Mathematics in India: From Vedic Period To Modern Times
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Lecture-11
Brahmasphutasiddhanta of Brahmagupta-Part 1

Okay, so it will be 3 lectures on Brahmasphutasiddhanta of Brahmagupta. So, 2 lectures will be delivered by me this is the first part.

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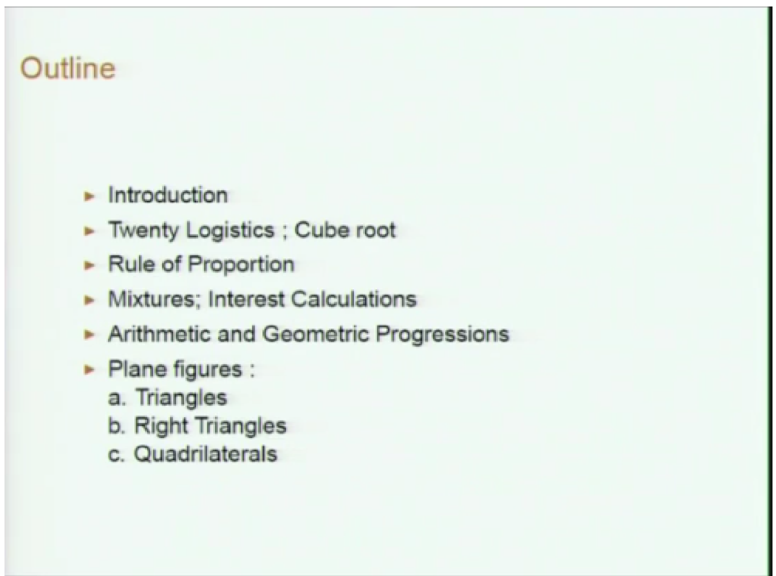

So, this is gives an outline I will give just an introduction about Brahmagupta then twenty logistics that it talks about especially cab cube root available dealer little bit then rule of proportion, then mixtures, interest calculations to an arithmetic and geometric progressions then geometry essentially some plane figures triangles, right triangles and quadrilaterals okay.

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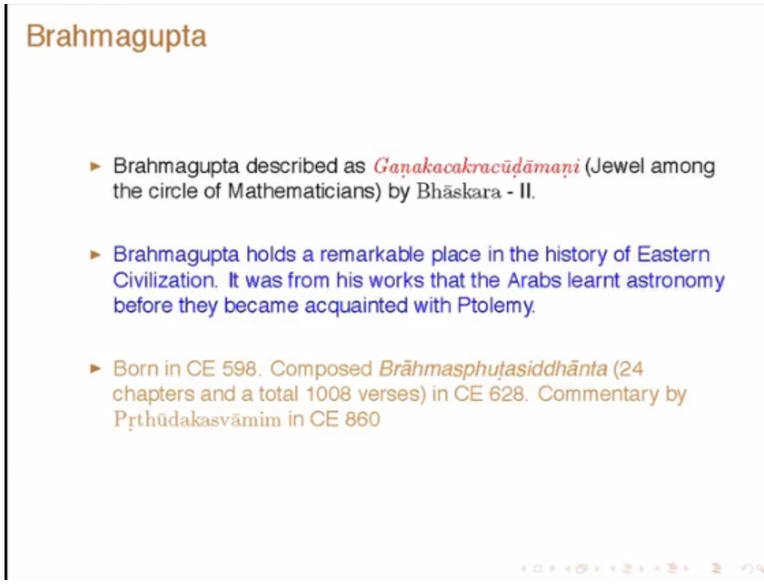

Brahmagupta is described as a (FL) jewel among the circle of mathematicians by Bhaskara-2 it was held in high esteem by most of the astronomer mathematicians in India would followed him. He holds a remarkable place in the history of eastern civilization. It was from his works that Arabs learnt astronomy before they became acquainted with Ptolemy. If for instance Brahmasphutasiddhanta in some far and went to Arabia I mean Arab speaking countries.

And it was transit as sin hin and similarly one more work of him (FL) also was translated as (FL) and that had a propend influence on developmental mathematics in Islamic Arab region it was born in 599 com 98 common era and it composed Brahmasphutasiddhanta in 24 chapters and a total of 1000 1008 verses in common era 628 there is a very elaborate common pre and dot and this work by to do the (FL) around about 2 and the half centuries later.

So, you can see that you know is a Aryabhattiya has only 121 verses is a much more elaborate work earlier as you would have seen some Aryabhattiya mathematics also is the part of astronomic hest Bramagupta also follow the same pattern they are 2 chapters on essentially on mathematics here one in arithmetic which includes actually geometry and other an algebra.

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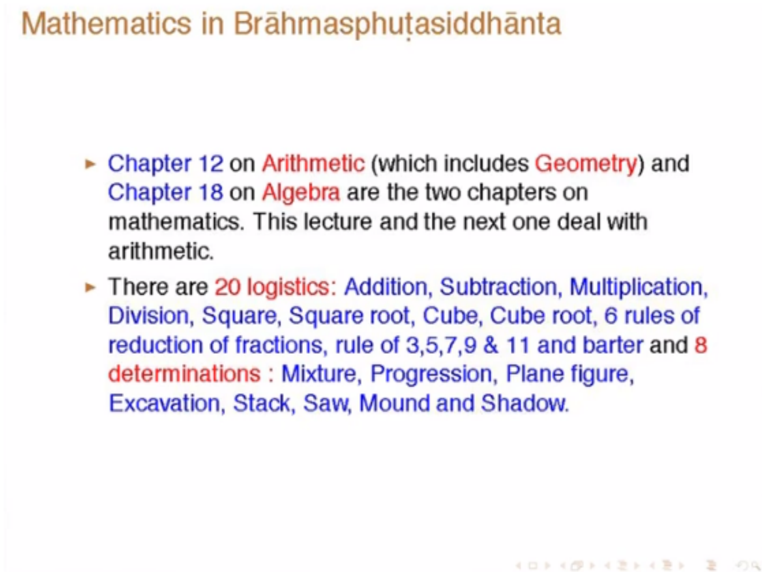

And of course as person M.D Srinivas pointed out it has chapter on changes also. This lecture and the next one deal with arithmetic, so in the arithmetic it talks about 20 logistics ; addition, subtraction, multiplication, division, square, square root, cube, cube root there are some 6 rules of reduction of fractions then rule of proportion rule of 3, 5, 7, 9 and 11 and barter and what you called 8 determinations; mixture, progression, plane figure, excavations, stacks, saw, mound and shadow, so that is how it divided it.

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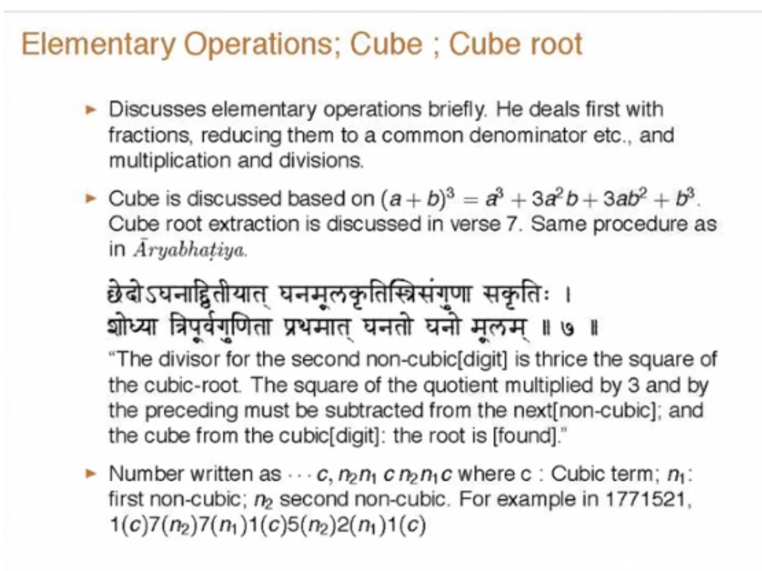

So, elementary operations most of the most elementary operations he will discuss only briefly as if you know people are expected to know already and he deals first with fractions, reducing them to a common denominator etc., and multiplication and divisions. Then when we talks about cube

I mean I will not discuss all the operations in detail when we talks about cube it is based on the formula a+b whole cube is equal to a cube+3 a square b+3ab square+b3.

And I will talk about cube root briefly cube root extraction is discussing verse 7 which is the same procedure as in Aryabhattiya which professor ramasubramaniyam has dealt with in great detail but anyway for completion I will say that (FL) the translation is the divisor for the second non-cubic digit is thrice the square of the cube root. The square of the quotient multiplied by 3 and by the preceding must be subtracted from the next non-cubic and the cube from the cubic digit that is the root which is already has been found.

So, anyway it has been discuss in great detail, so just late for square root you divide into (FL) here your dividing into groups of 3 by rotation is slightly difference of ramasubramaniyam but is a same thing in fact example the same thing. So, the number is written as c, c1 n2 n1cn2n1 like that c, so groups of 3 and c is the cubic term and n1 is the first non-cubic term and n2 is the second non-cubic.

So, groups of 3 are there for example in this 1771521, so you rate it is as 1 is the c 7, so you start from the right actually right 1 is c, 2 is n1, 5 is n2 and so on.

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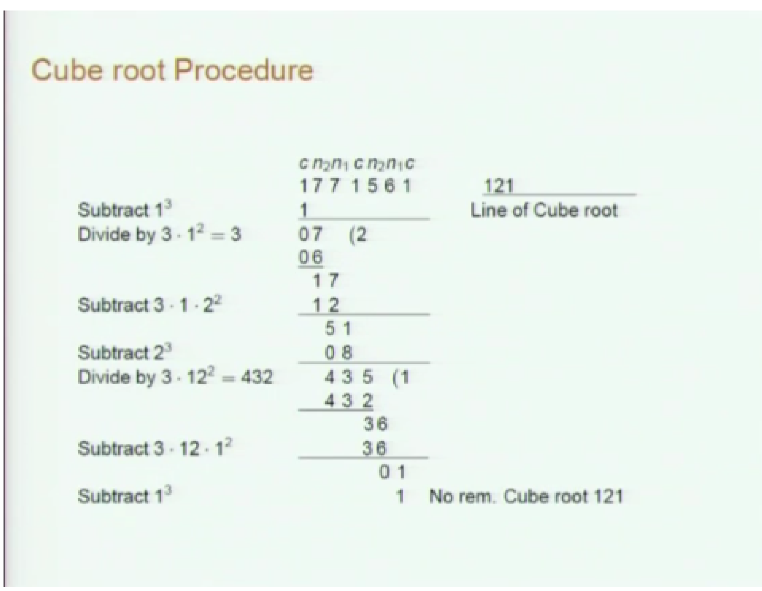

So, when you do, I do not have to explain already it has been same example has been done. So, I will keep the you know you just have to you know you have to first the first cubic term you take the term which is now some term which is you know cube is less than that. So, of course here only to c is 1 but sometimes it maybe as many as 3 digits okay. So, c that is last c you know left here it is 1 and the single digit of course it has to be only 1 or 2 the cube root will be 1 or 2.

The first digit of the cube root it maybe some 998, so then you have to that the first digit of the cube root will be 9, so like that so and then you proceed as he said you know the divide the first divide by the sorry subtract the cube root of the first digit cube of the first digit and then divide by 3 into square of that and then you subtract then the 3 into the fir first digit of the cube root and whatever quotients you have got.

You take the square of that and that to subtract and so on and rational for that also has been explained in great detail okay, now of course if you if you are perfect cubes of course it is very simple to teach but what you do you do not have of a cube it is not stated here but implicit in later techs is that you know essentially multiplies by you know powers of 10 cube like 1000 or 1000 square and all that.

And similarly the cube root is correspondingly divided by 10 or 10 square and so on. So, that is what the rate some of the rate has discuss okay.

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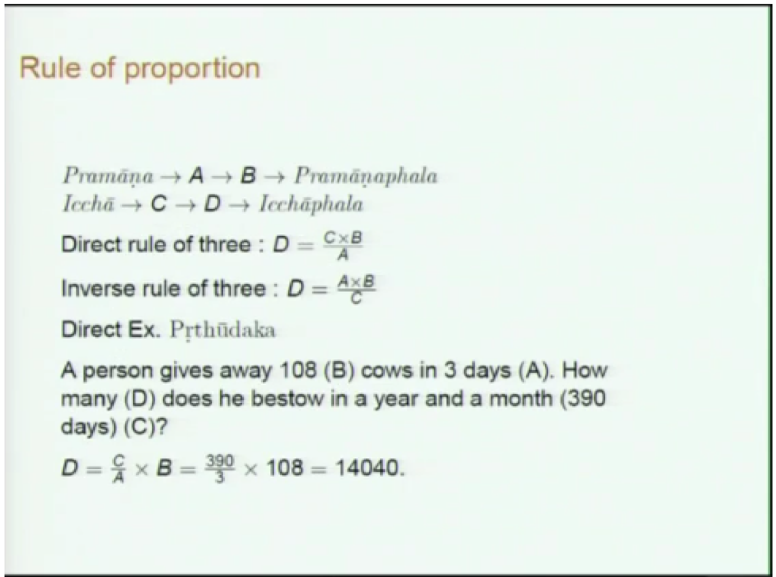

So, I do not have to go more about this cube root. So, then he talks about the rule of proportion is the very important thing (FL) and of course general relations of that 5, 7, 9 etc., So the problem is suppose you have Pranana A and the Pramanaphala is B then what is the (FL) for (FL) so c is Iccha then icchaphala is what okay. So, the direct rule of three will give D is equal to C*B/A okay. So, you take the pramanaphala then divided by the pramana.

And then multiply by the iccha for sometimes you have inverse rule of three, so then in that case you have to be numerator what we will get is the pramana and denominator you get the iccha. So, it gives an pruthudaka for instance gives an example the person gives away 108 cows in 3 days how many does he bestow in a year and a month that is 390 days years is taken to be **39**, 360 month is 30.

So, here it is a clearly a direct rule, so the first is the B is 108 the pramanaphala the pramana is 3 and iccha is 390, so 390/3 * 108, so this is the answer. Now inverse to a rule pruthudhaka in his commentary has given this examples.

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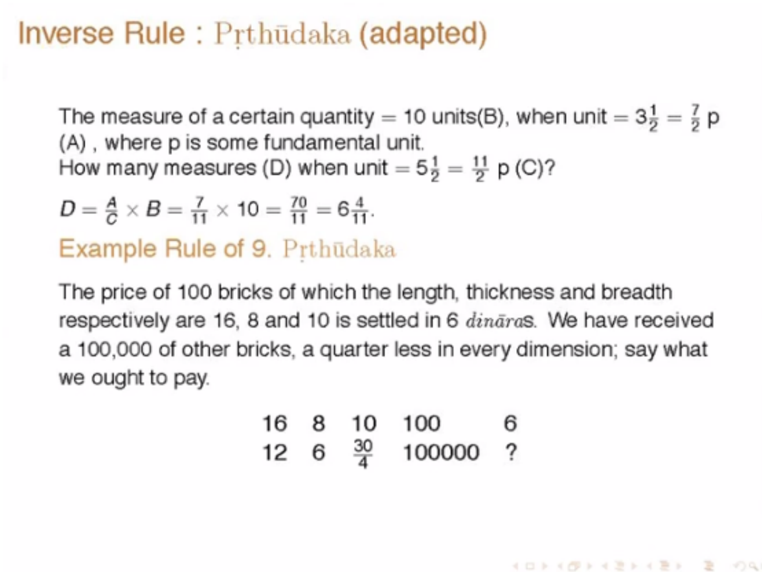

The measure of certain quantity is equal to 10 units that is B when the unit is 3 and half P, where P is some fundamental unit what is the measure when the unit is 5 and a half is equal to 11/2P which is C, see here pramana the first unit is 3 and half and pramanphala is the first measure which is 10, iccha the second unit is 5 and half. So, we have to find out the icchaphala the second measure, so we have to use the inverse rule A/C*B to obtain the icchaphala.

So, because if the unit is large the number of this things will be what you get is less right, I mean if you have some length the number of inches is much more the number of feet, so it is that is the unit is more than the what you get the measure will be measurement will be less. So, now for the proportions involving more quantities you know he gives an example. The price of 100 books bricks of a which the length thickness and breath respectively are 16, 8 and 10 is settled in 6 dinaras.

We have received a 100,000 of other bricks, a quarter in less in every dimension, say what we ought to follow, so the length thickness and breath are 16, 8 and 10 and price is 6 okay, then the for 100,000 bricks what is the amount okay where this each of the dimension is 1 quarter less okay 16 becomes 12, so the length is 12 instead of 16 the thickness is 6 instead of 8, and the breath is 30/4 instead of 10 so, we are dividing by3/4.

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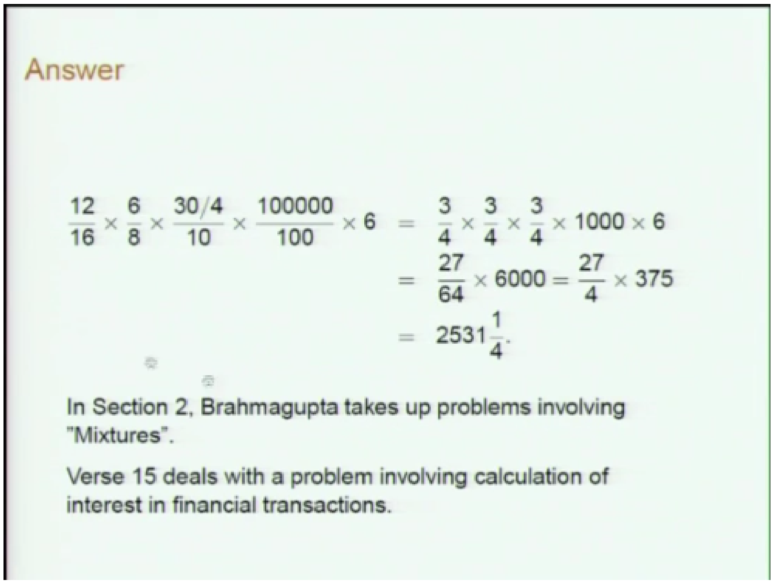

So, then in that case the answer will be so in the direct rule of proportion 12/16*6/8*30/4*10*100000/100 and the original price for 100 for this dimensions was 6, so you have to multiply all these things and the final answer is this 2531 ¼. One thing go on like this with examples involving rule of proportions, both direct and inverse indeed the text and commentary have many more examples.

So, in the section 2 Brahmagupta takes the problems involving mixtures, so for instance I to give you an example the 15th verse will give a problem involving a calculation of interest in financial transactions remember that interest is calculated in invention in Aryabhattiya also.

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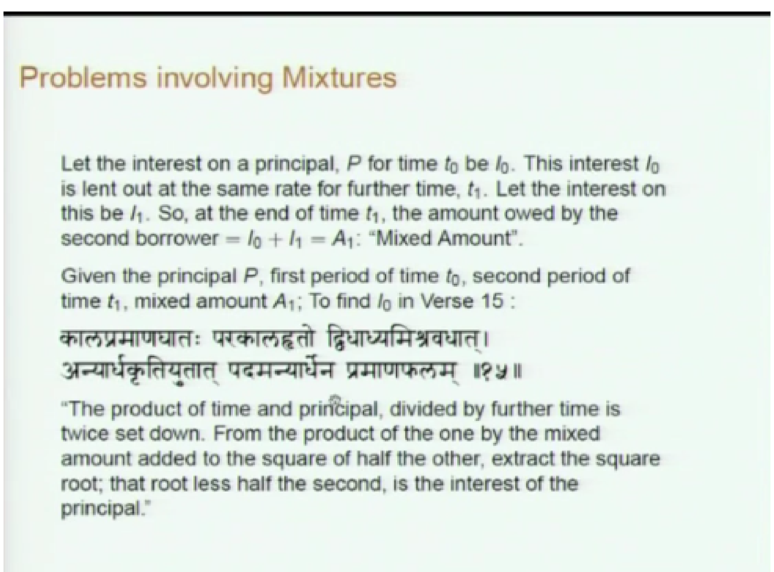

So, the what is discussed in this verse is the following let the interest on a principal, P for time t0 b I0. This interest I0 is lent out at the same rate for further time t1. Let the interest on this be /1. So, at the end of time t1 the amount owed by the second borrower is I0+I1 this A1, so when we are given A1 and P and t0 you have to find out I0, so what I am saying is that you know see.

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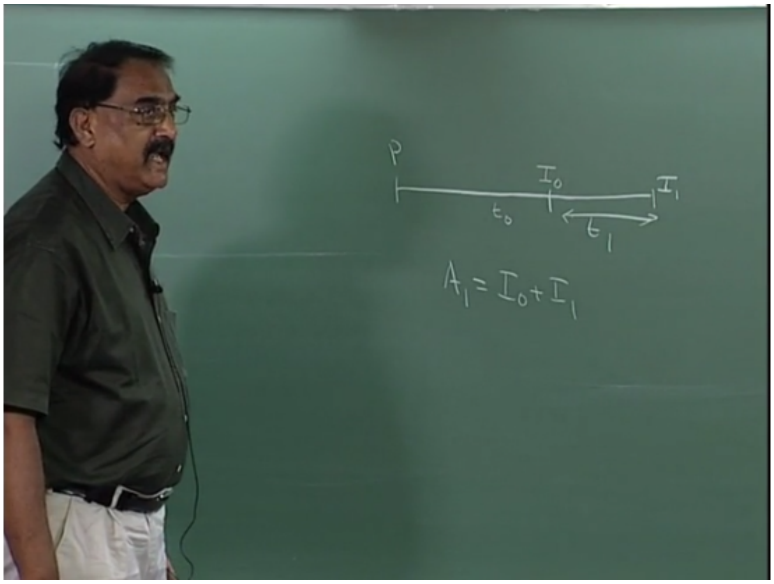

The first borrower has received an amount principal P and for time t is equal to t0 the interest for that is I0. Now a second borrower receives an amount I0 borrows an amount I0 from the first buyer borrower at the same interest and suppose after t1 the interest for this amount I0 is I1. So, now what is given is A1 is equal to I0+I1. So, this is the amount with the second borrower owes to the first borrower after time t1, so, that is principal+interest.

So, we have to find from this I0 and I1 separately (FL) the translation is the product of the time and principal, divided by further time is twice set down. From the product of the one by the mixed amount added to the square of half the other, extract the square root; that root less half the second, is the interest of the principal. So, if I express it in mathematically it will be clearer.

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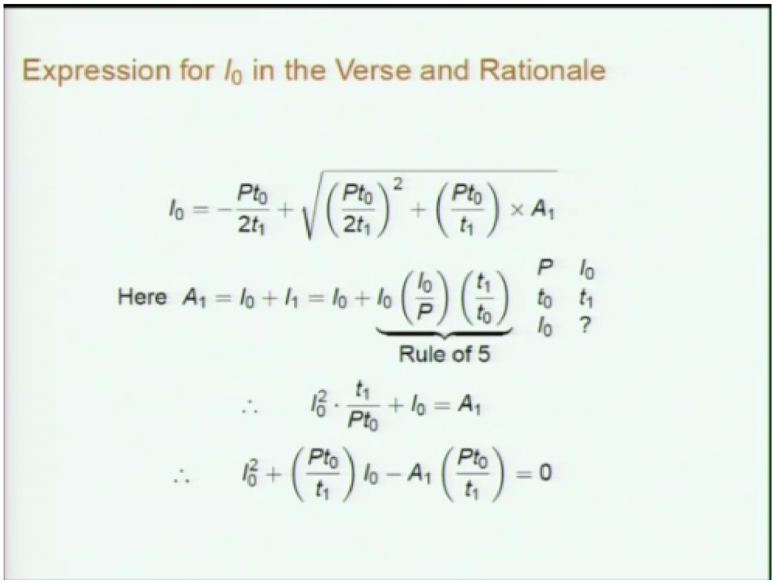

So, I0 the expression given is that I0 is-Pt0/2t1+square root of this quantity where A1 is I0+I1 and which is I0 you see but I1 see for interest is the principle P and for time t0 the interest is I0, so for principal I0 for time t1 the interest is this. So, I1 is I0 into I0/P into t1/t0. So, this is the rule of 5 we are doing, so this is A1 and A1 is given, so I0 square into t1/Pt0+I0 is A1, so this is the equation, so, this is the delta formatic equation coming.

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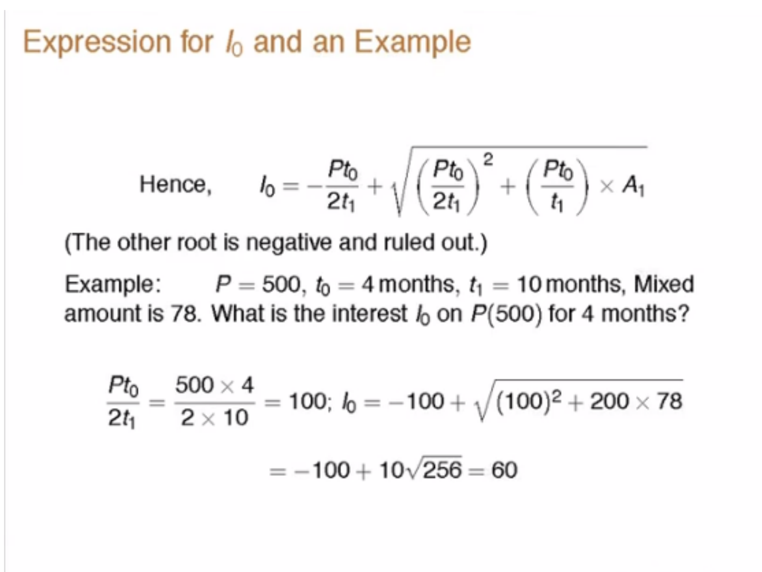

So, the quadratic equation the squared the solution is given by this, so because the solution of the quadratic equation is kind of a thing is even discuss in Aryabhattiya not for this problem but in the context of progressions. So, this is the solution is given, so which is correct, so of course it is another root as you know that there are 2 roots but here it is you know not meaningful, the second root is on meaningful.

So, it does not discuss it and an example is given in the commentary, so suppose P is 500 t0 is sorry P is 500 t0 is 4 months t1 is 10 months then mixed amount is 78 okay what the second borrower has to return to the first borrower that is the **va** amount he borrows+the interest and that, that is 78 then what is the interest I0 on P for 4 months, so clearly using this formula you get 60.

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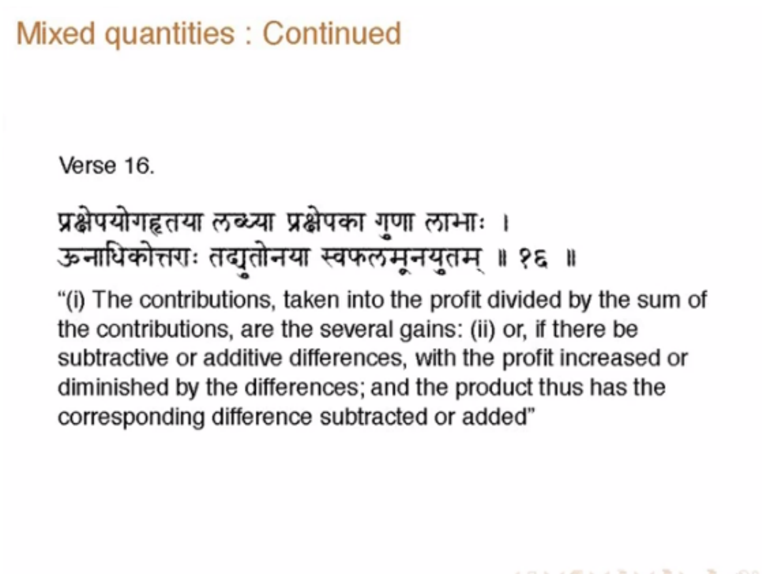

So, already some sophistication has come, so next he talks about this is all (FL) you know which will be useful for ordinary transactions you know in the today life. For instance what verse 16 says (FL) 16:42-16:58) the contributions taken into the profit divided by the sum of the contributions, are the several gains or if there be subtractive or additive differences, with the profit increased or diminished by the differences and the product thus has the corresponding differences subtracted or added.

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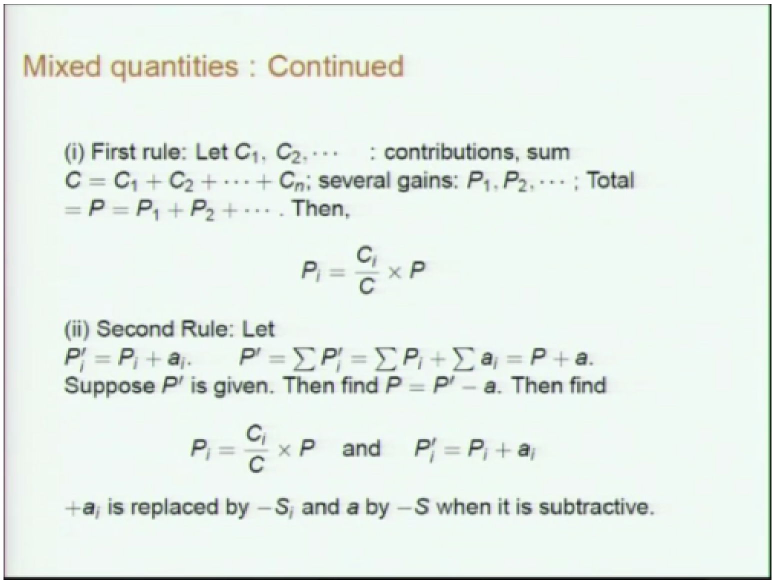

Again it is easier to discuss it you know using mathematics things you know with the mathematical notation the first rule he says you know that suppose for some transaction 1, 2, 3 etc., are contributing c1, c2 etc., and suppose the profit gain is you know total profit gain is P then how do you distribute it among 1, 2 etc., okay. So, then the there is a gains which has to be enjoyed by each of these peoples.

So, that let that be P1, P2 etc., then clearly Pi will be equal to Ci/C*P and the second rule says it is a it goes a little more complicated. So, suppose you are not given you know though total gain P but total gain+some quantity okay suppose that is how the problem disposed. So, then in that case similarly your Pi or to P primer i is Pi+Au define it like this and the total this this P prime will be P+a which is this sigma Pi+sigma ai then suppose P prime is given.

So, then find P is equal to P prime-a and some P you get Ci/C*P and P prime is Pi+ai if you discuss an example probably it will be clear.

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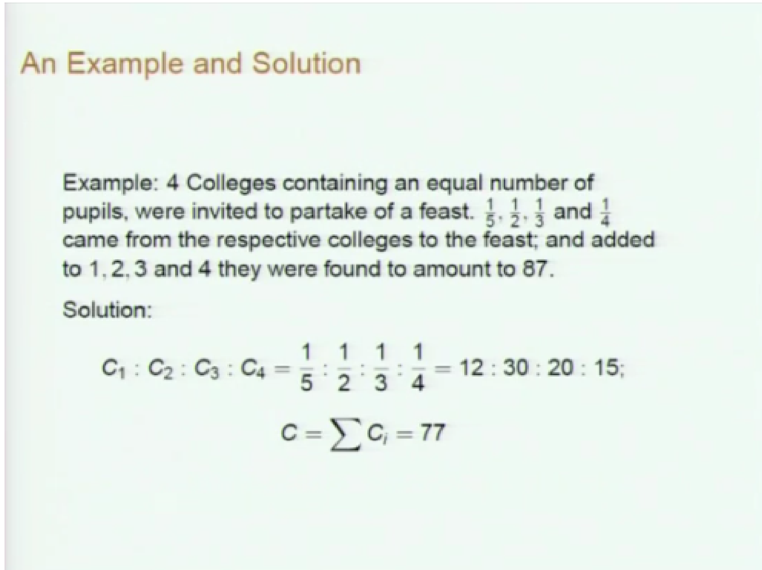

Four colleges containing a equal number of pupils were invited to partake of a feast, 1 fifth, half 1 third and 1 four came from the respective colleges to the feast and added to 1, 2, 3, 4 they were found to amount 87. So, then you have to find out the various numbers of course even that you know full statement of the problem is sometime not given. So, obviously this is what you have to find, so here the c1, c2, c3, c4 are in the ratio is given 1/5*:½:1/3:1/4. So, 12:30:20:15, so the c the total will be 77 but what is given is not that.

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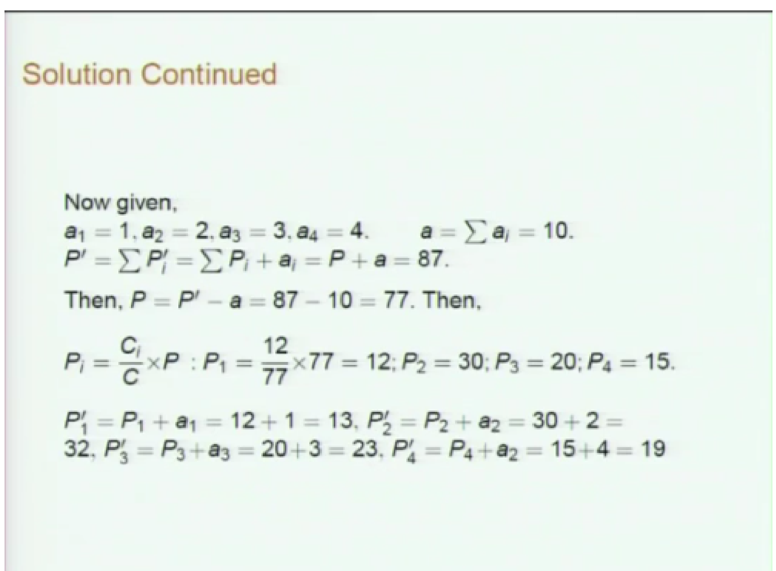

So, it is given a1 is 1, a2 is equal to 2, a3 is equal to 3, a4 is equal to 4, so a is 10 and this added quantity is total quantities 10, so then P prime is you know whatever this P+a, so that is 87 it take it as 87 and then P will be 77 because the P 87 is what is given and a is 10, so your P will be P prime – a will be 77 and then 77 you get this Pi is Ci/c*p, so P1 is 12/77*77 that is 12, P2 is 30 so what is coming here is this you know this contribution 12:30:20:15 right.

So, those are the corresponding quantities you have to multiply, so you get P2 is equal to 30 P3 is equal to 20 P4 is equal to 15. So, this is the number of four ins though are comes from the various colleges okay .

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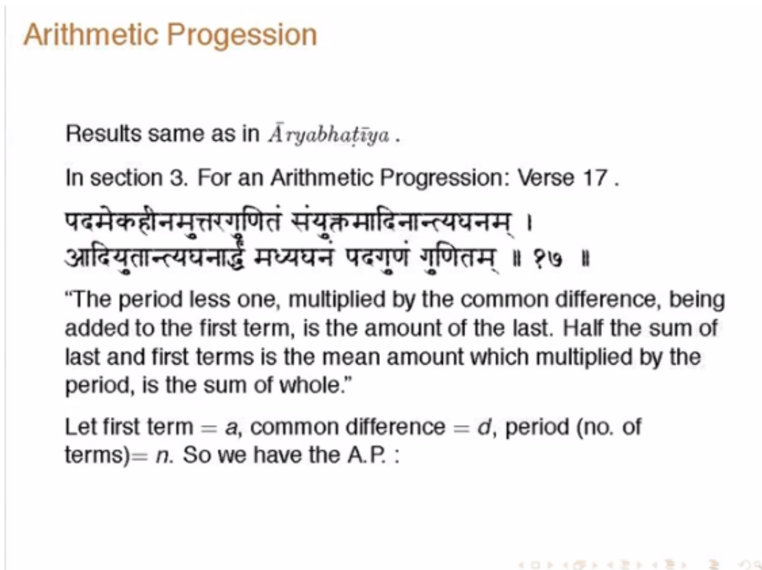

Now he goes to the arithmetic progression, so in the arithmetic progression is essentially result is the same as in Aryabhattiya, so the result is the sum of the arithmetic progression is given as (FL) the period less one, multiplied by the common difference, being added to the first term, is the amount of the last. Half the sum of the last and first terms is the mean amount which multiplied by the period is the sum of the whole.

Period means a number of terms and naturally the average of these you know quantities in the arithmetic this things. It is first and last divided by 2 right, so that is the mean amount, so then what is given is, so you got the arithmetic progression a, a+d, a+2d etc., a+n-1d.

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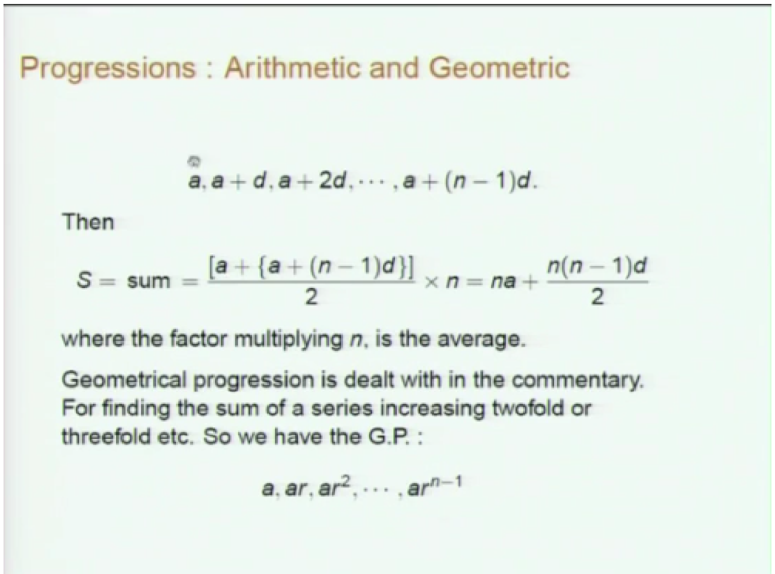

So, this is the arithmetic progression the sum is given by a+the first term then the last term you add them then divide by2 the multiple the number of calcu na+n(n-1)d/2, so so this is the average this one is average. So, geometrical progression is dealt with in the commentary Brahmasphutasiddhanta itself does not discuss it in the twelfth chapter. So, for finding the sum of a series it increasing 2 fold or 3 fold etc. he has the geometrical progression where suppose the factor is r, so then first term a then a, ar, ar square etc.,

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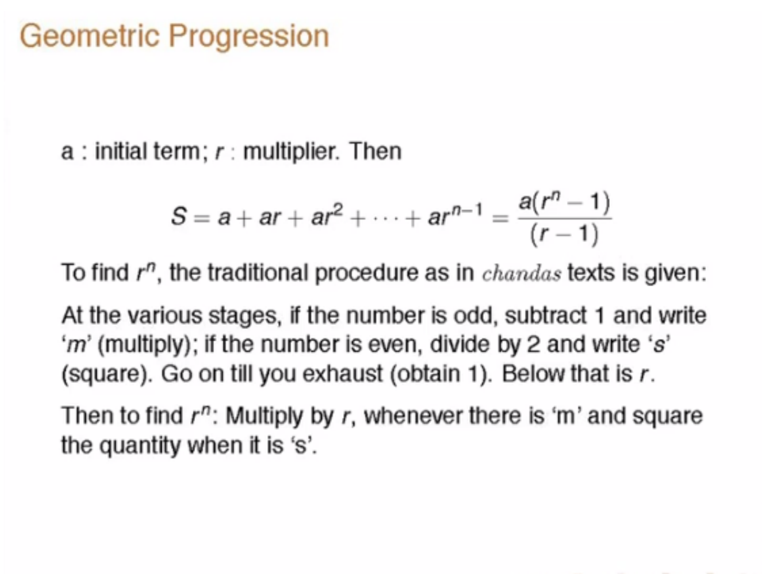

Ar to the power of n-1 is the number of terms is then so then the is a is the initial term r is the multiplier then the sum is a+ar+ar square etc.,+ar to the in the power of n-1, so is a into r to the power of n-1/r-1. So, to find r to the power of n the traditional procedure as in chandas texts is given. So, what is done is he stated is at the various stages in the commentary if the number is r subtract 1 and write m, if the number is even divide by 2 and write s, go on till you exhaust obtain 1, below that is r.

And then to find r to the power of n multiply by r whenever there is m, and square the quantity when it is s. So, this was nicely explained by professor Srinivas yesterday. So, this optimal method of finding the nth power of a number if suppose the number of terms is 17, so then 17 is the odd number.

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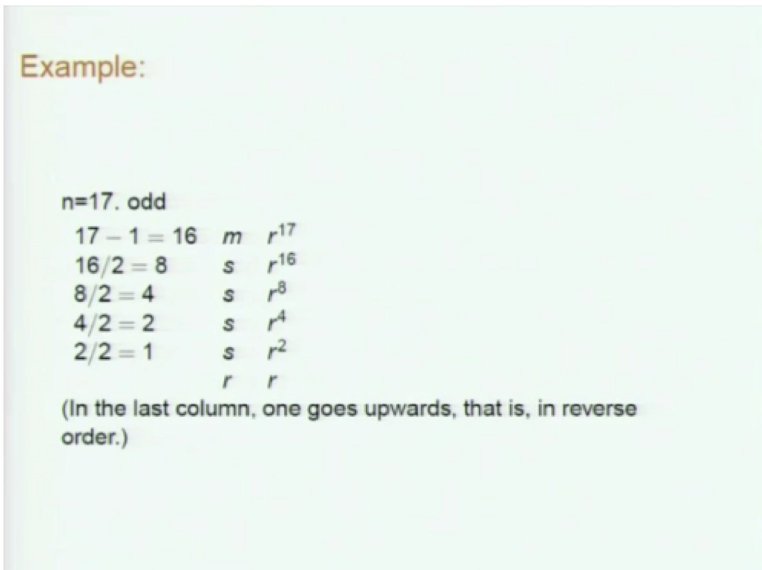

So, 17-1 16, so write m, so then 16 is divided by 2 is the is divisible by 2 8, so write s, 8 is divisible by 2 is 4 and write s, and 4 is divisible by 2 so 2 you write s and 2 is divisible by 2 is 1s, so we got 1 and then below this you stop okay r. So, now what is the next column you start with r, so whenever there is s you square it and whenever there is m you multiply by r. So, you start with r at the bottom now you are going reverse.

So, r next is s, so r square so next is again is s, so r to the power of 4 the next again is s, r to the power of 8, next is r to the power of 16 and then last is m so you have to multiply by r so r to the power of 17, so that is what it does okay.

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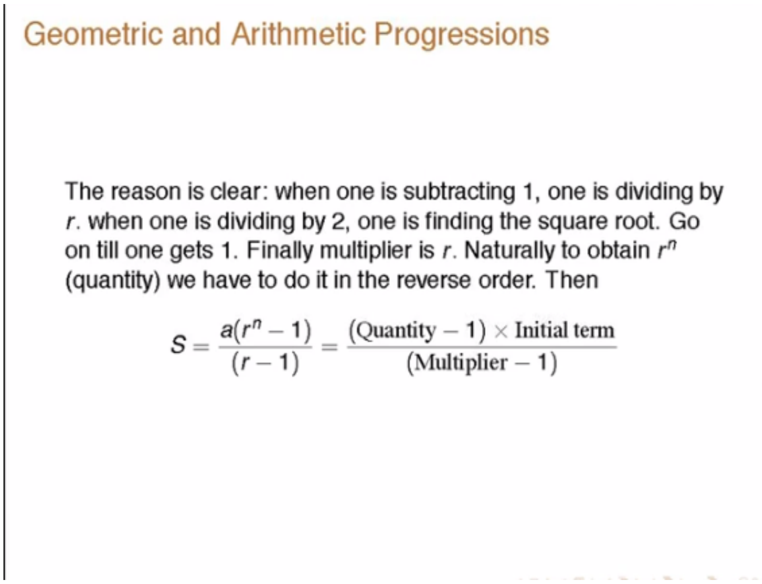

So, the reason is clear when 1 is subtracting 1, 1 is essentially dividing by r and 1 is and 1 is dividing by 2, 1 is finding the square root, so go on you till get 1, finally multiplier is r naturally to obtain r to the power of n you do have to you do have to do it in the reverse order. So, you have to square root and multiply it instead of dividing and this thing right. So, essentially the sum is this a into r to the power of after finding r to the power of n this is the sum a into r to the power of n-1/r-1. So, this quantity -1 * initial term/multiplier-1 okay.

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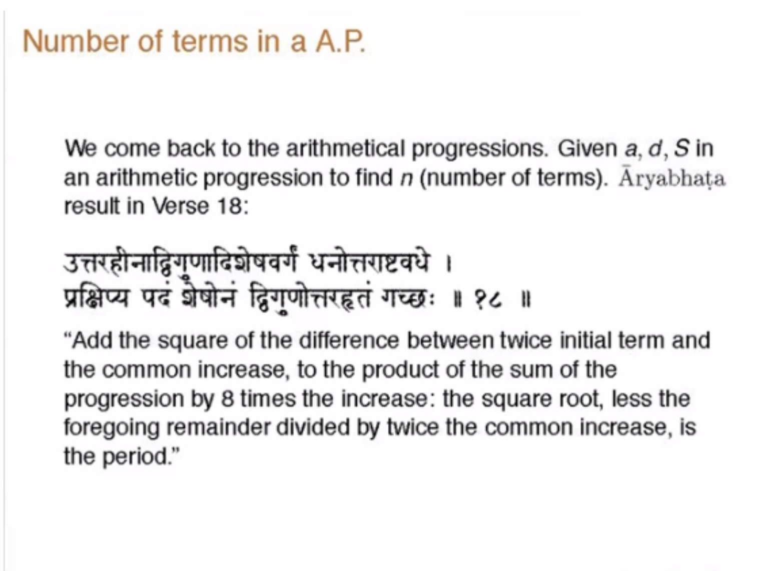

So, we come back to the arithmetic progression the number of terms is given again is the result which is given Aryabhattiya, so which probably professor ramasubramaniyam will explain in greater detail, so you clearly is you have seen earlier the expression for s right.

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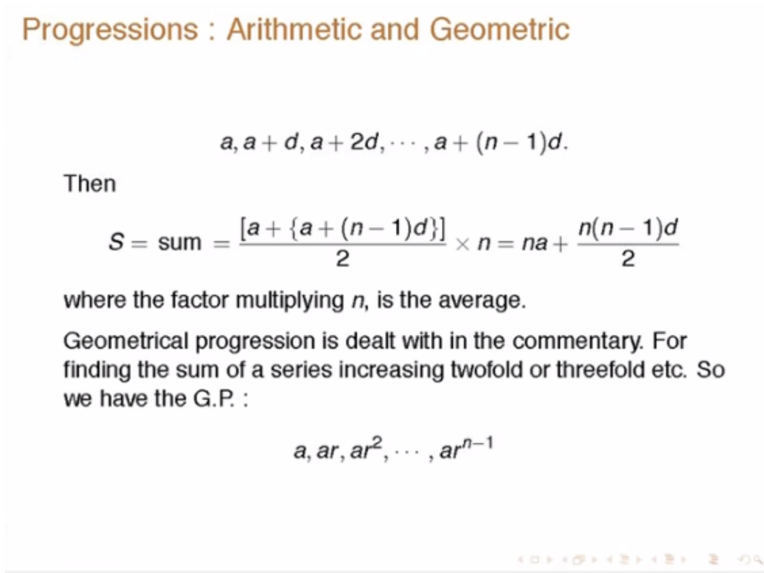

So, s is this, so s this quadratic in hour (()) (25:30) so, if know n, a and d you can find s but suppose you know s, d and a to find n clearly is the quadratic (()) (25:40) equation. So, such a solution is first given by Aryabhattiya in Aryabhattiya, so here is giving the same thing.

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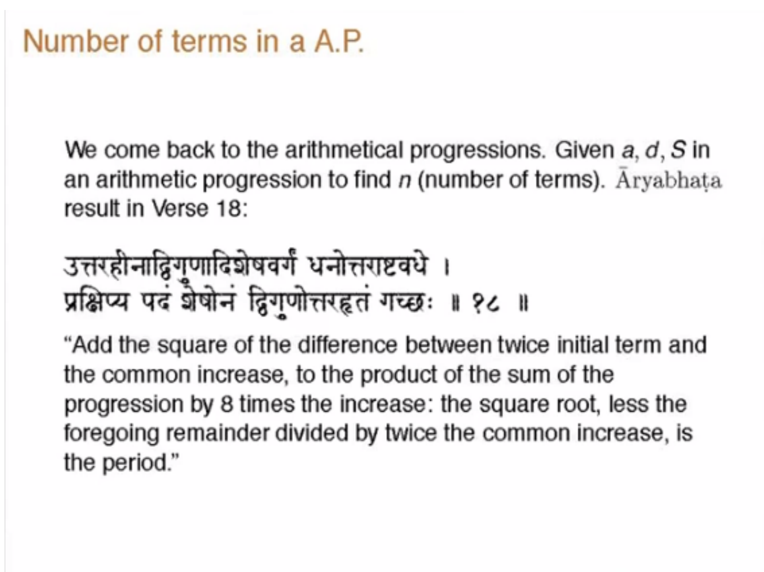

So, which is first (FL) so, this is clear enough again on talked about suppose you do the sum of sums you take okay. So, first sum is 1+the first sum is the then sum of the first end integer for 1+n square in n*n+1/2, the sum of sons sum of sums so that is you take r*r+1/2 that you sum from 1 to n and that is given by sum into period+2+period is remember is obtained.

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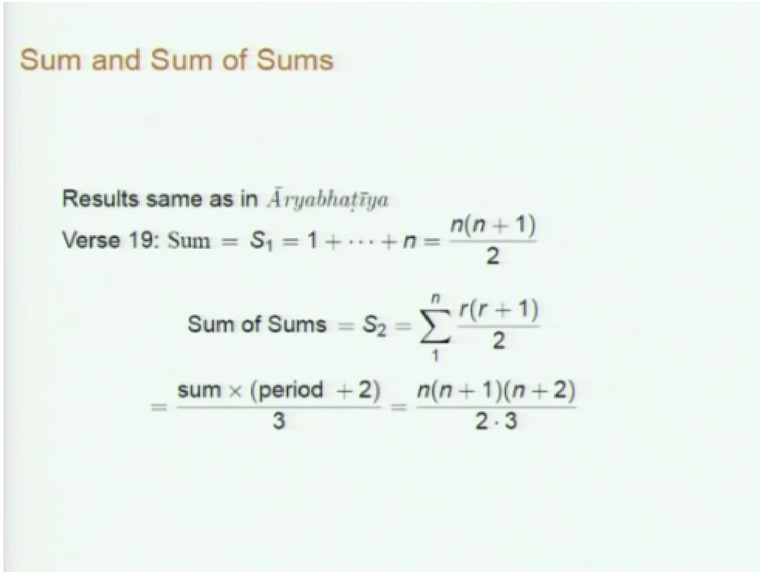

And n so sum itself is n into n+1/2, so this sum is n into n+1*n+2/2*3 general relation of that it will be given by narayana punitha later but this sum of sums is you know they stop here Aryabhattiya and Brahmasphutasiddhanta. And similarly for sum of squares and sum of cubes the results are the same as in Aryabhattiya, so which are given here.

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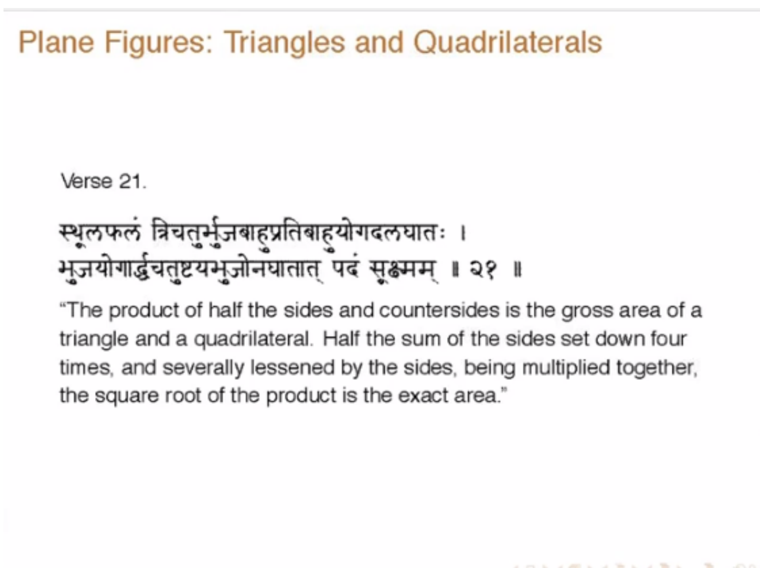

So, now you will go to some geometry, so where he discusses triangles and quadrilaterals in details and the first he will give the area of the this thing triangles and quadrilateral the product of half the sides and counter sides is the gross area of triangle and later than which is essentially quadrilateral half the sum of the sides set down 4 times and severally lessened by the sides being multiplied together, the square root of the product is the exact area.

So, here he first gives some approximate results which maybe at times and then he will give the exact result. So, if you have a quadrilateral like this.

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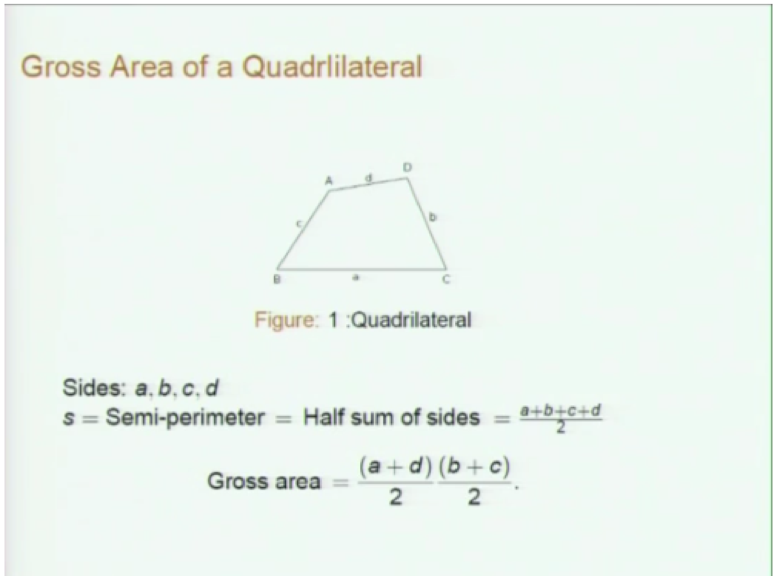

So, this sides a, b, c, d so then s is the semi-perimeter is half the sum of sides is a+b+c+d/2 the gross area is taken to be a+d/2 into b+c/2 it is called approximate area.

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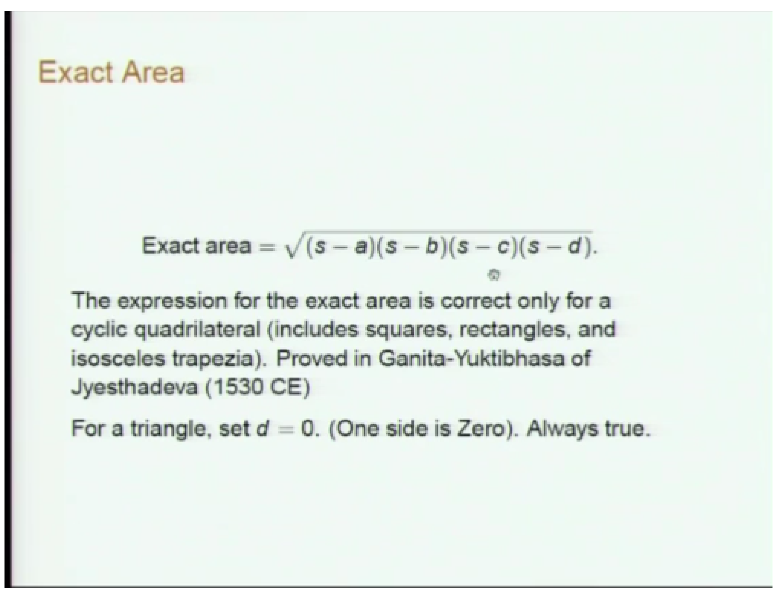

And the exact area is given by this expression square root of (s-a) (s-b) (s-c) (s-d) and the expression for the exact area as rude of remarked earlier is a correct only for a cyclic quadrilateral is not true for a general quadrilateral but this result is of course valid cyclic quadrilateral any square, rectangle, isosceles, trapezia is all for all them this result will be exact.

So, now the proof is not given in in the commentary are much later only it will come in (FL) which will be discuss later on it is a very elaborate proof of this result is given. Of course these for the quadrilateral cyclic quadrilateral for the triangle you take 1 besides to be 0, so then you will get the results for these. And for triangle it is always true because any try a triangle is cyclic because any triangle you can always know have a circle which is going through all the vertices.

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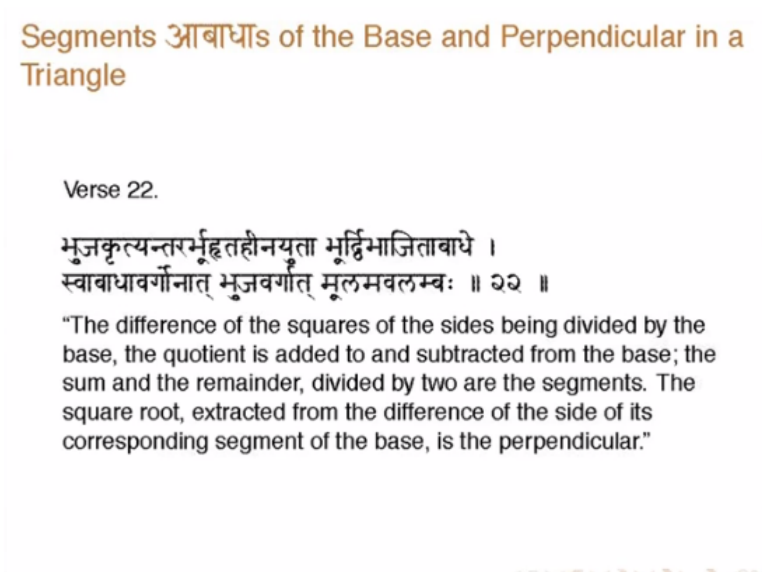

So, then he will talk about the (FL) base and perpendicular in the triangle, so is the interesting thing which I will discuss (FL) so, the difference of the squares of the sides being divided by the base, the quotient is added to and subtracted from the base the sum and the remainder, divided by two are segments and the square root extracted from the difference of the side of it is corresponding segment of the base, is a perpendicular.

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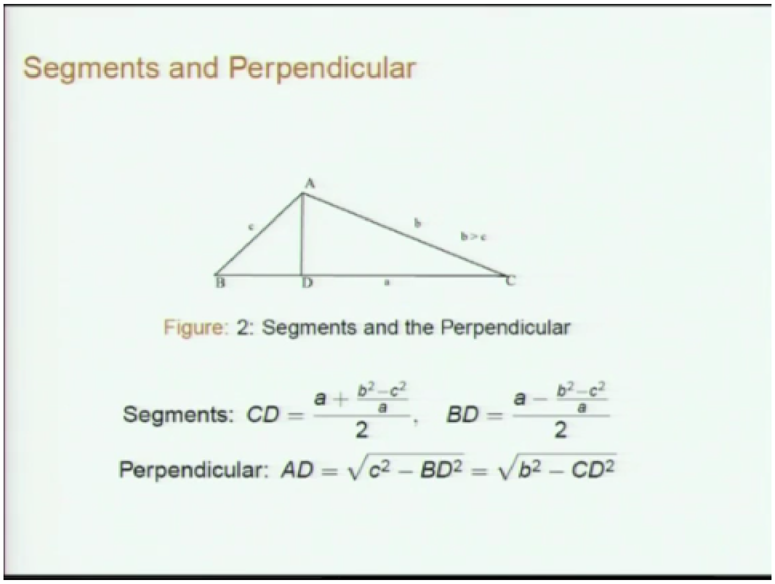

So, again which is much easier to explain with the diagram and the notation, so here A, B, C is a arbitrary triangle, so A is the base, C and B are the sides okay, so then you draw a perpendicular some A to the base BC, so AD is a perpendicular, so then segments is (FL) is you know BD and CD these are the segments okay. So, then he is giving the expressions for the segments CD and BD and perpendicular in terms of the side. So, the segments are CD is equal to a+b square-c square/a whole thing divided by 2 and BD is this okay.

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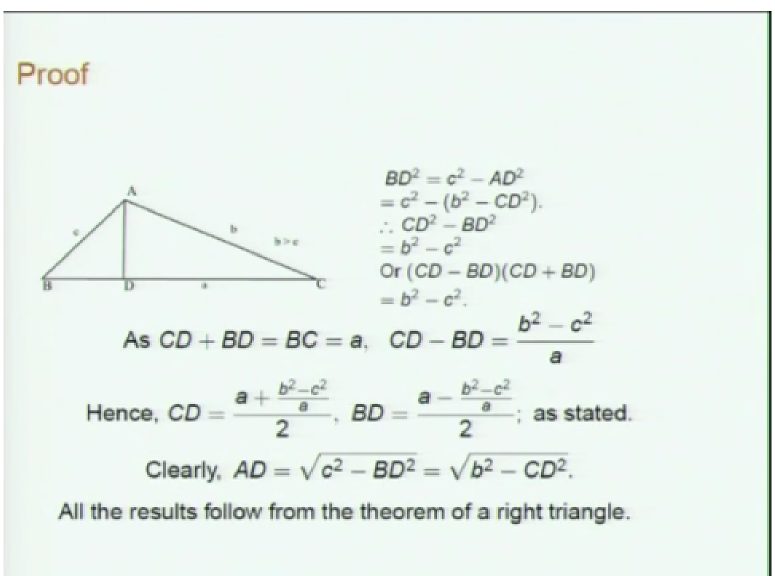

So, we can easily prove it, so because BD square for instance is AB square-AD square, so c square-AD square which is written as and AD it is again it is CD square- AD square, ad square is again b square-CD square. So, finally you get CD square-BD square is equal to b square-c square or CD-BD into CD+BD is b square-c square but CD+BD is equal to BC is equal to a.

So, then and CD-BD we are form to be b square-c square/a, so finally this CD and BD are found come to this equations. So, it is a+b square-c square/a whole thing divided by 2 and a-b square-c square/a whole divided by 2 as this how it is stated and AD is know you can found here some AD you know c square root of c square-this segments square or this side square-the corresponding segment square, square root of that.

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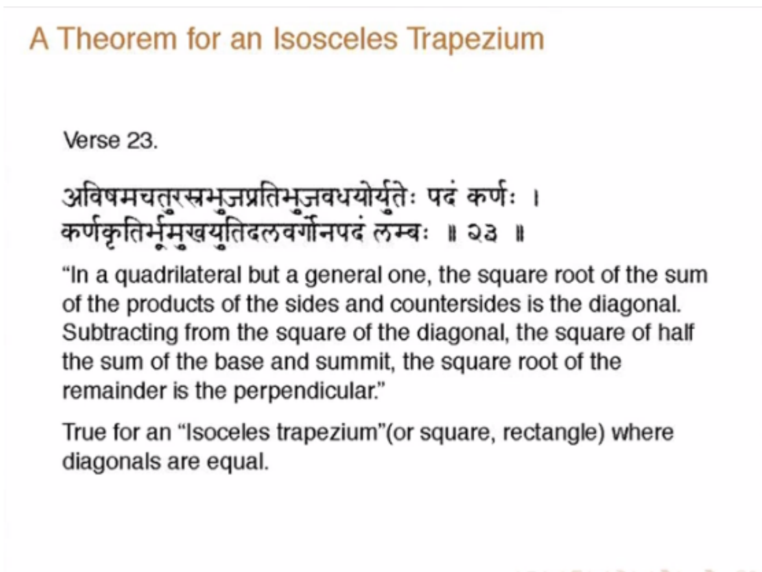

So, all the theorems things results are following from the theorem of the right triangle, so then for an Isosceles Trapezium is giving a result (FL) so in any again I think instead of reading now the translation what I am I will give explain it straight away.

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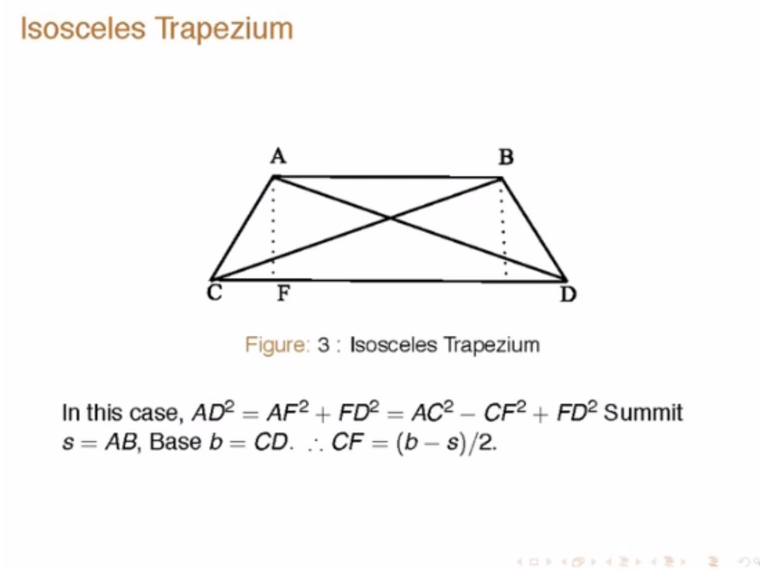

So, you have an what he does not say Isosceles trapezium, but it is actually applicable only to Isosceles trapezium see where the 2 flanks are equal. So, here CD is the base AD is summit, so AD and CB, BC are the diagonals, so for and Isosceles trapezium clearly they are equal. So, and then it gives AD square is you can find that AD square is AF square+FD square and so on it is a simple application of the scissors.

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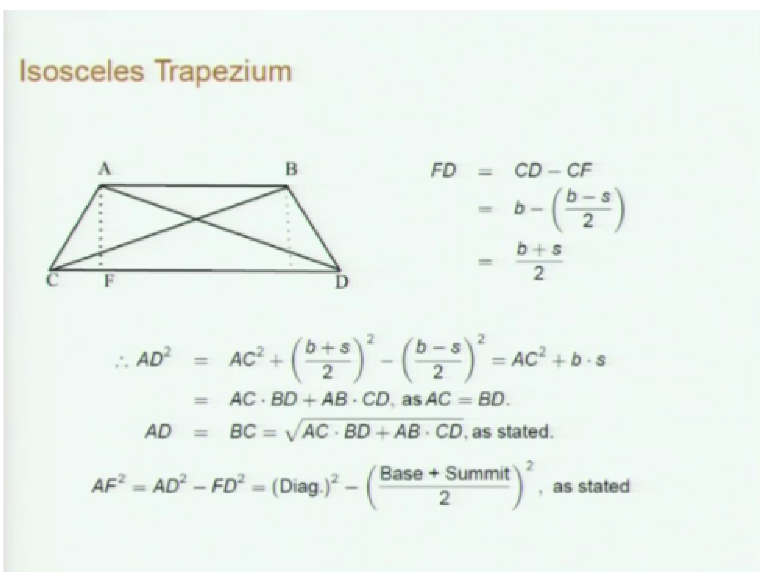

And the right triangle theorem so called Pythagoras theorem, so AD finally you get AD is equal to BC is square root of AC into BD because AB into CD, so it just matter of you know doing a few steps with the and A of square that is rumba right, so the perpendicular the that A A of square is given by diagonal square-base+summit/2 whole square okay. Because you found the diagonal AD the square it and then take the base+summit/2 whole square, so that is the.

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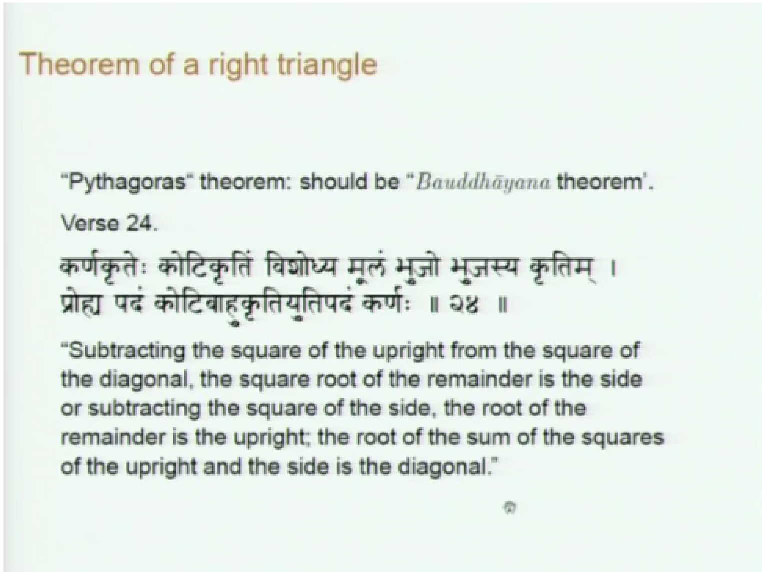

So, then is interesting that all this after giving all this results then the comes to the theorem of the right angle so called Pythagoras theorem, so probably should be called the (FL) theorem you know whatever it may in a everybody has strong reason this and but if it is known that most of the civilization knew about Pythagoras theorem before Pythagoras that is for (()) (34:12). So, it is he say that in a (FL).

Subtracting the square of the upright from the square of the diagonal, the square root of the remainder is the side of subtracting the square of the side the root of the remainder is the upright and the root of the sum of the squares of the upright and the side the diagonal. So, they are expressing within in this thing you know is there you get the side sum side is there upright is there is diagonal.

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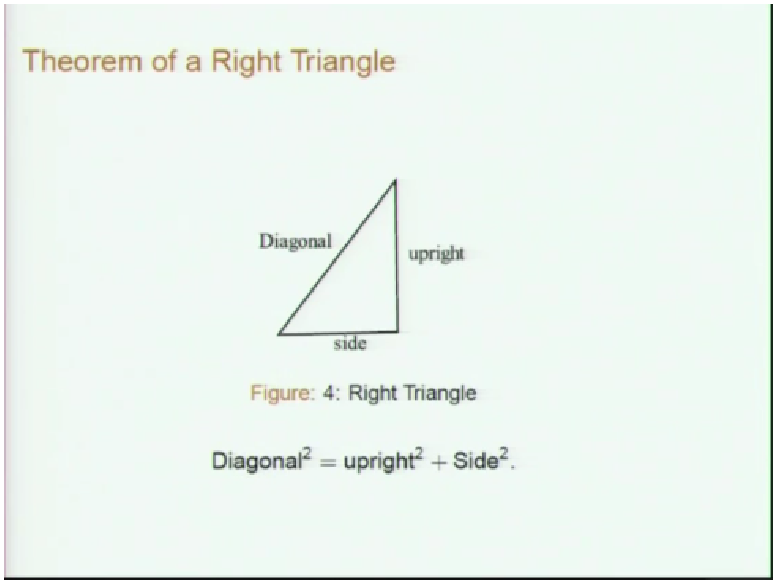

So, side is there diagonal is found from upright in side or side is found from upright in diagonal and so on but the essential is result is the diagonal square is upright side square.

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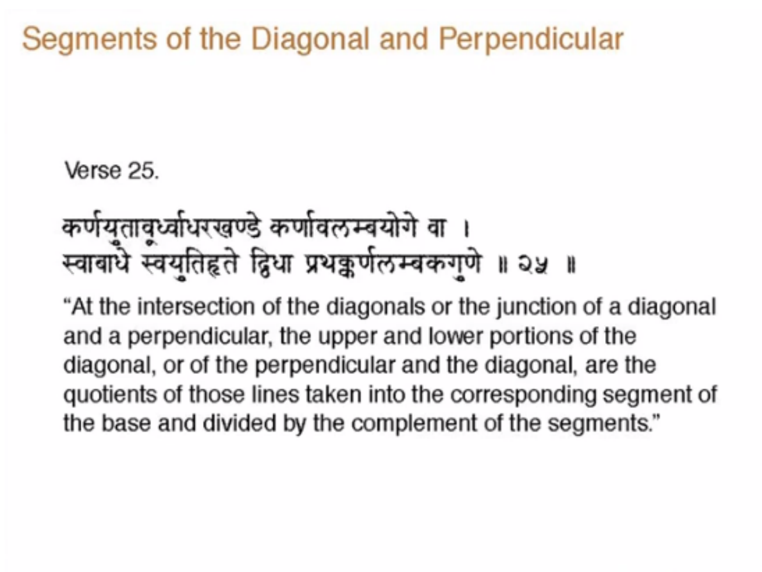

So, then he talks about the segments of the diagonal and the perpendicular, so again a go state the so this is the verse (FL) so, this is the result and a translation is this instead of that writing reading of the translation.

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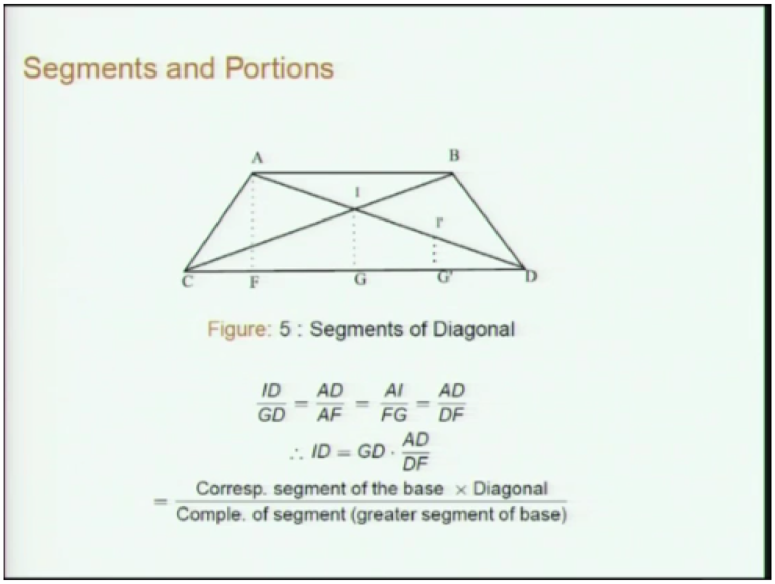

I am telling what he is trying to say so this is the trapezium like this okay, so the diagonals are intercepting in that I from the point of intersection you drop a perpendicular okay, so then essentially to find the segments AI and ID okay etc., from the diagonal and the segments and so on okay, say for instance ID is given by GD into AD/DF okay. So, that is corresponding segment of the base into diagonal/complement of segment.

So, it is all rule of proportion similar using similar triangles, so that what is doing but actually this is true not only we may I is an intersection for any other point almost it is true, so if you have I prime okay essentially if you want to find the I prime Dthe I prime D will be equal to G prime D into AD by instead of you know you will have this G prime D and AD/AF,so just a rule of proportion.

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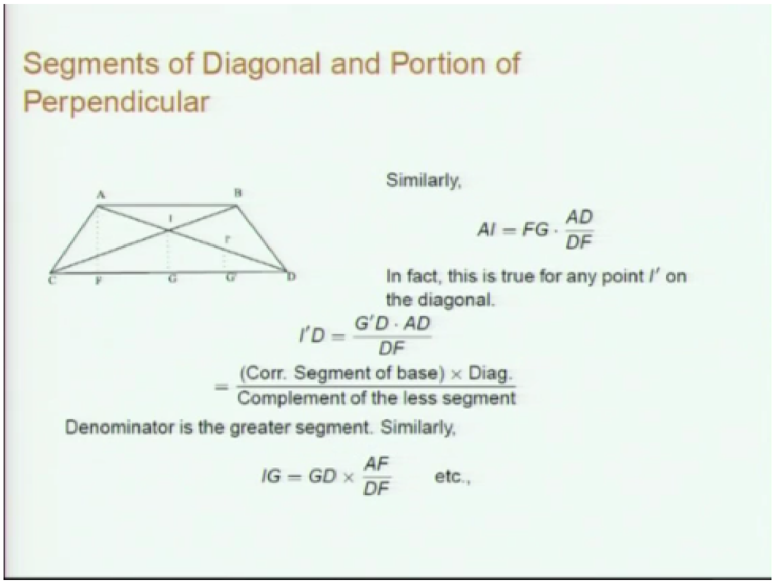

And similarly perpendicular you know IG and I prime G prime so these are the portions of the perpendicular you can say this is the real perpendicular and now you are getting some portions you know when you draw perpendicular sums of arbitrary points. So, there are given by the rule of proportion okay.

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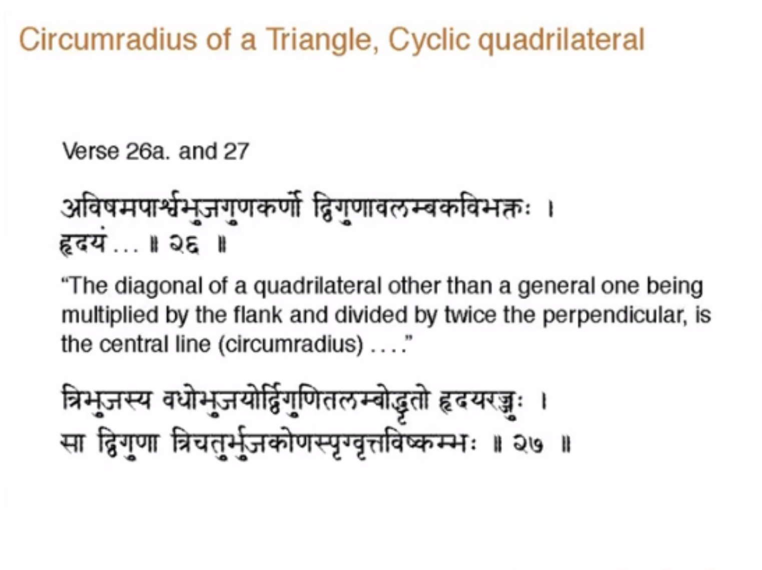

So, now he goes to the circumradius of a triangle and cyclic quadrilateral. So, the result is the diagonal of a quadrilateral other than the general one being multiplied by the flank and divided by twice the perpendicular is a central line that is the circumradius. So, these earlier verse part of the verse 26 in the chapter 12 then the other thing is the next verse gives for the circumradius of a triangle, so that is (FL), so this is the.

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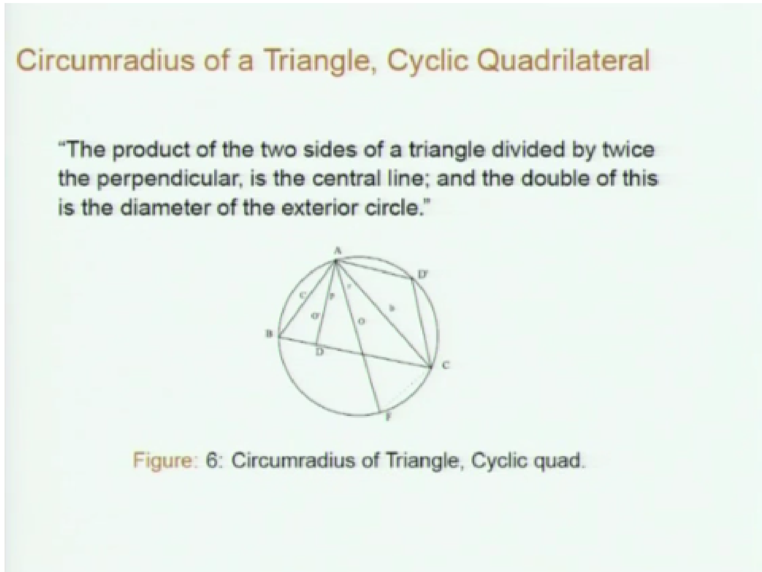

Figure: 6: Circumradius of Triangle, Cyclic quad.

So, what is he saying is the product of the two sides of a triangle divided by twice the perpendicular is the central line and the double of this is the diameter of the exterior circle. So, I will take this is for instance this is the ABC is a triangle and ABCD prime is the cyclic quadrilateral okay. So, both of them I am giving the same figure you have to find the circumradius.

So, what you do is I mean let us let OP the centre of this circle okay you draw the perpendicular AD from A to this side BC, so call it as P, so the sides of the triangle you know AB is C, AC is B okay.

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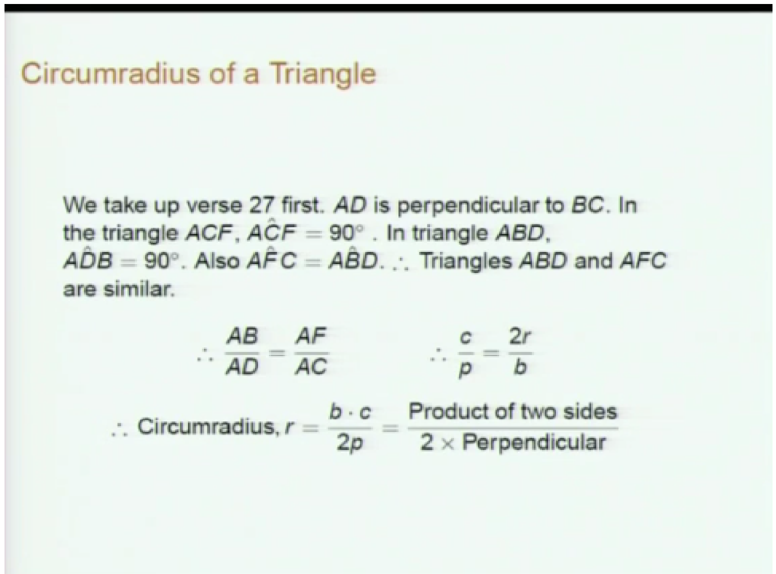

So, we take up is the triangle thing comes later but it is easier to handle, so here the side you have to find out this circumradius right AF. So, what you do is essentially you this line A from O you know you will just go up there where it touches the circle then you know join this okay. So, then we know that this angle will be equal to this angle ABC is equal to AFC. So, because of the standing on the same base, so in the same position the circle, so they will be equal.

So, then the triangles and this angle ACF you know that this will be semicircle, so CF will be essentially 90 degrees of course it does not give this I am giving the this thing. So, essentially ABD, so triangle ABD and ACF these triangles will be similar okay, so because this angle will be equal to this angle and essentially this angle will be equal to this angle okay. So, then sum this is thing you get AB/AD is equal to AF/AC.

So, the AB divided by AD is equal to AC by AF/AC, so c/p is 2r/b, so r is b*c/2p so product of sides divided by 2 * perpendicular. So, this circumradius I think is the first time he has has been got.

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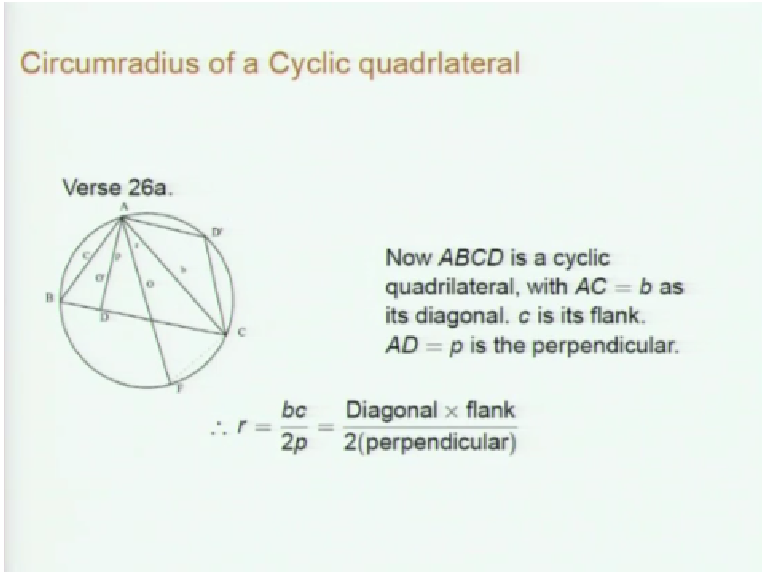

He does not prove it again it is proved in (FL) which will discuss there of course is a simpler is may circumradius on this you know the diagonals etc., taken up together in (FL) in one this thing, so it will come you know for a cyclic quadrilateral say again it is ABCD prime should not D and D prime ABCD prime is a cyclic quadrilateral. So, AC is B is diagonal okay and AD is equal to P is the perpendicular okay.

So, you make use of this portion triangular portion of the cyclic quadrilateral okay ABCD prime you take this so c into b the 2 sides will be of this triangle will be c that is one of the sides of the quadrilateral and b which is the diagonal of the cyclic quadrilateral, so then this will be equal to bc is equal to diagonal flank okay divided by the perpendicular okay. If the perpendicular has been found right using the earlier result, how to find out the perpendiculars and segments. So, using that you will get the result okay, so I think I will stop here thank you.