Measure and Integration

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Module No.#01

Lecture No. # 02

Algebra and Sigma Algebra of aSubsets of a Set

Welcome to today'slecture on Measure and Integration.This is the second lecture. Inthis lecture, we are going to cover the following topics.

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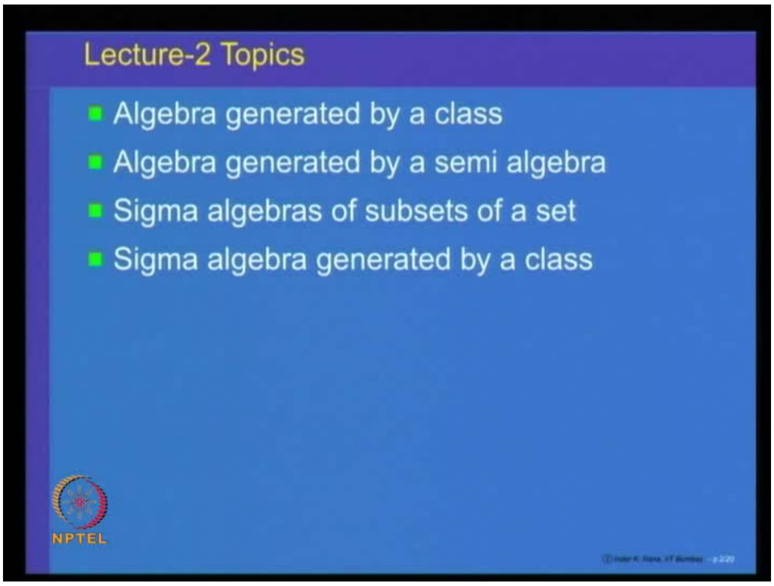

Algebra generated by a class; then we willlook at algebra generated by a semi algebra;we will also look at what is called the sigma algebra of subsets of X and sigma algebra generated by a class of subsets of a set X.

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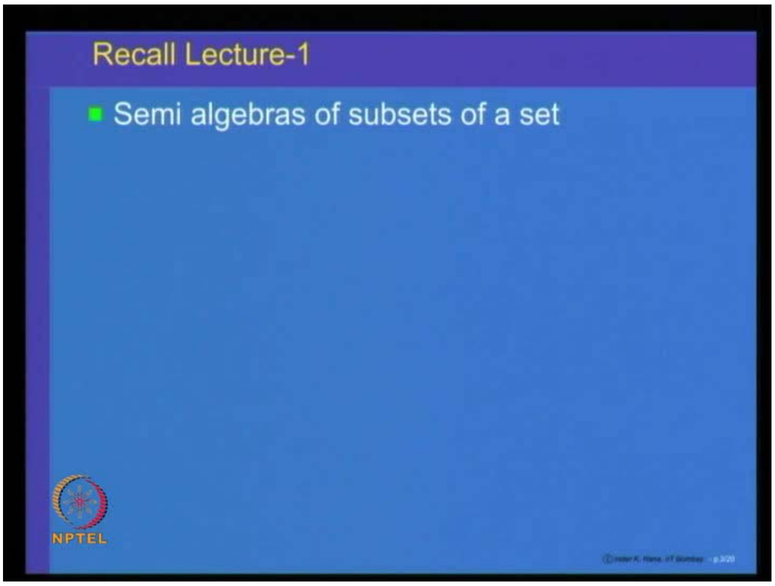

Let us just recall what we called as the semi algebra of subsets of a set X.It was a collection.

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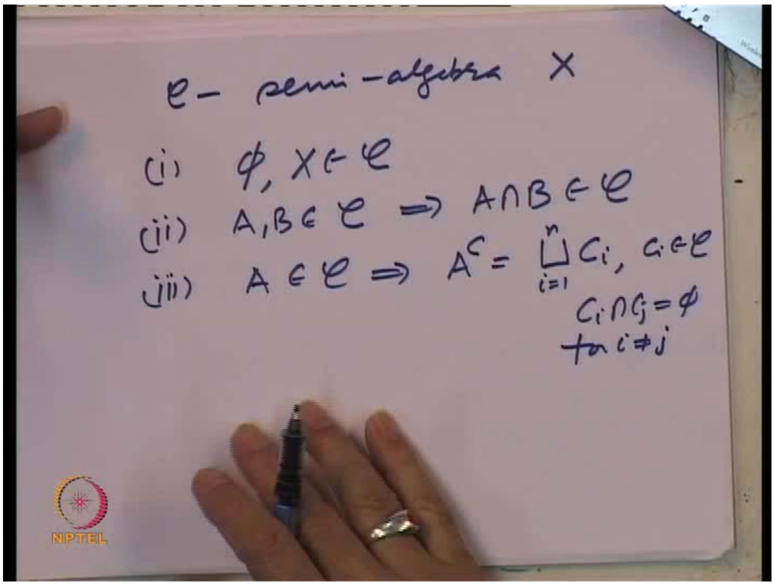

So, let me just recall– semi algebra C is thesemi algebraof subsets of a set X and ithad the following properties: one - empty set and whole space belong to it.Secondly,A and B belonging to C implies that the intersection of these two sets is also an element of C; that is the class C is closed under intersections.Third property was that if A belongs to C, then this implies that Acomplement can be represented as a finite disjoint union of elements ofthe class C; that is Acomplement is a union of elements Ci where each Ci belongs to C and Ci intersection C j is empty for i not equal to j. So, such a class was called semi algebra of subsets of X.

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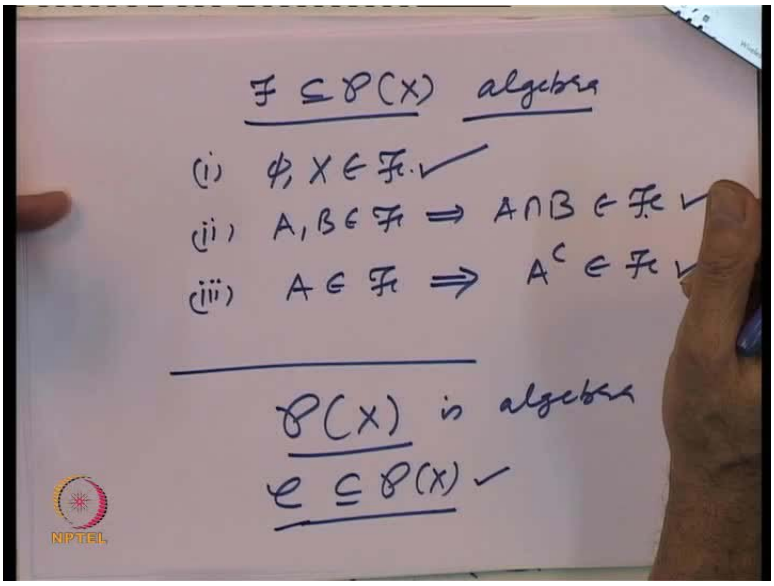

Then we looked at what we called as algebraof subsets of a set X.So,a class Fof subset of a set X is called an algebra, if it had the following properties, namely: empty set and the whole space belong to F, as in the case of semi algebra.Secondly,A and B belonging to F imply that A intersection B also belongs to F, as was the case for the semi algebra.Third property which is different from the semi algebra, which is a bit stronger than the semi algebra, iswhenever a set A belongs to F, this should imply that Acomplement also belongs to F.

So,algebra of subsets of a set X is a collection of subsets of X which includes the empty set and the whole space; it is closed under intersections and it is also closed undercomplements.The last timewe looked at some examples of algebrasandsemi algebras.

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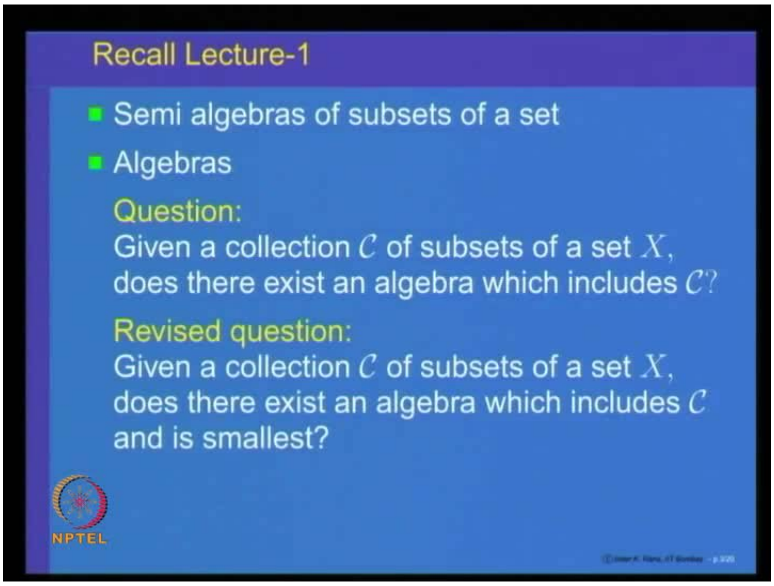

So, today, what we are going to look at is the question - given a collection C of subsets of a set X, does there exist an algebra which includes C?

So, the given collection C may not be algebra of subsets of X;it is an arbitrary collection.We would like to know, if we can find a collection of subsets ofX which includes this collection and this algebra.

Of course, there is one obvious answer -The power set.For example,the power set of X is always an algebra because it is collection of all subsets of X.So, it has all the properties namely: it is closed under intersection and complements, and includes the empty set in the whole space; obviously,C is a subset of p X. So, in some sense,pX is the largest algebra of subsets of X which includes any collection C.

So, we should modify our question - given a collection C of subsets of a set X, does there exists an algebra which includes C and is the smallest?So, to answer that question, let us look at the following.

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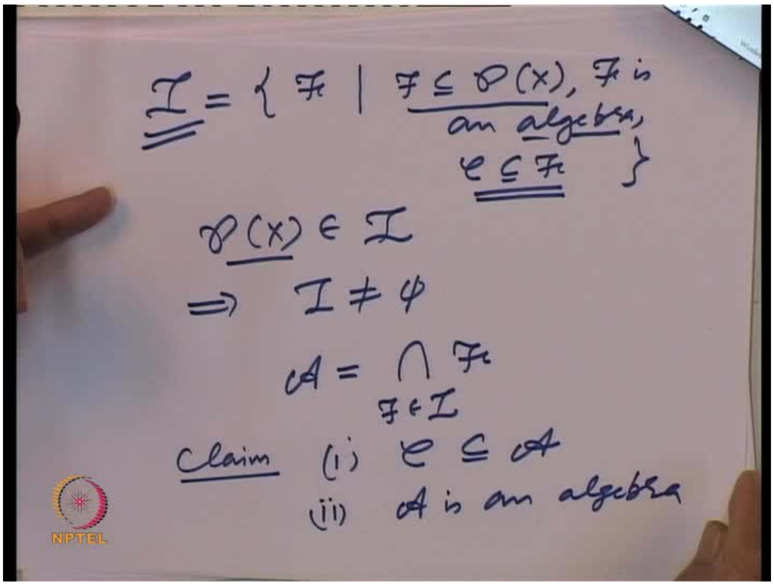

So, let us collect together, let us look at the collection of all F such that F is a subset of p X.It is a collection of subsets of X such that F is algebra and C is contained in F;C is the collection which is given to us, which may not be algebra. So, let us collect all those collections of subsets of X;so, call them as F.So,F isa collection of subsets of Xsuch thatF is algebra and includes C.

So, it has two properties: one -C is a subset of X; this collection C is inside the collection F and F is algebra. So, first of all, let us observe that p X is an element of this collection.

So, let us call this collection asI.So, this collection of all algebras which includes C is a non-empty collection because the power set of X, the collection of all subsets of X is an algebra and includesC.So, this is non-empty;so, implies that this collection is not empty.

Now,let us define A to be equal to intersection of all this F such that F belongs to I.So, let us take all the algebras which are members of this collection I and take their intersections. So, keep in mind that each F is a collection of subsets and it is algebra and we are taking the intersection of these collections.

So, the claim is that, one -C is a subset of this collection A which is obvious because the collection I of all the algebras F has that property;C is a subset of each member; so this property is obvious.Secondly, we claim that A is an algebra.So, to prove that A is an algebra, let us observe what we have to do.

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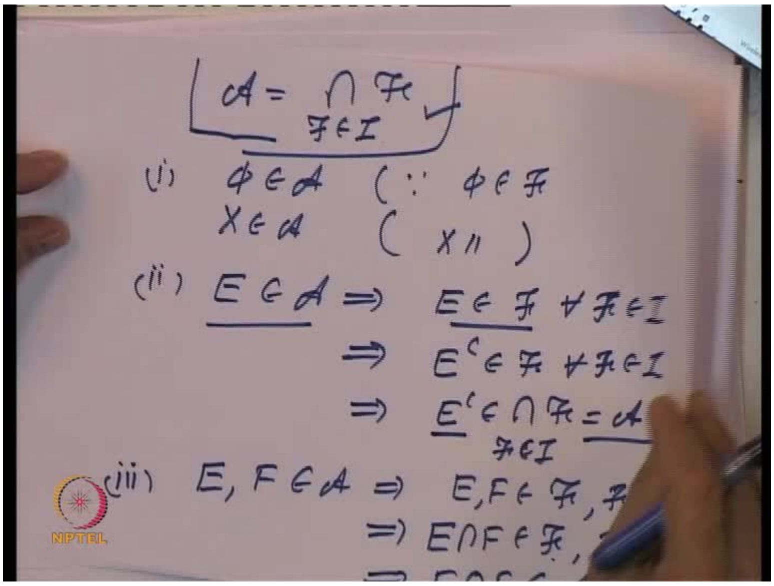

So, what is A?A is the intersection of all algebras F which are inside I.

So, first we want to show that empty set belongs to A.Why?Because we know empty set belongs to each F because each F is an algebra.Similarly,X belongs toA because of the same reason - because X belongs to F, and F is an algebra.Secondly, let us take a set E belonging to A, then that will imply by the very definition that E belongs to F, for every F inside the collection I, butthat implies because E belongs to F and F is an algebra, that implies that Ecomplement also belongs to F, for every F belonging to I. But then implies it is equivalent to saying this implies that E complement belongs to the intersection of all this F in I,and that is precisely our A.

So, we have shown that, if E belongsto A, then E complement also belongs to A.So, the class A is closed under complements.Let us finally show that it is closed under intersection also.

So,let us take two sets E and F belonging to A; then that implies that E and F both belong to each F,F belonging to I;that implies that E intersection F belongs to the collection F because F is an algebra.So, that is crucial.So, that is being used again and again,E and F belong to the collection F which is an algebra.So, the intersection also belongs and hence E intersection F belongs to A.

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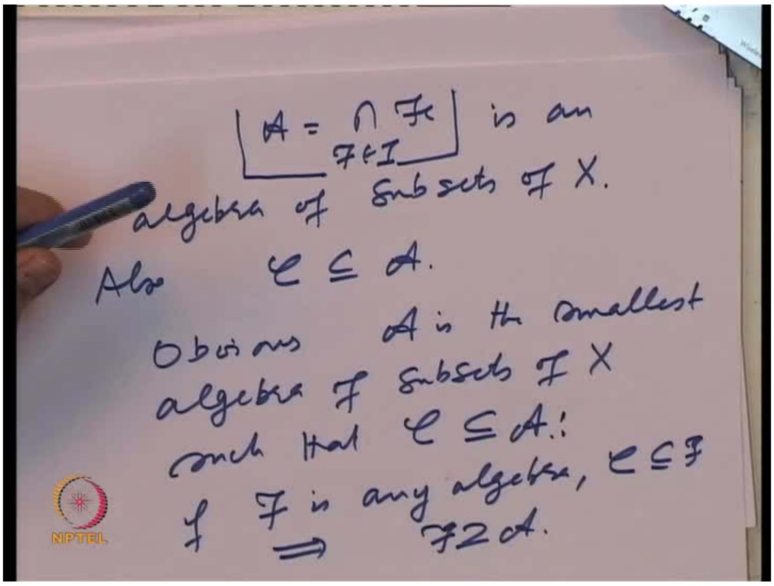

So, what we are showing is - this collection A which isintersection of all the algebras which include Cis an algebra. So,we have shown the second propertythat - this collection A of intersection of the all the algebras which includes C is an algebra of subsets of the set X; also,C is inside A.By the very nature because it is the intersection of all the algebras which include A, it should be an obvious property namely,A is the smallest algebra of subsets of X such that C is inside A.

What do we mean by that?That means if F is any algebra and C is inside F, that implies that F has to include A.So, what we have shown is that given any collection of subsets of set X, if we define A by this -A is the intersection of all the algebras including I, then this collection exists because there is at least one algebra which includes C namely the power set, and it is the smallest.

So, what we have shown is - given any collection of subsets of a set X, there is an algebra of subsets of Xwhich is smallest and includes A.

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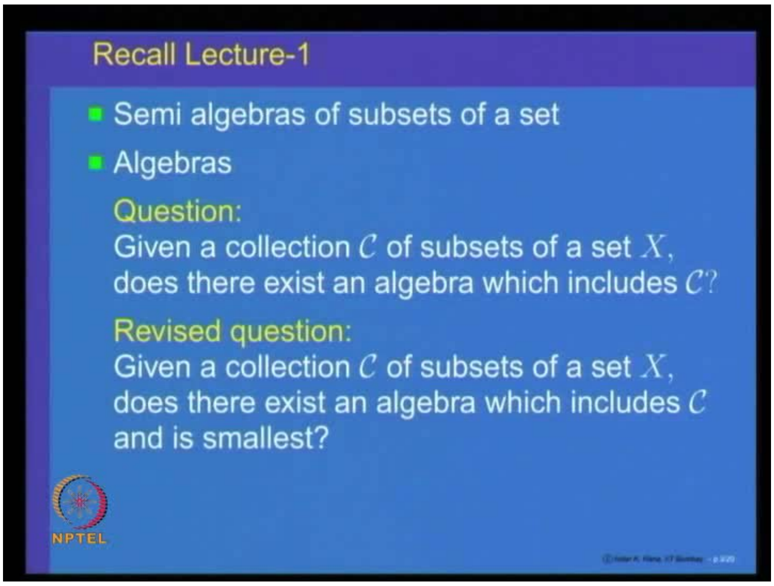

So, we have answered this question - given a collection C of subset of a set X, does there exists an algebrawhich include of C and is smallest?Yes; the answer is yes.

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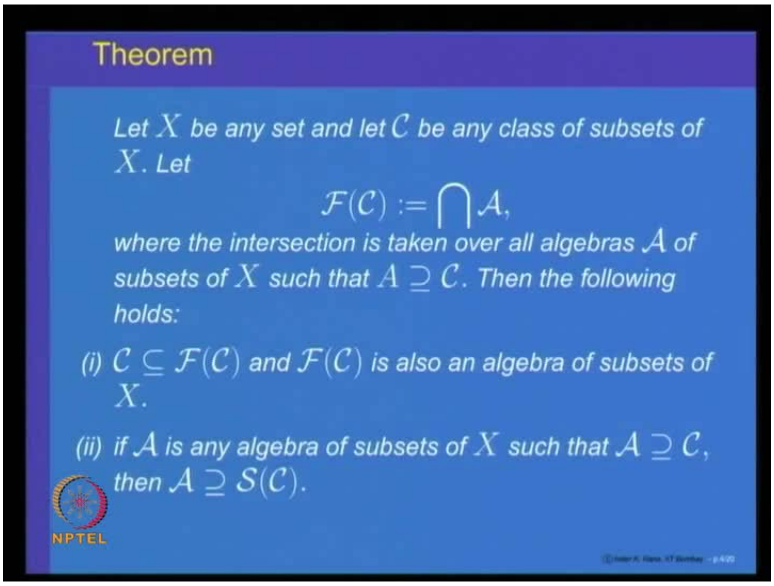

So, let us state this as a theorem:Let Xbe any set and let C be any class of subsets of a set X.LetF of C denote the intersection of all the algebras Awhich include the collection C.

So,F of C is the intersection of all the algebras that include the collection C.Then, what we have just now shown is that C is a subset of X that means F of C includes C and F of C is algebra of subsets of X. So,F of C is an algebra which includes X and it has an additional property.It is the smallest with that property; that is if A is any other algebra of subsets of X such that A includes C, then A must include F of C.So, this is not S of C; it is F of C.

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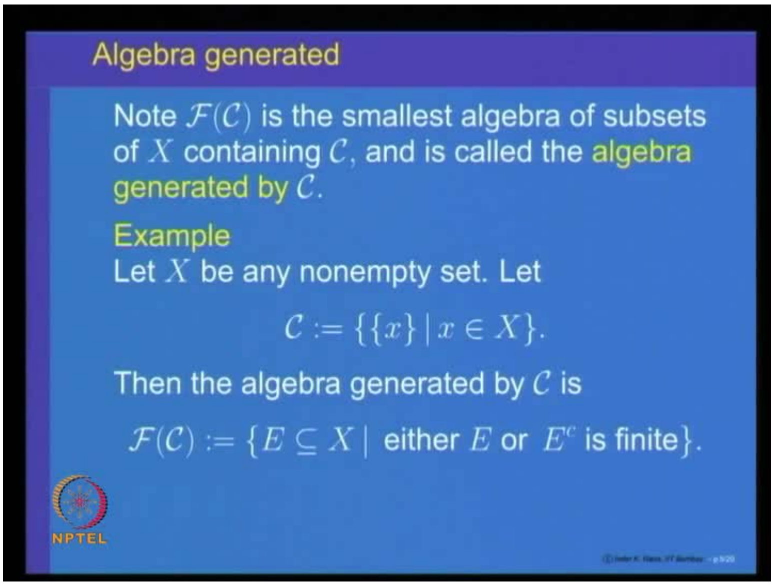

So,F of C is called the smallest sigma algebra of subsets of X containing C and is called the algebra generated by the class C.

So, what we have shown is that given any collection of subsets of a set X, we can always find algebra of subsets of X which is smallest and includes it.

Let us look at some examples.So, let us look at the collection:X is any nonempty set; let us look at the collection of all singletonsubsets of this collection X. So, C is the collection of all singleton sets where x belongs to X.We want to know what is the algebra generated by it.The claim is that the algebra generated by this C is nothing butall those sets E in Xsay that either E or E complement is finite.

So, let us prove this.How you prove such kind of asuggestion?

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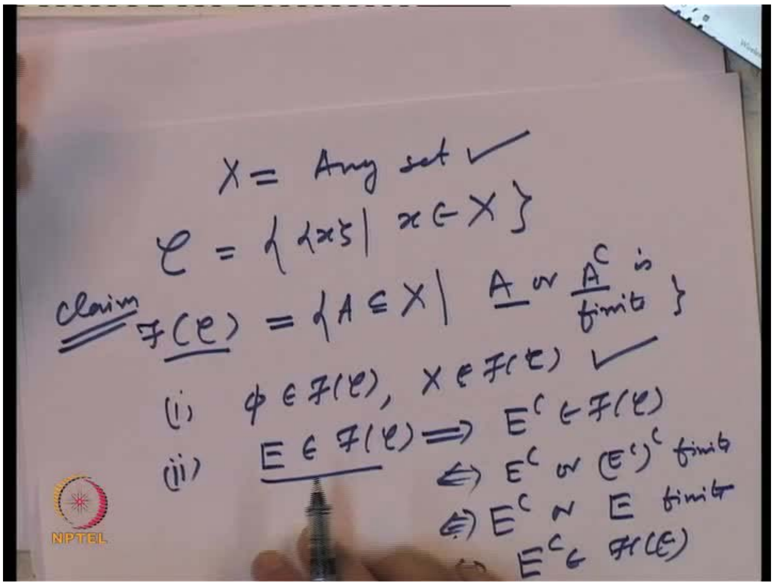

We have got X - any set;C is the collection of all singleton sets belonging to subsets of X.Then, we are looking at the claim.We claim that F of C is nothing butall sets of A contained in X say that A or A complement is finite.So, let us observe.

Does empty set belong to F of C?Yes.Because by definition, empty set is a finite set. Does X belong to F of C?

So, for X to be an element of F of C, either X should be finite which we do not know because it is any set; it may or may not be finite; but we know its complement which is empty set is finite and in F of C;so,by the second criteria, A or A complement.So,X may not be finite, butits complement is empty set which is finite.So, this property is true.

Let us check the second property:If E belongs to F of C, does this imply Ecomplement belong to F of C?Is this true?

For Ecomplementto belong to F of C, either E should be finite or E complement complement should be finite.So, this will be true if and only if E complement or E complement complement is finite; which is same as E complement or E complement complement is e finite; which is same as saying: this is if and only if;this is if and only if; this is if and only if E complement belongs to F of C (Refer Slide Time: 15:00). So,E belonging to F of C is true, if and only if E complement because our definition of F of Cis symmetric with respect to A and A complement. So, the collection F of C of all those sets for which A or A complement is finite, includes empty set - the whole space; it is closed under complements.

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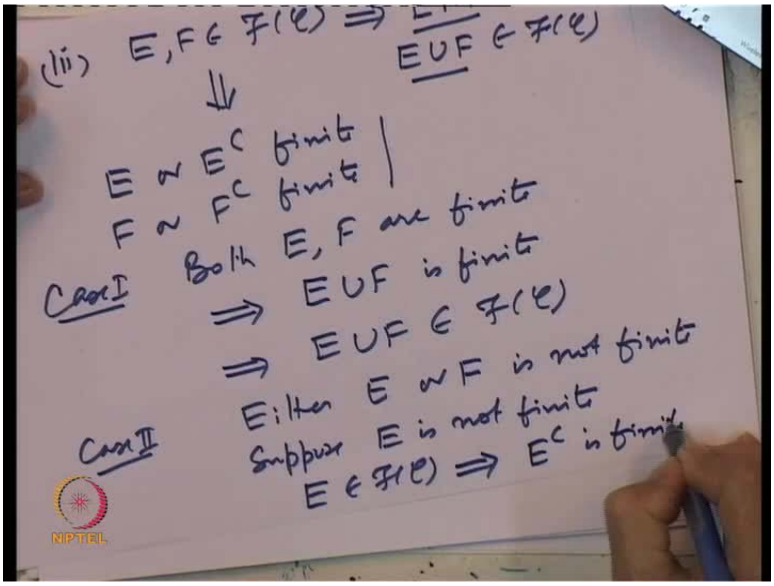

Let us show the third property: If E and F belong to F of C, then that implies their intersection also belongs to Fof C; or equivalently, this is equivalent to saying that E union F belong to F of C. Becausewe have observed that from union, you can go to intersection by complements because the class is already closed into complements. So,E and F belonging to F of C means E or E complement finite and F or F complement finite.

So,various possibilities arise.Let us take, case (1):Both E and F are finite.In that case, that will imply -E union F is finite because union of finite sets is a finite set and that will imply that E union F belongs to F of C - our collection.

So, whenever E and F are finite,that isso.What is the case 2?What is the other possibility?

Either E or F is not finite.

So, for the sake of definiteness,let us suppose thatE is not finite, butE belongs to the class F of C; so, that implies E belongs to F of C means E complement is finite.

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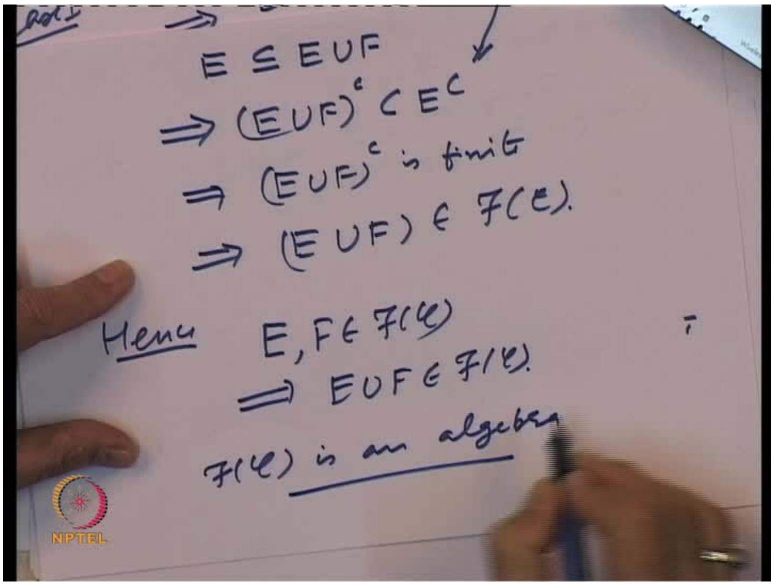

Now,E complement is finite andEis contained in E union F, but that implies that E union F complement is contained in E complement.E complement is finite; so, that implies E union F complementis finite.Hence, by definition, this implies E union F belongs to F of C.

So, we have shown in either case, hence whenever twosets E and F belong to F of C, that implies E union F belongs to F of C. So, thus we have shown that the collection F of C is an algebra.

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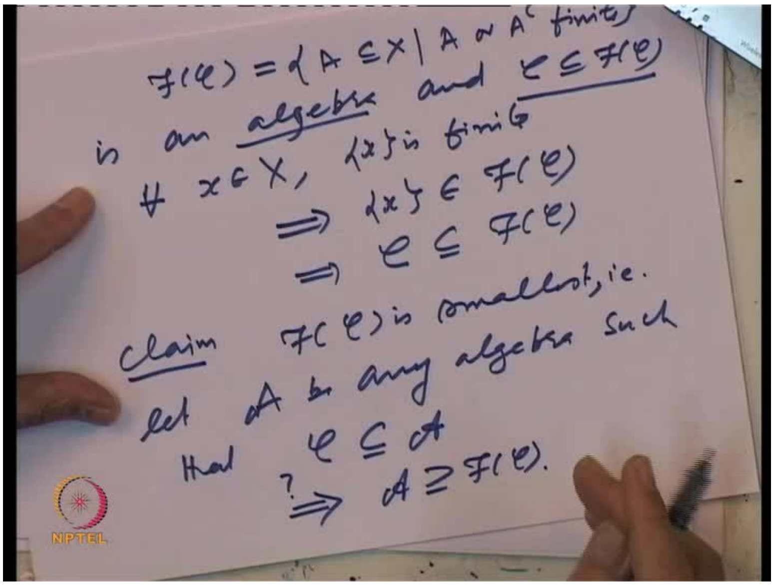

So,F of C is an algebra and F of C which was defined as all sets A contained in X such that A or A complement finite is an algebra andC is contained in F of C.

Is it clear why C is contained in F of C?Because for every x belonging to X, the singleton x is finite.So, implying the singleton x belongs to F of C;so, implying that C is a subset of F of C; so, thus F of C is an algebra and includes F of C.

Claim finallythat F ofC is smallest;that is let A be any algebra such that C is inside A.Then that should imply that A includesF of C.

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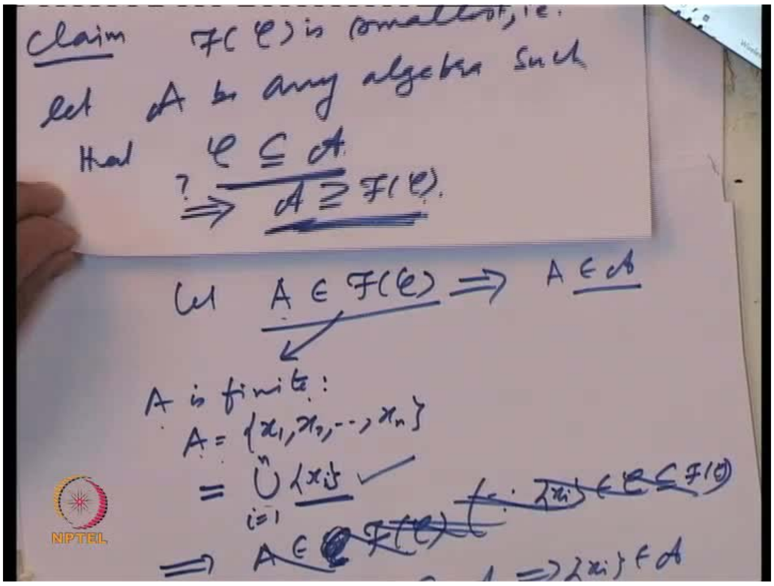

So, let us show how is that true.To prove this, let A belong to F of C.So,two possibilities:(1)A is finite.So, let us write A as x 1, x 2, some xn; that means this is equal to union of singletons xI,i equal to 1 to n and each singleton xi is an element of class Cand C is given to be inside A;so, this implies that A belongs to F of C.

So, let me just go through again.Because A is finite,A is written as x 1, x 2 up to xn.So,I can write as a finite union, each xi belongs to C; so, each xi belongs to F of C.So,because each xi belongs to C which is subset of F of C; so each one is inF of C. F of C is in algebra; so,A belongs to F ofC.

This is not that we wanted to prove.So, this is not true;of course,I am proving whatwas required.So, let Abe finite; so, we want to show that A isinside the class A.

So, each xibelong to C;C is contained in A; that means each xi belongs to A and A is an algebra.So, that implies that the union A belongs to A; same argument.

Basically using that C is inside A and A is an algebra.So, that will prove that if A belongs to F, then this implies that A belongs to A.So, that proves that Aalways includesF of C.

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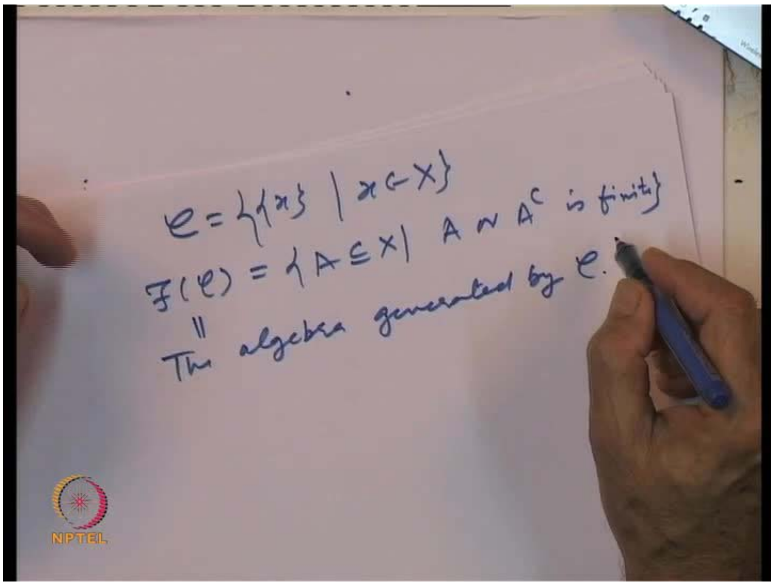

Hence,what we have shown is the following:If C is the collection of all singleton sets x belonging to X, then the algebra generated by it is nothing but all subsets A in Xsuch that A or A complement is finite. So, this is the algebra generated by C.So, we have computed we have described the algebragenerated by a collection C of singletons of a set X.

In general, it may not be possible togive a description of the algebra generated by a collection of subsets of a set X.In a special case, when the starting collection C is at least a semi algebra of subsets of X, it is possible to describe the algebra generated by it and that isour next theorem.So, we want to describe the algebra generated by a semi algebra.

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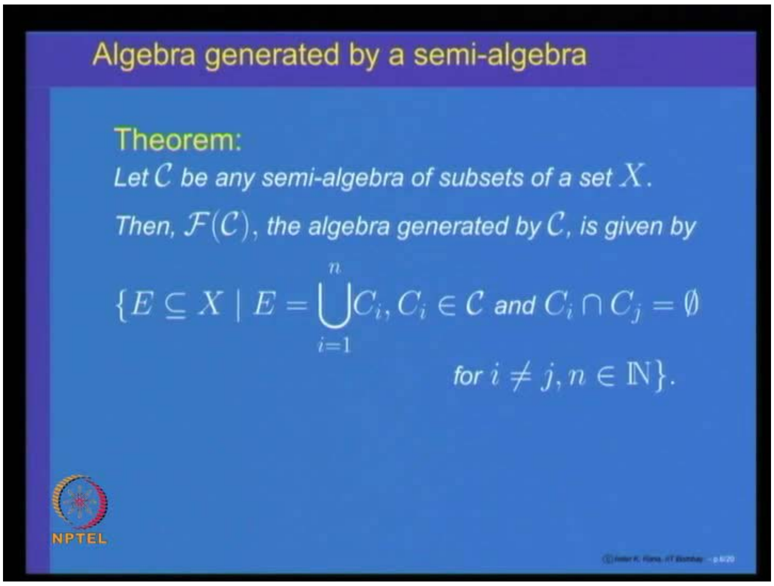

The theorems states the following: Let C be any semi algebra of subsets of a set X.Then,F of C - the algebra generated by C, is given by the following collection.So, it is a collection of all subsets E in X such that E can be written as a union of sets, finite number of sets Ci,iis 1to n, with each one of the elements Ci is in the collection in the semi algebra C.This Ci'sarepair wise disjoint.So, what we are claiming is that the algebra generated by a semi algebra is nothing, butthe finite disjoint union of elements of C.

We have seen an illustration of this in the previous lecture when we described;C was the collection of all intervals;we tookF of C, the finite disjoint union of intervals and showed that, that was algebra. So, that is a typicalmodel example or illustration ofdescribing the algebra generated by asemi algebra.So, let us prove that the algebra generated by the semi algebra is nothing butas described above.

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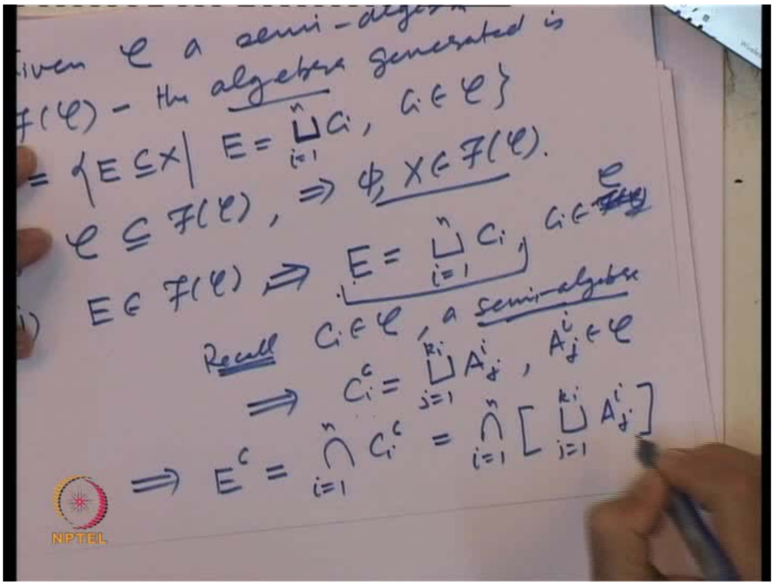

So, given C - a semi algebra,F of C - the algebra generated by C is equal to all sets such that I can write E as a finite disjoint union of elements is belonging to C; note this square bracket,square union means that the sets involved are pair wise disjoint.

So, let us observe first a few things.First of all,C is a subset of F of C; that should be obvious because every set in C is union of itself.So,E has a representation as union of itself.So, only one set is involved.So,E is a subset of it and that implies that the empty set and the whole space belong to F of C.

As a consequence, it implies the first property required for F of C to be an algebra and generated.We will see it later.

So, second thing:We are trying to show that F of C is an algebra.So, first property we have checked. let us takea set E belonging to F of C;that implies that E can be written as a finite disjoint union in Ci,i equal to 1to n where Ci’s belong to C.

Recall saying thatCi’s belong to Cand C is a semi algebra; this is important; this implies, by the property of the semi algebra that each Ci can be written as Ci complement can be written as a disjoint union of elements of C, again.Let us write it as Cij, j equal to 1 to some kI, where Aij’s belong to C againand they are disjoint.

So, together with the representation for this implies - look at E is a union of Ci’s;each Ci complement is equal to this(Refer Slide Time; 28:16); that means first of all

Ecomplement can be written as intersection of i equal to 1to n of Ci complement and each Ci complement is a finite disjoint union.So, this is i equal to 1to n of finite disjoint union j equal to 1 to ki of Ai j.

So, here we have used the property that -E complement by De Morgan laws is intersection ofC i because E is union of Ci complements and each Ci complement being an element of the semi algebracan be written as a disjoint union of elements Aji. So, that is the representation.

Now, we use the fact that intersection distributes over union.

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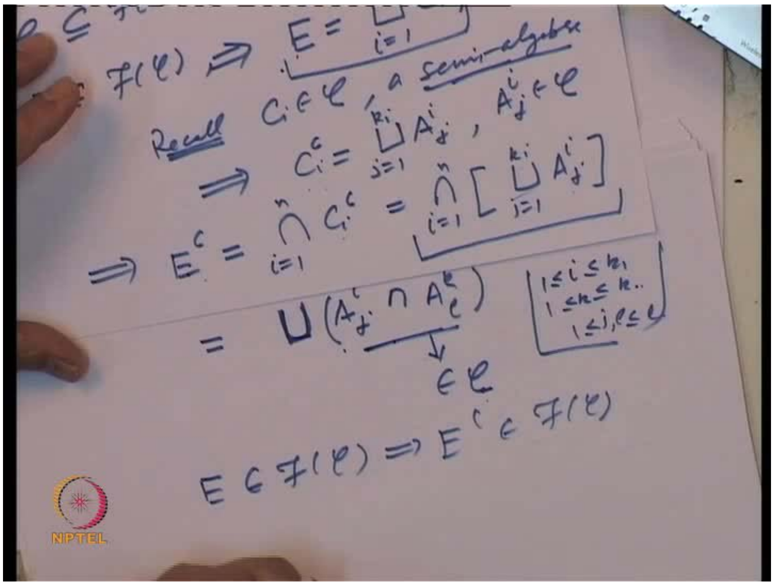

So, what kind of set is this? This can be written as a unionof finite number of sets which will involve Aij,sets of the following type, intersection Akl.

Because when I distribute it over,I will be getting 1 set Aij and some other set Akl and this is intersection of that and unions of those and these unions (Refer Slide Time; 29:47).So, each one of these sets belong to C.Why does it belong to C?Because the collection Cis a semi algebra, both of them are elements of the semi algebra.So, this intersection is element of the semi algebra and these sets are pair wise disjoint because if ikj and l - if anyone of the pairs are different, then those sets will be disjoint.

So,I can say this is a disjoint union of sets of the following type where i belong between 1 and k,i k between 1 and some k - some index, and j and l between 1 and l.

So, basically what we are saying is E complement can be represented again as a finite disjoint union of elements of C again.That is following because of the fact that E complement which is a union of setsis an intersection and that is intersection of distributive property and this.So,it is because the index is in order, so many,it is difficult to write all these things, but it should be clear that E complement is a finite disjoint union of elements of C.

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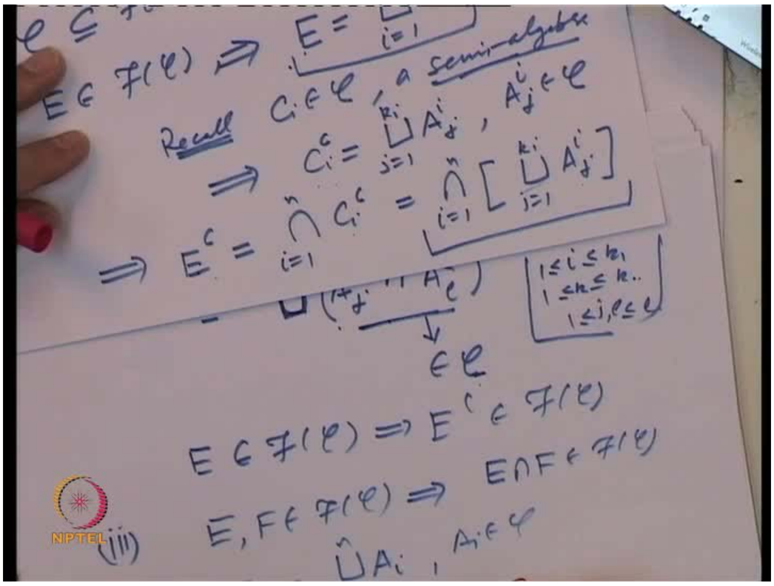

So, that means E belonging to F of C implies E complement belong to F of C.Let us look at the third property namely, if E and F belong to F of C - does this imply that Eintersection Fbelong to F of C?

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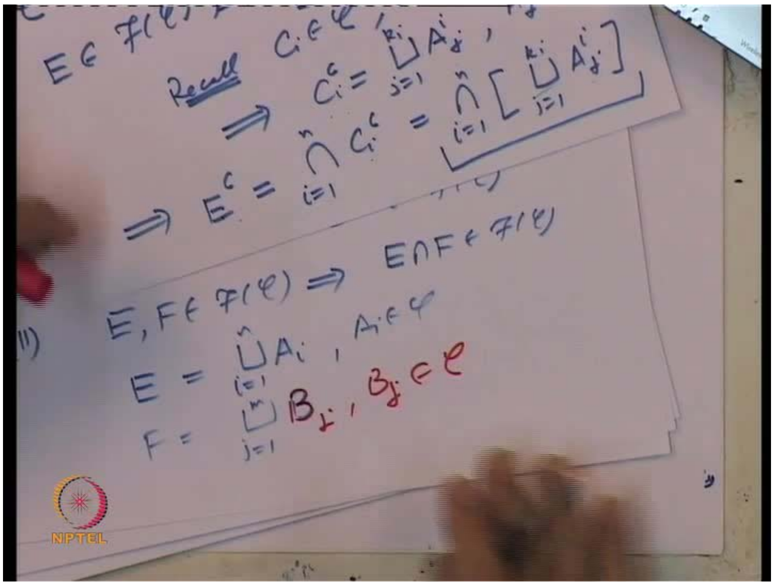

That should be obvious because E belonging to F of C means E is a disjoint union of some A I's1 to n;A i is belonging to C,F is a union of some j's 1 to m of some sets B j's j equal to 1 to m, where each B j belong to C.

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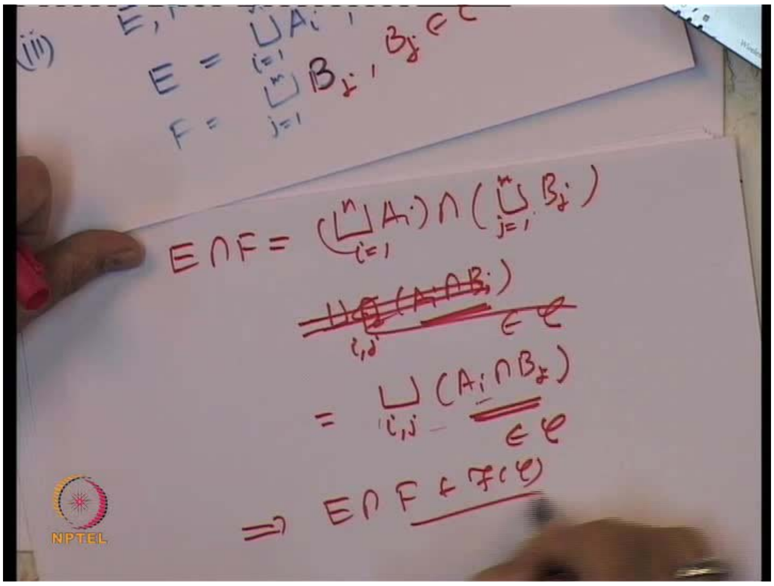

Let us conclude from here.So,E intersection F will be equal to this union Ais intersection of union B j's i equal to 1to n, j equal to 1 to m.Once again, using the distributed property, we got this intersection of Ai's intersection Bj,i and j. So, the intersectionand each one of them belongs to C;this is unionoveri and j.

So, by the distributive property,this will be union over i and j of Ai intersection Bj and these elements are elements of C (Refer Slide Time: 33:11) because C is a semi algebra A,i belong to C;Bj belong to C and these are again disjoint because they are Ai and Bj for disjointpairs.For different pairs,they will be disjoint by the property that these unions are disjoints.So, implies that an intersection F also belongs to F of C.

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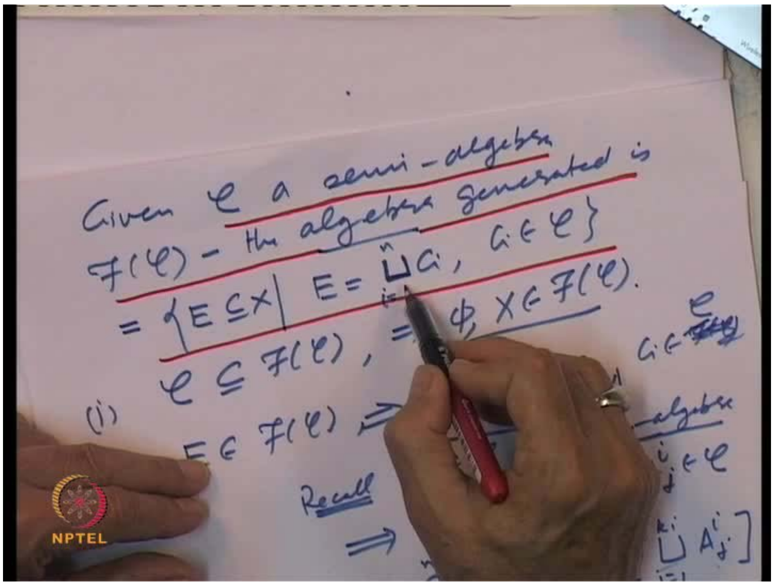

So, what we have shown is the following -C is any semi algebra and if I look at this collection, then subsets which are finite disjoint unions; then this includes C and it is an algebra.

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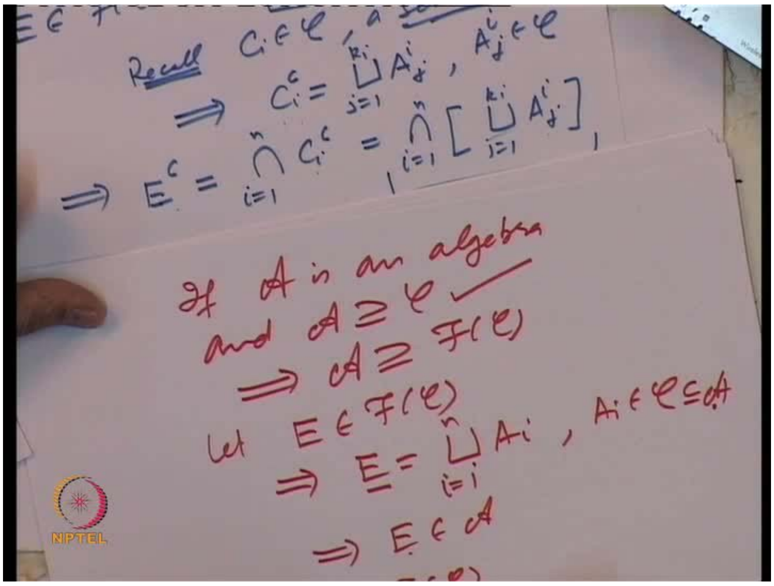

To show that it is algebra generated, we have only to show that ifA is an algebra and A include C; that should imply A includes F of C.

To prove that it isobvious, let us take let E belong to F of C, butthat will imply that Eis a disjoint union of elements Ai where AI’s belong to C and C is a subset of A;so,C inside A. So, that implies E is a union of some elements in A and A is an algebra; that means E belongs to A. Hence,F of C is a subset of A.So, what we have shown is that this collection F of Cof finite disjointunions of elements of the same algebra is indeed the algebra generated by the semi algebra.

So, we are able to describe completely the algebra generated by semi algebra.

Note one thing -we have described the algebra generated by a semi algebra explicitly.As remarked earlier, the algebra generated by a semi algebra is described, butin general, the algebra generated by any collection of subsets cannot be described explicitly.One may not be able to say what are the elements of that algebra generated by collection of subsets; so, that is notpossible always.It is only in the case when it is asemi algebra C,we are able to describe the algebra generated by it.

So, we have looked at a semi algebra,we have looked at algebra of collection of subsets of the X, and then we have looked at the algebra generated by semi algebra of subsets of X.

Next, we go to the next level ofcollection of subsets which are slightly stronger and that is called the sigma algebraof subsets of X.But before that, probably let us look at another property namely,how we generate morealgebras or sigma algebras out of a given collection?

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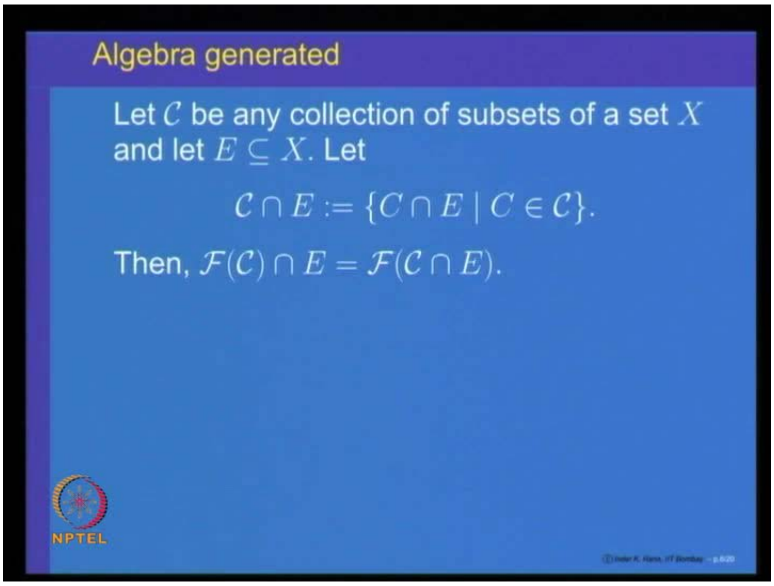

So, let us take,C - any collection of subsets of a set X, and let E be a fixed set in X.Let us write C intersection E to be the collection of sets of the type C intersection EC belonging to C. Note that C intersection EC is a collection of sets and E is a set.So,Cintersection E is just a notation for all sets of the type C intersection E; note that these are all subsets of the given set X.

So, the claims is that, given any collection C and given any set E, let us looks at C intersection E and generate the algebra by this collection of subsets. So,F of C intersection E is same as the algebra generated by Cintersecting with E. So, this is a very useful thing of restricting; in some sense, restricting the sigma algebras,restricting the algebras.

So, what we are saying is C any collection of subsets of a set X,E contained in X fix defined C intersection E to be C intersection E, where C belongs to C. So,what are these sets?So, note that it is C intersection E.So, this is a collection of subsets of the set E.

Now,I can look at F of C intersection E. So, what will be F of C intersection E?It is the algebra of subsets of Egenerated by the collection C intersection E and the claim is that F of C intersection E is same as you first generate the algebra by C.

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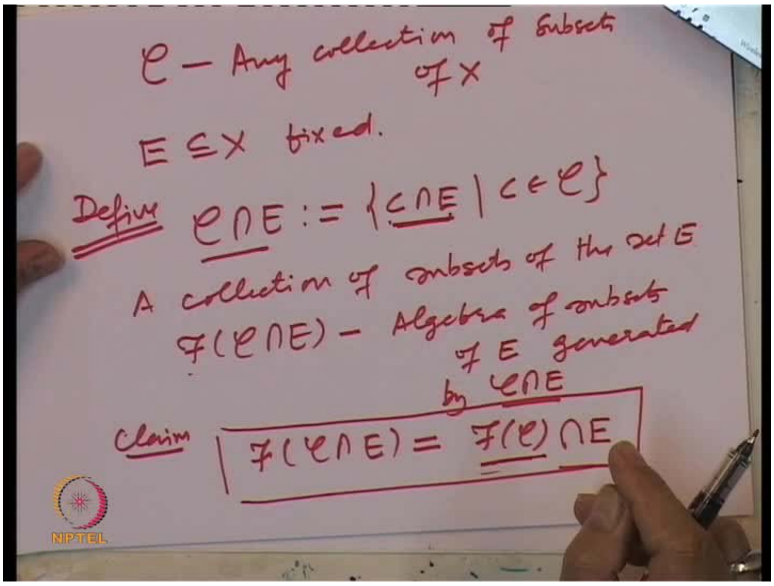

So, this will be the algebra generated by the collection of the subsets of X.So, this is algebra of subsets of X; take its restrictions to E; that is same as this.So, this is the theorem or this is the result that we say is true.

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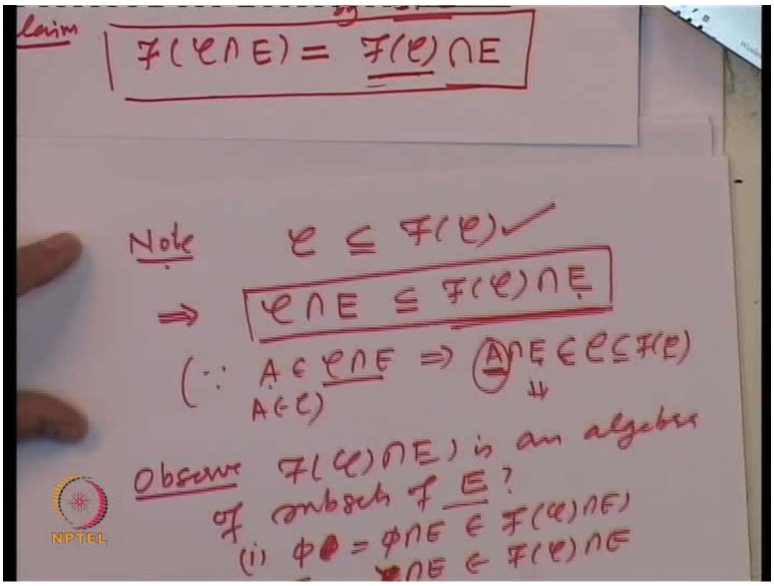

So, let us prove this.To prove this,let us first observe – note.So, first of all, we observe that C is contained in F of C.Given any collection C of subsets of a set X,F of C is the algebra generated by C.So, by the very definition,C is a subset of F of C.So, that implies that C intersection E is a subset of F of C intersection E. So, if I take a set in C intersection E,that is going to look like C intersection E, and C belongs to E.So, that is also an element in F of C; so is F of C intersection E.

So, let me writebecause if A belongs to C intersection E implies A intersection E belongs to Cwhich is in F of C, A is an element inC;so,C intersection E; so,A belongs to C; so, this is in C; so, that is in F of C.So, that implies this is an element of F of C intersected with E and that is precisely the meaning of this.So, we have got C intersection Ebecause of this is true.

Now, let us observe that F of C intersection E is analgebra of subsets of E; this is an algebra of subsets of E.Why?

Obviously, because empty set belongs to this and thiscan be written as empty set intersection E.So, that belongs to F of C intersection E; second, now note - this is a subsets of E.So,what is the whole space?That is E.So,E is equal to E intersection E;so, that is X intersection Eand that belongs to F of C intersection E.

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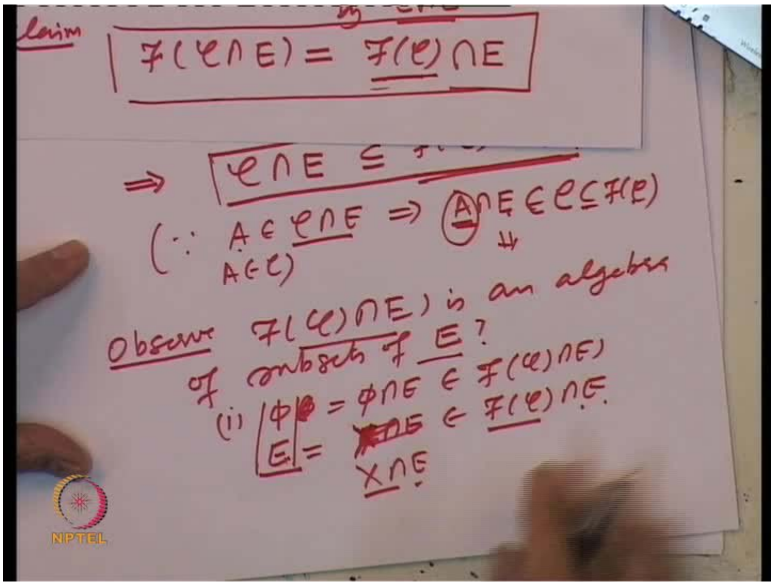

So I can write E as X intersection E; X is an element of F of C and E is here.So, that belongs to F of C. So, the empty set and the whole space that is E - they belong to this collection.What is the second observation?

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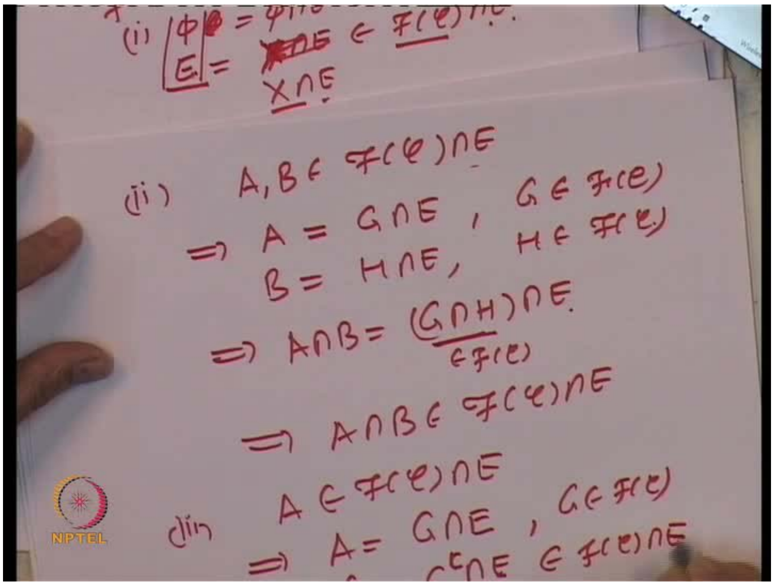

So, let us look at the second observation from here.Let us take two setsA and B belonging to F of C intersection E; that means if A belongs to C intersection E, that means A can be written as G intersection E, where G belongs toF of C and B is also in this.So,B is written as some Hintersection E,where H belongs to F of C.

So, that implies A intersection B is written as G intersection H intersection Eand now G and H both belong to F of Cso this belongs to F of C and that is in A.So, implies A intersection B belongs to F of C intersection of E.

Finally,let us look atthird - if A belongs to F of C intersection E; that means A can be written as some G intersection E, where G belongs to F of C.

So, now let us look at A complement, but keep in mind, the A complement is in E.So, this is nothing butG complement intersectionE.See this complement is in E;we are taking the complement inside set Ebecause we are looking at the collection of subsets of E.So, this belongs to F of C intersection E.

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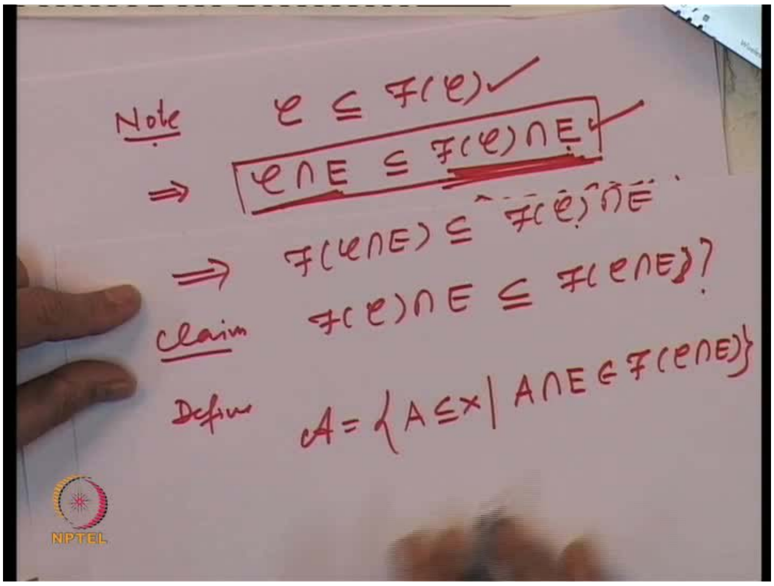

So, what we have shown is that C intersection E is subset of this andthis collection is an algebra; so, that implies the property. So, this implies that the algebra generated by C intersection E is a subset of F of C intersection E because this collection is inside this and this is algebra.So, the algebra generated by smallest must come inside.So, we get this property.

Now, we have to prove the other equality.So, claim that F of C intersection Eis a subset of F of C intersection E. So, this is what we have to prove.

So, to prove this, let us define a collection A to be all sets ifA contained in Xsuch that A intersection E belongs to F of C intersection E.Let us look at this collection A.

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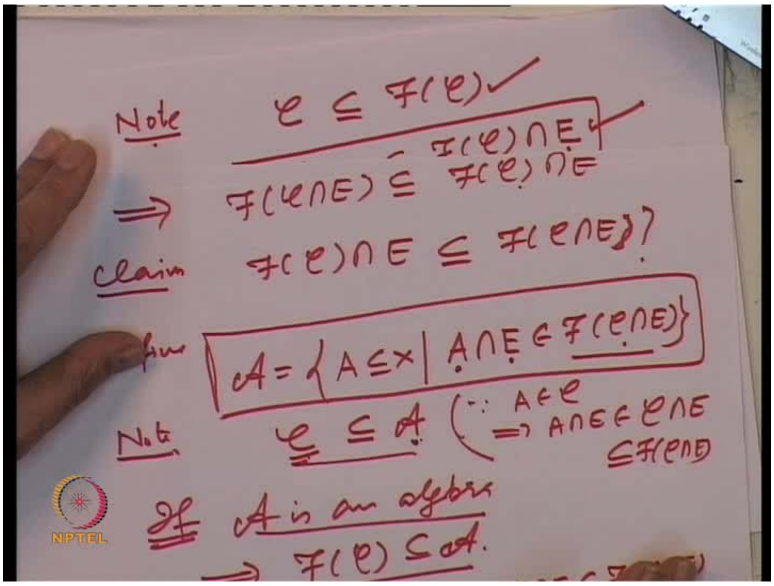

Note:C is contained in A; that is obvious because if I take an element in C, then that set A intersection E belongs to C intersection E which is inside this because A belonging to C implies A intersection E belongs to C intersection Ewhich is inside F of C intersection E; so,C is inside A.

So, ifA is an algebra, what will this imply?C is inside A.A is an algebra; that will imply F of C is inside A.What is the meaning of F of C is inside A?That means for every element F belonging to F of C.

Have a look at F intersection E that is belonging to F of C intersection E.So, that means that F of C intersection E is a subset ofF of C intersection E. So, to complete the proof, we only have to show that A is algebra of subsets of X.Now, let us look at this and show this is algebraof subsets of the set X.

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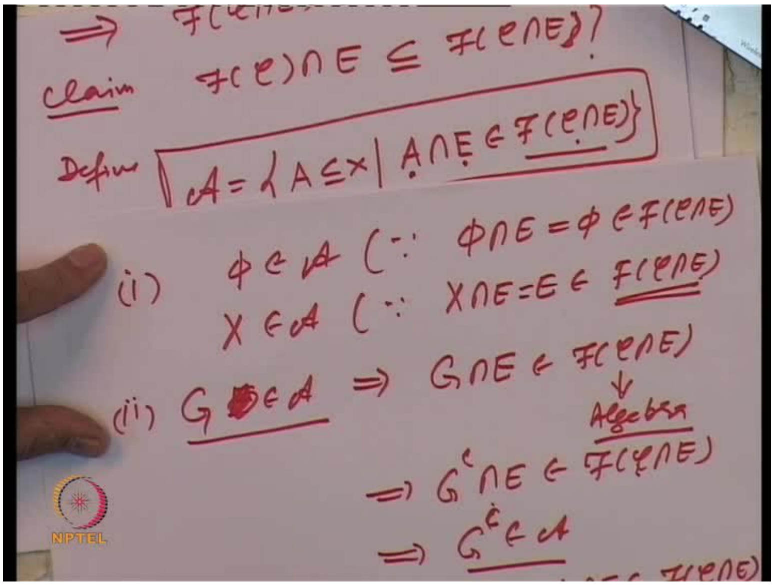

So, let us start observing.So, we want to show this is an algebra of subsets of a set X.So, to show that let us observe 1 empty set belongs to Abecause empty set intersection E is empty set which belongs to F of C intersection E. The whole space X belongs to A because the whole space intersection with E, which is E - that belongs to F of C intersection E.This is the algebra of subsets of E generated by this collection.

Secondly, let us take a set G belonging to A; that means Gintersection Ebelong to F of C intersection E and this is an algebra of subsets of E. So, this will imply that G complement intersection E also belongs to F of C intersection E because it is an algebra of subsets of E. So, its complement should also be inside that and that implies that G complement belongs to A.So,G belonging to A implies G complement belongs to A.

Finally,let us conclude that if G and H belong to A, that implies that G intersection E and H intersection E belong to F of C intersection E; that implies G intersection H intersection E belongs to F of C intersection E.

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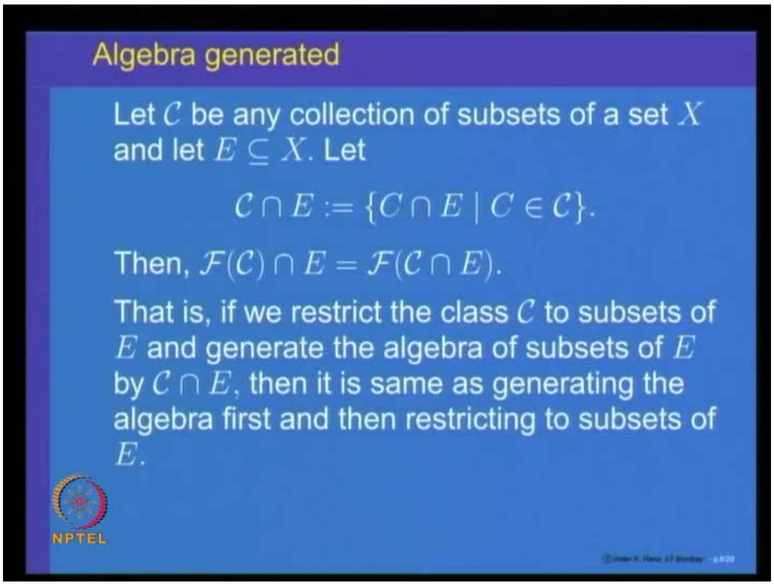

Once again, by the fact that this is an algebra,F of C intersection E - the collection A of subsets of X is an algebra; include C;so, that must include F of C and that proves the theorem that the algebra generated by a collection C that is F of C when restricted to A.The set E is same as first restrict the collection by which you are generating and then restrict and generate.

So, what we are saying is - given a collection C of subsets of X, if we restrict the class C to the subsets of E and then generate the algebra of subsets of E, that is same as first generating the algebra and then restricting that class of sets to that of E.

This is going to be very useful later on when we want to restrict collection of sets to subsets of it.

So, let us just conclude what we have done today.We started with recallingwhat is an algebra; what is a semi algebra; what is an algebra.Then, we started by observing that in general, if a collection C of subsets of a set X is not an algebra, we can always generate an algebra out of it.That means we can show the existence of a smallest algebra of subsets of the set X, which includes this collection C.Basically, the proof is by showing that if I take the intersection of all the algebras which include C, then that is the smallest one and that also had the important observation namely, intersections of algebras is again an algebra.

So, that was crucialproperty,a crucial observation that helped us to prove that the intersection of all the algebras that include C is also algebra, and that includes, because the intersection has to be smallestand hence that is the algebra generated by it.

Then, we gave examples of how to find algebra generated bya collection;for example,if you take singleton sets of any collection of any set X, then the algebra generated is the collection of all sets which are either finite or their complements or finite.

In general, if C is semi algebra, then the algebra generated by it is nothing but the collection of all finite disjoint unions of elements of that semi algebra.

So, thank you. We will continue next time.