Elementary Numerical Analysis
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Lecture No. # 39
Q R Decomposition

We are considering Q R decomposition of an invertible matrix. Today, we are going to show equivalence of Q R decomposition with Gram-Schmidt Orthonormalization process; then, by putting the condition that diagonal entries of R should be greater than 0 we will prove its uniqueness and then we are going to consider Q R decomposition by using reflectors.

That is going to be an efficient way of calculating a Q R decomposition; because, Q R method for finding Eigen values of a matrix A - the procedure will involve repeated Q R decomposition of certain matrices.

So, we want an efficient method for finding Q R decomposition of a matrix. Then, I will describe what is A Q R method and then we are going to consider some examples.

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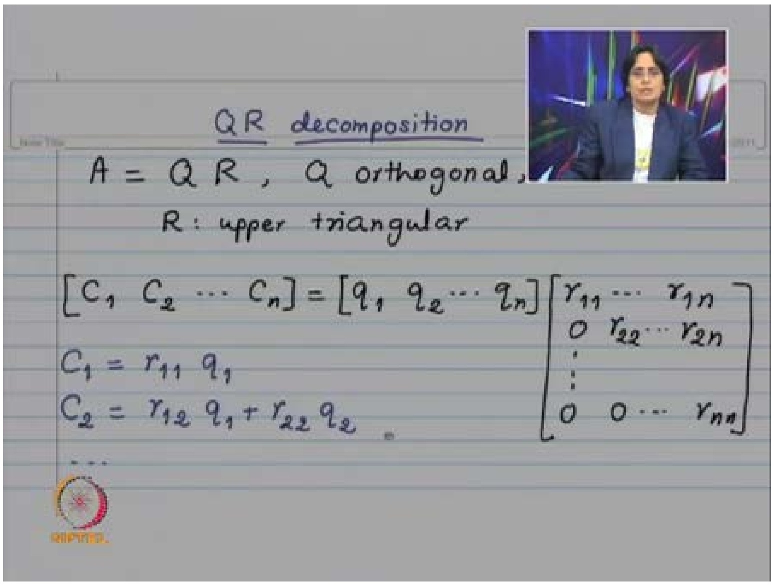

Our matrix A is invertible and we want to write it as Q into R, where Q is orthogonal matrix - that means, Q transpose Q is identity, R is upper triangular matrix; C 1, C 2 up to C n - these are columns of our matrix A; q 1, q 2 and q n - these are columns of matrix Q.

Yesterday, we saw that Q orthogonal means that the columns of q - they are going to form an ortho normal set; that means, each column vector is going to have Euclidean norm to be equal to 1 and if you consider inner product - standard inner product - on r n, q i comma q j - the inner product of q i with q j will be 0 if i not equal to j.

For simplicity, we are assuming A to be a real matrix - a real invertible matrix; r upper triangular - it means it is going to be of this form; below the diagonal all the entries they are going to be equal to 0.

Now, what we are going to do is multiply and equate the columns. We are going to have C 1 to be equal to r 11 multiplied by q 1; when you post multiply by such a matrix the first column q 1 will get multiplied by r 11; then, C 2 the second column will be r 12 times first column q 1 plus r 22 times second column q 2 and so on.

Now, what is given to us is matrix A; that means, C 1, C 2 and C n are known; what we need to find is columns of q and entries of this upper triangular matrix r. So, if I look at the first one - C 1 is equal to r 11 q 1, I know that Euclidean norm of q 12 norm norm q 2 norm q 12 norm is going to be equal to 1.Take norm of both the sides; you are going to have norm of C 12 norm is equal to modulus of r 11 into norm q 1 that is going to be equal to 1.

Mod r 11 is equal to norm of C 1; thus, for r 1 we have a choice - you can either choose it to be plus norm of C 1 or minus norm of C 1; thus, we have determined r 11. Once you determine r 11 q 1 is going to be C 1 divided by r 11.

You have determined q 1 and you have determined r 11; now, look at the second column. We need to determine r 12, r 22 and q 2; q 1 is already determined. What we will do is, we will use the fact that q 1 and q 2 are perpendicular; if I take inner product of C 2 with q 1, then the contribution from this term is going to be 0.

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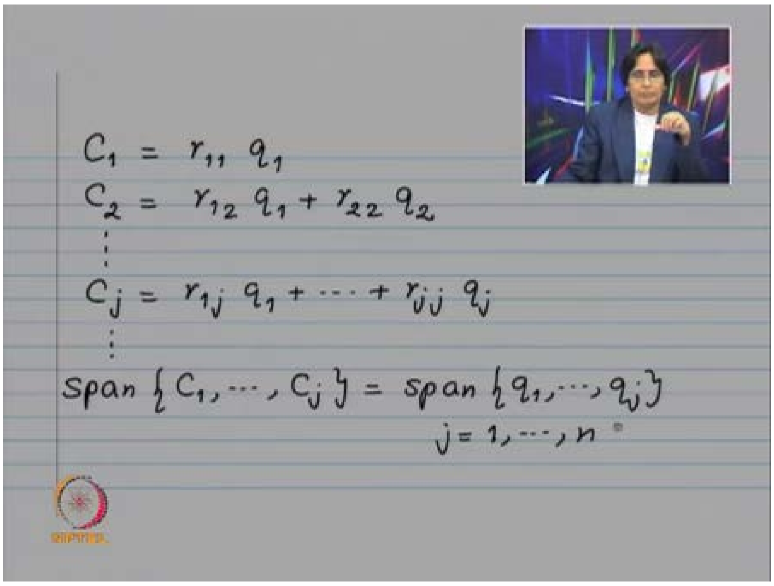

We have C 1 is equal to r 11 q 1; c 2 is equal to r 12 q 1 plus r 22 q 2, and in general the jth column C j will be equal to r 1j q 1; this is the first entry in the jth column of r.

Then, there will be r 2j q 2 and then r jj q j. So, from here we see that span of C 1, C 2 and C j is going to be equal to span of q 1, q 2 and q j, j going from 1 up to n.

C 1 q 1 they are multiples of each other; C 2 is a linear combination of q 1 and q 2; when I consider a linear combination of C 1 and C 2 that is going to be linear combination of q 1 and q 2 and so on. So, we have got this property.

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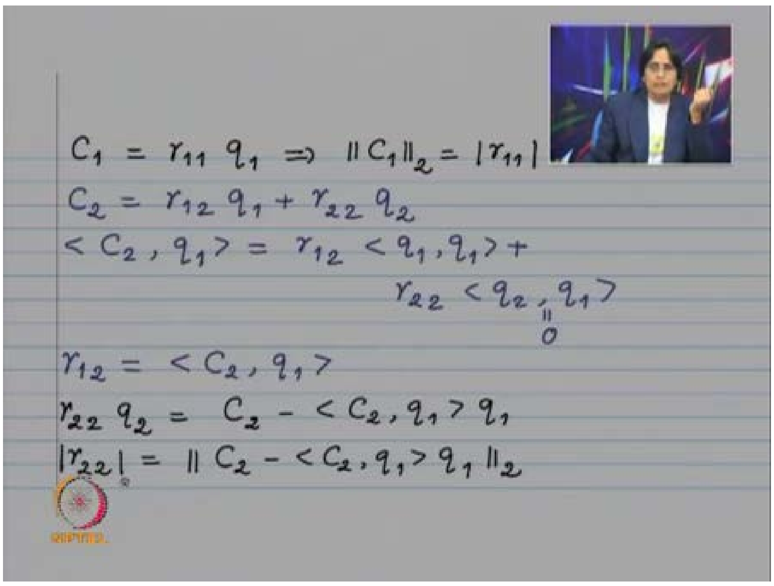

Then, C 1 is equal to r 11 q 1; take norm and then you are going to have modulus of r 11 to be equal to norm of c 1 2 norm; then, c 2 is r 12 q 1 plus r 22 q 2, take the inner product with q 1; inner product of c 2 with q 1 is equal to r 12 times inner products of q 1 with itself plus r 22 times inner product of q 2 with q 1; this is 0.

So, r 12 is going to be inner product of C 2 with q 1; we have already determined r 1 1 q 1 1 now we have determined r 12; what remain to determined are r 22 and q 2.

When you consider r 22 q 2 this is going to be C 2 minus r 12, which is inner product of C 2 with q 1 multiplied by q 1; take norm of the both the sides, norm q 2 is 1; modulus of r 22 is going to be Euclidean norm of this vector. Once again, for r 22, which is a real number, you have got choice - either you can choose it to be bigger than 0 or you can choose it to be less than 0.

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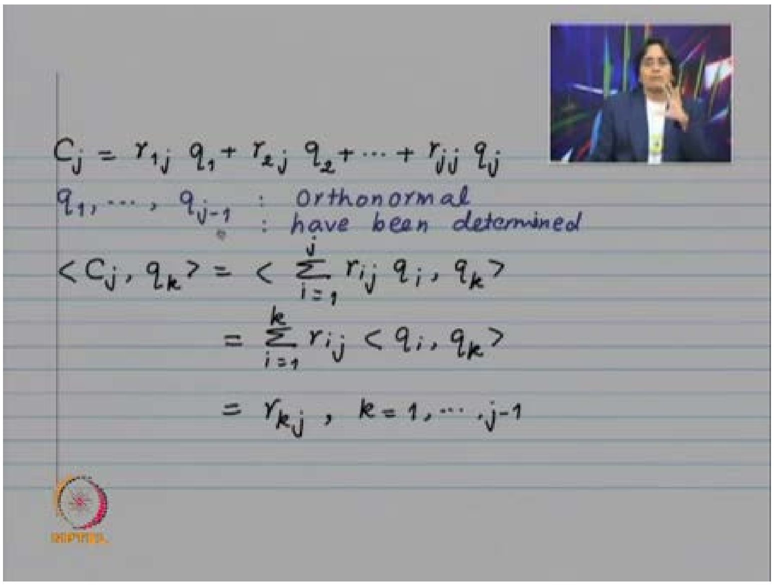

Next, a general case will be C j is equal to r 1j q 1 plus r 2j q 2 plus r jj q j.

We would have determined q 1 q 2 q j minus 1 - they are ortho normal vectors and they have already been determined. Look at inner product of C j with q k, this is going to be summation r ij q I - i goes from 1 to j; this expression I am writing in the compact form as a summation q k; by using linearity of the inner product in the first variable you get summation r ij inner product of q i with q k, the only term will remain when i is equal to k.

You will have r k j; thus, r 1 j r 2 j up to r j minus 1 j will be determined; now, what remains to be determined is the co efficient r j j and vector q j; vectors q 1 q 2 q j minus 1 we have already determined.

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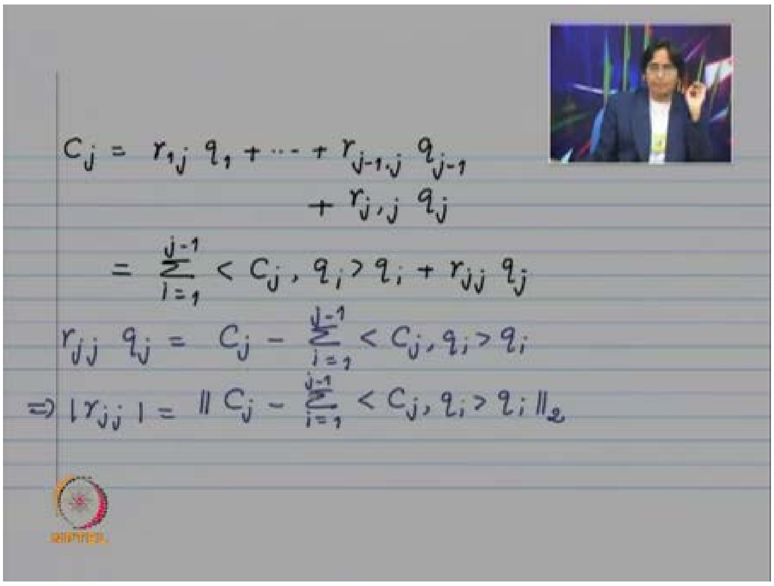

We have C j is equal to - just now we saw that r 1 j is going to be inner product of C j with q 1 and r ij will be inner product of C j with q I; so, I substitute plus r jjq j.

All these things they are known; take them on the other side and you are going to have j minus this. Take norm of both the sides - norm of this is going to be equal to modulus of r jj; r jj is going to be either bigger than 0 or less than 0 we can decide. Once you determine r jj q j will be determined from this expression.

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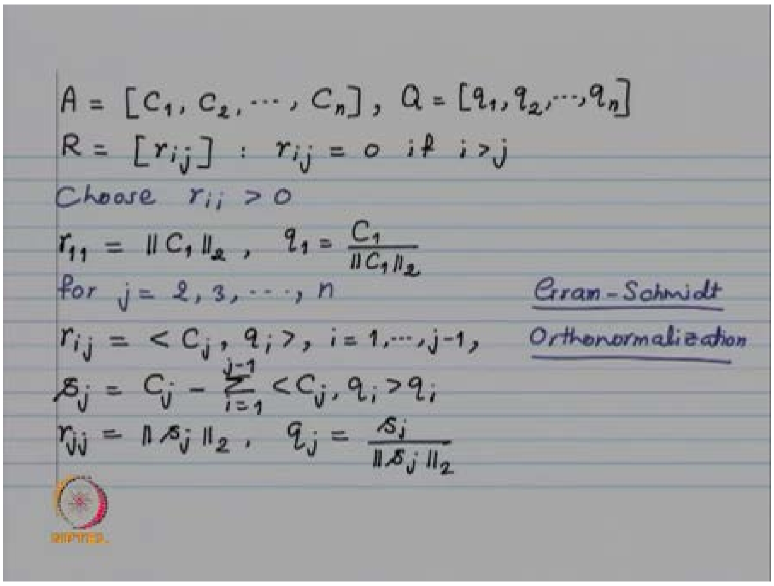

So, we have A is equal C 1, C 2…C n the columns of A columns of Q are q 1, q 2…q n; this is an orthogonal matrix.

R is equal to r ij upper triangular matrix. For the sake of definiteness, choose r ii to be bigger than 0. So, r 11 will be plus norm of C 1 2 and q 1 will be C 1 upon norm C 1.

Then, for j is equal to 2, 3 up to n, r ij's are going to inner product of C j with q I; let s j b C j b minus this summation; then, r jj will be 2 norm of this s j and q j is going to be equal to s j divided by its norm; this is nothing but Gram-Schmidt Orthonormalization process.

We had considered Gram-Schmidt process - if we have n linearly independent vectors then we can construct a set of n ortho normal vectors, which have the property that span of C 1, C 2, c j is same as span of q 1, q 2, q j.

Thus, Q R decomposition of A is nothing but Gram-Schmidt Orthonormalization process applied to columns of A. Whatever ortho normal vectors you get, they are going to form our orthogonal matrix Q and various coefficients r ij they form our upper triangular matrix R.

We have seen that at each stage when we want to consider or when we want to determine the diagonal entries of R we had a choice to choose our diagonal entry to be bigger than 0 or it to be less than 0, which will mean that the Q R decomposition of our matrix A is not unique; because, for each diagonal entry I have a choice - I can choose it either to be greater than 0 or less than 0.

Any invertible matrix A can be written as Q into R - the decomposition is not unique; if we want uniqueness then we have to fix or we have to put some condition on diagonal entries of our matrix R.

Let us put the condition to be that all the diagonal entries they should be bigger than 0 - it is just one of the conditions; I can as well put that all the diagonal entries should be less than 0 then such a decomposition will be unique or I can say that all even entries should be bigger than 0 or all odd diagonal entries should be less than 0; it is just that you have to put some conditions on the diagonal entry.

Let us prove uniqueness of Q R decomposition using - with the condition that the diagonal entries of R they are bigger than 0; now, let me recall earlier decompositions which we had considered - we had considered L u decomposition of a matrix.

Matrix A we were writing as L into u where L is lower triangular, u is upper triangular. Not all matrices have L u decomposition we had to put additional condition that when you consider the sub-matrix which is formed by first k rows and first k columns - leading principal sub-matrix, if its determinant is not equal to 0 for k is equal to 1 to up to n, then your matrix A you can write as L into u.

The converse is true; if such a case is there then the L u decomposition is unique; then, from L u decomposition we went to what is known as L d v decomposition. So, you have got L to b lower triangular in the L u decomposition again; for uniqueness we needed that the diagonal entries of L should be equal to 1 - so, it is unique lower triangular matrix.

In the L d v decomposition we had L to be unit lower triangular, v to unit upper triangular and d was diagonal; then, such L d v decomposition - it is unique.

From the L d v decomposition we considered Cholesky decomposition, which is valid for positive definite matrices; so, A is equal to g g transpose where g is a lower triangular matrix, then g transpose will be upper triangular matrix and the uniqueness is obtained provided you put some condition on the diagonal entries of g.

One of the conditions is - assume that all the diagonal entries of g they are bigger than 0; then, the Cholesky decomposition is unique; now, this uniqueness of Cholesky decomposition we are going to use to prove uniqueness of Q R decomposition with the condition that Q is orthogonal matrix and R is upper triangular matrix with diagonal entries to be bigger than 0.

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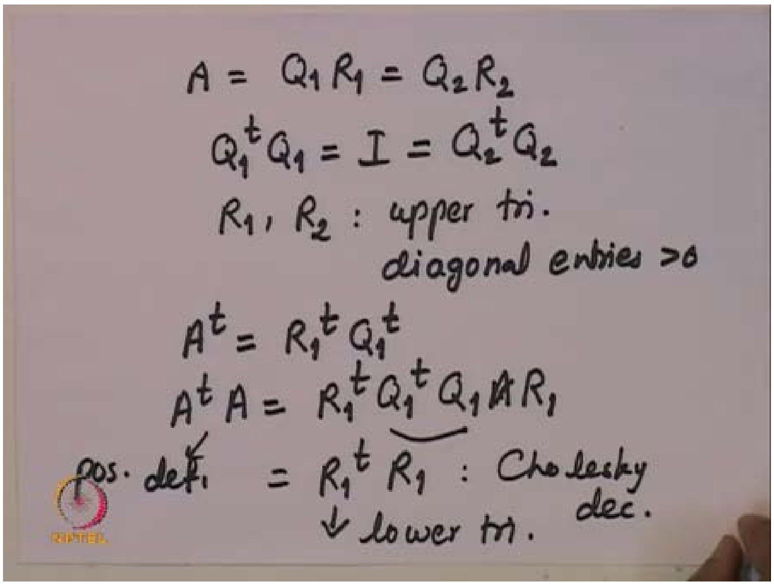

Suppose A is equal to q 1 r 1 is equal to q 2 r 2; what we are going to do is - we have A is equal to Q 1 R 1 is equal to Q 2 R 2.

We have got Q 1 transpose Q 1 is equal to identity is equal to Q 2 transpose Q 2; R 1 and R 2 these are upper triangular and diagonal entries are bigger than 0.

If I look at A transpose - A transpose will be nothing but R 1 transpose Q 1 transpose; if I look at A transpose A this will be R 1 transpose Q 1 transpose q 1 and R 1.

Now, Q 1 transpose Q 1 is identity and we have got R 1 transpose R 1; now, this R 1 transpose is going to be lower triangular because our R 1 is upper triangular. This is nothing, but Cholesky decomposition.

Our matrix A is invertible hence A transpose A is going to be positive definite matrix.

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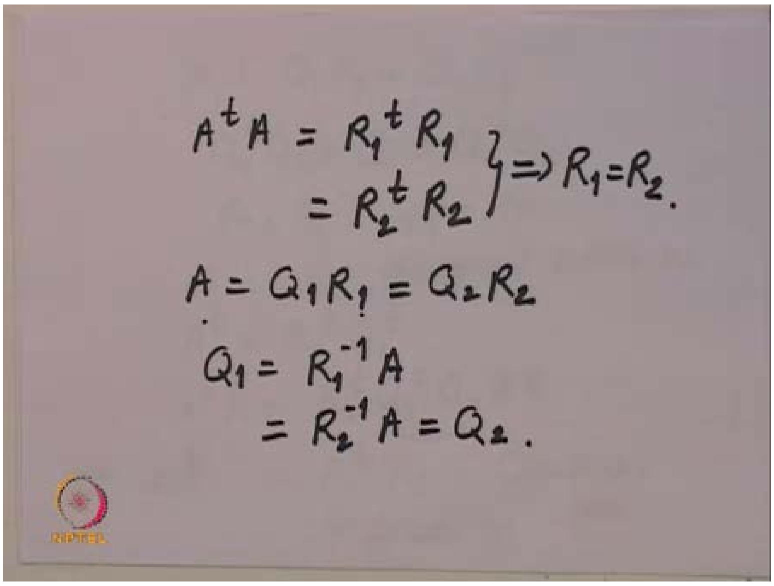

We have A transpose A to be equal to R 1 transpose R 1 and it will also be equal to R 2 transpose R 2. We are considering A is equal to Q 1 R 1 is equal to Q 2 R 2; now, by uniqueness of the Cholesky decomposition it will imply that R 1 has to be equal to R 2.

Now, if R 1 is equal to R 2 then Q 1 is going to be R 1 inverse A, because A is invertible; R 1 has - R 1 is a upper triangular matrix with diagonal entries bigger than 0, so it is going to be invertible; this is the same as R 2 inverse A which is Q 2.

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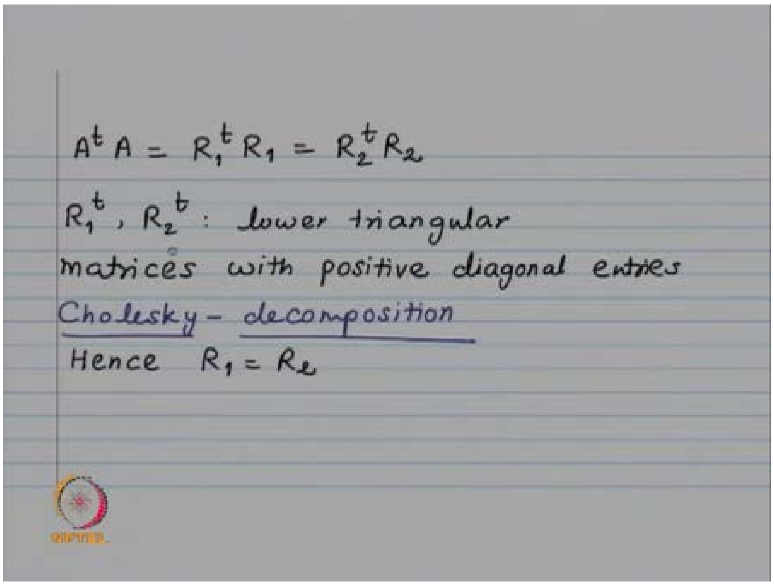

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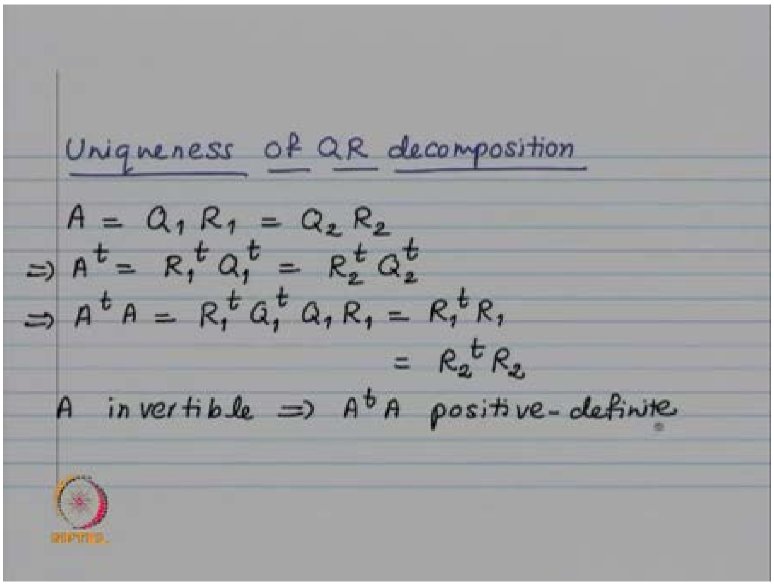

Thus, we have got A is equal to Q 1 R 1 is equal to Q 2 R 2, then A transpose A is equal to R 1 transpose R 1 which is same as R 2 transpose R 2; A invertible will imply A transpose A to be positive definite with positive diagonal entries.

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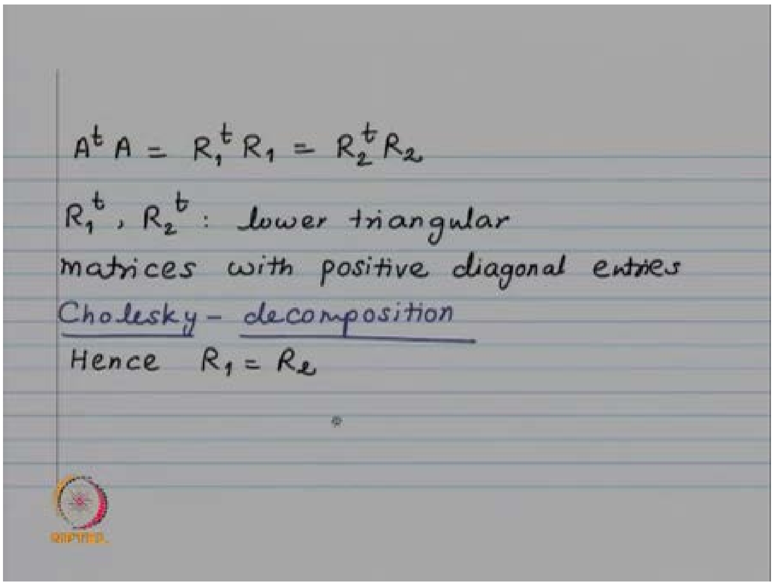

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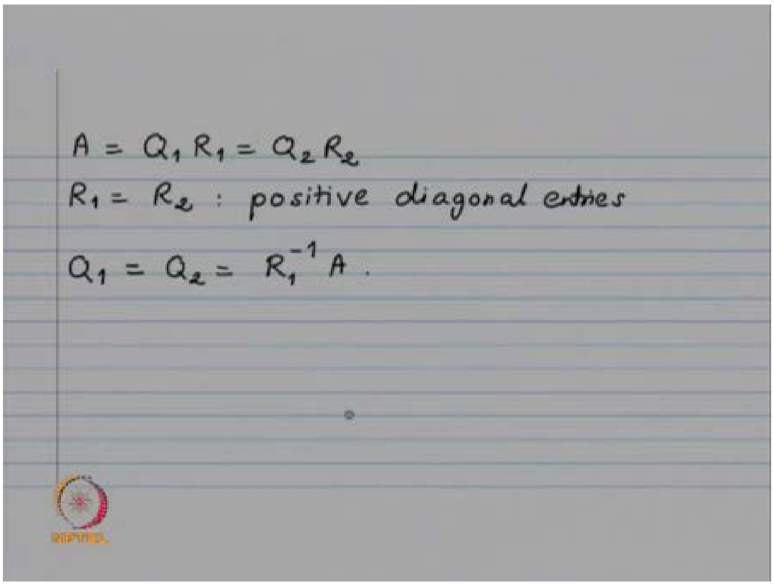

Since Cholesky decomposition is unique you get R 1 is equal to R 2; then, you get Q 1 is equal to Q 2.

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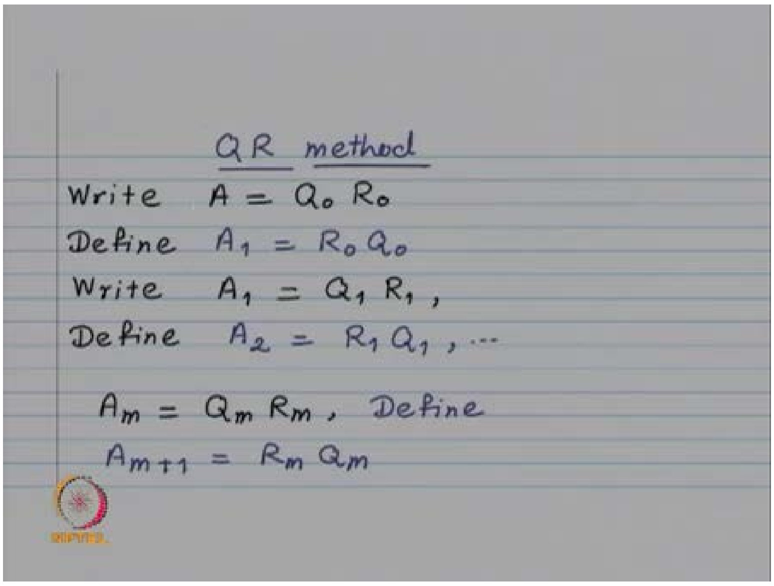

Now, we come to Q R method; the Q R method is easy to describe, what is difficult is to prove its convergence and to - there are many computational points which are involved.

We will not have time to consider all the intricacies of the Q R method; as I had said before, Q R method is one of the most popular methods for calculating Eigen values of a matrix at present. This Q R method can be considered as - or it can be interpreted as what is known as simultaneous iteration; in case of power method we were looking at only one vector and then we were defining iterate; instead of that one can consider several vectors and then one can define the Q R method or when one can interpret; that will give some idea has to why the Q R method works. So, let me describe what a Q R method is.

We are going to start with a matrix A; we know that invertible matrix A can be written as Q into R, where Q is going to be orthogonal matrix and R is going to be upper triangular matrix.

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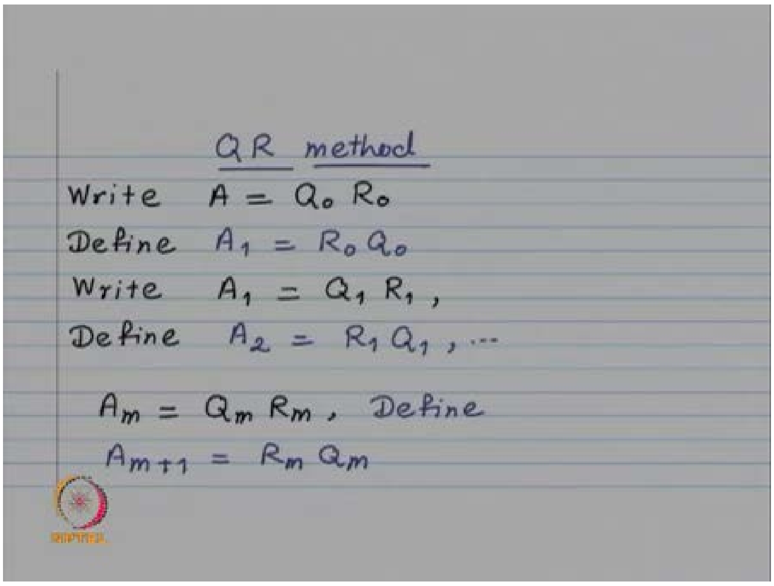

Here is Q R method: the first step is write your matrix A as Q 0 into R 1; that means, you calculate Q 1 decomposition of your matrix A; this is the first step.

In the next step what you do is you define a new matrix - you have obtained Q 0 and R 0; now, multiply them in the reverse order, so you consider R 0 into Q 0. The matrix multiplication is not commutative, so you are going to get a different matrix - A 1; after you have found A 1 now you find its Q R decomposition.

You look at the columns of A 1 orthonormalize and that will give you columns of Q 1; you have A 1 is equal to Q 1 R 1. Once you find these factors Q 1 and R 1 you multiply them in the reverse order and when you do that you are going to get a new matrix A 2.

You continue this procedure. When you consider A m - suppose you have to come up to mth step, that matrix A m - find its R Q decomposition multiply them in the reverse order and then obtain A m plus 1.

Under some conditions, this A m plus 1 it is going to converge to an upper triangular matrix. So, it looks like magic that you start with a matrix A, then you find its Q R decomposition, multiply it in a reverse order, get a new matrix, again you calculate the Q R decomposition of this new matrix and once you get the factors multiply them in the reverse order get a new matrix, you continue.

When you do that you are going to get a sequence of matrices; you are going to have A 1, A 2, A m plus 1; this A m plus 1 it converges to an upper triangular matrix and what I am going to show is whatever matrices we are constructing in the process all these matrices they are going to be unitarily equivalent; because, for Eigen values it is very important that the transformations which we are doing they preserve the Eigen values.

At each stage we are getting say A 1, A 2, A 3 and so on; we are going to show that they are all unitarily equivalent. If they are unitarily equivalent - that means, they are also similar matrices; that will preserve the algebraic - that will first of all it will have same set of Eigen values - algebraic multiplicities will be preserved, geometric multiplicities will be preserved .

If this sequence A m if, it is converging to upper triangular form or upper triangular matrix the Eigen values of this upper triangular matrix, which are nothing but the diagonal entries, they are going to be Eigen values of our matrix A. So, you construct a sequence A m, it converges to an upper triangular form; look at the diagonal entries of this upper triangular matrix those are going to be Eigen values you are interested in.

My plan is - first show that this sequence unitarily equivalent, then I want to show or I want to describe an efficient way of calculating Q R decomposition; efficient - that means, we have to take into consideration the stability.

As such we have got Gram- Schmidt Orthonormalization process, apply Gram-Schmidt Orthonormalization process and get Q R decomposition, but there is a better way of doing it and that is using reflectors. So, I am first going to show that all these matrices are unitarily equivalent and then consider reflectors.

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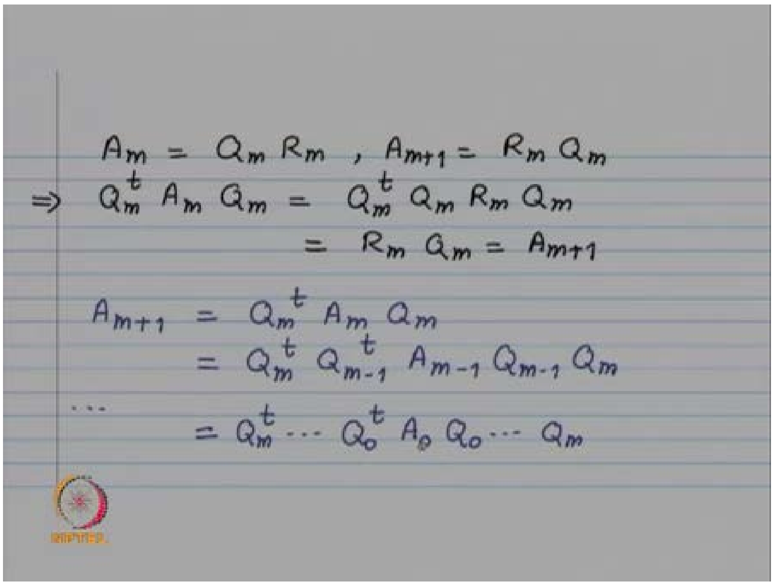

We have A m is equal to Q m R m; A m plus one is equal to R m Q m because that is our definition. If I pre multiply A m by Q m transpose, I am going to have - and post multiply by Q m - Q m transpose A m Q m is going to be equal to Q m transpose Q m R m q m; this Q m transpose Q m is going to be identity because it is an orthogonal matrix.

You get R m Q m. This will be equal to - that is nothing, but our A m plus 1. So, A m plus 1 is equal to Q m transpose A m Q m; Q m transpose is inverse of Q m, hence A m plus 1 and A m they are going to have same Eigen values with algebraic and geometric multiciplicities preserved.

Now, this one can substitute for A m; so, A m is going to be Q m minus 1 transpose A m minus 1 Q m minus 1, when you continue you are going to have Q m transpose Q m minus 1 transpose Q 0 transpose A 0 Q 0 Q 1 Q m; A 0 is our original matrix of which we want to find Eigen values.

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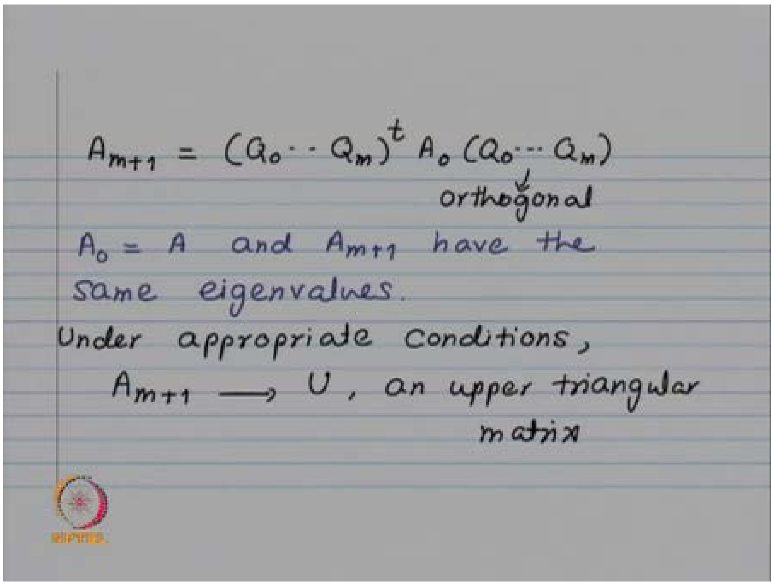

Thus, A m plus 1 is equal to Q 0 Q 1 Q m transpose A 0 Q 0 Q 1 Q m; now, this matrix is going to be orthogonal because the product of two orthogonal matrices is again orthogonal.

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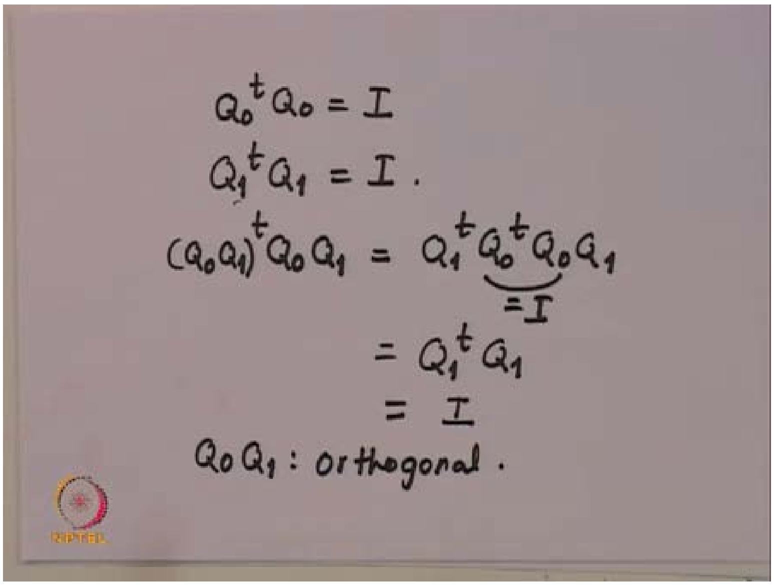

Suppose, you have got Q 0 transpose Q 0 is equal to identity and Q 1 transpose Q 1 is also equal to identity; I consider Q 0 Q 1 - multiplication, and then Q 0 Q 1 - transpose. This is going to be equal to - when you take the transpose the order gets reversed; so, you have Q 1 transpose Q 0 transpose Q 0 Q 1.

This is equal to identity, so you have got Q 1 transpose Q 1 and Q 1 transpose Q 1 is identity; you get Q 0 Q 1 also to be orthogonal.

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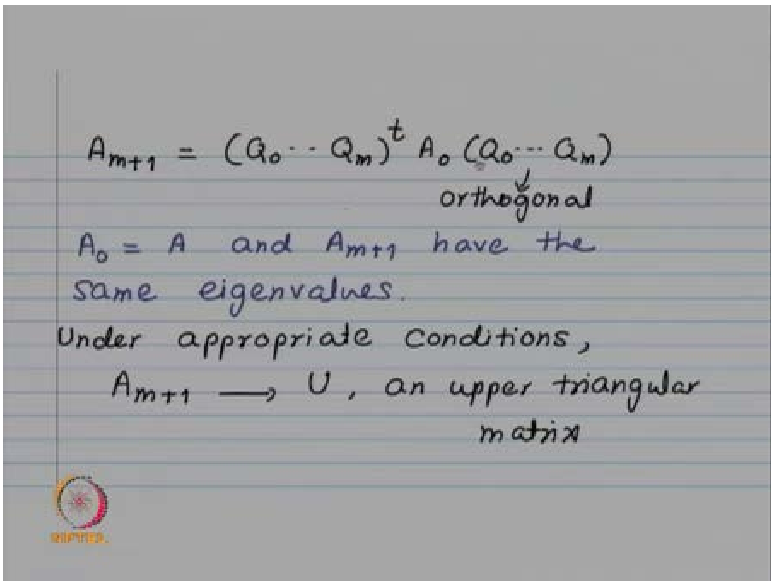

Thus, when you look at A m plus 1 then this Q 0 Q 1 Q m is going to be an orthogonal matrix, its transpose is going to be its inverse; A 0 and A m plus 1 they have the same Eigen values and under appropriate conditions A m plus 1 converges to U which is an upper triangular matrix.

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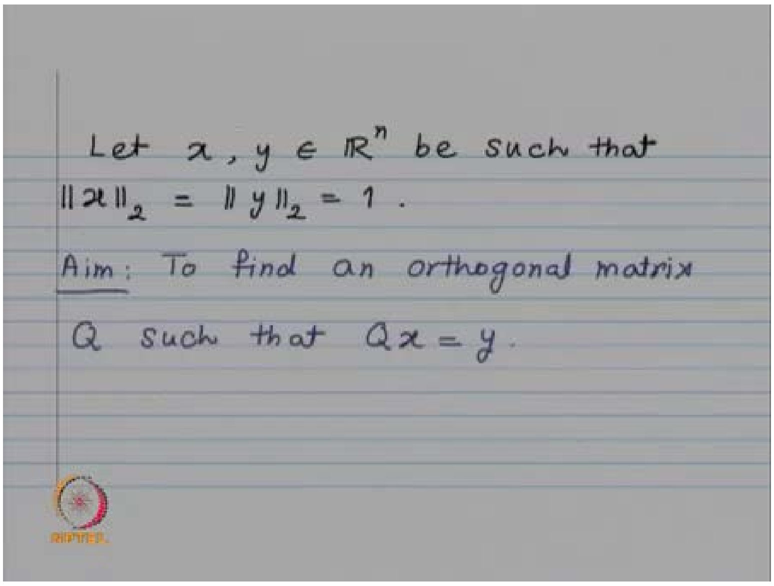

Next, we are going to look at Reflector. Suppose you have got two vectors x and y - we are looking at real vectors and suppose they have got the same Euclidean norm; so, x and y are distinct vectors with same Euclidean norm; we are going to show that there exists an orthogonal matrix Q such that Q into x is equal to y.

If I do this, how does it help me? We want to reduce our matrix A to an upper triangular form by using say similarity transformations; we have got matrix A, we will look at its first column; now, this first column is a vector, we want to transform this to a vector say sigma 1 and then remaining entry 0.

When we reduce a to upper triangular form, what we do is in the first column below the diagonal we introduce 0 (s); we want to do the same thing, but the transformations we are using we want them to be the unitary transformations so that the Eigen values are preserved.

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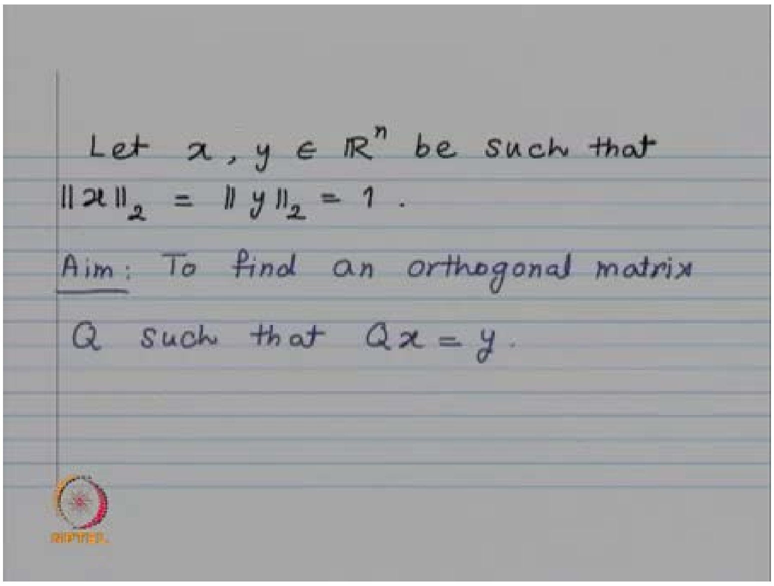

Here is our aim: x and y are vectors in R n with Euclidean norm to be equal to 1 and we want to find an orthogonal matrix Q such that Q x is equal to y.

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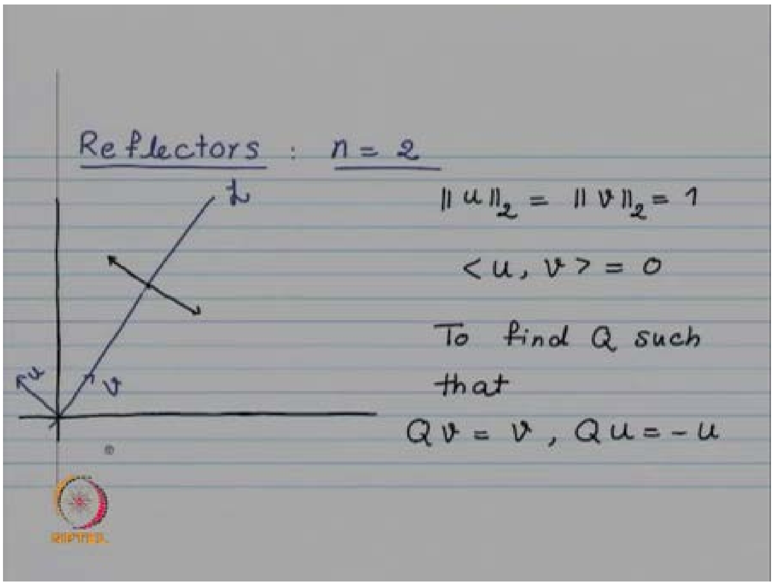

I am going to explain for the case n is equal to 2 and then we will see that this gets generalized.

You start with two unit vectors u and v, which are perpendicular; inner product of u and v it is equal to 0. We have a vector - unit vector - v here, then there is a perpendicular vector u which is also unit vector.

We look at line L in the direction of v and we want to consider a reflection in this line L; now, this reflection will be characterized by - we are considering reflection in the line L; Q of v should be equal to v and when you consider reflection of u it should be equal to minus u. We want to find such a Q with the property that Q v is equal to v Q u is equal to minus u.

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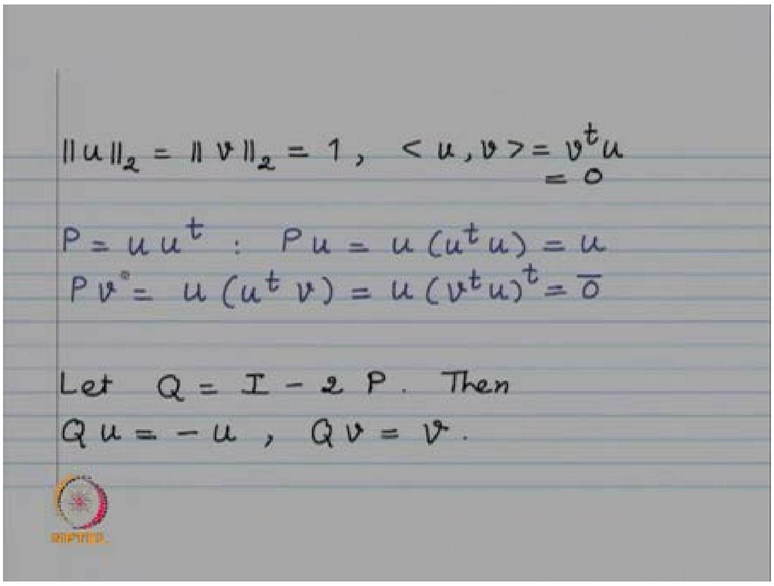

Now, look at P is equal to u u transpose.

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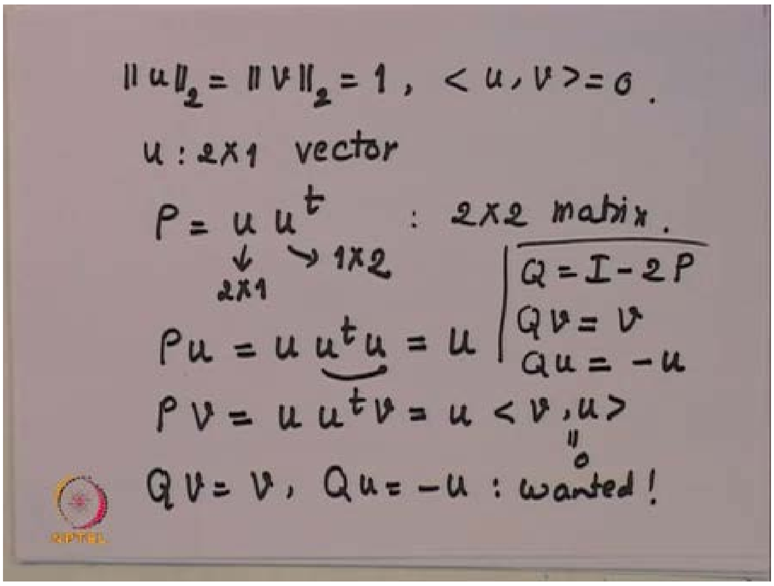

We have got our vectors are unit vectors norm u tra[nspose]- u 2 is equal to norm v 2 is equal to 1 inner product of u with v is equal to 0 u is a two by one vector; I look at P is equal to u u transpose; this is two by one u transpose will be one by two . So, P is going to be two by two matrix.

When I consider P of u this is equal to u u transpose u - u transpose u is nothing but norm u square, it is going to be equal to 1; so, P of u is equal to u and P of v is going to be equal to u; u transpose v - but this is going to be equal to u and then inner product of v with u; this is 0.

What I wanted was Q of v to be equal to v and q of u to be equal to minus u - this is what we wanted; what we have got is P of u is equal to u and P of v is equal to 0.

Now, what I am going to do is I am going do some manipulation; if I look at Q is equal to identity matrix minus two times P, then what will happen is Q of v will be equal to v because P of v is equal to 0; Q of u is going to be equal to u minus two times P u, but P into u is equal to u, so Q of u is equal to minus u.

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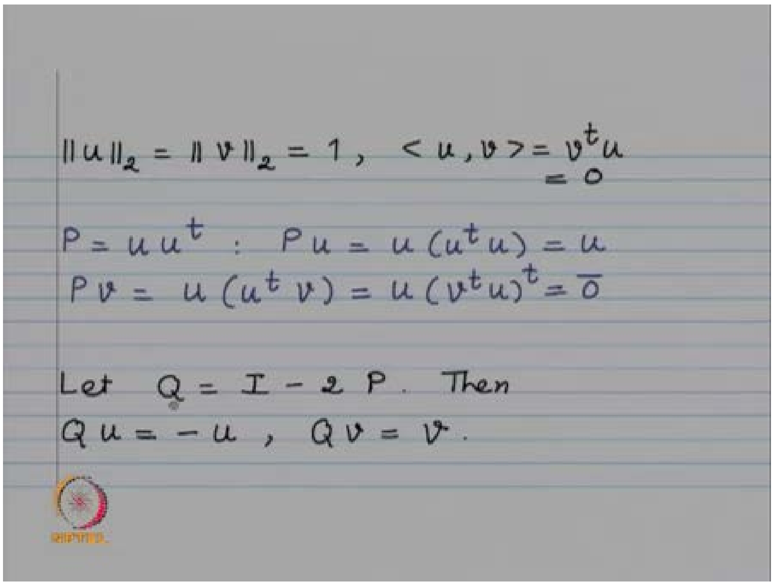

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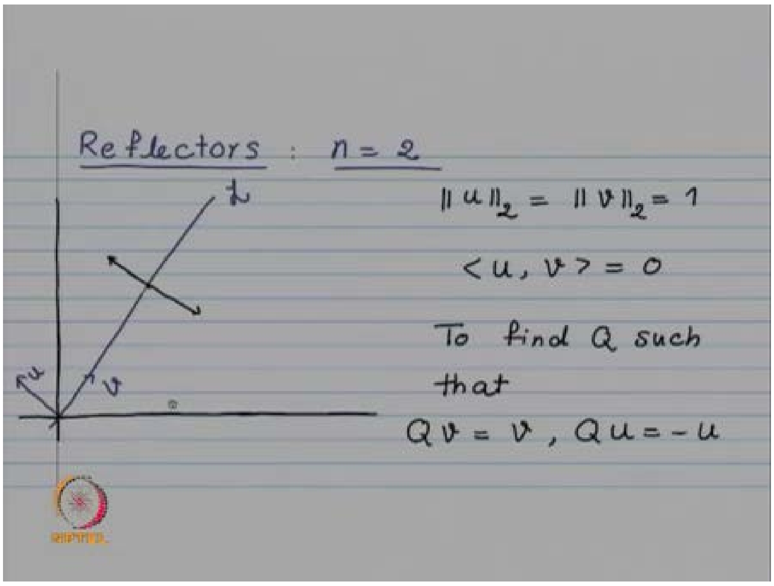

We have got a Reflector that Q - what we wanted was - these are our unit vectors which are mutually perpendicular; our aim is to look at the reflector - reflection - in the line L - Q of v is equal to v Q of u is equal to minus u.

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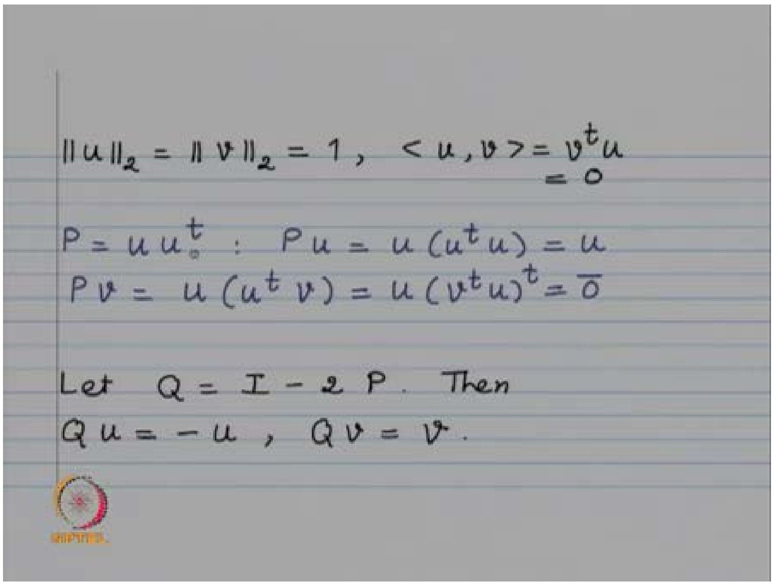

Consider, Q is equal to I minus 2, P where P is equal to u u transpose; then, its satisfies the desired condition. Now, let us look at the properties of P.

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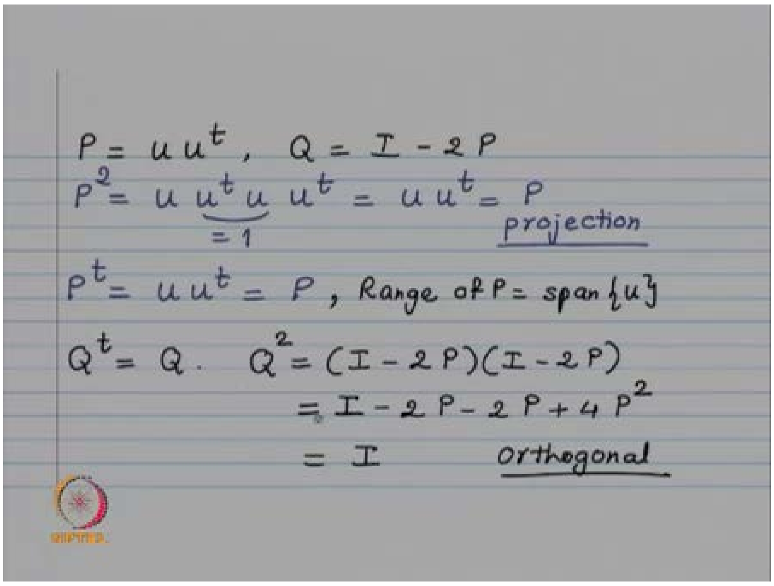

P is equal to u u transpose; when I consider P square, it will be u u transpose u u transpose - this is going to be equal to 1. So, P square is equal to P - a projection; then, P transpose is equal to P range of P is going to be equal to span of u.

Since, Q is equal to I minus 2 P Q transpose is equal to Q and when I consider Q square it will be I minus 2 P multiplied by I minus 2 P; it is I minus 2 P minus 2 P plus 4 P square; P square is equal to P, so you get Q square is equal to identity.

We have got - I started with something: I said that x and y are two vectors which have got the same Euclidean norm and I want to find the orthogonal matrix Q which will transform 1 vector x into y, this is what I wanted; then, we looked at something else - we looked at a reflector. We obtained an expression for reflection in a line L. How does it help me in getting a orthogonal matrix Q which we will transform vector x into vector y?

There, what we are going to do is - we have got two vectors x and y, they have got the same Euclidean length; now, you consider a parallelogram with one side as vector x and another side as vector y.

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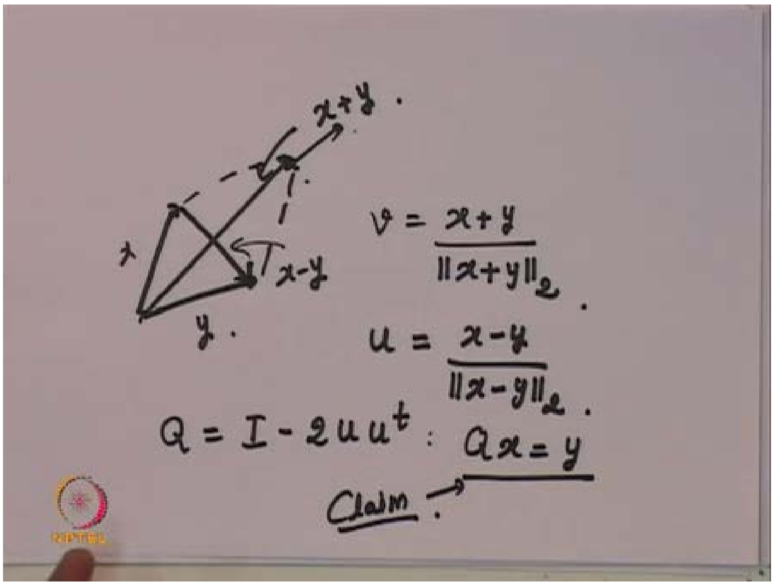

We have got, say - here is my vector x, vector y having the same length; now, I look at the - I complete the parallelogram; when I do that this vector is going to be vector x plus y.

Consider reflection in this line in the direction of x plus y; the reflection of x is going to be precisely our vector y. If we want to set consider the earlier notation v was unit vector in this direction; so, our v is going to be x plus y divided by norm of x plus y two norm. Our u was a vector which is perpendicular to our line; now, the diagonals of this parallelogram they are going to intersect or they are perpendicular; now, this diagonal - this was one diagonal x plus y another diagonal is going to be x minus y.

Your u is going to be x minus y divided by norm of x minus y 2 norm; then, if you consider Q is equal to I minus 2 times u u transpose, then this will transform our x to y - this is our claim. Let us work out the details of this.

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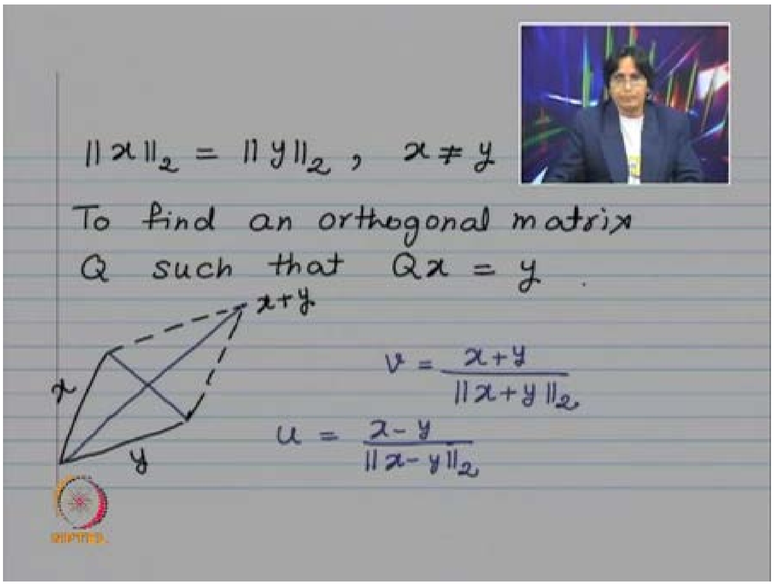

We have got norm x is equal to norm y, x not equal to y. We want to find an orthogonal matrix Q such that Q x is equal to y. Here is the parallelogram - v is x plus y divided by its Euclidean norm, u is x minus y divided by norm of x minus y; because, I am dividing by its norm we are going to get unit vectors.

We assume that x is not equal to y; your u is going to be non 0 vector. Now, we are going look at Q is equal to identity minus 2 u u transpose because when we wanted to look at the reflection in the line L that is what we have obtained that look at Q is equal to I minus 2 u u transpose; such a matrix was orthogonal matrix.

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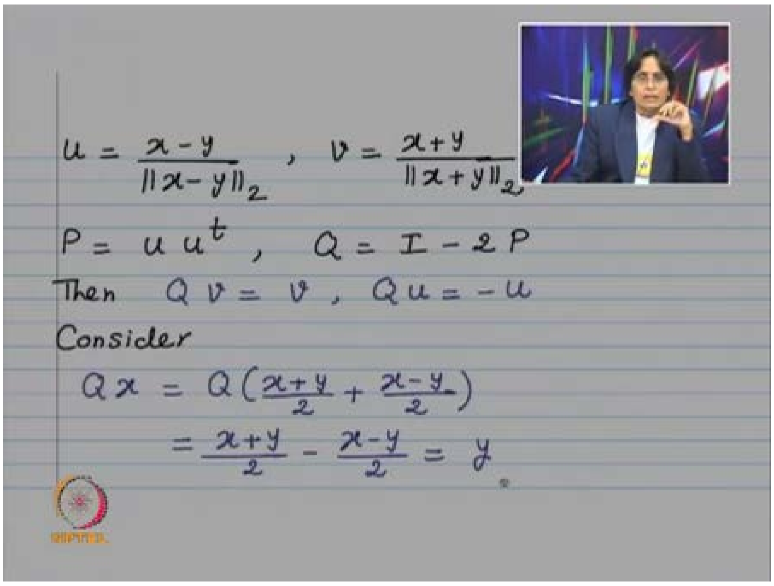

u is equal to x minus y upon norm of x minus y; v is equal to x plus y upon norm of x plus y; P is u u transpose and Q is I minus 2 P; Q of v is equal to v; Q of u is equal to minus u.

I look at Q x; this x I write as x plus y by 2 plus x minus y by 2 - y by 2 will get cancelled. I am adding and subtracting vector y by 2; x plus y by 2 it is going to be multiple of our vector v.

If Q of v is equal to v then q of alpha v also will be equal to alpha v. So, that is why Q of x plus y by 2 is going to be x plus y by 2; x minus y by 2 is a multiple of u; Q of u is equal to minus u.

Q of x minus y by 2 will be minus x minus y by 2; now, x will get cancelled or other x by 2 will get cancelled and you will get Q x is equal to y; so, we have got a way to construct Q.

Once the vectors x and y are given to us., what you have to do is construct these unit vectors and then look at Q is equal to I minus 2 times u u transpose; such an orthogonal matrix will transform the vector x to vector y.

Let us use this to construct or rather to - what we are trying to do is writing a is equal to Q into R; now, I am now going to look at the first column of A - it is an invertible matrix so the column will be non 0.

This column I want to transform in a column vector whose first entry is non 0 and all the other entries they are 0; if I do this I do not have much choice for this vector y - the first column is given to us - that is given; now, for the second vector it has only one non 0 entry and the Euclidean norms, both of them, they have to be the same.

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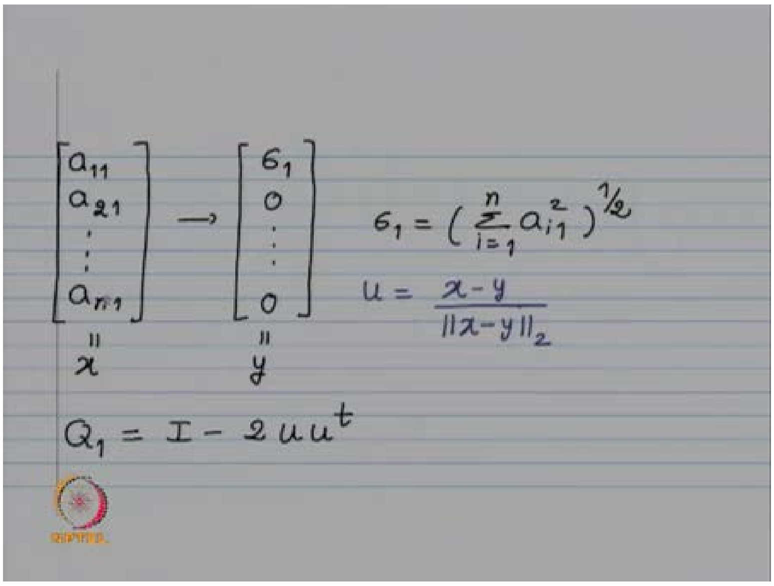

We have - this is our first column a 11 a 21 a n 1; this is my vector x, vector y will be of the form sigma 1 0 0 0; since, x and y should have same Euclidean norm - that is an important condition; this sigma 1 will be nothing but summation a i1 square - i going from 1 to n and its square root. For this sigma 1 the choice is either it should be positive square root or negative square root that is it; u is going to be x minus y divided by its Euclidean norm; this is our x and this is our y then you look at u and then your Q 1, it is going to be identity minus 2 u u transpose.

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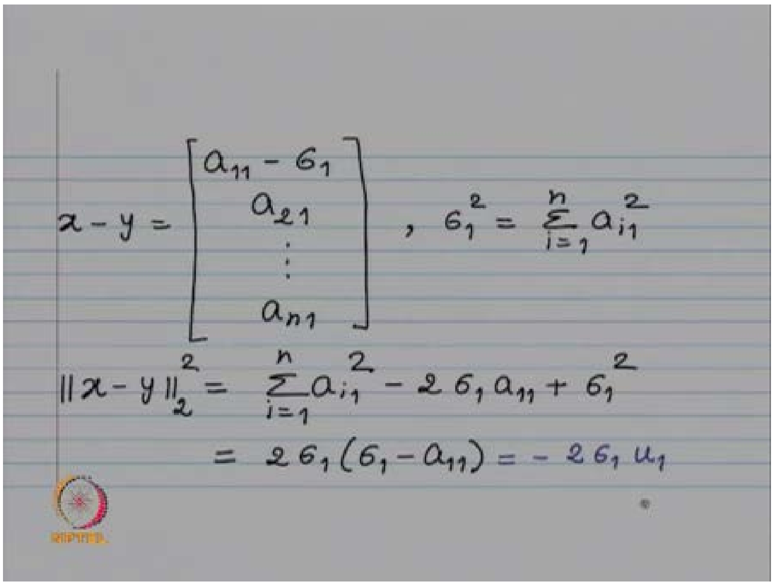

You have x minus y; this is going to be a 11 minus sigma 1 a 21 a n1; sigma 1 square is summation a 1 square i goes from 1 to n because we want to have y and x to have the same Euclidean norm; when I calculate norm of x minus y 2 norm square it will be summation a i1 square minus 2 sigma 1 a 11 plus sigma 1 square - this is sigma 1 square; it becomes 2 sigma 1 into sigma 1 minus a 11 and that is minus 2 sigma 1 u 1.

We can transform the first column into a column, which has only 1 non 0 entry and that is the first entry. In my next lecture we will see how this - when you continue this procedure how we get Q R decomposition of our matrix; then, we are going to consider L e square problem. Thank you.