Elementary Numerical Analysis

Prof. Rekha P.Kulkarni

Department of Mathematics

Indian Institute Of Technology,Bombay

Module No.# 01

Lecture No. # 09

Tutorial 1

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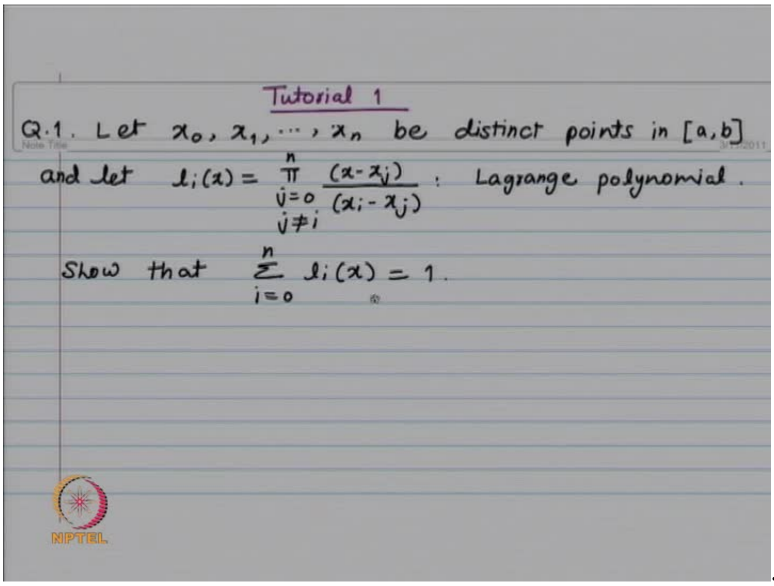

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So, before we start new topic of numerical integration we are going to solve some problems. I will be giving you solutions but may be you can find some alternate solutions which may be a better look at the problem that in the interval a, b we have got n plus 1 distinct points look at the lagrange polynomial which is based on this n plus 1 points.

So, the lagrange polynomial has the property that a l_i at x_j is equal to 1_i if i is equal to j and 0 if i not equal to j we want to show that summation i goes from 0 to n ,l_i of x is equal to 1. Since the lagrange polynomial, they are not defined recursively induction may not be a good idea.

In order to prove this we are going to use the property that if the function f is a polynomial of degree m which is less than or equal to n then when we try to fit a polynomial of degree n or bigger than or equal to n then the interpolating polynomial is going to be function itself here the interpolating polynomial we know that it is given by p_n of x is equal to summation f of x i l_i of x , i goes from 0 to n.

We are interested in showing the summation i goes from 0 to nl_i of x is equal to 1. So, the coefficients of l_i of x should be equal to 1. So, the simplest choice is look at f of x is equal to 1.

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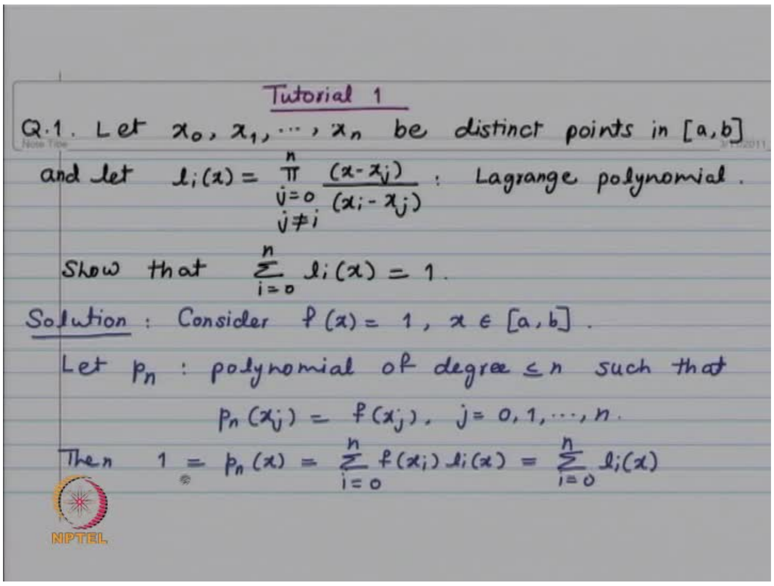

For f of x is equal to 1_it is a constant polynomial. So, when you look at p_n to be a polynomial of degree less than or equal to n which interpolates the constant function then that is going to be the function itself. So, that is the idea. So, choose f x is equal to 1 for x belonging to a, b and p_n be the interpolating polynomial of degree less than or equal to n such that p_n x_j is equal to f of x_j p_n x general form is summation i goes from 0 to n, f of x_i l_i of x.

P_n of x is going to be equal to 1 function is f of x is equal to 1. So, f of x_i will be equal to 1 and hence we get summation i goes from 0 to n ,l_i of x is equal to 1.

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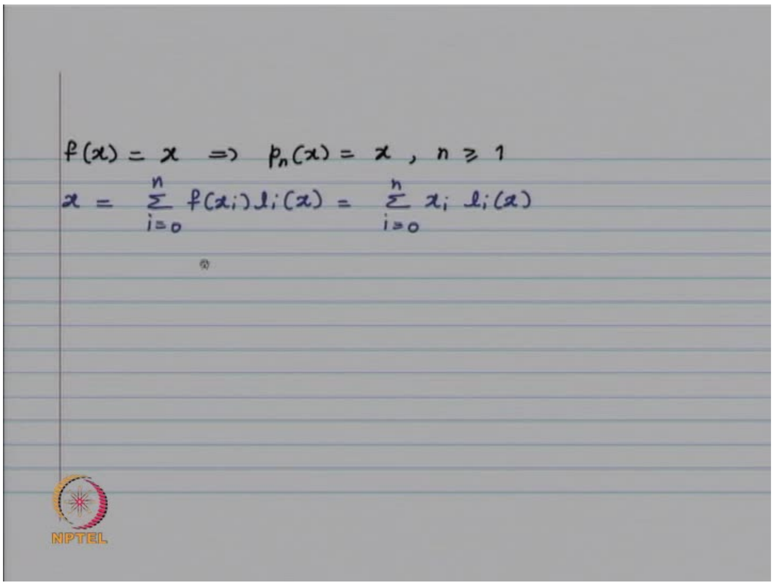

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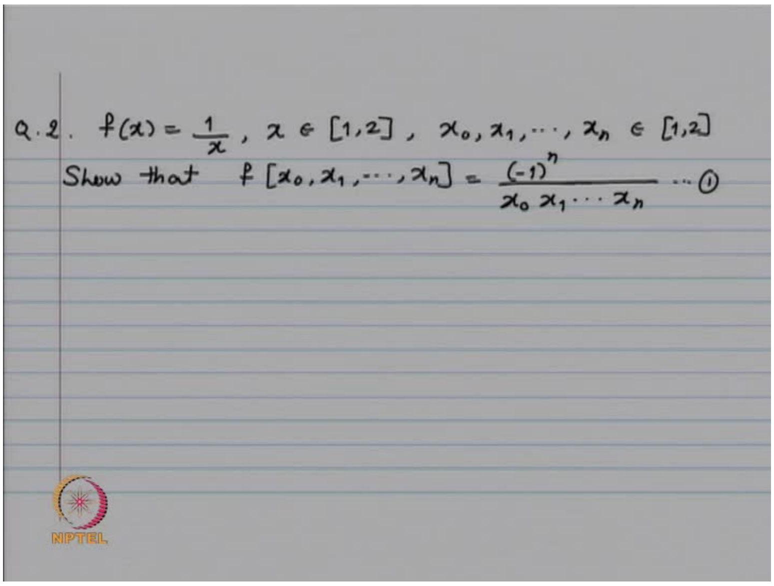

If instead of f of x is equal to 1, we look at f of x is equal to x in that case for n bigger than or equal to 1, p_n will be equal to function itself and that gives us a relation summation x_i li of x i goes from 0 to n is going to be equal to x in a similar fashion for n bigger than or equal to 2 summation x_i square l_i of x ,i goes from 0 to n will be x square and so on. Now, look at the second problem. So, we are looking at f of x is equal to 1 by xx in the interval 1 to 2 then we want to prove this expression that the divided difference f of x_0 x_1 x_n is given by minus 1 raise to n divided by x_0 x_1 x_n .

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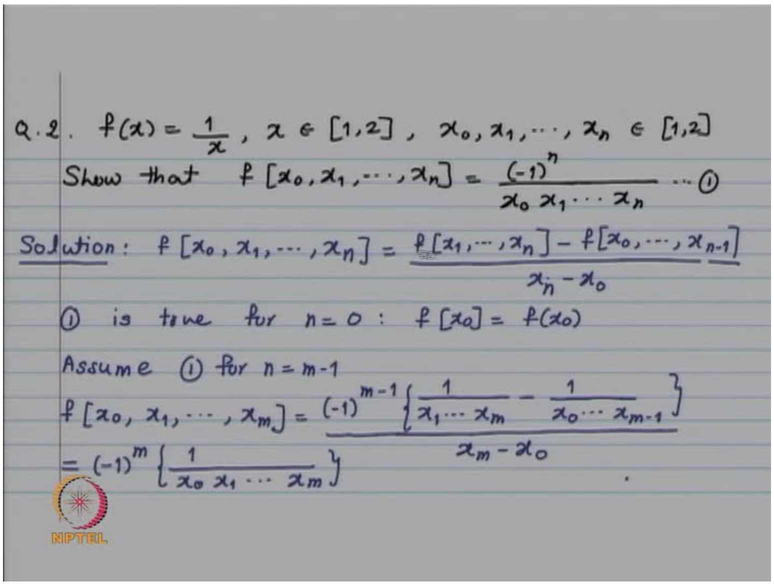

In this problem the induction is going to work we will be using the recurrence formula for the divided differences. So, for n is equal to 01 quickly sees that the result is true. Assume the result to be true for n is equal to m minus 1 and then using the recurrence formula for the divided differences one proves the result to be for n is equal to m you are to look at the recurrence relation f of x_0 x_1 n to be given by the divided difference based on x_1 x_2 x_n minus divided difference based on x_0 x_n minus 1 and then whole thing divided by x_n minus x_0 .

We are assuming that x_0 x_1 x_n these are all distinct points when you put n is equal to 0 on the left hand side you have got f of x_0 and on the right hand side you have got 1 up on x_0 and hence the result is true for n is equal to 0 now assume the result for n is equal to m minus 1 consider divided difference f based on x_0 x_1 x_n we have got this recurrence relation we are assuming the result to be true for n is equal to m minus 1.

And hence we will have f of x_1 x_2 x_m to be minus 1 raise to m minus one divided by 1 up on or multiplied by 1 up on x_1 x_2 x_m f of x_0 x_1 x_m minus 1 will be 1 up on x 0 x n minus 1 and then divided by x_m minus x_0 now when you simplify then you are going to get in the numerator x_0 minus x_m in the denominator you have x_m minus x_0 and then multiplied by x_0 x_1 x_m .

So, that adds one more minus one and then we have got minus 1 raise to m1 up on x_0 x_1 x_m. So, for some functions we can have the divided difference to be given explicitly.

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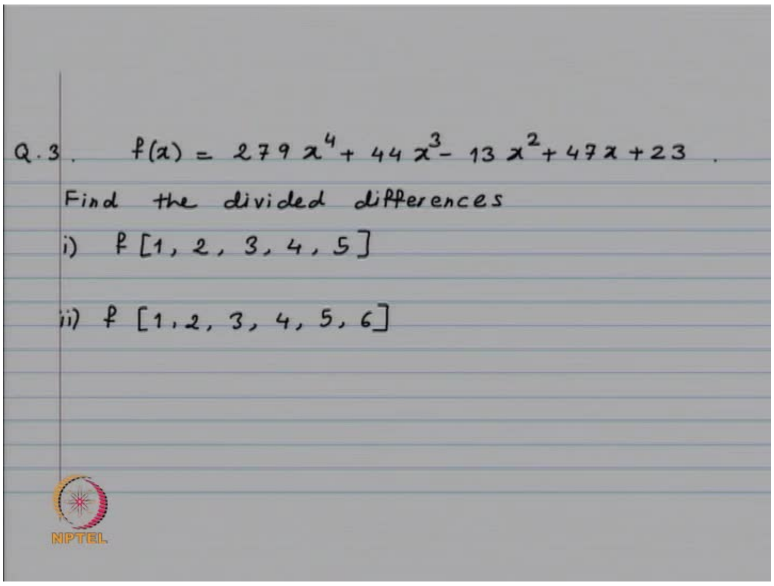

Our next problem is you are given a polynomial of degree 4 and you are to find 2 divided differences one is based on 5 points and another is based on 6 points. So, what one can do is construct the divided difference table you are given the function. So, construct the divided difference table and then find the divided differences.

But here one can observe that f is a polynomial of degree 4 we are looking at divided difference based on 5 points our definition of divided difference is look at the interpolating polynomial which interpolates these 5 points and look at the coefficient of x raise to 4 in that polynomial.

Our function is a polynomial of degree 4. So, interpolating polynomial of degree 4 is going to be the function itself.

And hence coefficient of x raise to n x raise to 4 in our function that will be the divided difference based on 5 points if we look at the 6 points then we have to look at polynomial of degree 5 which interpolates the given function at those 6 points the function is a polynomial of degree 4 so a polynomial of degree less than or equal to 5 that is going to be the function itself.

So, treat our function which is a polynomial of degree 4 as a polynomial of degree 5; that means, you add plus 0 into x raise to 5 and now we have to look at the coefficient of x raise to 5 which is going to be 0.

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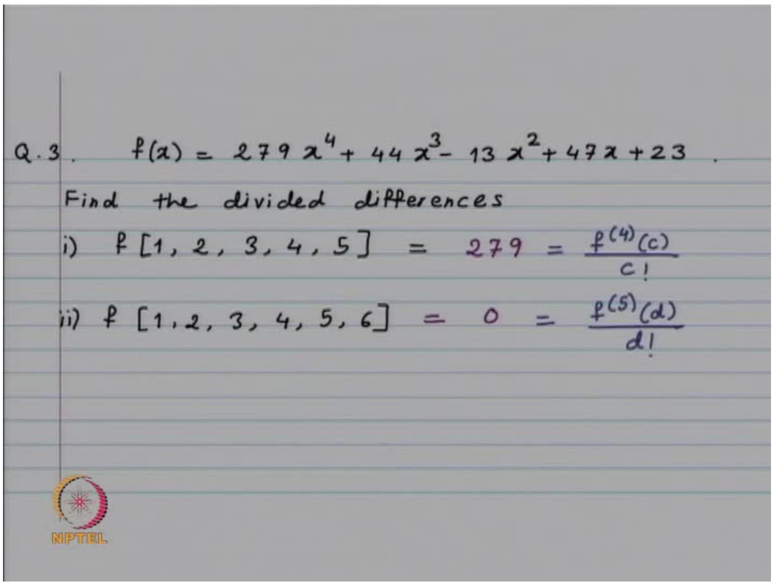

Now, the same result one can obtain by using the fact that our function f is sufficiently differentiable then when you look at the divided difference based on 5 points it is equal to f 4 of c divided by c factorial divided by it should be 4 factorial not c factorial then the function is a polynomial of degree 4 and hence when you take the fourth derivative it is going to be constant and that will be nothing, but 279 when you look at the fifth derivative that is going to be 0 and hence the divided difference is equal to 0.

So, you can either use the definition or you can use the formula in terms of the derivative if you try to write down the divided difference you are going to get the same result but the computations they are going to be missing.

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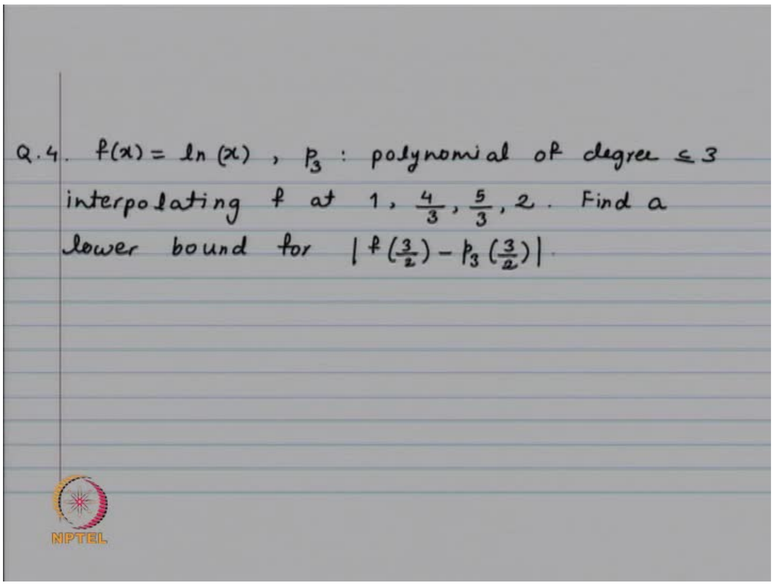

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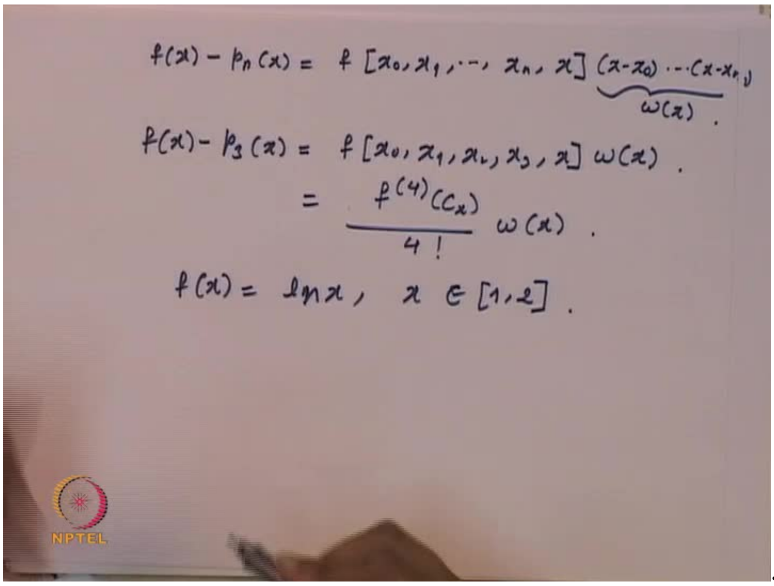

So, far we had obtained upper bounds now let us look at anexample where f of x is equal to ln x and look at a cubic polynomial of degree less than or equal to 3 interpolating at 4 points and we want to find a lower bound. So, we have got f of x minus p_n of x is equal to f of x_0 x_1 x_n x and then multiplied by x minus x_0 x minus x_n. So, this we denote by w of x.

We are looking at f of x minus p_3 of x. So, that will be f of x_0 x_1 x_2 x_3 x multiplied by w of x and this will be f4 of c_x divided by 4 factorial w of x our function is f of x is equal to ln of x, x belonging to 1 by 2. So, we will calculate the fourth derivative of this ln x and now because we want to be consider the lower bound for the derivative f4 c_x we will look at the least value and for the w of x we are going to x_0 x_1 x_2 x_3 these are given to us we are going to substitute x to be equal to 3 by 2 and then we are going to get a lower bound throughout.

So, far we had always considered upper bound. So, I wanted to show that one can also consider a lower bound. So, that tells us that there is going to be at least this much of error now the computations are straight forward.

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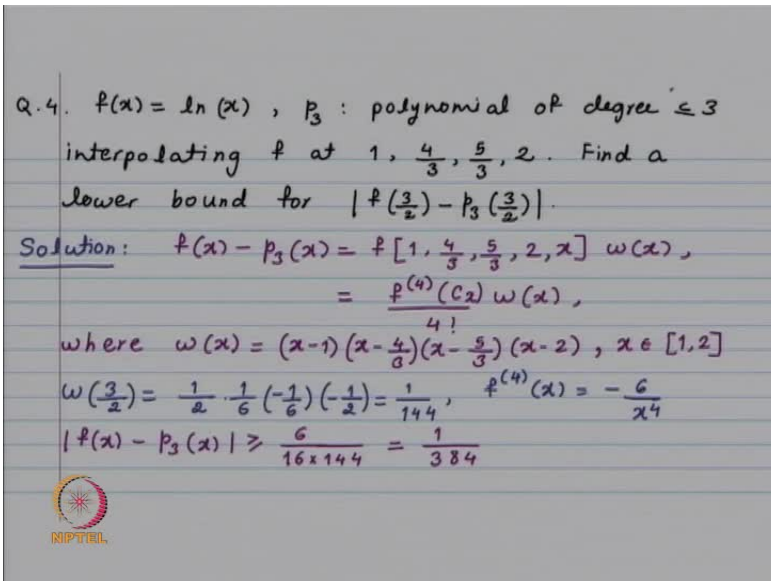

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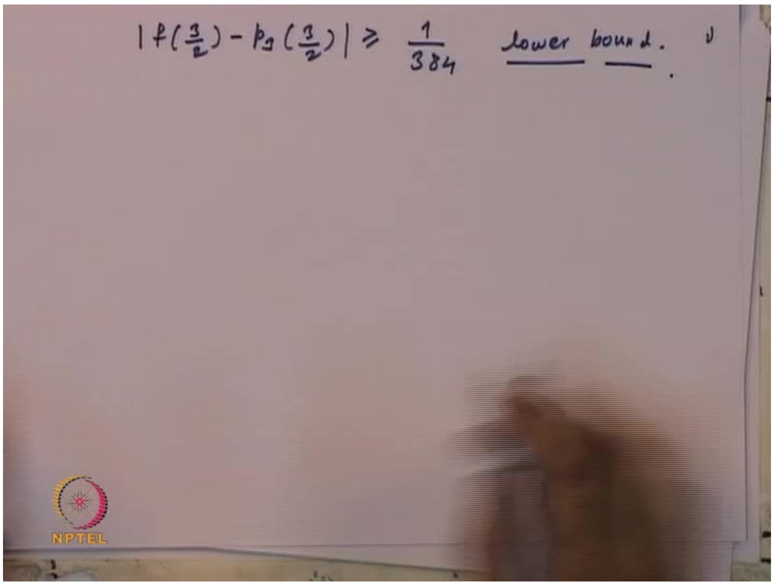

Look at f of x minus p_3 of x. So, that is f4 of x c_x divided by 4 factorial into w of x then you can verify that the fourth derivative at x will be given by minus 6 by x raise to 4 and we have to look the least value of this. So, the least value will be obtained when x is equal to 2 so the least value for f^4(x) is going to be 6 up on 2 raise to 4. So, that is going to be 6 up on 16 and then you calculate the value of w of 3 by 2 and that gives you the lower bound for x is equal to 3 by 2. So, we have modulus of f of 3 by 2 minus p_3 of 3 by 2 is going to be bigger than or equal 1 up on 384. So, it is a lower bound.

Then now consider we are again going to consider interpolating polynomial but now our polynomial is going to interpolate the function and a derivative values. So, suppose our function has a double 0 at some point x_0. So, we have got f of x_0 is equal to f dash of x_0 is equal to 0 and at some other distinct point x_1 we have got f of x_1 is equal to f dash of x_1 is equal to f double dash of x_1 to be equal to 0.

Now, what we want to do is we want to fit a polynomial of degree less than or equal to 5 which is going to interpolate the function value derivative value at x_0 function value derivative value and second derivative value at x_1 and we want a polnomial of degree less than or equal to 5. So, we need one more condition. So, let x to be another point which is distinct from x_0 and x_1 .

And it should interpolate the function at this point then we want to find such a polynomial so our interpolation points are x_0 is repeated twice x_1 is repeated thrice x_2 is repeated once.

So, the usual way of constructing such a polynomial is you form the divided difference table once you form the divided difference table then you look at the value f x_0 then f of x_0 x_1 divided difference and so on.

And then that is the newton form that is how we construct the polynomial but here we can give a different proof our function our polynomial it is going to match with the function value and derivative value at x 0 and the function value and the derivative value both of them they are 0.

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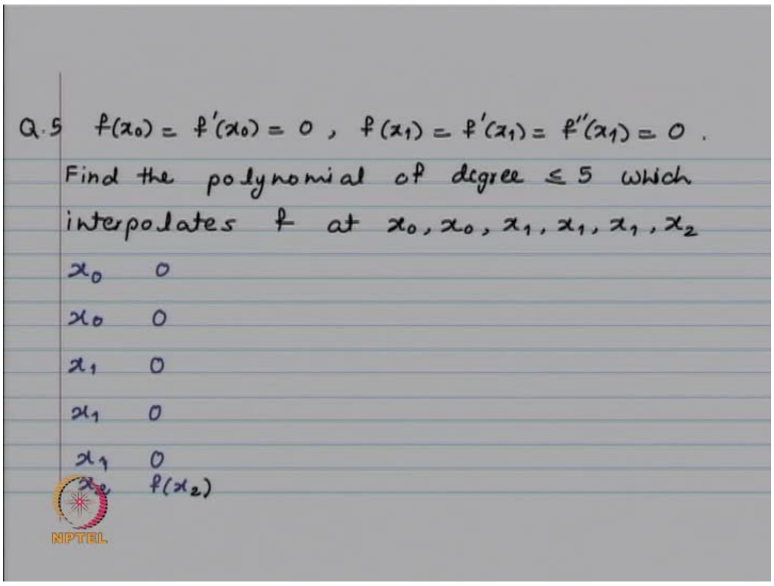

That means the polynomial p_5 is going to have a factor x minus x_0 square it is a double you factorize similarly ,you look at the point x_1 our polynomial should be such that it vanishes at x_1 its vanishes its derivative vanishes at x_1 and second derivative vanishes at x_1 that means in our polynomial there should be a factor x minus x_1 cube. Now, look at the polynomial we have got a factor x minus x_0 square x minus x_1 cube and it is a polynomial of degree less than or equal to 5.

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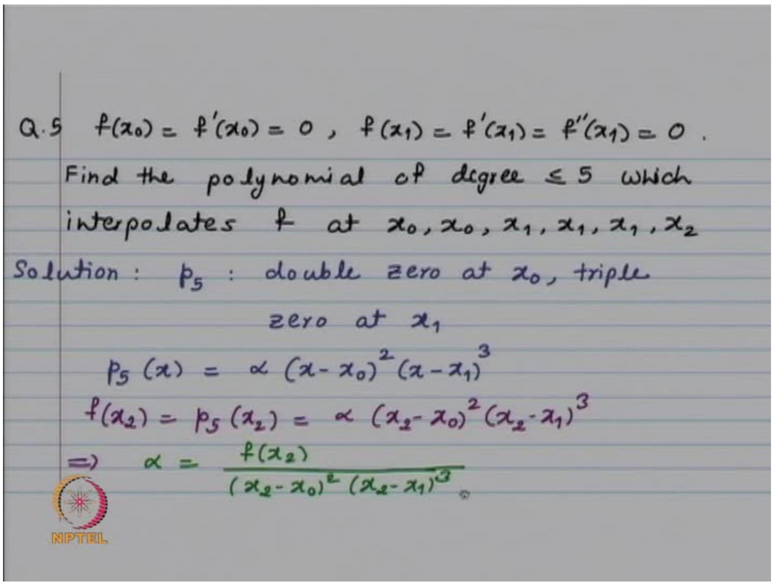

So, now what will be there will be a coefficient that has to be constant and we can we are going to determine that constant so we can as I said either form divided difference table or you look at the fact that p 5 it has got the double 0 at x_0 triple 0 at x_1. So, we have got look at the factors x minus x_0 square x minus x_1 cube since it is a polynomial of degree less than or equal to 5 you have to multiply only by a constant.

Otherwise if it was a higher degree polynomial here it would have been a function of x and hence you get this polynomial form now we have to determine alpha. So, far we have not use the fact that our polynomial is going to interpolate the given function at x_2 also so we have got x_2 is equal to p_5 x_2 is equal to alpha times put x is equal to x_2. So, x_2 minus x_0 square x_2 minus x_1 cube and that will give you alpha to be equal to f x_2 divided by x_2 minus x_0 square and x_2 minus x_1 cube.

It will be a good exercise to try to complete the divided difference table get the various divided differences which are going to come into picture for the p_5 of x and see that you are going to get the same result.

Now, we will consider one more example or one more problem and then we will go to the next topic of numerical integration.

Now, it is a simple example we are going to look at f of x is equal to x square a simple function and then look at the interval n to n plus 1. So, n is a integer or a natural number and what we want to do is we want to find an upper bound now when we consider interpolating polynomials and then we know that the error is in terms of the appropriate derivative of the function evaluated at a point now that point is not known but look at function f of x is equal to x square.

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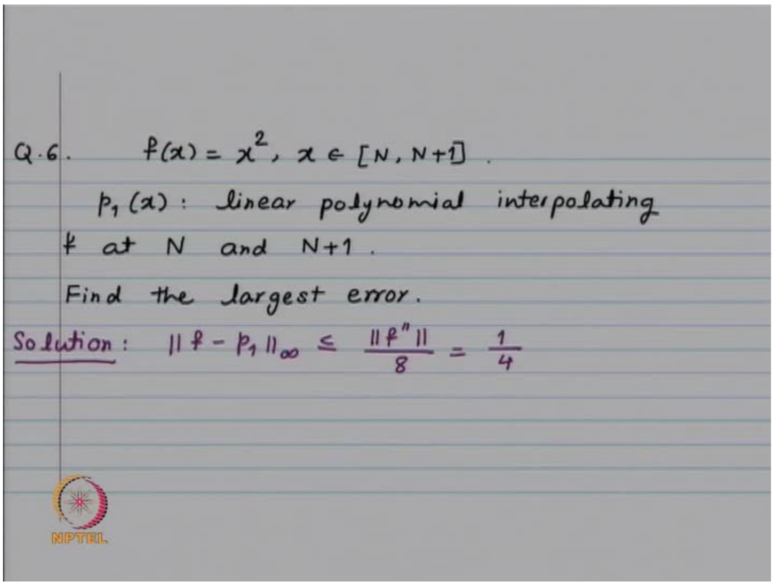

We are looking at linear polynomial. So, in the error the second derivate comes into picture and second derivative for x square it is going to be constant. So, using that fact one just calculates what is an upper bound. So, we have norm of f minus p_1 infinity this will be less than or equal to norm f double dash infinity divided by 8 and then you have got this will be equal to 1 by 4.

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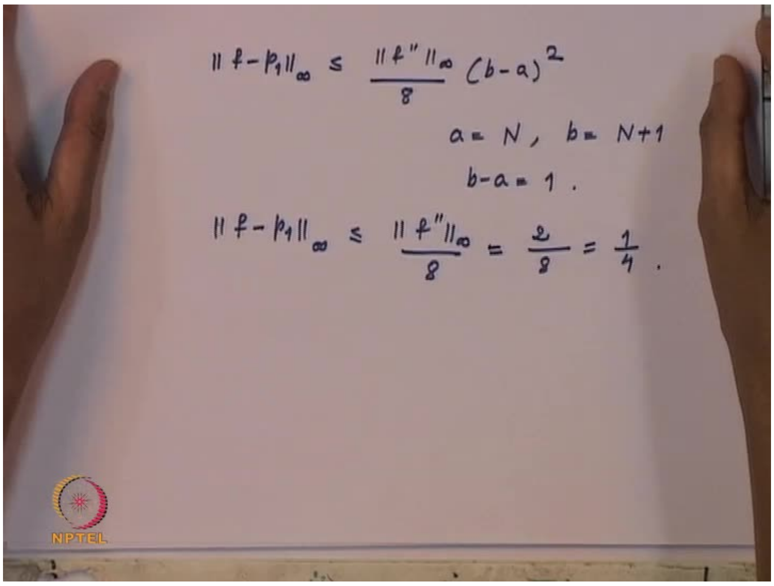

Because we have when we look at norm of f minus p_1 infinity norm to be less than or equal to it is norm f double dash infinity divided by 8 into b minus a square our a is capital N b is N plus 1. So, b minus a is equal to 1. So, that is why norm f minus p 1 infinity norm is less than or equal to norm f double dash infinity by 8 and f double dash being equal to2 it will be 2 by 8 which will be equal to 1 by 4.

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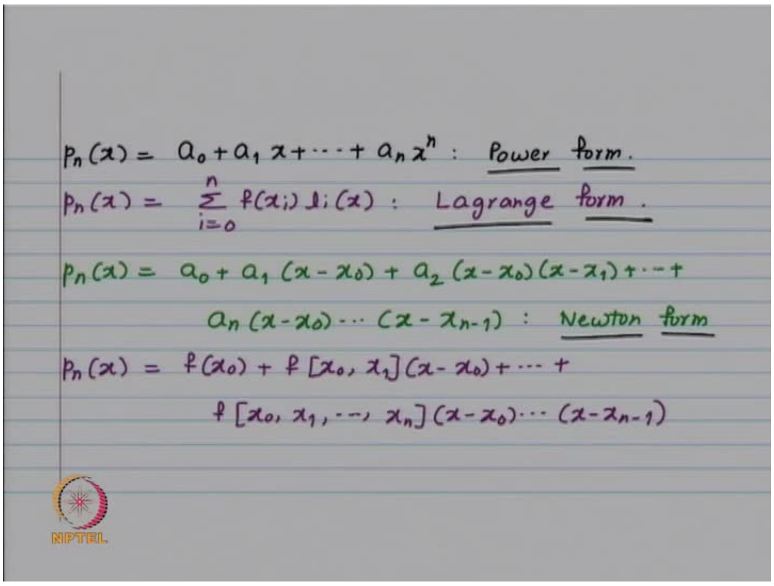

So, on any interval n to n plus 1 this is going to be the bound now I want to recall what forms of the polynomials we have consider. So, we have power form which is a 0 plus a

1 x plus a n x raise to n we have got lagrange form which involves lagrange polynomials and we have got newton form.

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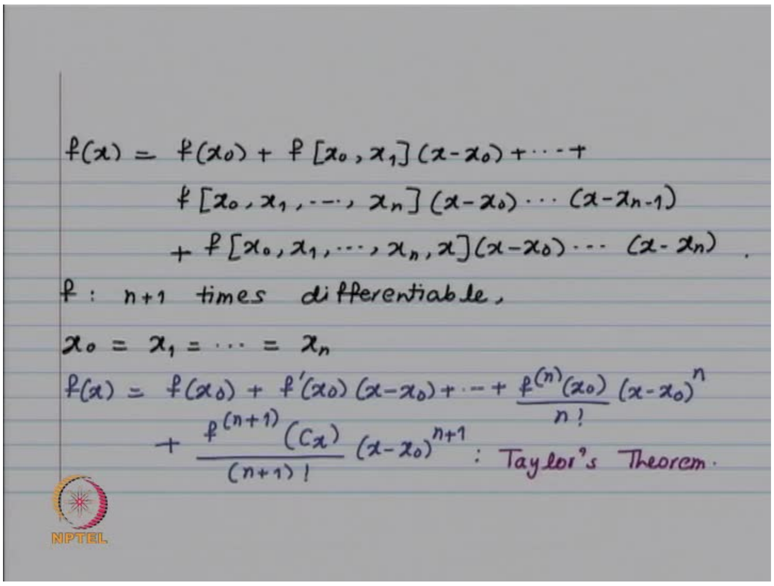

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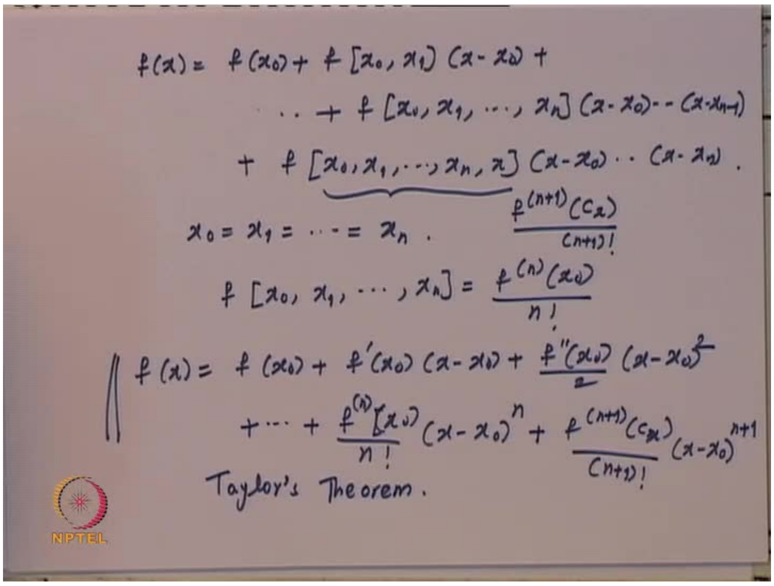

Then we had this error. So, we have f of x is equal to f of x_0 plus f of x_1 x_1 x minus x_0 plus f of x_0 x_1 x_n x minus x_0 x minus x_n minus 1 and then the error f of x_0 x_1 x_n x and then x minus x_0 x minus x_n suppose our points are all identical then we have proved that f of x_0 x_1 x_n in this case we define it be equal to f_n of x_0 divided by n factorial.

So, thus our this result becomes f of x is equal to f of x_0 plus f dash x_0 into x minus x_0 plus f double dash of x_0 by 2 x minus x_0 square plus f of f_n of x_0 divided by n factorial x minus x_0 raise to n plus this divided difference is equal to f n plus 1 c_x divided by n plus 1 factorial. So, it will be f n plus 1 c_x divided by n plus 1 factorial multiplied by x minus x_0 raise to n plus 1.

Now, you are familiar with this this is nothing but Taylor's theorem so, this is an observation now we are going to start the new topic of numerical integration suppose your function f is defined on interval a, b and it is continuous.

In that case one defines the riemann integration for that what one does is looks at the Riemann sum, upper Riemann sum, lower Riemann sum, and in terms of those one define the integral for a continuous function we can show that take the interval a,b subdivide it into 2 equal parts form the riemann sum by taking any. So, what you have to do is in each interval you look at value a take a point any point and then you look at summation f at c_i multiplied by length of the interval. So, the length of the interval will be h submit over take its limit as n tends to infinity and that is going to be our value of integral a to b f of x_d .

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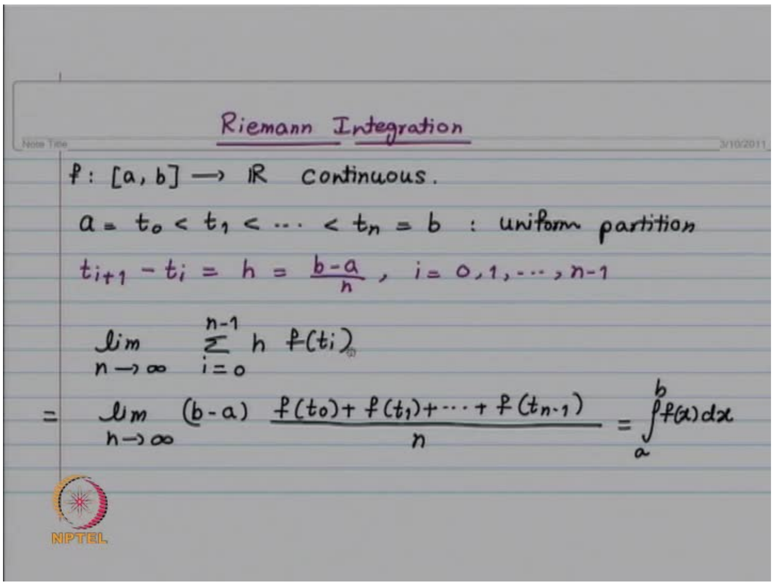

So, we have this for a continuous function limit n tending to infinity summation i goes from 0 to n minus one h times f of t_i, I had told you that you can take any point in the interval t_i to t_i plus 1. So, let us choose it to be n point t_i .

Form this sum and then take its limit as n tends to infinity our h is b minus a by n. So, what you are doing is your looking at an average and then you are multiplying by b minus a and then you get the limiting value is equal to integral a to b f of x dx. So, using this definition one proves many important results and in numerical analysis now suppose I give you a continuous function all continuous function they are going to be integrable. So, if you are given a continuous function and you are ask tell me what is value of integral a to b f of x dx.

If not exact value may be some approximate value then you are not going to look at this riemann sum take limit as n tends to infinity. So, this is not how one finds the value of the integral. So, then there is a class of functions for which one know how to integrate. So, if you know anti-derivative of your function; that means, I want to find integral a to b f of x dx. So, suppose I know another function capital F such that capital F its derivative is going to be my function f if I do that if I know that then we have got integral a to b f of x dx is capital F of b minus capital F of a.

In fact, that is how one calculates the integration that you know integration and differentiation these are opposite process. So, there are some functions for which you can calculate anti derivative. So, one big class is to polynomial functions. So, for polynomial functions we know there anti-derivatives there are also trigonometric functions we know sin x cos x how to integrate exponential x. So, there are there is some class of problems or class of functions rather one can calculate the integral a to b f of x dx.

So, our idea is that our function we know how to approximated by polynomials. So, consider approximation of function f by a interpolating polynomial integrate that polynomial whatever value you get that you are you will be able to do even using computer. In fact, when we started the polynomial approximation that is what we said that the polynomials they are nice functions you can derivative using computer you can find the integral using the computer so you calculate the value of the integral and then you are going to get an approximate value of the integration. So, in order to have sufficient accuracy you have choose and begin up but we have seen that if your function is only given to be continuous then we do not have a sequence of interpolating polynomials which is going to converge to our function f.

So, then that is where now we went to piecewise polynomial approximation. So, why not look at piecewise polynomial approximation.

So, on each interval or on each subinterval I will have a polynomial I will integrate that I will add it up and then I will get value of the function. So, that is how one is going to do when you consider a single polynomial.

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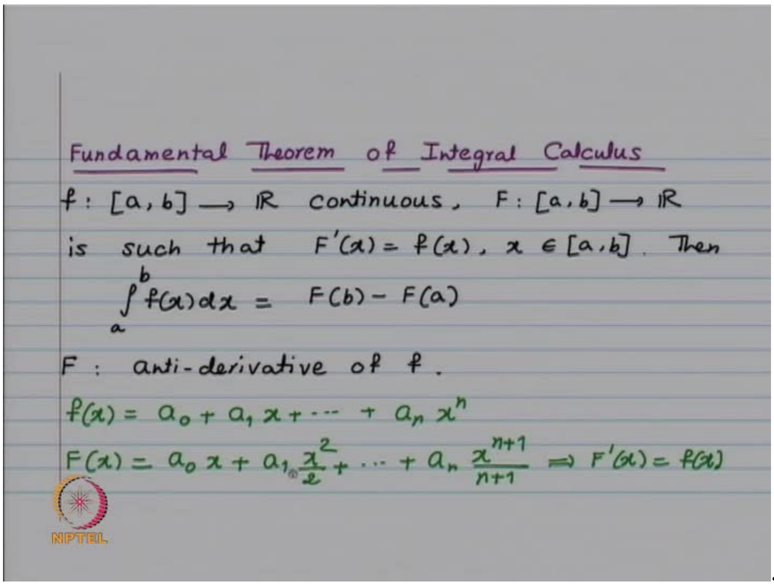

Then you are going to get basic rule when you are going to look at piecewise polynomials you are going to get what are known as composite rules. So, we are going to consider some of the standard rules which are used in practice. So, here is if your function is a polynomial a_0 plus a_1 x plus a_n x raise to n.

Then if you look at capital F of x to be equal to a_0 x plus a_1 x square by 2 plus a_n x raise to n plus 1 by n plus 1 then it satisfies the property that F dash x is equal to small f of x.

And then this is one of the fundamental theorem of integral calculus that integral a to b f x dx will be nothing but capital F of b minus capital Fof a.

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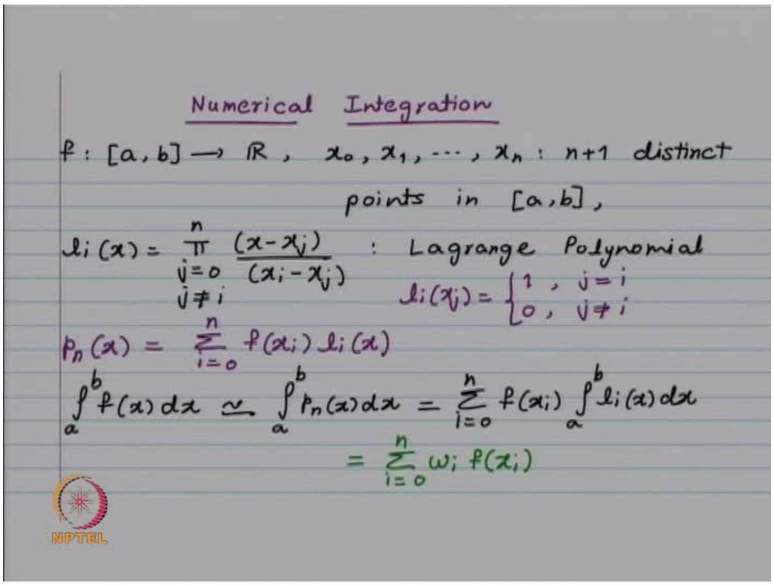

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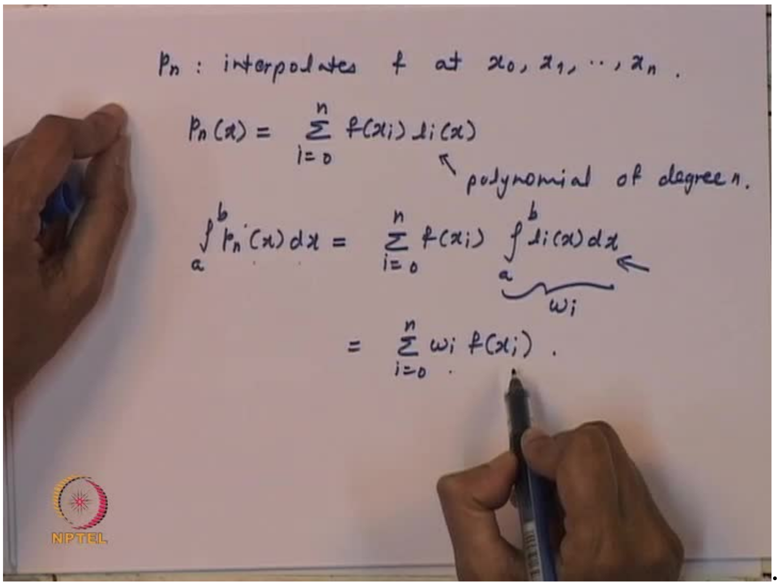

So, now let us look at the interpolating polynomial. So, we have this form we have p_n interpolates f at x_0 x_1 x_n then p_n of x is going to be summation f of x_i l_i of x, i goes from 0 to n where l_i is a polynomial of degree n.

So, integral a to b p_n of x dx will be equal to summation i goes from 0 to n f of x_i integral a to b l_i of x dx. So, this is going to give us some real number wi. So, you have summation i goes from 0 to n, wi f of x_i note that here there is no dependence on the function. So, this we will do once for all and then we get a formula for a integral a to b p_n of x dx in terms of these wi's and the function values $x_{i.}$

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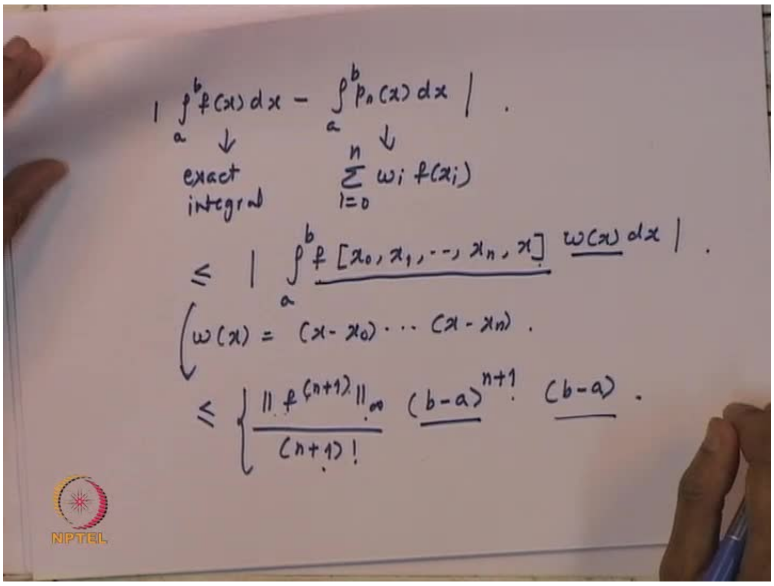

Now, look at the error. So, here we know the error in the interpolating polynomial. So, we have f of x is equal to p_n of x plus a error term the error term is a function of x. So, you integrate so we are going to have integral a to b f of x dx is equal to integral a to b p_n of x dx plus integration of the error now for the error we will be calculating only an upper bound. So, we have modulus of integral a to b f of x dx minus integral a to b p_n of x dx modulus.

So, this is exact integral this will be of the form summation wi f of x_i, i goes from 0 to n this will be less than or equal to modulus of integral a to b f of x_0 x_1 x_n x into w of x dx where w of x is x minus x_0 x minus x_n. So, this will be less or equal to norm of f of n plus one infinity divided by n plus 1 factorial and then we have got b minus a raise to n plus 1 and into b minus a.

So, norm of f of n plus 1 infinity is coming from this factor this I am going to dominate by this. So, it will come out of the integration sign modulus of w of x will be less than or equal to b minus a raise to n plus 1so that also come out the integration sign and then you will have a integral a to b of 1 or a to b dx that is going to give me b minus a.

Now, this error which we have obtained it have got n plus first derivative of the function its norm divided by n plus 1 factorial and then we have got b minus a raise to n plus 2 now here what has happened is the x_0 x_1 x_n they sort of we have lost we have used crude approximation that x_0 x_1 x_n they all lie the interval a, b. So, if our x is varying over the interval a b x minus x g also will vary over the interval a, b.

Now, the choice of interpolation points that is important. So, we do not want to lose them. So, what will be a better estimate is that w of x is a polynomial it is x minus x_0 x minus x_1 x minus x_n we can integrate it. So, that integration value that will retain the information about x_0 x_1 x_n .

But then w of x in the error w of x is multiplied by n plus first derivative of the function divided by n plus 1 factorial. So, I cannot just take it out and integrate. So, for that purpose in order to get a finer bound we are going to prove what is known as mean value theorem for integral also there will be cases when our if the points are such that for example suppose you are looking at 1 point integral and that 1 point is the midpoint then integral a to b x minus x_0 dx that is going to be 0 and using that property one will get a higher order of convergence or a better order of a convergence for the corresponding composite rule.

So, that is why we do not want to be satisfied with this upper bound it is an upper bound but we want to have some better upper bound and for that we prove mean value theorem for integrals. So, what it tells is that suppose you are integrating 2 functions or you are looking at the product of 2 function f of x into g of x over the interval a, b our function f suppose it is continuous.

The other function g what we want is it should be integrable continuous functions they are integrable if your function has finite number of discontinuities then also it is integrable. So, our g x should be a integrable function riemann integration I am always talking about riemann integration and it should be either bigger than or equal to 0 or less than or equal to 0 throughout the interval. So, one function should be continuous other function should be either bigger than or equal to 0 or less than or equal to 0 if that happens then integral a to b f x into g x dx will be equal to value of f at some point c into integral a to b g of x dx.

So, remember our error in the integration it was consisting of 2 parts one was the divided difference based on x_0 x_1 x_n x and we have proved continuity of this function the other term was w of x that is x minus x_0 x minus 1 x minus x_n. So, depending on the choice of x_0 x_1 x_n it can happen that w x is also is bigger than or equal to 0 or less than or equal to 0.

Look at the linear polynomial which interpolates at the 2 end points in that case our w x is going to be x minus a into x minus b our x varies over the interval a to b. So, x minus a will be bigger than or equal to 0 x minus b will be less than or equal to 0. So, there product will be less than or equal to 0.

So, now if I can take out the divided difference outside the integration by evaluating at a at a point then I will have a integral a to b w of x dx and that value we can calculate

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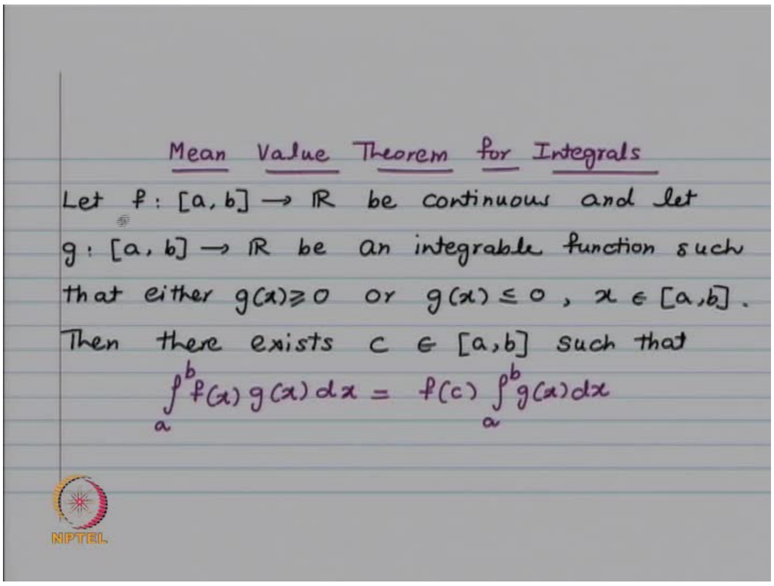

So, let us prove this mean value theorem for integrals. So, we have f is a map from a b to R which is continuous and g is an integrable function such that either g of x bigger than or equal to 0 or g x less than or equal to 0.

Then there exist apoint c in the interval a b such that integral a to b f of x g of x dx is equal to f of c into integral a to b g of x dx. So, this is the theorem we are going to prove now.

The proof of this theorem it is going to be best on property of continuous function our function f is defined on interval a b closed interval closed and bounded interval a b and takes real values then such a function is going to be bounded it is going to attain its infimum and supremum. So, supremum is nothing but least upper bound amongst the upper bounds the one which is the smallest when its attains its upper bound then that is known as maximum and similarly ,it is going attain its minimum that is greatest lower bound among the lower bounds whichever is the biggest. So, our function f will have absolute minimum small m and absolute maximum capital M. So, small m will be less than or equal to f of x less or equal to capital M.

And then we have intermediate value property that between this small m and capital M our function is going to assume each value. So, this is a crucial property which we are going to use in the proof of this theorem. So, what let mere fresh the property this continuous functions our function f is defined on interval it will be interval a b it will be its range will be small m to capital M. So, it is going to be on 2 functions.

And any point in the interval small m to capital M that point that is going to be attain. So, our we want to show that integral a to b f of x g of x dx is equal to f of c into integral a to b g of x dx. So, if integral a to b g x dx is not 0 we divide. So, we look at integral a to b f x g x dx divided by integral a to b g of x dx and show that it lies between small m and capital M so.

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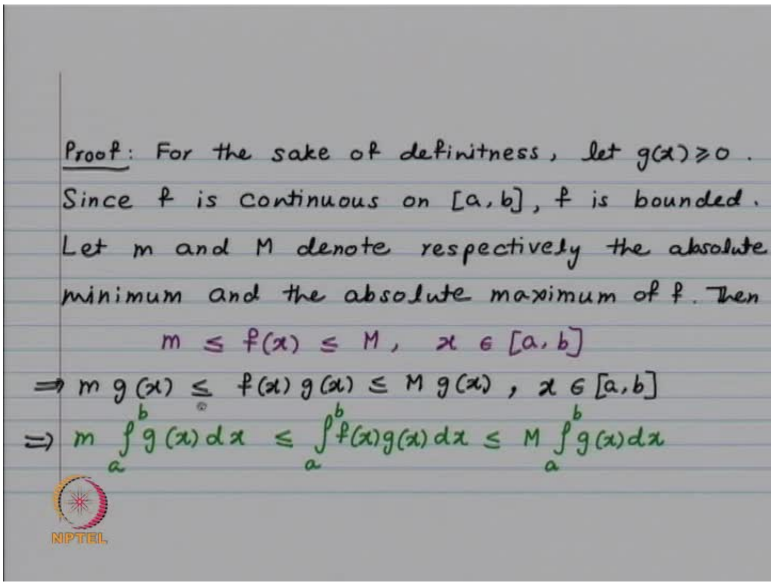

Now, for the sake of definiteness we will assume g of x to be bigger than or equal to 0 if it is less than or equal to 0 some inequality is they will change but the proof is very much similar. So, let us assume that g of x is bigger than or equal to 0 and let small m and capital M denote the absolute minimum and absolute maximum of function f.

So, we have m less than or equal to f x less than or equal to capital M g of x is bigger than or equal to 0. So, m into g of x is less than or equal to f of x into g of x less than or equal to capital M into g of x.

Now, you integrate using the property of integration we will have the inequality science will be preserved and then small m and capital M they do not depend on x. So, they come out of the integration sign and we have small m into integral a to b g of x dx less than or equal to integral a to b f x g x dx less than or equal to capital M into integral a to b g of x dx.

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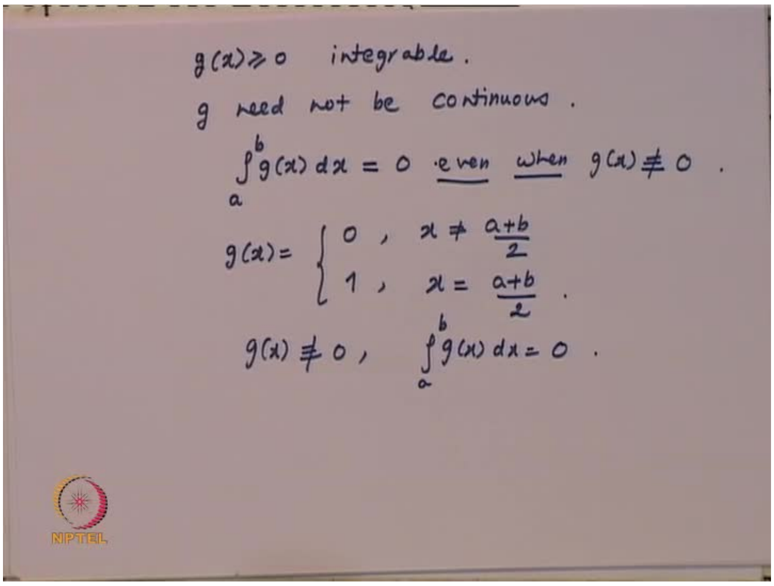

Now, our g we are assuming it to be bigger than or equal to 0 and integrable. So, g need not be continuous so, integral a to b g of x dx it can be equal to 0.

Even when g of x is not identically 0 if g of x is identically 0 then of course, integral a to b g of x dx is 0 but because g need not be continuous it can happen that integral a to b g of x dx is equal to 0 when g of x is not identically 0 like look at function g of x which is equal to 0 if x is not equal to a plus b by 2 and is equal to 1 if x is equal to a plus b by 2 so g of x is not identically 0 and integral a to b g of x dx is equal to 0.

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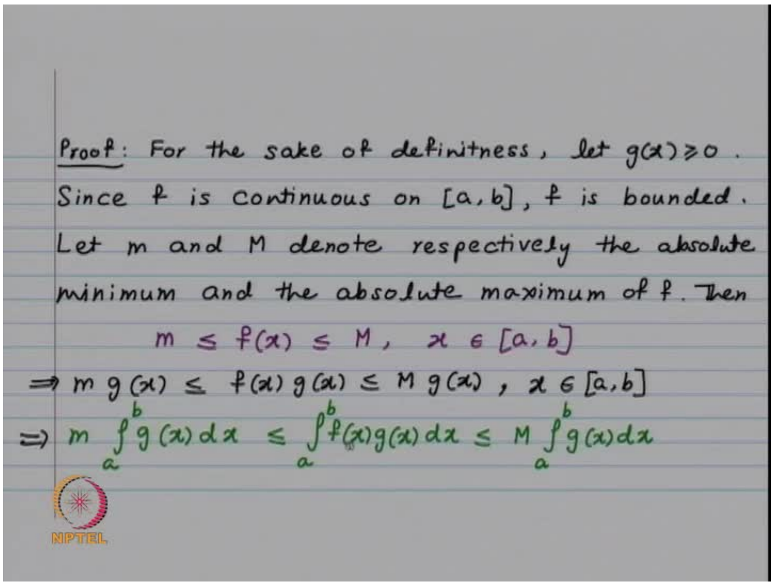

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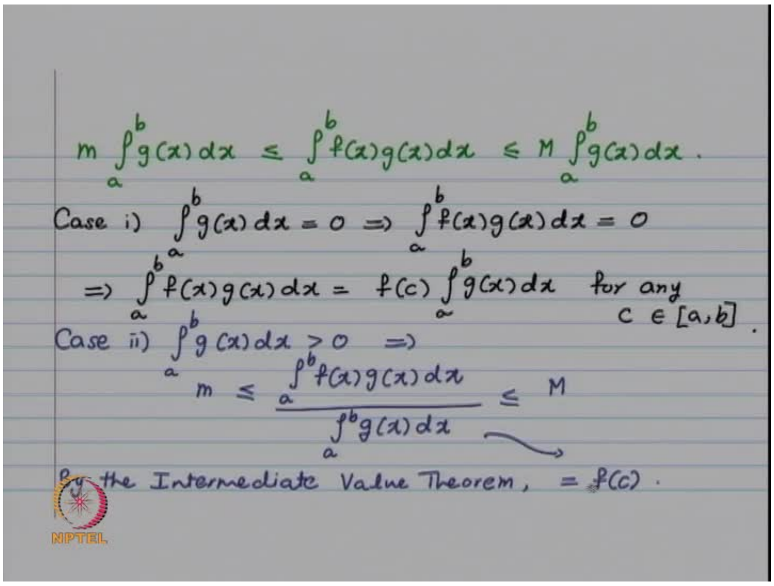

So, we have small m into integral a to b g of x dx this inequality we will look at 2 cases once when integral a to b g of x dx is equal to 0 and other when it is not equal to 0 if this is equal to 0 then we have got 0 here 0 here which will mean integral a to b f of x g of x dx is equal to 0. So, you can choose c to b any point the second case will be integral a to b g of x dx is strictly bigger than 0 in that case I divide by integral a to b g of x dx then the quotient is lying between small m to capital M. So, it has to be equal to f of c.

So, that is by the intermediate value theorem and this will mean that integral a to b f of x g of x dx is equal to f of c multiplied by integral a to b g of x dx that is the mean value theorem for integral and this mean value theorem for integrals we are going use for calculating the error bounds in various rules now what we are going to do in the next lecture is we will first consider some basic rules which are coming from what are known as newton cotes formula.

Look at the interval a, b subdivide it into equal parts fi tan interpolating polynomial integrate it and then we get a numerical quadrature rule you are familiar with these rules such as if you choose a constant polynomial and choose the interpolating point to be the end point then what you get is what is known as a rectangle rule if you take the midpoint and consider the constant polynomial what you get is a midpoint rule.

Then if you consider polynomial of degree less than or equal to 1 choose 2 end points of the interval as your interpolating point you get trapezoidal rule then you have got simpson rule in which case you are interpolating points are three points a band midpoint a plus b by 2 you are fitting a parabola and then are integrating. So, that gives you simpson's rule and then we are going to look at the error in all these formally then from these we go to composite rules. So, these will be the basic rules then we go to composite rules where each of these basic rule instead of applying them at the whole of interval a, b we will apply them on the sub interval that and then the gaussian rules.

So, this thing we will continue next time thank you!