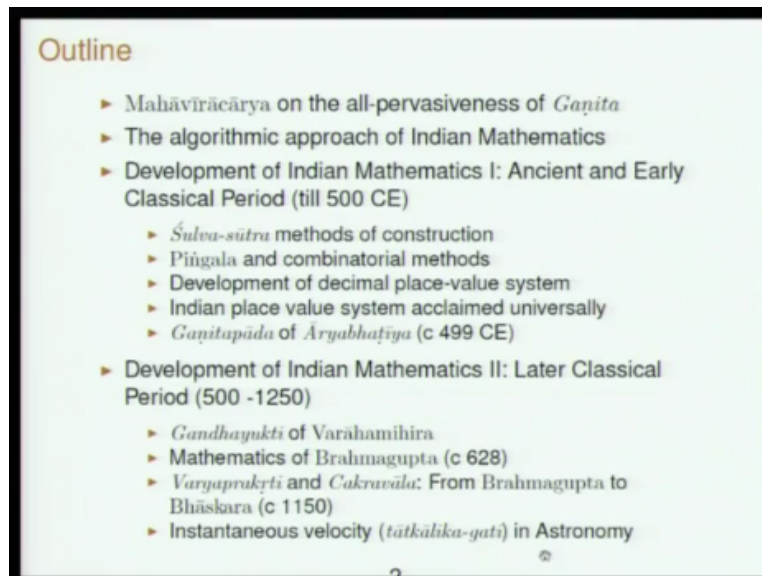


**Mathematics in India: From Vedic Period to Modern Times**  
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**Lecture-1**  
**Indian Mathematics: An Overview**

Good morning, this is the first lecture of this course, which is being given on mathematics in India from Vedic period to modern times, it is a novel course which tries to trace the way mathematics developed in India, the first talk is an overview talk. In this I will try to highlight those periods in which there was a significant development of mathematics in India. I will also try to summarise the special nature of mathematics as it developed in India. I would like to emphasize the algorithmic way in which most problems in mathematics were considered in the Indian tradition.

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So I am flashing the outline not going to read it out we can just see the kind of topics we are going to follow up , we will cover the development of Indian mathematics in the ancient period indicate some highlights during that period. Then the early classical period say 500 BCE to 500 CE which culminated in the birth of Aryabhata. Then the development mathematics in the latest classical period from 500 in the common era to 1250.

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Outline
<ul style="list-style-type: none"> <li>▶ Development of Indian Mathematics III: Medieval Period (1250 - 1850)               <ul style="list-style-type: none"> <li>▶ <i>Vārusaṅkalita</i> of Nārāyaṇa Paṇḍita (c.1356)</li> <li>▶ Folding method of Nārāyaṇa for the construction of magic squares</li> <li>▶ Kerala School and development of calculus (1350-1825)</li> <li>▶ Mādhyama Series for <math>\pi</math> and end-correction terms</li> <li>▶ A history of approximations and exact expressions for <math>\pi</math></li> <li>▶ Nīlakaṇṭha's formula for instantaneous velocity (c.1500)</li> </ul> </li> <li>▶ Proofs in Indian Mathematics               <ul style="list-style-type: none"> <li>▶ Bhāskara on <i>Upapatti</i></li> <li>▶ <i>Upapatti</i> and "Proof"</li> </ul> </li> <li>▶ The genius of Srinivasa Ramanujan (1887-1920)</li> <li>▶ Lessons from History</li> <li>▶ Summary</li> </ul>

We will then go to some uses of the highlights of what happened during the mediaeval period till about 1850, towards then we will discuss something about the nature of mathematics in India how mathematics, how was results like that and then about the contemporary period we will speak a little bit about the Srinivasa Ramanujan.

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**Mahāvīrācārya on the all-pervasiveness of *Gaṇita***

लौकिके वैदिके वापि तथा सामायिकेऽपि यः ।  
 व्यापारस्तत्र सर्वत्र संख्यानमुपयुज्यते ॥  
 कामतन्त्रेऽर्थशास्त्रे च गान्धर्वे नाटकेऽपि वा ।  
 सूत्रशास्त्रे तथा वैदो वास्तुविद्यादिवस्तुषु ॥  
 छन्दोऽलङ्कारकाव्येषु तर्कव्याकरणादिषु ।  
 कलागुणेषु सर्वेषु प्रस्तुतं गणितं परम् ॥  
 सूर्यादिग्रहचारेषु ग्रहणे ग्रहसंयुतौ ।  
 त्रिप्रभे चन्द्रवृत्तौ च सर्वत्राङ्गीकृतं हि तत् ॥

Finally know that history, so some find out pervasive mathematics in India is this following statement from the Ganita Sara sangraha of mahaviracharya, if along six packs statement again when mahaviracharya, it got by saying that mathematics is important in all areas when finally concludes by saying (FL) (0) (02:30) to (0) (02:36) that is not provided by mathematics.

So that is kind of statement that mahaviracharya is beating, weather is it in Astronomy or beat in architecture or beat in conjunction of granite position kind course of moon logic quite the grammar, so it says all purpose statement of mahaviracharya that mathematics provides all aspects, all subject this quotation from the 9 century world called Ganita Sara sangraha or mahaviracharya.

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### Mahāvīrācārya on the all-pervasiveness of *Gaṇita*

“The number, the diameter and perimeter of the islands, oceans and mountains; the extensive dimensions of the rows of habitations and halls belonging to the inhabitants of the world, of the interspaces between the worlds, of the world of light, of the world of the Gods and of the dwellers in hell, and other miscellaneous measurements of all sorts all these are understood by the help of *gaṇita*. The configuration of living beings therein the length of their lives, their eight attributes and other similar things, their staying together, etc. – all these are dependent on *gaṇita*.

Why keep talking at length? In all the three worlds involving moving and non-moving entities, there is nothing that can be without the science of calculation (*gaṇita*).<sup>2</sup>

<sup>2</sup> *Gaṇitasārasaṅgraha* of Mahāvīrācārya (c.850), 1.9-16.

So Ganita stands for calculation competition (FL) (03:22) to (( )) (03:25) is a statement due to Ganesa Daivajna is a commentator of Lilavati.

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### *Gaṇita*: Indian Mathematics of Computation

गण्यते संख्यायते तद् गणितम्। तत्प्रतिपादकत्वेन तत्संज्ञं  
शास्त्रमुच्यते।

As noted by Gaṇeśa Daivajña, in his commentary *Buddhivilāsini* (c.1540) on *Līlāvati* (c.1150), *Gaṇita* (Indian Mathematics) is the science (art) of computation. Indian Mathematical Texts give rules to describe systematic and efficient procedures of calculation.

Here is an ancient rule for squaring as cited by Bhāskara I (c.629 AD)

अन्त्यपदस्य वर्गं कृत्वा द्विगुणं तदेव चान्त्यपदम्।  
शेषपदैर्गहन्यात् उत्सार्योत्सार्यं वर्गविधौ ॥

In the process for calculating the square, the square of the last digit is found (and placed over it). The rest of the digits are multiplied by twice the last digit (and the results placed over them). Then (omitting the last digit), moving the rest by one place each, the process is repeated again and again.

And therefore we can expect that Indian mathematical text really abound in rules to describe systematic and efficient procedure for calculation. Just to give you an example we will go to a very ancient rule this is given by Bhaskara I, this is in this commentary to Aryabhatiya, it is

just a rule for calculating the square of a number (FL) (03:59) to (04:10) so we can see the kind of calculation was talking about.

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**Ganita: Indian Mathematics of Computation**

An Example: To calculate  $125^2$

$\begin{array}{r} 1\ 5\ 6\ 2\ 5 \\ \hline \end{array}$	$5^2 = 25$
$\begin{array}{r} \phantom{1\ 5\ 6\ 2\ 5} \\ \phantom{1\ 5\ 6\ 2\ 5} \\ \phantom{1\ 5\ 6\ 2\ 5} \\ \phantom{1\ 5\ 6\ 2\ 5} \\ \phantom{1\ 5\ 6\ 2\ 5} \\ \hline \end{array}$	$2^2 = 4, 5.2.2 = 20$
$\begin{array}{r} 1\ 4\ 10 \\ \hline 1\ 2\ 5 \end{array}$	$1^2 = 1, 2.2.1 = 4, 5.2.1 = 10$

**Note:** This ancient rule for squaring, uses  $\frac{n(n-1)}{2}$  multiplications for squaring an  $n$ -digit number.

The modern word **algorithm** derives from the medieval word *algorism*, which referred to the Indian methods of calculation based on the place value system. The word *algorism* itself is a corruption of the name of the Central Asian mathematician al Khwarizmi (c.825) whose *Hisab al Hindi* was the source from which the Indian methods of calculation reached the Western world.

To take any number 125, first you square the last number multiply the other numbers by 2 and the last number. So you get this row. Then move away remove one digit square the next number multiply by 2 and that number the next row and finally square the last number, remove one number, add all of them. The important thing to realise even this very ancient rule written in 1619 AD.

Actually uses  $n \cdot (n-1) / 2$  multiplication to calculate the square of a number, a  $n$  digit number multiplied by another  $n$  digit number we will  $n$  square multiplication, but since we are squaring the same number is  $2ab$  like that comes in and so you are having an optimal algorithm for square. This is the Indian mathematicians always right to give the best possible way of the best possible procedure for doing a calculation.

Now the algorithm itself as a history it was the name given to the Indian methods of doing calculation when we coordinates when the name of Central Asia Mathematician call (05:29) who in the ninth century wrote a book on Indian methods of calculation that is methods of calculation using the decimal place value system and that book was called algorithm Latin version of that book is available.

The original Arabic version is not available. And this was the book that introduced the decimal place value notation to the Arabic world and later on to the European world and so



the people who followed this was calculation were called algorithm and the algorithm comes from (FL ) and this algorithm factor is not something very specific to mathematics impact it provides all Indian Sciences.

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**Śāstras: Present Systematic Procedures**

Most of the canonical texts on different disciplines (*śāstras*) in Indian tradition do not present a series of propositions; instead they present a series of rules, which serve to characterize and carry out systematic procedures to accomplish various ends.

These systematic procedures are variously referred to as *vidhi*, *kriyā* or *prakriyā*, *sādhana*, *karma* or *parikarma*, *karāṇa*, etc., in different disciplines.

These rules are often formulated in the form of *sūtras* or *kārikās*.

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Most of Indian disciplines sastras as we call them, they do not present a series of propositions, they normally give you a set of rules, a set of procedure which tell you how to systematically accomplish something. So the rules given in sastras are usually called as vidhi, kriya, prakriya, sadhana, parikarma, karana, these are the names and these rules are what are usually formulated as Sutras.

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**Śāstras: Present Systematic Procedures**

Pāṇini's *Aṣṭādhyāyī* is acknowledged to be the paradigmatic example of a canonical text in Indian tradition. All other disciplines, especially mathematics, have been deeply influenced by its ingenious symbolic and technical devices, recursive and generative formalism and the system of conventions governing rule application and rule interaction. In recent times it has had a deep influence on modern linguistics too.

"Modern linguistics acknowledges it as the most complete generative grammar of any language yet written and continues to adopt technical ideas from it".<sup>3</sup>

<sup>3</sup>P. Kiparsky, Pāṇinian Linguistics, in *Encyclopaedia of Language and Linguistics*, VI, 1994

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So disciplines the ethical disciplines in India they provide systematic rules of procedure rather than a set of propositions. And the for the most canonical such systematic text in India

is the great grammar written by Panini called Astadhyayi. In fact most other disciplines and especially mathematics is extensively influenced by the method of Panini. Please use symbolic and technical devices recursive and generative formalism.

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### Pāṇini and Euclid

"In Euclid's geometry, propositions are derived from axioms with the help of logical rules which are accepted as true. In Pāṇini's grammar, linguistic forms are derived from grammatical elements with the help of rules which were framed ad hoc (i.e. *sūtras*)....

Historically speaking, Pāṇini's method has occupied a place comparable to that held by Euclid's method in Western thought. Scientific developments have therefore taken different directions in India and in the West....

In India, Pāṇini's perfection and ingenuity have rarely been matched outside the realm of linguistics. Just as Plato reserved admission to his Academy for geometricians, Indian scholars and philosophers are expected to have first undergone a training in scientific linguistics...."<sup>4</sup>

**Note:** The word "derived" means "demonstrated" in the case of Euclidean Geometry; it means "generated" in the case of Pāṇini's Grammar (*upapatti* and *niṣpatti*)

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<sup>4</sup>J. F. Staal, Euclid and Pāṇini, Philosophy East and West, 15, 1965, 99-116

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And this system of convention that govern rule application and rule interaction all these go back to Panini and it has deep influence you are not the modern discipline of linked list. In fact many scholar actually acknowledge that place Panini holds in Indian tradition plays taht something analogous to the place you could hold in the Ecuclidean tradition and here is a quotation from stall where he saying Panini is also dividing systematically Sanskrit occurrences from a set of rules.

And Euclid is also deriving a set of propositions from a collection of axioms, but the world deriving will have 2 different mean means in this 2 context, Panini is actually generating valid occurrences of Sanskrit is not proving theorems. Euclid is demonstration is proving theorems in mathematics from a set of postulates.

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## Development of Indian Mathematics I

### Ancient Period (Prior to 500 BCE)

- ▶ *Śulvasūtras* (prior to 800 BCE): The oldest texts of geometry. They give procedures for construction and transformation of geometrical figures and alters (*vedī*) using rope (*rajju*) and gnomon (*śaṅku*).
- ▶ The ancient astronomical *siddhāntas* are from this period.

### Early Classical Period (500 BCE - 500 CE)

- ▶ Pervasive influence of the methodology of Pāṇini's *Aṣṭādhyāyī*
- ▶ Piṅgala's *Chandaḥsūtra* (c.300 BCE) and the development of binary representation and combinatorics
- ▶ Mathematical ideas in Bauddha and Jaina Texts
- ▶ The notion of zero and the decimal place value system
- ▶ Mathematics and Astronomy in *Āryabhaṭīya* (c.499 CE): Most of the standard procedures in arithmetic, algebra, geometry and trigonometry are perfected by this time.

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So in ancient period the ugliest text in mathematics available are the text on construction of higher alters the vedis. These are the sulvasutras, these are the oldest texts of geometry in teh world. They give procedures for construction in transformation of geometrical figures. Then there are ancient astronomical siddhantas which deal with astronomy. When we come to the classical period starting from Panini we then have the chandahsutra of Pingala which initiated combinatorics.

We have some mathematics in the Jaina tradition in the Jaina. Then more crucially the idea of 0 and decimal place value system developed in this period and all these terminated in the mathematics and astronomy that is found in the text Aryabhatiya Aryabhata which was written in 499 of the common error, most of the standard procedure in Arithmetic algebra geometry trigonometry were perfected.

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## Baudhāyana-Śulvasūtra (Prior to 800 BCE)

- ▶ Units of measurement (*Bhūmiparimāṇa*)
- ▶ Marking directions and construction of a square of a given side (*Samacaturaśra-karaṇa*)
- ▶ Construction of a rectangle and isosceles trapezium of given sides
- ▶ Construction of  $\sqrt{2}$  (*Dvikaraṇā*),  $\sqrt{3}$  and  $\left(\frac{1}{\sqrt{3}}\right)$  times a given length
- ▶ The square of the diagonal of a rectangle is the sum of the squares of its sides (*Bhujā-Koṭi-Karṇa-Nyāya* – Oldest Theorem in Geometry)

दीर्घचतुरश्रस्याङ्गयारज्जुः पार्श्वमानी तिर्यङ्मानी च यत् पृथग्भूते कुरुतस्तद्भुजं करोति।

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And many more things which was used in astronomy like the indeterminate equations sign tables, all these things were perfected by the time of Aryabhata. So the ancient sulvasutras deal with lot of things, units of measurement, marking directions, construction of rectangle, square, trapezium, transformation of square, and it has the first oldest statement of geometry the theorem which we attribute commonly, it is called the Bhuja-Koti-Karna-Nyaya in later Indian mathematical text.

It is the sum of the two sides of a rectangle, the square, sum of the squares of two sides of a rectangle is equal to the square of the diagonal. This is the rule I stated in Baudhayana sulvasutras (FL) (()) (10:18) to (()) (10:27).

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## Baudhāyana-Śulvasūtra

- ▶ Construction of squares which are the sum and difference of two squares
- ▶ Transforming a square into a rectangle, isosceles trapezium, isosceles triangle and a rhombus of equal area and vice versa
- ▶ Approximate conversion of a square of side  $a$  into a circle of radius

$$r \approx \left(\frac{a}{3}\right) (2 + \sqrt{2}). [\pi \approx 3.0883]$$

- ▶ An approximation for  $(2)^{\frac{1}{2}}$  (*dvikaraṇā*):

$$\sqrt{2} \approx 1 + \frac{1}{3} + \frac{1}{3.4} - \frac{1}{3.4.34} = 1.4142156$$

- ▶ Positions, relative distances and areas of altars. Shapes of different altars and their construction.

There are even more complicated rules of adding squares, then there is a rule for approximate conversion of square into a circle which leads to a value of 5 around 3.08. Then there is a very interesting formula for square root of 2 is called dvikarana in it is accurate of 2 several decimal places that you can see. Finally all this geometry is used in constructing all types.

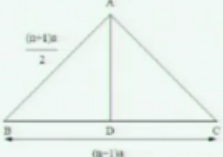
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**Kātyāyana-Śulvasūtra**

**To construct a square which is  $n$ -times a given square**

यावत्प्रमाणानि समचतुरश्राण्येकीकर्तुं चिकिर्षत् एकोनानि तानि भवन्ति तिर्यक् द्विगुणान्येकत एकाधिकानि। त्र्यसिर्भवति तस्येपुस्तत्करोति। (कात्यायनशुल्वसूत्रम् ६.७)

As many squares as you wish to combine into one, the transverse line will be one less than that. Twice the side will be one more than that. That will be the triangle. Its arrow (altitude) will produce that.



$$AD^2 = AB^2 - BD^2$$


$$= \left[ \frac{(n+1)a}{2} \right]^2 - \left[ \frac{(n-1)a}{2} \right]^2$$

$$= na^2.$$

This is the rule in Katyana sulvasutra the problem is how to construct a square which is  $n$  times the area of a given square and Katyana sulvasutra gives a very interesting geometrical formula  $n+1a/2$  whole squared- $n-1a/2$  whole square is  $na$  square. It is using this very interesting algebra it result to calculate the side of a square which is  $n$  times in area of the given square.

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**Varṇa-Merū of Piṅgala**



The number of metrical forms with  $r$  gurus (or laghus) in the *prastāra* of metres of  $n$ -syllables is the binomial coefficient  ${}^n C_r$ .

Halāyudha's commentary (c.950) on *Piṅgala-sūtras* (c.300 BCE) explains the basic rule for the construction of the above table, which is the recurrence relation

$${}^n C_r = {}^{n-1} C_{r-1} + {}^{n-1} C_r$$

Pingala sutra are the combinatorics and these are a very interesting diagram known as the Meru prastara which appears in Pinglas (FL) it gives you the what we now call as the binomial coefficients NCR. They arise very naturally when you want to count how many metres are there which are n syllables but in which are number of groups appears that is NCR as we shall see later.

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### Decimal Place Value System

The Indian Mathematicians developed the decimal place value system along with the notion of the zero-number.

The place value system is essentially an algebraic concept:  
 $5203 = 5 \cdot 10^3 + 2 \cdot 10^2 + 0 \cdot 10 + 3$  is analogous to  $5x^3 + 2x^2 + 0x + 3$

It is this algebraic technique of representing all numbers as polynomials of a base number, which makes all the calculations systematic and simple.

The algorithms developed in India for multiplication, division and evaluation of square, square-root, cube and cube-root, etc., have become the standard procedures. They have contributed immensely to the simplification and popularisation of mathematics the world over.

Sometimes, the Indian texts also discuss special techniques of calculation which are based on the algebraic formalism underlying the place value system. For instance, the *Buddhivilāsini* (c.1540) commentary of Gaṇeśa Daivajña discusses the "vertical and cross-wise" (*vajrabhyāsa*) technique of multiplication.

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The decimal place value system arose in the ancient period the main thing about the decimal place value system is that is an essential in algebraic concept, the number 52038 written as 5 times 10 cube, 2 times 10 square and 0 times 10+3 is something I think to a algebraic polynomial  $5x^3 + 2x^2 + 0x + 3$ , it is this algebraic future of place value system that enabled the Indian mathematicians to give systematic and very interesting procedure for making calculations.

And they became the standard methods of calculation all the world over. Sometimes the Indian books do give some special techniques also which are essentially originating out of the place value systems, for instance the forming the buddhivilasini of Ganesa Daivajna Lilavati. It discusses what is currently popularly known as the vajrabhyasa method of multiplication vertical and cross wise method multiplication.

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## Development of Decimal Place Value System

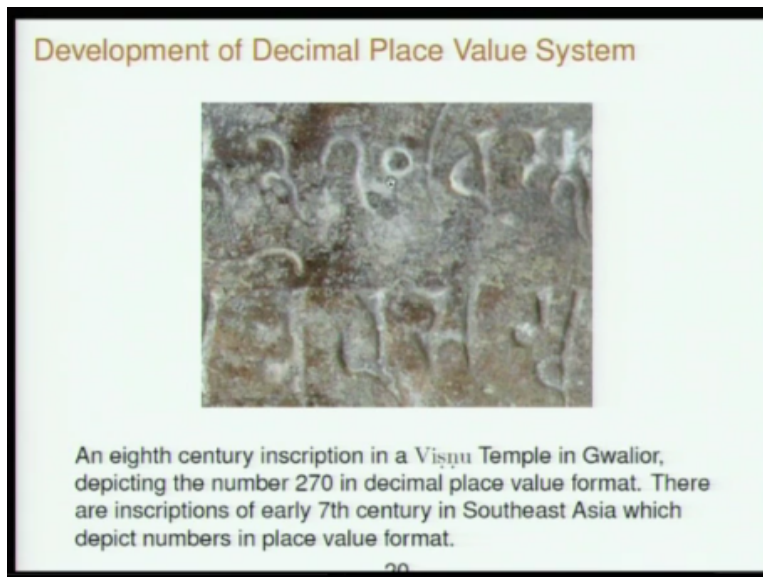
- ▶ The *Yajurveda-Saṃhitā* talks of powers of 10 up to  $10^{12}$  (*parārdha*).
- ▶ The *Upaniṣads* talk of zero (*śūnya*, *kha*) and infinity (*Pūrṇa*).
- ▶ Pāṇini's *Aṣṭādhyāyī* uses the idea of zero-morpheme (*lopa*).
- ▶ The Bauddha and Naiyāyika philosophers discuss the notions of *śūnya* and *abhāva*.
- ▶ Piṅgala's *Chandaśhāstra* uses zero as a marker (*Rupe śūnyam*).
- ▶ Philosophical works such as the works of Vasumitra (c.50 CE) and *Vyāsaśhāstra* on *Yogasūtra* refer to the way the same symbol acquires different meanings in the place value system.  
यथैका रेखा शतस्थाने शतं दशस्थाने दश एका च एकस्थाने यथा चैकत्वेपि स्त्री माता चोच्यते दहिता च स्वसा चेति।
- ▶ Amongst the works whose dates are well established, decimal place value system occurs for the first time in the *Vṛddhayanajātaka* (c.270 CE) of Sphūjīdhvaja.
- ▶ *Āryabhaṭīya* (499 CE) of Āryabhaṭa presents all the standard methods of calculation based on the place value system.

The history of decimal place value system goes back to the Vedas, they use the system to the base 10 very naturally. The upanisads talk of zero and infinity panini's Astadhyayi has a notion of lopa which is I think what is called as (FL) this idea of (FL) in Bauddha philosophy, the idea of abhava in the naiyayika philosophy. Pingala's Chandahsustra uses a 0 as a marker which not a clear whether at that time the idea of 0 as a number was no.

Now soon enough the idea of place value system became so common that philosophical works such as vasumitras with this text and even (FL) and yogasutra started explaining the speciality of the place value system. There is a quotation from the vyasabhasya on Yogasutra (FL) (()) (14:00) to (()) (14:14) just as NAD is understood as a mother, daughter-in-law or a sister LI which appears at different places that is number 1 which appears at different places will have different values and they got 10.

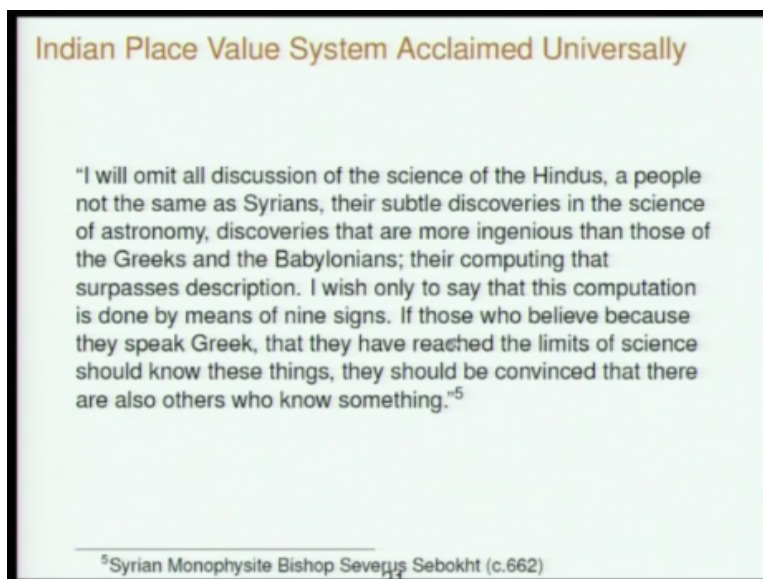
So like this, this issue became well known in the circles of philosophy also and got discussed and one of the oldest place value system explicitly is in a book called Vrdhayanajataka written by Sphujidhvaja around 270 in the common era and by the time of Aryabhata Aryabhatiya all calculations are formulated with the place value system.

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Here an inscription in Gwalior which is giving the number 270 it means it is appearing here and the many other inscriptions in southeast Asia in Gwalior in various other places around early 7th century which give numbers in the place value system with 0 also.

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Now this Indian place value system acclaimed universally this statement in the 7th Century by a Syrian (FL) who is this out of saying that the Greek seem to think too much of themselves but they really do not know the basic methods of calculation that Indians have discovered and they better know that the others also know something of science.

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**Indian Place Value System Acclaimed Universally**

“By the time I was ten I had mastered the Koran and a great deal of literature, so that I was marveled at for my aptitude. . . Now my father was one of those who has responded to the Egyptian propagandist (who was an Ismaili); he, and my brother too, had listened to what they had to say about the Spirit and the Intellect, after the fashion in which they preach and understand the matter. . . Presently they began to invite me to join the movement, rolling on their tongues talk about philosophy, geometry, Indian arithmetic; and my father sent me to a certain vegetable-seller who used the Indian arithmetic, so that I might learn it from him.”<sup>6</sup>

<sup>6</sup>From *The Autobiography* of the Islamic Philosopher Scientist Ibn Sina (980-1037)

Here is (FL) a very famous philosopher in Asian region in 10th century he is saying that he learn the methods of calculation the Indian methods of calculation from a vegetable vendor. So this was the day that the place value system really revolutionize calculation all over the world.

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**Indian Place Value System Acclaimed Universally**

“It is India that gave us the ingenious method of expressing all numbers by means of ten symbols, each symbol receiving a value of position as well as an absolute value; a profound and important idea which appears so simple to us now that we ignore its true merit. But its very simplicity and the great ease which it has lent to all computations put our arithmetic in the first rank of useful inventions; and we shall appreciate the grandeur of this achievement the more when we remember that it escaped the genius of Archimedes and Apollonius, two of the greatest men produced by antiquity.”<sup>7</sup>

“To what height would science now have been if Archimedes made that discovery [place value system]!”<sup>8</sup>

<sup>7</sup>Pierre-Simon Laplace  
<sup>8</sup>Carl Friedrich Gauss

This more modern quotation by Laplace and Gauss saying that this is indeed one of the most wonderful discoveries in the history of mathematics.

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### *Gaṇitapāda of Āryabhaṭīya (499 CE)*

The following topics are dealt with in 33 verses of *Gaṇitapāda* of *Āryabhaṭīya*:

- ▶ *Samkhyāsthāna*: Place values.
- ▶ *Vargaparikarma, ghanaparikarma*: Squaring and cubing.
- ▶ *Vargamūlānāyana*: Obtaining the square-root.
- ▶ *Ghanamūlānāyana*: Obtaining the cube-root.
- ▶ Area of a triangle and volume of an equilateral tetrahedron.
- ▶ Obtaining the area of a circle, volume of a sphere.
- ▶ Obtaining the area of a trapezium.
- ▶ Chord of a sixth of the circumference.
- ▶ Approximate value of the circumference ( $\pi \approx 3.1416$ )

24

Now by the time when you come to Aryabhatiya in 500 CE, it discusses the what is called as what is called as parikarma logistics methods of calculation where square, square root, cube, cube root, areas of triangle, circle, trapezium, approximate value of Pi computing sine tables.

**(Refer Slide Time: 16:41)**

### *Gaṇitapāda of Āryabhaṭīya*

- ▶ *Jyānāyana*: Computing table of Rsines
- ▶ *Chāyā-karma*: Obtaining shadows of gnomons.
- ▶ *Karṇānāyana*: Square of the hypotenuse is the sum of the squares of the sides.
- ▶ *Śarānāyana*: Arrows of intercepted arcs
- ▶ *Śreḍhī-gaṇita*: Summing an AP, finding the number of terms, repeated summations
- ▶ *Varga-ghana-saṅkalanānāyana*: Obtaining the sum of squares and cubes of natural numbers.
- ▶ *Mūlaphalānāyana*: Interest and principal
- ▶ *Trairāśika*: Rule of three

25

Problems to do with interceptor arch in a circle, progressions, rule of three.

**(Refer Slide Time: 16:45)**

## Gaṇitapāda of Āryabhaṭīya

- ▶ *Jyānāyana*: Computing table of Rsines
- ▶ *Chāyā-karma*: Obtaining shadows of gnomons.
- ▶ *Karṇānāyana*: Square of the hypotenuse is the sum of the squares of the sides.
- ▶ *Śarānāyana*: Arrows of intercepted arcs
- ▶ *Średhī-gaṇita*: Summing an AP, finding the number of terms, repeated summations
- ▶ *Varga-ghana-saṅkalanānāyana*: Obtaining the sum of squares and cubes of natural numbers.
- ▶ *Mūlaphalānāyana*: Interest and principal
- ▶ *Trairāśika*: Rule of three

25

Arithmetic of fractions and finally something very interesting called as kuttakara which was Aryabhata one invention is the method of solving linear indeterminate equation.

(Refer Slide Time: 17:04)

### Āryabhaṭa's Sine Table

θ in min.	R sin θ according to		
	Āryabhaṭīya	Govindaswami	Mādhava(also Modern)
225	225	224 50 23	224 50 22
450	449	448 42 53	448 42 58
675	671	670 40 11	670 40 16
900	890	889 45 08	889 45 15
1125	1105	1105 01 30	1105 01 39
1350	1315	1315 33 56	1315 34 7
1575	1530	1530 28 22	1530 28 35
1800	1719	1718 52 10	1718 52 24
2025	1910	1909 54 19	1909 54 35
2250	2093	2092 45 46	2092 46 03
2475	2267	2266 38 44	2266 39 59
2700	2431	2430 59 54	2430 54 15
2925	2585	2584 37 43	2584 38 06
3150	2728	2727 29 29	2727 29 52
3375	2859	2858 22 31	2858 22 55
3600	2978	2977 19 09	2977 19 34
3825	3084	3083 12 51	3083 13 17
4050	3177	3175 03 23	3176 03 50
4275	3256	3255 17 54	3255 18 22
4500	3321	3320 36 02	3320 36 30
4725	3372	3371 41 01	3371 41 29
4950	3409	3408 19 42	3408 20 11
5175	3431	3430 22 42	3430 23 11
5400	3438	3437 44 19	3437 44 48

26

So this is the kind of sine table that Aryabhata came up with and details later on being systematically including India, this is by Govindaswamy in 9 century and this improved table is due to Madhava.

(Refer Slide Time: 17:17)



### Gandhayukti of Varāhamihira

Chapter 76 of the great compilation *Bṛhatsaṃhitā* of Varāhamihira (c.550) is devoted to a discussion of perfumery. In verse 20, Varāha mentions that there are 1,820 combinations which can be formed by choosing 4 perfumes from a set of 16 basic perfumes ( ${}^{16}C_4 = 1820$ ).

षोडशके द्रव्यगणे चतुर्विकल्पेन भिद्यमानानाम्।  
अष्टादश जायन्ते शतानि सहितानि विंशत्या॥

In verse 22, Varāha gives a method of construction of a *meru* (or a tabular figure) which may be used to calculate the number of combinations. This verse also very briefly indicates a way of arranging these combinations in an array or a *prustāru*.

पूर्वेण पूर्वेण गतेन युक्तं स्थानं विनान्त्यं प्रवदन्ति सङ्ख्याम्।  
इच्छाविकल्पैः क्रमशोऽभिनीय नीते निवृत्तिः पुनरन्यनीतिः॥

Bhaṭṭotpala (c.950) in his commentary has explained both the construction of the *meru* and the method of *loṣṭaprustāru* of the combinations.

When we come to the later period we have luminaries like (FL) (()) (17:20) to (()) (17:25) Aryabhata, brahmagupta one of the most celebrated 25th in India.

(Refer Slide Time: 17:31)

### Gandhayukti of Varāhamihira

16			
15	120		
14	105	560	
13	91	455	1820
12	78	364	1365
11	66	286	1001
10	55	220	715
9	45	165	495
8	36	120	330
7	28	84	210
6	21	56	126
5	15	35	70
4	10	20	35
3	6	10	15
2	3	4	5
1	1	1	1

In the first column the natural numbers are written. In the second column, their sums, in the third the sums of sums, and so on. One row is reduced at each step. The above *meru* is based on the relation.

$${}^nC_r = {}^{n-1}C_{r-1} + {}^{n-2}C_{r-1} + \dots + {}^{r-1}C_{r-1}$$

In varahamihira which is a compendium there is a chapter on human and there he is introducing combinatorics idea and he is explaining that 1820 various perfumes can be formed by choosing 4 out of a collection of 16 and to calculate the  ${}^{16}C_4$  he gives a different kind of (FL) he is giving a different kind of a table here the first column natural integers, the second column some of natural integers.

The third column is the sum of sums of natural integers, fourth column is and its based upon the reconciliation which is equivalent to (FL).

(Refer Slide Time: 18:19)



### Brāhmasphuṭasiddhanta of Brahmagupta

Topics dealt with in Chapter XVIII, *Kuṭṭakādhyāya* (Algebra)

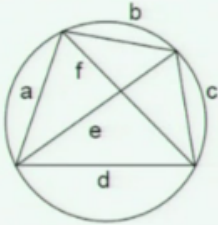
- ▶ Solutions of linear indeterminate equations by *kuṭṭaka* process and its applications in astronomical problems
- ▶ Rule of signs and arithmetic of zero
- ▶ Surds (*karṣṇi*)
- ▶ Operations with unknowns (*varṇa-śaḍvidha* or *avyakta-śaḍvidha*)
- ▶ Equations with single unknown (*ekavarṇa-śamīkaraṇa*)
- ▶ Elimination of middle term in quadratic equations (*madhyamāharaṇa*)
- ▶ Equations with several unknowns (*anekavarṇa-śamīkaraṇa*)
- ▶ Equations with products of unknowns (*bhāvita*)
- ▶ *Vargaprakṛti*: Second order indeterminate equation  $x^2 - Dy^2 = 1$ . *Bhāvanā* and applications to finding rational and integral solutions.
- ▶ Various problems

22

Brahmasphutasiddhanta of Brahmagupta is a text on astronomy, it has 2 chapter in mathematics, chapter 12 when 13 is called ganitagya and chapter 17 is called (FL), chapter 12 is ganitadhyaya, 17 is (FL) ideas with most ideas in Algebra, in brahmagupta for the first time find the Arithmetic of negative qualities calculations with zero and then detailed statement of equations and even introduction of complicated equations known as the vargaprakrti which became a very important equation in the Indian mathematical tradition.

**(Refer Slide Time: 18:59)**

### Brahmagupta's Formulae for Cyclic Quadrilaterals



The diagonals  $e, f$  are given in terms of the sides  $a, b, c, d$ , by the formulae

$$e = \sqrt{\frac{(ab + bc)(ac + bd)}{ab + cd}}, \quad f = \sqrt{\frac{(ab + cd)(ac + bd)}{ab + bc}}$$

The area is given by

$$A = [(s - a)(s - b)(s - c)(s - d)]^{\frac{1}{2}} \text{ with } s = \frac{(a + b + c + d)}{2}$$

24

Brahmagupta also gave very interesting result such as this equation of the diagonals of a cyclic quadrilateral, a quadrilateral which is inscribed in a circle, he gave a formula for the diagonals of a cyclic quadrilateral in terms of the sites and an expression for the area of the circle quadrilateral which is a generalization of the formula that perhaps all of you know as the heroines formula for the area of a triangle.

(Refer Slide Time: 19:36)

**Brahmagupta's *Bhāvanā***

मूलं द्विघेष्टवर्गाद् गुणकगुणादिष्टयुतविहीनाच्च ।  
आदावधो गुणकगुणः सहान्त्यघातेन कृतमन्त्यम् ॥  
वज्रवधैकां प्रथमं प्रक्षेपः क्षेपवधतुल्यः ।  
प्रक्षेपशोधकहृते मूले प्रक्षेपके रूपे ॥

If  $X_1^2 - D Y_1^2 = K_1$  and  $X_2^2 - D Y_2^2 = K_2$  then

$$(X_1 X_2 \pm D Y_1 Y_2)^2 - D(X_1 Y_2 \pm X_2 Y_1)^2 = K_1 K_2$$

In Particular given  $X^2 - D Y^2 = K$ , we get the rational solution

$$[(X^2 + D Y^2)/K]^2 - D[(2XY)/K]^2 = 1$$

Also, if one solution of the equation  $X^2 - D Y^2 = 1$  is found, an infinite number of solutions can be found, via

$$(X, Y) \rightarrow (X^2 + D Y^2, 2XY)$$

25

Brahmagupta course mentioned that this formula is applicable to quadrilateral in triangle. He gave some interesting properties of equations of the kind, these are called the varga prakryathi equation. He was the first person to call me later property called as Bhavana, the given 1 solution you can go to another solution we will discuss this later during the course.

(Refer Slide Time: 19:52)

***Cakravāla* Algorithm of Bhāskarācārya II (c.1150)**

To solve  $X^2 - D Y^2 = 1$

Set  $X_0 = 1, Y_0 = 0, K_0 = 1$  and  $P_0 = 0$ .

Given  $X_i, Y_i, K_i$  such that  $X_i^2 - D Y_i^2 = K_i$

First find  $P_{i+1}$  so as to satisfy:

(I)  $P_i + P_{i+1}$  is divisible by  $K_i$

(II)  $|P_{i+1}^2 - D|$  is minimum.

Then set

$$K_{i+1} = \frac{(P_{i+1}^2 - D)}{K_i}$$

$$Y_{i+1} = \frac{(Y_i P_{i+1} + X_i)}{|K_i|}, X_{i+1} = \frac{(X_i P_{i+1} + D Y_i)}{|K_i|}$$

These satisfy  $X_{i+1}^2 - D Y_{i+1}^2 = K_{i+1}$

Iterate till  $K_{i+1} = \pm 1, \pm 2$  or  $\pm 4$ , and then use *bhāvanā* if necessary.

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But this enabled later on Indian mathematician to work out every systematic algorithm, this one of the most famous algorithms in Indian mathematics called as Cakravala.

(Refer Slide Time: 20:04)

### Bhāskara's Example: $X^2 - 61Y^2 = 1$

i	$P_i$	$K_i$	$a_i$	$\varepsilon_i$	$X_i$	$Y_i$
0	0	1	8	1	1	0
1	8	3	5	-1	8	1
2	7	-4	4	1	39	5
3	9	-5	3	-1	164	21

To find  $P_1$  :  $0 + 7, 0 + 8, 0 + 9 \dots$  divisible by 1. Of them  $8^2$  closest to 61. Hence,  $P_1 = 8, K_1 = 3$

To find  $P_2$  :  $8 + 4, 8 + 7, 8 + 10 \dots$  divisible by 3. Of them  $7^2$  closest to 61. Hence,  $P_2 = 7, K_2 = -4$

After the second step, we have:  $39^2 - 61 \times 5^2 = -4$

Since  $K = -4$ , we can use *bhāvanā* principle to obtain

$$X = (39^2 + 2) \left[ \left( \frac{1}{2} \right) (39^2 + 1)(39^2 + 3) - 1 \right] = 1,766,319,049$$

$$Y = \left( \frac{1}{2} \right) (39 \times 5)(39^2 + 1)(39^2 + 3) = 226,153,980$$

$$1766319049^2 - 61 \times 226153980^2 = 1$$

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And it enables you to solve equations. This is a very famous problem  $x^2 - 61y^2 = 1$ . You have to solve for X and Y in integers and as you can see these solutions are about  $1.766 \times 10^9$  and 226 million. So these are very high numbers, this lowest solution of this equation, after Bhaskaracharya who in his book 1150 solve this equation by a very simple method this table tells you the method.

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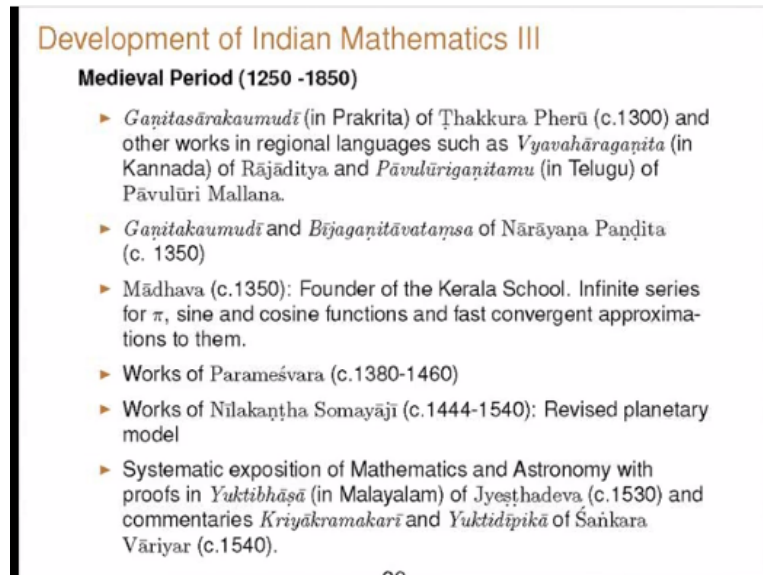
### *Tātkālika-gati*: Instantaneous Velocity of a Planet

- ▶ Approximate formula for velocity (*manda-gati*) in terms of Rsine-differences was given by Bhāskara I (c.630) and he also comments on its limitation (*Laghu-bhāskarīya* 2.14-15).
- ▶ True velocity (*sphuṭa-manda-gati*) in terms of Rcosine (as the derivative of Rsine) is given in *Laghu-mānasa* of Muñjāla (c. 932) and *Mahā-siddhānta* of Āryabhaṭa II (c. 950).
- ▶ Bhāskara II (c.1150) discusses the notion of instantaneous velocity (*tātkālika-gati*) and contrasts it with the so-called true daily motion. He also evaluates the *manda-gati* and *śighra-gati* (*Vāsanā* on *Siddhānta-Śiromaṇi* 2.37-39).
- ▶ Bhāskara II notes the relation between maximum equation of centre (correction to displacement) and the vanishing of velocity correction (*Vāsanā* on *Siddhānta-Śiromaṇi*, *Gola* 4.3).

This problem again came up 500 years later when bharma post this as a problem to the British mathematician. Ideas of calculus started developing and they arose in the context of astronomy, the idea instantaneous velocity became important because especially to understand the motion of moon one needed to know the rate of variation of its position and one found that even the rate of change of its position was continuously changing.

And the idea of instantaneous velocity arose this way. Now there is a common misconception 6 years ago in modern times that Bhaskaracharya II 11 AD was the last important mathematician in Indian mathematics afterwards the people were just repeating what was done in earlier books or they forgotten mathematics all together. It is only in the last 56 years that works of later mathematicians have been studied.

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**Development of Indian Mathematics III**

**Medieval Period (1250 -1850)**

- ▶ *Gaṇitasāraśaṅgī* (in Prakṛita) of Ṭhakkura Pherū (c.1300) and other works in regional languages such as *Vyavahāraṅgī* (in Kannada) of Rājāditya and *Pāvulūriṅgī* (in Telugu) of Pāvulūri Mallana.
- ▶ *Gaṇitakaumudī* and *Bījagaṇitāvataṃsa* of Nārāyaṇa Paṇḍita (c. 1350)
- ▶ Mādhava (c.1350): Founder of the Kerala School. Infinite series for  $\pi$ , sine and cosine functions and fast convergent approximations to them.
- ▶ Works of Parameśvara (c.1380-1460)
- ▶ Works of Nilakaṇṭha Somayāji (c.1444-1540): Revised planetary model
- ▶ Systematic exposition of Mathematics and Astronomy with proofs in *Yuktibhāṣā* (in Malayalam) of Jyeṣṭhadeva (c.1530) and commentaries *Kriyākramakarī* and *Yuktidīpikā* of Śaṅkara Vāriyar (c.1540).

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And understood and actually the picture is quite different , first of all around 1200 works in mathematics started at caring in regional language ganitasarakaumudi in Prakrita, Vyavaharaganita in Kannada, Pavuluriganitamu in Telugu. These are very important was written around 13 century, 12<sup>th</sup> century. Ganitakaumudi and Narayana Pandita is a great advance of bhaskaracharya Lilavati a large part of course will be devoted to study of that.

Then there arose a school in Kerala which had been special contributions to make the Kerala school of astronomy initiated by Madhava, then parameswaran then nilakantha somayaji. They revise the older astronomical model and came up with a new astronomical model, but Madhava is more well known for his discovery of infinite series for PI, sine and cosine and their the proofs of all these results of Madhava written down in a very famous Malayalam book called Yuktibhasa written in 1530.

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### Development of Indian Mathematics III (contd.)

**Medieval Period (1250 - 1850)**

- ▶ Works of Jñānarāja (c.1500), Gaṇeśa Daivajña (b.1507), Sūryadāsa (c.1541) and Kṛṣṇa Daivajña (c.1600): Commentaries with *upapattis*
- ▶ Works of Munīśvara (b.1603) and Kamalākara (b.1616)
- ▶ Mathematics and Astronomy in the Court of Savai Jayasimha (1700-1743). Translation from Persian of Euclid and Ptolemy.
- ▶ Works of later Kerala astronomers Acyuta Piṣārati (c.1550-1621), Putumana Somayajī (c.1700) and Śaṅkaravarman (c. 1830)
- ▶ Candrasekhara Sāmanta of Orissa: All the major lunar inequalities (1869)

Mathematics continues in Maharashtra and Kasi with scholars such as Jnanaraja, Ganesa Daivajna, Suryadasa. They wrote proofs on Bhaskara result trigonometrically result were discovered by Munisvara Kamalakar, Savai Jayasimha in Jaipur he built this 5 observatories which was very important at that time to correct the older astronomical calculations. The Kerala school also continued the last back was sankaravarman (FL) in 1830.

**(Refer Slide Time: 23:23)**

### Nārāyaṇa's Folding Method

**To construct 4x4 square adding to 40**

*Sampuṭikaraṇa* (folding) gives

2+15	3+10	2+0	3+5	=	17	13	2	8
1+0	4+5	1+15	4+10		1	9	16	14
3+15	2+10	3+0	2+5		18	12	3	7
4+0	1+5	4+15	1+10		4	6	19	11

Nārāyaṇa also displays the other square which is obtained by interchanging the coverer and the covered.

**Note:** This method leads to a pan-diagonal magic square. That is, the broken diagonals also add up to the same magic sum.

There was an astronomer called Candrasekhara Samanta in Orissa who impact the game by traditional method. All the major lunar inequalities in 1869. So just to tell you the kind of that Narayana Pandita even considered topics like magic squares as serious mathematical topics and came up with very interesting way of constructing magic squares several very new algorithms.

**(Refer Slide Time: 23:46)**



### Mādhava Series for $\pi$ and End-correction Terms

The following verses of Mādhava are cited in *Yuktibhāṣā* and *Kriyākramakarī*, which also present a detailed derivation of the relation between diameter and the circumference:

व्यासे वारिधिनिहते रूपहृते व्याससागराभिहते।  
 त्रिशरादिविषमसङ्ख्याभक्तमृणं स्वं पृथक् क्रमात् कुर्यात् ॥ १ ॥  
 यत्सङ्ख्यायाऽत्र हरणे कृते निवृत्ता हतिस्तु जामितया।  
 तस्या ऊर्ध्वगता या समसङ्ख्या तद्वलं गुणोऽन्ते स्यात् ॥ २ ॥  
 तद्वर्गो रूपयुतो हारो व्यासाब्धिघाततः प्राग्वत्।  
 ताभ्यामाप्तं स्वमृणे कृते धने क्षेप एव करणीयः ॥ ३ ॥  
 लब्धः परिधिः सूक्ष्मो बहुकृत्वो हरणतोऽतिसूक्ष्मः स्यात् ॥ ४ ॥

The first verse gives the Mādhava series

$$Paridhi = 4 \times Vyāsa \times \left( 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots \right)$$

This is what is called is the folding method of calculating magic squares. This is the infinite series for the ratio of the circumference to diameter discovered by by Madhava the Kerala mathematician.

(Refer Slide Time: 23:54)

### Mādhava Series for $\pi$ and End-correction Terms

The Mādhava series for the circumference of a circle (in terms of odd numbers  $p = 1, 3, 5, \dots$ ) can be written in the form

$$C = 4d \left[ 1 - \frac{1}{3} + \dots + (-1)^{\frac{(p-1)}{2}} \frac{1}{p} + \dots \right]$$

This is an extremely slowly convergent series. In order to facilitate computation, Mādhava has given a procedure of using end-correction terms (*antya-saṃskāra*), of the form

$$C = 4d \left[ 1 - \frac{1}{3} + \dots + (-1)^{\frac{(p-1)}{2}} \frac{1}{p} + (-1)^{\frac{(p+1)}{2}} \frac{1}{a_p} \right]$$

In fact, the famous verses of Mādhava, which give the relation between the circumference and diameter, also include an end-correction term

$$C = 4d \left[ 1 - \frac{1}{3} + \dots + \dots (-1)^{\frac{(p-1)}{2}} \frac{1}{p} \right. \\ \left. + (-1)^{\frac{(p+1)}{2}} \frac{\left\{ \frac{p+1}{2} \right\}}{\{(p+1)^2 + 1\}} \right]$$

He not only discovered the infinite series that is a slowly convergent series that 1-1/3+1/5/1/7. If we calculate 50 tons of that series you get only one decimal place in the expansion of Pi. So Madhava at the same time gave what are known as the end correction terms. So this is the first end connection term due to Madhava, then there is another end correction term.

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## Mādhava Series for $\pi$ and End-correction Terms

Mādhava has also given a finer end-correction term

अन्ते समसङ्खादलवर्गः सैको गुणः स एव पुनः ॥  
युगगुणितो रूपयुतः समसङ्खादलहतो भवेद् हारः ।

$$C = 4d \left[ 1 - \frac{1}{3} + \dots + \dots (-1)^{\frac{(p-1)}{2}} \frac{1}{p} \right] \\ + (-1)^{\frac{(p+1)}{2}} \frac{\left[ \left( \frac{p+1}{2} \right)^2 + 1 \right]}{\left[ ((p+1)^2 + 5) \left( \frac{p+1}{2} \right) \right]}$$

17

It is this end correction terms which give you more accurate and more accurate result even if you sum only 50 terms in the Madhava series incidentally that series due to Madhava is also known as the (FL) series.

(Refer Slide Time: 24:32)

## Mādhava Series for $\pi$ and End-correction Terms

The following verses of Mādhava are cited in *Yuktibhāṣā* and *Kriyākramakarī*, which also present a detailed derivation of the relation between diameter and the circumference:

व्यासे वारिधिनिहते रूपहृते व्याससागराभिहते ।  
त्रिशरादिविषमसङ्खाभक्तमृणं स्वं पृथक् क्रमात् कुर्यात् ॥ १ ॥  
यत्सङ्ख्याऽत्र हरणे कृते निवृत्ता हतिस्तु जामितया ।  
तस्या ऊर्ध्वगता या समसङ्खा तदलं गुणोऽन्ते स्यात् ॥ २ ॥  
तद्वर्गो रूपयुतो हारो व्यासाब्धिघाततः प्राग्वत् ।  
ताभ्यामाप्तं स्वमृणे कृते धने श्लेष एव करणीयः ॥ ३ ॥  
लब्धः परिधिः सूक्ष्मो बहुकृत्वो हरणतोऽतिसूक्ष्मः स्यात् ॥ ४ ॥

The first verse gives the Mādhava series

$$Paridhi = 4 \times Vyāsa \times \left( 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots \right)$$

15

Because it discovered by (FL) 1674.

(Refer Slide Time: 24:40)

## Mādhava Series for $\pi$ and End-correction Terms

To Mādhava is attributed a value of  $\pi$  accurate to eleven decimal places which is obtained by just computing fifty terms with the above correction.

विबुधनेत्रगजाहिहृताशनत्रिगुणवेदभवारणवाहवः ।  
नवनिखर्वमिते वृतिविस्तरे परिधिमानमिदं जगदुर्बुधाः ॥

The  $\pi$  value given above is:

$$\pi \approx \frac{2827433388233}{9 \times 10^{11}} = 3.141592653592\dots$$

40

So using his connection Madhava was able to give the value of pi correctly to 30 11 decimal places just by using 50 terms in his series with that end correction term.

(Refer Slide Time: 24:52)

## A History of Approximations to $\pi$

	Approximation to $\pi$	Accuracy (Decimal places)	Method Adopted
Rhind Papyrus - Egypt (Prior to 2000 BCE)	$\frac{256}{81} = 3.1604$	1	Geometrical
Babylon (2000 BCE)	$\frac{25}{8} = 3.125$	1	Geometrical
<i>Sūlvasūtras</i> (Prior to 800 BCE)	3.0883	1	Geometrical
Jaina Texts (500 BCE)	$\sqrt{(10)} = 3.1623$	1	Geometrical
Archimedes (250 BCE)	$3\frac{10}{71} < \pi < 3\frac{1}{7}$	2	Polygon doubling (6.2 <sup>4</sup> = 96 sides)
Ptolemy (150 CE)	$3\frac{17}{120} = 3.141666$	3	Polygon doubling (6.2 <sup>6</sup> = 384 sides)
Lui Hui (263)	3.14159	5	Polygon doubling (6.2 <sup>9</sup> = 3072 sides)
Tsu Chhung-Chih (480?)	$\frac{355}{113} = 3.1415929$	6	Polygon doubling (6.2 <sup>9</sup> = 12288 sides)
Āryabhaṭa (499)	$\frac{62832}{20000} = 3.1416$	4	Polygon doubling (4.2 <sup>9</sup> = 1024 sides)

40

So we can briefly sketch this history of pi as typical of the way mathematics developed across different cultures please see Aryabhata's value 3.1416 which is activate up to 4 decimal places that sulvasutra values which is activated up to 1 decimal places, the Jaina text use root 10 Archimedes give this standard is equality 3 10/71 less than Pi, less than 3 1/7, the Chinese mathematician Tsu Chhung Chih had this 355/113 which is accurate up to nearly 7 6-7 decimal places.

(Refer Slide Time: 25:29)

### A History of Approximations to $\pi$

	Approximation to $\pi$	Accuracy (Decimal places)	Method Adopted
Mādhava (1375)	$\frac{2827433388235}{9 \cdot 10^{11}} = 3.141592653592\dots$	11	Infinite series with end corrections
Al Kasi (1430)	3.1415926535897932	16	Polygon doubling (6.2 <sup>27</sup> sides)
Francois Viete (1579)	3.1415926536	9	Polygon doubling (6.2 <sup>16</sup> sides)
Romanus (1593)	3.1415926535...	15	Polygon doubling
Ludolph Van Ceulen (1615)	3.1415926535...	32	Polygon doubling (2 <sup>62</sup> sides)
Wilhebrod Snell (1621)	3.1415926535...	34	Modified Polygon doubling (2 <sup>30</sup> sides)
Grienberger (1630)	3.1415926535...	39	Modified Polygon doubling
Isaac Newton (1665)	3.1415926535...	15	Infinite series

But fact to this please see Madhava coming up 11 decimal places between Aryabhata to Madhava. Then Madhavas result was based upon infinity series, all these most of these results are actually based upon root force calculation with approximating the area of a circle by polygon Al Kasi etc. Newton again came up with an infinite series around 1665.

**(Refer Slide Time: 25:56)**

### A History of Approximations to $\pi$

Abraham Sharp (1699)	3.1415926535...	71	Infinite series for $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$
John Machin (1706)	3.1415926535...	100	Infinite series relation $\frac{\pi}{4} = 4\tan^{-1}\left(\frac{1}{5}\right) - \tan^{-1}\left(\frac{1}{239}\right)$
Ramanujan (1914), Gosper (1985)		17 Million	Modular Equation
Kondo, Yee (2010)		5 Trillion	Modular Equation

Then the various other thing, but we can see in recent time Ramanujan in 1914 came up with a very interesting series for Pi using modular equation and that created a small recorded that act at 1980s that people calculated Pi to about 17 million decimal places, today's achievement is about 5 trillion.

**(Refer Slide Time: 26:19)**

## A History of Exact Results for $\pi$

Mādhava (1375)	$\pi/4 = 1 - 1/3 + 1/5 - 1/7 + \dots$ $\pi/\sqrt{12} = 1 - 1/3 \cdot 3 + 1/3^2 \cdot 5 - 1/3^3 \cdot 7 + \dots$ $\pi/4 = 3/4 + 1/(3^3 - 3) - 1/(5^3 - 5) + 1/(7^3 - 7) - \dots$ $\pi/16 = 1/(1^5 + 4 \cdot 1) - 1/(3^5 + 4 \cdot 3) + 1/(5^5 + 4 \cdot 5) - \dots$
Francois Viète (1593)	$\frac{2}{\pi} = \sqrt{1/2} \sqrt{1/2 + 1/2 \sqrt{1/2}}$ $\sqrt{1/2 + 1/2 \sqrt{1/2 + 1/2 \sqrt{1/2}}}$ ... (infinite product)
John Wallis (1655)	$\frac{4}{\pi} = \left(\frac{3}{2}\right) \left(\frac{3}{4}\right) \left(\frac{5}{4}\right) \left(\frac{5}{6}\right) \left(\frac{7}{6}\right) \left(\frac{7}{8}\right) \dots$ (infinite product)
William Brouncker (1659)	$\frac{4}{\pi} = 1 + \frac{1^2}{2^2} - \frac{3^2}{2^2} + \frac{5^2}{2^2} - \dots$ (continued fraction)
Isaac Newton (1665)	$\pi = \frac{3\sqrt{3}}{4} + 24 \left[ \frac{1}{12} - \frac{1}{5 \cdot 32} + \frac{1}{28 \cdot 128} - \frac{1}{72 \cdot 512} - \dots \right]$

But equal important of this exact results of Pi you can see Madhava all these exact result which was later on repeated by others James Gregory Tan inverse series with series, short series, all these are contained in Madhavas paper.

**(Refer Slide Time: 26:36)**

## A History of Exact Results for $\pi$

James Gregory (1671)	$\tan^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots$
Gottfried Leibniz (1674)	$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$
Abraham Sharp (1699)	$\frac{\pi}{\sqrt{12}} = 1 - \frac{1}{3 \cdot 3} + \frac{1}{3^2 \cdot 5} - \frac{1}{3^3 \cdot 7} + \dots$
John Machin (1706)	$\frac{\pi}{4} = 4 \tan^{-1}\left(\frac{1}{5}\right) - \tan^{-1}\left(\frac{1}{239}\right)$

Ramanujan (1914)

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801} \sum_{k=0}^{\infty} \frac{(4k)!(1103 + 26390k)}{(k!)^4 396^{4k}}$$

This is the series given by Ramanujan in his 1914 paper.

**(Refer Slide Time: 26:43)**

## Nīlakaṇṭha's Formula for Instantaneous Velocity

चन्द्रबाहुफलवर्गशोधितत्रिज्यकाकृतिपदेन संहरेत् ।  
तत्र कोटिफललिसिकाहतां केन्द्रभुक्तिरिह यच्च लभ्यते ॥  
तद्विशोध्य मृगादिके गतेः क्षिप्यतामिह तु कर्कटादिके ।  
तद्भवेत्स्फुटतरा गतिविधोः अस्य तत्समयजा रवेरपि ॥

Nīlakaṇṭha gives the derivative of the second term above in the form

$$\left[ \frac{\left(\frac{r_0}{R}\right) R \cos(m - \alpha)}{\left(R^2 - \left(\frac{r_0}{R}\right)^2 R \sin^2(m - \alpha)\right)^{\frac{1}{2}}} \right] \left[ \left(\frac{d}{dt}\right) (m - \alpha) \right]$$

--

The idea of instantaneous velocity also lead to more complicated derivative, the derivative of sine function as a cosine function was well known by the time of Bhaskara, Nilakantha is formulating that derivative of sine inverse function as 1/square root of 1-x square in this words.

(Refer Slide Time: 27:00)

## Nīlakaṇṭha's Formula for Instantaneous Velocity

चन्द्रबाहुफलवर्गशोधितत्रिज्यकाकृतिपदेन संहरेत् ।  
तत्र कोटिफललिसिकाहतां केन्द्रभुक्तिरिह यच्च लभ्यते ॥  
तद्विशोध्य मृगादिके गतेः क्षिप्यतामिह तु कर्कटादिके ।  
तद्भवेत्स्फुटतरा गतिविधोः अस्य तत्समयजा रवेरपि ॥

Nīlakaṇṭha gives the derivative of the second term above in the form

$$\left[ \frac{\left(\frac{r_0}{R}\right) R \cos(m - \alpha)}{\left(R^2 - \left(\frac{r_0}{R}\right)^2 R \sin^2(m - \alpha)\right)^{\frac{1}{2}}} \right] \left[ \left(\frac{d}{dt}\right) (m - \alpha) \right]$$

55

Now again till about 56 years ago people had study only I mean the modern scholars, had studies only the basic text of Indian mathematics.

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### *Upapattis* in Indian Mathematics

While there have been several extensive investigations on the history and achievements of the Indian mathematics, there has not been much discussion on the Indian mathematicians' and philosophers' understanding of the nature and validation of mathematical results and procedures, their views on the nature of mathematical objects, and so on.

Traditionally, such issues have been dealt with in the detailed *bhāṣyas* or commentaries, which continued to be written till recent times and played a vital role in the traditional scheme of learning. It is in such commentaries that we find detailed *upapattis* or "proofs" of the results and procedures, apart from a discussion of methodological and philosophical issues.

Amongst the available texts of Indian mathematics, a discussion of the way of validating the results (*pratyayakaraṇa*), or of demonstrating them (*upapatti*) is found first in the *Āryabhaṭīyabhāṣya* of Bhāskara I (c.629)

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So they had the sort of idea that Indian somehow enhance lot of results, but they did not seem to have any method for arriving at this result or at least those messages were very obscure. So it is only in the last 56 years that many of the common trees to the original text people started studying traditionally such issues that how to obtain results, how to understand them etc. have been dealt with in detail Bhaskara commentary.

This is not just super mathematics that if you pick up any basic text even Bhagavad Gita to understand it in a very serious manner you have to take requests to the detail common trees which are written on them and this commentaries continue to be returned till recent times they played a very vital role in the traditional schema learning. As per mathematics is concerned.

It is in this comment that we find what are known as upapatti or uptis. They are something similar to demonstration a rational of proofs in mathematics, you one of the oldest words available words which has upapatii is a Bhaskara 1 commentary on Aryabhata.

**(Refer Slide Time: 28:16)**



### *Yuktibhāṣā* of Jyeṣṭhadeva (c.1530)

The most detailed exposition of *upapattis* in Indian mathematics is found in the Malayalam text *Yuktibhāṣā* of Jyeṣṭhadeva, a student of Dāmodara.

At the beginning of *Yuktibhāṣā*, Jyeṣṭhadeva states that his purpose is to present the rationale of the procedures given in the *Tantrasaṅgraha*. Many of these rationales have also been presented (mostly in the form of Sanskrit verses) by Śaṅkara Vāriyar (c.1500-1556) in his commentaries *Kriyākramakārī* (on *Līlāvati*) and *Yuktidīpikā* (on *Tantrasaṅgraha*)

*Yuktibhāṣā* comprising 15 chapters is naturally divided into two parts, Mathematics and Astronomy.

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But of course the most detailed exposition of *upapatti* is found in the Malayalam text *Yuktibhasa* written in 1530.

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### *Bhāskara on Upapatti* (c.1150)

In *Siddhāntaśiromaṇi*, Bhāskarācārya II (1150) presents the *raison d'être* of *upapatti* in the Indian mathematical tradition:

मध्यादां द्वासदां यदत्र गणितं तस्योपपत्तिं विना  
प्रौढिं प्रौढसभासु नैति गणको निःसंशयो न स्वयम्।  
गोले सा विमला कगमलकवत् प्रत्यक्षतो दृश्यते  
तस्मादस्म्युपपत्तिबोधविधये गोलप्रबन्धोदात्तः ॥

Without the knowledge of *upapattis*, by merely mastering the calculations (*ganita*) described here, from the *madhyamādhikāra* (the first chapter of *Siddhāntaśiromaṇi*) onwards, of the [motion of the] heavenly bodies, a mathematician will not be respected in the scholarly assemblies; without the *upapattis* he himself will not be free of doubt (*nīhsamsāya*). Since *upapatti* is clearly perceivable in the (armillary) sphere like a berry in the hand, I therefore begin the *Golādhyāya* (section on spherics) to explain the *upapattis*.

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Now as you text *upapatti* what was the *upapatti*'s suppose to do, what is the nature of this, this was captured by this words of Baskaran, *upapatti* mean (FL) without the proof a mathematician will not be considered as a scholarly mathematician in any assembly of mathematician (FL) any doubt regarding the result that he is enunciating. So for this reason that I am going to discuss *upapatti*'s are ture.

(Refer Slide Time: 29:12)

**Gaṇeśa on *Upapatti* (c.1540)**

The same has been stated by Gaṇeśa Daivajña in the introduction to his commentary *Buddhivilāsini* (c.1540) on *Līlavatī* of BhāskaraĀcārya

व्यक्ते वाव्यक्तसंज्ञे यदुदितमखिलं नोपपत्तिं विना तत्  
निर्भ्रान्तो वा ऋते तां सुगणकसदसि प्रौढतां नैति चायम्।  
प्रत्यक्षं दृश्यते सा करतलकलितादर्शवत् सुप्रसन्ना ॐ  
तस्मादग्र्योपपत्तिं निगदितुमखिलम् उत्सहे बुद्धिवृद्धौ ॥

Thus, according to the Indian mathematical texts, the purpose of *upapatti* is mainly:

- i To remove confusion and doubts regarding the validity and interpretation of mathematical results and procedures; and,
- ii To obtain assent in the community of mathematicians.

This is very different from the ideal of "proof" in the Greco-European tradition which is to irrefutably establish the absolute truth of a mathematical proposition.

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That is what Bhaskara is explaining in his commentary on Siddhanta Shiromani. The same point is repeated across Ganesa is follower in the tradition of Bhaskara is writing a commentary on Leelavati in 1540 explaining this proves. Again (FL) (()) (29:18) to (()) (29:28) that person who does not know upapatti will not be without confusion normally be considered as a serious mathematician.

Now so the basic purpose of a upapatti is sort of clearly stated to be to remove confusion and doubt regarding the validity and to obtain ascent in the community or something like sending a paper and getting it period and getting it publish. It does it mean that result is going to stand for all times for all ages that was the ideal of proof in the European tradition, that does not seem to be the kind of ideal that the Indians are initiated by doing mathematics.

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***Upapatti* and "Proof"**

The following are some of the important features of *upapattis* in Indian mathematics:

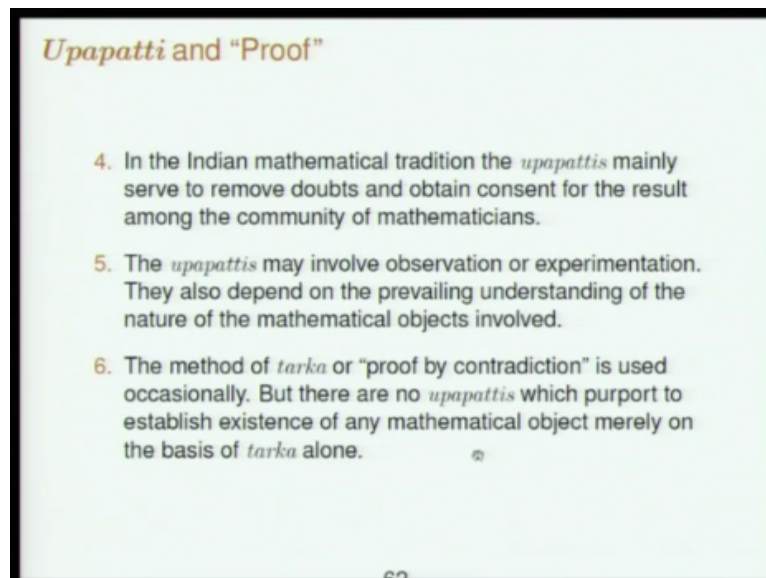
1. The Indian mathematicians are clear that results in mathematics, even those enunciated in authoritative texts, cannot be accepted as valid unless they are supported by *yukti* or *upapatti*. It is not enough that one has merely observed the validity of a result in a large number of instances.
2. Several commentaries written on major texts of Indian mathematics and astronomy present *upapattis* for the results and procedures enunciated in the text.
3. The *upapattis* are presented in a sequence proceeding systematically from known or established results to finally arrive at the result to be established.

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In fact the detail study of proofs in Indian mathematics shows that there are the differences between the idea of proof as we know from the Greek or European tradition and the idea of upapatti in Indian mathematics. First of all the Indian mathematician is a very clear that proofs are needed upapatti are needed result even if verified in 100s of cases, does not mean that it is proved in mathematic.

So only when you can give some logical argument or some other argument you can you say that it is a valid mathematical result and several commentary are written listing such upapattis. When the upapattis like as we know proofs in modern arithmetic they are written in a sequence that to go from known result to new result and from them to let other result. So you will have a sequence of establishing results.

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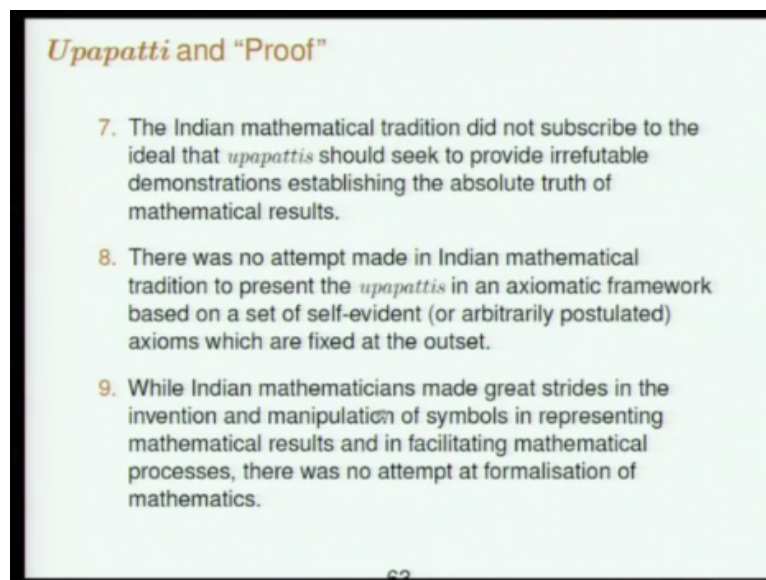


And the understanding is that it is by giving proves we are clear how the result is to be applied and understood. The proofs may many times depend upon experiment, this something which is new we may be doing it another mathematics teaching but Euclidean ideal of proof is that to prove something is very abstracted should not be dependent on experimentation.

Should not be depend on even our understanding what is the nature of the mathematical object, but the Indian proofs were always they could involve experimentation, they could involve an understanding of the explicit use of the nature of the object and another crucial things is that what is called the proof by contradiction the (FL) which is called in Indian mathematics that was employed.

The (FL) was employed to understand the non existence of certain mathematical quantities, but it was not employed to established the existence of a mathematical object whose existence would not otherwise be accessible to us by any other means. So (TL) non considered as a independent from (FL). So existence of quantities cannot be established by nearly proofing that their non existence is inconsistent with whatever we know.

**(Refer Slide Time: 32:31)**



But by giving a means as an access to the way there existence can be understood by us which is something known as the constructive philosophy which is mathematic and there is no ideal the proofs will give it that curable demonstration or will give this the absolute truth of mathematical proposition. There was no idea that you fix one set of postulates once in for all in derive all the results.

And by so many symbolism and symbolic techniques were used formalization of mathematics was not something that is attempted in Indian mathematic.

**(Refer Slide Time: 33:02)**

### The Genius of Srinivasa Ramanujan (1887-1920)

In a recent article commemorating the 125<sup>th</sup> birth-day of Ramanujan, Bruce Berndt has presented the following overall assessment of the results contained in his notebooks (which record his work prior to leaving for England in 1914):

"Altogether, the notebooks contain over three thousand claims, almost all without proof. Hardy surmised that over two-thirds of these results were rediscoveries. This estimate is much too high; on the contrary, at least two-thirds of Ramanujan's claims were new at the time that he wrote them, and two-thirds more likely should be replaced by a larger fraction. Almost all the results are correct; perhaps no more than five to ten are incorrect."

Now coming to more contemporary kinds this issue of proof we tell something very crucial in understanding the mathematic of Srinivasa Ramanujan. Then Ramanujan sent his result in 1930 in a long method to high if it is 100, 120 results, hard immediately response by saying this all kind looking very interesting but where are the proofs, you please send me the proofs of all these results of course they was not so trivial that hardly could prove it for himself the straight away on a piece of paper or something like that.

When did the proofs be given and Ramanujan there is a very famous let he send hardly in 1930 saying that he has a systematic method for deriving all the results but that cannot be explained in a short communication and he thinks that he has a new methodology for doing things and he anywhere but he says that why do not you just some of this results and can we check what I am writing his really and that should convince you that there is something interesting in what I am doing.

**(Refer Slide Time: 34:03)**

### Ongoing Work on Ramanujan's "Lost Notebook"

The manuscript of Ramanujan discovered in the Trinity College Library (amongst Watson papers) by G. E. Andrews in 1976, is generally referred as Ramanujan's "Lost Notebook". This seems to pertain to work done by Ramanujan during 1919-20 in India. This manuscript of about 100 pages with 138 sides of writing has around 600 results. Profs G. E. Andrews and B. Berndt have embarked on a five volume edition of all this material. They note in the preface of the first volume that:

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### The Genius of Srinivasa Ramanujan

"The topics examined by Ramanujan in his notebooks fall primarily under the purview of analysis, number theory and elliptic functions, with much of his work in analysis being associated with number theory and with some of his discoveries also having connections with enumerative combinatorics and modular forms. Chapter 16 in the second notebook represents a turning point, since in this chapter he begins to examine the q-series for the first time in these notebooks and also to begin an enormous devotion to theta functions."<sup>9</sup>

<sup>9</sup>B. Berndt, Notices of AMS 59, December 2012, p.1533.

Now issues important because finds where there is this notebook of Ramanujan which is set of all results that he noted here to going to England and later analysis in the last 2530 shows that there are more than 3000 results this notebooks contain are the initially thought that two thirds of them were already well known but now the understanding is more than two thirds was not known at the time Ramanujan was recording this results in the notebook.

And almost all the results are correct and there is no more than 5 to 10 or incorrect, this is the current assessment of the results that Ramanujan wrote down in his notebook, there is of course a notebook of the work that he was doing in the last year of his life 1919 to 20 which was lost seemingly and it was recorded in 1975 in the Trinity college library by Mr. G.E. Andrews it is called the Lost Notebook.

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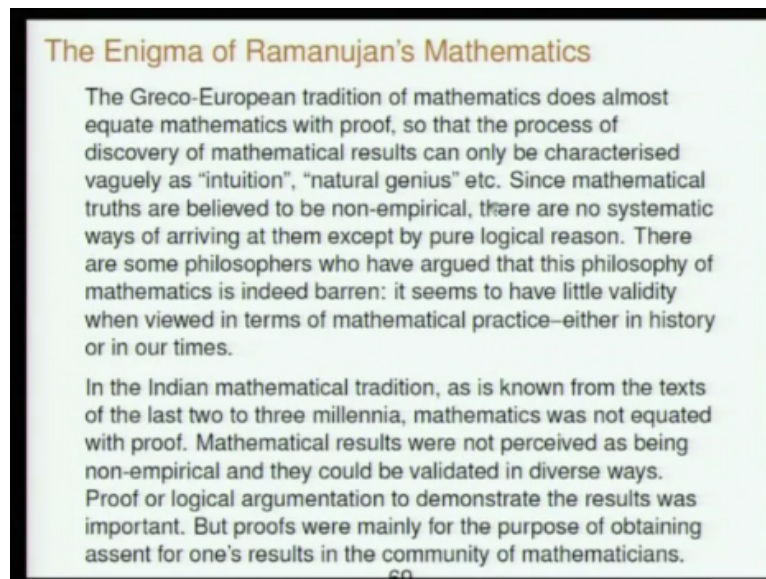
### Ongoing Work on Ramanujan's "Lost Notebook"

"...only a fraction (perhaps 5%) of the notebook is devoted to the mock theta functions themselves. A majority of the results fall under the purview of q-series. These include mock theta functions, theta functions, partial theta function expansions, false theta functions, identities connected with the Rogers-Fine identity, several results in the theory of partitions, Eisenstein series, modular equations, the Rogers-Ramanujan continued fraction, other q-continued fractions, asymptotic expansions of q-series and q-continued fractions, integrals of theta functions, integrals of q-products, and incomplete elliptic integrals. Other continued fractions, other integrals, infinite series identities, Dirichlet series, approximations, arithmetic functions, numerical calculations, Diophantine equations, and elementary mathematics are some of the further topics examined by Ramanujan in his lost notebook."

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And result in that are still being established by the mathematician of present day and this contains full lot of results like this.

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So what I was trying to say was that Greco-European tradition of mathematics almost equate mathematics with proof and the way mathematical results of discovery therefore is hardly understood which may be termed as intuition natural genius etc. and there is an understanding that mathematical results of non empirical and therefore there is no access to them except by logical argumentation.

Of course there are philosopher of mathematics to do argue that this philosophy of mathematics is confusing the barrel . This philosophy does not explain most of history of mathematics, today mathematics is done either it was done in earlier life or even mathematics is being done present. In Indian tradition the understanding was that proof is only one of the aspects of mathematics important . Mathematical result does not part of to be nonempirical.

Mathematics was not thought out to be a science different from other sciences, it results were equally contestable and falsifiable and they could be validated in diverse days, the proof was important but they were more for obtaining as an for once result .

**(Refer Slide Time: 36:44)**

## Ramanujan: Not a Newton but a Mādhava

In 1913, Bertrand Russell had jocularly remarked about Hardy and Littlewood having discovered a "second Newton in a Hindu clerk". If parallels are to be drawn, Ramanujan may indeed be compared to the legendary Mādhava.

It is not merely in terms of his methodology and philosophy that Ramanujan is clearly in continuity with the earlier Indian tradition of mathematics. Even in his extraordinary felicity in handling iterations, infinites series, continued fractions and transformations of them, Ramanujan is indeed a successor, a very worthy one at that, of Mādhava, the founder of the Kerala School and a pioneer in the development of calculus.

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So the process of mathematical discovery in the mathematical justification or in some unicell in where the Indian have understood mathematics. Long time ago when Ramanujan letter arrived in England the conclusion that it and it would had but (FL) one of his friends that (FL) have discovered second Newton in a Hindu. But if some comparison is to be made regarding Ramanujan he is more in the line up Madhava.

Both in the kind of topics like infinity transformations of them and continued fractions and transformations of them handling iteration and indeed success of the great genius Madhava who was one of the Pioneer of calculation.

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## Lessons from History

"It is high time that the full story of Indian mathematics from vedic times through 1600 became generally known. I am not minimizing the genius of the Greeks and their wonderful invention of pure mathematics, but other peoples have been doing math in different ways, and they have often attained the same goals independently. Rigorous mathematics in the Greek style should not be seen as the only way to gain mathematical knowledge. In India where concrete applications were never far from theory, justifications were more informal and mostly verbal rather than written. One should also recall that the European enlightenment was an orgy of correct and important but semi-rigorous math in which Greek ideals were forgotten. The recent episodes with deep mathematics flowing from quantum field theory and string theory teach us the same lesson: that the muse of mathematics can be wooed in many different ways and her secrets teased out of her. And so they were in India..."<sup>10</sup>

<sup>10</sup>David Mumford, Review of Kim Plofker, *Mathematics in India*, Notices of AMS 2010, p.390.

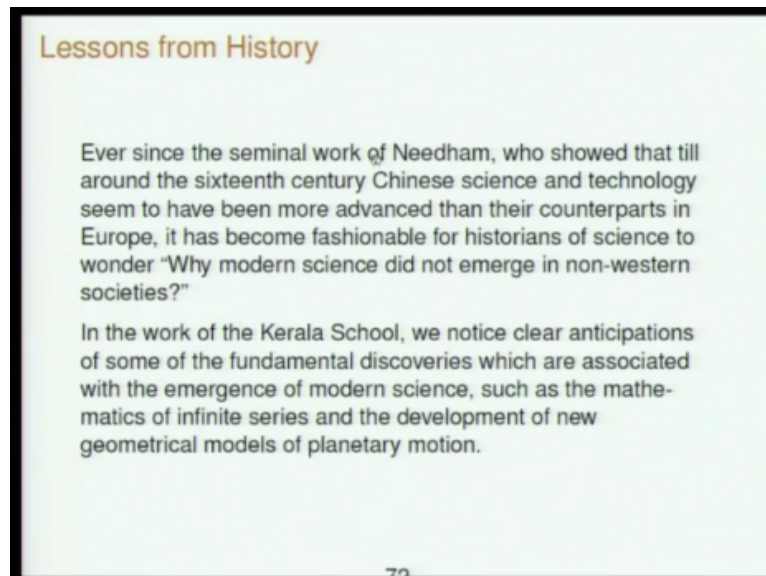
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I tried to extract review of recent book on mathematics in India by David Mumford the well known (FL) the main point was emphasize that by studying Indian mathematics of the history

of mathematics in India what one can understand is that Indian mathematics can be done in different ways the views of mathematics can be boot in many different ways and have secret is out of that and so they were in India.

And one should not just confused the fact that absence of rigorous mathematics in the Greek style means that the rest is not mathematics at all. When he caution that most of interesting mathematics that we used today which was developed in 16, 17, 18 century was indeed done by abandon the Greek term of doing mathematics , this is the kind of understanding that scholars are arriving at the importance of knowing a different tradition of mathematics like the Indian tradition.

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Another interesting in this the question of the history of science in recent times. That ever seen the work of Needham, it has generally been understood that till 16 century the Chinese science and technology seem to be considerable advanced over science and technology in Europe and then Needham showed the question that Needham almost made it an important focus was why modern science did not emerge in China and did not emerge in non-western societies.

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## Lessons from History

It seems therefore more appropriate to investigate "Why science did not flourish in non-western societies after the 16th Century?"

It would be worthwhile to speculate "What would have been the nature of modern science (and the modern world) had sciences continued to flourish in non-western societies?" In this way we could gain some valuable insights regarding the sources and the nature of creativity of geniuses such as Srinivasa Ramanujan, Jagadish Chandra Bose, Prafulla Chandra Roy, Chandrasekhara Venkata Raman, and others, in modern India.

Now when we study mathematics in India for instance notice that many of hallmarks of modern science such as development of Calculus, infinite series etc. are development of you astronomical models of the planet which system they were all there in Kerala that of 14, 15, 16 century. So a very crucial question that we should understand is why science did not flourish in non-western societies that is 16<sup>th</sup> century.

And it is even more important but today's purpose to have some idea how science would have developed, how the science today would have been if the non-western societies had continued developing science along the lines that they have laying down for themselves earlier maybe with many modifications, maybe with some transformations in interaction with modern science developed in Europe and subsequent time.

It is only by that kind of speculation we can come to some understanding on of the great genius of modern times in India such as Ramanujan, Bose, Prafulla Chandra Roy, Raman and many others.

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## Summary

The most striking feature of the long tradition of Indian mathematics is the efficacy with which complex mathematical problems were handled and solved.

The basic theorems of plane geometry had already been discovered in *Sūlvasūtras*.

By the time of *Āryabhaṭīya* (c.499), a sophisticated theory of numbers including the concepts of zero, and negative numbers had also been established and simple algorithms for arithmetical operations had been formulated using the place-value notation. By then, the Indian tradition of mathematics was aware of all the basic mathematical concepts and procedures that are today taught at the high school level and much more.

By the 11th century sophisticated problems in algebra, such as quadratic indeterminate equations, were solved.

By the 14th century, infinite series for trigonometric functions like sine and cosine were written down. By the same time, irrational character of  $\pi$  was recognised, and its value was determined to very high levels of approximation.

So to summarise the development of mathematics in India that the main thing was that the complex mathematical problems were not send, even if complete solutions to them were not found approximate less than perfect solutions were accepted and then developed into better and better solution and the idea was always was in simplicity of mathematical procedure and by this Indians were able to do quite a bit.

They could get the basic (FL ) geometry by the time of sulvasutras they could establish most of our arithmetic algebra geometry and trigonometry by the time of Aryabhata by the time of Bhaskara II they could solve complicated quadratic indeterminate equations or by 14, 15 century calculus exact series were sine and very accurate sine tables which all very important perform.

So the crucial thing is explicitly algorithmic and computational nature of Indian mathematics and this seems to have persisted that till recent times and to some extent of Srinivasa Ramanujan as I told you could be thought of as a (FL) traditional Indian methodology and perhaps is important that we should have a detailed understanding of the development of mathematics in India to understand the way Indians approached many complex problems.

Even in other sine and we let to say it very important that we should teach the highlights of this great tradition of mathematics to our students in schools and colleges and I think courses like this will help in sort of formulating that kind of a work. So with that I I complete this initial overview and thank you very much.