

Project Management

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Week: 1

Lecture 05 : Examples of Project Selection Models

Dear students, in the previous lecture, I have discussed about project selection model. There I have explained numeric and non-numeric models. In this lecture, I am going to show you some examples of numerical models. By using the numeric models, how to use, how to select various projects. Just to recollect where we are now, see initially I talked about the project selection model on two lectures, then I have discussed about agile project management. The previous lecture I discussed about project selection model.

Phase-I
Project Initiation

Course outline

- Introduction to project management-I
- Introduction to project management-II
- Agile project management
- Project selection models
- Examples of Project selection Model
- Project manager
- Attributes of Effective Project Manager
- Managing for stakeholders
- Resolving Conflicts
- Negotiation
- Project in the organization structure
- Human factors and the project team



Currently, I am going to give some example of project selection model. The agenda for this lecture is net present value, weighted scoring model, internal rate of return and AHP. These are the techniques used for project selection model. After explaining each techniques, I am going to use Excel to give you a demo on how to use Excel for selecting your project.

Agenda

- The Net Present Value (NPV)
 - Calculation of NPV using Excel
- Weighted Scoring Model
- Internal Rate of Return (IRR)
 - Calculation of IRR using Excel
- Analytic Hierarchy Process (AHP)
 - Calculation of AHP using Excel



First, I am going to explain the net present value. As I explained previously, the net present value of your project is the sum of present value of all the cash flows. Generally, we use positive as well as negative. The cash inflows is taken as positive and cash outflows is taken as negative that are expected to occur over the life of the project. The general formula for NPV is

$$NPV = \sum_{t=1}^n \frac{C_t}{(1+r)^t} - \text{Initial Investment}$$

The Net Present Value (NPV)

The net present value (NPV) of a project is the sum of the present values of all the cash flows - positive as well as negative - that are expected to occur over the life of the project. The general formula for NPV is:

$$NPV = \sum_{t=1}^n \frac{C_t}{(1+r)^t} - \text{Initial investment}$$

where C_t is the cash flow at the end of year t , n is the life of the project, and r is the discount rate.



Here C_t is the cash flows at the end of the year “ t ” and “ n ” is the life of the project and “ r ” is the discount rate. Now, I am going to explain how to use NPV with the help of an example. The example one is Siddharth bought a house of 750,000 dollar and sold it a year later for 990,000 dollar after deducting any realtors fees and taxes. Now, calculate

the NPV if the rate of return is 5%. So, what are the data is given to us? Investment on buying the house 750,000 dollar, monetary received from sale a year later is 990,000 dollar that is C_t .

The Net Present Value (NPV)

Example 1:

- Siddharth bought a house for \$750,000 and sold it a year later for \$990,000 after deducting any realtor's fees and taxes.
- Calculate net present value if the rate of return is 5%
 - Investment on buying the house = \$750,000
 - Money received from sale a year later = $C_t = \$990,000$
 - Rate of return (r) = 5% = 0.05



The rate of return r is 5%. Now, we will use NPV formula.

$$NPV = \sum_{t=1}^n \frac{C_t}{(1+r)^t} - \text{Initial Investment}$$

What is C_t ? Cash value at the time period 1 plus rate of return to the power of time period. So, the cash value at time period that is after selling how much is getting 990,000 $\frac{990000}{(1+0.05)}$.

The Net Present Value (NPV)

Using net present value formula,

- Present value, $PV = \frac{C_t}{(1+r)^t} = \frac{\text{cash value at time period}}{(1+\text{rate of return})^{\text{time period}}}$
- $PV = \frac{\$990,000}{(1+0.05)^1}$



So, when you simplify that we are getting first present value. So, present value is 942,857 dollar that is a present value. Now, we have to find out the net present value. So, when you when you subtract from this present value minus over initial investment that is

750,000 dollar you are getting 192,857 dollar. Therefore, for 5% rate of return the investment has NPV of 192,857 dollar.

The Net Present Value (NPV)

- $PV = \$990,000/1.05$
- $PV = \$942,857.143$
- $\text{Net Present Value} = \$942,857.143 - \$750,000 = \$192,857.143$
- Therefore, for 5% rate of return, investment has $\text{NPV} = \$192,857.143$



More example which is related to the project. I have your project the cost of capital “r” for that form is 10%. The net present value of the proposal is like this. So, 0th year you are investing 1 million dollar. See, I have put that 1 million in the bracket that implies negative because that is a cash outflow which goes away from the pocket.

The Net Present Value (NPV)

Example 2:

- The cost of capital r , for the firm is 10 percent. The net present value of the proposal is:

Year	Cash Flow
0	₹(1,000,000)
1	200,000
2	200,000
3	300,000
4	300,000
5	350,000



So, I instead of using negative I put bracket sign. So, first year cash inflow is 200,000 dollar, second year 200,000, third year 300, fourth year 300,000 dollar and fifth year 350,000 dollar. Now, we are going to find out the net present value of this proposal. The NPV is because that 1 million is our initial investment. So, “-1000000” that minus sign is more important plus the cash inflows for next 5 years.

The Net Present Value (NPV)

$$\text{NPV} = -\frac{1,000,000}{(1.10)^0} + \frac{200,000}{(1 \cdot 10)^1} + \frac{200,000}{(1 \cdot 10)^2} + \frac{300,000}{(1.10)^3} + \frac{300,000}{(1 \cdot 10)^4} + \frac{350,000}{(1 \cdot 10)^5}$$
$$= -\text{₹}5,272$$

- The net present value represents the net benefit over and above the compensation for time and risk.

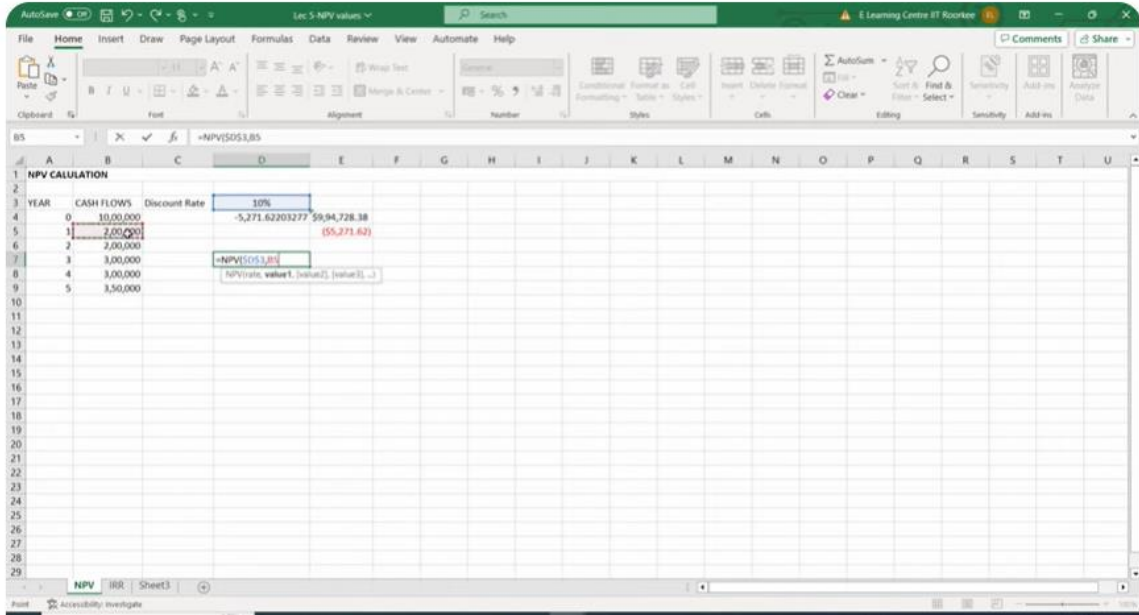


So, $200,000/(1.10)$, here r is 10%. The cost of capital is 10%. So, the net present value is minus 5,272 rupees. Here we are getting NPV is negative in the sense it is not a good proposal for us. So, the net present value represents the net benefit over and above the compensation for time and risk.

The screenshot shows an Excel spreadsheet with the following data:

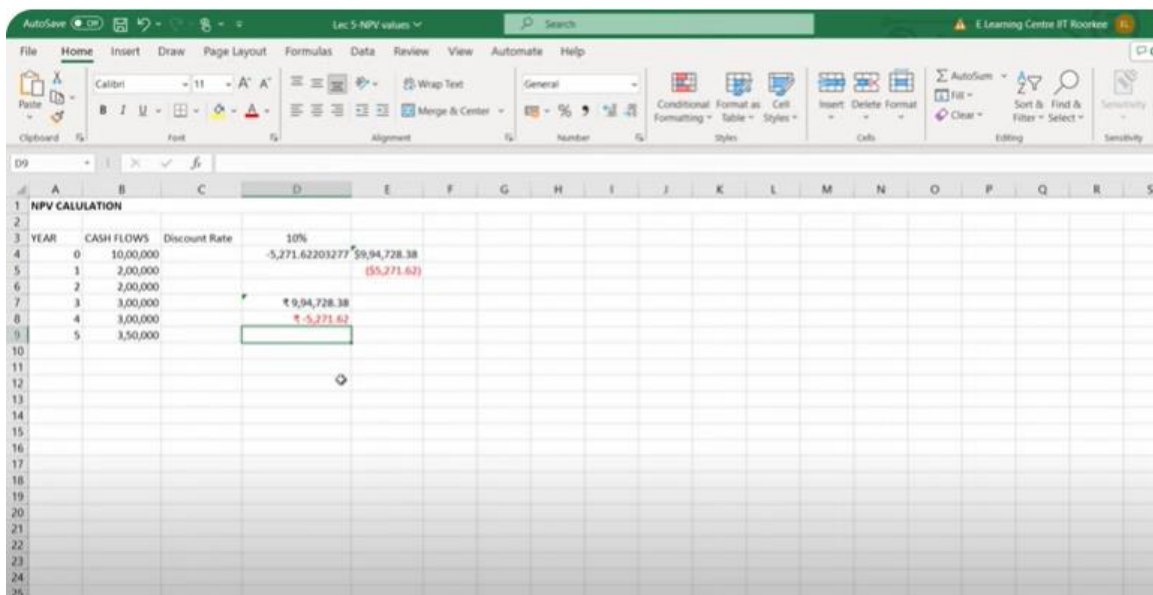
YEAR	CASH FLOWS	Discount Rate	
0	10,00,000	10%	-5,271.62203277
1	2,00,000		59,94,728.38
2	2,00,000		(55,271.62)
3	3,00,000		
4	3,00,000		
5	3,50,000		

So, we are getting negative value. So, this is not advisable proposal. Now, I will explain how to use Excel. So, in this problem it is given in the 0th year the investment is 1 million rupees. So, we are getting cash inflow for next 5 years 200,000, 200,000, 300,000, 300,000 again 350,000.



So, the discount rate is 10%. So, already I have calculated but I am going to redo it for your understanding. So, you type equal to sign. So, NPV so it is asking internal rate of return.

So, it is a 10%. Then you have to freeze this. So, F4, all the values. So, this is my present value. If I want to do the net present value, this value equal to this value minus the investment. So, it implies that the net present value is negative.



So, if it is a net present value is negative, we will not go for that proposal. So, the decision rule associated with net present value criterion is like this. Accept the project if

the net present value is positive. In our example, our net present value has come negative. So, we should not choose that proposal.

The Net Present Value (NPV)

Hence, the decision rule associated with the net present value criterion is:

- Accept the project if the net present value is positive
- Reject the project if the net present value is negative
- (if the net present value is zero, it is a matter of indifference)



The second reject the project if the net present value is negative. If the NPV is 0, it is a matter of indifferent. Next, we will go to another project selection model weighted scoring model is a very popular model. Consider buying a house as a project, you have different criteria by which you are going to make the decision. So, for buying a house, the location is one criteria that is the neighborhood and proximity to amenities, work and schools.

Weighted Scoring Model

Consider buying a house as a project and you have different criteria by which you are going to make a decision

- **Location:** The neighbourhood and proximity to amenities, work, and schools.
- **Price:** The cost of the home and associated expenses.
- **Size:** The square footage and number of rooms in the house.



Weighted Scoring Model

- **Safety:** The security of the area and crime rates.
- **Resale Value:** The potential for the home's value to appreciate over time.
- **School Quality:** The quality of schools in the area for families with children.



The price is another criteria, the cost of the home and associated expenses, size, the square footage and the number of rooms in the house is another criteria. Safety, the security of the area and the crime rate is one criteria. Then resale value, the potential for the homes value to appreciate over time. Then the school quality, the quality of schools in the area for the families with children. So, these are the some criteria for choosing a house.

Here choosing a house is a project. We can have some other criteria, this I have given only for the illustration purpose. So, we got the weightage from the expert for all six criteria. For example, location, the criteria is 10%, price 7%, size is 17% and safety is 12%, resale value 24%. You see the highest weightage is given by the expert is school quality that is a 30%.

Weighted Scoring Model

Following are the weights associated with the criteria

1. **Location** : 0.10
2. **Price** : 0.07
3. **Size** : 0.17
4. **Safety** : 0.12
5. **Resale Value** : 0.24
6. **School Quality**: 0.30



So, we have the criteria and the weightage and there are some alternatives by using this

criteria and their alternative, I am going to find the weighted sum. So, here there are five alternatives there. So, first step is multiplying the score for the new house with respect to criteria one that is a location. The score is 3, then multiplied by corresponding weightage.

Weighted Scoring Model

Alternatives	Criteria and weights						
	Location (0.10)	Price (0.07)	Size (0.17)	Safety (0.12)	Resale value (0.24)	School quality (0.30)	$\sum s_{ij}w_j$
New House 1	3*0.10 =0.30	1*0.07 =0.07	4*0.17 =0.68	2*0.12 =0.24	1*0.24 =0.24	3*0.30 =0.90	3.06
New House 2	3*0.10 =0.30	3*0.07 =0.21	2*0.17 =0.34	5*0.12 =0.60	4*0.24 =0.96	4*0.30 =1.20	3.73
New House 3	2*0.10 =0.20	1*0.07 =0.07	4*0.17 =0.68	4*0.12 =0.48	3*0.24 =0.72	3*0.30 =0.90	3.05
New House 4	5*0.10 =0.50	4*0.07 =0.28	3*0.17 =0.51	2*0.12 =0.24	2*0.24 =0.48	2*0.30 =0.60	2.61
New House 5	4*0.10 =0.40	5*0.07 =0.35	5*0.17 =0.85	2*0.12 =0.24	1*0.24 =0.24	5*0.30 =1.50	3.58



So, 3 multiplied by 0.10, it is 0.3. Then with respect to price, the criteria for the new house is 1. So, here, so this number is a score, for example, 3, 1, 4, 2, 1, 3.

Weighted Scoring Model

Alternatives	Criteria and weights						
	Location (0.10)	Price (0.07)	Size (0.17)	Safety (0.12)	Resale value (0.24)	School quality (0.30)	$\sum s_{ij}w_j$
New House 1	3*0.10 =0.30	1*0.07 =0.07	4*0.17 =0.68	2*0.12 =0.24	1*0.24 =0.24	3*0.30 =0.90	3.06
New House 2	3*0.10 =0.30	3*0.07 =0.21	2*0.17 =0.34	5*0.12 =0.60	4*0.24 =0.96	4*0.30 =1.20	3.73
New House 3	2*0.10 =0.20	1*0.07 =0.07	4*0.17 =0.68	4*0.12 =0.48	3*0.24 =0.72	3*0.30 =0.90	3.05
New House 4	5*0.10 =0.50	4*0.07 =0.28	3*0.17 =0.51	2*0.12 =0.24	2*0.24 =0.48	2*0.30 =0.60	2.61
New House 5	4*0.10 =0.40	5*0.07 =0.35	5*0.17 =0.85	2*0.12 =0.24	1*0.24 =0.24	5*0.30 =1.50	3.58



The other one is 0.10, 0.07, this is the weightage. So, I am multiplying the score with their corresponding weightage. Then I am summing it. So, when I sum it, it is 3.06 for first alternative house one. For second alternative, there is a score and multiplying by weight. So, I am getting 3.73.

Weighted Scoring Model

Alternatives	Criteria and weights						$\sum s_{ij}w_j$
	Location (0.10)	Price (0.07)	Size (0.17)	Safety (0.12)	Resale value (0.24)	School quality (0.30)	
New House 1	3*0.10 =0.30	1*0.07 =0.07	4*0.17 =0.68	2*0.12 =0.24	1*0.24 =0.24	3*0.30 =0.90	3.06
New House 2	3*0.10 =0.30	3*0.07 =0.21	2*0.17 =0.34	5*0.12 =0.60	4*0.24 =0.96	4*0.30 =1.20	<u>3.73</u>
New House 3	2*0.10 =0.20	1*0.07 =0.07	4*0.17 =0.68	4*0.12 =0.48	3*0.24 =0.72	3*0.30 =0.90	3.05
New House 4	5*0.10 =0.50	4*0.07 =0.28	3*0.17 =0.51	2*0.12 =0.24	2*0.24 =0.48	2*0.30 =0.60	2.61
New House 5	4*0.10 =0.40	5*0.07 =0.35	5*0.17 =0.85	2*0.12 =0.24	1*0.24 =0.24	5*0.30 =1.50	3.58



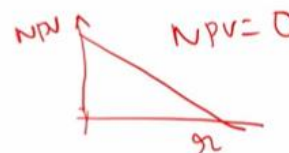
For third one, we are getting 3.05. For fourth one, it is 2.61. The fifth one is 3.58. So, now, we have to make the decision by looking at the highest weighted sum.

So, 3.73 is the highest weighted sum. So, we are finalizing that the alternative two that is a new house alternative two. So, this is the way to use weighted scoring model for choosing a project. The third example which I am going to discuss now is internal rate of return. So, there is a close connection between NPV and internal rate of return. Remember, in the NPV formula, it will be like this.

Internal Rate of Return (IRR)

- The internal rate of return (IRR) of a project is the discount rate, which makes its NPV equal to zero.
- Put differently, it is the discount rate which equates the present value of future cash flows with the initial investment.
- It is the value of 'r' in the following equation:

$$\text{Investment} = \sum_{t=1}^n \frac{C_t}{(1+r)^t}$$



where, C_t is the cash flow at the end of year t , 'r' is the internal rate of return (IRR), n is the life of the project.



So, what will happen? This in x-axis is the interest rate, y-axis is NPV. So, what will happen? It will go like this. When you increase the interest rate, the value of NPV will decrease. So, when you decrease the interest rate, value of NPV will 0, NPV will increase. So, we want to know a particular interest rate where your NPV is 0.

So, the interest rate at which the NPV is 0 is nothing but your internal rate of return. So, the internal rate of return of your project is the discount rate, which makes an NPV equal to 0. Put differently, it is a discount rate which equates the present value of future cash flows with the initial investment. It is the value of r in the following equation.

Investment is equal to your present value. So, what interest rate will bring your present value to the investment? So, that interest rate is nothing but internal rate of return. In this equation, C_t is the cash flow at the end of the year t , r is the internal rate of return. Here the objective is we have to find out what is the value of r . Otherwise, what is the interest rate that will make the present value is equal to 0, net present value is equal to 0. In the NPV calculation, we assume that the discount rate is known and we have determined the NPV.

Internal Rate of Return (IRR)

- In the NPV calculation we assume that the discount rate (cost of capital) is known and determine the NPV
- In the IRR calculation, we set the NPV equal to zero and determine the discount rate that satisfies this condition.



In the IRR calculation, we set the NPV equal to 0 and determine the discount rate that satisfy this condition. This is the connection between NPV and IRR. To illustrate the calculation of IRR, consider the cash flows of your project being considered by some company. So, the initial investment day is say 100,000 dollar and every year for next 4 years, we are getting cash inflow 30,000, 30,000, 40,000, 45,000. Now, we have to find out the internal rate of return.

Internal Rate of Return (IRR)

- To illustrate the calculation of IRR, consider the cash flows of a project being considered by Techtron Limited:

Year	0	1	2	3	4
Cash	100,000	30,000	30,000	40,000	45,000



The internal rate of return is the value of r which satisfy the following equation for the given problem. So, we know our initial investment is 100,000 and there is a cash inflow for next 4 years. So, in this equation, we have to find out the value of r . The calculation of r involves a process of trial and error, because it is a non-linear relationship. We try different values of r till we find the right hand side of the above equation is equal to 100,000.

Internal Rate of Return (IRR)

- The IRR is the value of “ r ” which satisfies the following equation:

$$100,000 = \frac{30,000}{(1+r)^1} + \frac{30,000}{(1+r)^2} + \frac{40,000}{(1+r)^3} + \frac{45,000}{(1+r)^4}$$

- The calculation of ‘ r ’ involves a process of trial and error
- We try different values of ‘ r ’ till we find that the right-hand side of the above equation is equal to 100,000
- Let us, to begin with, try ‘ r ’ = 15 per cent.

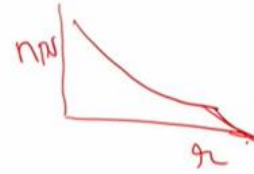


Randomly, let us try to begin with r equal to 15%. So, when I make r equal to 15%, when I find the present value, it is getting 100,801. It is just above the your 100,000. So, if you want to decrease this, as I told you, there is a interest rate and the NPV is having the inverse relationship.

Internal Rate of Return (IRR)

- try $r = 15$ percent. This makes the right-hand side equal to:

$$\frac{30,000}{(1.15)^1} + \frac{30,000}{(1.15)^2} + \frac{40,000}{(1.15)^3} + \frac{45,000}{(1.15)^4} = 100,801$$



So, it will be like this. So, where it is touching here. So, now we have to find out the r when NPV become 0. So, now, if I increase the r value, the NPV will decrease. So, what I am going to do from 15% I am going to for 16%.

Internal Rate of Return (IRR)

- This value is slightly higher than our target value of 100,000
- So, we increase the value of ' r ' from 15 per cent to 16 per cent
- (In general, a higher ' r ' lowers, and a smaller ' r ' increases the right-hand side value)



So, we increase the value of r from 15 to 16%. As I told you in general, the higher r lowers and the smaller r increases the right hand side value. So, when I use 16%, so I am getting 98,636. Now, we got an idea that interest rate is going to be between your 15 and 16%. Since the value is now less than 100,000, we conclude that the value of r lies between 15% and 16%.

Internal Rate of Return (IRR)

- try $r = 16$ percent. The right-hand side becomes:

$$\frac{30,000}{(1.16)^1} + \frac{30,000}{(1.16)^2} + \frac{40,000}{(1.16)^3} + \frac{45,000}{(1.16)^4} = \underline{98,636}$$



Many times this approximate value is enough. For most of the purposes, this indication is enough. But if you want to know an accurate value, there is one methodology that has to be followed. If a more refined estimate of r is needed, use the following procedure. Determine the net present value of the two closest rates of return.

Internal Rate of Return (IRR)

If a more refined estimate of ' r ' is needed, use the following procedure:

1. Determine the net present value of the two closest rates of return.
 - (NPV@15 percent) 801
 - (NPV@16 percent) -1,363
2. Find the sum of the absolute values of the net present values obtained in step 1

$$801 + 1,363 = 2,164$$



For example, find the NPV when interest rate is 15%. We are getting 801. Then find the NPV when the interest rate is 16%. We are getting minus 1363. Find the sum of absolute values of the net present value obtained in step 1.

That is sum of 801 plus 1363. We are getting 2164. Calculate the ratio of net present value of the smaller discount rate identified in step 1 to the sum obtained in the step 2. See the smaller discount rate. The smaller NPV is $801 / 2164$.

Internal Rate of Return (IRR)

3. Calculate the ratio of the net present value of the smaller discount rate, identified in step 1, to the sum obtained in step 2:

$$801 / 2,164 = 0.37$$

4. Add the number obtained in step 3 to the smaller discount rate:

$$15 + 0.37 = 15.37 \text{ percent}$$



When you simplify, we are getting 0.37. So add the number obtained in step 3 to the smallest discount rate. That is a 15% is the smallest discount rate plus 0.

37. So the refined value is 15.37. So the decision rule for IRR is as follows. Accept if the IRR is greater than the cost of capital. What is the meaning of cost of capital here? Suppose you are borrowing some money from banks, say 10%.

Internal Rate of Return (IRR)

The decision rule for IRR is as follows:

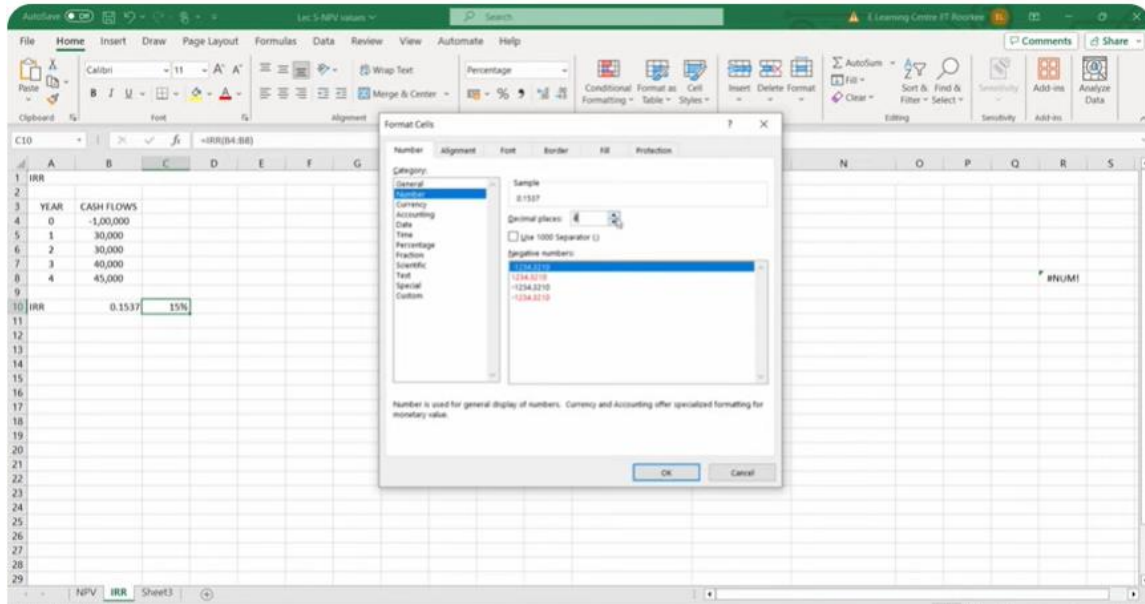
- **Accept** : If the IRR is greater than the cost of capital
- **Reject** : If the IRR is less than the cost of capital

$$\begin{aligned} &10\% \\ \text{IRR} &= 12\% \\ &= 9\% \end{aligned}$$



Your IRR is 12%. Then we can accept the project. If the IRR is less than 10%, for example 9%, we cannot accept the project. So reject if the IRR is less than the cost of capital. So these are the decision criteria for internal rate of return. Now I am going to explain with the help of Excel how to find out internal rate of return.

Now already I have calculated 15.37%. I am going to redo it for your understanding. So equal to IRR, internal rate of return. So select all the values from the 0 onwards.



So you are getting 15%. You have to make it more decimal. So here make it to decimal places 4. Yeah, you are getting 15.37%. So this is the way to use Excel to find out the internal rate of return. Another popular method, a multi-criteria decision making techniques for choosing a project is analytic hierarchy process.

Analytic Hierarchy Process (AHP)

- Analytic Hierarchy Process (AHP) is a Multi-Criteria Decision-Making Technique introduced by T. L. Saaty
- Different studies have been carried out in the past to determine the optimal size, location, and technology by using other MCDM methods



Analytic hierarchy process is a multi-criteria decision making techniques introduced by Professor T.L. Saaty. Different studies have been carried out in the past to determine the optimal size, location, technology for by using other multi-criteria decision making methods.

But this AHP is most popular and easy to follow. Now I have assumed that you have your project. So that project is selecting a car, easy project. You are going to use an AHP method to select a car by considering certain criteria. The advantage of AHP is we are trying to make a well-informed decision by analyzing different criteria in order to

compare three cars. So there are three alternative A, B, C and determine which is the best car for the selection.

AHP Example for selection of Car

- Suppose selecting a car is a project selection problem.
- Here, we are trying to make a well-informed decision by analyzing different criteria in order to compare three cars, A, B and C and determine which is the best car for selection



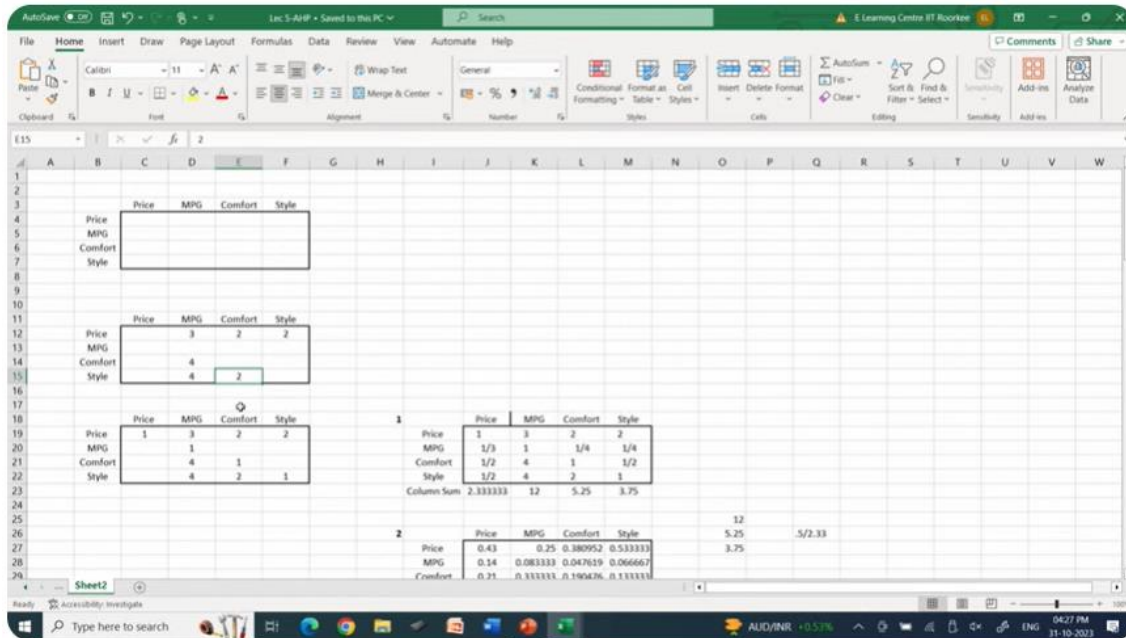
source: Meredith, J. R., Shafer, S. M., & Mantel Jr, S. J. (2017). *Project management: a strategic managerial approach*. John Wiley & Sons.

This methodology also for the purpose of illustration, you can take a real-time projects and we can have a criteria which is relevant to the project that can be used for choosing the project by using AHP. But here only for the explaining the methodology of AHP, I have taken a very simple example selecting a car as a project. I am going to explain this AHP methodology by using Excel. So for selecting the car, assume that there are four criteria. One is the price, another one is miles per gallon, the third one is comfort and style.

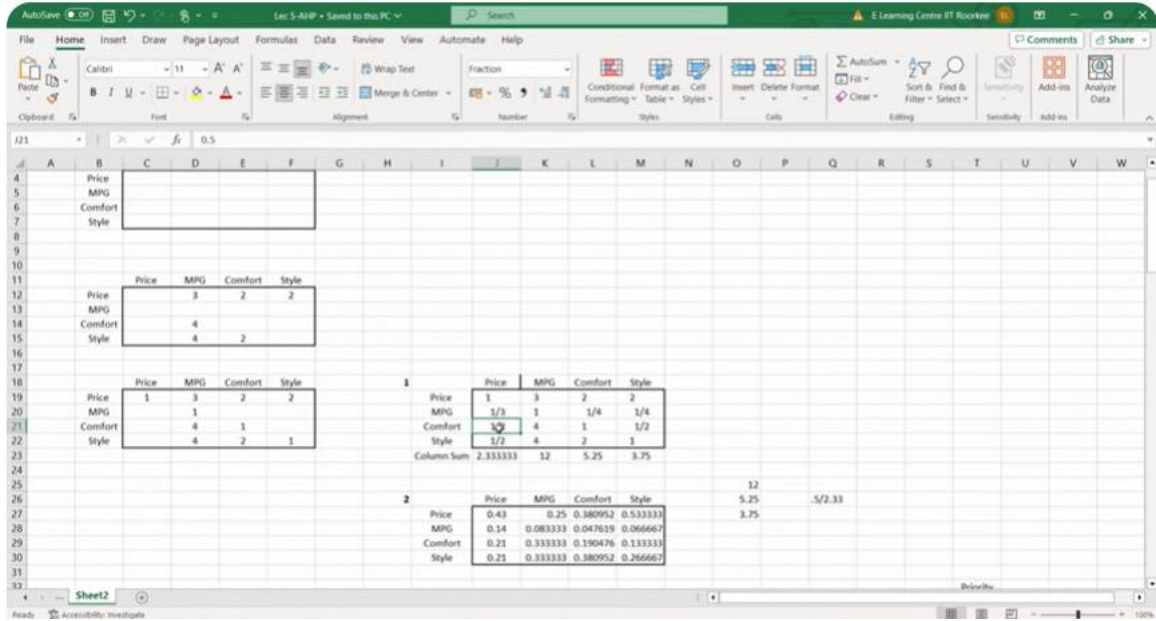
	Price	MPG	Comfort	Style
Price	1	3	2	2
MPG	1/3	1	1/4	1/4
Comfort	1/2	4	1	1/2
Style	1/2	4	2	1
Column Sum	2.333333	12	5.25	3.75
Row Sum	12	5.25	5/2.33	
Priority Vector	0.43	0.25	0.380952	0.533333
MPG	0.14	0.083333	0.047619	0.066667
Comfort	0.21	0.333333	0.190476	0.133333

The same value I have written in row-wise and column-wise. Now I am going to get some value pair-wise comparison value from the person who is willing to buy a car. So I

have entered three. So what is the meaning of this three is, if the person compares price versus miles per gallon, so the price is three times more important than the miles per gallon. For example, there is a four. What is it four implies? That comfort is four times more important than the miles per gallon.



For example, here two, style and comfort. So style is two times more important than the comfort. So this value is three, two, two, four, four, two, which I got from the experts, the person who is going to make the decision. You see the diagonal value, obviously it will be one, one. So after entering this, now you see, for example, when you compare price and miles per gallon, it is three.



If you compare miles per gallon versus price, it should be inverse of that three. So it will be one by three. So if you compare price versus comfort, it is two. If you compare comfort versus price, it should be one by two. Just the inverse of the upper triangle has to be written here in the bottom triangle. Now, after writing the inverse, then you will be filling the whole tables, the pairwise comparison.

First you have to do the column sum. For example, the price column, one, one by three, one by two, one by two. So first I have done the column sum. Similarly for miles per gallon, column sum.

	Price	MPG	Comfort	Style	Priority
1	1	3	2	2	0.40
MPG	1/3	1	1/4	1/4	0.09
Comfort	1/2	4	1	1/2	0.22
Style	1/2	4	2	1	0.30
Column Sum	2.333333	12	5.25	3.75	

Similarly for comfort, column sum. Similarly for style, column sum. The next step is each element, each element, for example, one has to be divided by the column sum. So if you divide one upon 2.33, you are getting 4.285, for example, 4.3, if you consider only two decimal. So everywhere we will consider only two decimal here. Okay. Similarly for the second column, 3 upon 12, 3 upon 12 is 1 by 4, it is 0.25. So, 1 upon 12, 0.08. One upon, 4 upon 12, one by three. So it is a 0.33. So each element has to be divided by its column sum, that is a step two. In step three, we are going to find out the row average. You see that this row is there. For example, price is one row.

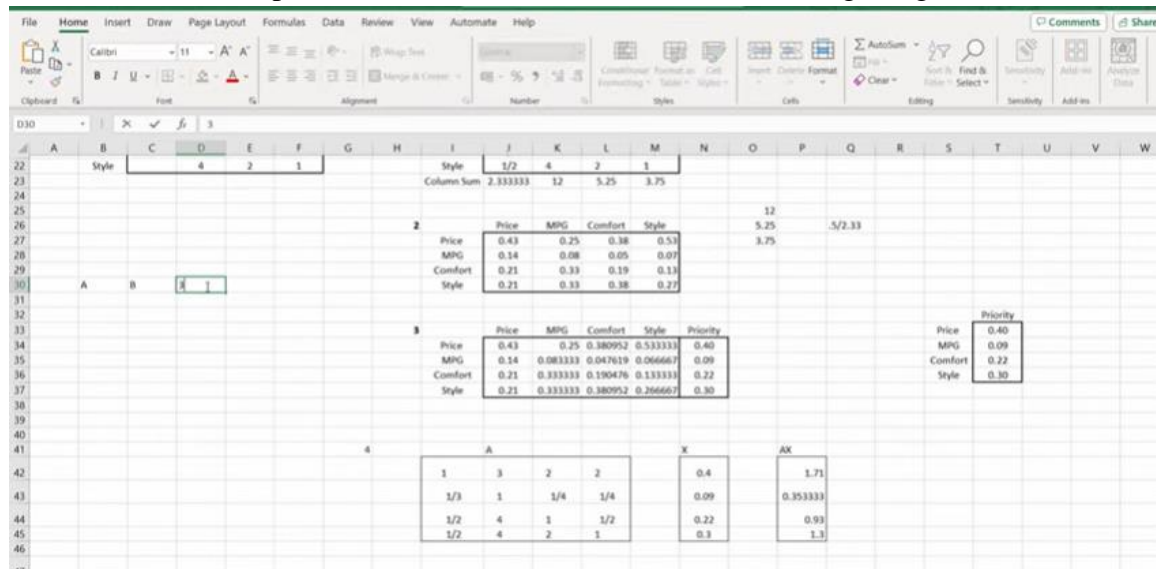
	Price	MPG	Comfort	Style	Priority
1	1	3	2	2	0.40
MPG	1/3	1	1/4	1/4	0.09
Comfort	1/2	4	1	1/2	0.22
Style	1/2	4	2	1	0.30
Column Sum	2.333333	12	5.25	3.75	

So I have found the average of this row, where the price is written. Similarly, the average of this miles per gallon.

So this average value represents the priority 0.4, 0.09, 0.22, 0.30. So what it implies, is

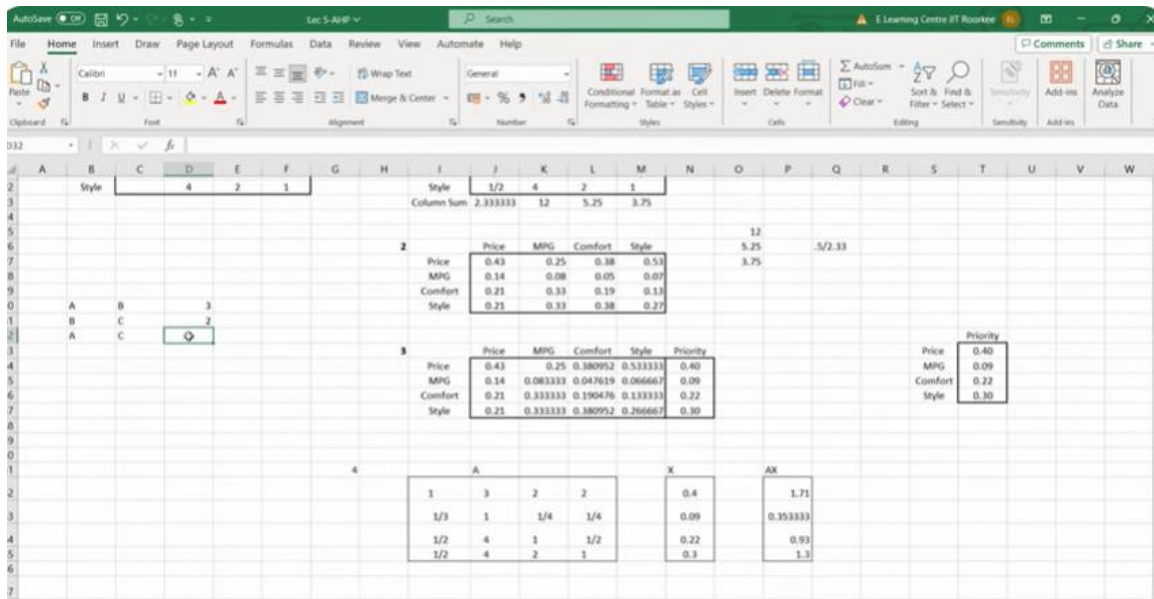
if I compare these four criteria, the highest weightage for criteria is given to the price, 40% weightage is given to the price. The next highest weightage is to style. The next highest weightage is comfort. The least weightage is only 9% which was given by the experts.

So this is a way to get the weightage for each criteria. Before proceeding further, we have to check a concept called consistency ratio. What is the meaning of consistency ratio? When the respondent is given some pairwise comparison value, we have to compare, we have to identify whether the respondent is given a consistent value or not. For example, A is one criteria and B is one criteria. Suppose when I compare A and B, for example, I am getting three.



When I compare A and three, so A is three times more important than B. Similarly, if I compare B and C, for example, when I compare B and C, I have written two. So B is two times more important than C. Suppose if you want to compare A and C, it should be greater than six because when I compare pairwise comparison A and B, A is three times more important. When I compare B and C, B is three times more important than C.

If I compare A and C, it should be six and above. It should be minimum six and above. In case the respondent is given weightage say five, so that means that the respondent is inconsistent in giving the value for the pairwise comparison. This is the meaning of this consistency that I am going to find out for each matrix, each pairwise comparison matrix. Here the pairwise comparison matrix is a criteria. We have considered four criteria.



So how to find out that pairwise comparison matrix? I go back to my presentation now. How to check the consistency? So consistency of data is checked according to the procedure suggested by TLCRT. The consistency check is done by performing consistency ratio check. How to find out consistency ratio? Consistency index upon random index. So the consistency ratio = (consistency index/ random index).

AHP Example for selection of Car

Check for consistency

- Consistency of data is checked according to the procedure suggested by T. L. Saaty
- The consistency check is done by performing the “consistency ratio check”

$$\text{Consistency ratio} = \frac{CI}{RI}$$

rc: Meredith, J. R., Shafex, S. M., & Mantel jr, S. I. (2017). *Project management: a strategic managerial approach*. John Wiley & Sons.

Random index is mean consistency indices for randomly generated pairwise comparison matrix. What is the random index? Random index is mean consistency indices which was obtained for a randomly generated pairwise comparison matrix. That value can be directly obtained from the table. But now how to find out consistency index? For that you see the formula = $\frac{(\lambda_{max}-n)}{n-1}$.

n is the size of the matrix, for here it is a 4. We have to find out the λ_{max} . So that λ_{max} I am going to calculate it. Please look at this. Look at this portion of the excel. A is the initial pairwise comparison matrix which was given by the respondent.

The screenshot shows an Excel spreadsheet with the following data:

	Price	MPG	Comfort	Style	Priority
Price	0.43	0.25	0.380952	0.533333	0.40
MPG	0.14	0.083333	0.047619	0.066667	0.09
Comfort	0.21	0.333333	0.190476	0.133333	0.22
Style	0.21	0.333333	0.380952	0.266667	0.30

	A	X	AX		
1	3	2	2	0.4	1.71
1/3	1	1/4	1/4	0.09	0.353333
1/2	4	1	1/2	0.22	0.93
1/2	4	2	1	0.3	1.3

4.275
3.925926
4.227273
4.333333
Average= 4.190383

X is the criteria of the weightage which we have obtained for each criteria. Then we have to multiply Ax. Ax is a matrix multiplication. So when you do the matrix multiplication excel there is a function M-M-U-L-T().

Similarly, you have to select cell matrix 1, matrix 2. Then we will get this value. Then so for example 0.4 upon 1.71 that is a P. So next we have to find out 1.71 that is a "P42" /N42 cells.

This screenshot shows the same Excel spreadsheet as above, but with the product matrix (AX) values updated to reflect the calculations:

	A	X	AX		
1	3	2	2	0.4	1.71
1/3	1	1/4	1/4	0.09	0.353333
1/2	4	1	1/2	0.22	0.93
1/2	4	2	1	0.3	1.3

4.275
3.925926
4.227273
4.333333
Average= 4.190383

Similarly, how we got 3.92, 0.3533 upon 0.79. Then 0.93 upon 0.22 we are getting 4.22. Then 1.3 upon 0.33 we are getting 4.33. So the average value of these four values is nothing but your lambda max. So the lambda max is 4.190. Now we will go back to my

presentation. We know that consistency ratio is (consistency index / random index). How to find out the consistency index by using $= \frac{(\lambda_{max}-n)}{n-1}$.. With the help of excel I have explained how to find out the lambda max. n is the size of the matrix. By using these values first we will find out consistency index. So consistency index is lambda max which I got from our excel minus 4 because our size of the matrix is 4 X 4 is 4 upon 3.

AHP Example for selection of Car

$$\text{Consistency ratio} = \frac{CI}{RI}$$

n	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49

- where CI is the consistency index $= \frac{(4.190383-4)}{(4-1)}$
 $CI = 0.063333$
- **Consistency ratio** $= \frac{0.063333}{0.90} = 0.07037$



Source: Meredith, J. R., Shafer, S. M., & Mantel Jr, S. J. (2017). *Project management: a strategic managerial approach*. John Wiley & Sons.

So consistency index is 0.063. Next we have to find out the random index. This value which you can directly get from the table but already I have explained the meaning of random index that is the average value of consistency for randomly generated pair wise comparison matrix. Here for 4 cross 4 matrix the value of random index is 0.90. So when you divide consistency index upon your 0.90 you are getting 0.07. This 0.07 is less than 10 % that is less than 0.10. We can conclude that that pair wise comparison matrix is consistent. Once we conclude that it is a consistent we can go for further calculation. If it is not consistent for example consistency ratio is more than 10 percentage then we have to ask the respondent to provide new pair wise comparison matrix values then we have to redo all our procedures what I have done it so far.

The screenshot shows an Excel spreadsheet with the following data:

Criteria	Alternative	A	B	C	Priority
1 Price	A	1	1/3	1/3	0.141667
	B	3	1	1/3	0.291667
	C	4.5	2	1	0.566667
1 Comfort	A	1	2	8	0.593432
	B	1/2	1	6	0.341161
	C	1/8	1/6	1	0.065407
1 Style	A	1	1/3	4	0.26638
	B	3	1	7	0.60251
	C	1/6	1/7	1	0.073909

So we got consistency ratio is 0.07 so we have accepted. Now we have to do the pair wise comparison of these three alternative with respect to each criteria. For example the price is one criteria there are three types of cars available A, B, C then we are doing pair wise comparison. After pair wise comparison we are finding the column sum then I am dividing each element by column sum then that is this element then I am finding the row average. So for alternative A with respect to price the weightage is 14 percentage for B it is 29 percentage for C it is 56 percentage. So what is the meaning of that? Suppose the price is only criteria the best alternative car is car C because it is a 56 percentage.

Similarly for comfort I am repeating the same procedure. What is that one? I have the pair wise comparison matrix I am finding the column sum then each element is divided by the column sum then I am finding the row average so I am getting 0.59, 0.34, 0.06 what it implies if the comfort is only criteria the best car option is car A.

The screenshot shows an Excel spreadsheet with the following data:

Criteria	Alternative	A	B	C	Priority
1 Price	A	1	1/3	1/3	0.141667
	B	3	1	1/3	0.291667
	C	4.5	2	1	0.566667
1 Style	A	1	1/3	4	0.26638
	B	3	1	7	0.60251
	C	1/6	1/7	1	0.073909
1 MPG	A	1	1/4	1/6	0.086948
	B	4	1	1/3	0.273718
	C	6	3	1	0.639335

Similarly miles per gallon if the miles per gallon is only criteria the best alternative is car C. If the style is only criteria the best alternative is car B. So I got priority of all the

alternatives for each criteria so that value I have brought it so with respect to price this is the criteria which I got it with respect to miles per gallon I got this criteria.

	A	B	C	Priority
A	0.090909	0.058824	0.111111	0.086948
B	0.363636	0.235294	0.222222	0.279718
C	0.545455	0.705882	0.666667	0.639335

	Price	MPG	Comfort	Style	Priority
A	0.141667	0.086948	0.599432	0.26638	0.40
B	0.291667	0.279718	0.341161	0.660251	0.09
C	0.566667	0.639335	0.065407	0.073369	0.22

Already here I know what is the criteria weight already we know the weightage for the our criteria so 0.4, 0.09. So when you do the matrix multiplication see that I have used M, M-U-L-T so I am getting this 0.27. So my final conclusion is that the car B that we are getting 41% that is the best alternative for selecting the car. So for this set of criteria for this set of pair wise comparison matrix so we are suggesting the car B is the best option for that respondent.

Summary

- The Net Present Value (NPV)
 - Calculation of NPV using Excel
- Weighted Scoring Model
- Internal Rate of Return (IRR)
 - Calculation of IRR using Excel
- Analytic Hierarchy Process (AHP)
 - Calculation of AHP using Excel



Dear students in this lecture I have discussed about different project selection methodology. First the technique which I have discussed is net present value then calculation of this net present value using excel then I have used weighted scoring model for selecting the project then we have discussed about internal rate of return then I have

discussed about calculation of internal rate of return using excel. Finally I have used a multi-criteria decision making technique called AHP then also I have explained calculation of AHP priority values using excel. Thank you.