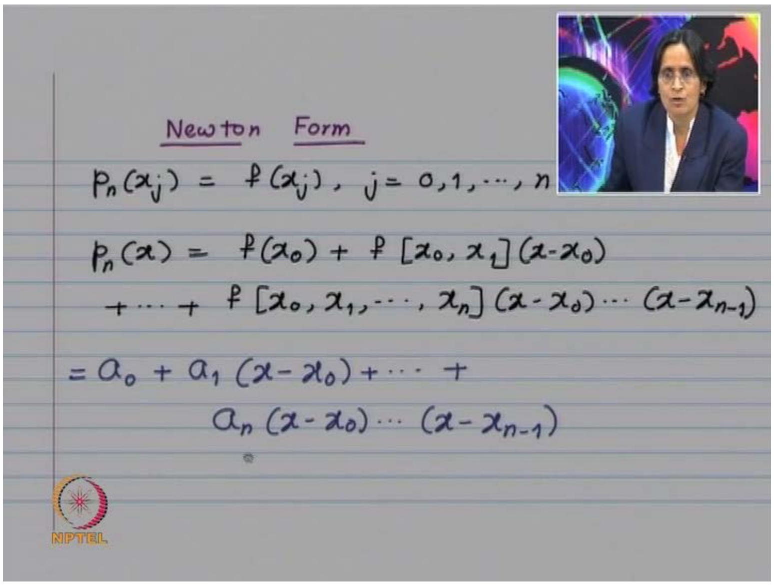
Elementary Numerical Analysis
Prof. Rekha P. Kulkarni
Department of Mathematics
Indian Institute of Technology, Bombay

Module No. # 01
Lecture No. # 06
Cubic Hermite Interpolation

Today, first we are going to look at a number of computations needed to evaluate polynomial p_n interpolating polynomial in the Newton's form. Next we will look at cubic hermite interpolation where not only function values are interpolated, but the derivative values are also interpolated. So, we will look at the existence and uniqueness of a cubic hermite polynomial, the error in it and then I will state a result about the convergence of interpolating polynomial.

(Refer Slide Time: 01:12)

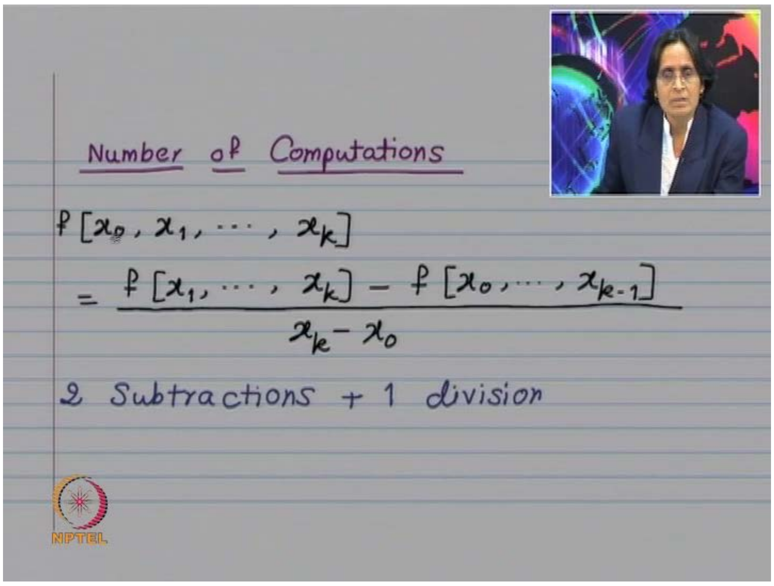


After that we are going to look at piece wise polynomial. So, in this lecture we will consider mainly piece wise linear polynomial. So, a polynomial in Newton's form interpolating polynomial it is given by $p_n(x)$ is equal to $f(x_0)$ plus divided difference based on x_0 x_1 into x minus x_0 plus divided difference based on x_0 x_1 x_n into x minus x_0 x minus $x_{(n \text{ minus } 1)}$.

So, it is a polynomial of degree less than or equal to n which matches with f at x_j j going from 01 upto n. So, it is of the form a_0 plus a_1 (x minus x_0) plus a $_n$ x minus x_0 into x minus x $_{(n \text{ minus } 1)}$.

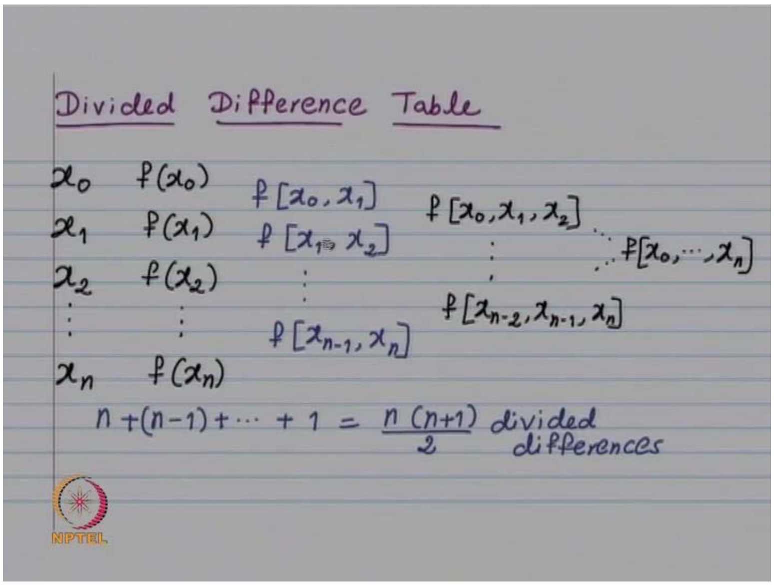
So, in order to compute the coefficients a_0 a_1 a_n we will need to calculate the divided differences. So, we will first look at the number of computations needed to calculate these divided differences and then we will consider an efficient way of evaluating p_n at a point once we have computed a_0 a_1 up to a_n.

(Refer Slide Time: 02:28)



Now, the recurrence formula for the divided difference based on k plus 1 points is given by f (x_1 x_2 x_k) minus f (x_0 x_1 x $_{(k \text{ minus } 1)}$) . So, here are k plus 1 points. So, here are k points divided by x_k minus x_0. So, in case we have the divided differences based on k points available then what we need is one subtraction here ,one subtraction here. So, total two subtractions and then one division now this is for one divided difference.

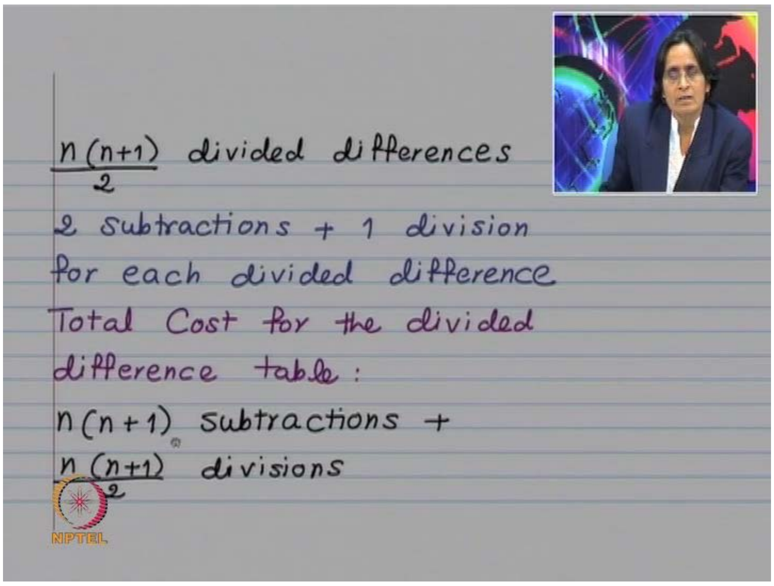
(Refer Slide Time: 03:16)



So, let us look at the divided difference table the divided difference table is given by:

These are our points x_0 x_1 x_2 x_n these are the function values then f (x_1) minus $f(x_0)$ divided by x_1 minus x_0 will give us this divided difference and so on ,we continue . So, the total number of divided differences which we will need to calculate will be in first column there will be n divided differences ,in the second column there will be n minus 1 and in the last column 1.Thus, total is n into n plus 1 by 2 divided differences.

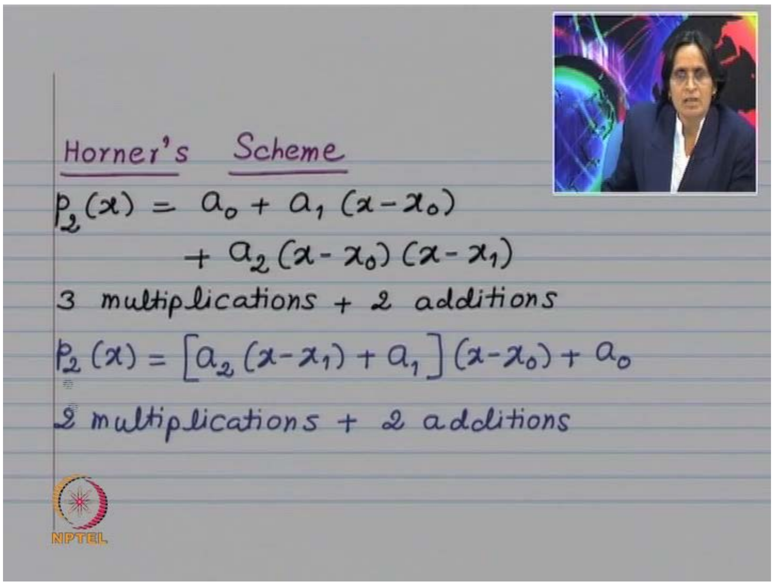
(Refer Slide Time: 04:05)



And for each divided difference we need two subtractions and one division and thus the total cost for the divided difference table will be, for each divided difference 2 subtraction and 1 division and we need to calculate total n into n plus 1 by 2 divided differences. So, it is going to be n into n plus 1 subtractions plus n into n plus 1 by 2 divisions.

So, what is important is the power is going to be of the order of n square. Now, once we have calculated the divided difference we have got the coefficients a_0 a_1 a_n and now, let us look at the number of computations for evaluating our interpolating polynomial.

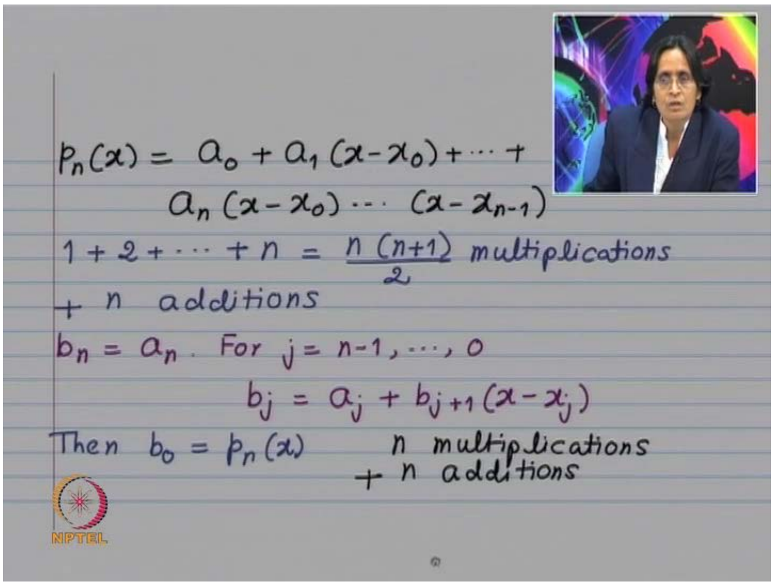
(Refer Slide Time: 04:52)



So, let me first look at a polynomial of degree 2. So, p_2 (x) is a $_0$ plus a_1 (x minus x_0) plus a_2(x minus x_0)(x minus x_1) .Let us ignore these subtractions x minus x_0 then x minus x_0 x minus x_1.Assume that they are already done. So, here if we directly do it there will be one multiplication here, two multiplications here so two multiplications plus two editions.

Now, in Horner's scheme what we do is we write the same polynomial in the form a_2 into x minus x_1 plus a_1 into x minus x_0 plus a_0. So, it is the same polynomial, I am just writing differently. Now I will have 1 multiplication here 1 addition here then I will have one multiplication here. So, instead of three multiplications I will have two multiplications and there will be two additions as before. So, for the quadratic polynomial instead of three multiplications we have got two multiplications, but for a polynomial of degree n the saving is going to be much more.

(Refer Slide Time: 06:18)

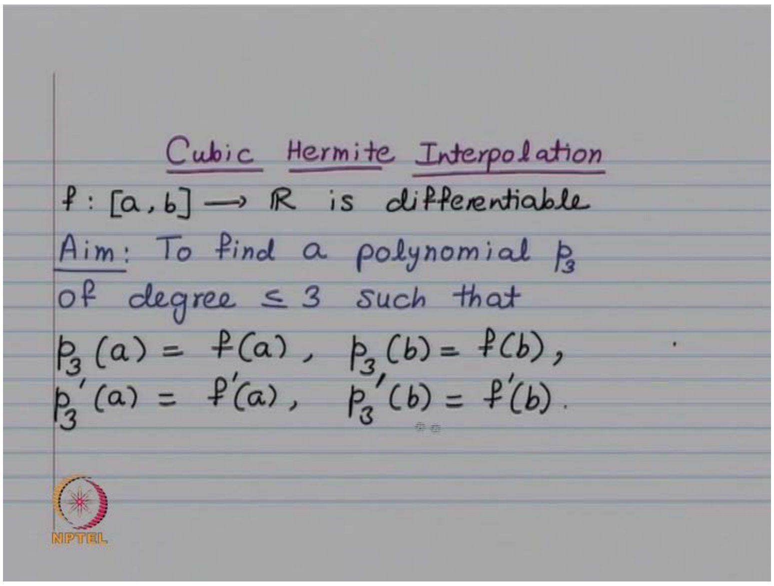


So, here if you have got p_n to be a polynomial in the Newton form. Here ,if I evaluate p_n directly I will have one multiplication here, the next one, next term will have two multiplications and last term will have n multiplications. So, total n into n plus 1 by 2 multiplications and n additions. Now, as we did before, if we consider the algorithm b_n is equal to a_n and then for j equal to n minus 1 up to 0 you are reducing the values of j to be b_j is equal to a_j plus b_j plus 1 into x minus x_j then when we reach b_0 that is going to be $p_n(x)$ here you see that you are doing it for j is equal to n minus 1 up to 0.

So, there are going to be n multiplications and n additions. So, instead of n square by two multiplications you have reduced it to n multiplications. So, that is a considerable saving and that is known as "Hornor's scheme". Now, next what we want to do is we want to consider cubic hermite interpolation. So, this is part of what is known as oscillatory interpolation .What we are going to do is consider a function defined on closed interval a ,b and let us try to find a polynomial of degree less than or equal to 3 which interpolates the given function at 'a' and 'b' the two end points and also the polynomial it is derivative should approximate or if p_3 is the polynomial p_3 dash at 'a' should be equal to f dash at 'a' where dash means derivative and p_3 dash at 'b' is equal to f dash (b).

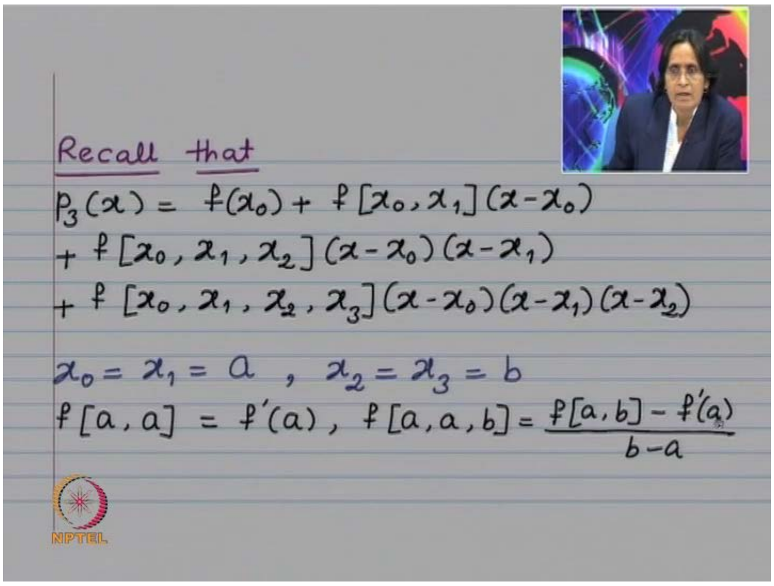
So, we will be interpolating function values and the derivative values at the two end points and that is cubic hermite interpolation. We will first prove its existence and then we will prove its uniqueness and after that we will look at the error .Now, the way we are going to write this p_3 it is if our points x_0 x_1 x_2 x_3 if these are distinct points then we know how to write p_3 (x) in the Newton form.

(Refer Slide Time: 09:28)



Now, the difference here is it will be instead of distinct points you will have x_0 is equal to x_1 is equal to a. So, x_0 and x_1 are identical and x_2 and x_3 are identical, but we will try to write a similar formula as in the case of distinct point and see whether it works. So, assume that f to be defined on interval a,b taking real values and assume that it is it to be differentiable ((??)CHECK) . is to find a polynomial p_3 of degree less than or equal to 3 such that p_3 at 'a' is equal to f of a ,p3 at 'b' equal to f of b .The derivative of p_3 at 'a' is equal to f dash of a and derivative of p_3 at 'b' is equal to f dash of b.

(Refer Slide Time: 10:00)

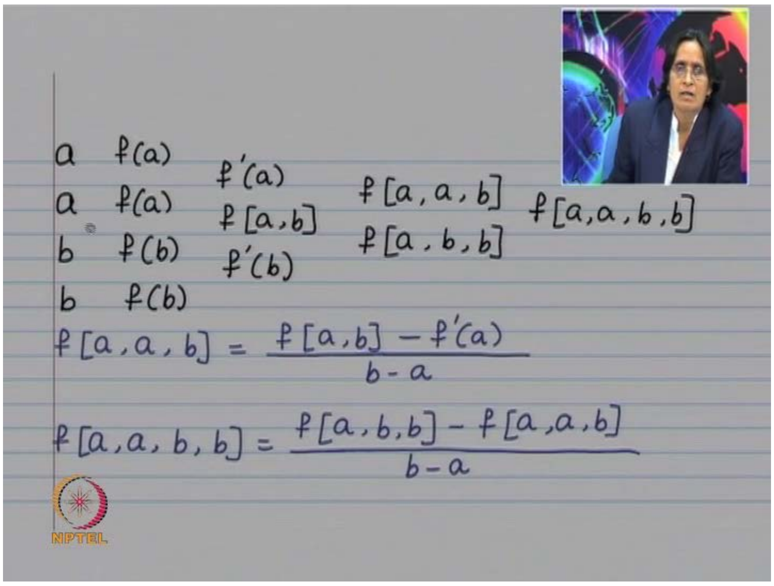


Now, let us recall that if x_0 x_1 x_2 x_3 are distinct points then the interpolating polynomial p_3 (x) is given by value of f at x_0 plus divided difference of f based on x_0 x_1 into x minus x_0 then divided difference based on x_0 x_1 x_2 into x minus x_0 x minus x_1 and the last term is divided differences based on the four interpolation points multiplications by x minus x_0 x minus x_1 x minus x_2.

Suppose x_0 is equal to x_1 is equal to 'a' and x_2 is equal to x_3 is equal to 'b' then this f of x_0 x_1 that is going to be f of a,a and then we have, we are defining it to be equal to f dash of a.

Here this divided differences will be f of a, a, b. So, that will be f of a ,b 'a' and 'b' are distinct minus f of a ,a which is f dash 'a divided by 'b' minus 'a'. So, f of a ,a make sense f of a, a, b make sense and f of a, a, b, b will be defined in a similar manner.

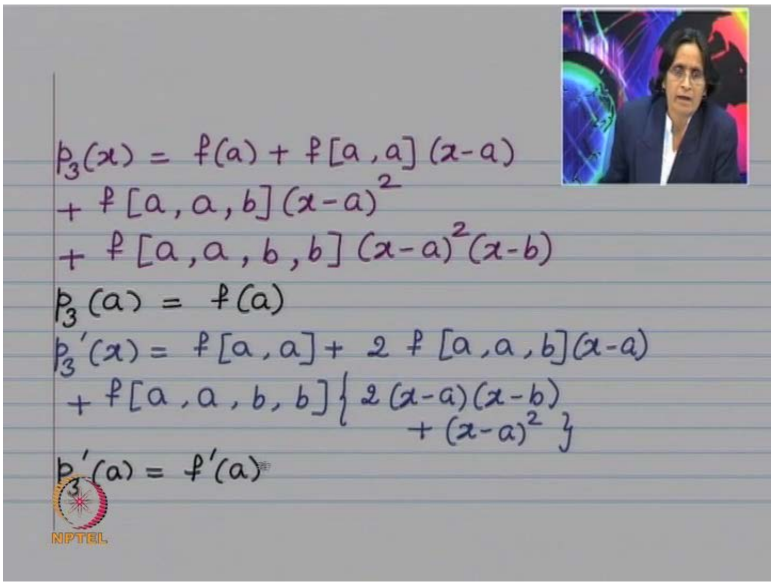
(Refer Slide Time: 11:35)



So, here is a divided difference table ‘a’ repeated twice ‘b’ repeated twice these are the function values the divided difference correspond into this will be f dash of ‘a’ the divided difference based on a b will be f of a ,b and divided difference based on b, b that is f dash b.

Next f of a, a, b is f of a ,b minus f dash ‘a’ divided by ‘b’ minus ‘a’. Similarly f of a, b, b will be f dash ‘b’ minus f of a ,b divided by ‘b’ minus ‘a’ and the last term will be given by f of a ,b, b minus f of a, a, b divided by ‘b’ minus ‘a’. So, this is divided difference table corresponding to ‘a’ repeated twice ‘b’ repeated twice.

(Refer Slide Time: 12:47)



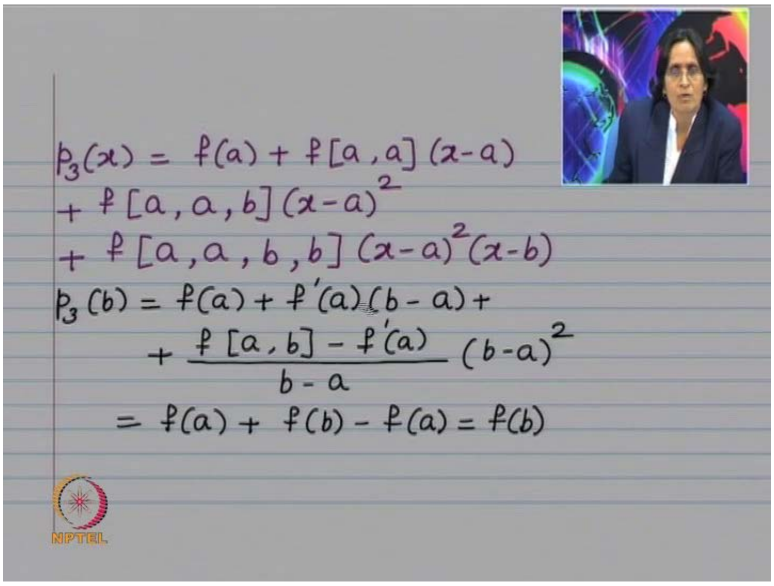
So, here one needs to be given function value at a function value at ‘b’ derivative of f at ‘a’ and derivative of f at b’’ and then our p_3 (x) is f of a plus f of a, a into x minus a plus f of a, a, b into x minus a^2 .

So, this I write, but now I want to show that this is our desired polynomial which interpolates the function and derivative values at two end points. So, first of all put x is equal to a when I put x is equal to a there is x minus a term here x minus a term here x minus a term here. So, p_3 at a will be equal to f at ‘a’.

Take the derivative of p_3. So, this being a constant term its derivative will be 0. So, you will have f of a, a then derivative of this will be 2 times x minus a into f of a,a ,b plus this divided difference and then by product tool this will be 2 into x minus a into x minus b plus x minus a^2 .

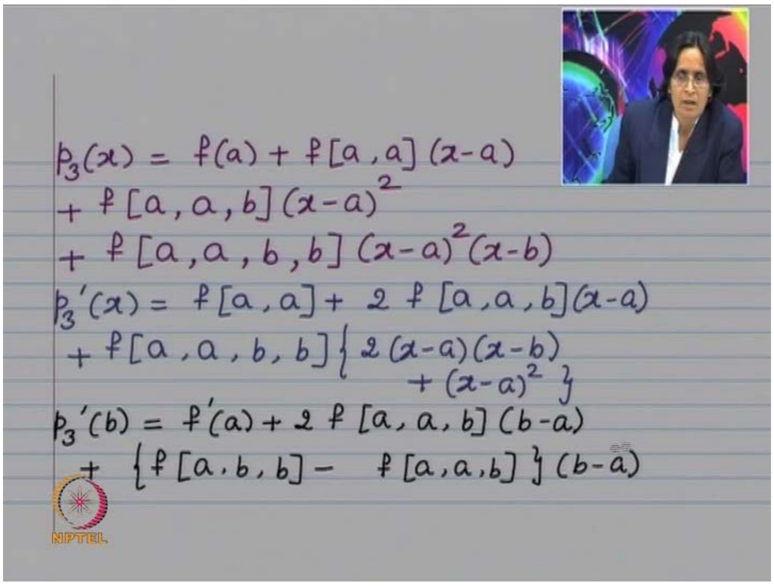
When I put x is equal to ‘a’ there is x minus a term here x minus a term here and x minus a term here. So, p_3 dash at ‘a’ is equal to f dash at ‘a’. So, this p_3 it interpolates the function value at ‘a’ and its derivative at ‘a’.

(Refer Slide Time: 14:30)



Now, let us look at the case when x is equal to b. So, when I put x is equal to b ,p_3 of b will be f of a this is f dash a x is equal to b.(??CHECK) So, b minus a plus this will be s a b minus f dash a by b minus a into b minus a square. So, one b minus a will get cancelled then f dash a into b minus a gets cancelled f of a, b is f of b minus f of a upon b minus a. So, this will be f of a plus f of b minus f of a. So, it is equal to f of b.

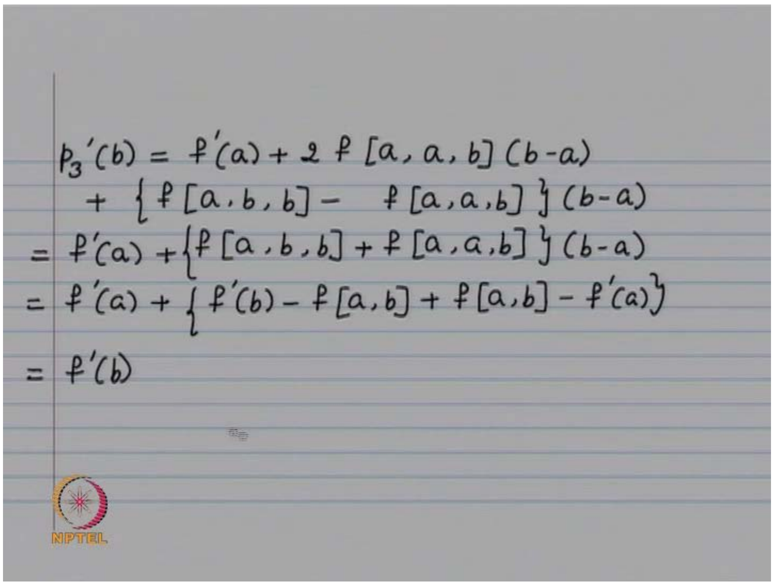
(Refer Slide Time: 15:20)



So, thus p_3 interpolates the function value at ‘b’ and now what remains is whether it interpolates the derivative value at point ‘b’. So, this is our p_3 dash x .We have seen earlier put x is equal to b and then you will have f dash a, plus 2 times f of a, ,a, b into b minus a plus this this will be nothing, but f of a ,b ,b minus f of a ,a, b and then divided by b minus a. So, we are putting x is equal to b. So, no contribution from this term and then you have got b minus a.

So, thus p_3 interpolates the function value at ‘b’ and now what remains is the whether it interpolates the derivative value at point ‘b’. So, this is our p_3 dash x we have seen earlier put x is equal to ‘b’ and then you will have f dash a, plus 2 times f of a, a, b into b minus a plus this this will be nothing but, f of a, b, b minus f of a, a, b and then divided by b minus a. So, we are putting x is equal to b. So, no contribution from this term and then you have got b minus a.

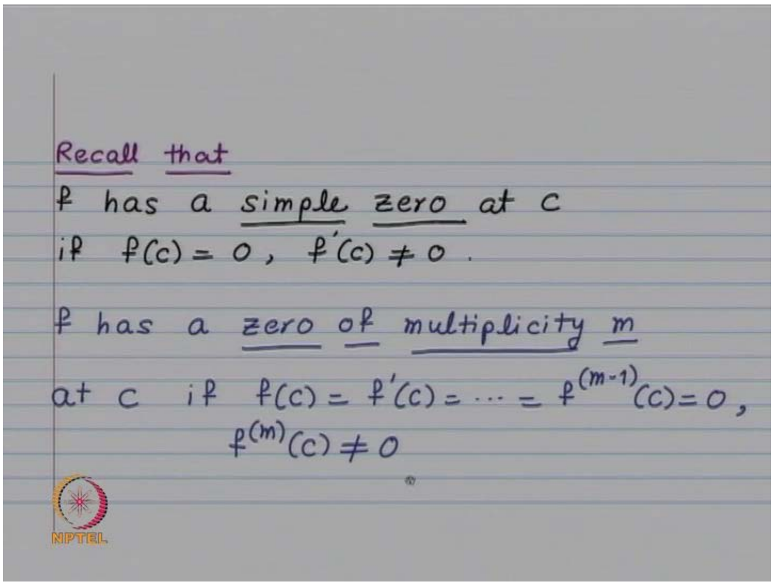
(Refer Slide Time: 15:53)



So, this was our p_3 dash b now we simplify it this will be f dash a then here you have 2 times f of a, a, b into b minus a and here it is minus. So, that is going to be f of a ,a, b with plus sign and then this plus f of a,b,b into b minus a this will be nothing, but f dash b minus f of a,b divided by b minus a. So, it will get cancelled here and it is f of a ,b minus f dash a. So, again b minus a gets cancelled and then you have f dash b.

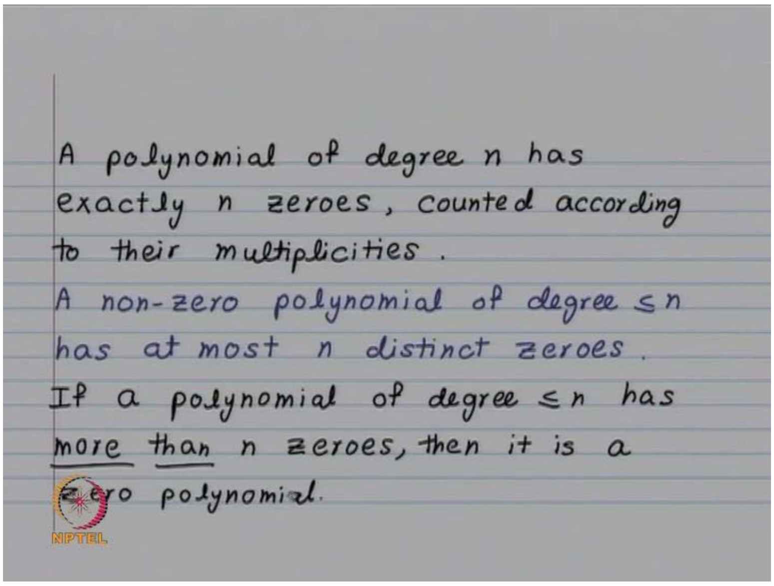
So, thus the polynomial of degree less than or equal to 3 which we wrote in an analogous fashion when the interpolation point where distinct we saw that it is a desired polynomial that means it interpolates the function and derivative values at 2 end points a and b.

(Refer Slide Time: 17:34)

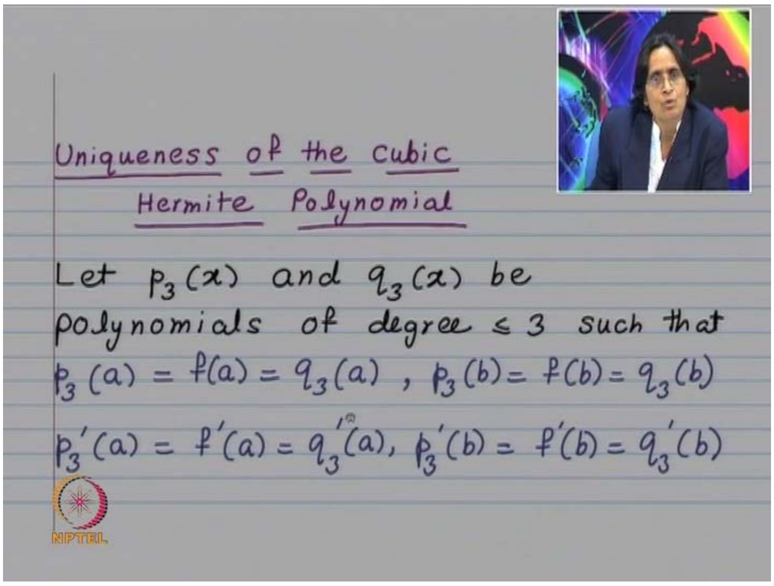


Now, we want to show uniqueness. So, for the uniqueness we are going to use the result from the fundamental theorem of algebra that if you have got a polynomial of degree less than or equal to n and if it has got n plus 10 now these n plus 0; that means, counted according to their multiplicities then such a polynomial has to be 0 polynomial .So, we have let me recall that a simple 0 means f of c is equal to 0 f dash c not equal to 0 a zero of multiplicity m; that means, the function value and its derivatives up to m minus 1 they are 0, but f m c not equal to 0 when m is equal to 2 we call it to be a double zero.

(Refer Slide Time: 17:59)

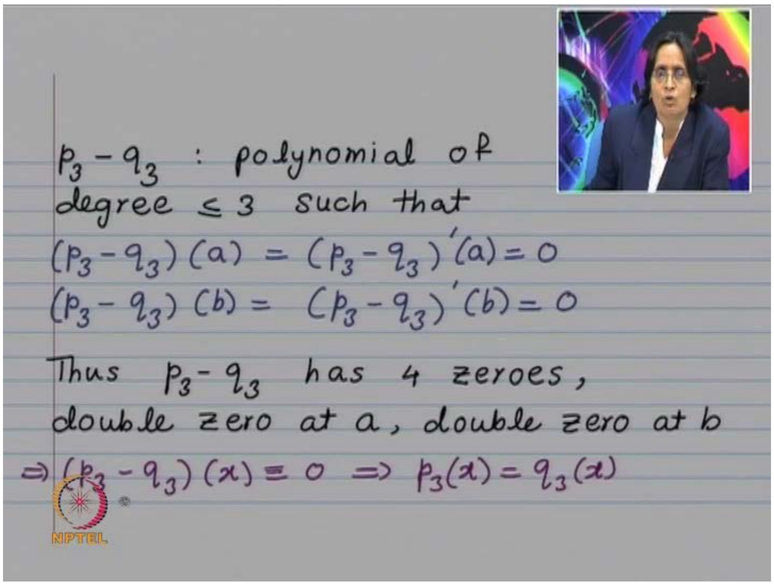


(Refer Slide Time: 18:47)



Now, this is the fundamental theorem or rather consequence of fundamental theorem that a polynomial of degree n has exactly n zeros counted according to their multiplicities as a consequence of this a non-zero polynomial of degree less than or equal to n it has got at most n distinct zeros and if a polynomial of degree less than or equal to n has more than n zeros then it is going to be a zero polynomial so it is this result that we are going to use to show the uniqueness of cubic hermite polynomial .So, let p_3 and q_3 be two polynomials of degree less than or equal to 3 which interpolate the given function under derivative and at points a and b.

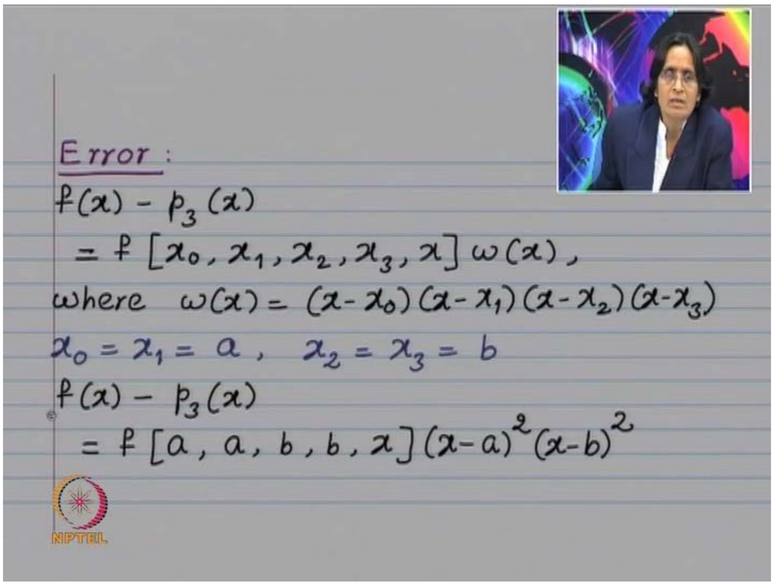
(Refer Slide Time: 19:53)



So, if I look at p_3 minus q_3 that will be a polynomial of degree less than or equal to 3 which will vanish at 'a' vanish at 'b' and also their derivatives will vanish at 'a' and at 'b'. So, if you are looking at p_3 minus q_3 it will be a polynomial of degree less than or equal to 3 such that it has got double zero at 'a' and double zero at 'b',that means, when we count according to the multiplicity p_3 minus q_3 has got four zeros, but it is a polynomial of degree less than or equal to 3 so, that means, it has to be a zero polynomial .So p_3 minus q_3 a polynomial of degree less than or equal to three which has got four zeros that means a double zero at 'a' and double zero at 'b'. So, p_3 minus q three x has to be identically zero and that implies that p_3(x) is equal to q_3 (x).

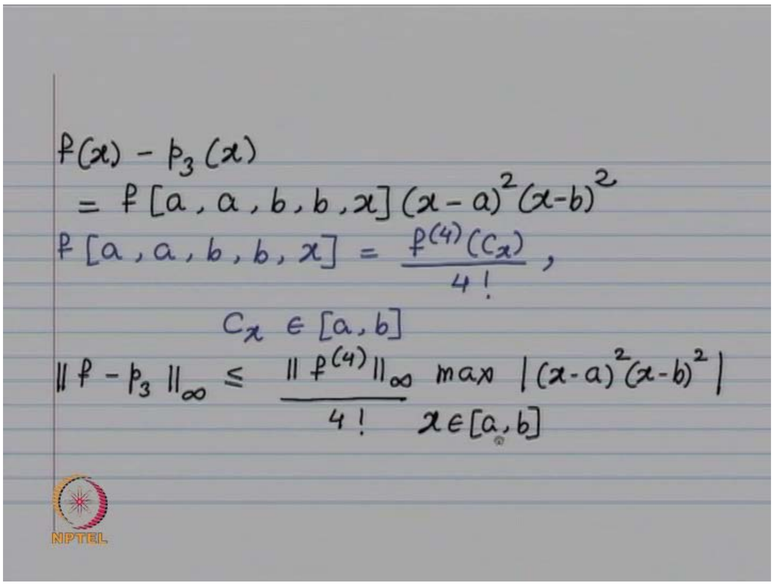
So, now we have proved the existence and uniqueness of cubic hermite polynomial now we want to look at the error in it when we had considered the interpolating polynomial and the interpolation points being distinct points we have got an error formula the same formula is going to hold even when the points are repeated when we proved the error formulae for interpolating polynomial what we had used was Rolle's theorem. So, the proof is similar and what I am going to do is, I am going to write directly the formula to proof is being very much similar I am going to skip it.

(Refer Slide Time: 21:07)

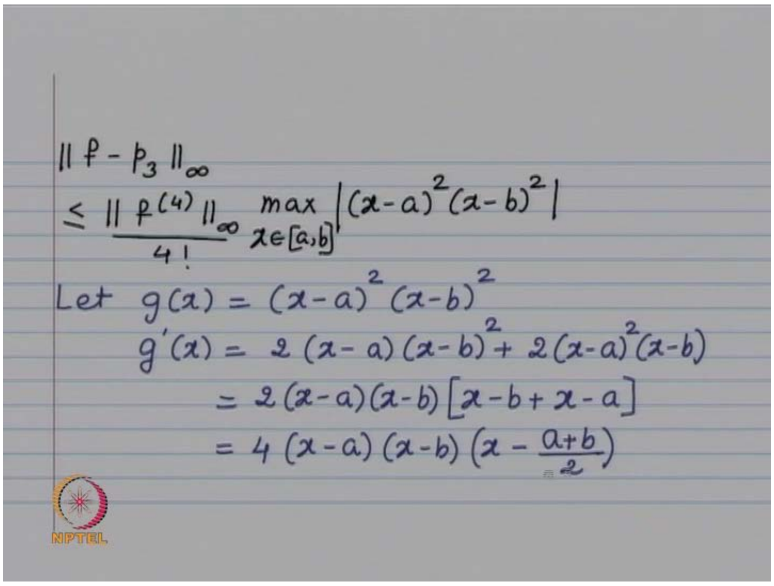


So, when the interpolation points are distinct say p_3 a polynomial interpolating at x_0 x_1 x_2 x_3 then f of x minus p_3 of x is the error term the divided difference based on x_0 x_1 x_2 x_3 x and multiplied by function w x where w x is x minus x_0 into x minus x_1 into x minus x_2 into x minus x_3 we have got x_0 is equal to x_1 is equal to ‘a’ and x_2 is equal to x_3 is equal to ‘b’ so, that means, the error will be the divided difference and then x minus a^2 and x minus b^2. Now , let us calculate the infinity norm of f minus p_3 that is the maximum norm and that is modulus of f of x minus p_3 x its maximum over the interval a,b.

(Refer Slide Time: 22:13)

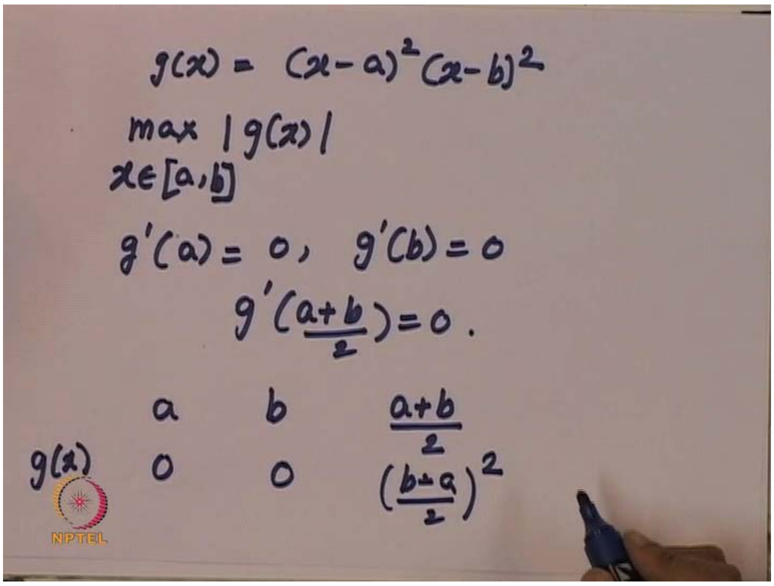


(Refer Slide Time: 22:56)



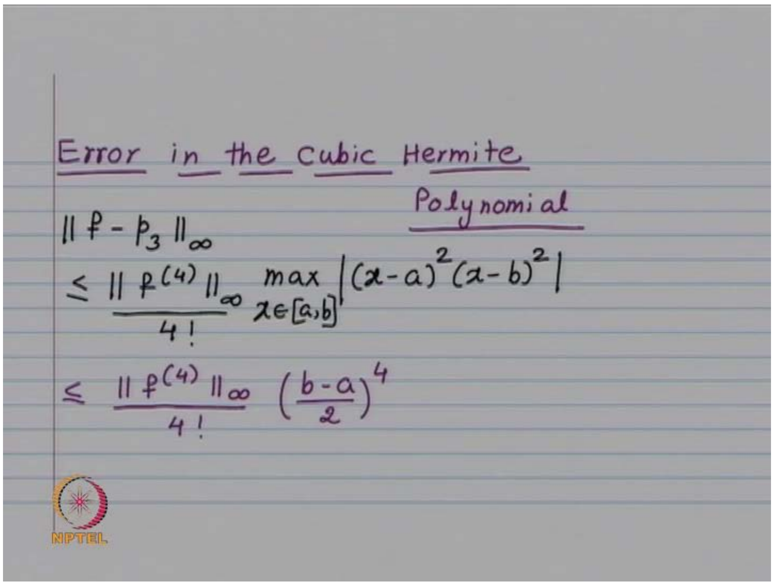
This divided difference is going to be fourth derivative evaluated at some point c which depends on x divided by 4 factorial and hence norm of f minus p_3 infinity take the modulus of both the sides and take maximum over interval a, b. So, this will be less than or equal to norm f 4 infinity divided by a 4 factorial and then maximum of this function. Now, when we want to calculate maximum of this function we want to calculate absolute maximum. So, the absolute maximum is obtained by considering the two end points and the critical point. So, the critical points are the points where the derivative vanishes or it does not exist. So, let me call this x minus a square x minus b square to be function g of x it is derivative is given by using product rule it is 2 into x minus a x minus b square plus 2 into x minus a square into x minus b. So, take common x minus a into x minus b and simplify. So, you will get 4 x minus a x minus b x minus a plus b by 2.

(Refer Slide Time: 23:43)



So, we have got g of x is equal to x minus a^2 x minus b^2 we are trying to calculate maximum of modulus of g x ,x belong into interval a ,b g dash at ‘a’ is equal to 0 g dash at ‘b’ is equal to 0 and g dash of a plus b by 2 is 0. So, we look at the value of g at these three points a ,b and a plus b by 2 and whichever is the maximum that is the absolute maximum.

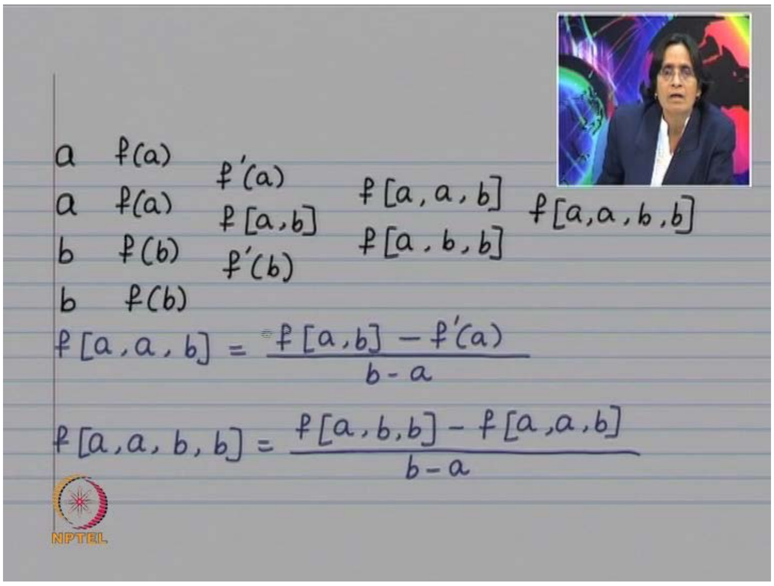
(Refer Slide Time: 24:43)



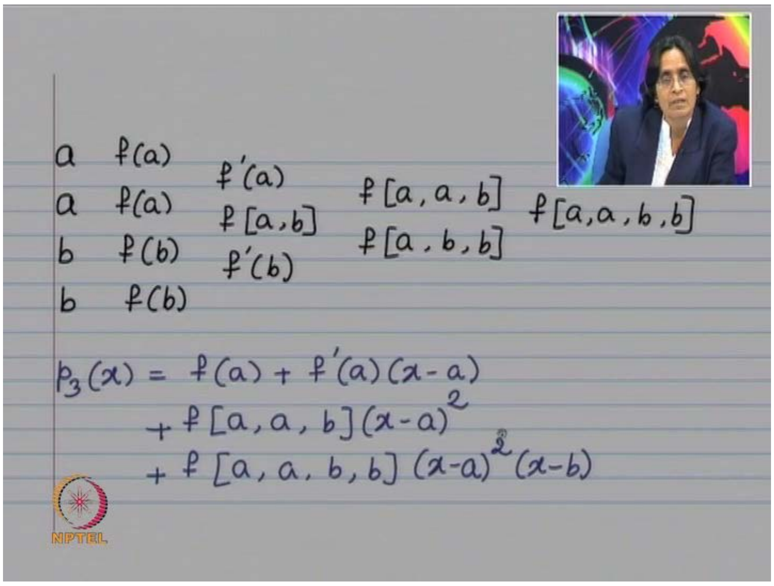
So, when I look at g x g at ‘a’ will be 0 g at ‘b’ will be 0and g at ‘a’ plus b by 2 will be b minus a by2 square and hence we will have error in the cubic hermite polynomial to be less than or equal to norm f 4 infinity divided by4 factorial into b minus a by 2 raise to 4.

So, what I am going to do is I am going to look at a example. So, we will consider a specific example calculate it is cubic hermite polynomial and then afterwards we will consider the convergence of interpolating polynomial.

(Refer Slide Time: 25:21)

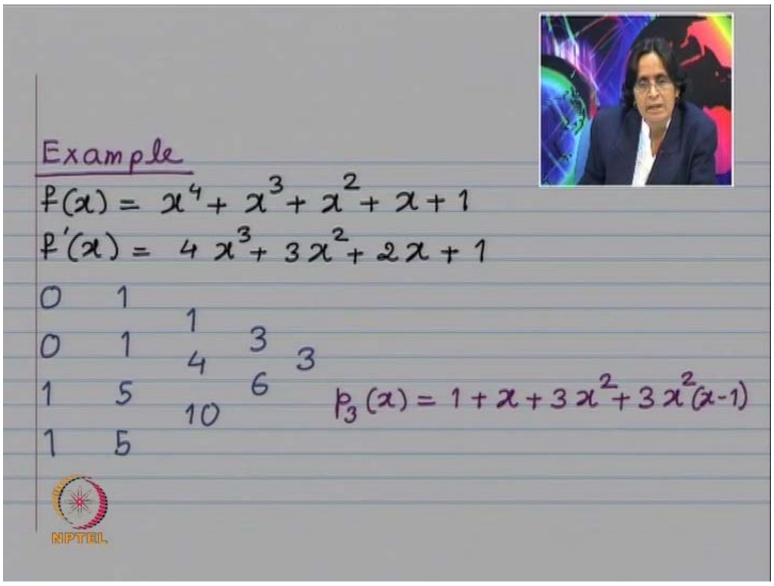


(Refer Slide Time: 25:48)



So, let me just recall that if I want to calculate the cubic hermite interpolating polynomial I need to prepare such a divided difference table here a is repeated. So, this entry is f dash a b is repeated. So, this entry is f dash b and f of a b is the usual recurrence formula once we prepare such a divided differences table then p 3 x is f a plus f dash a into x minus a plus f of a ,a, b x minus a square and f of a, a, b, b x minus a square x minus b this term corresponds to x minus x_0 x minus x_1 our x_0 is equal to x_1 is equal to ‘a’.

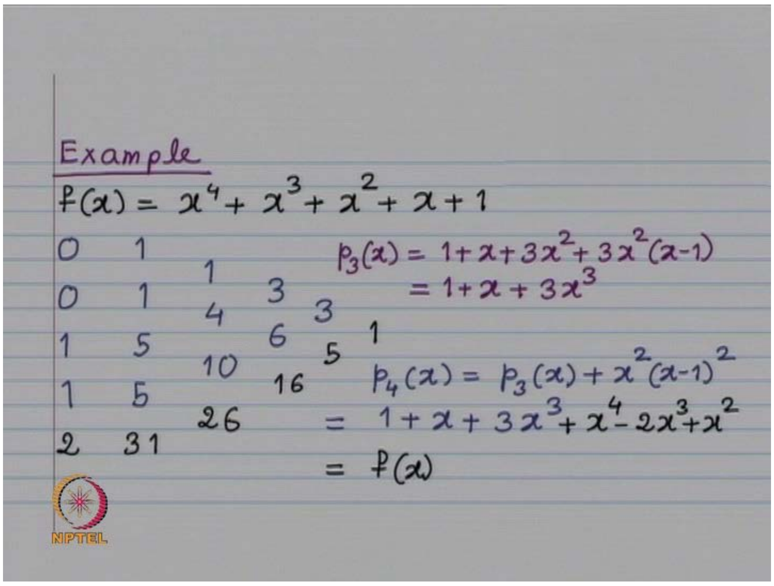
(Refer Slide Time: 26:14)



So, look at function f x to be x raise four plus x cube plus x square plus x plus 1 its derivative will be given by 4 x cube plus 3 x square plus 2 x plus 1 the value at 0 it is going to be f of 0 will be 1 f of 1 is going to be 5 this entry here this is going to be f dash at 0 from this formulae it is 1 then this will be f of 11. So, it is five minus one divided by 1 minus 0. So, that is 4.

This entry is f dash of 1. So, f dash of 1 is going to be equal to 10 put x equal to 1in this formula now this will be 4 minus 1 divided by 1 minus 0. So, that is 3 then 10 minus 4 divided by1 minus 0. So, that is 6 and finally, this entry is 6 minus 3 divided by 1 minus 0. So, that is 3 now our polynomial p_3 x will be 1 plus 1 into x because that is x minus 0 plus 3 into x minus 0 square. So, that is x square plus 3 into x minus 0 square and then x minus one.

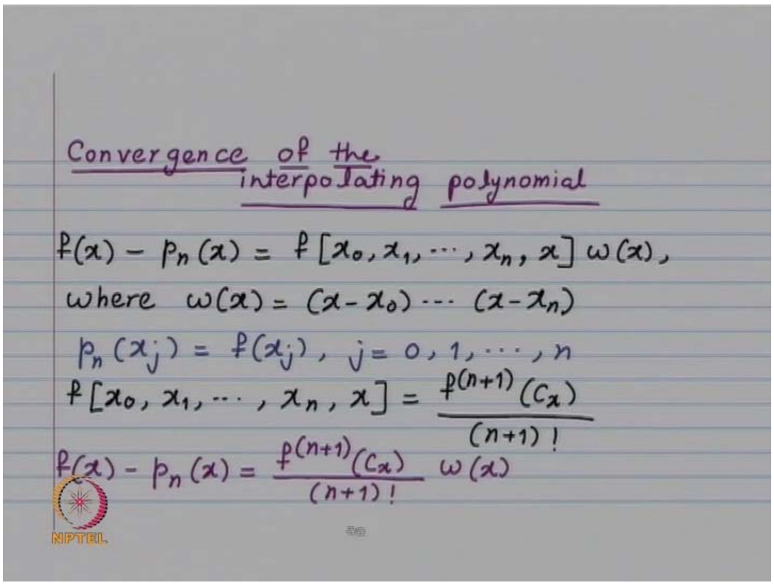
(Refer Slide Time: 28:25)



So, this is going to be cubic hermite polynomial for function f of x now let me add one more interpolating point. Suppose, I want to find a polynomial which interpolates the given function at 0 repeated 1 repeated and 2. So, then this divided difference table we had already calculated. So, we just need to add these entries. So, here is 2 f of 2 is 31 this entry will be 31 minus 5 divided by 2 minus 1 this entry is 16 minus 20 divided 2 minus 1. So, it is 16 then you have 16 minus 6 divided by 2 minus 0. So, that is why 5 and 5 minus 3 divided by 2 minus 0. So, that is 1 and now p_4 x will be this p_3 x and then you add one more term the coefficient of that term is 1 and then your interpolation points are 0011.

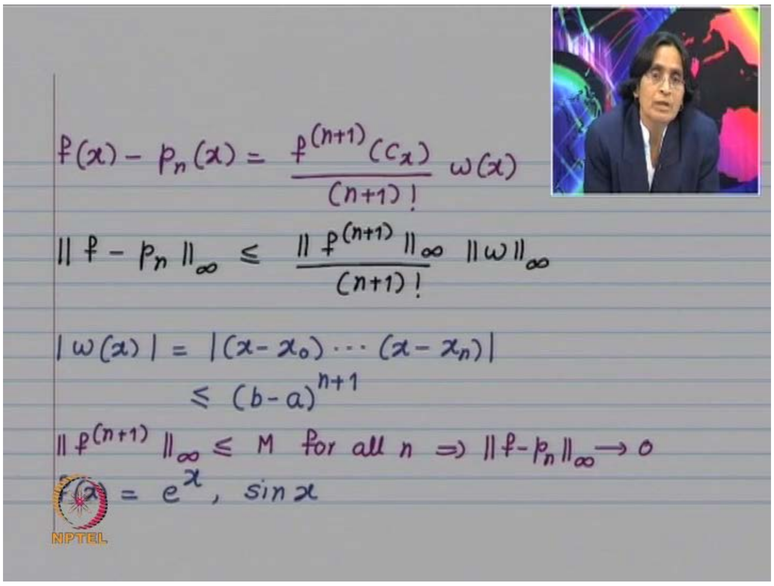
So, that is why x square into x minus 1 square when you simplify. So, this was our p_3 x 1 plus x plus 3 x cube x square into x minus 1 square will be x raise to 4 minus 2 x cube plus x square. So, you get back f of x which is what it should be you have got a function f to be a polynomial of degree 4 and you are considering the interpolation from a space of polynomial of degree less than or equal to 4.

(Refer Slide Time: 30:47)



So, we have seen that the interpolation it reproduces polynomials. So, even when some of the interpolation points are repeated the property will hold and you will get the interpolating polynomial to be the function itself because we had a polynomial of degree 4 and we had 5 interpolation points 0 repeated twice 1 repeated twice and 2. Now, that brings me to convergence of interpolating polynomial. So, we have f x minus p_n of x to be the divided difference multiplied by w of x where w of x is product of x minus x of j, j going from 01 up to n .p_n is the polynomial which interpolates the function at n plus 1 points and which has got degree to be less than or equal to n if your function f is sufficiently differentiable in this case. Suppose it is n plus 1 times differentiable then the divided difference will be f n plus 1 evaluated at some point c divided by n factorial.

(Refer Slide Time: 31:41)

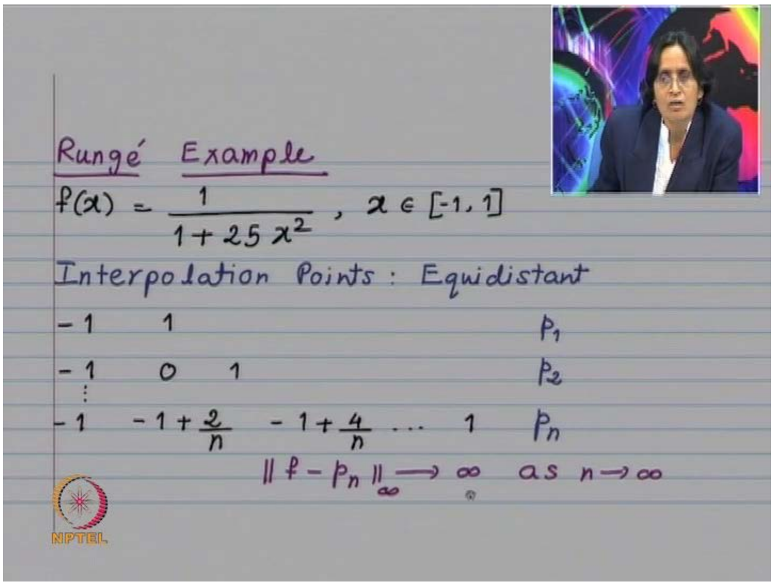


So, norm of f minus p_n will be less than or equal to norm of f n plus 1 divided by n plus 1 factorial into norm of w w is given by x minus x_0 into x minus x_n x varies over the interval a ,b x_0 x_1 x_n they are also in the interval a, b . So, x minus x_j will be at the most b minus a and hence modulus of w of x will be less than or equal to b minus a raised to n plus 1.

So, look at now the error suppose that this n plus first derivative is less than or equal to n for all n this is the same bound for all the derivatives then we will have m into b minus a raise to n plus 1 divided by n plus 1 factorial. Now, no matter how big b minus a is b minus a raise to n plus 1 divided by n plus one factorial that will tend to 0 and hence, if you have got this condition that your function f is infinitely many times differentiable and the derivatives they are bound by a constant in that case your interpolating polynomials p_n they will converge to f in the uniform norm as n tends to infinity for example, the function f of x is equal to e raise to x sin x the entire functions which are defined on interval a, b.

Now, this is a very strong condition. What a Weierstrass theorem tells that if you have got f to be a continuous function then it is a uniformly approximated by polynomials. Now, the Weierstrass theorems says that it is uniformly approximated by polynomials it does not say by interpolating polynomial and in fact , the result is not true.

(Refer Slide Time: 33:52)



So, here you have got well known example which is known as Runge' example that f of x is equal to 1 upon to 1 plus 25 x square x belonging to minus 1 to 1 and suppose you choose interpolation points to be equidistant point, if you want to consider p1 you will need two points. So, take them to be minus 11 for p_2 you will need three points. So, take them to be minus 1 zero 1 for p_n you will need n plus 1 points so divided minus 1 to 1 into n equal parts and look at the partition points.

Then norm of f minus p n infinity norm tends to infinity as n tends to infinity. So, not only it does not tend to 0, but it tends to infinity. So, then one feels that may be something is wrong with the equidistant points that I chose interpolating points to be equidistant points and then my interpolation is interpolating polynomials they do not converge in the norm. So, why not take the Chebyshev points those were in some way optimal and it minimized the norm of w, but then there is a result that no matter how you choose your interpolation points there will always exist a continuous function for which you will not have convergence.

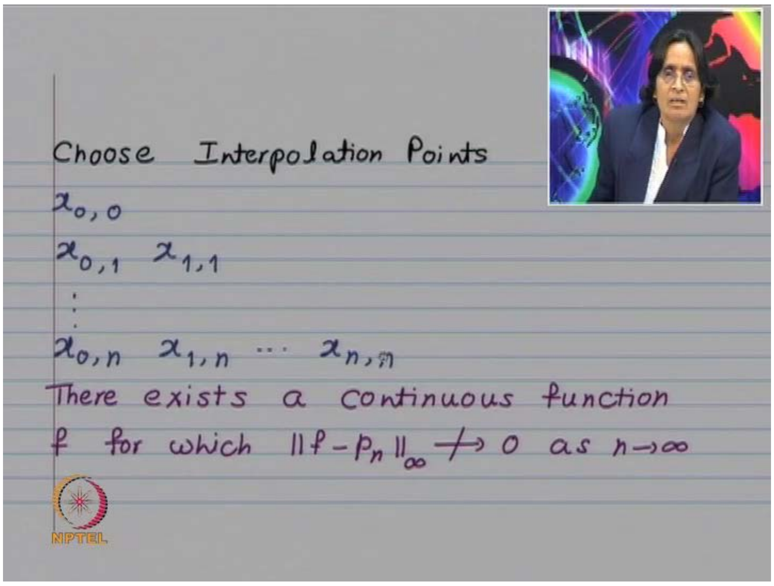
So, here is a negative result that if I want to approximate f by interpolation points and I am ready to increase the degree, but then no matter how I will choose my interpolation points there will always exist a continuous function for which I will not have convergence.

So, this is the drawback of interpolating polynomials, but we had seen that as such one should not go for higher degree polynomials because they have got stability problem. So, then what one does is instead of increasing the degree of the polynomial subdivide your interval a, b into n parts and on each interval you approximate or you consider a interpolating polynomial of fix degree for example, polynomial of degree less than or equal to 1. So, what you are doing is you are fixing the degree of the polynomial on each subinterval and you are ready to increase the number of intervals.

Now, if you do such a thing then under relatively modest conditions you are going to have convergence. So, from polynomials you are going to piece wise polynomials now there is always a trade off in this case we gain in convergence, but there is a loss in smoothness of polynomials and they are infinitely many times differentiable whereas, piece wise polynomials they will their differentiability properties they will get reduced for example, if you want piece wise linear polynomials then at the most you can have linear overall continuity.

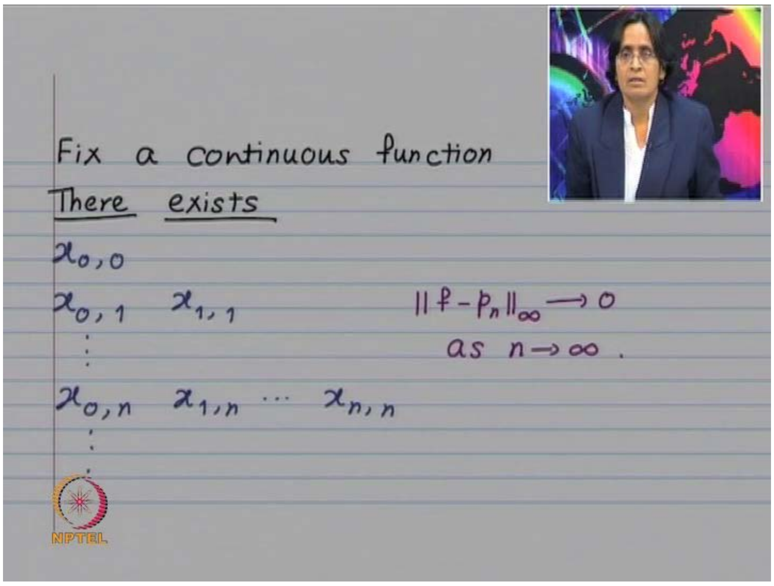
If you are considering piece wise quadratic then you can have c1 continuity. So, what I am saying is on each interval it is a quadratic polynomial or polynomial of degree less than or equal to 2 and then overall it will be differentiable.

(Refer Slide Time: 38:54)



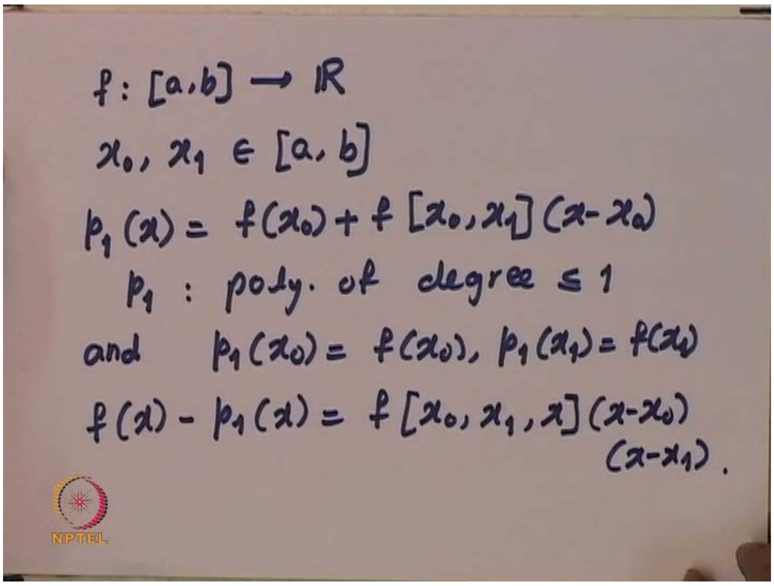
When you consider in the interior of sub interval there will be no problem it will be infinitely many times differentiable, but you are going to join two quadratic polynomials when you look at the adjoining intervals you are going have different quadratic polynomials. So, at the joining point there is going to be you can say only or you can demand only the c1 continuity if you say it should be a c2 continuity it will be a single polynomial on the two intervals.

(Refer Slide Time: 39:25)



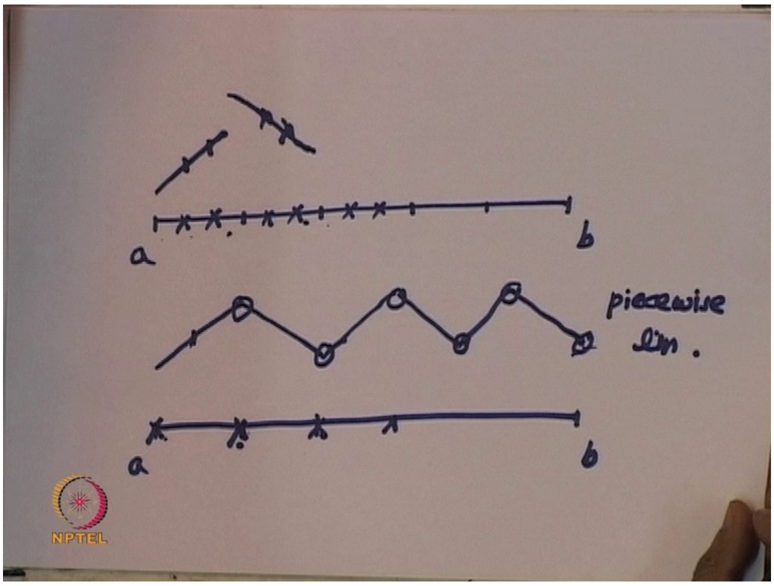
So, here is the feber's result that you choose interpolation points in any manner you want whatever rule you do there exist a continuous function f for which the interpolation point interpolating polynomials will not converge to f in the maximum norm now this is a negative result there is also positive result and that result is you fix a continuous function then there will always exist a set of points if you choose those set of points and consider interpolating polynomials then norm of f minus p_n infinity norm will tend to 0 as n tends to infinity now there is a catch and a catch is their exist; that means, it difficult to know like given a continuous function I do not know what there exist, but I do not have a recipe for such interpolation point, but anyway it is a positive result that there exist a set points for which a interpolation points they are going to converge to f in the norm.

(Refer Slide Time: 40:47)



Now, we are going to consider the piece wise polynomials. So, the first thing as I said we are going look at a piece wise linear polynomial. So, let me first recall the single linear polynomial if you have got f to be defined on interval a, b taking real values x_0 and x_1 these are points in interval a ,b then p_1 of x is equal to f of x_0 plus f of x_0 x_1 x minus x_0 this is polynomial of degree less than or equal to 1. So, p_n polynomial of degree less than or equal to 1 and p_1 x_0 is equal to f of x_0 ,p_1 of x_1 is equal to f of x_1 the error is given by f of x minus p_1 of x is equal to f of x_0, x_1, x into x minus x_0 into x minus x_1

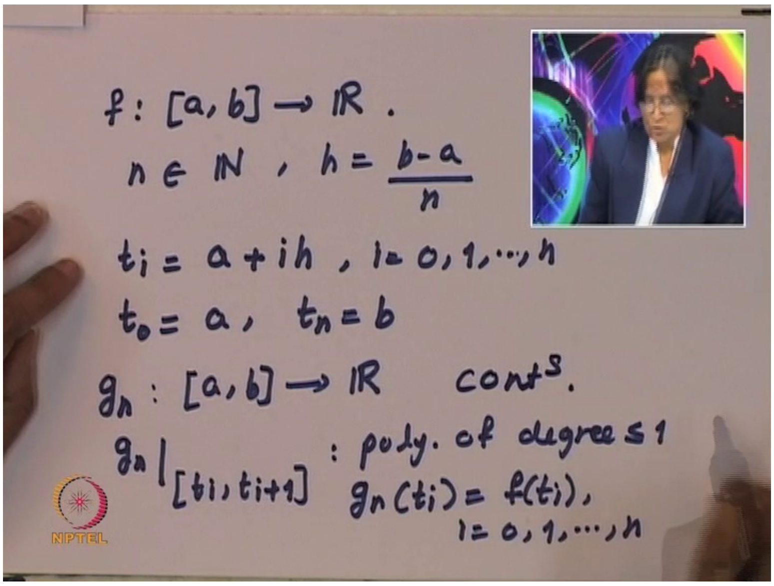
(Refer Slide Time: 42:19)



So, this is for one polynomial now what I am going to do is I am going to subdivide my interval into n equal parts. So, I have interval a, b I will subdivide into smaller interval now in each interval I will choose interpolation points. So, suppose I am taking two interpolation points here and so on. Now, suppose these are the function values then this is going to be linear polynomial interpolating the given function at these two points if I look at the linear polynomial interpolating the given function at these two points it can be something like this it depends on the function value.

So, I may not even have continuity. So, if I want continuity then let me do like this way that this is my interval a, b. So, when I consider this point this interval take the interpolation points to be the end point. So, I will have some function now in this interval take these two as the interpolating points. So, this interpolation point is common. So, you are going to have a polynomial here then a polynomial here and so on. So, this will be piece wise linear.

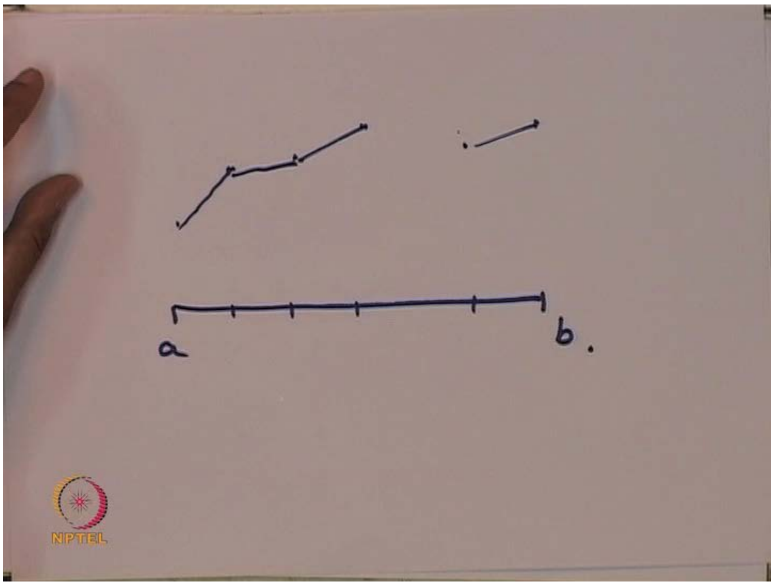
(Refer Slide Time: 44:39)



Now, these are the corner points wherever function will not be differentiable if I consider a point here then no problem. So, overall we are going to have only continuity. So, this will be piece wise linear. So, we have say f is from a ,b to R then consider n to be a natural number h to be b minus a divided by n and ti to be equal to a plus IhI going from 01 up to n. So, t_0 is going to be equal to a and t_n is going to be equal to b and we try to look at a function g_n which is defined on interval a ,b taking real values such that g_n

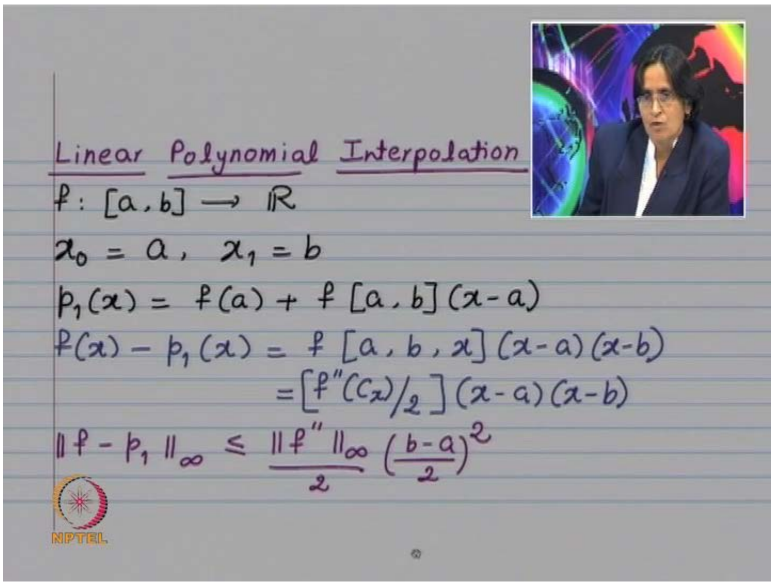
restricted to ti to ti plus 1 is a polynomial of degree less than or equal to 1. So, on each interval it should be a polynomial degree less than or equal to 1 g_n at ti should be equal to f at ti i going from 0 1 up to n.

(Refer Slide Time: 46:27)

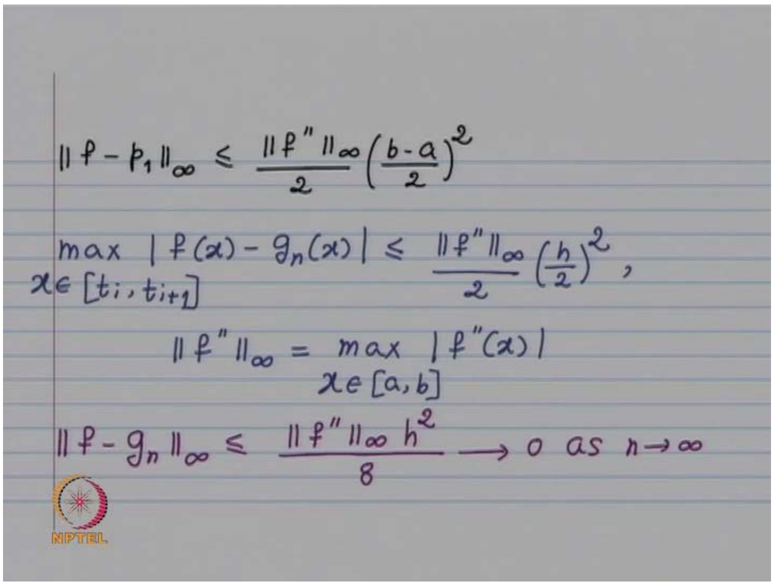


Now, the choice of these interpolation points they will guarantee that g_n will be continuous we have our function defined on interval a,b we are looking at the n plus 1 points in the interval a ,b. So, you are going to have some points n plus 1 points if there are n plus 1 points I can fit a polynomial of degree less than or equal to n instead of that we go to piece wise linear and then we join by straight line. So, that is the piece wise linear interpolation.

(Refer Slide Time: 47:11)



(Refer Slide Time: 47:55)



Now, we want to look at the error in the interpolating polynomial. So, we have in case of linear polynomial interpolation the error is given by norm f double dash infinity divided by 2 into b minus a by2 square the way we had seen that the maximum of modulus of x minus a square x minus b square it was attended at the midpoint at a plus b by 2 . Similarly the modulus of this function it is maximum will be again at the midpoint and then you have got norm of f minus p 1 infinity to be less than or equal to this.

Now, when I consider the piece wise linear interpolation then you have g_n on the interval ti to ti plus 1 it is polynomial of degree less than or equal to 1. So, using this bound you will get norm f double dash infinity by 2 b minus a is length of the interval now the length of the interval is h. So, it is h by 2 square now what I am doing is here I could have dominated by maximum of the second derivative on ti to ti plus 1, but I dominated by maximum over the whole interval a, b. So, that this bound becomes independent of the interval and we will get norm f minus g n infinity norm which is maximum over interval a b that will be less than or equal to norm f double dash infinity by 8 h square and h being b minus a divided by n that will tend to 0 as n tends to infinity that is about the piece wise linear interpolation.

In the next lecture we will consider other piece wise polynomials such that piece wise quadratic interpolation then piece wise cubic hermite interpolation and lastly, we will consider cubic spline interpolation which consists of piece wise polynomials of degree less than or equal to 3. So, on each interval it will be a polynomial of degree less than or equal to 3 and overall you will have c2 continuity; that means, our piece wise cubic polynomial will be 2 times continuously differentiable. So, that we are going to do in the next lecture. So, thank you!