Mathematics in India: From Vedic Period to Modern Times
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Lecture-10
Aryabhatiya of Aryabhata-Part 4 and Introduction to Jaina Mathematics

So far we had 3 lectures on Aryabhatta. So in this fourth part phone I will be definitely concluding Aryabhatiya and then we will move on to bit of Jaina mathematics.

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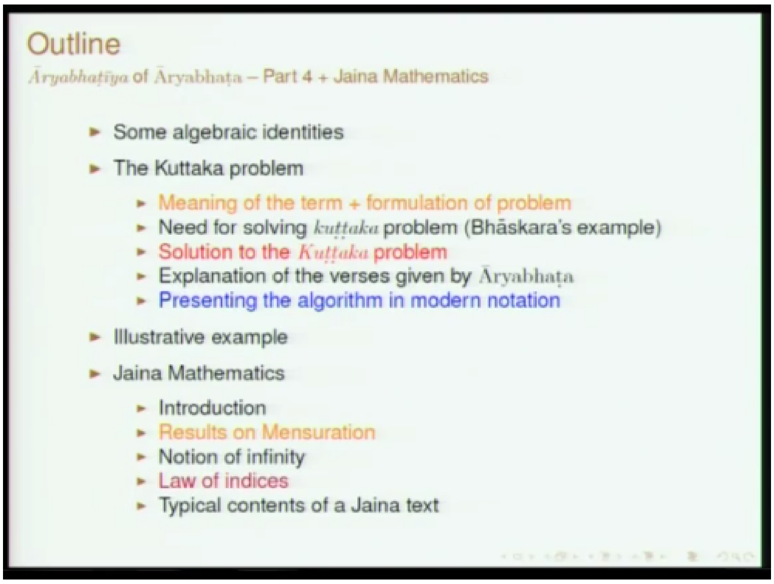

In the last lecture on Aryabhatta I was explaining to you about the methods by which Aryabhatta has derived sin table, so he has suggested and then we also (FL) problems. So problems can be handled with (FL) and in this lecture I will basically start with series in fact I touched up on the arithmetic progression which has been built by Aryabhatta. So in fact there are few other problems we can skipping on Ganitapada.

So it because highlighting two more problems and then I will move on to the last part of Ganitapada which displaces the kuttaka problem. So kuttaka is a very interesting terminology which has been used to refer to a certain process. So by this we keep on reducing the numbers. So (FL) is basically hitting on head (FL) so it is basically founding up pulverising, so things are brought down ok, so that is the set of the origin of the term kuttaka.

So formulation of the problem then I will give you an example of how it has been used in Astronomy. In fact the very purpose of the kuttaka seems to have been to solve pattern problems in Astronomy. We will see a single examples of Bhaskara's commentary and then I will explain in great detail in the couple of versus. So I think through which Aryabhatta has presented this kuttaka already.

So then I have an example, then you move on to Jaina mathematic, even I say Jaina mathematics, so I refer to the earlier part of the Jaina mathematics, the earlier part of Jaina mathematics has not been in samaskritham, so (FL) so it is difficult to decipher the text themselves and text themselves of not available and some of the commentaries are taken as the resources for deciphering what happening there in the Jaina earlier Jaina text.

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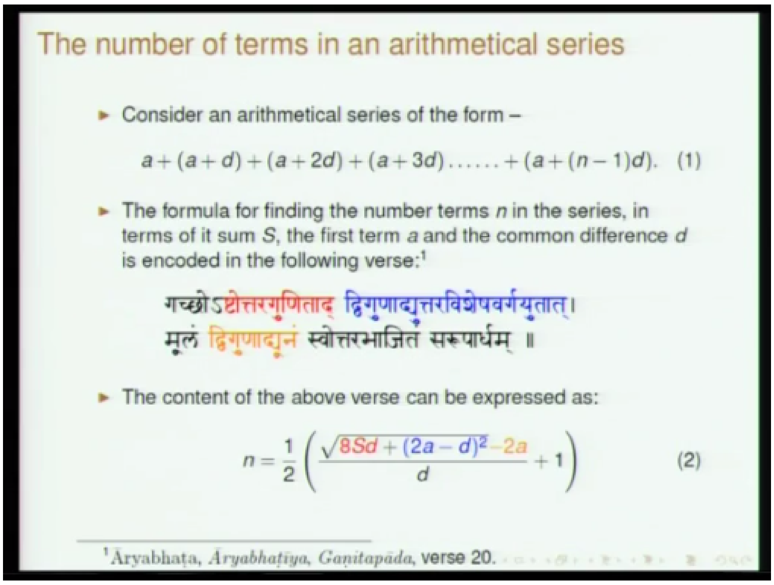

So with this I will first of all introduce an interesting verse of Aryabhatta. This verse is presenting a formula by which will be able to find out the number of terms in Arithmetic series, given that you know the sum of the series and first term and the common difference. So the verse goes like this (FL) refers to the number of terms in Arithmetic series (FL) in fact in the previous verse, so we had the sum mention the sum of the series.

The formula for finding the sum mention and therefore in this verse is not explicitly stated but since Aryabhatta as I was mentioned earlier is more or less composed in sutra style. In sutra style so we have something call (FL) so whatever is not present in this sutra and it is available in the previous sutra, we borrow it. So this is the kind of thing which seems to have been done here and since you are speaking about the sum in the previous verse.

So here we have to exam, so (FL) is the common difference, so (FL) is the first term, (FL) difference. So (FL) 2a-d and then varga is square, (FL) is addition, so this term 8Sd in order

2a-d square. So this is what (FL) then what is to be done (FL) take the square root of that. (FL) -2a (FL) so sum here of this, so series as I said, (FL) is the common difference, the common difference is (FL) that forms the divisor. Then the last term is (FL) 1.

(FL) add 1 to that and then he says (FL) divided by 2. So this is basically gives the number of terms in an arithmetic series provided you know Sd and a, the formulas, an interesting thing to see why Aryabhata has specified this in this particular form, it has been commented by Nilakantha wherein he has shown certain way of moving this result, not the alzebra, but by geometrical construction.

So you can think of (FL) in fact the series can be considered to be ready, ready is step by step. So a, a+b and so on. So similar kind of construction can be used to actually shows this result. So this result as such looks complicated and the if you going to show this kind of course in various ways, so right hand side can be written in various ways and this particular way in which has been written seems to be coming from the fact that they might have had a certain way of looking at it.

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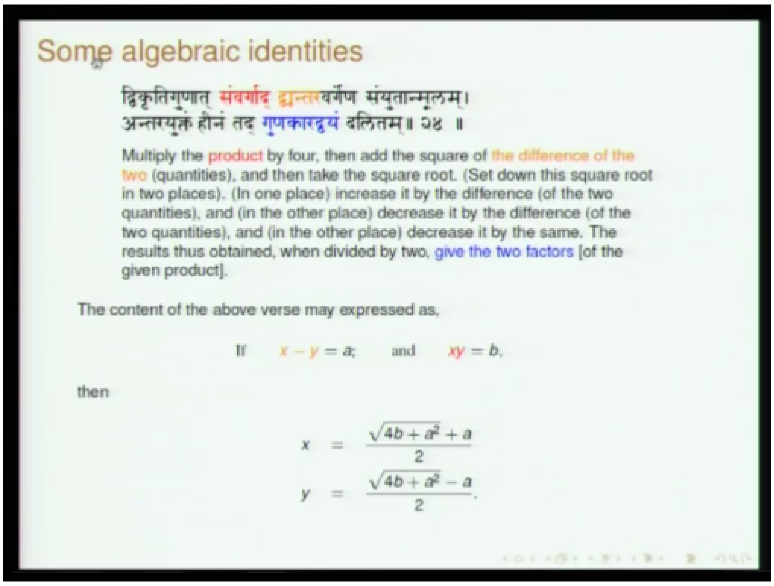

And that has been shown by Neelakanta, so which I m not discussing right now. Certain algebraic identities have also been presented by Aryabhatta. So just sample which I want to give Aryabhatta has also discussed rule of three of course Professor Sriram, so in his talks will be covering all that in great detail, but this mean the earliest text so I wanted to present more or less so on various issues which Aryabhatta has discussed.

Suppose you know the product of 2 number and you also know the difference. So this is the kind of problem which Aryabhatta has chosen here. We will you be able to find out those two numbers. So there is a question, if x-y is a and xy=b, then how do you find xy. So this is the relation which has been given by Aryabhatta in this verse. He says (FL) 2 square which is 4 (FL) is multiplication, (FL) to design a product.

So suppose you know the product of number numbers and that product he refers (FL) xy let us say b, so (FL) so if you look at this expression, so which gives the value for x and y the first term is (FL) and then (FL) ok four times b, (FL) so the 2 numbers which is considered (FL) is difference (FL) is square. So (FL) you have to add this, and then you take the square root (FL) half of it.

So this is trace out, this is one of the algebraic identify present and it useful in various context. So this algebraic identities have also been presented in the form of verse some like to we memorise what is the value of x in a quadratic equation 2-b (FL) so that is kind of thing, so this essentially in the form of verses. Now I move on to the kuttaka algorithm.

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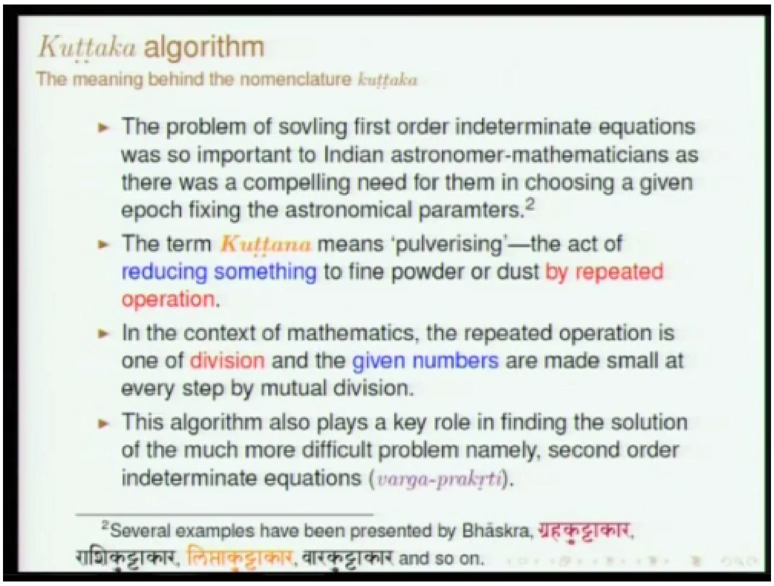

So this Kuttakara algorithm is primarily a method by which you will be able to find out solution to the first order indeterminate equation ok. The indeterminate equation we have 2 variables in this and this first order we have just 2 variables and step by step procedure has been delineated by Aryabhatta in 2 versus for solving this equation and why were they worried about solving this.

I will show you an example the context in which they had to necessarily find a solution for this. So this finally to solve astronomical problems, so kuttaka is referring to the process of pulverizing something. So basically a repeated operation, so even hammering so something is a repeated operation and here what we do is, so given two numbers will keep on successively divided one by the other. So this can be thought of it is similar to the repeat in algorithm.

And by mutual division we will keep on reducing the magnitude of those two number and at some stage will stop this and it can be stopped at any stage and then you can go and construct what the two numbers x and y for. This algorithm also plays a very key role in solving (FL) which I will also be discussing little later when we discuss about Brahmagupta and it like to be touched up on (FL).

This kuttakara occurs in various contact, so therefore Bhaskara classification (FL) if it is arising the context of dealing in the revolution number of planets we call it (FL). This can arise in various occasions.

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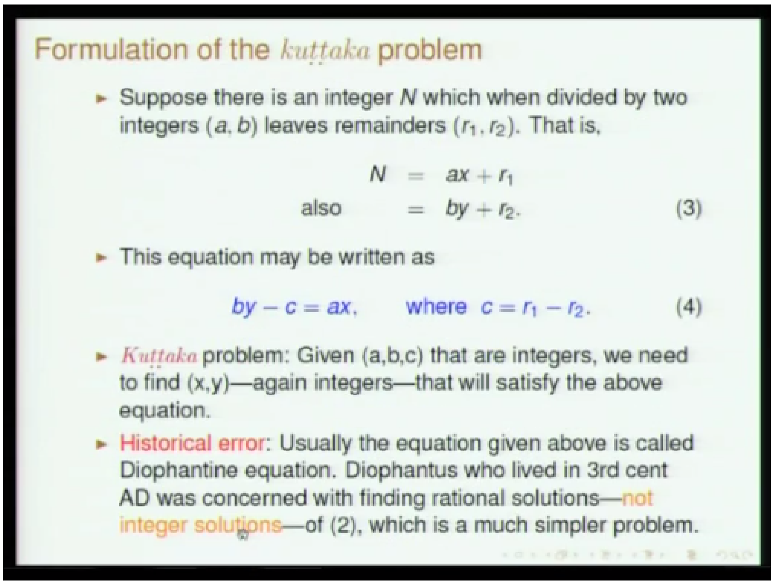

Suppose you want to express some magnitude in terms of degree is sum in some other magnitude, so if you have to handle those two. So again there will be a kuttakara, so (FL) is basically some number. So (FL) so all that requires kuttakara method. So to base the is the problem in more concrete way. So let us consider a number n, so this is how we can easily understood the different ways in which this kuttakara problem can even be presented.

So we choose one particular way which is very close to the method which has been described by Aryabhatta. Suppose this number n and this number when divided by a it gives some value, and when divided b it give some other value and then plus reminder (FL) plus reminder. So n=ax+r1 is also equal to b/r2. So we it can see that so this can be represent in this particular form, we choose this by-c+ax where c is the difference of the two reminders r1 and r2.

So the problem is given a, b and c, so we have to find interior solutions for x and y. So this is called kuttakara is all about. So this the this is indefinite equation because we have only one equation we have two variables. So therefore it is not suitable to find a solution in all the cases, in certain cases we can easily guess the solution but in most of the cases it will not be trivial and we need a certain systematic procedure by which will be able to solve this equation.

From new point of history this equation has been generally referred to as diophantine equation, so which is not quite correct, so if where to look at from the historical viewpoint in fact the diaphanous also attempted a slightly different kind of a problem, he was not even trying to find integer solution to this problem. So he was trying to find rational solutions and rational solutions of all more easier than getting integer solutions for this problem.

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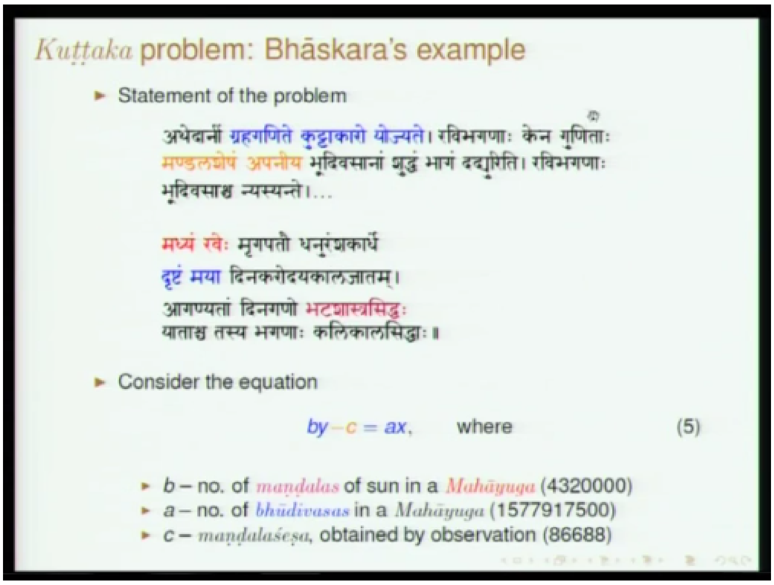

So for so will give you the example one of the examples which has been presented by Bhaskara to give you your flavour of the context in which this kuttakara problem arises in Astronomy in fact Bhaskara says (FL) so all that he says, so (FL) I am going to present you how kuttakara arises, (FL) this is a problem. So (FL) is basically the number of revolution made by the sun (FL) multiplied by which factor.

(FL) in fact is a very important problem in the sense that Indian astronomer have presented a very large period. So this for you to have a much general picture I just wanted to spend a couple of minutes period. So you have a large period so you can call (FL) period that you want to define late to define, let us define this and this large period to see that the various planets which are moving in various orbits.

So they complete some integral number of revolutions, so there will be some reminder here and some reminders there. So now you have to see to it that you will be able to get a second period by which all of them can make integral number of revolution. So it is in this context with kuttakara naturally arises ok. And here Bhaskara says (FL) is basically the number of (FL).

So number of (FL) mean the number of sunrises that take place in a given period. See even sun start moving or moon starts moving, so I started particular point, so it will make some n number of solutions and when it comes back so it may not be the same time. So can we see that that after making integral number of revolutions, can fix certain period by with this is also company integral number of solution and something else also will do.

So that is kind of context in which these arises. He says (FL) meaning of this period, so (FL) so this is the kind of problem which Bhaskara wants to attempt with reference to the number of revolutions made by the planet.

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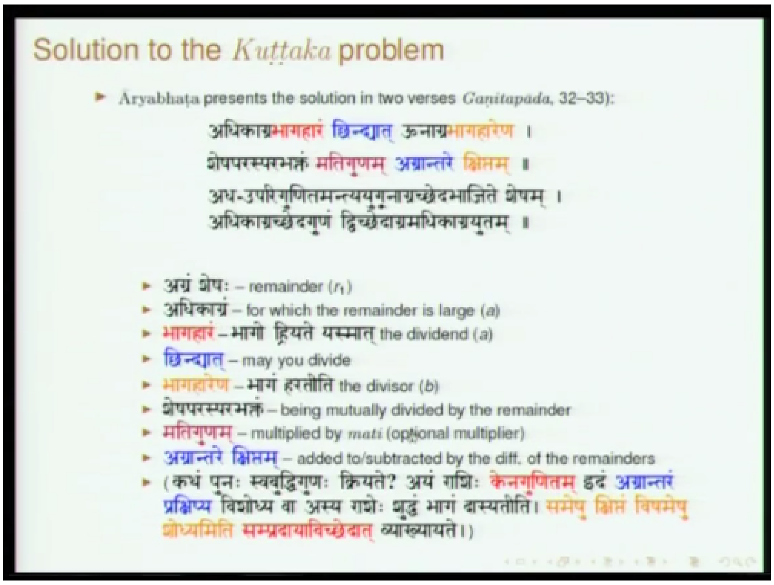

So now we move on to the kuttakara problem message so and the solution which Aryabhatta gives. These verses are not very easy so and have follow it carefully because the kind of compounds which Aryabhatta has formed and the terminology which he is employed may not be that familiar given to those who wore fairly conversation with Sanskrit. So (FL) this is the first half of the algorithm which has been given by Aryabhatta.

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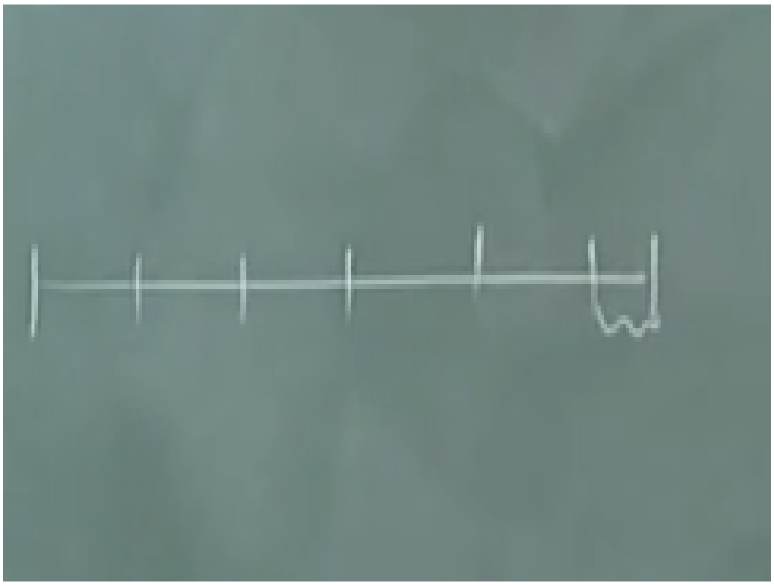

So then the latter half completes the whole algorithm. The term (FL) normally means, so tip kind of a thing. So here this has been used in the sense of getting a reminder, see suppose you think of representing a certain number, so by this magnitude, so if you keep on dividing this there will be some reminder which comes. So this is the tip that means. So (FL) means reminder, if you go back and then see this.

See N=ax+r1 and by+r2, so r1 and r2 are (FL) greater than r2, so if you call r1 as (FL) and r2 as (FL) fine, so (FL) may you divide (FL) cutting is referring to the process of division here. So in fact Bhaskara starts his commentary by saying (FL) in common language we call it (FL) where Aryabhata has used the term (FL). So (FL) the remainder is large (FL) so here so this is a compound (FL). So that is how we need to understand.

So the number so which is being divided ok in Sanskrit confirm various kinds of compounds. So rameshwara when you say, so rama is eshwara is one way of saying or you can say (FL) so that he says, so here what (FL) understood as (FL) removing the compound from which we are removing the part. So which means it is dividend. So (FL) may you divide. When you say divide, divide by what.

So (FL) by the other number for which the reminder is small. So (FL) so the process is end there, this is the initial state in which you get the first question. Then you will get some reminder, take the reminder and then you so divide the number by which you divided before for the process is going to be repeated. So for you to see this is the kind of the thing.

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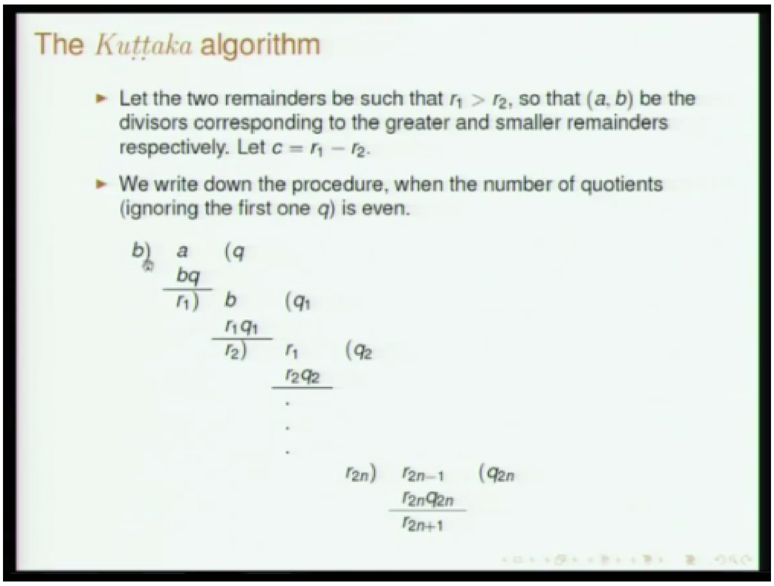

Suppose a is (FL) b (FL) so you have to do this, so this q is first portion and then so r1 is a reminder, so once again you divide b. So you will have something r2 is a reminder, if you keep on doing the process.

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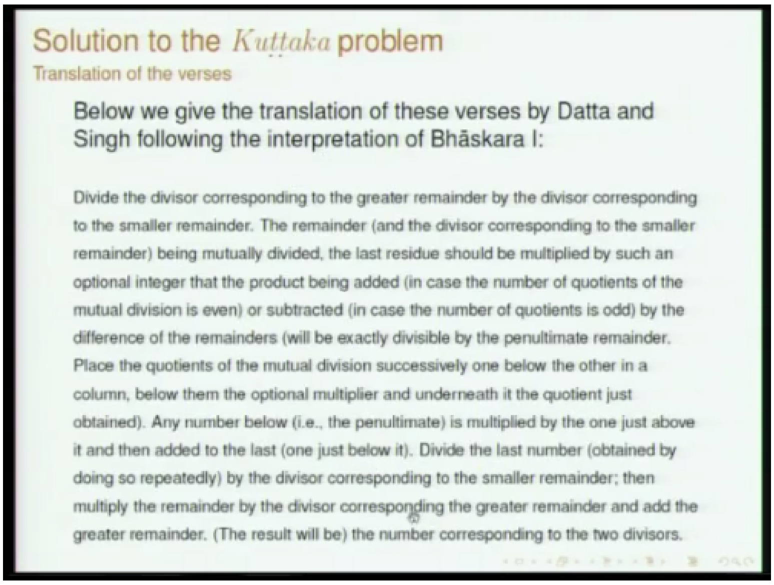

So this is what is referred to the Aryabhatta (FL) he says (FL) is reminder, so (FL) is so the previous reminder is going to become the dividend now and this reminder so very few divide will become the dividend the next stage. So you have to do it continuously (FL). Then when you reach a certain stage so this can be terminated and at that stage. So he says you had to do a certain process that process is (FL).

So (FL) multiplication, so (FL) means multiplied by (FL) so that is (FL) so what is this (FL) so here use that word to refer to a certain number which you are going to guess by making use of your budhi, that is why we are called he is calling it as mathi, so mathi gunam means a number which will guess. So you have to make a guess, so that the product of this mathi and the reminder that you have so minus or plus something will be the device.

That will exactly be multiple of something else ok, just keep it in mind as see algorithm clear but mathi is basically referring to a number which you have to guess at a particular stage. So you can stop the division at any stage in fact, so if you carefully analyse the algorithm there is good algorithm. So mathi gunam multiplied by mathi. So this is called an optional number (FL) is remainder (FL) is difference of the remainder.

So that is what is denoted as c, c refers to r1-r2 (FL) all that he says is so you have get mathi, so in this mathi has to be multiply by some number and to that this (FL) basically it could be either addition or subtraction ok you have to do either addition or subtraction (FL) is referring to either of it, that can understand only from the commentaries (FL) this number (FL) multiply by what (FL) see you have to multiply this (FL) by some number.

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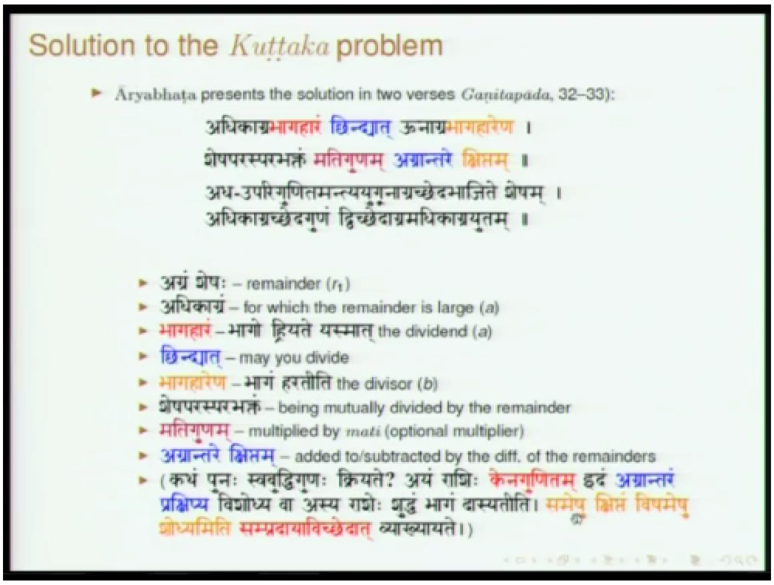

So and then +/-c (FL) so then he also says (FL) when should you add c, when should you remove c, (FL) is even (FL) is odd. So when the number of portions that you have see depending upon the process, so you will get a certain number of portions, the number of portion is odd or have to do something. If it is even you had to and he says (FL) if it is even (FL) you have to add and when it is (FL) you have to remove it (FL).

So how do you know this where he says (FL) so Aryabhatta has worked out, this is how it seems to be. So it is a (FL) which has been handed down to it okay, traditionally they have discovered this to be the method and therefore we interpret it that way. So (FL) this is how I explain, so I move on to the second part, see in fact this stage of the verse we have to see the commentary because what is to be done all this.

So that has been very expressively stated by Aryabhatta in the verse and therefore (FL) ok (FL) means to be guest. So (FL) ok if you divide so you will get a set of portions (FL) so you have to arrange them in a particular way. This has not been set in the verse. So that is why the commentator say something has to be (FL) means you have to make a guess. So (FL) so I will quickly tell you we can keep this in mind and then you can understand the verse.

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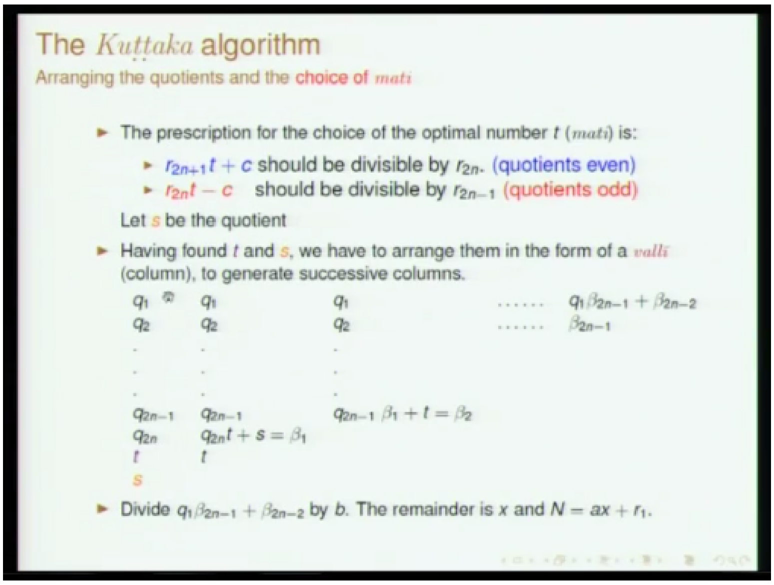

So we got a we get a series of division, so we got a series of portions, so all that has to be done is you have to arrange them one below the other, say q1, q2, bla bla, 2n let us say up to that, so one by one below the other you have this is generally only means (FL) a sequence ok. So all the portions on the sequence and then below that you have to place this mathi and then some other number yes.

So which is actually the portion which is obtained by doing this operation. So I said you have to make a guess of mathi, so mathi into what is to be done is the last reminder that you get. So this time denoting as this for instance so you have this division process be noted here and the last reminder is let us say r2n+1. Now r2n+1*mathi ok, so this is guest, so the number will be guess, so how should we guess, if it guess in such a way that r2n+1Xt+cis divisible by r2 by the previous ok.

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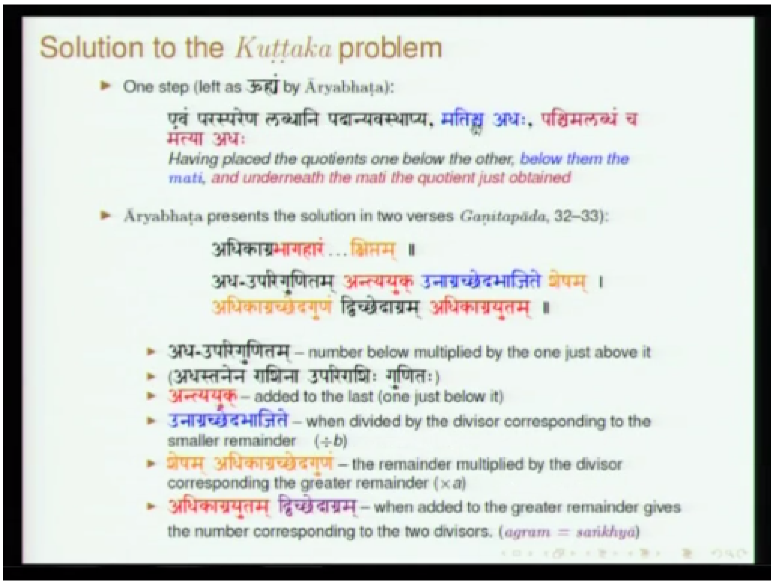

So that is something which has not been stated in the, so that particular mathi also has to be placed over start a one below the other how to place (FL) you have to place the mathi button below and (FL) and then +/-c if it is divisible by some other r2n and then that portion is what is referred to as (FL) means after making use of mathi and after doing this division whatever be the portion, so that is the ultimate thing that you find out in the process.

After that it is a matter of multiplication and getting the value of x and y. So as the process end here, and therefore (FL) something later (FL) you have to place that also below. so that is how it is stick, get all the portion one together place mathi and place the portion that we get by doing this process ok (FL). Then what should be done. so this is the arrangement. So till that is over now I am going to the second verse which is given by Aryabhatta.

So (FL) so having created this (FL) now so what is to be done, (FL) so the number which is there an alternate number has to be multiplied by the previous number (FL) means you multiply the number which is about that and then (FL) below that you place the portion so we have to add that also. So to show you here if you have done this, for this stage what to do if you have to take a product of P and qn q2n, this is (FL) multiply.

And then whatever is below you have to add that. So this is how he describes the algorithm (FL) more or less over (FL) so he says so this has to be divided by v, (FL) smaller reminder, so in fact both A and B are referred to as (FL) so you may think that AC for instance if you look at this. So ae/b, how can this b called (FL) in that both are (FL) in the sense. So the main problem if we consider.

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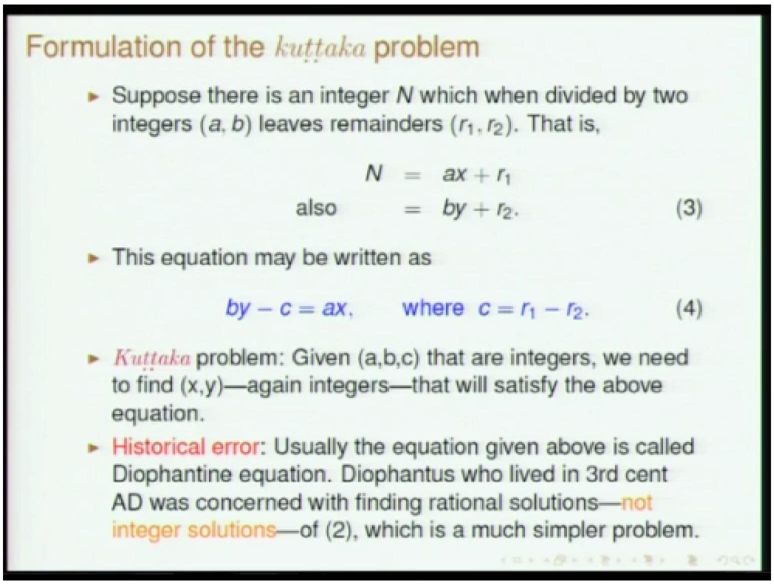

See (FL) is the main problem, so a is division for this b is also devise you have to get x and y ok. So therefore both are refer to as (FL) here. So (FL) so whatever comes to the reminder (FL) ok so this must be multiplied by the (FL) you find that (FL) being used here as well as here one not get confused. So (FL) is also (FL) refers to number n, where it later says so b becomes the divisor and a becomes another dividend that is a different thing ok.

So (FL) so when you add to that then we will be able to get x, y. So this is the algorithm it has been described in (FL) by Aryabhatta. So I will leave this so this is just the way the process of division with has to be done and all the portions are obtained. So you have to leave the first portion, so when you count the number of portions see this is something which has to be kept in mind. The prescription is so this c has to be added so that the portion are even.

And c has to be subtract so the portion are r, so in doing this once should not coming the mistake of taking q also in top, so it is only after this, so the number of portion have to be counted, so this division of b should be left out.

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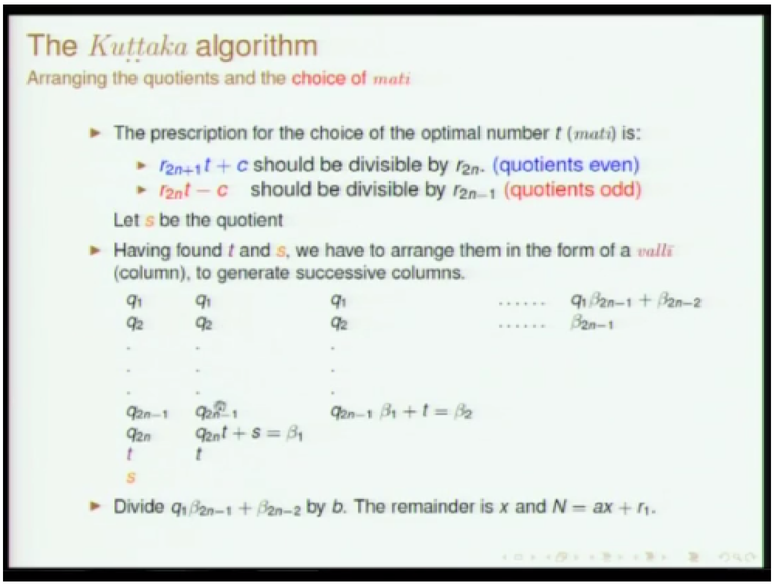

So this is how things are arranged and q1 q2, qn-1, 2n t is mathi, and then this see now what is to be done having created this (FL) so you have to multiply that by this. So qn*q+s, so let us call this is beta 1 and then the rest have to be this place like this. When we move on to next stage, so this will be multiplied by this and then t will be added and then this will go on c, so qn-1*beta1 this quantity+t.

So this is algorithm and then when reach the state where you have only 2 there is nothing more to be added you understand. So at this stage you stop this multiplication and creating the table (FL). So this is a serious of police finally he says if you look at the verse, so (FL) once you do that (FL) means so this has to be divided by b, b was referred to as (FL) so a was (FL) so divide by that.

So the reminder whatever you get that is basically x ok, so we wanted to find out x and y, so he described certain process and this is after you reach the store numbers. So then you make the product of this the last number with b and when you divide that so whatever reminder that you get is basically yx ok. So N=ax+r1.

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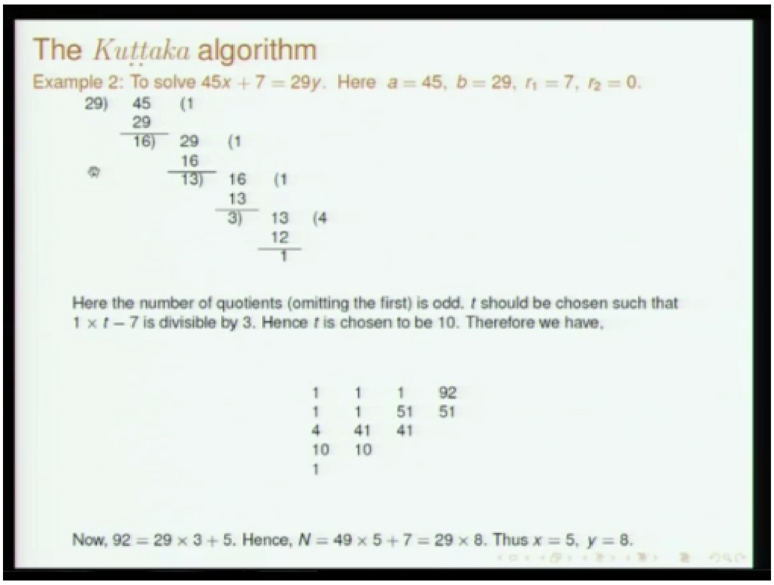

So let us take an example, so let us have this equation 45x+7=29y. So what do we do, so we start this division process, so 45/29 what you get 1, and then remainder is 16, so you take this as the dividend now. So divide, so you get 1 here, 15, 16, remainder is 13 and so the 16 become divider now. So you have 1, so we have 3 as reminder, so then 13 becomes dividend, so you have 4 as portion fine.

So how many portions we have in this it is odd or even, we have 1,2,1, 3, so this is the case of having odd number of portions, so what are the prescription, so prescription is when you have odd number of portion, the c factor has to be subtracted. Before that we need to make a guess of t, so this, this number so now we have arrived at the situation where we can found (FL). So forming (FL) I just give q1 and q2 so on.

I have to make a guess of this mathi now, chine -7 to be divisible by the previous reminder North-eastern project question precaution recognize 12063 so these remainder last remainder is 1, so one time t-7 should be divisible by the previous remainder, so it is 30, then you had this, so t is 10 for that so I just put 10. So when I divide by 3, the portion I get is 1, so because it is 3, so you just have 1 as the last. So this is how it has to be arranged.

So at this stage to having found this it is just a matter of simple multiplication and then creating for the (FL) so 10x4+1, so that will be this number. So this has to be taken as it is, then 41x1+10 that will be this number, so 41 has to be taken as it and then 51 will be taken as it is. So 51x1+ 41 is 92 over. So all the value of (FL) so finally having reach this stage what is to be done take the last number 92.

And then it was said if you look at the verse so he says (FL) so this is basically dividing by (FL) so 92 has to be divided by 29, so the portion will be 3, define so having obtain this we have a algorithm, so when we look at this, so the last thing you have to divide by e, (FL) so when you do that so this basically saying 92 has to be divide by 29, so it has 3 as portion and 5 a reminder. So whatever is the remainder is basically x ok.

So now let us see this equation, so this is x so n the given number n is 49 times x, so +7 and the shows y also has 8 so finished, so x is 5 and y is 8, so one thing which we need to understand with reference to this algorithm is so once you have one solution you have infinite number of solutions.

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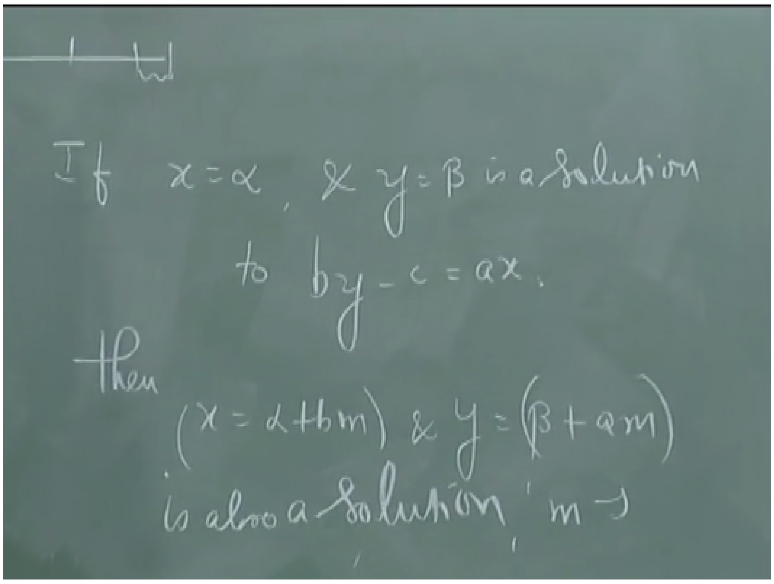

So this is pretty evident se for instance if x-alpha and y=beta is a solution **is a solution** 2 the equation that we had was by-c=ax then so x=alpha+bm and y=beta+am is also a solution m is any integer, so this can you easily check. So where m is integer,m so any choice of m has to satisfy and therefore it is pretty evident that we have infinite number of solutions to this fine, this is all the algorithm all about kuttaka algorithm.

So now I have to need kuttaka algorithm and then move on, in fact what we have done is later of course brahmagupta has discuss this, Aryabhatta is the first to discuss this kuttaka algorithm, so brahmagupta has also discussed this, so Bhaskara has done, Mahavira has done, so all was later have provided certain kind of modified version of the kuttaka and those will be covered when we discuss those text.

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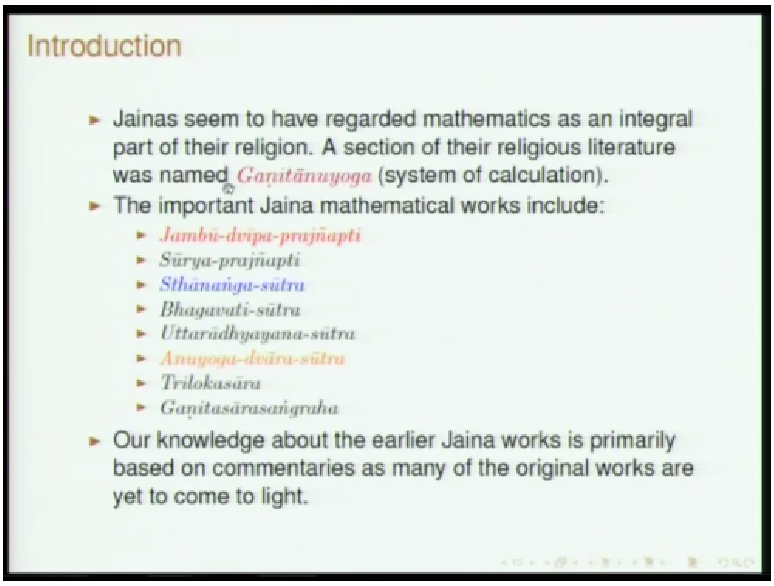

Now I move on to give you a flavour of how this Jaina mathematics has been before of course this mahaviracharya Jaina tradition. In Jaina tradition it seems that this mathematics was also thought of considered as a part of their religious literature, so (FL) we have this jothisum as a part of vedhjangam so in fact it is called vedhanga jyotisha, so basically it gives some kind of mathematics.

So there it I think it is in a much more in fact they say there is a section called (FL) in the religious literature itself. So some of the ancient Jaina mathematical works are listed here (FL) is a nice work. So all (FL) of early Jaina works are primarily based upon commentaries are some of the original work and not even come to light. So these 2 are many of these (FL) was very unfortunate that even today that all the sophistication that we have.

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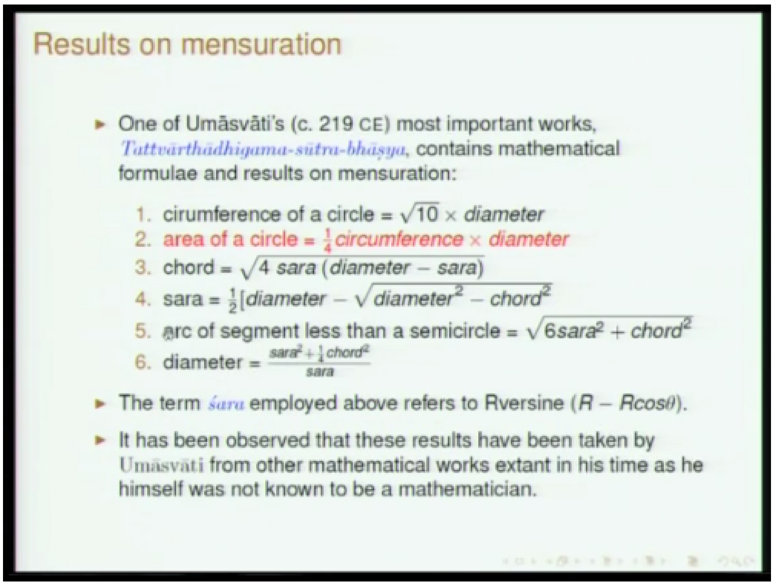

So not much k has been taken to preserve this manuscripts ok. So results of mensuration I just list a few of them, if I have information the (FL) we do not have good manners clips Aryabhattiya fortunately we have plenty of (FL) and its commentary (FL) is only one and even that is not acceptable. So that is how things are, so it is a very sad state of affairs. Anyway so let us come to this Jaina literature.

And some of this formula related to mensuration, so I am just listening here, so for instance they say the circumference of circle is root 9, root 10 x metre. So in fact brahmagupta out of uses this value. So this is somewhat (FL) as you know compare to the value which has been specified by Aryabhatta. Anyway but for practical purposes so this is how they have been using this. So then area as a circle.

So you one fourth circumferencexdiamter, so this is all right but the value of circumference is not by that great, so then there is a chord and today morning we had a discussion on great length of the Aryabhatta Sutra right, (FL) so that is all this is, so basically the product of chord they are say so (FL) so this is how it will be.

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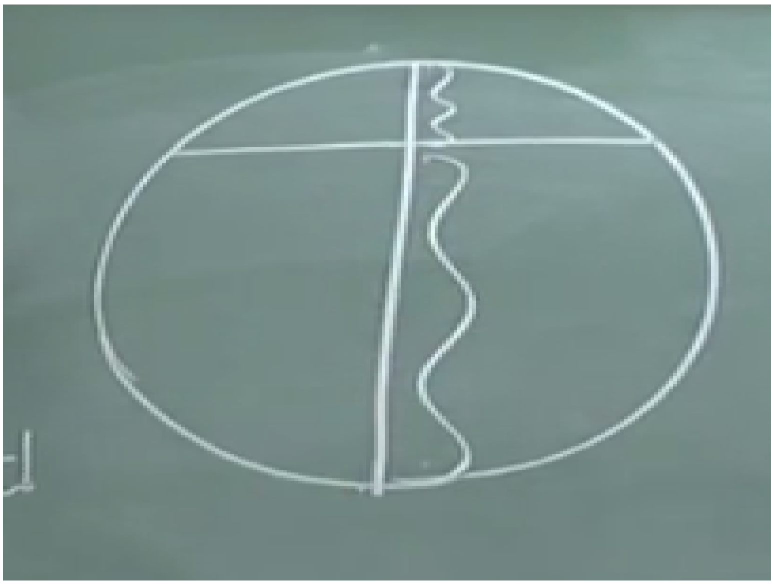

If you consider this as chord, this is one (FL), this is another (FL), so (FL) as far as this is (FL) bow and you consider this as bow, this is (FL) so the product of this will be the same as a product of these two. So that is what it amounts to, so this stated in different way, but that is what it amounts to. so that is why 4 comes here, so then sara see once you know the diameter on chord you will be able to make the sara. So which this a cycle in certain way.

And there is various approximations which people have been trying see to obtain sara given this (FL) sara and so on, there are various formulas has been given by Jaina mathematicians. So by sara so if you should understand that basically r-rcos theta, ok.

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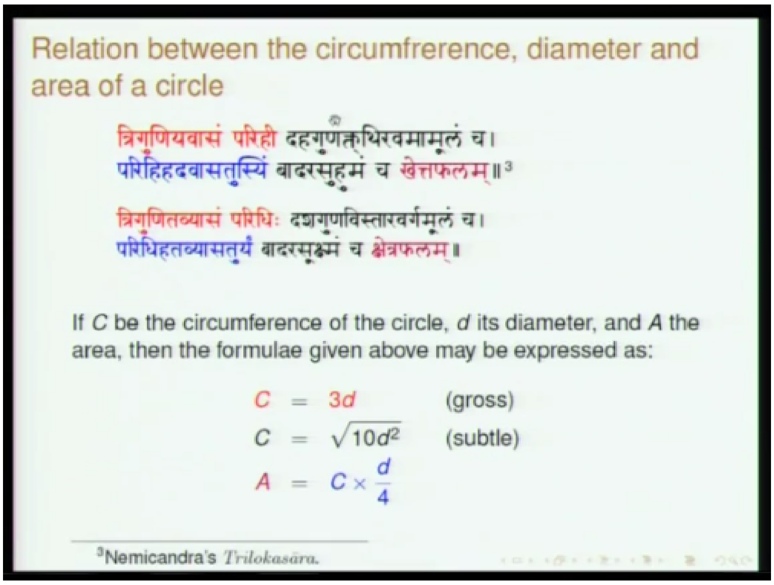

So give you a flavour of how the original verses are, so I have just coated 2 verses, so this is may be in (FL) but definitely not in Sanskrit and I have tried to make a Sanskrit rendering of this, so that we try to understand. So see here (FL) so this is (FL) ok 3 times the diameter (FL) is circumference ok, so c is 3xd, then (FL) so this is (FL) 2 expressions, one is (FL) the other is (FL) 10 multiply by (FL) is diameter ok.

So (FL) d square, so root of 10 times d square is also that is what they have stated 10 times the diameter, so this the expression for the circumference once you know the diameter, so these are the 2 different (FL). Then (FL) in philosophical literature we will see there are 4 states, so one is (FL) and they call (FL) is the fourth state. So (FL) means the sense here uses one fourth understand.

So (FL) is multiplication (FL) is diameter (FL) one fourth of that. So that gives the (FL) ok (FL) what is a bit difficult to (FL) more accurate ok, so (FL) so this is the form in which we find some of these verses have been cited by Bhaskara also in his commentary 2 Aryabhattiya and most of the earlier (FL) in this language which not for different from Sanskrit, but it requires a certain familiarity to figure out it.

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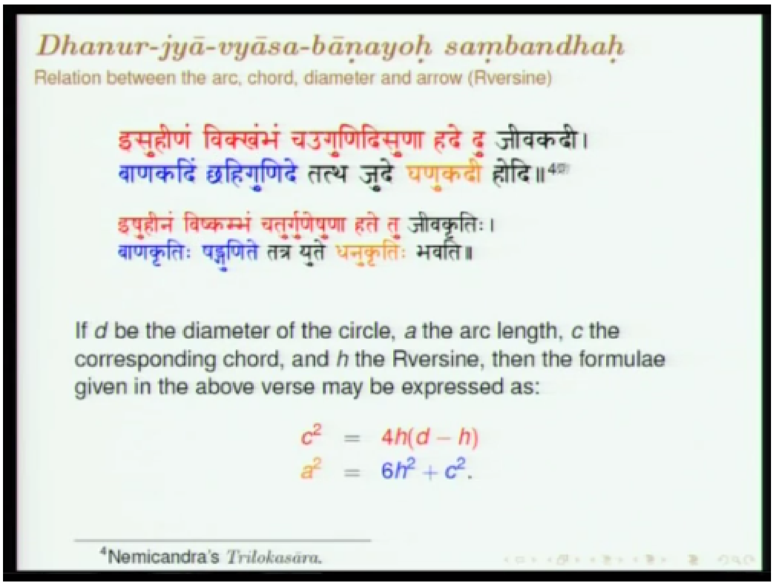

And this literature (FL) similarly one more was so (FL) so if you look at this, so if you know the formula it easy to square root, otherwise it is not so easy, so this (FL) so multiply by 4 and (FL) suppose this issue (FL) then this (FL) should be (FL) so chord square, we can have this approximation and that is what he says, then (FL) arc square this, so this all sudden approximations, so which have been presented in Jaina works.

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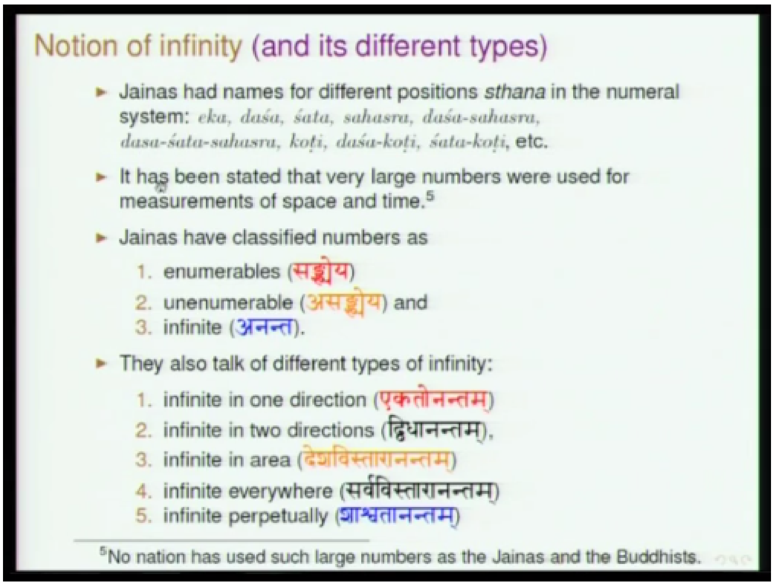

So finally so this Jaina also has this notion of infinity express in 20 different ways in fact they speak of different kinds of infinity the countess motion of infinity in our mind and then what they have spoken I mean that is not a good way then what is interesting is so they have spoken of different kinds of infinity ok. So we today we have to infinite number of infinity in case, so that is a different way.

So we have gone much more advanced way of analysing infinity, here so they actually classify the number into (FL) is numerable (FL) is unnumerable and then (FL) infinite and infinite also they say (FL) and then (FL) infinite in terms of area one direction, 2 direction and so on and infinite everywhere (FL) so this is true perhaps is the something is eternality fine, so eternality in terms of space in terms of time, in terms of direction.

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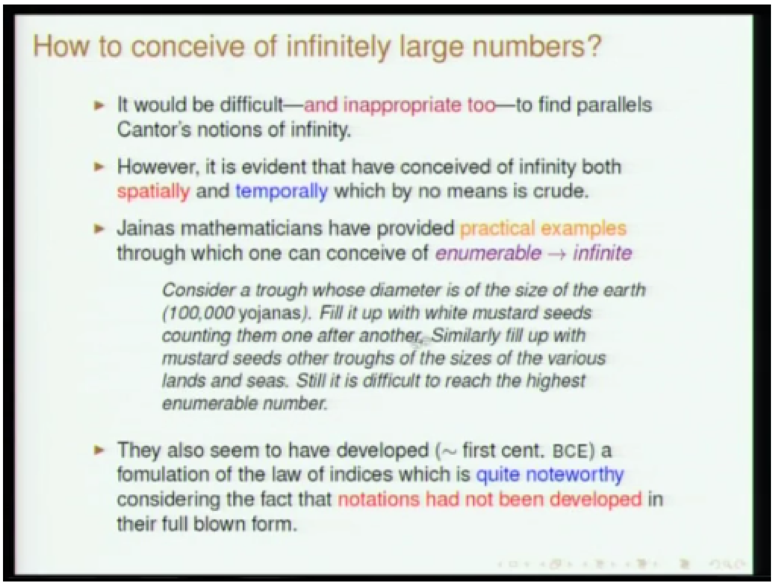

Fine this are classify using various ways and some interesting examples which Jaina have chosen to convey what infinity is, so for instance they say consider a trough whose diameter is of the size of the earth ok, so considerate a trough so which is 100000 (FL) so fill it up with white mustard seeds, so feeling a tough of the size of the earth and then the mustard seed you have to fill it up and keep counting them.

Similarly fill up with mustard seeds other trough of the size of various lands and seeds, ok, so one is size of the earth still it is difficult to reach the highest numeral. So this just give you a certain conception of what we are talking about and we see large numbers, ok so then we have also this (FL) very interesting thing. So (FL) discuss about logarithm also, but it is very difficult to figure out.

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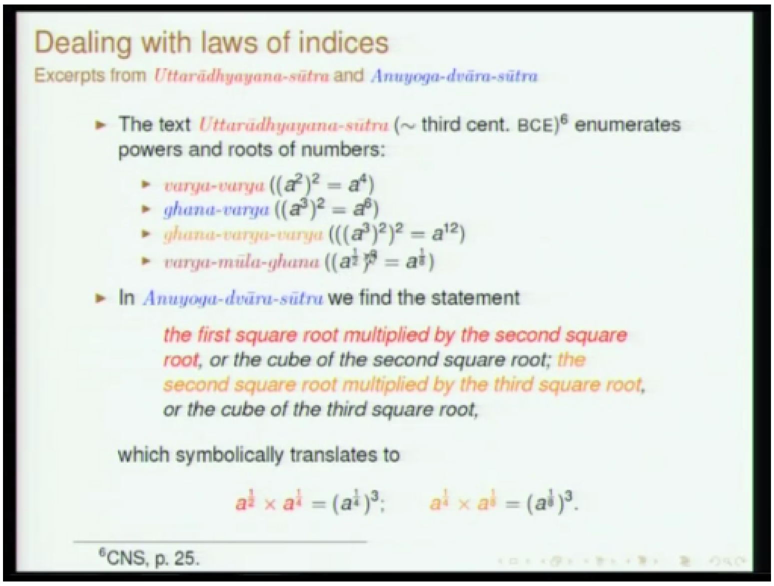

See first of all I must admitted I am that conversion and secondly anyone could additions and interpretations very very clearly written things are not available, certain articles are there, but it is also based on certain other countries. So point I want to convey is so they (FL) so all that they discuss about various kinds of indices. So it is in this connection, so people say that they also talked about this logarithms.

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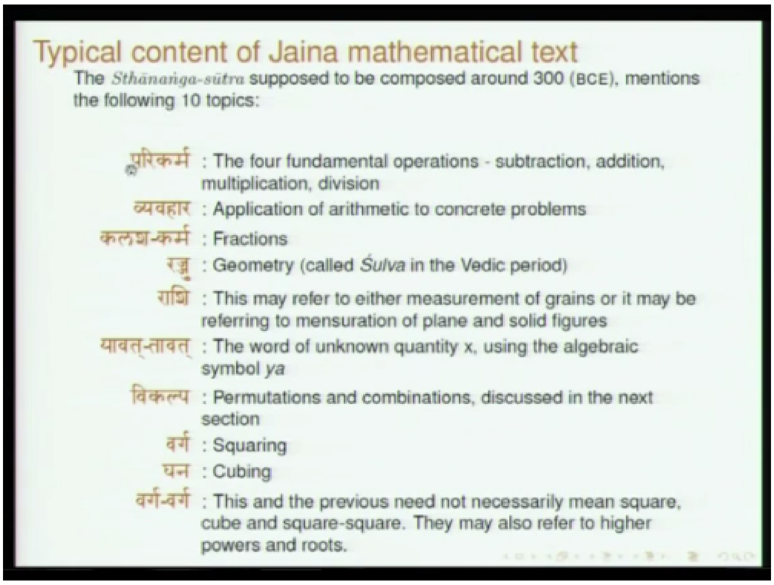

And I just skip this and to give you an idea so what are the various topics which all discussed so in any difficulty in a text (FL) so this might you have seen (FL) professor Sriram had discussed about mahaviracharya, so (FL) combination so were gone so on, so with this I conclude my session on Aryabhatta as well as Jaina mathematics ancient Jaina mathematics ok. So then you will have discussion on brahmagupta and so on, thank you.