Mathematics in India: From Vedic Period to Modern Times
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Lecture-1
Indian Mathematics: An Overview

Good morning, this is the first lecture of this course, which is being given on mathematics in India from Vedic period to modern times, it is a novel course which tries to trace the way mathematics developed in India, the first talk is an overview talk. In this I will try to highlight those periods is there was a significant development of mathematics in India. I will also try to summarise the special nature of athletics as a developed in India. I would like to emphasize the algorithmic way in which most problems in mathematics was considered in the Indian tradition.

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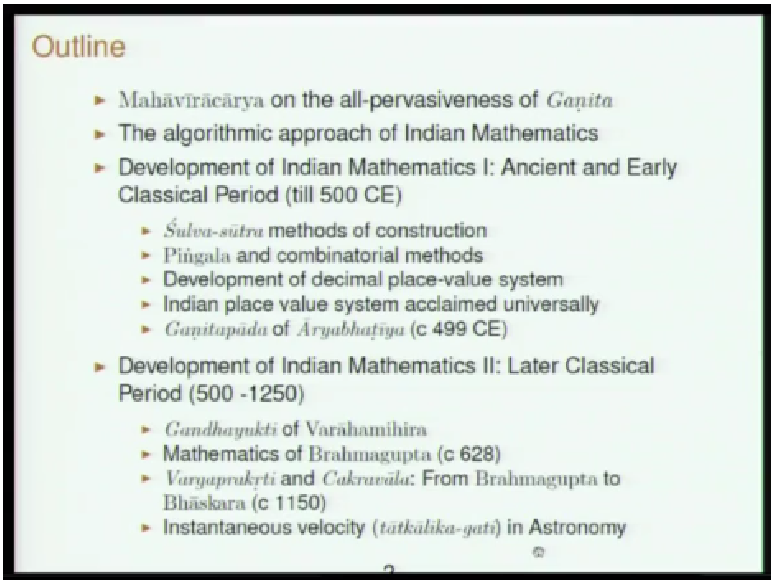

So I am flashing the outline not going to read it out we can just see the kind of topics we are going to follow up , we will cover the development of Indian mathematics in the ancient period indicate some highlights during that period. Then the early classical period say 500 BCE to 500 CE which culminated in the birth of Aryabhatta. Then the development mathematics in the latest classical period from 500 in the common era to 1250.

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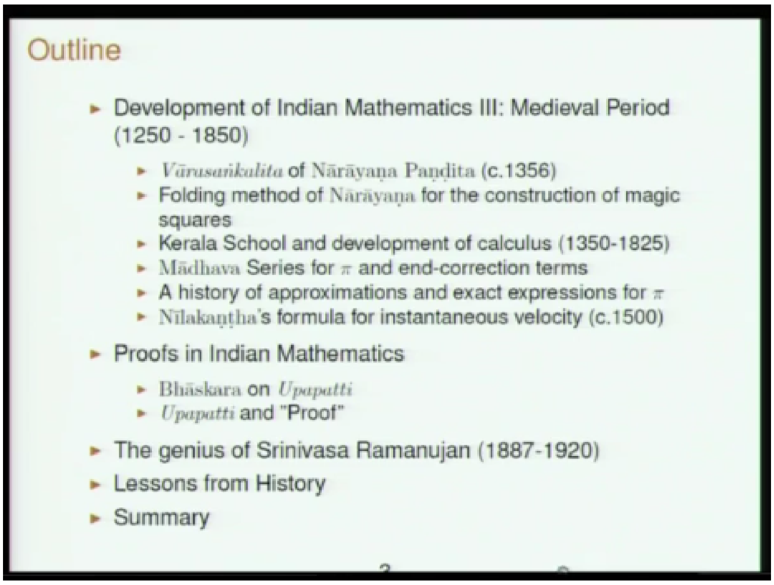

We will then go to some uses of the highlights of what happened during the mediaeval period till about 1850, towards then we will discuss something about the nature of mathematics in India how mathematics, how was results like that and then about the contemporary period we will speak a little bit about the Srinivasa Ramanujan.

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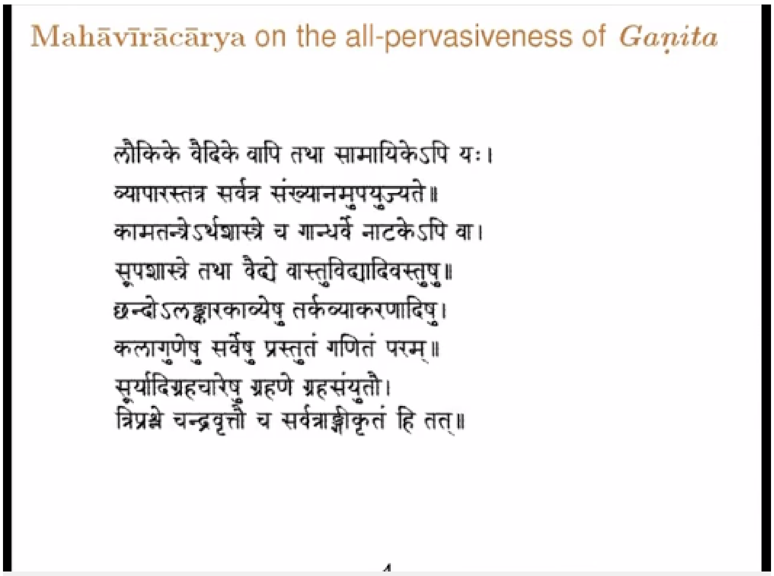

Finally know that history, so some find out pervasive mathematics in India is this following statement from the Ganita Sara sangraha of mahaviracharya, if along six packs statement again when mahaviracharya, it got by saying that mathematics is important in all areas when finally concludes by saying (FL) (()) (02:30) to (()) (02:36) that is not provided by mathematics.

So that is kind of statement that mahaviracharya is beating, weather is it in Astronomy or beat in architecture or beat in conjunction of granite position kind course of moon logic quite the grammar, so it says all purpose statement of mahaviracharya that mathematics provides all aspects, all subject this quotation from the 9 century world called Ganita Sara sangraha or mahaviracarya.

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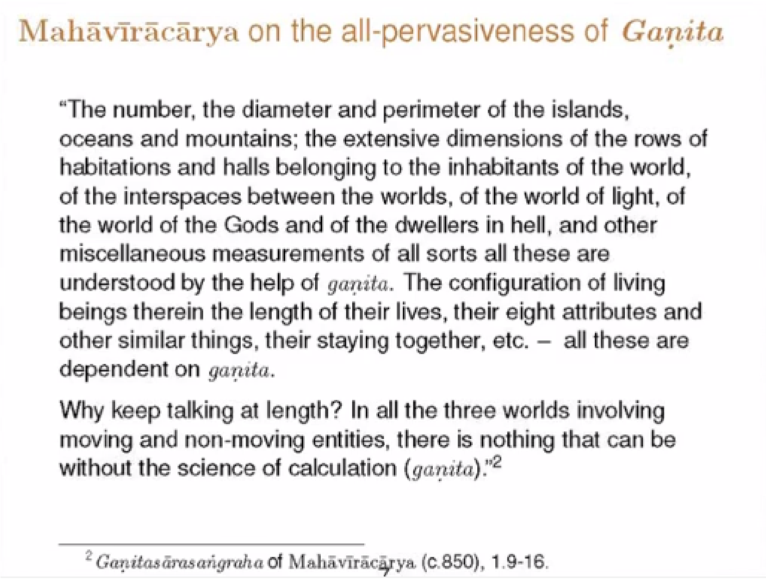

So Ganita stands for calculation competition (FL) (03:22) to (()) (03:25) is a statement due to Ganesa Daivajna is a commentator of Lilavati.

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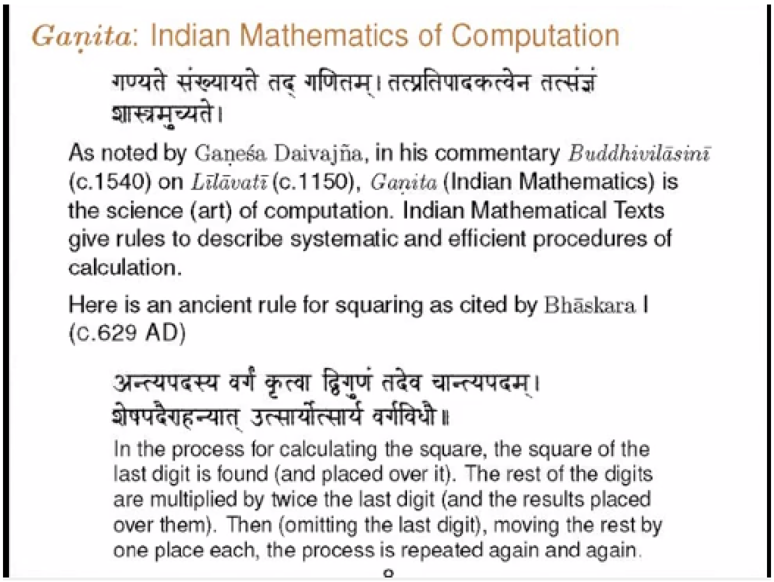

And therefore we can expect that Indian mathematical text really abound in rules to describe systematic and efficient procedure for calculation. Just to give you an example we will go to a very ancient rule this is given by Bhaskara I, this is in this commentary to Aryabhatiya, it is just a rule for calculating the square of a number (FL) (()) (03:59) to (()) (04:10) so we can see the kind of calculation was talking about.

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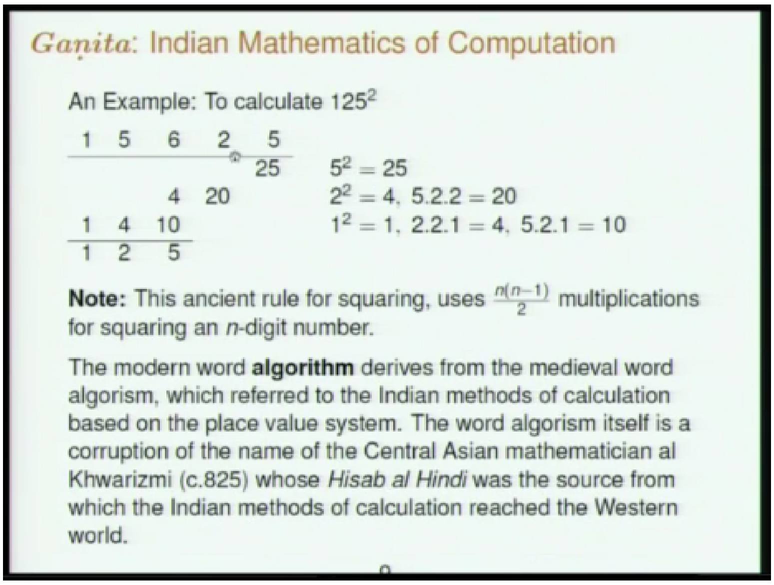

To take any number 125, first you square the last number multiply the other numbers by 2 and the last number. So you get this row. Then move away remove one digit square the next number multiply by 2 and that number the next row and finally square the last number, remove one number, add all of them. The important thing to realises even this very ancient rule written in 1619 AD.

Actually uses n*n-1/2 multiplication to calculate the square of a number, a n digit number multiplied by another n digit number we will n square multiplication, but since we are squaring the same number is 2ab like that comes in and so you are having an optimal algorithm for square. This is the Indian mathematicians always right to give the best possible way of the best possible procedure for doing a calculation.

Now the algorithm itself as a history it was the name given to the Indian methods of doing calculation when we coordinates when the name of Central Asia Mathematician call (()) (05:29) who in the ninth century wrote a book on Indian methods of calculation that is methods of calculation using the decimal place value system and that book was called algorithm Latin version of that book is available.

The original Arabic version is not available. And this was the book that introduced the decimal place value notation to the Arabic world and later on to the European world and so the people who followed this was calculation were called algorithm and the algorithm comes from (FL) and this algorithm factor is not something very specific to mathematics impact it provides all Indian Sciences.

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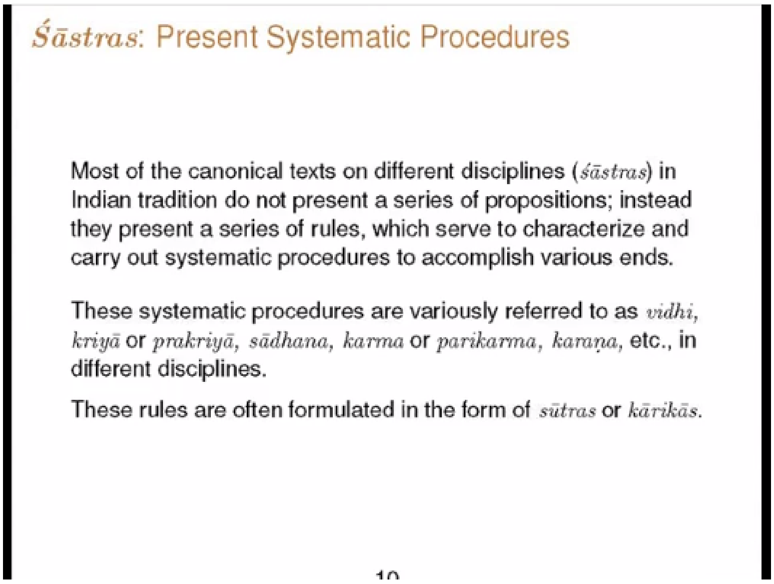

Most of Indian disciplines sastras as we call them, they do not present a series of propositions, they normally give you a set of rules, a set of procedure which tell you how to systematically accomplish something. So the rules given in sastras are usually called as vidhi, kriya, prakriya, sadhana, parikarma, karana, these are the names and these rules are what are usually formulated as Sutras.

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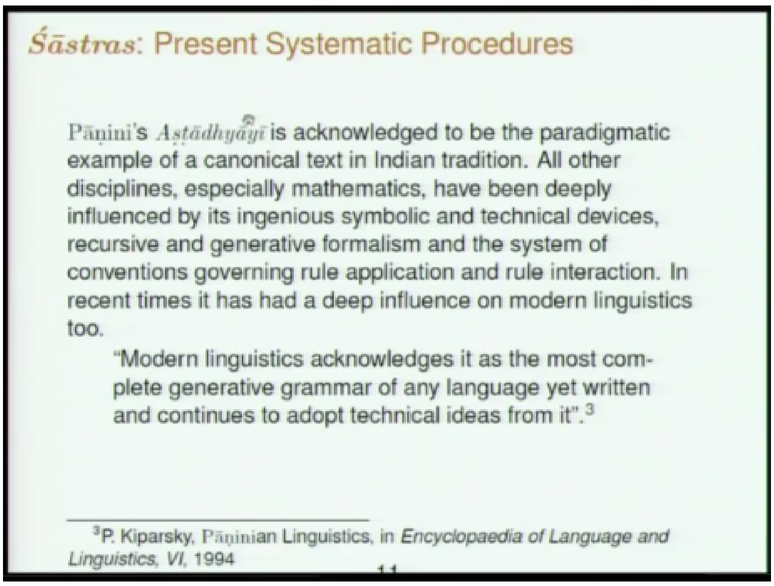

So disciplines the ethical disciplines in India they provide systematic rules of procedure rather than a set of propositions. And the for the most canonical such systematic text in India is the great grammar written by Panini called Astadhyayi. In fact most other disciplines and especially mathematics is extensively influenced by the method of Panini. Please use symbolic and technical devices recursive and generative formalism.

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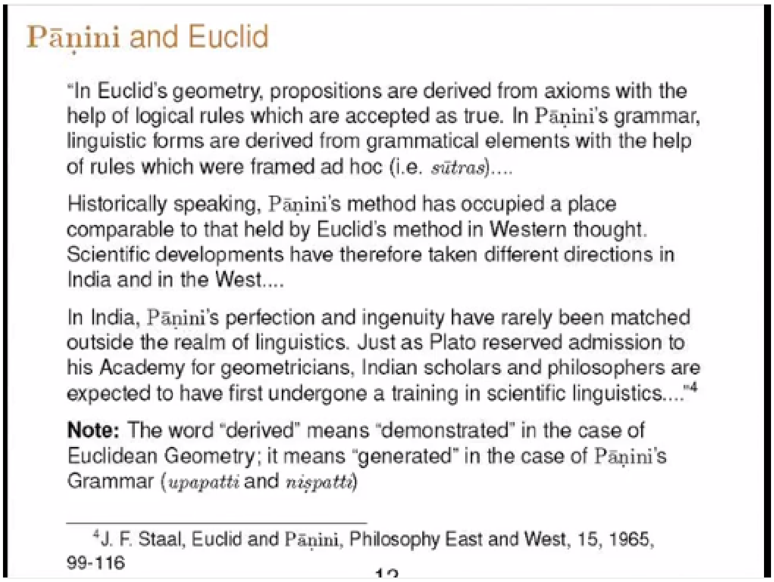

And this system of convention that govern rule application and rule interaction all these go back to Panini and it has deep influence you are not the modern discipline of linked list. In fact many scholar actually acknowledge that place Panini holds in Indian tradition plays taht something analogous to the place you could hold in the Ecuclidean tradition and here is a quotation from stall where he saying Panini is also dividing systematically Sanskrit occurrences from a set of rules.

And Euclid is also deriving a set of propositions from a collection of axioms, but the world deriving will have 2 different mean means in this 2 context, Panini is actually generating valid occurrences of Sanskrit is not proving theorems. Euclid is demonstration is proving theorems in mathematics from a set of postulates.

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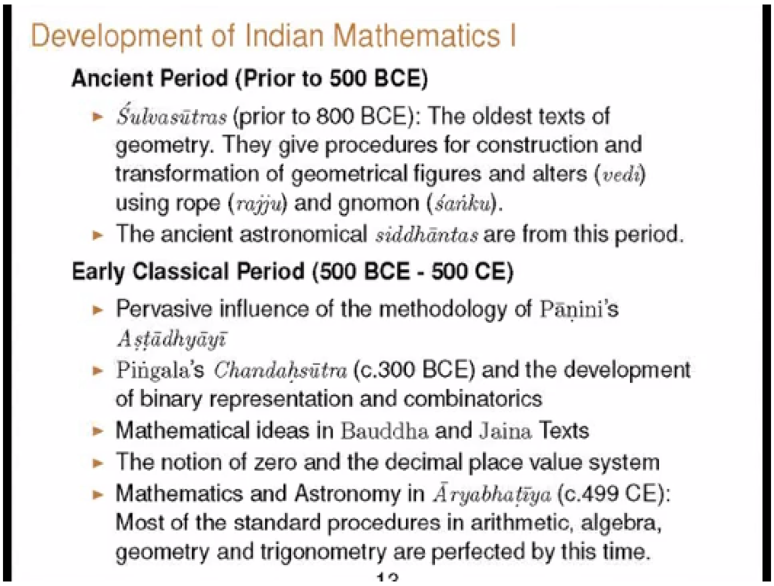

So in ancient period the ugliest text in mathematics available are the text on construction of higher alters the vedis. These are the sulvasutras, these are the oldest texts of geometry in teh world. They give procedures for construction in transformation of geometrical figures. Then there are ancient astronomical siddhantas which deal with astronomy. When we come to the classical period starting from Panini we then have the chandahsutra of Pingala which initiated combinatorics.

We have some mathematics in the Jaina tradition in the Jaina. Then more crucially the idea of 0 and decimal place value system developed in this period and all these terminated in the mathematics and astronomy that is found in the text Aryabhatiya Aryabhatta which was written in 499 of the common error, most of the standard procedure in Arithmetic algebra geometry trigonometry were perfected.

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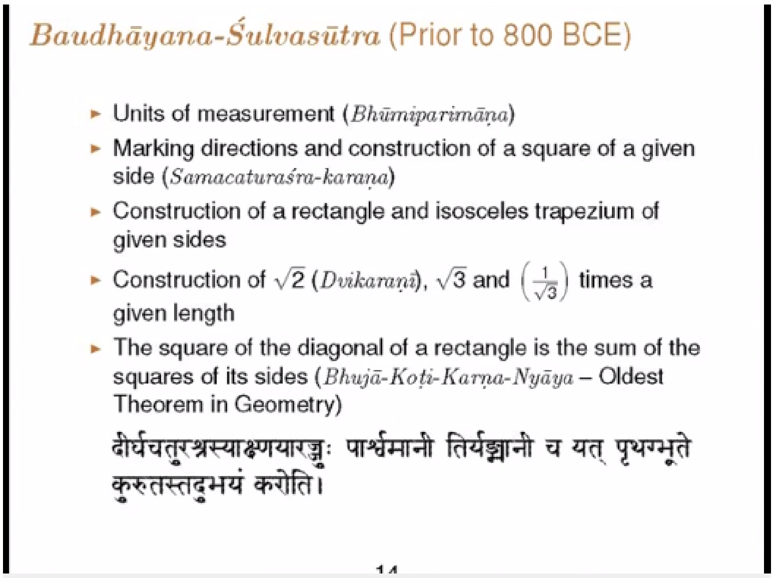

And many more things which was used in astronomy like the indeterminate equations sign tables, all these things were perfected by the time of Aryabhatta. So the ancient sulvasutras deal with lot of things, units of measurement, marking directions, construction of rectangle, square, trapezium, transformation of square, and it has the first oldest statement of geometry the theorem which we attribute commonly, it is called the Bhuja-Koti-Karna-Nyaya in later Indian mathematical text.

It is the sum of the two sides of a rectangle, the square, sum of the squares of two sides of a rectangle is equal to the square of the diagonal. This is the rule I stated in Baudhayana sulvasutras (FL) (()) (10:18) to (()) (10:27).

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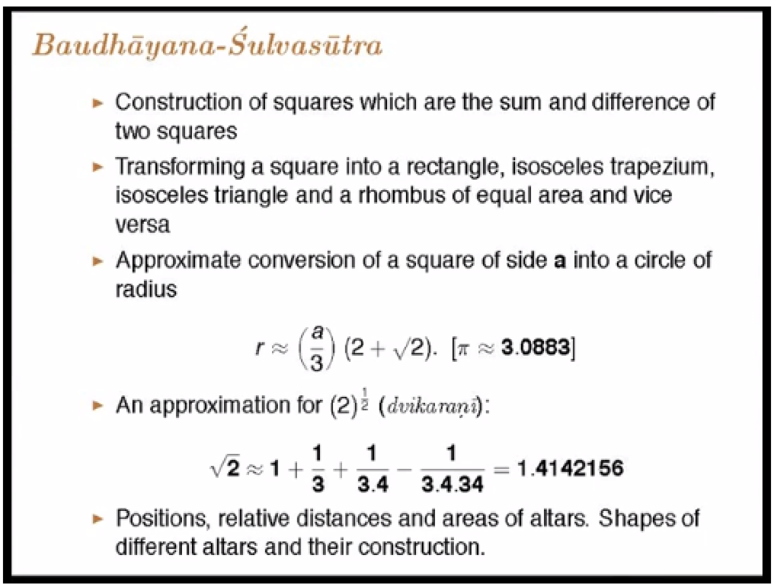

There are even more complicated rules of adding squares, then there is a rule for approximate conversion of square into a circle which leads to a value of 5 around 3.08. Then there is a very interesting formula for square root of 2 is called dvikarana in it is accurate of 2 several decimal places that you can see. Finally all this geometry is used in constructing all types.

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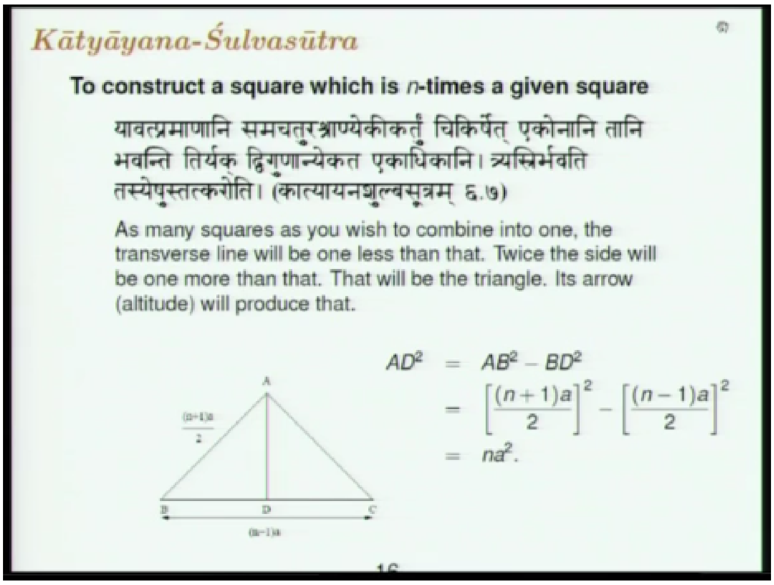

This is the rule in Katyana sulvasutra the problem is how to construct a square which is n times the area of a given square and Katyana sulvasutra gives a very interesting geometrical formula n+1a/2 whole squared-n-1a/2 whole square is na square. It is using this very interesting algebra it result to calculate the side of a square which is n times in area of the given square.

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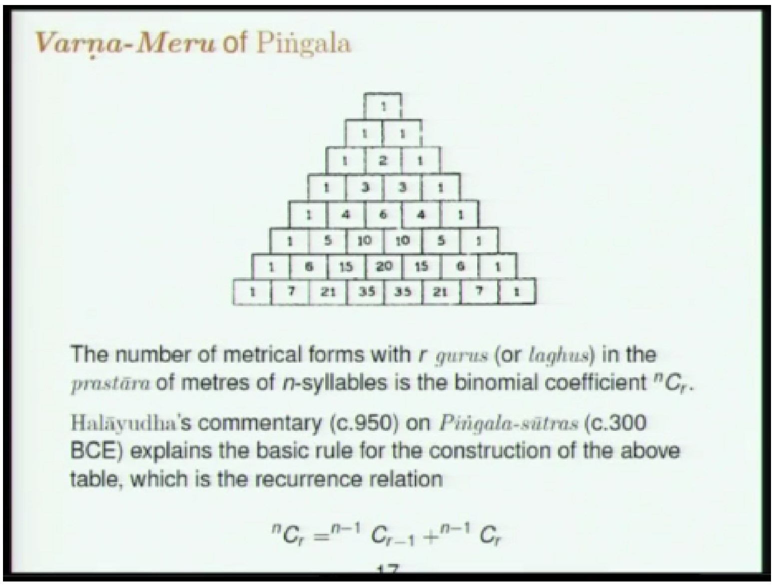

Pingala sutra are the combinatorics and these are a very interesting diagram known as the Meru prastara which appears in Pinglas (FL) it gives you the what we now call as the binomial coefficients NCR. They arise very naturally when you want to count how many metres are there which are n syllables but in which are number of groups appears that is NCR as we shall see later.

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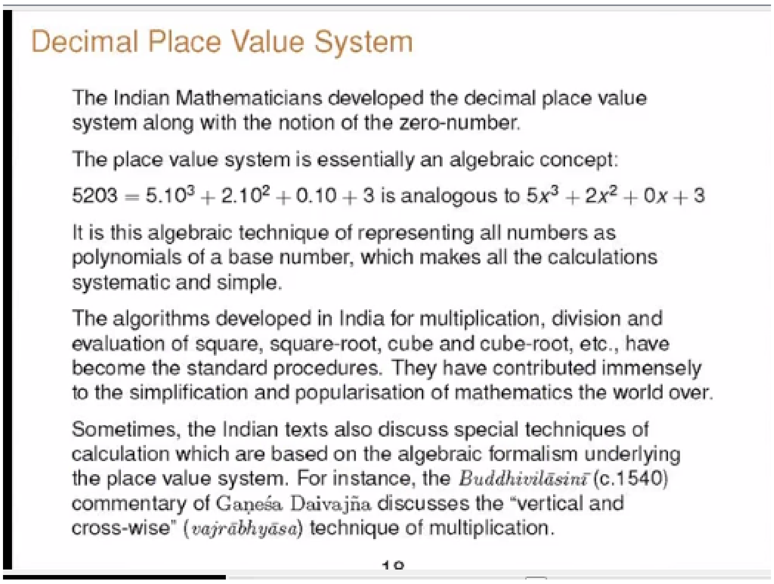

The decimal place value system arose in the ancient period the main thing about the decimal place value system is that is an essential in algebraic concept, the number 52038 written as 5 times 10 cube, 2 times 10 square and 0 times 10+3 is something I think to a algebraic polynomial 5x cube 2z+square+ 0x+3, it is this algebraic future of place value system that enabled the Indian mathematicians to give systematic and very interesting procedure for making calculations.

And they became the standard methods of calculation all the world over. Sometimes the Indian books do give some special techniques also which are essentially originating out of the place value systems, for instance the forming the buddhivilasini of Ganesa Daivajna Lilavati. It discusses what is currently popularly known as the vajrabhyasa method of multiplication vertical and cross wise method multiplication.

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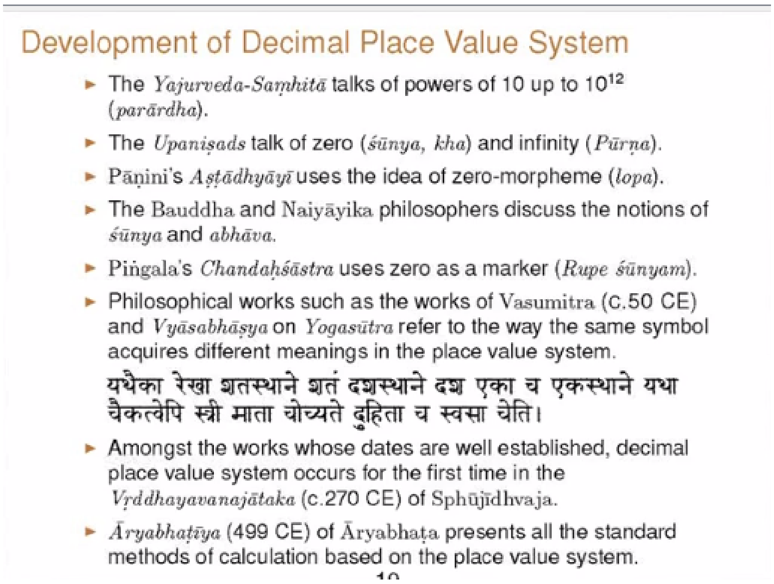

The history of decimal place value system goes back to the Vedas, they use the system to the base 10 very naturally. The upanisads talk of zero and infinity panini's Astadhyayi has a notion of lopa which is I think what is called as (FL) this idea of (FL) in Bauddha philosophy, the idea of abhava in the naiyayika philosophy. Pingala's Chandahsustra uses a 0 as a marker which not a clear whether at that time the idea of 0 as a number was no.

Now soon enough the idea of place value system became so common that philosophical works such as vasumitras with this text and even (FL) and yogasutra started explaining the speciality of the place value system. There is a quotation from the vyasabhasya on Yogasutra (FL) (()) (14:00) to (()) (14:14) just as NAD is understood as a mother, daughter-in-law or a sister LI which appears at different places that is number 1 which appears at different places will have different values and they got 10.

So like this, this issue became well known in the circles of philosophy also and got discussed and one of the oldest place value system explicitly is in a book called Vrddhayanajataka written by Sphujidhvaja around 270 in the common era and by the time of Aryabhatta Aryabhatiya all calculations are formulated with the place value system.

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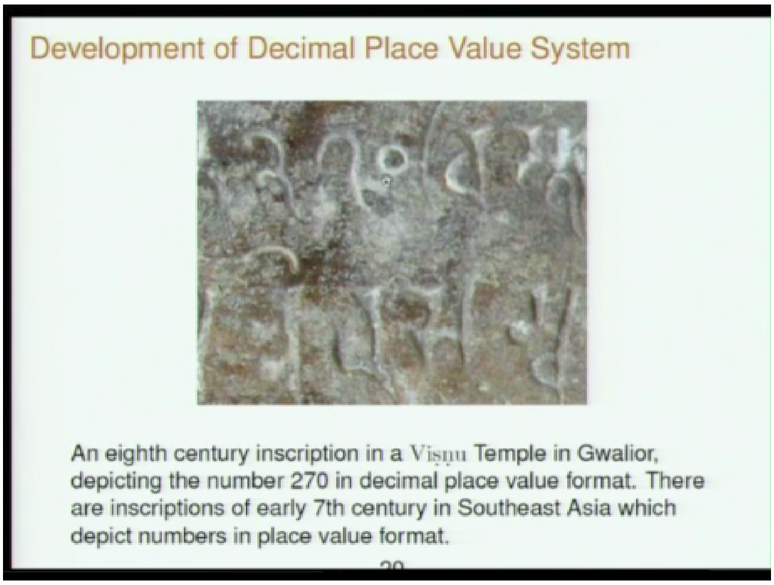

Her an inscription in Gwalior which is giving the number 270 it means it is appearing here and the many other inscriptions in southeast Asia in Gwalior in various other places around early 7 century which give numbers in the place value system with 0 also.

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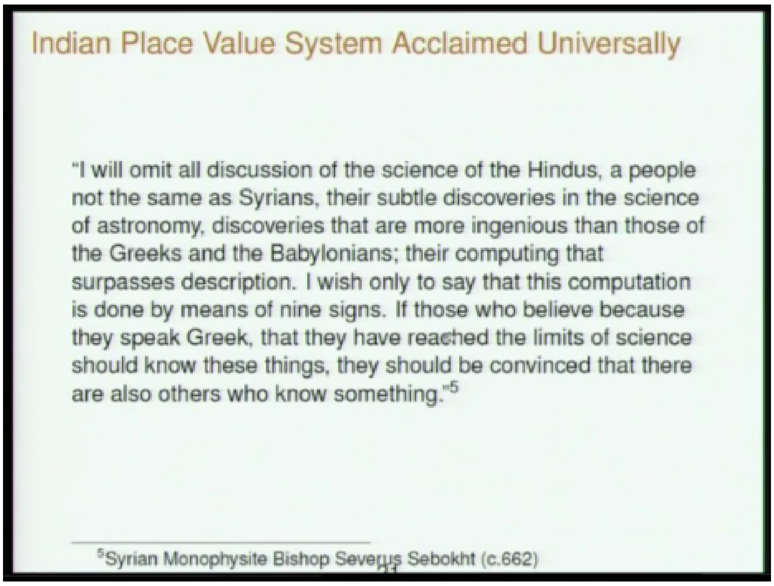

Now this Indian place value system acclaimed universally this statement in the 7th Century by a Syrians (FL) who is this out of saying that the Greek seem to think too much of themselves but they really do not know the basic methods of calculation that Indians have discovered and they better know that the others also know something of science.

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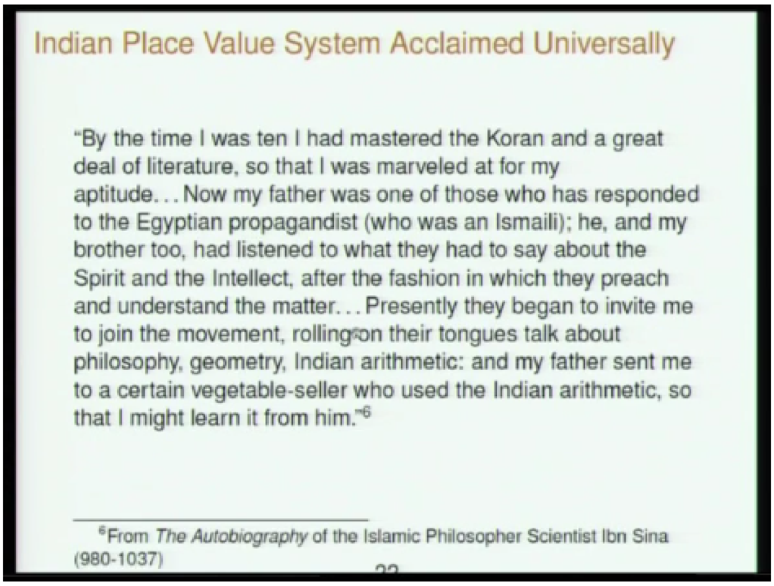

Here is (FL) a very famous philosopher in Asian region in 10th century he is saying that he learn the methods of calculation the Indian methods of calculation from a vegetable vendor. So this was the day that the place value system really revolutionize calculation all over the world.

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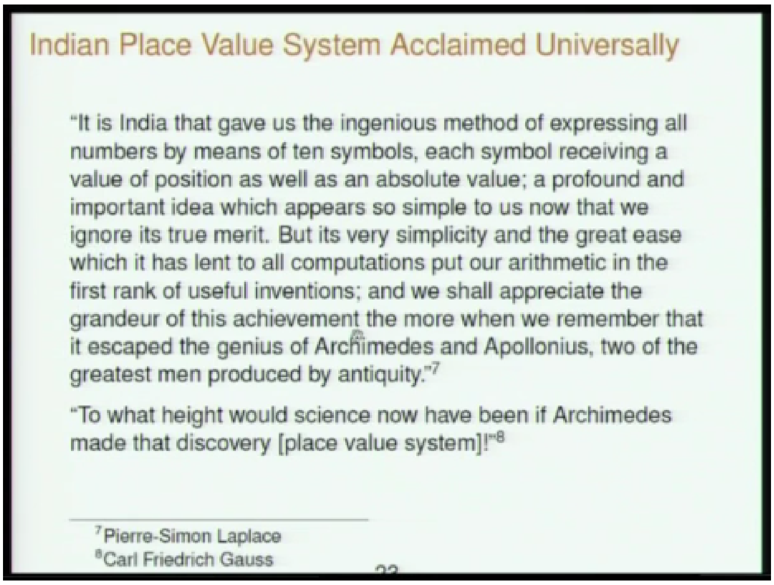

This more modern quotation by Laplace and Gauss saying that this is indeed one of the most wonderful discoveries in the history of mathematics.

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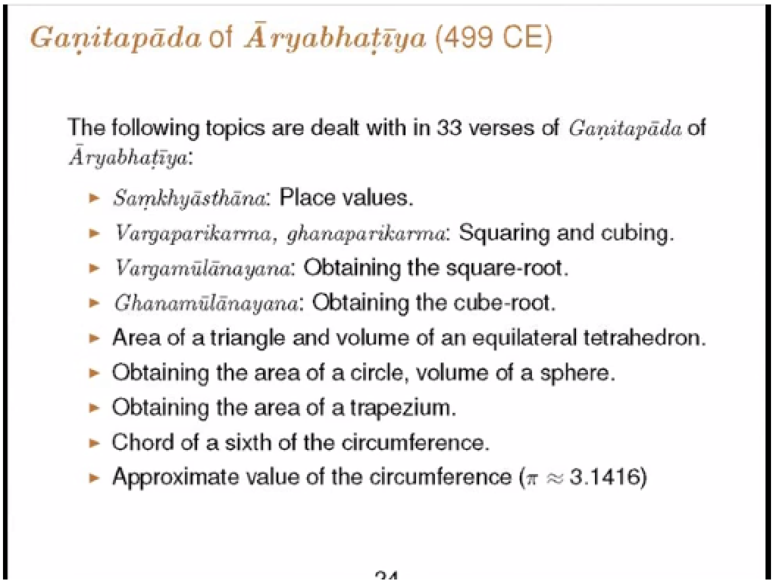

Now by the time when you come to Aryabhatiya in 500 CE, it discusses the what is called as what is called as parikarma logistics methods of calculation where square, square root, cube, cube root, areas of triangle, circle, trapezium, approximate value of Pi computing sine tables.

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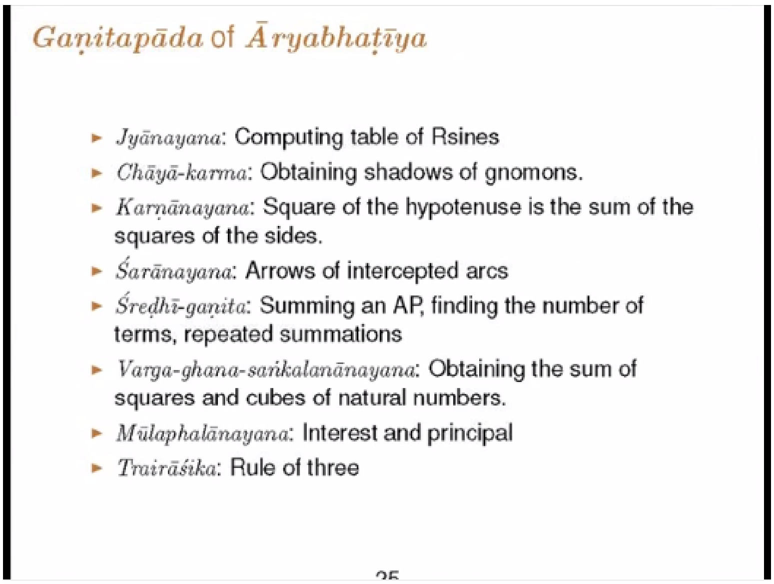

Problems to do with interceptor arch in a circle, progressions, rule of three.

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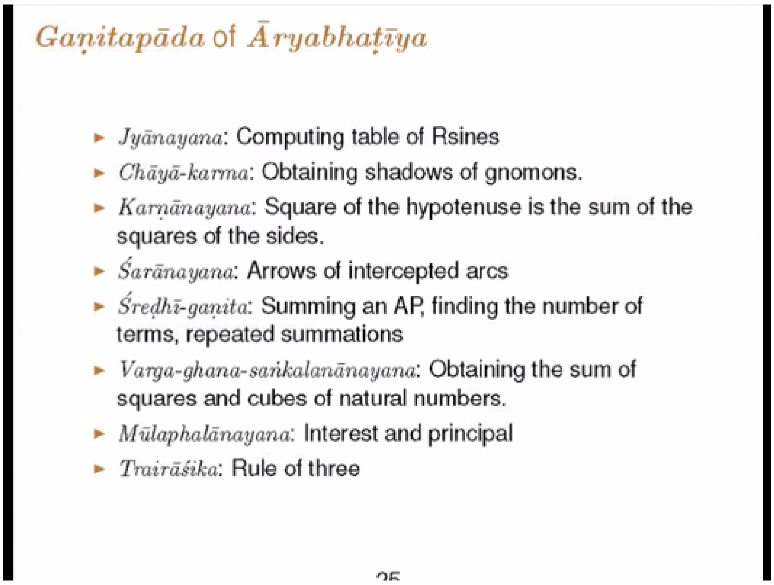

Arithmetic of fractions and finally something very interesting called as kuttakara which was Aryabhatta one invention is the method of solving linear indeterminate equation.

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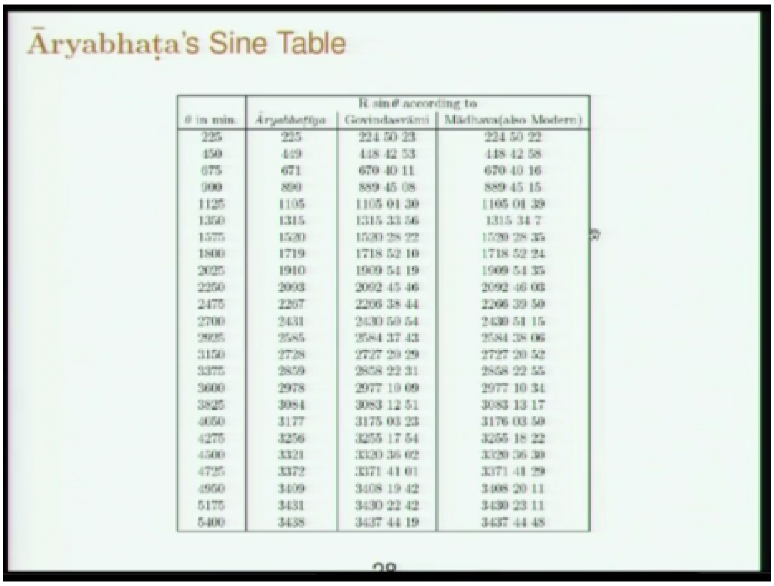

θ in min.	R sinθ according to Āryabhaṭīya	Govindasvāmi	Mādhava(also Modern)
225	225	224 50 23	224 50 22
450	449	448 42 53	448 42 58
675	671	670 40 11	670 40 16
900	890	889 45 08	889 45 15
1125	1105	1105 01 30	1105 01 39
1350	1315	1315 33 56	1315 34 7
1575	1520	1520 28 22	1520 28 35
1800	1719	1718 52 10	1718 52 24
2025	1910	1909 54 19	1909 54 35
2250	2093	2092 45 46	2092 46 03
2475	2267	2266 38 44	2266 39 50
2700	2431	2430 50 54	2430 51 15
2925	2585	2584 37 43	2584 38 06
3150	2728	2727 20 29	2727 20 52
3375	2859	2858 22 31	2858 22 55
3600	2978	2977 10 09	2977 10 34
3825	3084	3083 12 51	3083 13 17
4050	3177	3175 03 23	3176 03 50
4275	3256	3255 17 54	3256 18 22
4500	3321	3320 36 02	3320 36 30
4725	3372	3371 41 01	3371 41 29
4950	3409	3408 19 42	3408 20 11
5175	3431	3430 22 42	3430 23 11
5400	3438	3437 44 19	3437 44 48

So this is the kind of sine table that Aryabhatta came up with and details later on being systematically including India, this is by Govindaswamy in 9 century and this improved table is due to Madhava.

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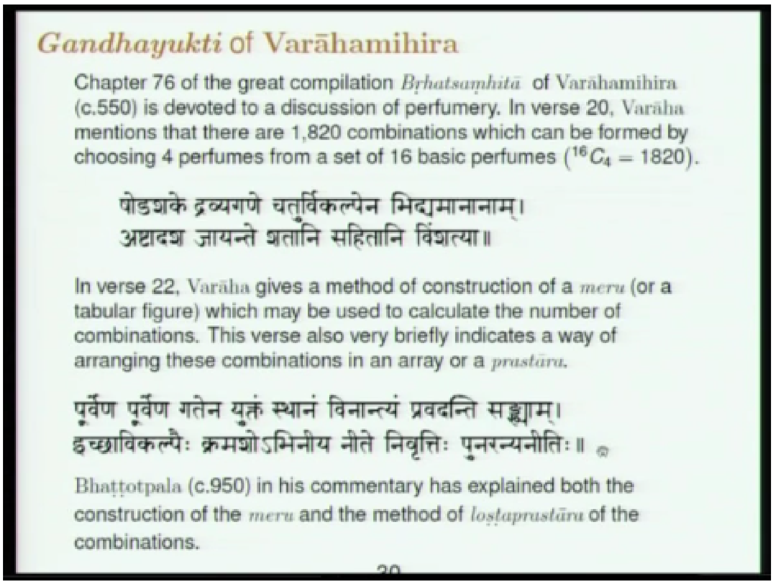

When we come to the later period we have luminaries like (FL) (()) (17:20) to (()) (17:25) Aryabhatta, brahmagupta one of the most celebrated 25th in India.

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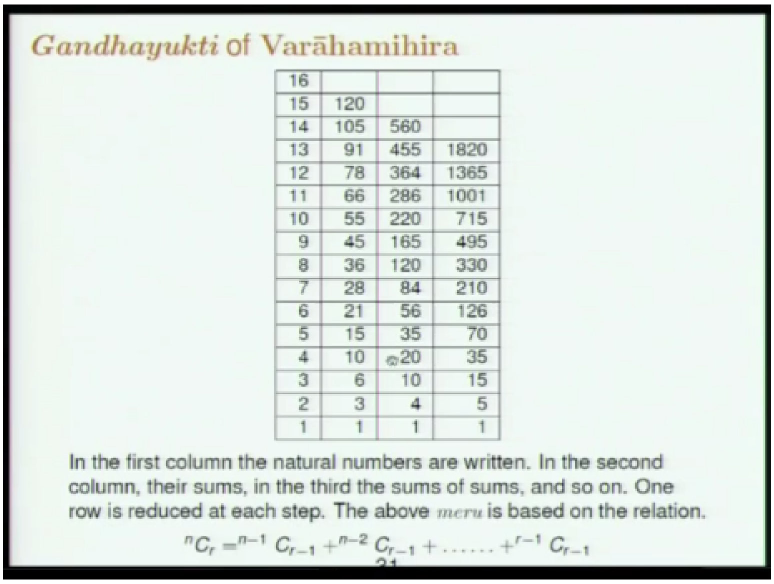

In varahamihira which is a compendium there is a chapter on human and there he is introducing combinatorics idea and he is explaining that 1820 various perfumes can be formed by choosing 4 out of a collection of 16 and to calculate the 16 C4 he gives a different kind of (FL) he is giving a different kind of a table here the first column natural integers, the second column some of natural integers.

The third column is the sum of sums of natural integers, fourth column is and its based upon the reconciliation which is equivalent to (FL).

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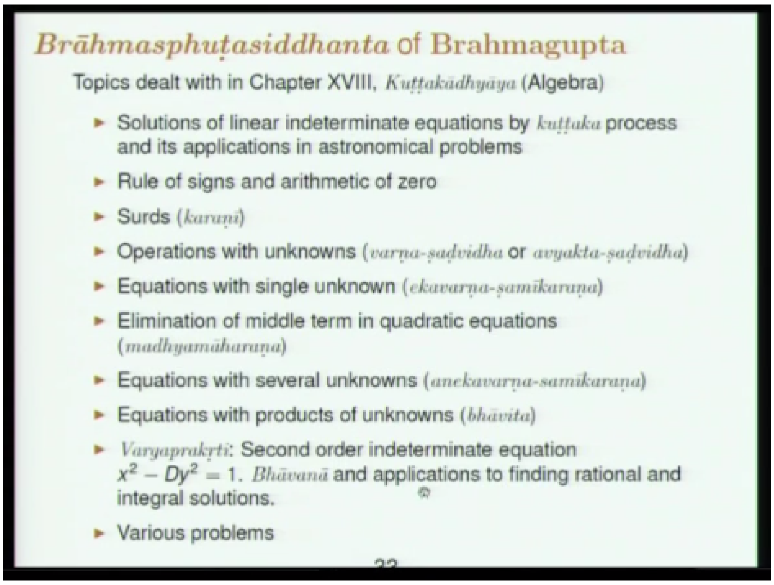

Brahmasphutasiddhanta of Brahmagupta is a text on astronomy, it has 2 chapter in mathematics, chapter 12 when 13 is called ganitagya and chapter 17 is called (FL), chapter 12 is ganitadhyaya, 17 is (FL) ideas with most ideas in Algebra, in brahmagupta for the first time find the Arithmetic of negative qualities calculations with zero and then detailed statement of equations and even introduction of complicated equations known as the vargaprakrti which became a very important equation in the Indian mathematical tradition.

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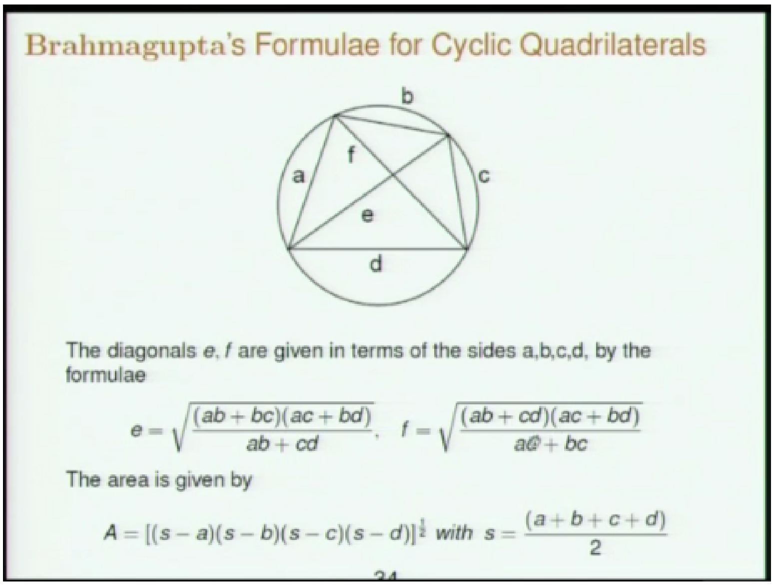

Brahmagupta also gave very interesting result such as this equation of the diagonals of a cyclic quadrilateral, a quadrilateral which is inscribed in a circle, he gave a formula for the diagonals of a cyclic quadrilateral in terms of the sites and an expression for the area of the circle quadrilateral which is a generalization of the formula that perhaps all of you know as the heroines formula for the area of a triangle.

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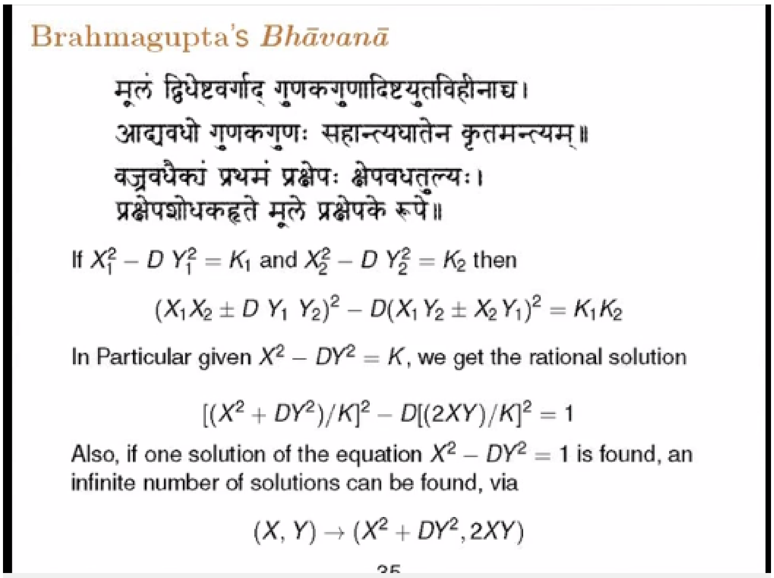

Brahmagupta course mentioned that this formula is applicable to quadrilateral in triangle. He gave some interesting properties of equations of the kind, these are called the varga prakyathi equation. He was the first person to call me later property called as Bhavana, the given 1 solution you can go to another solution we will discuss this later during the course.

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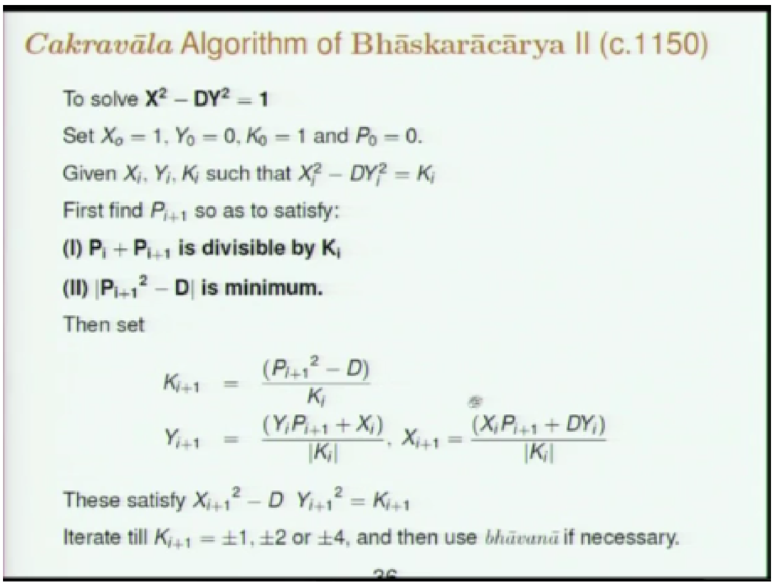

But this enabled later on Indian mathematician to work out every systematic algorithm, this one of the most famous algorithms in Indian mathematics called as Cakravala.

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Bhāskara's Example: $X^2 - 61Y^2 = 1$

i	P_i	K_i	a_i	ε_i	X_i	Y_i
0	0	1	8	1	1	0
1	8	3	5	-1	8	1
2	7	-4	4	1	39	5
3	9	-5	3	-1	164	21

To find P_1 : $0+7, 0+8, 0+9 \ldots$ divisible by 1. Of them 8^2 closest to 61. Hence, $P_1 = 8, K_1 = 3$

To find P_2 : $8+4, 8+7, 8+10 \ldots$ divisible by 3. Of them 7^2 closest to 61. Hence, $P_2 = 7, K_2 = -4$

After the second step, we have: $39^2 - 61 \times 5^2 = -4$

Since $K = -4$, we can use *bhāvanā* principle to obtain

$X = (39^2 + 2)\left[\left(\frac{1}{2}\right)(39^2+1)(39^2+3) - 1\right] = \mathbf{1,766,319,049}$

$Y = \left(\frac{1}{2}\right)(39 \times 5)(39^2+1)(39^2+3) = \mathbf{226,153,980}$

$1766319049^2 - 61 \times 226153980^2 = 1$

And it enables you to solve equations. This is a very famous problem x square-61, y squared=1. You have to solve for X and Y in integers and as you can see this solutions are about 1P 1.7 P and 226 million. So these are very high numbers, this lowest solution of this equation, after Bhaskaracharya who is book 1150 solve this equation by a very simple method this table tells you the method.

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Tātkālika-gati: Instantaneous Velocity of a Planet

- Approximate formula for velocity (*manda-gati*) in terms of Rsine-differences was given by Bhāskara I (c.630) and he also comments on its limitation (*Laghu-bhāskarīya* 2.14-15).
- True velocity (*sphuṭa-manda-gati*) in terms of Rcosine (as the derivative of Rsine) is given in *Laghu-mānasa* of Muñjāla (c. 932) and *Mahā-siddhānta* of Āryabhaṭa II (c. 950).
- Bhāskara II (c.1150) discusses the notion of instantaneous velocity (*tātkālika-gati*) and contrasts it with the so-called true daily motion. He also evaluates the *manda-gati* and *śīghra-gati* (*Vāsanā* on *Siddhānta-Śiromaṇi* 2.37-39).
- Bhāskara II notes the relation between maximum equation of centre (correction to displacement) and the vanishing of velocity correction (*Vāsanā* on *Siddhānta-śiromaṇi, Gola* 4.3).

This problem again came up 500 years later when bharma post this as a problem to the British mathematician. Ideas of calculus started developing and they arose the context of astronomy, the idea instantaneous velocity become important because especially to understand the motion of moon one needed to know the rate of variation of its position and one found that even the rate of change of its position was continuously changing.

And the idea of instantaneous velocity arouse this way. Now there is a common misconception 6 years ago in modern times that Bhaskaracharya II 11 AD was the last important mathematician in Indian mathematics afterwards the people were just repeating what was done in earlier books or they forgotten mathematics all together. It is only in the last 56 years that works of later mathematicians have been studied.

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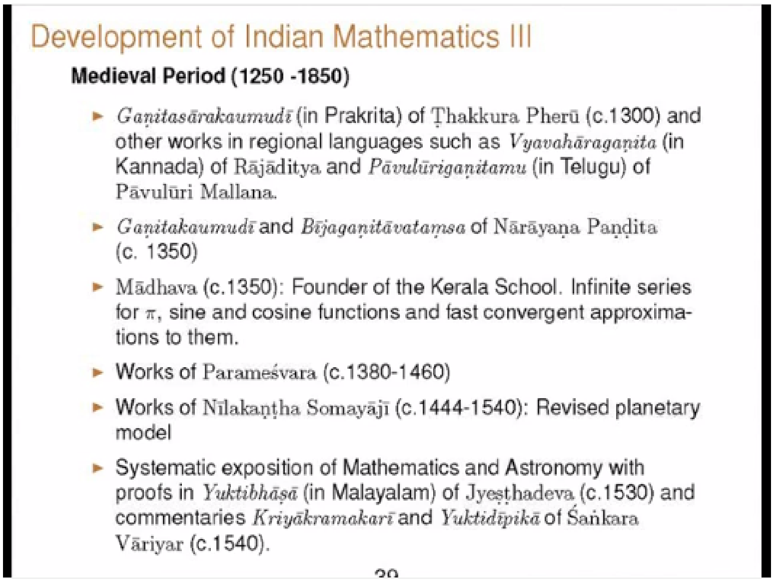

And understood and actually the picture is quite different , first of all around 1200 works in mathematics started at caring in regional language ganitasarakaumudi in Prakrita, Vyavaharaganita in Kannada, Pavuluriganitamu in Telugu. These are very important was written around 13 century, 12^{th} century. Ganitakaumudi and Narayana Pandita is a great advance of bhaskaracharya Lilavati a large part of course will be devoted to study of that.

Then there arouse a school in Kerala which had been special contributions to make the Kerala school of astronomy initiated by Madhava, then parameswaran then nilakantha somayaji. They revise the older astronomical model and came up with a new astronomical model, but Madhava is more well known for his discovery of infinite series for PI, sine and cosine and their the proofs of all these results of Madhava written down in a very famous Malayalam book called Yuktibhasa written in 1530.

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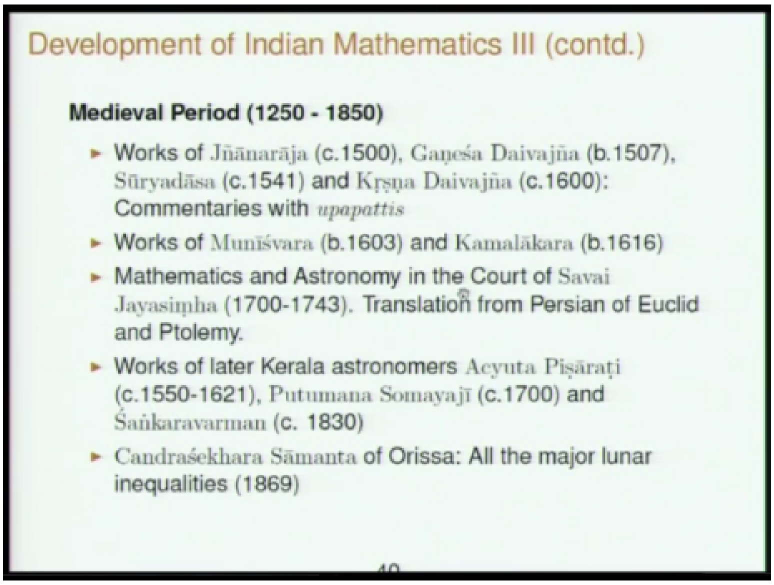

Mathematics continues in Maharashtra and Kasi with scholars such as Jnanaraja, Ganesa Daivajna, Suryadasa. They wrote proofs on Bhaskara result trigonometrically result were discovered by Munisvara Kamalakar, Savai Jayasimha in Jaipur he built this 5 observatories which was very important at that time to correct the older astronomical calculations. The Kerala school also continued the last back was sankaravarman (FL) in 1830.

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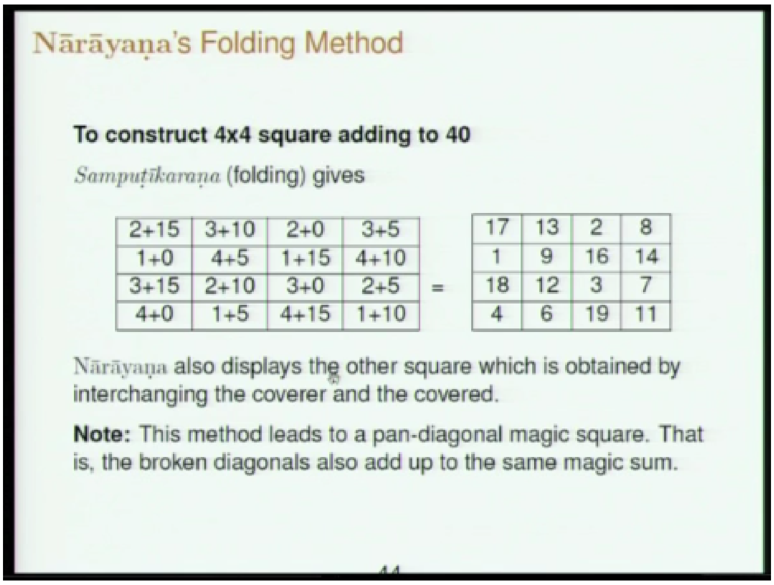

There was an astronomer called Candrasekhara Samanta in Orissa who impact the game by traditional method. All the major lunar inequalities in 1869. So just to tell you the kind of that Narayana Pandita even considered topics like magic squares as serious mathematical topics and came up with very interesting way of constructing magic squares several very new algorithms.

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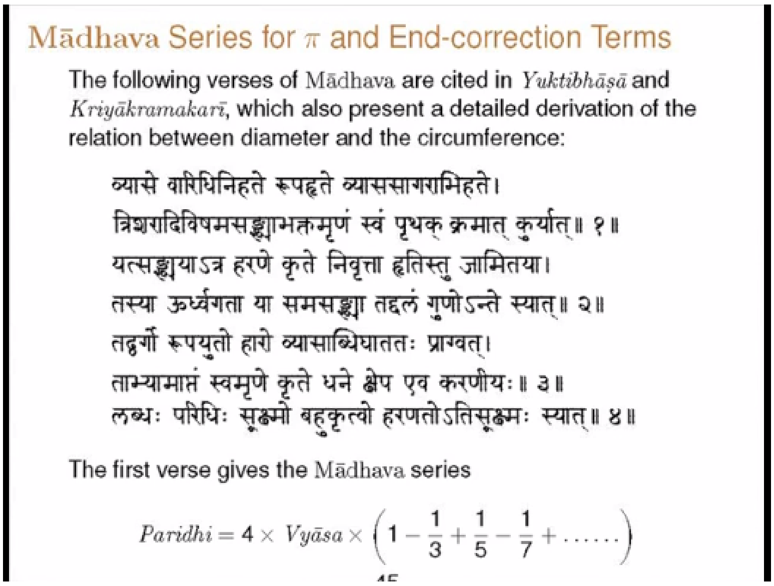

This is what is called is the folding method of calculating magic squares. This is the infinite series for the ratio of the circumference to diameter discovered by by Madhava the Kerala mathematician.

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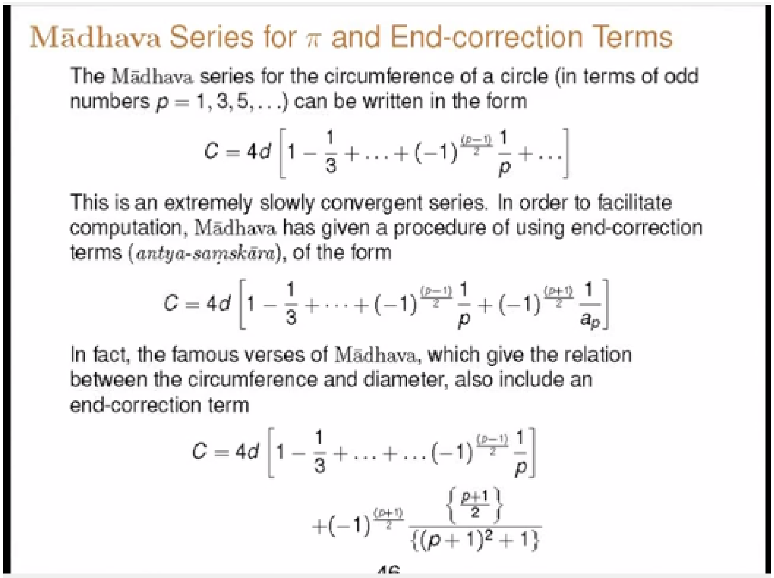

He not only discovered the infinite series that is a slowly convergent series that 1-1/3+1/5/1/7. If we calculate 50 tons of that series you get only one decimal place in the expansion of Pi. So Madhava at the same time gave what are known as the end correction terms. So this is the first end connection term due to Madhava, then there is another end correction term.

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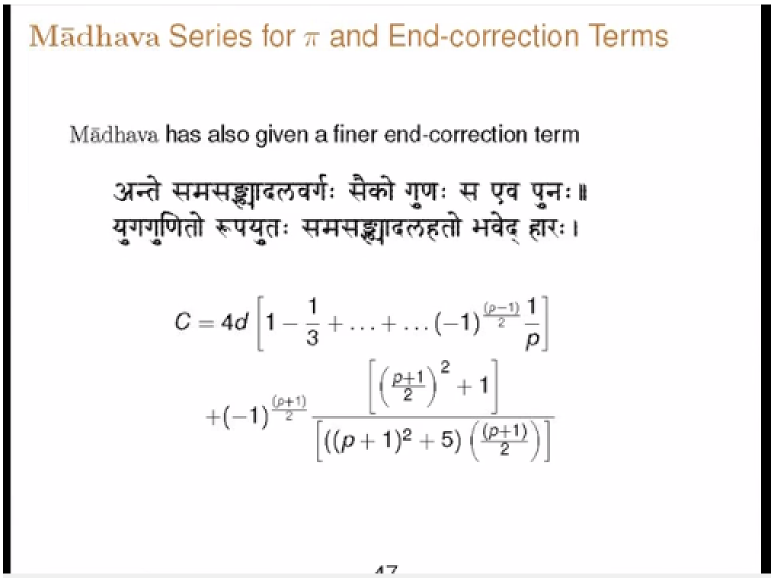

It is this end correction terms which give you more accurate and more accurate result even if you sum only 50 terms in the Madhava series incidentally that series due to Madhava is also known as the (FL) series.

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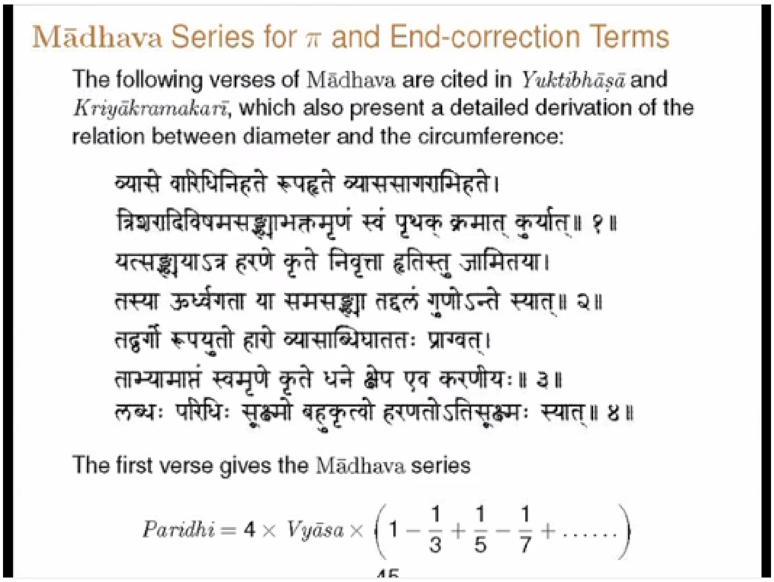

Because it discovered by (FL) 1674.

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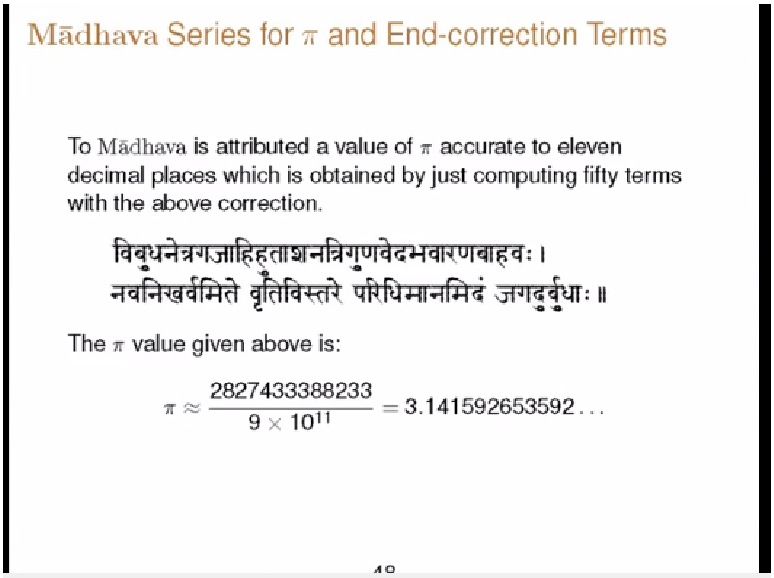

So using his connection Madhava was able to give the value of pi correctly to 30 11 decimal places just by using 50 terms in his series with that end correction term.

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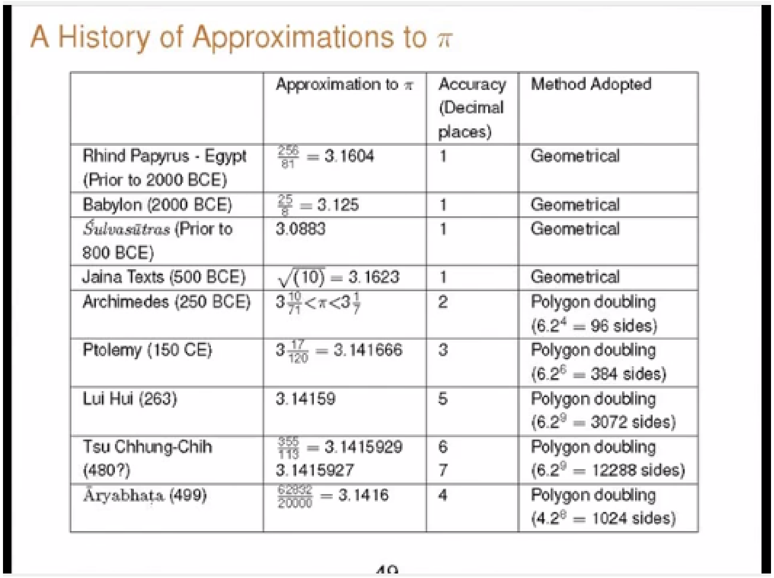
A History of Approximations to π

	Approximation to π	Accuracy (Decimal places)	Method Adopted
Rhind Papyrus - Egypt (Prior to 2000 BCE)	$\frac{256}{81} = 3.1604$	1	Geometrical
Babylon (2000 BCE)	$\frac{25}{8} = 3.125$	1	Geometrical
Śulvasūtras (Prior to 800 BCE)	3.0883	1	Geometrical
Jaina Texts (500 BCE)	$\sqrt{(10)} = 3.1623$	1	Geometrical
Archimedes (250 BCE)	$3\frac{10}{71} < \pi < 3\frac{1}{7}$	2	Polygon doubling ($6.2^4 = 96$ sides)
Ptolemy (150 CE)	$3\frac{17}{120} = 3.141666$	3	Polygon doubling ($6.2^6 = 384$ sides)
Lui Hui (263)	3.14159	5	Polygon doubling ($6.2^9 = 3072$ sides)
Tsu Chhung-Chih (480?)	$\frac{355}{113} = 3.1415929$ 3.1415927	6 7	Polygon doubling ($6.2^9 = 12288$ sides)
Āryabhaṭa (499)	$\frac{62832}{20000} = 3.1416$	4	Polygon doubling ($4.2^8 = 1024$ sides)

So we can briefly sketch this history of pi as typical of the way mathematics developed across different cultures please see Aryabhatta's value 3.1416 which is activate up to 4 decimal places that sulvasutra values which is activated up to 1 decimal places, the Jaina text use root 10 Archimedes give this standard is equality 3 10/71 less than Pi, less than 3 1/7, the Chinese mathematician Tsu Chhung Chih had this 355/113 which is accurate up to nearly 7 6-7 decimal places.

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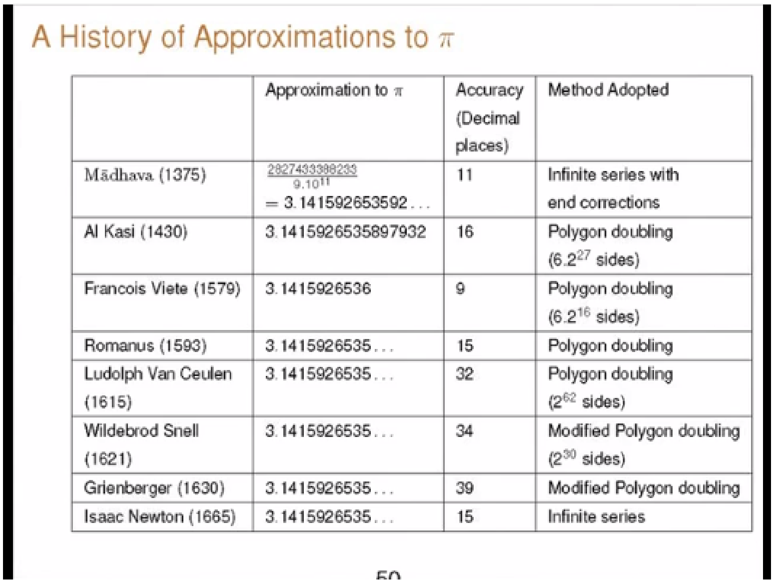

A History of Approximations to π

	Approximation to π	Accuracy (Decimal places)	Method Adopted
Mādhava (1375)	$\frac{2827433388233}{9.10^{11}}$ $= 3.141592653592\ldots$	11	Infinite series with end corrections
Al Kasi (1430)	3.1415926535897932	16	Polygon doubling (6.2^{27} sides)
Francois Viete (1579)	3.1415926536	9	Polygon doubling (6.2^{16} sides)
Romanus (1593)	3.1415926535...	15	Polygon doubling
Ludolph Van Ceulen (1615)	3.1415926535...	32	Polygon doubling (2^{62} sides)
Wildebrod Snell (1621)	3.1415926535...	34	Modified Polygon doubling (2^{30} sides)
Grienberger (1630)	3.1415926535...	39	Modified Polygon doubling
Isaac Newton (1665)	3.1415926535...	15	Infinite series

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But fact to this please see Madhava coming up 11 decimal places between Aryabhatta to Madhava. Then Madhavas result was based upon infinity series, all these most of these results are actually based upon root force calculation with approximating the area of a circle by polygon Al Kasi etc. Newton again came up with an infinite series around 1665.

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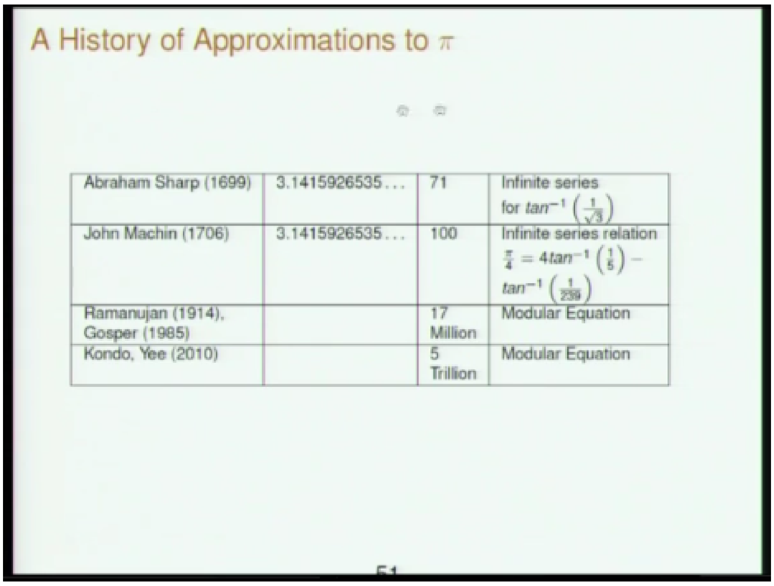

A History of Approximations to π

Abraham Sharp (1699)	3.1415926535...	71	Infinite series for $tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$
John Machin (1706)	3.1415926535...	100	Infinite series relation $\frac{\pi}{4} = 4tan^{-1}\left(\frac{1}{5}\right) - tan^{-1}\left(\frac{1}{239}\right)$
Ramanujan (1914), Gosper (1985)		17 Million	Modular Equation
Kondo, Yee (2010)		5 Trillion	Modular Equation

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Then the various other thing, but we can see in recent time Ramanujan in 1914 came up with a very interesting series for Pi using modular equation and that created a small recorded that act at 1980s that people calculated Pi to about 17 million decimal places, today's achievement is about 5 trillion.

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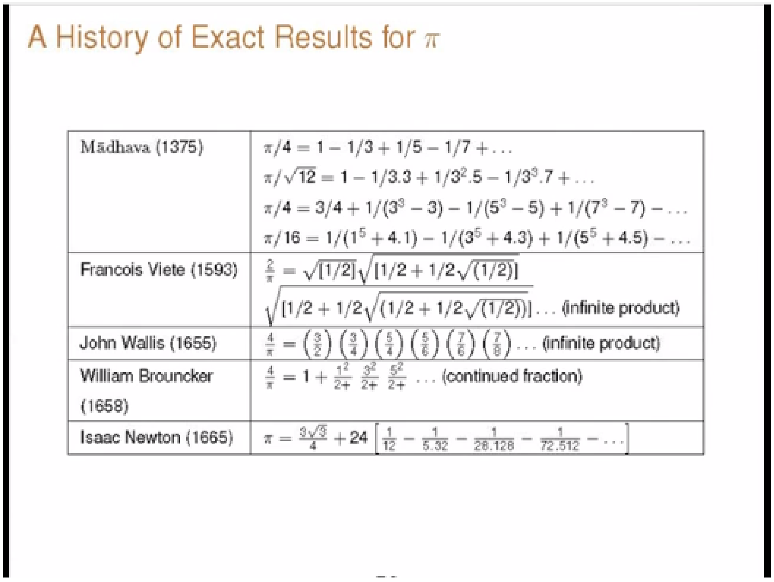

But equal important of this exact results of Pi you can see Madhava all these exact result which was later on repeated by others James Gregory Tan inverse series with series, short series, all these are contained in Madhavas paper.

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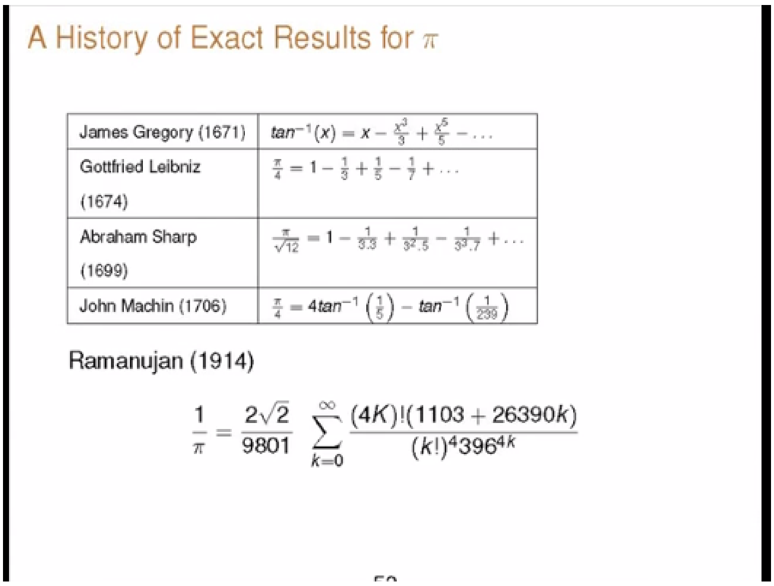

This is the series given by Ramanujan in his 1914 paper.

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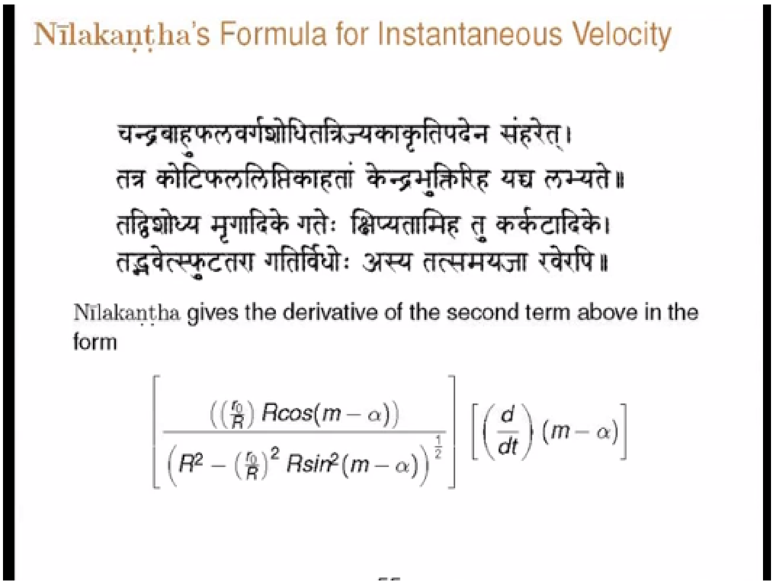

The idea of instantaneous velocity also lead to more complicated derivative, the derivative of sine function as a cosine function was well known by the time of Bhaskara, Nilakantha is formulating that derivative of sine inverse function as 1/square root of 1-x square in this words.

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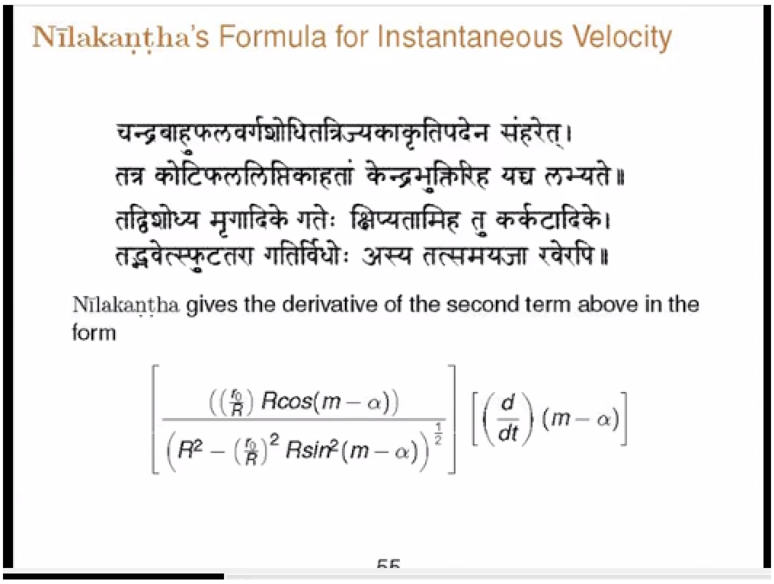

Now again till about 56 years ago people had study only I mean the modern scholars, had studies only the basic text of Indian mathematics.

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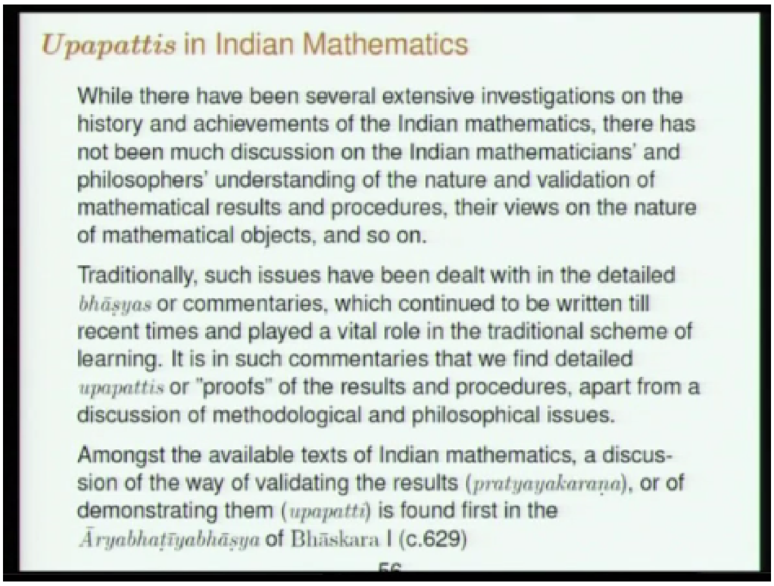

So they had the sort of idea that Indian somehow enhance lot of results, but they did not seem to have any method for arriving at this result or at least those messages were very obscure. So it is only in the last 56 years that many of the common trees to the original text people started studying traditionally such issues that how to obtain results, how to understand them etc. have been dealt with in detail Bhaskara commentary.

This is not just super mathematics that if you pick up any basic text even Bhagavad Gita to understand it in a very serious manner you have to take requests to the detail common trees which are written on them and this commentaries continue to be returned till recent times they played a very vital role in the traditional schema learning. As per mathematics is concerned.

It is in this comment that we find what are known as upapatti or uptis. They are something similar to demonstration a rational of proofs in mathematics, you one of the oldest words available words which has upapatii is a Bhaskara 1 commentary on Aryabhatta.

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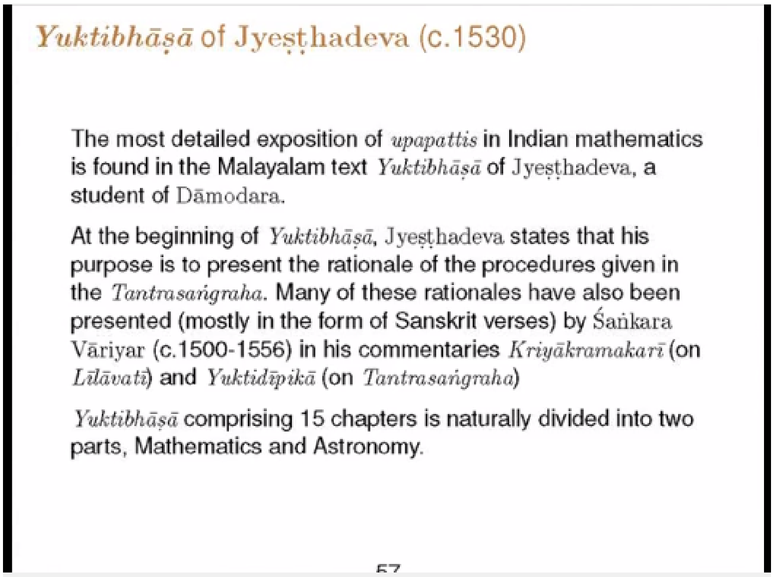

But of course the most detailed exposition of upapatti is found in the Malayalam text Yuktibhasa written in 1530.

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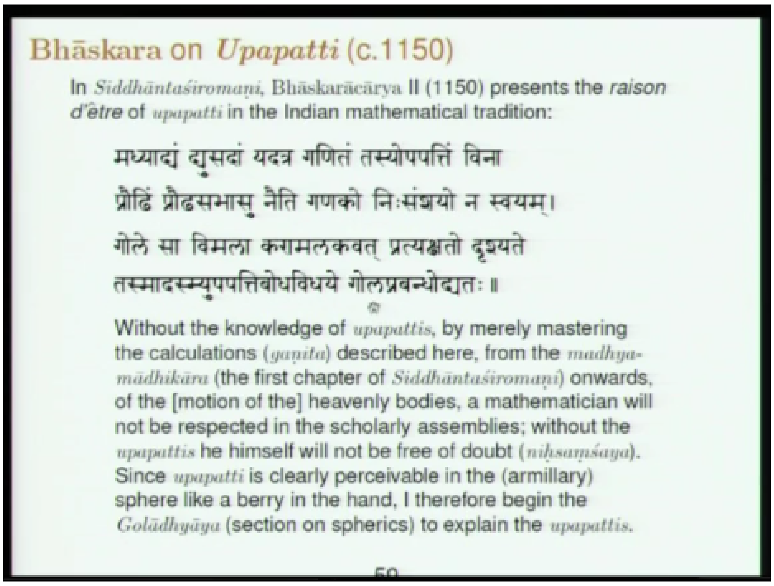

Now as you text upapatti what was the upapatti's suppose to do, what is the nature of this, this was captured by this words of Baskaran, upapatti mean (FL) without the proof a mathematician will not be considered as a scholarly mathematician in any assembly of mathematician (FL) any doubt regarding the result that he is enunciating. So for this reason that I am going to discuss upapatti's are ture.

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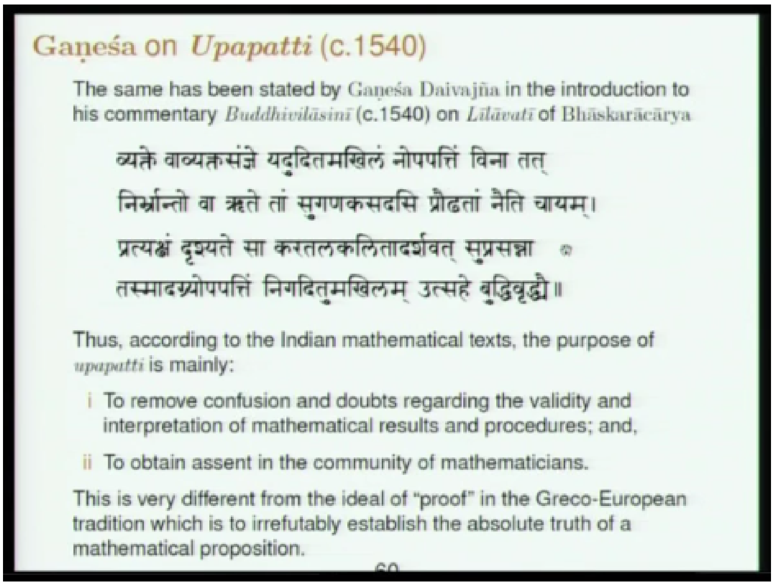

That is what Bhaskara is explaining in his commentary on Siddhanta Shiromani. The same point is repeated across Ganesa is follower in the tradition of Bhaskara is writing a commentary on Leelavati in 1540 explaining this proves. Again (FL) (()) (29:18) to (()) (29:28) that person who does not know upapatti will not be without confusion normally be considered as a serious mathematician.

Now so the basic purpose of a upapatti is sort of clearly stated to be to remove confusion and doubt regarding the validity and to obtain ascent in the community or something like sending a paper and getting it period and getting it publish. It does it mean that result is going to stand for all times for all ages that was the ideal of proof in the European tradition, that does not seem to be the kind of ideal that the Indians are initiated by doing mathematics.

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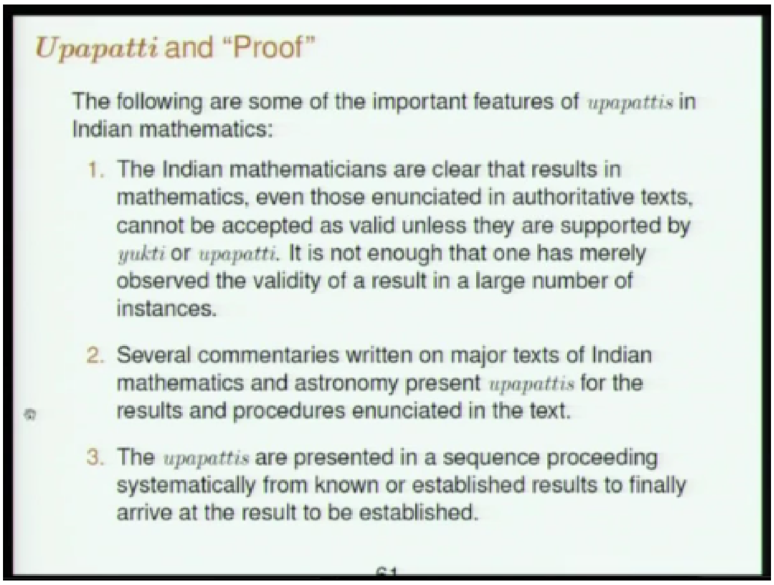

In fact the detail study of proofs in Indian mathematics shows that there are the differences between the idea of proof as we know from the Greek or European tradition and the idea of upapatti in Indian mathematics. First of all the Indian mathematician is a very clear that proofs are needed upapatti are needed result even if verified in 100s of cases, does not mean that it is proved in mathematic.

So only when you can give some logical argument or some other argument you can you say that it is a valid mathematical result and several commentary are written listing such upapattis. When the upapattis like as we know proofs in modern arithmetic they are written in a sequence that to go from known result to new result and from them to let other result. So you will have a sequence of establishing results.

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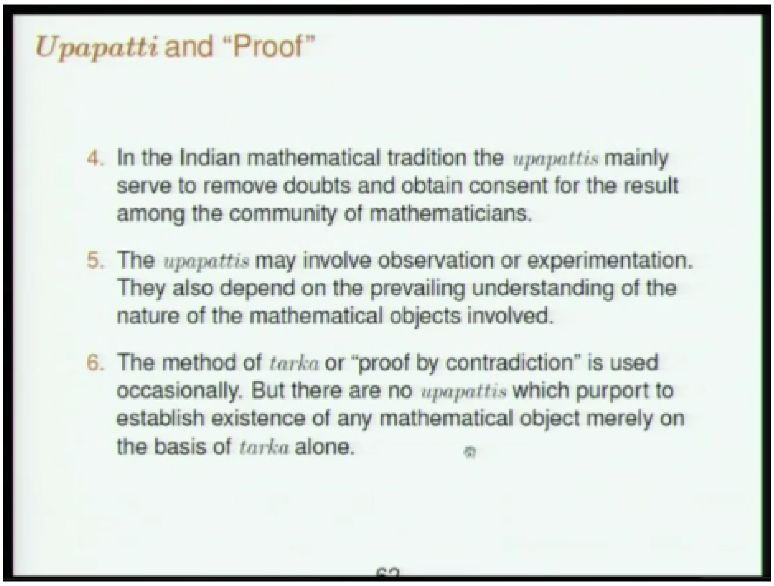

And the understanding is that it is by giving proves we are clear how the result is to be applied and understood. The proofs may many times depend upon experiment, this something which is new we may be doing it another mathematics teaching but Euclidean ideal of proof is that to prove something is very abstracted should not be dependent on experimentation.

Should not be depend on even our understanding what is the nature of the mathematical object, but the Indian proofs were always they could involve experimentation, they could involve an understanding of the explicit use of the nature of the object and another crucial things is that what is called the proof by contradiction the (FL) which is called in Indian mathematics that was employed.

The (FL) was employed to understand the non existence of certain mathematical quantities, but it was not employed to established the existence of a mathematical object whose existence would not otherwise be accessible to us by any other means. So (TL) non considered as a independent from (FL). So existence of quantities cannot be established by nearly proofing that their non existence is inconsistent with whatever we know.

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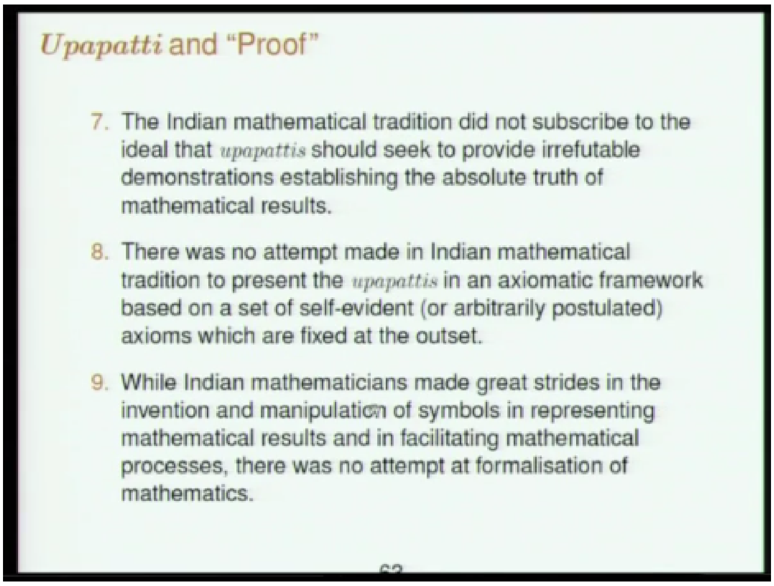

But by giving a means as an access to the way there existence can be understood by us which is something known as the constructive philosophy which is mathematic and there is no ideal the proofs will give it that curable demonstration or will give this the absolute truth of mathematical proposition. There was no idea that you fix one set of postulates once in for all in derive all the results.

And by so many symbolism and symbolic techniques were used formalization of mathematics was not something that is attempted in Indian mathematic.

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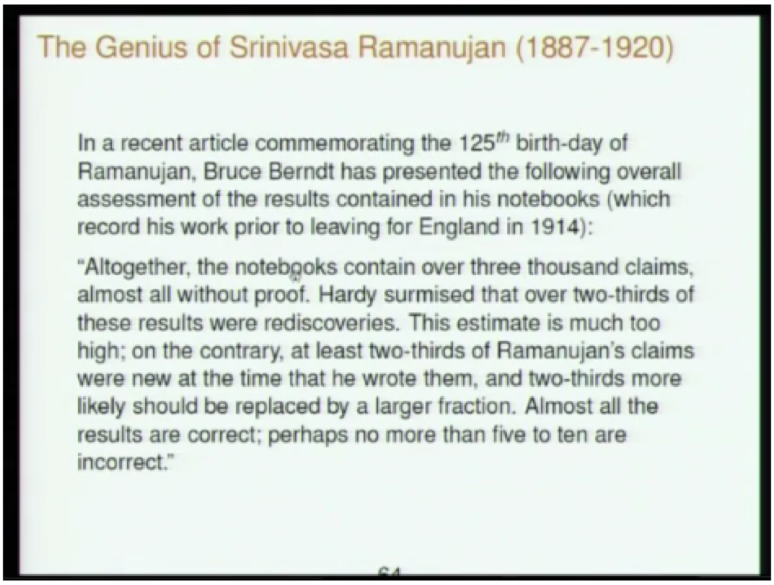

Now coming to more contemporary kinds this issue of proof we tell something very crucial in understanding the mathematic of Srinivasa Ramanujan. Then Ramanujan sent his result in 1930 in a long method to high if it is 100, 120 results, hard immediately response by saying this all kind looking very interesting but where are the proofs, you please send me the proofs of all these results of course they was not so trivial that hardly could prove it for himself the straight away on a piece of paper or something like that.

When did the proofs be given and Ramanujan there is a very famous let he send hardly in 1930 saying that he has a systematic method for deriving all the results but that cannot be explained in a short communication and he thinks that he has a new methodology for doing things and he anywhere but he says that why do not you just some of this results and can we check what I am writing his really and that should convince you that there is something interesting in what I am doing.

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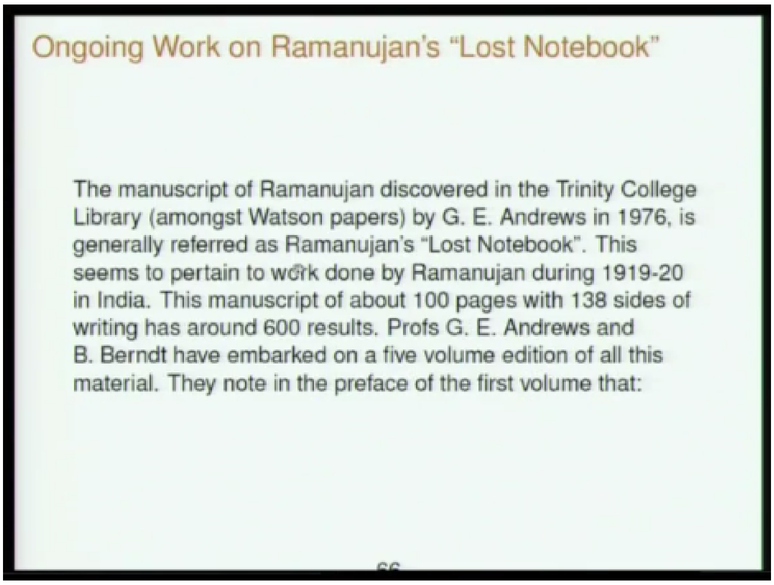

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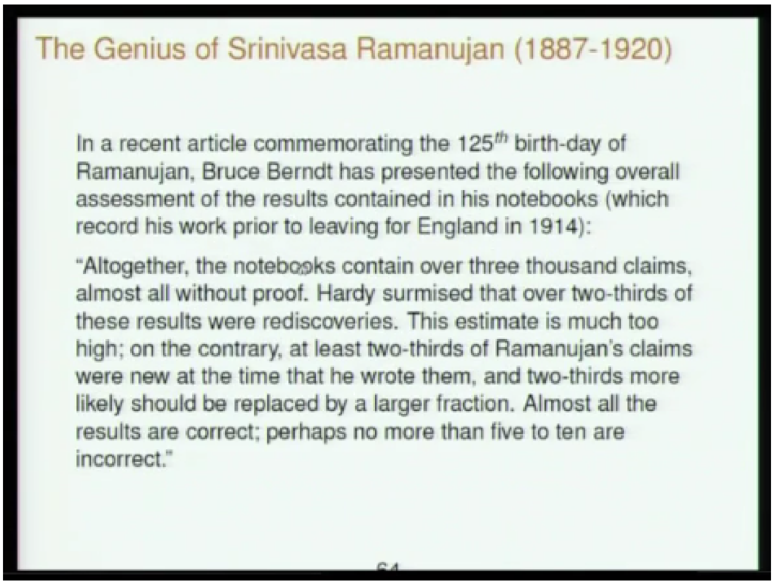

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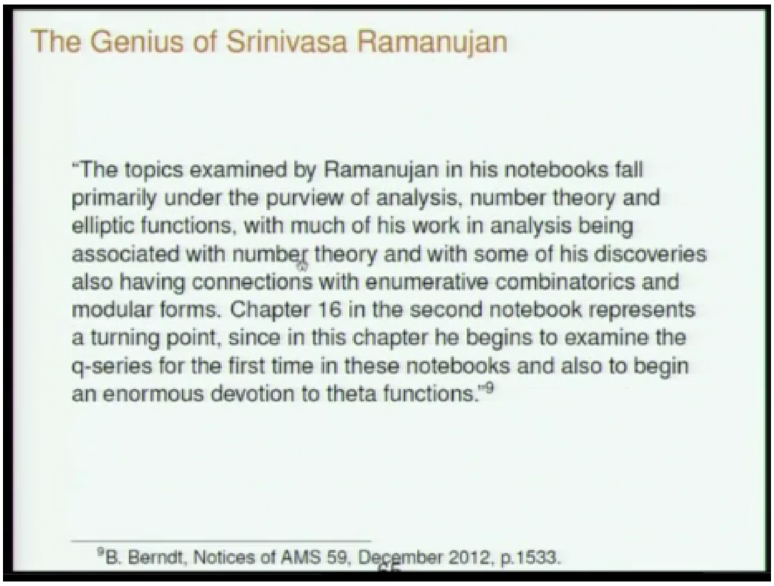

Now issues important because finds where there is this notebook of Ramanujan which is set of all results that he noted hire to going to England and later analysis in the last 2530 shows that there are more than 3000 results this notebooks contain are the initially thought that two thirds of them where already well known but now the understanding is more than two thirds was not known it the time Ramanujan was recording this results in the notebook.

And almost all the results are correct and there is no more than 5 to 10 or incorrect, this is the current assessment of the results that Ramanujan wrote down in his notebook, there is of course a notebook of the work that he was doing in the last year of his life 1919 to 20 which was lost seemingly and it was recorded in 1975 in the Trinity college library by Mr. G.E. Andrews it is called the Lost Notebook.

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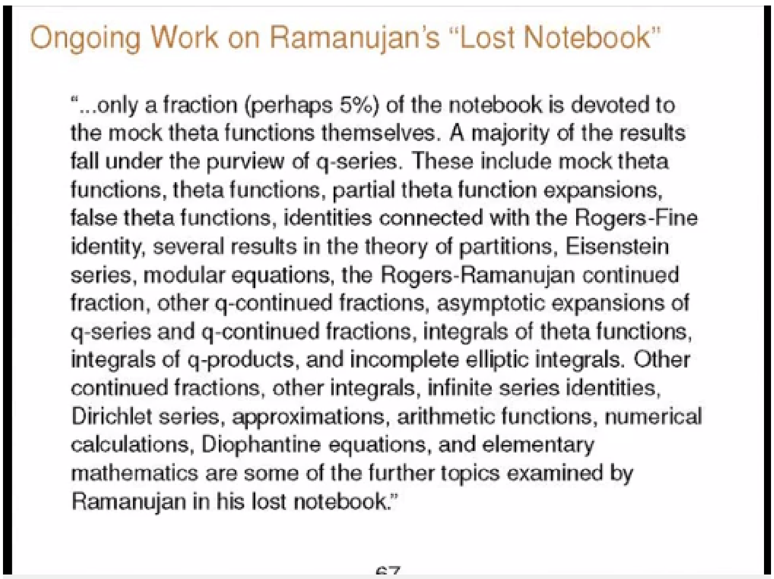

And result in that are still being established by the mathematician of present day and this contains full lot of results like this.

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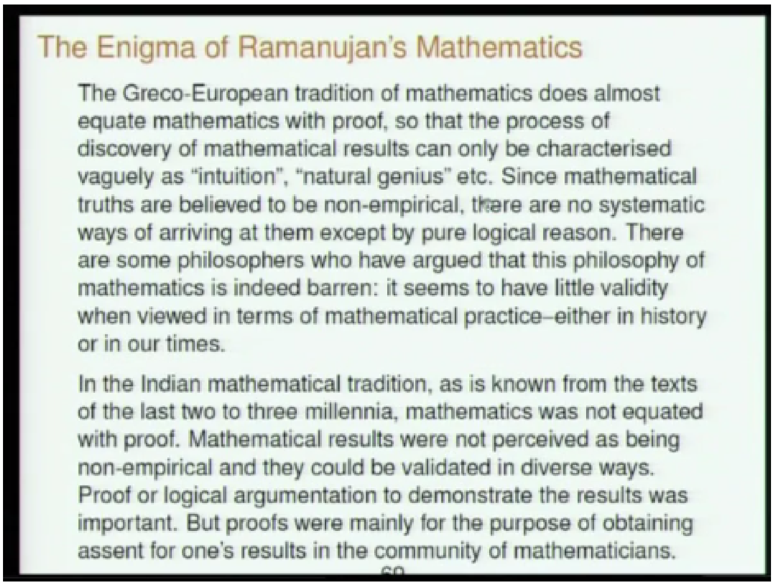

So what I was trying to say was that Greco-European tradition of mathematics almost equate mathematics with proof and the way mathematical results of discovery therefore is hardly understood which may be termed as intuition natural genius etc. and there is an understanding that mathematical results of non empirical and therefore there is no access to them except by logical argumentation.

Of course there are philosopher of mathematics to do argue that this philosophy of mathematics is confusing the barrel . This philosophy does not explain most of history of mathematics, today mathematics is done either it was done in earlier life or even mathematics is being done present. In Indian tradition the understanding was that proof is only one of the aspects of mathematics important . Mathematical result does not part of to be nonempirical.

Mathematics was not thought out to be a science different from other sciences, it results were equally contestable and falsifiable and they could be validated in diverse days, the proof was important but they were more for obtaining as an for once result .

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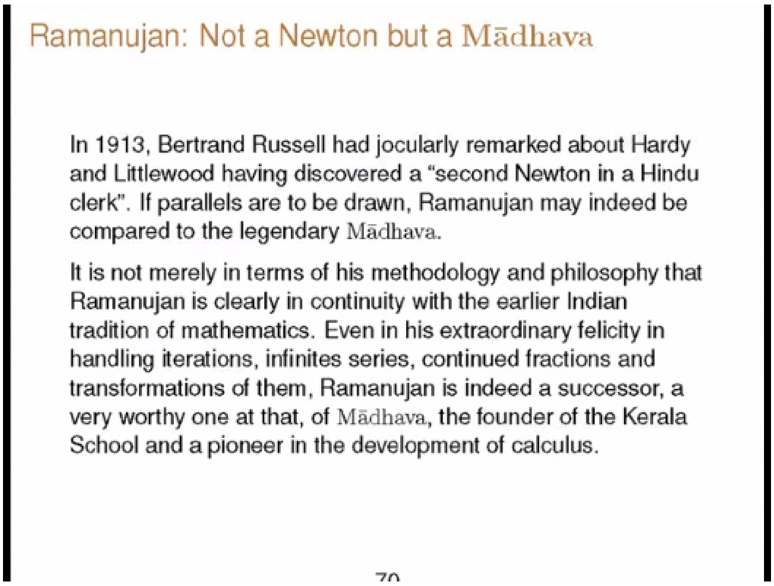

So the process of mathematical discovery in the mathematical justification or in some unicell in where the Indian have understood mathematics. Long time ago when Ramanujan letter arrived in England the conclusion that it and it would had but (FL) one of his friends that (FL) have discovered second Newton in a Hindu. But if some comparison is to be made regarding Ramanujan he is more in the line up Madhava.

Both in the kind of topics like infinity transformations of them and continued fractions and transformations of them handling iteration and indeed success of the great genius Madhava who was one of the Pioneer of calculation.

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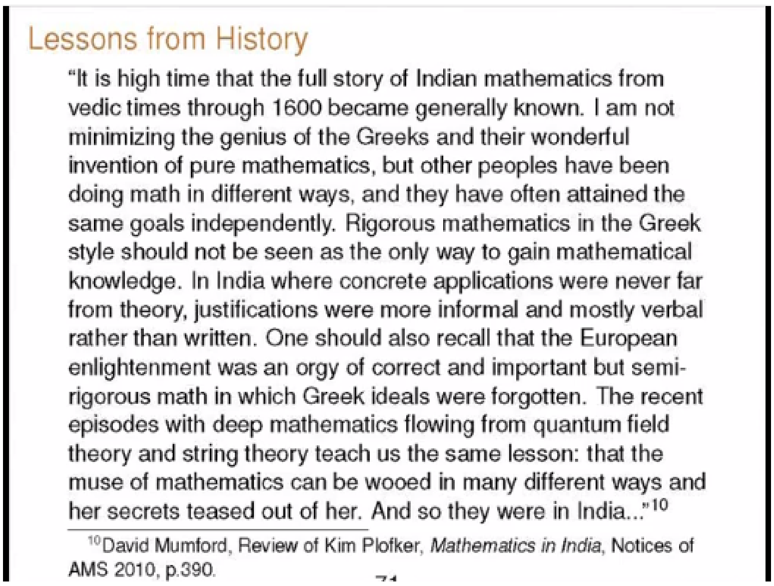

I tried to extract review of recent book on mathematics in India by David Mumford the well known (FL) the main point was emphasize that by studying Indian mathematics of the history of mathematics in India what one can understand is that Indian mathematics can be done in different ways the views of mathematics can be boot in many different ways and have secret is out of that and so they were in India.

And one should not just confused the fact that absence of rigorous mathematics in the Greek style means that the rest is not mathematics at all. When he caution that most of interesting mathematics that we used today which was developed in 16, 17, 18 century was indeed done by abandon the Greek term of doing mathematics , this is the kind of understanding that scholars are arriving at the importance of knowing a different tradition of mathematics like the Indian tradition.

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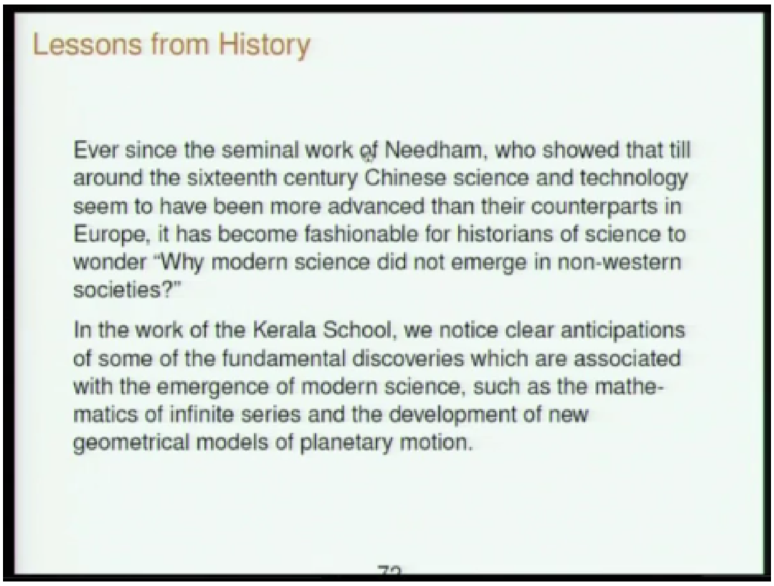

Another interesting in this the question of the history of science in recent times. That ever seen the work of Needham, it has generally been understood that till 16 century the Chinese science and technology seem to be considerable advanced over science and technology in Europe and then Needham showed the question that Needham almost made it an important focus was why modern science did not emerge in China and did not emerge in non-western societies.

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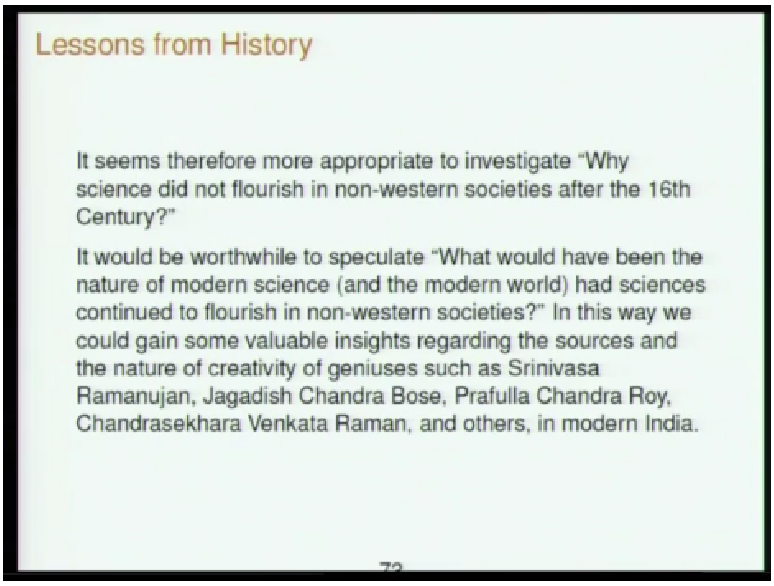

Now when we study mathematics in India for instance notice that many of hallmarks of modern science such as development of Calculus, infinite series etc. are development of you astronomical models of the planet which system they were all there in Kerala that of 14, 15, 16 century. So a very crucial question that we should understand is why science did not flourish in non-western societies that is 16th century.

And it is even more important but today's purpose to have some idea how science would have developed, how the science today would have been if the non-western societies had continued developing science along the lines that they have laying down for themselves earlier maybe with many modifications, maybe with some transformations in interaction with modern science developed in Europe and subsequent time.

It is only by that kind of speculation we can come to some understanding on of the great genius of modern times in India such as Ramanujan, Bose, Prafulla Chandra Roy, Raman and many others.

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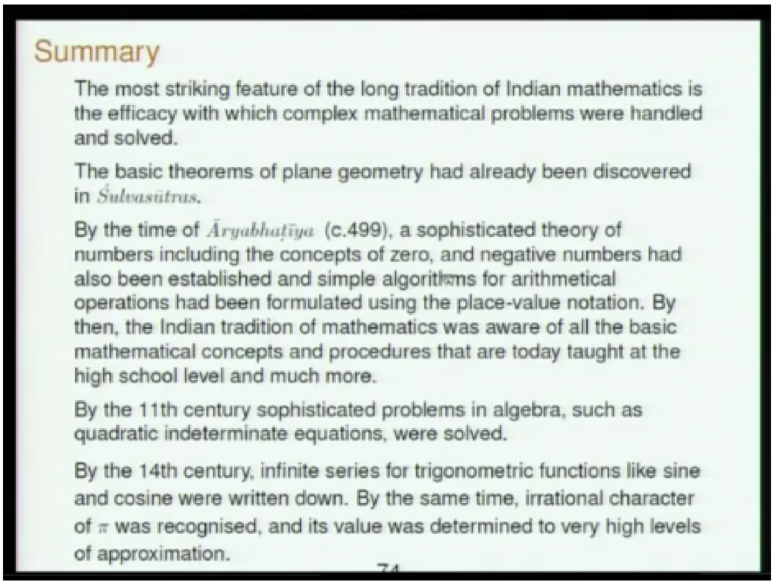

So to summarise the development of mathematics in India that the main thing was that the complex mathematical problems were not send, even if complete solutions to them were not found approximate less than perfect solutions were accepted and then developed into better and better solution and the idea was always was in simplicity of mathematical procedure and by this Indians were able to do quite a bit.

They could get the basic (FL) geometry by the time of sulvasutras they could establish most of our arithmetic algebra geometry and trigonometry by the time of Aryabhatta by the time of Bhaskara II they could solve complicated quadratic indeterminate equations or by 14, 15 century calculus exact series were sine and very accurate sine tables which all very important perform.

So the crucial thing is explicitly algorithmic and computational nature of Indian mathematics and this seems to have persisted that till recent times and to some extent of Srinivasa Ramanujan as I told you could be thought of as a (FL) traditional Indian methodology and perhaps is important that we should have a detailed understanding of the development of mathematics in India to understand the way Indians approached many complex problems.

Even in other sine and we let to say it very important that we should teach the highlights of this great tradition of mathematics to our students in schools and colleges and I think courses like this will help in sort of formulating that kind of a work. So with that I **I** complete this initial overview and thank you very much.