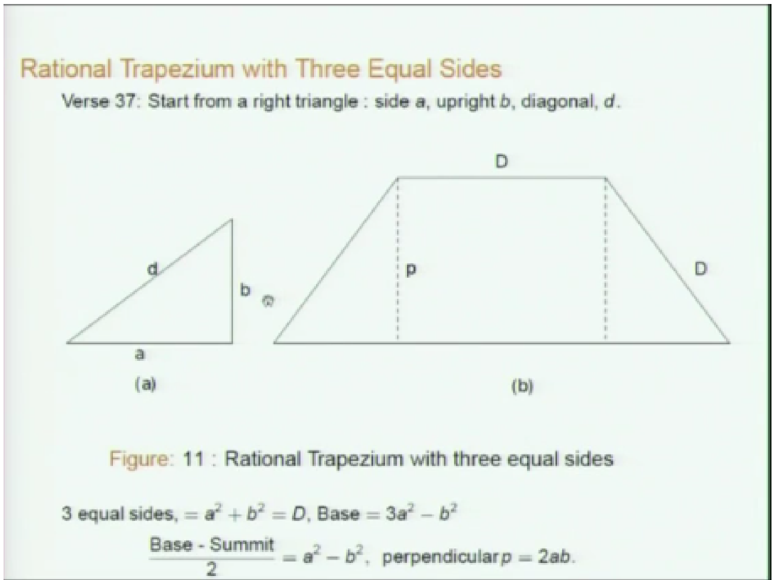

Now a more non trivial thing is a rational Trapezium with 3 equal sides, so this is not that easy, so start with a triangle with side a, upright b and diagonal D you start with that, so then you construct the 3 equal side is constructed like this the 3 equal sides capital D is a square +b square, so this side this one, this one and this one a square+b square and b is 3 a square 3 a square-b square.

And is base-summit/2 so this will be a square- b square, essentially this is a square-b square, so this is a square+ b square and this will be necessarily be 2ab, so all 3 sides will be equal and the base should be the different all rational.

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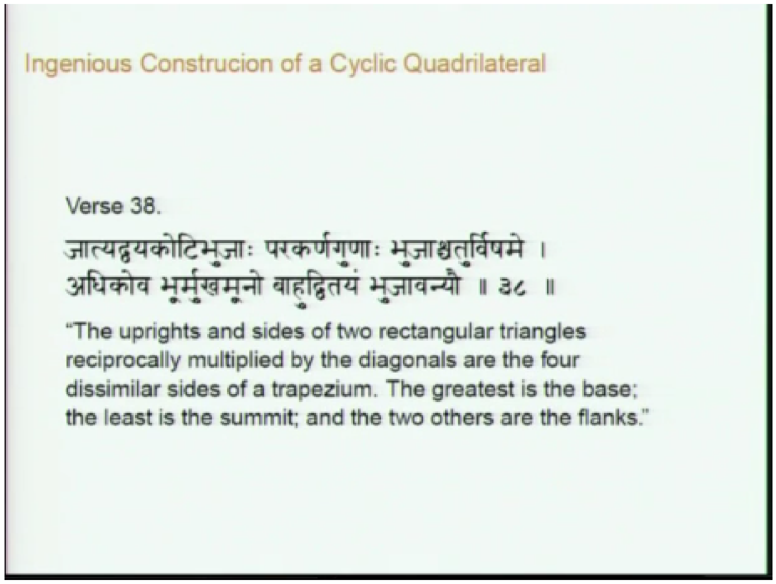

So, next it constructs a cyclic Quadrilateral in a very engineers manners, so in such a very interesting construction that comes in Brahmasphutasiddhanta verse 38 says ((FL) the uprights and sides of two rectangular triangles reciprocally multiplied by the diagonals are the four dissimilar sides of trapezium. The greatest is the base the least is the summit; and the two others are the flanks.

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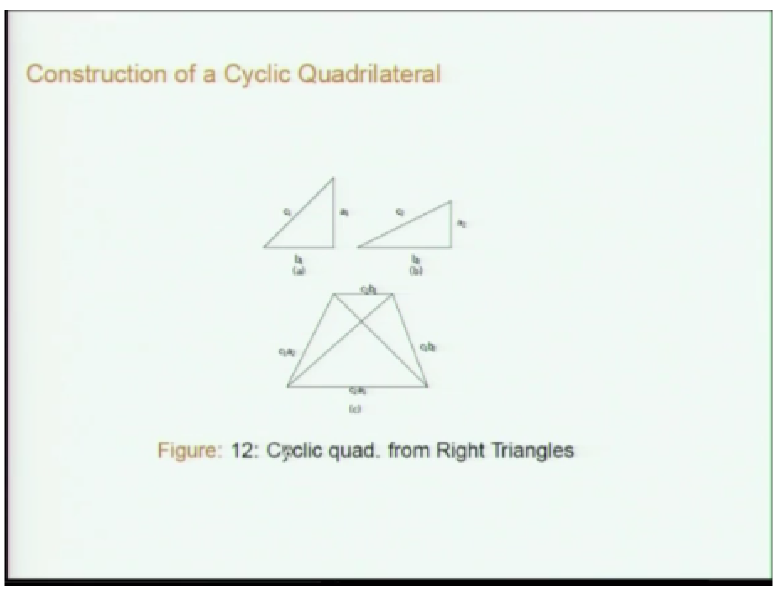

So, what is doing is he considers 2 right angle rational right angle triangles, so that is a1 upright to a1, b1, c1 all rational similarly a2, b2, c2. So, then what it as is c2, c2 into b1 is one side c1 into b2 is other side then c1 into a2 and c2 into a1 you are your reciprocabally multiplied the sides and the diagonals of other triangle okay. So, this is the cyclic Quadrilateral from right triangles and one can show that actually of course he just giving the sides.

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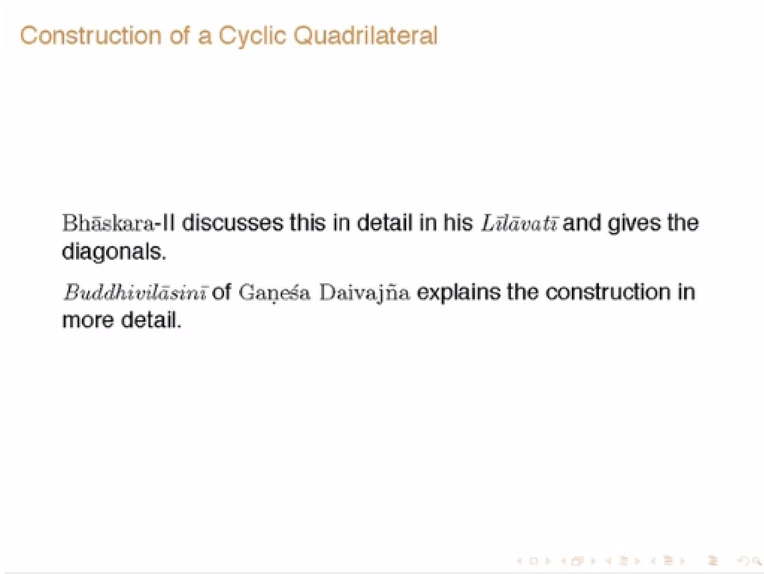

But once we should see what exactly at the segments also *Bhaskara*-2 discuss as this detail in this Lilavati and gives the diagonals Buddhivilasini of *Ganesa Daivajna* explains the construction in more detail.

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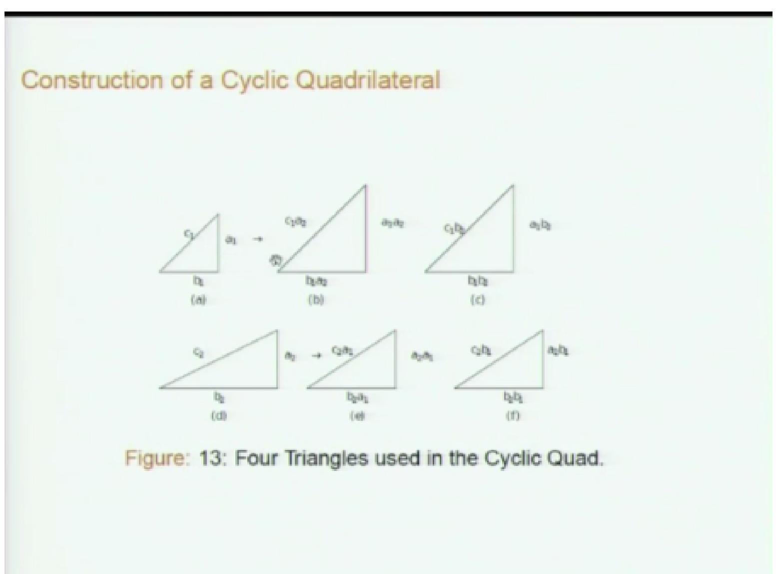

So, what it does is this c1, a1, b1 okay, so is one right triangle, so what you do is you multiply all the sides by the operate of the other triangle. So, you will get this then multiply all the distinct quantities c1, b1, a1 by the side of that. So, that is other thing okay so, these are the two triangles we generated from the c2, b2, a2 so, these are the four triangles.

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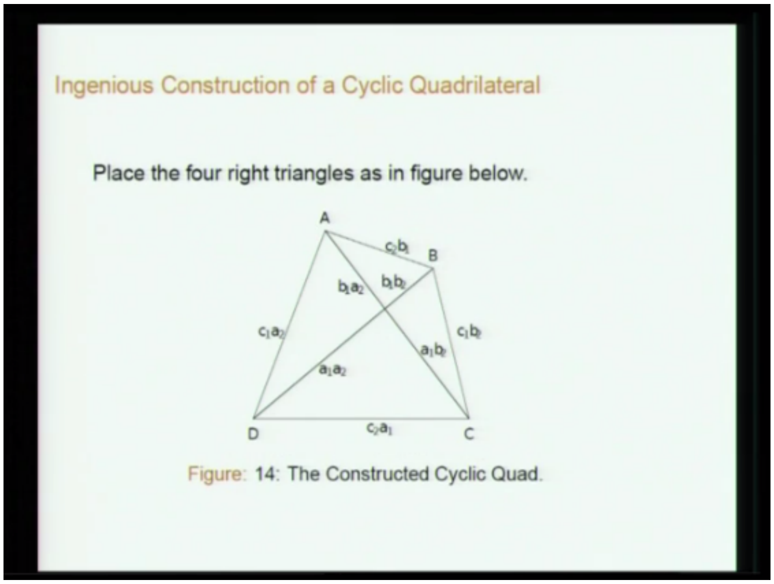

And you place them together you have to place them properly so, what you do is you know you these triangle c2, b1, one b1, b2 should come here. You know c2b1, b1a2, b1b2 that will be one triangle as you can check the other triangle will be one more triangle beside b1b2 you know operate b1b2 and like this. So, if you put all these things all together. Then it will be the cyclic quadrilateral in fact one can see that it is the diagonals and meeting perpendicularly here.

You see the two diagonals are intersecting right angles, so this is Brahmagupta does not give these fully here given the sides only. But this so, it can be constructed so, these one of the methods you know various interesting methods. But we frequently come across in Indian mathematics you know generation of various kinds of things. You know using some simple concepts so, not just generalities, but construction of various things with some specific properties.

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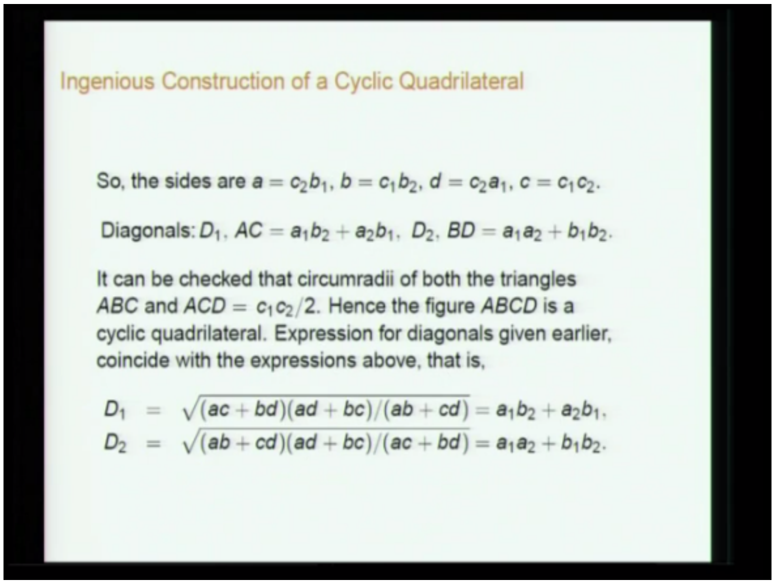

So, here the sides are as I have mentioned c2b1 c1b2 and their other sides are c2a1 c is equal to c1a2 the diagonals will be a1b2+a2b1. So, this particular conception will give the parts of the diagonal also. And it can be check the circum radii of both the triangles ABC and ABD so, they are c1c2/2 so, that we because if it is cyclic quadrilateral then the circumradius of the two triangles which are involve. They should be equal so, that is guaranteed.

So, the figure is indeed a cyclic quadrilateral then the expression for the diagonals by constructed itself we have got this diagonal. But we are got the expression for the diagonals from the Brahmagupta in a general**.** So, they should coincide one can check that the diagonal d1 is ac+bd into ad+bc/ab+cd that a1b2+a2b1, a1a2+b1b2. So, all these are so, this is the very good construction.

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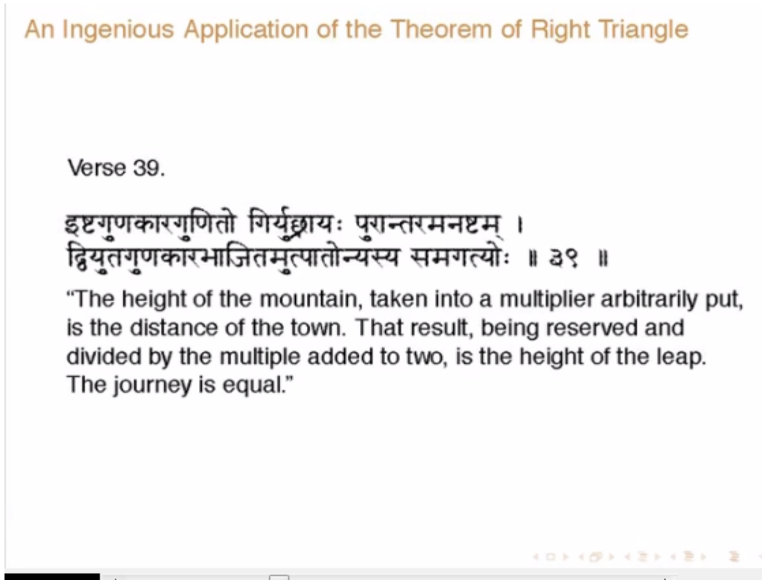

Baskara will actually generate this so, here using the four triangles you are constructed one cyclic quadrilateral whose diagonals are intersecting perpendicularly later Baskara will give another cyclic quadric using the same triangles. But you know but two triangles are interchange kind of a thing. So, then the diagonals will not intersect perpendicularly so, another thing is so, then another very interesting kind of a problem is a discussed very briefly in as characteristic of many books.

It is just given in one was but it contains a lot of information but it does not give the details. He says (FL) the height of the mountain, taken into a multiplier arbitrarily put, is the distance of the town. The result, being reserved and divided by the multiple added to two, is the height of the leap. The journey is equal okay I mean if it just somewhat mysterious in a varies amount and varies the lead and all that but together for me will a commentator will give some detail.

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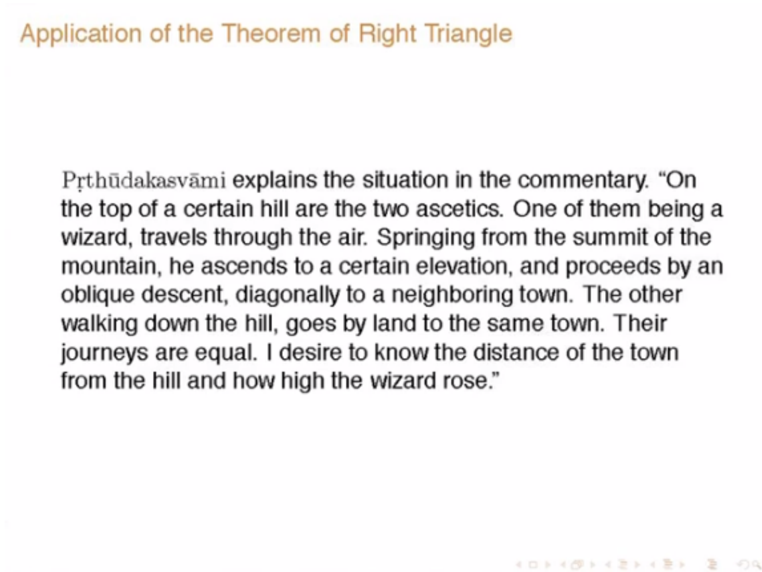

He explains the situation in commentary on the top of the certain hill are two ascetics one of them being a wizard, travels through the air. Springing from the summit of the mountain, he ascends to a certain elevation, and proceeds by an oblique descent, diagonally to a neighbouring town. The other walking down the hill goes by land to the same town. Their journeys are equal. I desire to know the distance of the town from the hill and how high the wizard rose.

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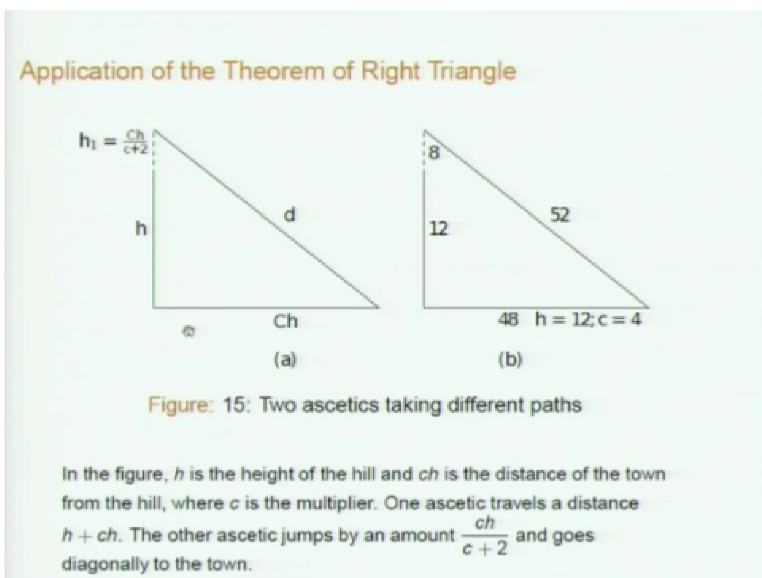

So, what is being discuss in the following, so here you know this is a hill h you know, so these are base and this is the emit of the hill there is down here okay. So, 2 ascetics are here okay one of the ascetics who is a not so interesting, so he will just get down from this sorry, he will get down the hill and then walked to the town. The other wizard you know he will go up he will leap and then (FL) he will go down you see using yogi power, so he will read that.

And it is given at the both the journeys are equal okay so, then what is the height and what the related, so that is what we have to find. So, in this similar h is the height of the hill and Ch is the distance of the top from the hill, and c is a multiplier okay, just assume that the distance of the town is the c time height of the hill. So, ascetic clearly he travels the distance h+ch right he goes down and then perpendicularly, he goes from the base of the hill to the town.

The other ascetic who is more (FL), so he will go up by this height jumps by an amount ch/c+2 and he goes diagonally to this. So, we will come to the other figure later.

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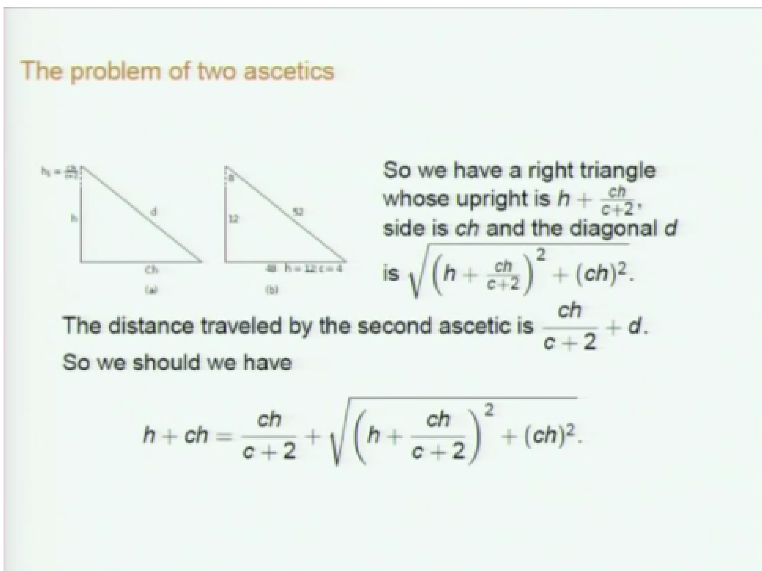

So, we have a right angle triangle, so whose right upright is h+ch/c+2 you see these height here jump right second ascetic and the side is ch and the diagonal d is clearly, so h+ you have to add this also h1, h+ch/C+2 whole square + ch whole square. So, this is the square root that that will be this diagonal and the distance travel by the ascetic is ch/c+2+d naturally the second ascetic it jumps and then goes along the diagonal.

So, we should have this h=ch+p equal to ch/c+2 +this result okay. So, is there any restriction on c or h or whatever it is in fact one can, see there is no restriction for every c and h this satisfies, so that is for every c and h is you can check that.

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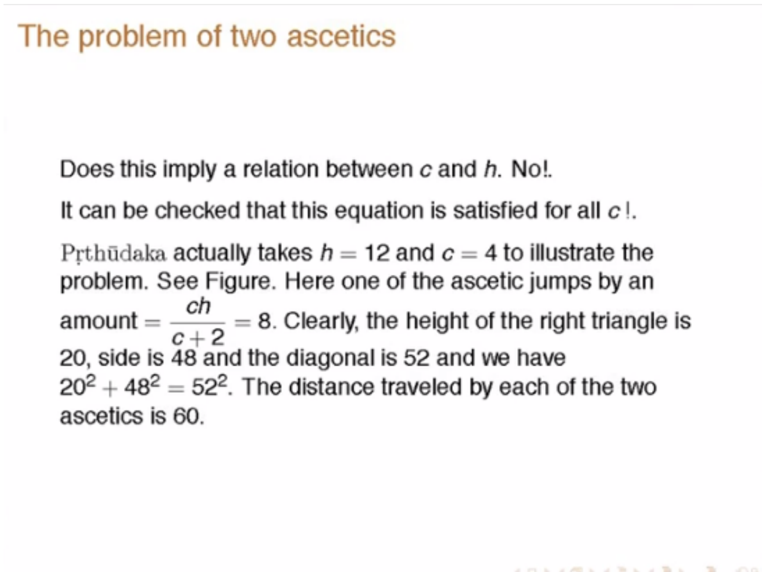

And Pruthudaka actually takes h is equal to 12 and c is equal to 4 illustrates the problem. So, the height is 12 and the ascetic is jumping by amount ch/c+2 is 4 is things c is equal to 4, so 12 into 4/6, so this is a 8. So, clearly the height of the right triangle is 20 okay, so here 12 is the height of the height hill 8 is the amount by which is jumps and the distance of the tone from the base of the hill is 48 okay. So, 48 and this is 20 and in 52 which is diagonal and one can see that 1 **1** ascetic is travelling 12+48 other is travelling 8+52, so they are equal. So, there is a nice application of the right angle triangle.

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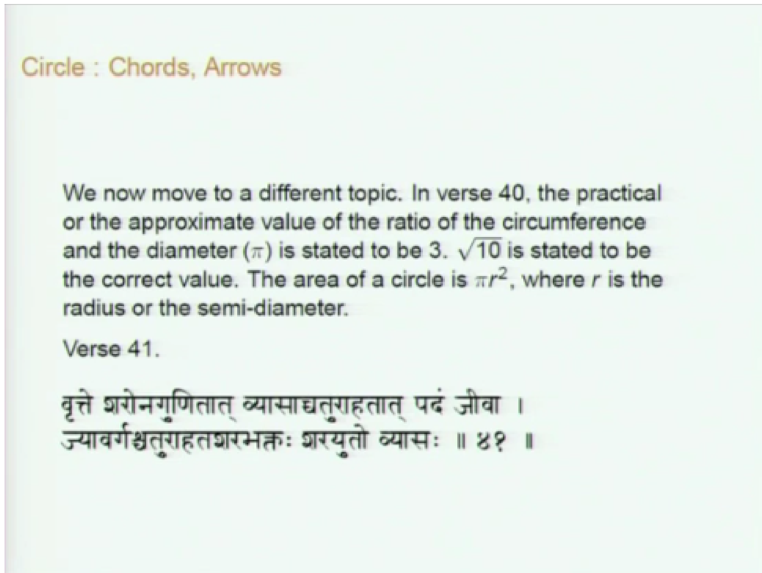

So, the next application that Brahmagupta considers continuing with the this thing is plane figures is a circle and come examples results related to that. So, in worst part is the practical or approximate value of the ratios circumference and the diameter pie is stated to be 3 and root 10 is stated to be the correct value. So, he is given an approximate a practical value which is used several times in many situations.

So, that is taken to be 3 and these root 10 is a has value for pie which is there in many books especially in works and all that they always few this root 10 as the approximate value of pie but I do not know why though Brahmagupta is follow I mean after Aryabhatta and you were aware of his work somehow he does not give the value of pie which is given Aryabhatta he does not prove itself.

And area of the circle is stated to be correct pie r square where r is the radius and the semi diameter I mean it is not stated this way obviously it is not stated this way it is ay that half of circumference into semi diameter as a it was told in the last lecture that is what is stated. So, now he will discuss some chords and arrows ((FL)**.**

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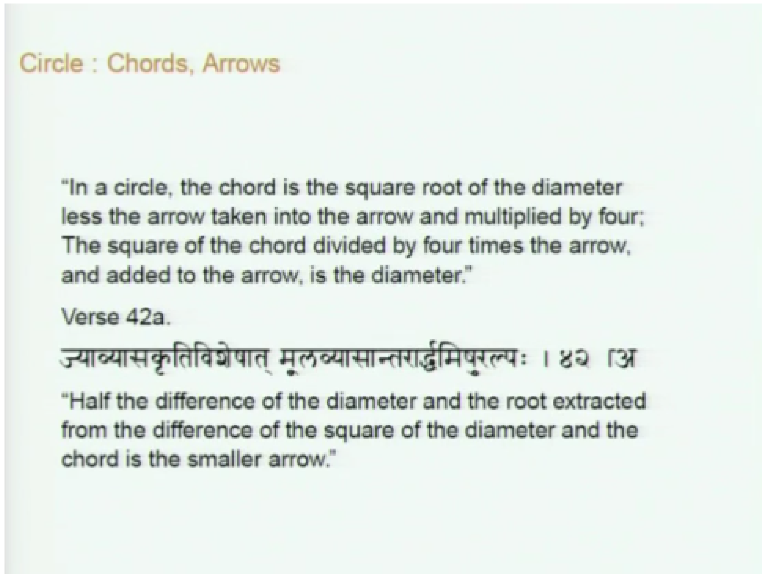

So, in a circle the chord is the square root of the diameter less the arrow taken into the arrow and multiplied by four; the square of the chord divided by four times the arrow and added to the arrow is the diameter. And in the next verse of part of it will say (FL) half the difference of the diameter and the root extracted from the difference of the square of the diameter and the chord is the smaller arrow.

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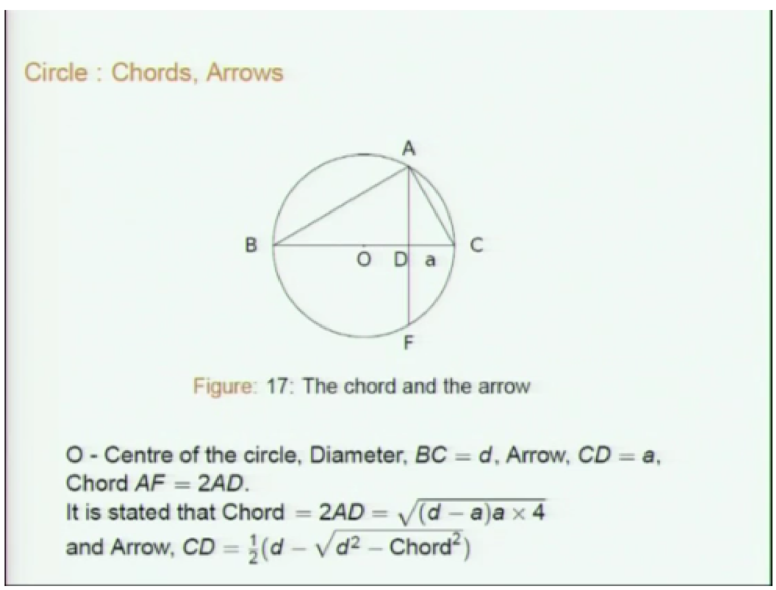

So, essentially what is the doing is these a circle with centre O, so you have this triangle A, B, C okay. So, this is right angle triangle at A O is centred circle diameter is BC, so these called arrow (FL) which Ramasubramaniyam mentioned there this thing you know worse sign he called it is related to that and the jaw is chord AF full jaw you see and this AD is essentially the sign of this angle AOD, so that is (FL) right.

So, jaw is the whole jaw is this chord AF is 2 into AD and arrow is CD okay suppose you call it by A and the chord is AF is equal to 2AD and it is stated that chord is 2AD is square root of d-a into a into 4. So, d if you know a and d you can find out for and arrow of course it is half of the few give the chord and diameter you can find the arrow is half of the d-square root of d square-chord square.

So, these also a non trivial result because you have to use the similarity of these triangle you see ABD and then you know some this A, DC and everything and they are both of them similar to the original triangle ABC I mean I must say go in the correct order but that is how it is because of the similar triangles only.

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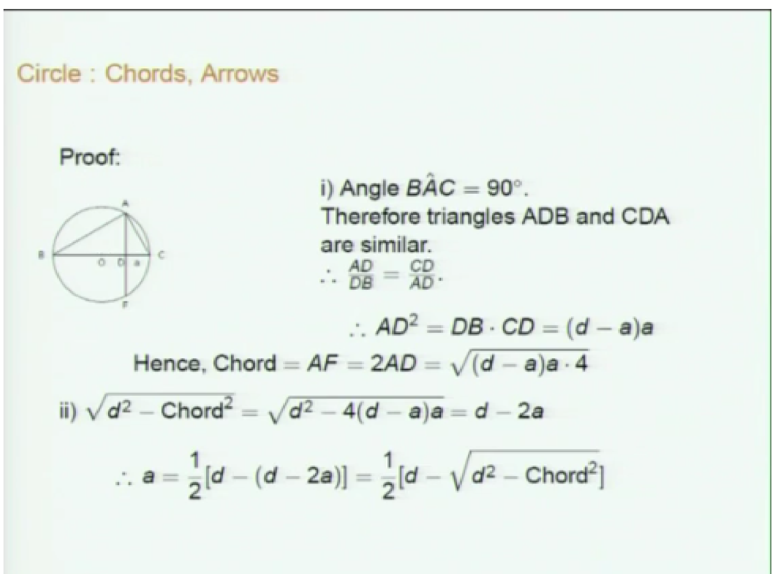

So, may in fact we will see that now see in this triangle ABC and the angle BAC is 90 degrees as I said right because BC is the diameter. So, triangles ADB and CDA okay, so ADB and CDA, so this triangle is equal to this triangle right sorry this triangle is equal to this triangle and this angle P is equal to this triangle here. So, they are similar so ADB and CDA similar, so AD/DB is equal to CD/AD, so therefore you can see that AD square is equal to DB into CD into d-a into a.

And the chord AD is equal to 2AD is very clearly from this square root of d-a into 4 square root of 4, because is you have to get this 2. So, this is I hope it is start and screws now because in early b certainly had this result given to us in our high school. So, similarly if you take square root of d square-chord square will get d-2a because is this, so the is half of d-d-2a half of d-this, so these see the stated result.

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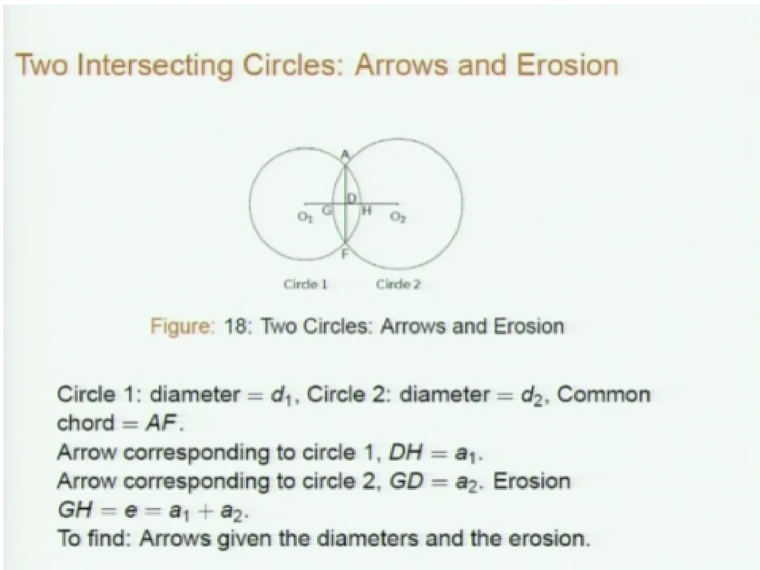

So, now then he talks about intersecting circles some arrows and erosion as a call, two circles arrows and erosion. So, circle 1 is having diameter d1 this is circle 1 a circle 2 has having diameter d2 and common chord is AF and arrow corresponding to circle 1 is DH, so this is the one right this is the arrow is a1 and arrow corresponding to the other circle GD is a2 the erosion GH called erosion e is equal to a1+a2 see this kind of a thing comes in for instance eclipse, eclipses.

These kind of a situation will come okay when sun is partially covered by the moon is partially covered by the earth okay . So, then this kind of a situation will come and we will be all this will be actually one has one can calculate these things in there of importance how much is you know eroded at given time after the eclipse starts. So, that is the remember the Brahmasphutasiddhanta is a astronomy work.

So, naturally some things related to astronomy will be there , so fine now to given the arrows the arrows to find the arrows given the diameter and the erosion suppose you are given the diameters and the erosion GH, so then you can find the arrows.

So, the result for this is (FL) the erosion being subtracted from the both the diameters, the remainders multiplied by the erosion and divided by the sum of the remainders are the arrows. So, it is you know you divide these diameter are d1 and d2 to subtract the erosion from there from these and then the multiply with erosion and divide by the sum of the diameters-by the erosion, so this is the result.

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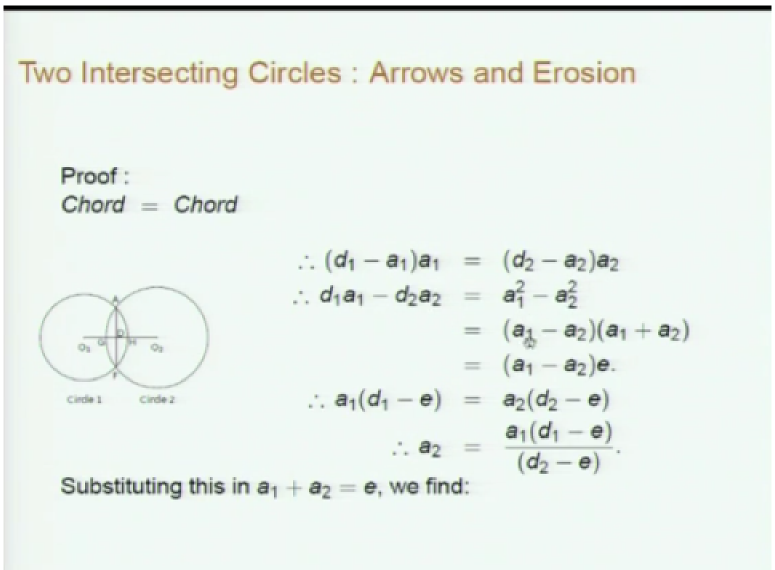

So, one can prove it again all essentially right angle triangles only, so here the chord, chord is the same or both circle 1 and circle 2 these are chord, so that is square root of 4 into you know that, so d1-a1 into a1 must be equal to d2-a2 into a2, so from this one find that d1a1-d2a2 is a1square-a2 square, so this a1-a2 into a1+a2 and a1+a2 is the sum of the arrow that is the erosion how much is you know 1 eclipse in the other basically a1-a2 into e. So, now you get a1into d1-e this, so therefore a2 is equal to a1 into d1-e/d2-e from very simple and substituting this from a1+a2 is equal to e.

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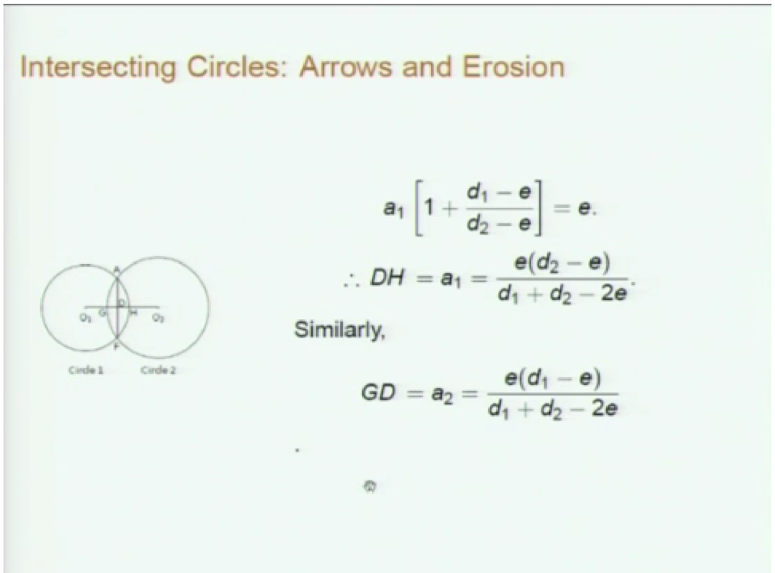

So, we find so we got a relations a1+a2 is e and we got a relation between a1 and a2, so using this we get a1 into 1+d1-e/d2-e is e, so DH is a1 is this and GD a2 is e into d1-e but d1+d2 -2e.

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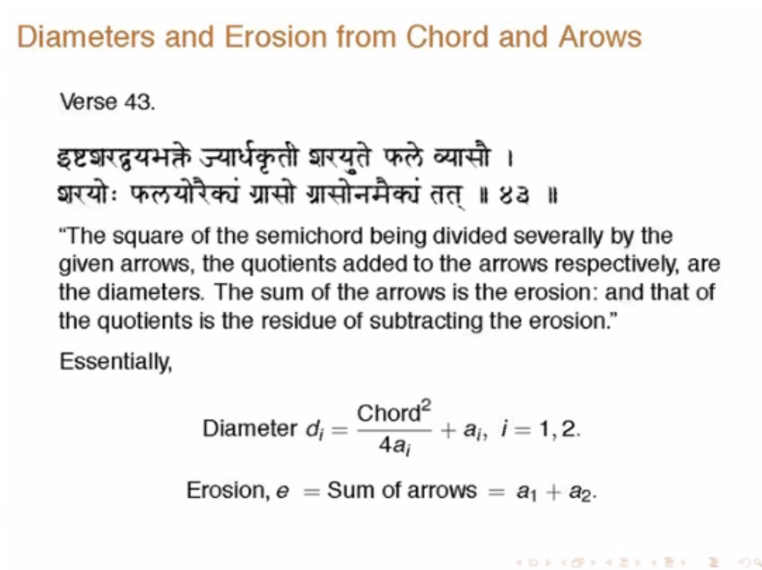

So, these are the results, so now you can do the other way also, so some chord suppose if you are given instead of the diameters and the erosion suppose you are given the arrows and the chord then you can find out the diameters and erosion, so that is what is said next (FL) the square of the semichord being divided severally by the given arrows, the sum of the arrows is the erosion: and that of the quotients is the residue of subtracting the erosion.

So, this is, so essentially you have to same result is there you can use this similar arguments to get these there is nothing very a single mode essentially as use that result you know where chord is equal to square root of d-a into 4 that what is done and this relation a1and a2 get got okay.

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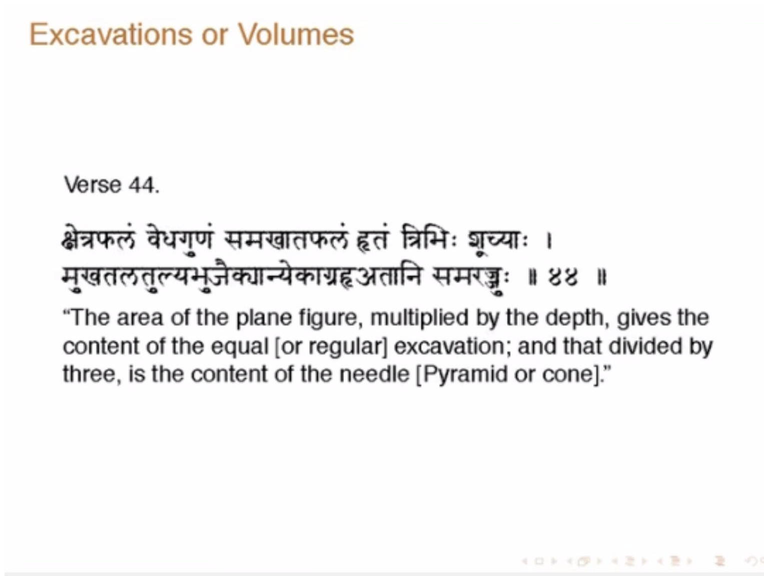

So, the next topic he will discuss is excavations or volume, so is talks about the calculations the various kinds of volumes, so he says in the verse 44 (FL) or of the plane figure, multiplied by the depth, gives the content of the equal or regular excavation and that divided by three, is the content of the needle or pyramid or cone.

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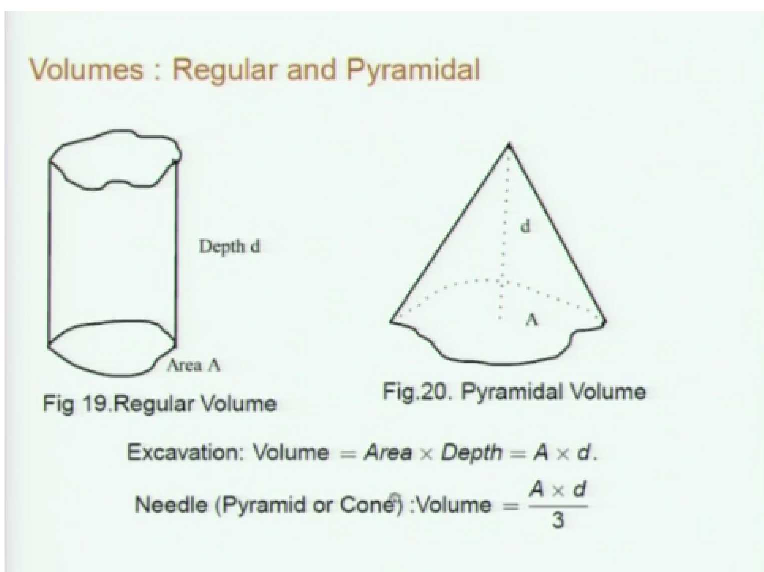

So, this is discussing 2 situations, so this is a this thing where is the uniform cross section I mean the cross section itself maybe irregular but it is uniform throughout okay, it is raised by height d and then the pyramidal value volume okay that is it is tapering to them point okay. So, these are the 2 kinds of things and is or this is say the excavation volume is area into depth, so that is A*d where are this called needle term is called needle also in this thing. Pyramid or cone the volume is A*d/3, so this is the result which is stated.

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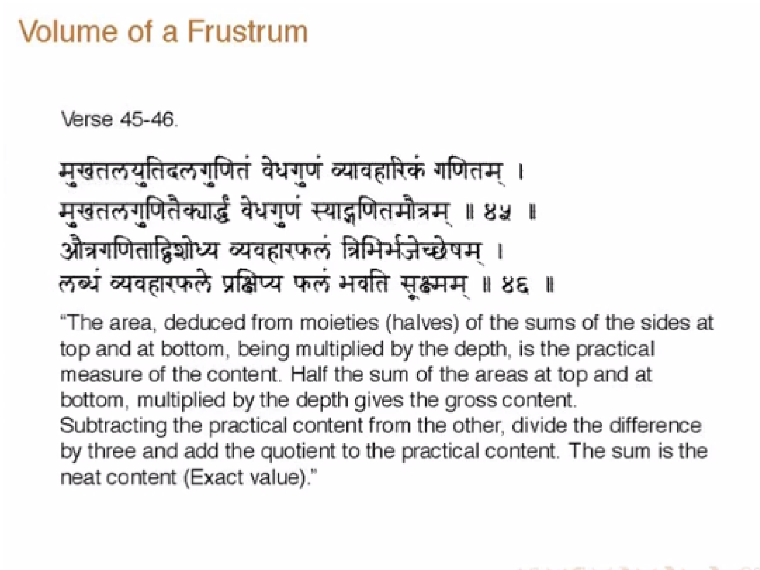

So, we will discuss this now in some of the detail and for some other things also is discussed (FL) the area deduced from moieties halves of the sums of the sides are top and at bottom, being multiplied by the depth, is the practical measure of the content half the sum of the areas at top and at bottom, multiplied by the depth gives the gross content okay. So, forget about it moieties that is the Victorian English okay because is whole nook translation it is half only.

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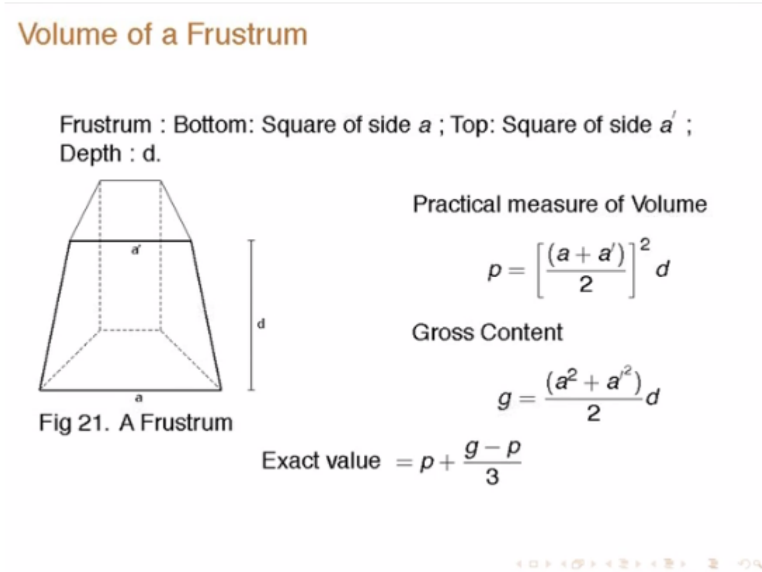

Fig 21. A Frustrum

So, see essentially what he saying that he gives various kinds of approximation and then the exact result and subtracting the practical content from the other divide the difference by 3 and add the quotient to the practical content the sum is the neat content or exact value, so what is the talking about that we should you know understand. So, this is so called frustum okay, this also is an old verb we do not use it now a days.

But anyway so what is see here, it is not tapering off to a point it is tapering up to some value only okay. So, it is some see base is let us say take the base to be sum square of side a okay. then it is tapering up to some point where the side is a prime okay and uniformly it is decreasing uniformly it is decreasing the side of the square from a to a prime, so that is called a frustum. So, he says for this kind of a thing the practical measure of a volume is a+aprime/2 whole square into depth is the practical volume.

And a gross content somewhat better this thing is a square+aprime square/2 into d that is essentially a here you had averaging the sides. And then taking the square and multiplying the depth whereas the next where averaging the areas, area of this a square area of this a prime square averaging that and multiplying with it and the exact value is supposed to be P+ g-p/3. So, that is the result is giving.

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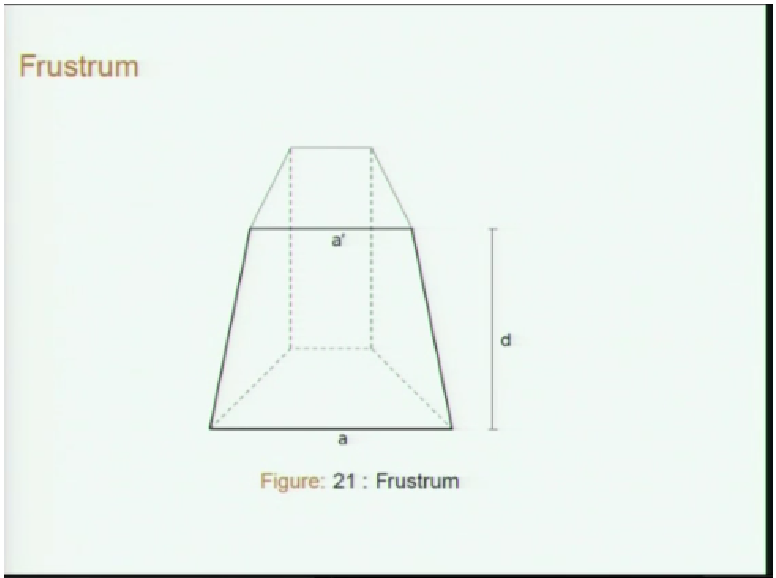

So, this is the full you know figure for this.

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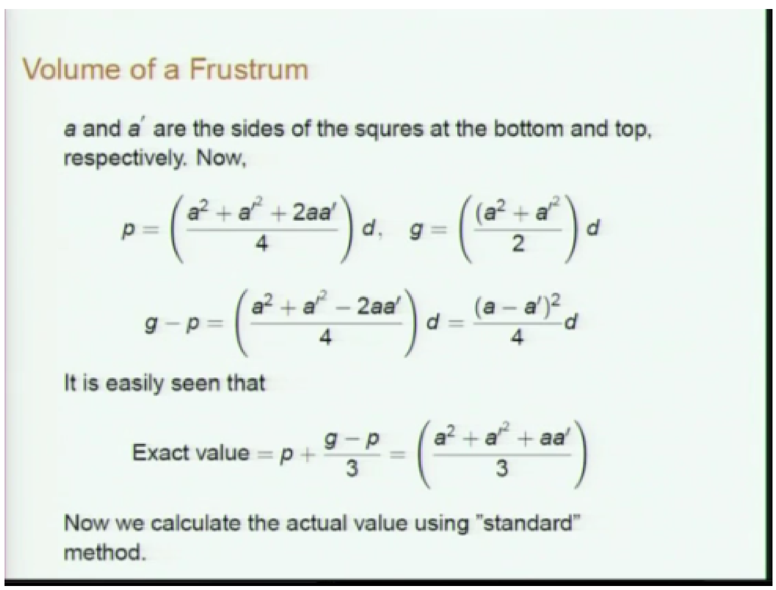

One can find this for a square kind of cross section, so remember the p comes you know this a+aprime whole square by 4 right that comes a square+aprime square+2 a prime by 4 into d this is the practical is the gross is this, so g-p is this one can show that exact value is a square+aprime square+a aprime by 3 okay. So, now we can calculate the actual value using standard method.

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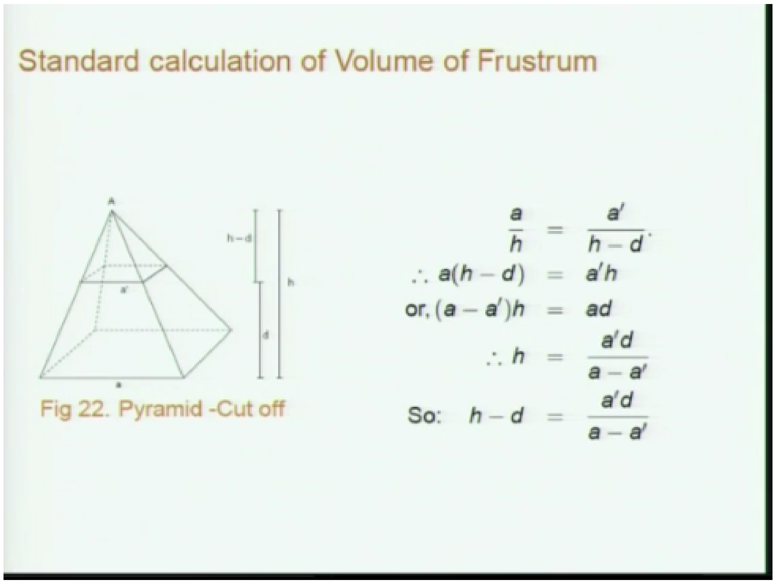

This is a stated result he is not giving any proof, so how do we get this, how do we see you can do it like this the a/h see suppose it is tapering uniformly see at a constant rate. So, then a/h this is this h is the total height a/h is a prime/h-d okay by proportion because this you know corresponds them perpendicular h-d and this corresponds to perpendicular h. So, as it is you know growing uniformly or it is tapering uniformly this proportion can be used. So, then a*h-d is equal to a prime h, so from this one get you know h is a prime d/a-a prime, so h-d is this.

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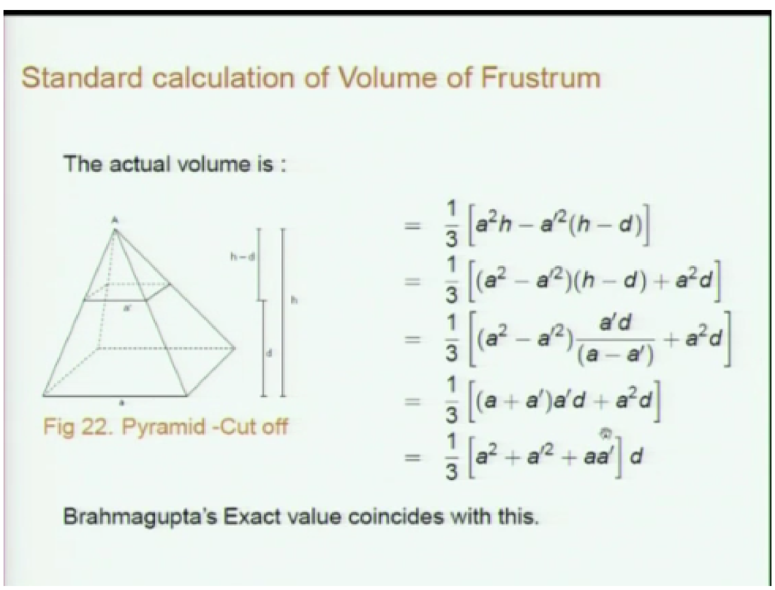

So, the actual volume will be so the total volume of this is 1/3 a square h using this same kind of a thing what we are got for the earlier thing, so full volume you know measure it is tapering volume it is 1/3 of the area of the base into the height. So, 1/3 a square h is a total volume from this, you have to subtract this volume right top volume that is a prime square is the area and h prime-h-d is the height.

So, if subtract this, so then you will get 1/3 a square+a prime square+a prime into d, so this Brahmagupta's exact value coincide with this. So in fact why this 1/3 see there is a in fact he is saying that is valid for all kinds of situations you know he is restraining contagious thing you know but probably he may be thinking of some square or rectangle kind of a thing but actually it is valid for all kinds of situations.

So, how do we understand it how we how does one get it in the modern using so called modern way you see or even if you do not want to do the modern way suppose you know you want to (FL) you know understand the volume suppose you have a square okay.

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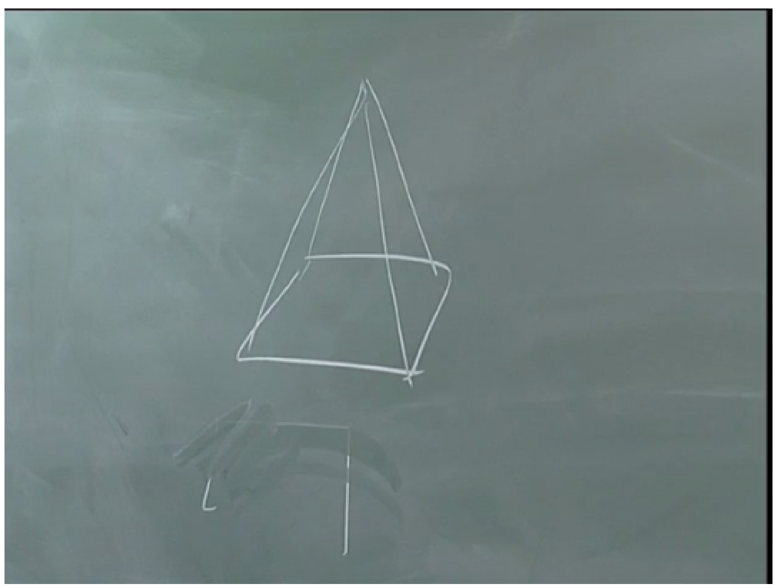

Square is there and it is tapering of two a point okay, so then you construct a cube you know with this side and 3 of them can be fitted in that so that is why it is 1/3 factor will come. So, one can show 1 has to think a little bit 3 of them can be fitted, so the volume or 3 the so this volume multiplied 3 must be the volume of the cube which is the area into height, so 1/3 factor. In general how you will get using the so called modern method.

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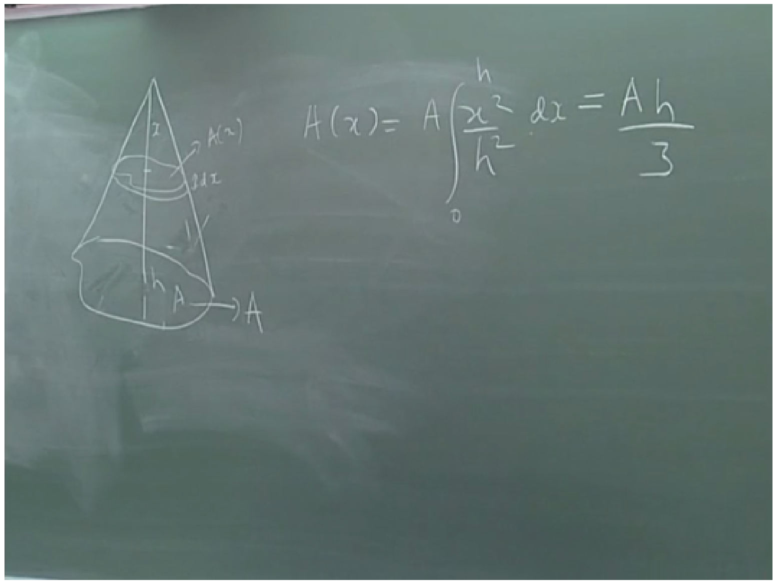

See suppose you have in arbitrary area okay and which is tapering of uniformly okay. So, then see suppose at this so these are total height h okay and this is the area A here and suppose at this point it is x you know from here top to this thing okay. So, then in this case okay the volume will be you can consider discs okay. So, then this area see this A at x you can call it and this is A see the length is increasing in x, x is increasing right.

So, at this point the linear dimension will be proportional to x, the linear dimension of the area will be proportion to x and the area itself be proportional to be x square. So, you can say that at this point A of x is equal to A by into x square/h square and this disc this thickness dx okay. So, you integrate between 0 and h, so integral x square dx is x cube/3, so integral is sum 0 to h you get by x square is A, so Ah/3 right.

So, this is how we get 1/3 okay but what Indians would have done, so that is you know nobody has stated it but we can guess it from what they doing (FL) the exactly calculate the volume of a spear using a similar method, so what they would have I am saying you know this is not given into the particular book okay. So, some westerns call I will scream you know Oh it is not given a new book but I am saying is you know given that method how would have done.

So, what they would have done is they will not write A, x, dx and all this the dx and all the they will not write what they will do would have done and which is what precisely is done in (FL) for finding the volume of a spear.

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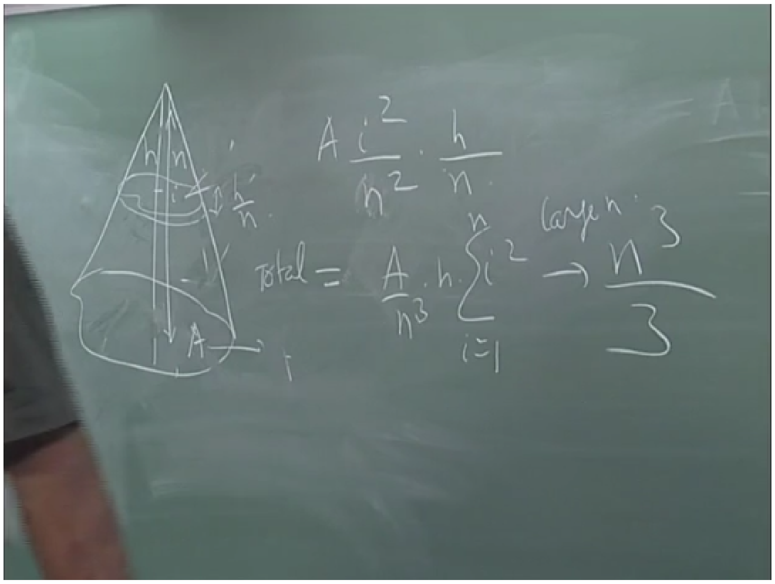

So, what they will do is you divide this linearly into n parts okay, so suppose this is at high height level okay height level okay. So, then yeah so they would have also notice that the area will be proportional to the you know the length square kind of a thing you know square of the dimension will what will come okay. So, this area so this is the area here the area here would be A into i square/n square.

And this is divided into n parts right n so h is h is divided into n this thing, so this each segment will be h/n okay. So, so this will be h/n okay, so now it is a square+n square so you have to sum you have to sum more all this lap, so this is the area of this lap you know this area of this lap is sorry the volume of this lap is A area is A into i square/n square and the thickness of the slab is h/n, so this will be equal to this.

So, essentially you are getting A/n cube into h into i square now you have to sum over i is equal to 1 to n total okay. And sigma all over i square you have got the result sum of the squares okay and not only that they will go they will also estimate you know the for large n it is stated this is very stated very explicitly that for large n it is n cube/3 and you will come across this thing when you discuss.

So, then finally will get Ah/3 so this would be the Indian way of calculating the volume per an arbitrary kind of cross section so, that is the or if you are not satisfied what you have to do is sigma is squared equal to you know that you know what is the n into n+1 into this single bit 2n+1/6 yeah. So, and finally you have to take for large n only you have to take the limit n going to infinity okay for large n so, only these other terms will be very small.

Because you have taking large ends so, they 1/n, 1/n square will come so, finally get this so, these the so, essentially similar kind of below know from Brahmagupta might have give this kind of okay.

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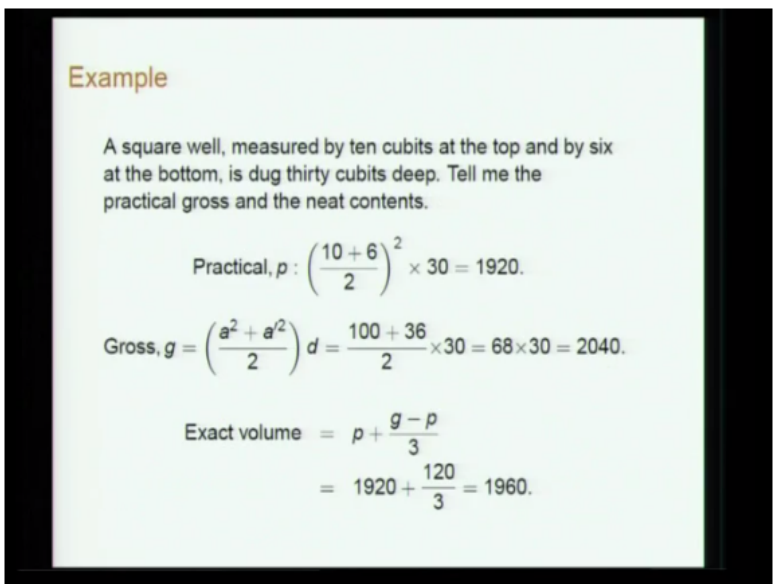

Example is given a square well measured by ten cubits at the type top and by six at the bottom is the thirty cubits deep. Tell me the practical gross and neat contents okay 10+6/2 whole square into **thrit** 30 you can easily this is the these the average the sides square into the depth. So, this is the average areas into the depth so, there are coming out more or less the same. But not always is it true okay.

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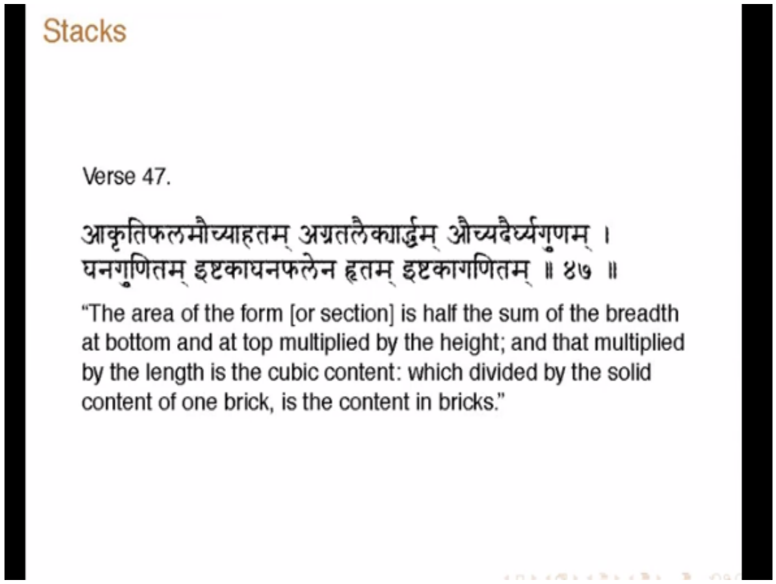

And then it talks about stacks so, this all you know these are all things which have to be discussed kind of a things something like that. You know seems to be there volumes and stacks, because for a practical applications it is there. So, is yes the area of the form or section is half the sum of the breadth at bottom and at top multiplied by height. And that multiplied by the length is the cubic content which divided by the solid content of the brick is the content in bricks.

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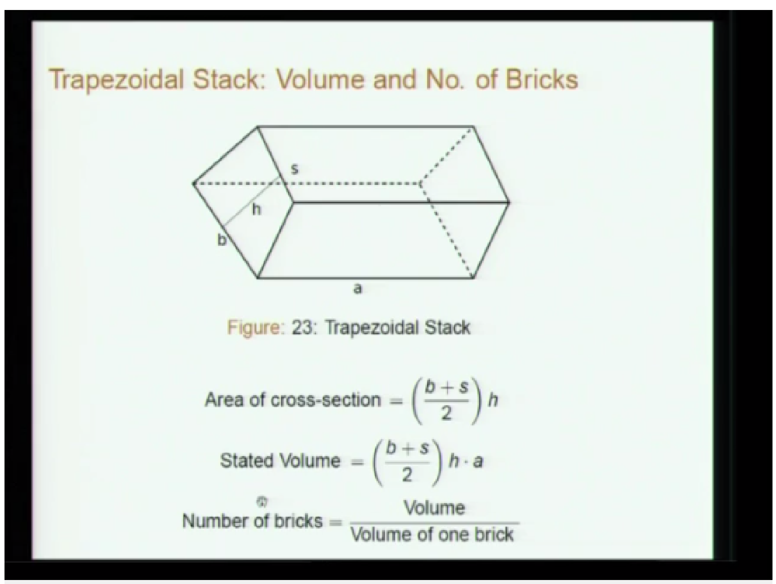

So, what is essentially referring to is a situation like this is a trapezoidal cross section okay. So, the basis a submit is s and height is h and the length is a. So, is a called tripletail stacks which is constructed out of bricks okay. So, then in that case the area of cross section is b+s/2 into h remember there previous lecture. So, it **wellt** upon t length so, the trapezoid it trapezium you remember the base+submit/2 into the height of the thing.

So, that is the cross section or so, the volume is uh essentially this into a right and then **num** so, that is what is stated okay stated volume s this. And then number of bricks suppose obviously volume/volume of one brick right tell me the number okay. So, then measure of shadows (FL) so, the distance between the foot of the light and the bottom of the gnomon multiplied by the gnomon of a given length.

And divided by the difference between the height of the light and the gnomon is a shadow oh so, that each situation **situa** considering the situation with gnomon.

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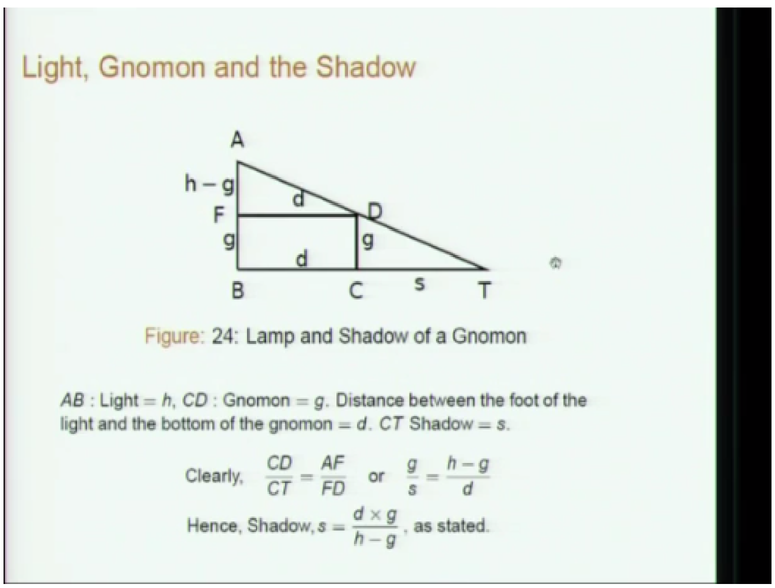

Yourself (FL) right two days back so, that is essentially these are gnomon gnomon (FL) so, this is the some **thising** height you know lamp is here. So, it is calling the shadow so, the CT the shadow right CD is the gnomon (FL) CD and CT is the shadow okay. So, the chaya so, ab is the light height so, CD is gnomon g and distance between the foot of the light and the bottom of the gnomon is g.

This is the g and shadow is s okay so, then shadow is clearly CD/CT is equal to AF these two triangles are similar AFD and DCT so, CD/CT is equal to AF/AD so, g/h is h-g/d. So, shadow s is equal to d*g/h-g okay. So, this is this distance into this divided by h-g okay. So, this is very this precisely what is the formula stated by Aryabhatta for finding the length of the art shadow in a lunar eclipse besidely same thing.

Because here it is the sun okay suppose ab is the sun okay so, earth this is the same and then this is the shadow which is there of course moon will be travelling somewhere here okay. Lunar eclipse when it enters this region of course it will be a eclipse right. So, **so** the shadow easy know distance between the earth, and the sun into the radius of the earth divided by the radius of the sun-the earth kind of it is. So, this is straight away applied to eclipse.

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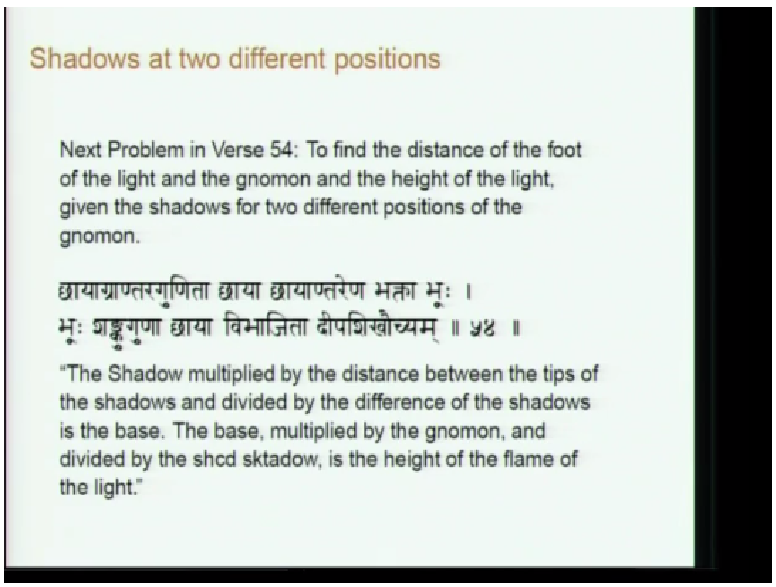

And then it is **shkadow** are two different positions

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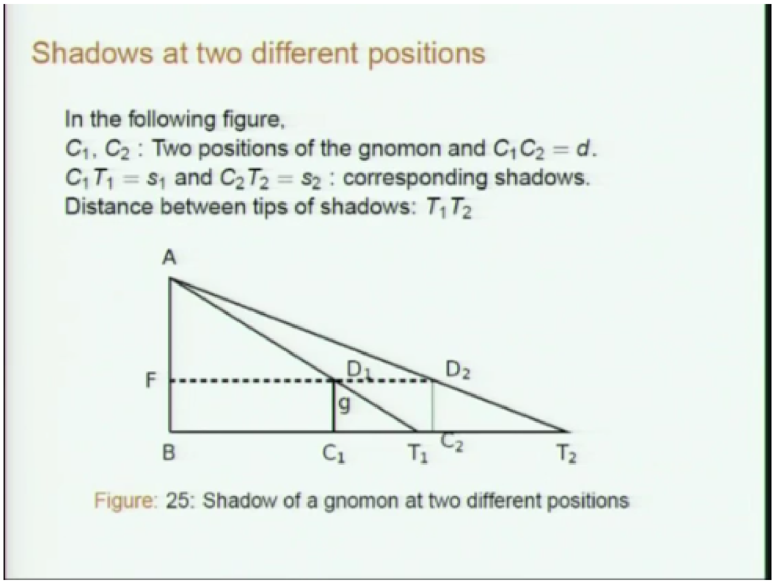

So, essentially you have got this is one same gnomon so, C1, D1, C2, D2 they are the same expect there placed at different positions with respect to the light source AB okay. So, then when it is here the shadow is C1, T1 when it is here the shadow is C2, T2. So, then we have to there some relations **between** between them. So, that is what is talking about so, to defined a distance the foot of the light and a gnomon and a height of the light given the shadows are two different positions of the gnomon.

If the given the shadows for two different positions you can find the distance between the light and the gnomon and the height is the light (FL) the shadow multiplied by distance between the tips of the shadows and divided by the difference of the shadows is the base. The **mase** base multiplied by the gnomon and divided by the shadow that is the height of the flame of the light.

So, essentially base is something straightly is falling this BT1 at the base. So, C1, C2 are two positions gnomon the distance between the positions of these shangu is D. And then C1, T! Te shadow one shadow is s1 and the other shadow is s2. You have the corresponding shadows and the distance between the shadows is T1, T2 so, it is again elementary similar triangles you have to do.

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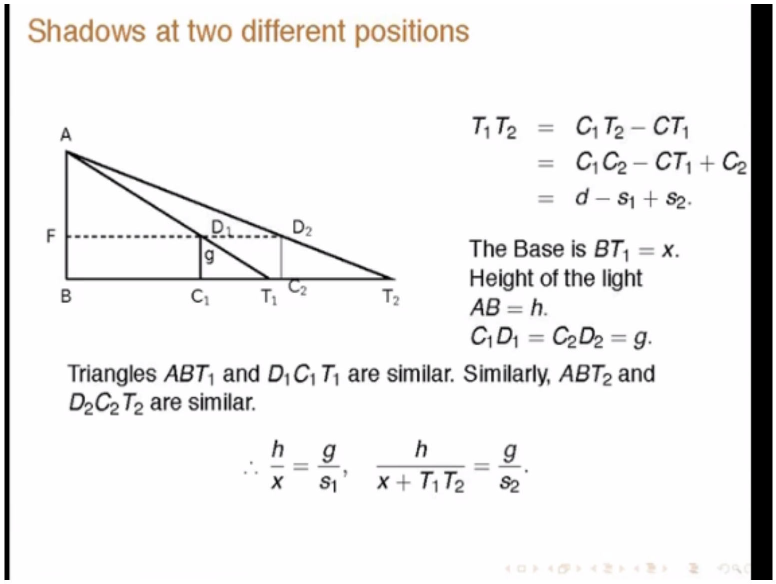

And finally what to get this that h/x T1, T2 is the C1T2-CT1 so, various C1T1 so, various things you can you will all **all** these similar triangles are there. Triangles ABT1 and D1, C1, T1 they are yeah similar ABT1 and D1C1T1 similar and similarly AB, C2 and D2C2T2 they are similar. So, you get h/x is this and so on.

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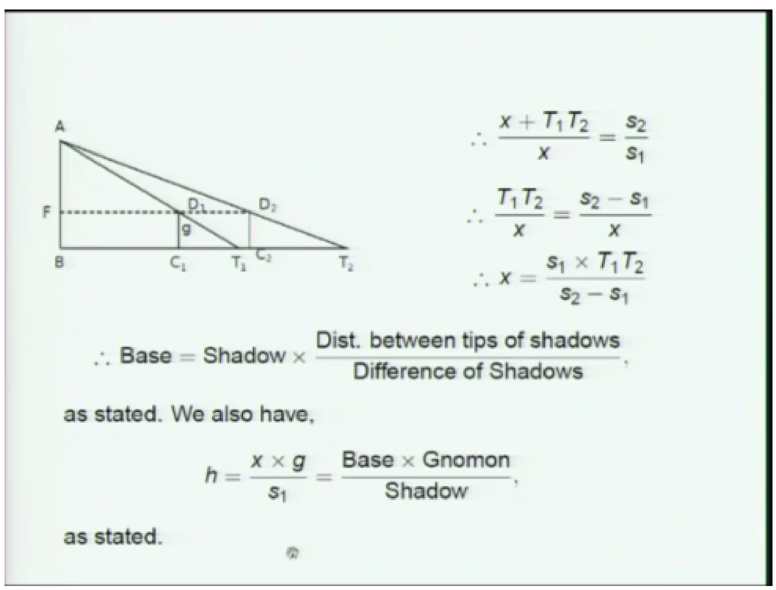

The finally you get x this base this base BT1 x is equal to s1*T1T2/s2-s1 and height h is shadow*distance between tips of shadows/ difference of shadows, so base*Gnomon/shadow.

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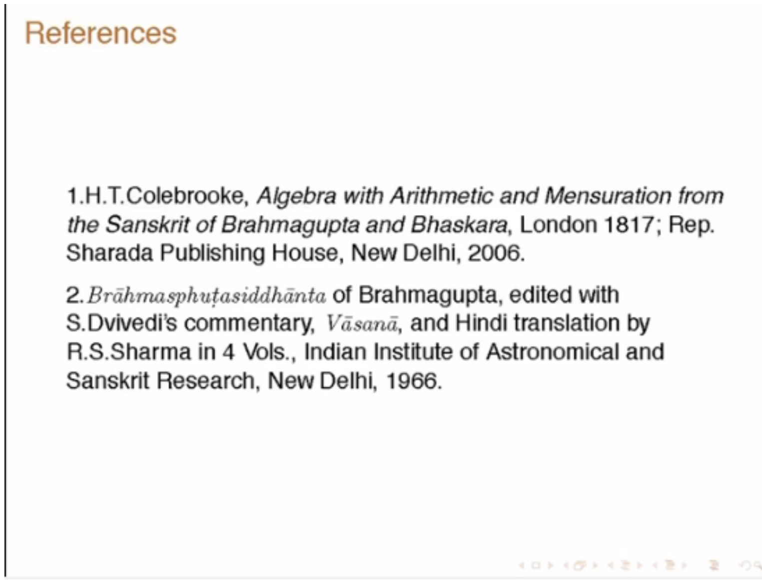

So, this are the some of the things which are discussed in Brahmasphutasiddhanta of course the one more chapter as I called an algebra, so that is contains a lot of interesting results including this (FL) and then all this Bhavana principle and all that which professor M.D. Srinivas rocks till (()) (50:59) about he will even more elaborately in the lectures to come. So, various things are there and there many other apart from that **aqueous** (FL) he has got his (FL) where he has got some very interesting results that second order interpretation formula and other.

There is also input a short work an astronomy moves mainly for calculations and that also has some interesting second order interpretation formula and other the Brahmagupta is really a very creative mathematician as a very genius kind of a person, so he could various results and this are some of the things only I have presented here. The references are given here, so thank you.