Project Management

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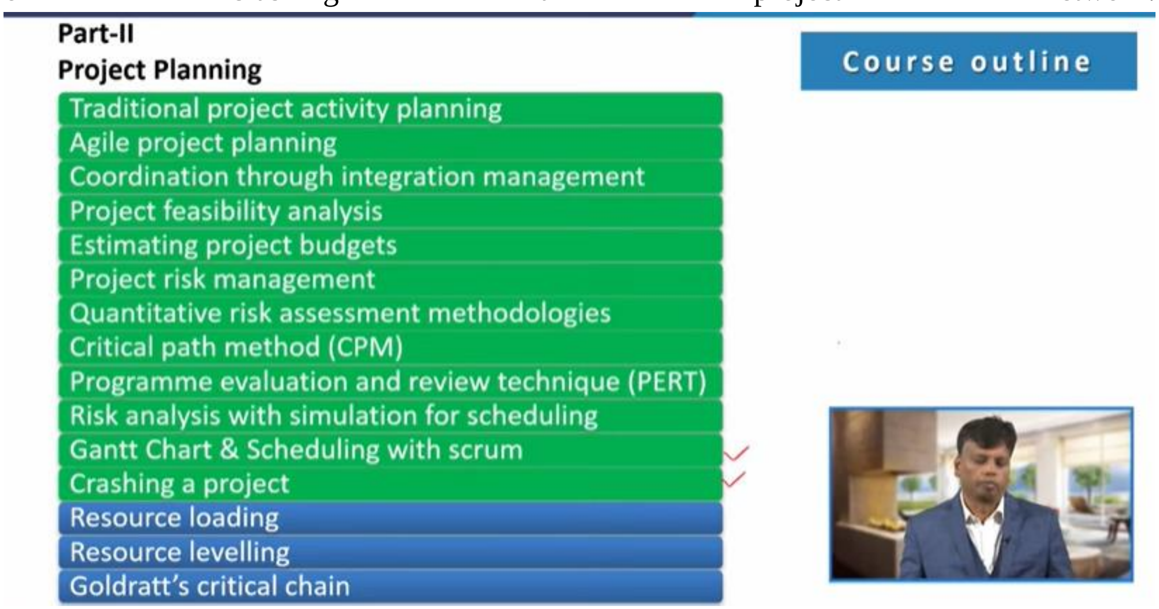
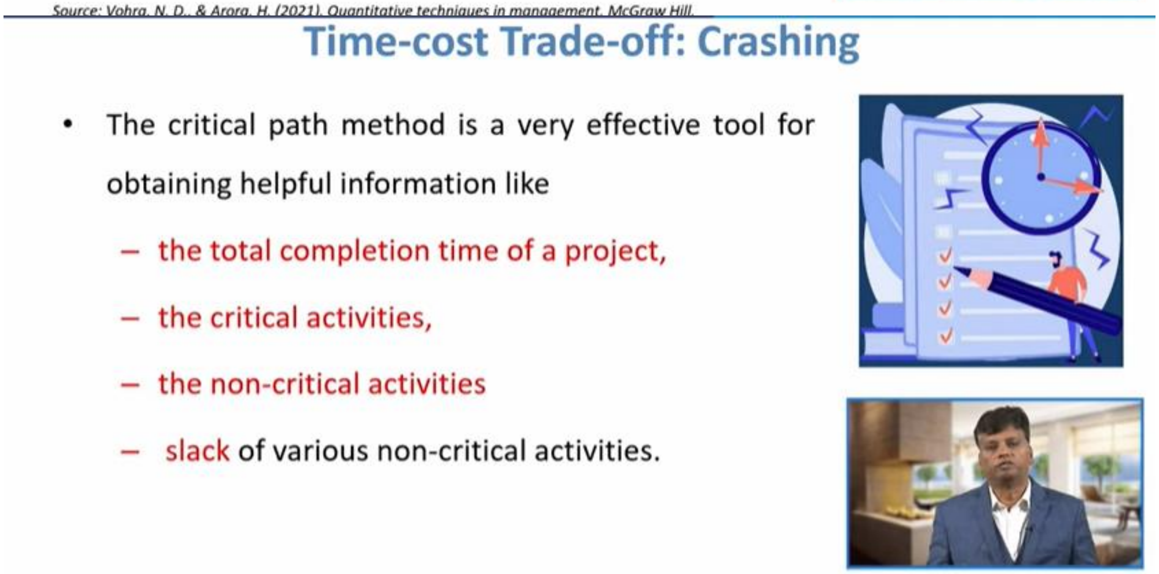
Week: 5

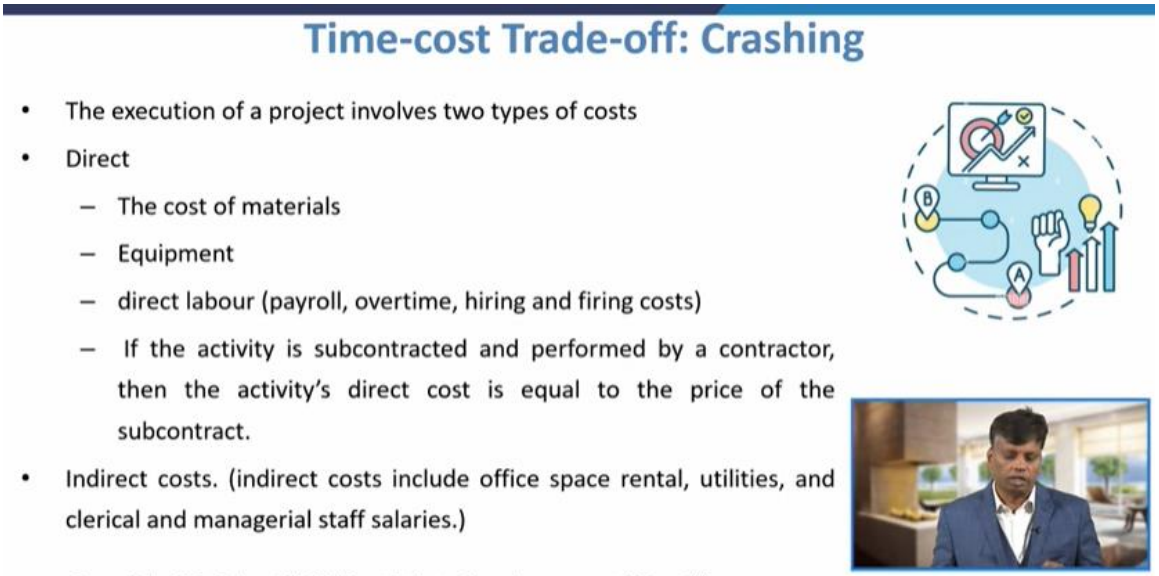
Lecture 24 Crashing a project

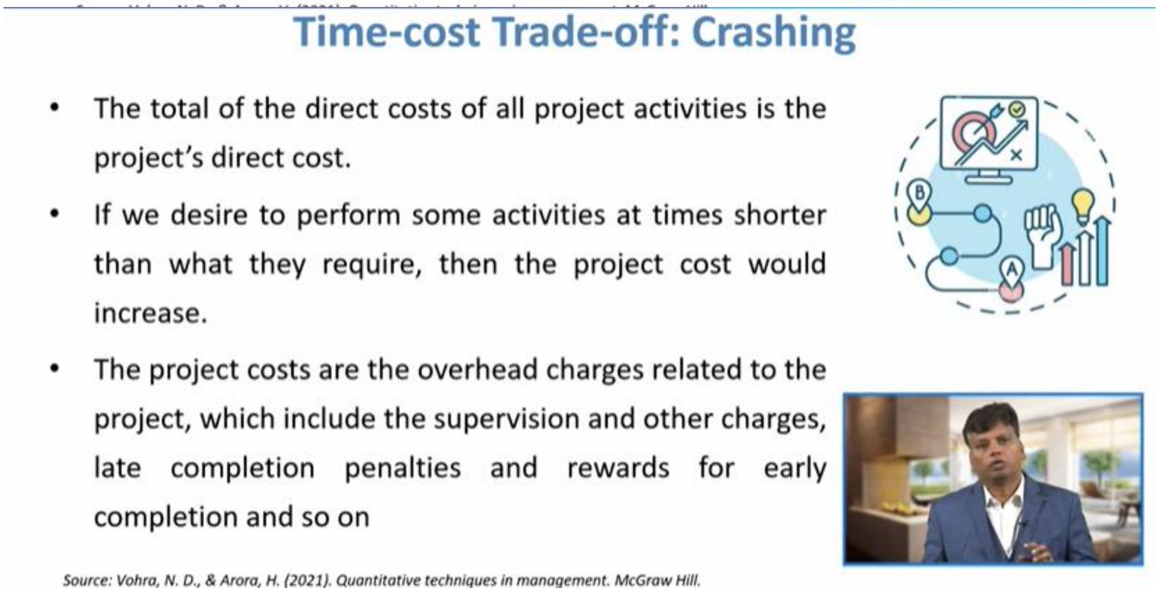
Dear students, in this lecture, I am going to explain about time cost trade-off in project management. Another name is called project crashing. In the previous lecture, I have discussed about GANTT chart and scheduling with Scrum. In this lecture, I am going to start a new topic called crashing a project. So, the agenda for this lecture is time cost trade-off called crashing, then what is the crash time and crash cost, then what is a normal cost, then there is a relationship between time cost relationship that we will discuss. I have taken a sample example, there I have explained step by step methodology of crashing a project network.



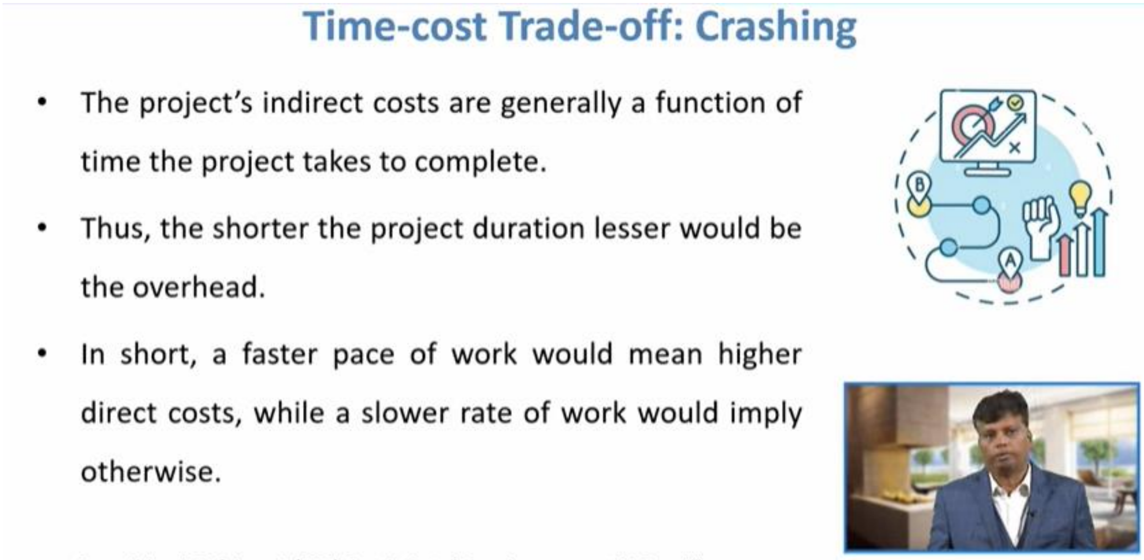


The critical path method is very effective tool for obtaining helpful information like total completion time of your project, critical activities and non-critical activities and slack of various non-critical activities. The advantage of knowing this information is, for example, from the first information, we know what is the total completion time, then we focus on what are the critical activities that has to be carefully managed, then what are the non-critical activities, in that activities, we can delay the time, we can utilize the slack. So, these are the advantages of answering this question. So, the execution of project involved two type of cost, one is direct cost, another one is indirect cost.

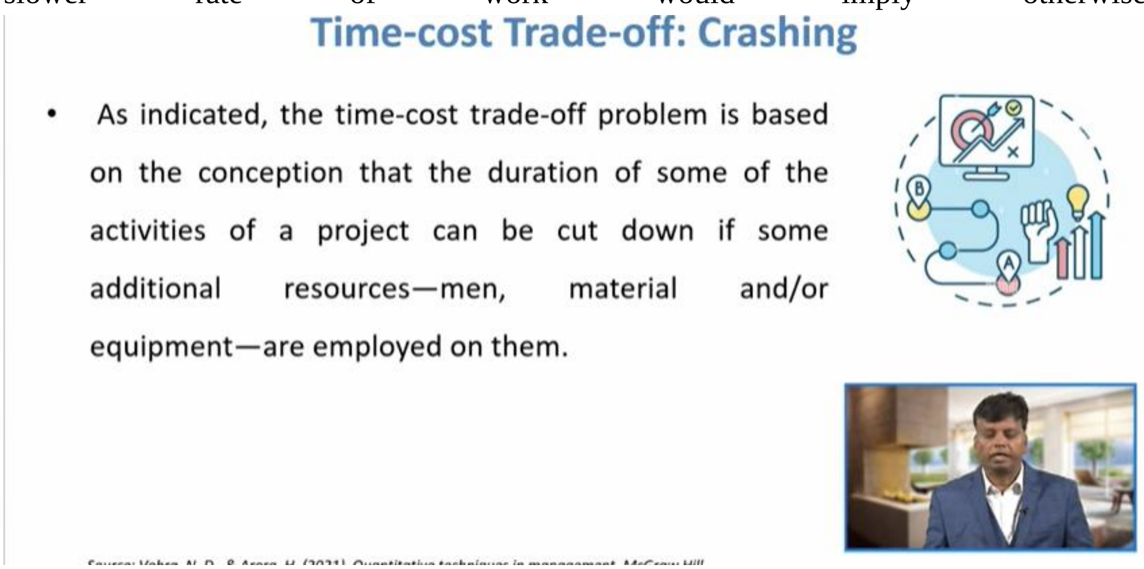


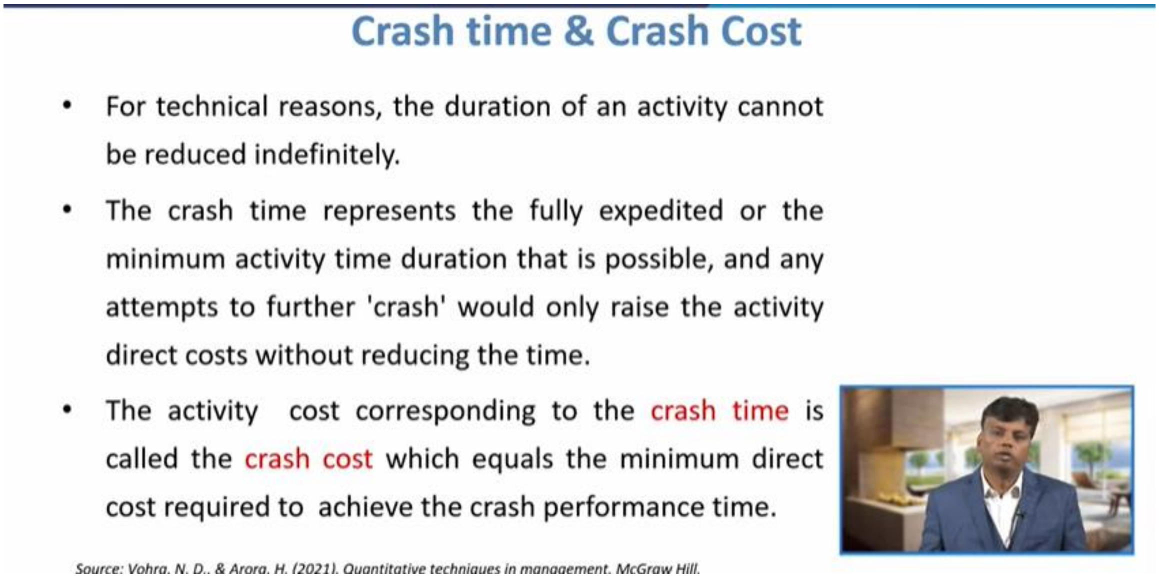


So, direct cost, which has directly is going to affect the project duration, for example, cost of materials, equipment, direct labor cost, sometime if the activity is subcontracted and performed by a contractor, then activities direct cost is equal to the price of subcontract. There is one more cost is involved in project is called indirect cost, that cost includes office space, rental, utilities and clerical and managerial stop salaries, there is not going to affect directly to the duration of the project. Now, we will talk about time cost trade off, the total cost, the total of the direct cost of all project activities is called project direct cost. So, in this lecture, what we are focusing is reducing the direct cost. So, if you desire to perform some activities at times shorter than what they require, then the project cost would increase, say the project is 10 days, if you want to compress the time to 5 days, so the cost of that activity will increase.

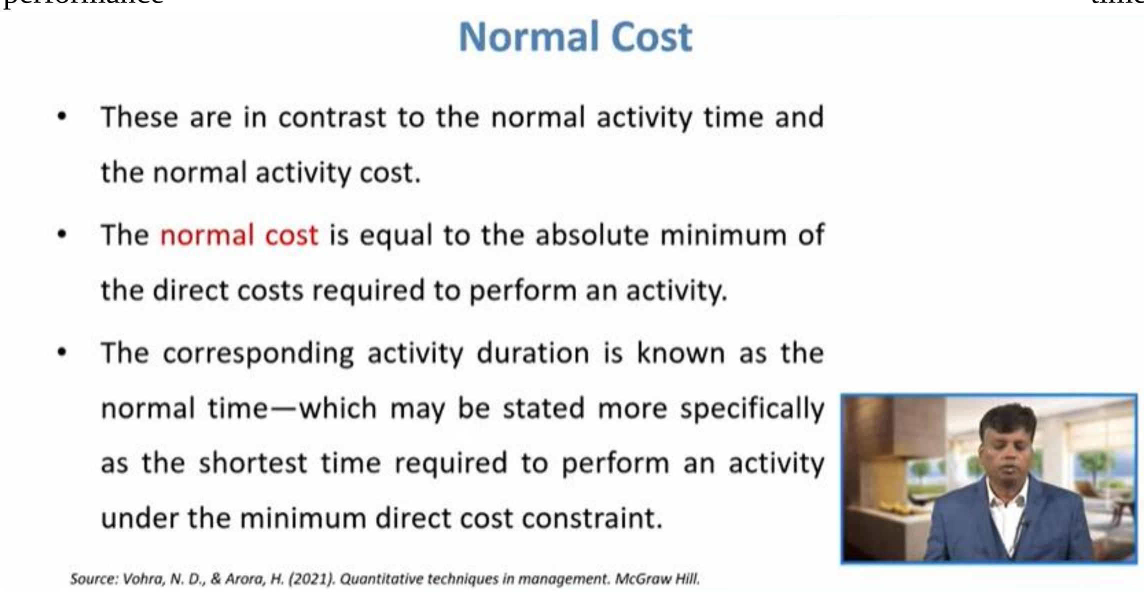


So, the project cost are the overhead charges related to the project, which include the supervision and other charges and late completion penalties and rewards for early completion and so on. So, there may be a possibility when you delay the project duration, there may be a penalty cost, when you do it early, you will get a reward cost, there in the sense doing at the right time as per the expectation of the client. So, these are the cost element that we need to consider. Now, the project's indirect cost generally a function of time of the project takes to complete, thus the shorter the project duration lesser would be the overhead. In short, the faster pace of work would mean higher direct cost, while slower rate of work would imply otherwise.



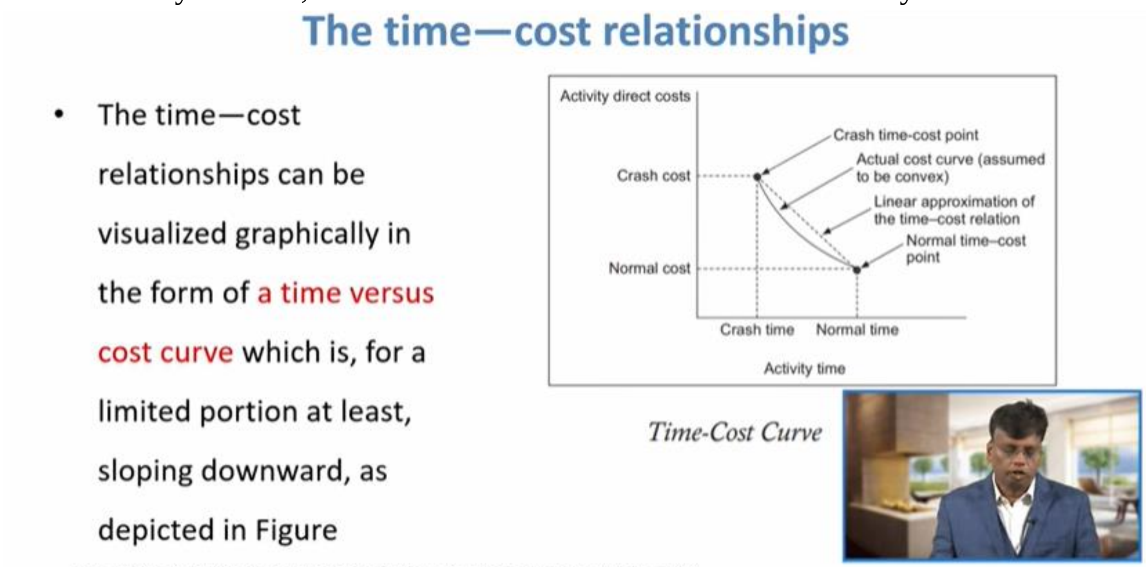


So, we have to consider when we consider project overall cost, you have to consider total direct cost and total indirect cost that will explain with the help of this one example, which I am going to explain after few minutes. So, as indicated the time cost trade off problem is based on the conception that the duration of some of the activities of a project can be cut down if some additional resources like men material and other equipment are employed on them. For technical reasons, the duration of an activity cannot be reduced indefinitely and there will be a partial reduction in the time also not possible. So, the crash time represents the fully expedited or minimum activity time duration that is possible and any attempts to further crash would only raise the activity direct cost without reducing the time. So, the activity cost corresponding to the crash time is called the crash cost, which equal to the minimum direct cost required to achieve the crash performance time.

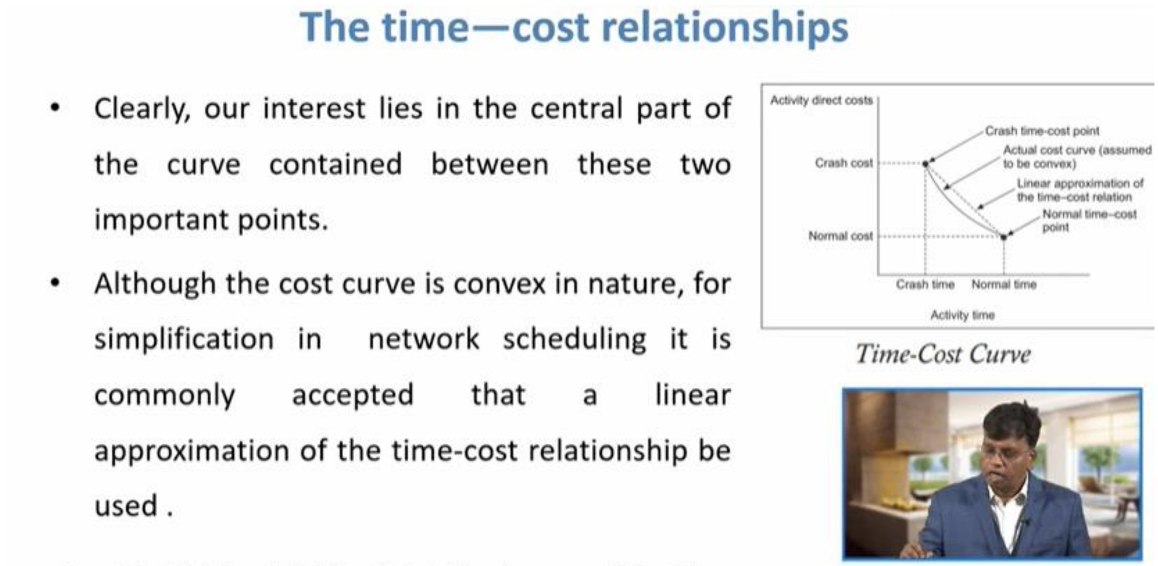


These are in contrast to the normal activity time and normal activity cost. So far, we discussed about the crash time and crash cost. Now, we will discuss about normal time and normal cost. So, the normal cost is equal to the absolute minimum of the direct cost required to perform an activity. So, the corresponding activity duration is known as normal time, which may be stated more specifically as the shortest time required to perform an activity under the minimum direct cost constraint that is the normal cost.

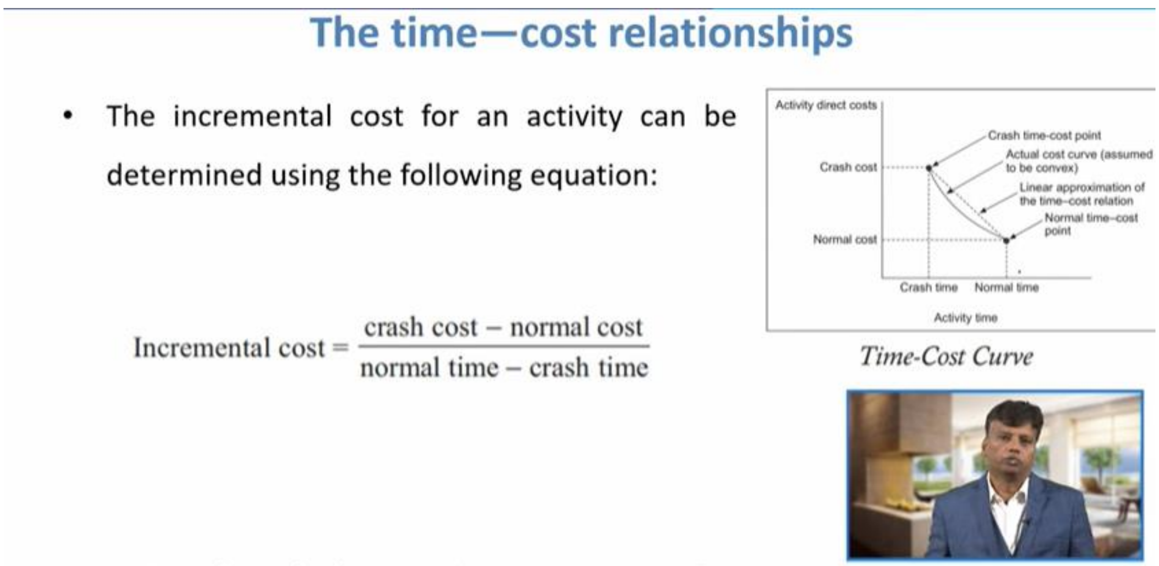
So, when you decrease that the cost will increase. Now, I have brought an example the time cost relationship. So, the time cost relationship can be visualized graphically in the form of time versus cost curve. Look at the figure on right hand side. So, here we have the activity time, here we have the activity direct cost.



So, this point look at this point, this is the normal time and normal cost and this point is crash time that is a smaller than the normal time and higher than the normal cost. So, generally the actual cost curve will be the convex, but we are approximating to linear form just for our convenient because our concern is only these two points not along the path for our simplification. So, this time versus cost curve for a limited portion of portion at least sloping downwards as shown in the figure. So, the widely accepted convex shape of the curve between the crash time and the cost, cost point and the normal time and cost point indicates that it is marginally costlier to induce the last percentage of reduction in activity time duration than the first percentages. So, what will happen? So, in this you see these point it is marginally costlier to induce the last percentage of reduction of activity.



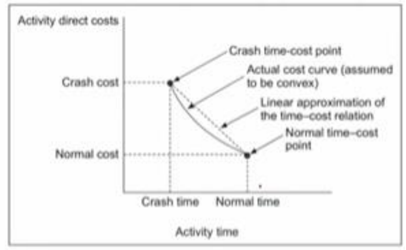
So, what is the if you are trying to reduce the time on this point, the cost will be high. Clearly our nearest interest lies in the central part of the curve contained between these two important point. One is the normal time and cost, crash time and crash cost. Although the cost curve is a convex in nature for simplification then in the network scheduling it is commonly accepted that the linear approximation of the time cost relationship is used. This is shown in the figure by two point approximation from the normal time cost point to the crash time cost point, the points being shown joined by the dotted line.



So, the linear approximation represents a linear incremental cost per unit of time saved under conditions of crashing. So, the incremental cost of an activity can be determined using the following equation. So, incremental cost is difference in the crash cost and normal cost. Similarly, the difference in normal time and crash time. For example, if the crash cost and time are 700 and 8 days, if the normal cost and time are 500 and 12 days.

The time—cost relationships

- For an activity, if the crash cost and time are ₹ 700 and 8 days, and if the normal cost and time are ₹ 500 and 12 days, we have



Time-Cost Curve

$$\text{Incremental cost per day} = \frac{700-500}{12-8} = \frac{200}{4} = ₹\ 50$$

So, what is the difference in cost 700 minus 500, difference in time normal time minus crash time 12 minus 8 so 4, 200 upon 4. So, we are getting 50 rupees. This indicates if any activity if you reduce that duration of that activity by one day, the cost will increase by 50 rupees. So, this is the implication of this incremental cost per day. So, I have taken a sample problem.

An Example for Crashing

- Table shows for each activity needed to complete the project the normal time, the shortest time in which the activity can completed of a building contract and the cost per day for reducing the time of each activity.

Activity	Normal Time in Days	Shortest Time in Days	Cost of Reduction per Day (₹)
1–2	6	4	80
1–3	8	4	90
1–4	5	3	30
2–4	3	3	–
2–5	5	3	40
3–6	12	8	200
4–6	8	5	50
5–6	6	6	–

So, in that there are activities there. The example is taken from Arora, “quantitative techniques in management”. So, the table shows for each activity needed to complete the project normal time, shortest time which the activity can be completed of building a contract and cost per day for reducing the time of each activity. So, the activity is given here. The normal time is given and shortest time that is the crash time also given the cost per reduction per day.

An Example for Crashing

- Table shows for each activity needed to complete the project the normal time, the shortest time in which the activity can completed of a building contract and the cost per day for reducing the time of each activity.

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1–4	5	3	30
2–4	3	3	–
2–5	5	3	40
3–6	12	8	200
4–6	8	5	50
5–6	6	6	–

Source: Vohra, N. D., & Arora, H. (2021). Quantitative techniques in management. McGraw Hill

So, that is the slope of that curve is given directly here. In case if it is not given you have to find out but in this problem it is given to you. So, what it means that activity 1 to 2 you can do it 6 days but when you reduce it to 4 days. So, the per day cost increase will be 80 rupees that is the meaning of this cost of reduction per day. Now, for this problem I am going to explain the concept of crashing.

An Example for Crashing

- The contract includes a penalty clause of per day over 17 days.
- The overhead cost per day is ₹ 160.
- The cost of completing the eight activities in normal time is ₹ 6,500

So, the other information related to the problem is the contract includes a penalty clause of per day over 17 days. If you completed the project beyond 17 days for each day there is a cost, there is a penalty. So, the overhead cost per day is 160 rupees. The cost of completing the 8 activities in normal time is 6500. So, because they have to provide normal cost and normal time and crash cost and crash time but in this problem the slope of for each activity is already given to you.

So, that is why the cost of normal cost also given as directly as a 6500. Otherwise you have to add for all activities what are the normal cost the sum of that cost is directly given here as 6500. So, what is required? Calculate the normal duration of the project, its cost and the critical path. Next, calculate and plot the graph cost time function for the project and state the lowest cost and associated time and the shortest time and the associated cost. So, this I am going to explain.

Questions

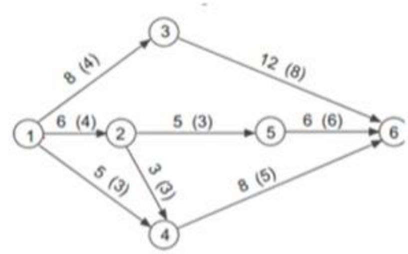
- Calculate the normal duration of the project, its cost and the critical path.
- Calculate and plot on a graph the cost/time function for the project and state:
  - the lowest cost and associated time.
  - the shortest time and associated cost.

Source: Vohra, N. D., & Arora, H. (2021). Quantitative techniques in management. McGraw Hill.

The network is drawn and shown in the figure where the normal and the shortest time for various activities are given. So, here so activity 1, 2, 3 the normal time is 8, the maximum allowable time is 4 days. For example, activity 1, 2, 2 normal time is 6 days and maximum. So, this says maximum time for crashing. Now, when I look at this figure, now we have to find out critical path.

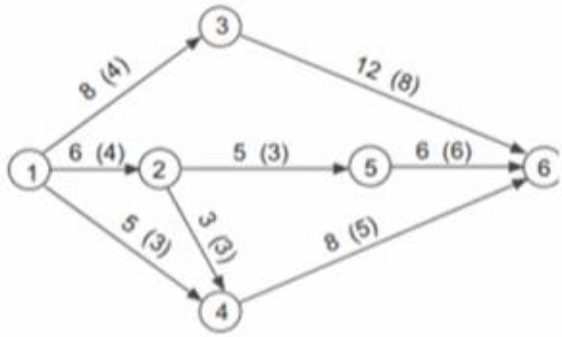
An Example for Crashing

- The network is drawn and shown in Figure, where the normal and the shortest (crash) times for various activities are given.



Source: Vohra, N. D., & Arora, H. (2021). Quantitative techniques in management. McGraw Hill.

An Example for Crashing



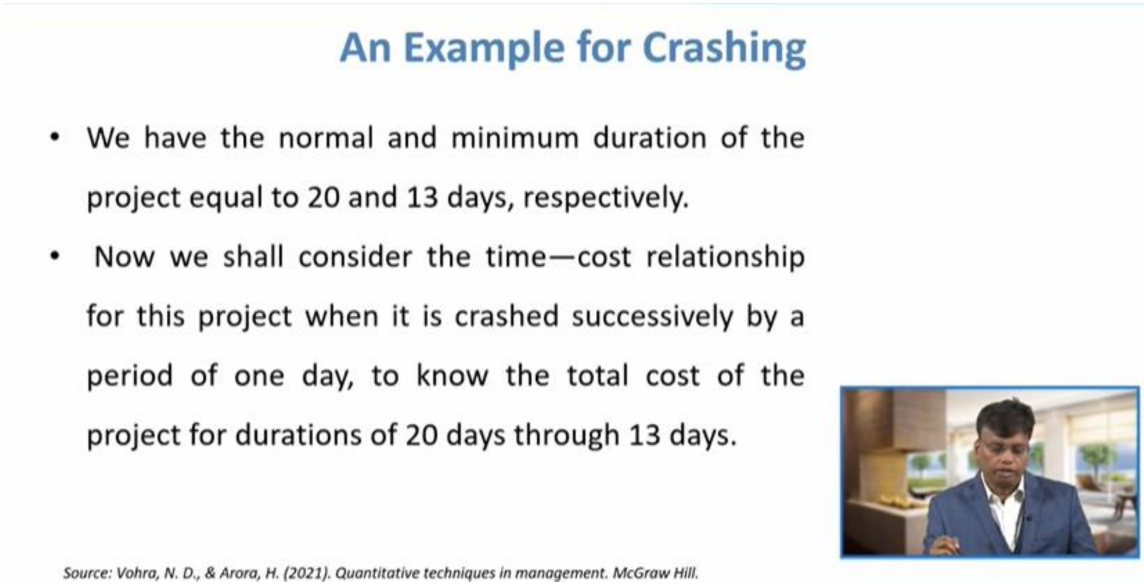
Path	*Normal Length*	*Crash Length*
1–3–6	20 (critical)	12
1–4–6	13	8
1–2–4–6	17	12
1–2–5–6	17	13 (critical)

Source: Vohra, N. D., & Arora, H. (2021). Quantitative techniques in management. McGraw Hill.

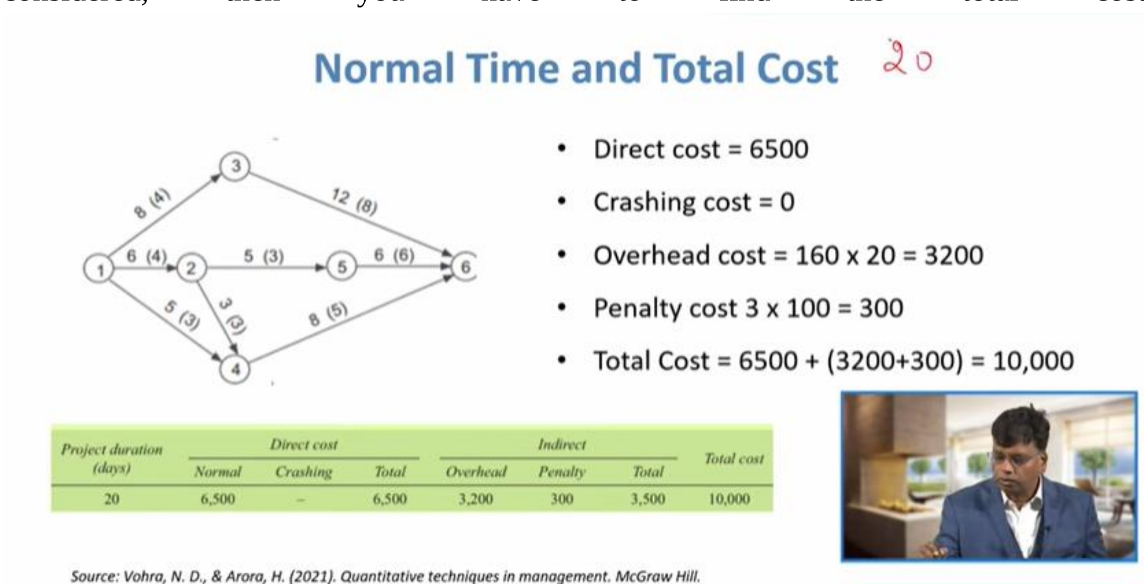
So, the critical path actually you have to find out the earliest start time, earliest finishing time, latest start time, latest finishing time that way you can do that. But at present since the network is very small, we can see all possibilities 1, 2, 6. So, 1, 3, 6 is one possibility is there, 1, 4, 6 is one possibility is there, then 1, 2, 4, 6 is one possibility is there, then 1, 2, 5, 6 is another possibility is there. So, for normal time say 1, 3, 6 is 20. So, the largest path is this 1, 3, 6.

So, that is our critical path. Suppose it is in the crash length. So, you see this activity if you start with 4 days, it will become 4 plus 8, 12. So, 1, 4, 6. So, this will be 8, then 1, 2, 4, 6, this is 12, so here it is 13. So, the maximum crash length is 13.

So, what we are understanding from here, so from 20 days, this project can be reduced up to 13 days if you are crashing it. So, we have to find out the cost for what is optimal cost for each day, that is what you are going to do that. So, if it is 20 days, we are going to find out what is the total completion cost. If it is 13 days, we are going to find out what is the total completion cost. So, the first step is we have the normal and minimum duration of the project equal to 20 and 13 days respectively.



Now, we shall consider the time cost relationship for this project, when it is crashed successively by period of 1 day and to know the total cost of project for duration of 20 days through 13 days. So, without doing crashing, first we will see if it is a 20 days, now we know that the critical path is 20 days that is 1, 3, 6. So, without doing before crashing, let us find out what is the total cost. So, 20 days, see this table, a project duration is given, direct cost, there is a normal cost, crashing cost, when I add it, you will get the total cost. For indirect cost, it is per day cost is given, then penalty also has to be considered, then you have to find the total cost.

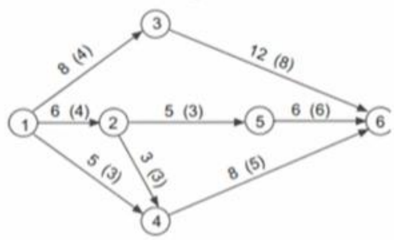


Project duration (days)	Direct cost			Indirect			Total cost
	Normal	Crashing	Total	Overhead	Penalty	Total	
20	6,500	–	6,500	3,200	300	3,500	10,000

So, this is a grand, that is overall total. So, the summation of total direct cost and total indirect cost. So, for this figure, you will find out what is the direct cost. So, direct cost is given here, we are not at all crashing 0, what is the overhead cost? So, per day it is 160 rupees, it is given in the problem. So, if you are making this project for 20 days, it will be 20, this is given to us, 20 X160, so 3200 and if the project is exceeding more than 17 days, because it is 20 days, so there will be a penalty cost is equal to 3 multiplied by 100, per day penalty is 100, so 300.

Crashing-The First Crashing

From 20 days to 19 Days



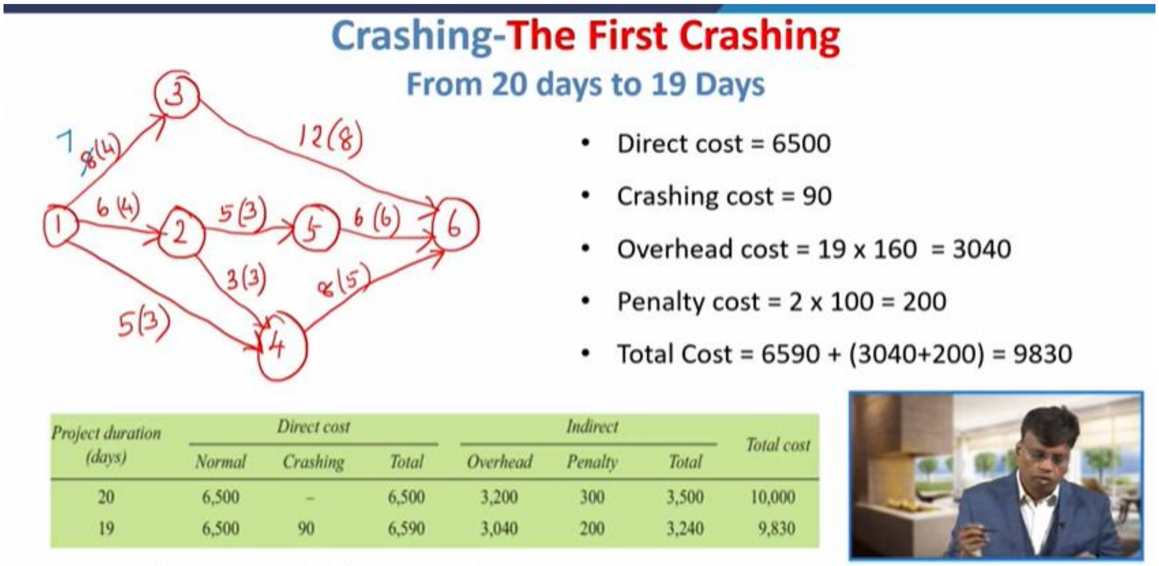
- To begin with, the critical activities are 1—3 and 3—6 for which the cost of reduction per day is 90 and 200.
- We would decide to crash the activity 1—3. Crashing it by a day, the project length is reduced to 19 days and the total cost equals 9,830.

Project duration (days)	*Direct cost*			*Indirect*			*Total cost*
	Normal	*Crashing*	*Total*	*Overhead*	*Penalty*	*Total*	
20	6,500	–	6,500	3,200	300	3,500	10,000
19	6,500	90	6,590	3,040	200	3,240	9,830

Source: Vohra, N. D., & Arora, H. (2021). Quantitative techniques in management. McGraw Hill.

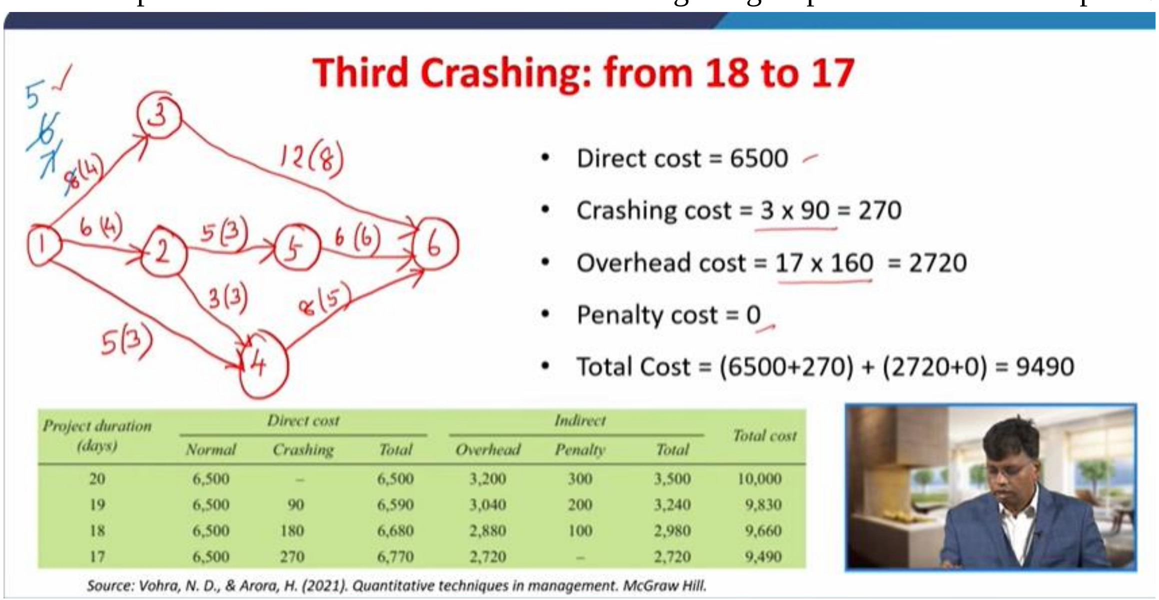
So, the overall cost is total direct cost is 6500 plus overhead cost is indirect cost is 3200 plus 300, so it is a 10000. So, if you complete this project without crashing and it will take 20 days, the corresponding cost is 10000. Now, we will start with the crashing. Now, the first step is from 20 days to 19 days. So, to begin with, first you have to look out the critical activities.

So, the critical activities are 1, 3 and 3, 6 this activity and the cost of reduction per day, 90 rupees for 1, 2, 3 and 3, 2, 6 it is a 200 rupees. So, we would decide to crash the activity 1, 2, 3, so crashing it by a day, the project length is reduced to 19 days and the total cost is equal to 9000 into 130. How it has come? Suppose if it is 19 days, so normal cost is 6500, so per day reduction for 1, 2, 3 is 90 rupees, so the total direct cost is 6590. Then what is indirect cost? 19 multiplied by per day overhead cost, so that will be this much and it goes beyond 17, so 2 days more per day 100, 2 day 200, when you add it, it will be 3240. So, when you add this total direct cost and total indirect cost, we are getting 9830.



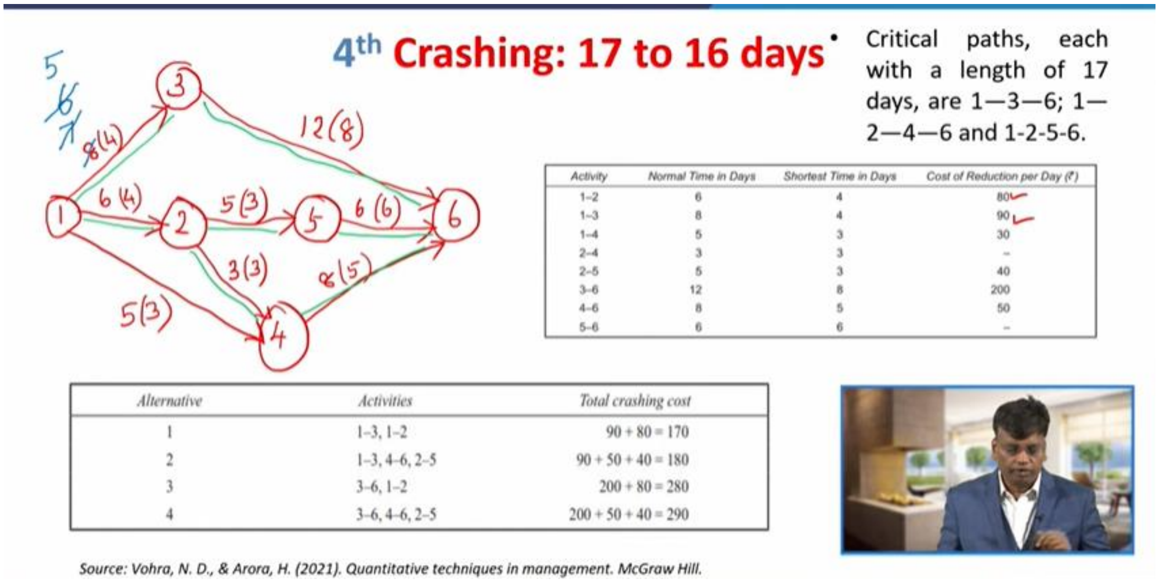
Project duration (days)	Direct cost			Indirect			Total cost
	Normal	Crashing	Total	Overhead	Penalty	Total	
20	6,500	–	6,500	3,200	300	3,500	10,000
19	6,500	90	6,590	3,040	200	3,240	9,830

This I have explained, what I have done it? So, I have possibility 1, 2, 3, 3, 2, 6, but the cost of reducing per day 1, 2, 3 is lower, so we have started to reduce from 8 days to 7 days. So, the calculation says this crashing cost, overhead cost, penalty cost, when you add it, it is giving 9830. Now, we will go to second crashing, that is a crashing from 19 to 18 days. So, what you will do? Again, see that again we are reducing 7 to 6, so what will happen? Direct cost is 6500, now we have reduced 2 days, so 2 multiplied by 90, 180 and now overhead cost is per day 160, so 18 days, 18 multiplied by 160, 2880 and penalty cost is because it is 18 days more than 17, so 1 multiplied by 100, it is 100, so the total cost is 9630. What is the point here is that you have to keep on reducing that critical path until otherwise we are getting parallel critical paths.

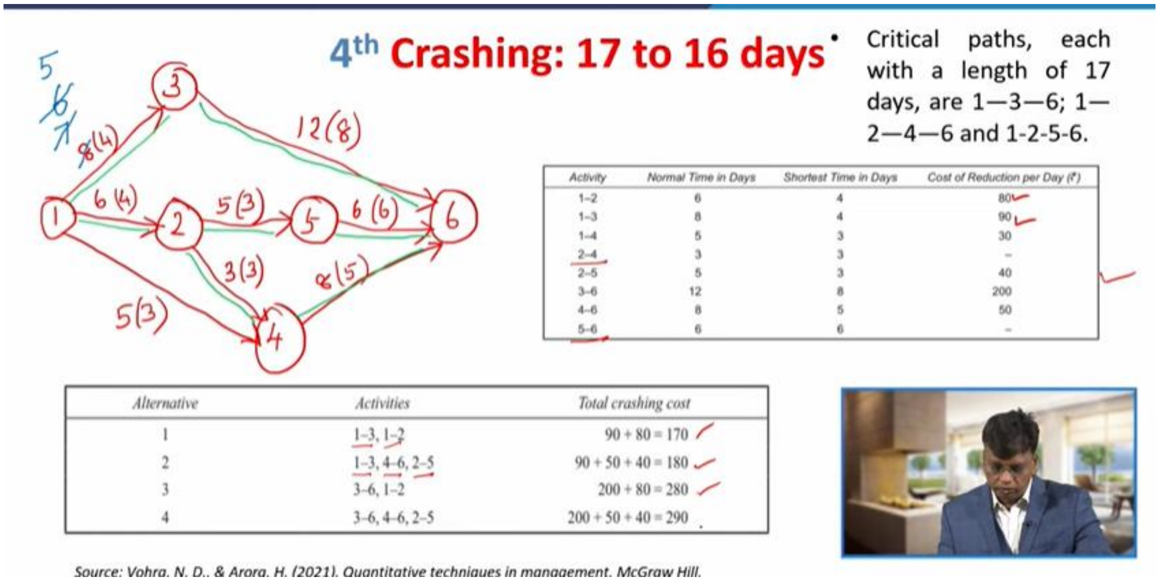


Project duration (days)	Direct cost			Indirect			Total cost
	Normal	Crashing	Total	Overhead	Penalty	Total	
20	6,500	–	6,500	3,200	300	3,500	10,000
19	6,500	90	6,590	3,040	200	3,240	9,830
18	6,500	180	6,680	2,880	100	2,980	9,660
17	6,500	270	6,770	2,720	–	2,720	9,490

So, at present still 1, 3, 6 is a critical path. Now, we will go for 18 to 17 days, third crashing. Again from here see that 5 to 6, I have reduced it, so now from 8 to 5, 3 days I have reduced, so direct cost will be as it is per day crashing is 90, so 3 days 270, overhead cost is now it becomes 17 days. Now, there is no penalty cost because it is exactly at 17 days, so the total cost is, so this one total direct cost plus total indirect cost, so we are getting 9490. You see that when we reduce the duration, the cost is decreasing, so that means it is giving an indication further we can reduce the cost.



Now, we will go to 17 days to 16 days that is the fourth crashing. Now, look at this figure now, so when we do in the third crashing itself, there are 3 critical path has come, one is 1, 3, 6, 17 days, then 1, 2, 4, 6, 17 days, then 1, 2, 5, 6, 17 days. Now, we have to see all possibilities along these 3 path for choosing which path has to be chosen for crashing. So, the first alternative is we can reduce 1, 3 by 1 day and 1, 2 by 1 day, so what will happen now 1, 3, 6 and what will happen when you reduce this one, so what will happen all these 3 path will be reduced to 16 days, so what will happen this will be 5, 5, 10, 16, 5 plus 8, 8, 16, so that is one option. Another option is reduce 1, 3, then we can reduce 4, 6, then we reduce 2, 5, remember that we cannot do reduction in 2, 4 and 5, 6 because 2, 4's normal time also 3 days, crash time also 3 days.

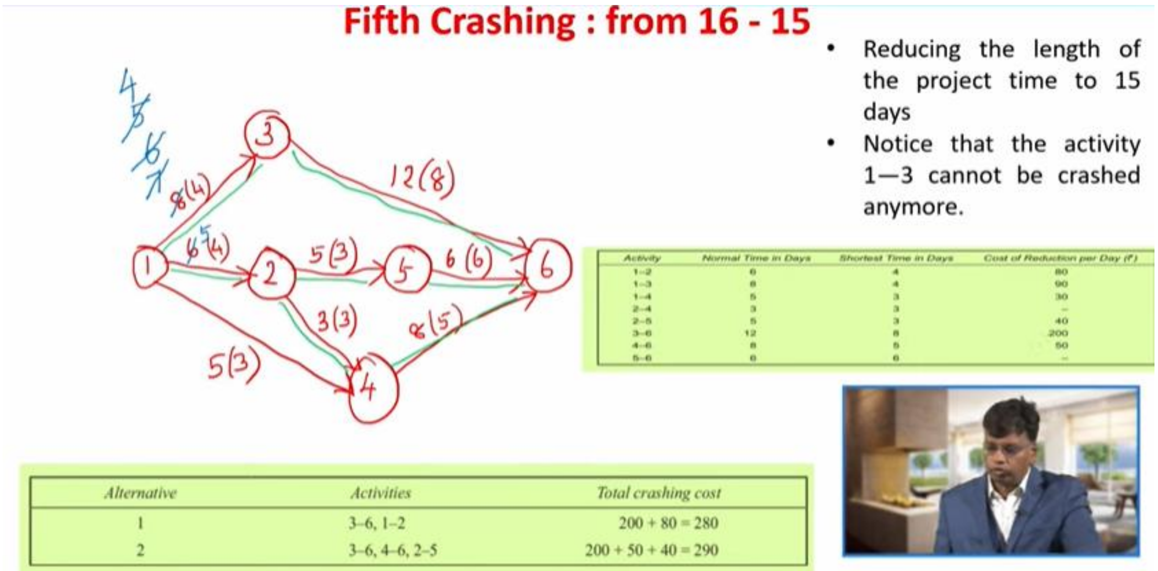


Activity	Normal Time in Days	Shortest Time in Days	Cost of Reduction per Day (₹)
1–2	6	4	80
1–3	8	4	90
1–4	5	3	30
2–4	3	3	–
2–5	5	3	40
3–6	12	8	200
4–6	8	5	50
5–6	6	6	–

Alternative	Activities	Total crashing cost
1	1–3, 1–2	90 + 80 = 170
2	1–3, 4–6, 2–5	90 + 50 + 40 = 180
3	3–6, 1–2	200 + 80 = 280
4	3–6, 4–6, 2–5	200 + 50 + 40 = 290

Similarly, for 5, 6 also normal time also 6 days, crash time also 6 days, so this is the second possibility. The third possibility is we can reduce 3, 6 by 1 day, then 1, 2 by 1 day. The fourth possibility is 3, 6 by 1 day, then 4, 6 by 1 day, then 2, 5 by 1 day, so all these alternatives we have to find out the total crashing cost. So, what will happen the first alternative 90 plus 80, 170, in the second alternative 90 plus 50 plus 40, 180, so this value I got from this table, then third alternative 3, 6 and 1, 2 you get 280, so fourth alternative we are getting 290. So, what is happening, first time see 1, 3, I am seeing all other possibilities, so there is a 2 possibility we got it, then first time we reduce 3, 6, then you see all other possibilities, I got another 2 possibilities, so I got 4 possibilities.

I am seeing the cost, total crashing cost all possibilities, the lowest one is the first option. So, what I am going to do, I am going to choose the first option reducing, so 5 in mid to 4, then 6 here you see 5 to 4, then 6 to 5, then what will be total direct, direct cost there would not be change, so the crashing cost is, so from 8 days I have reduced to 4 days, so 4 X 90, then here per day 1 to 2, per day crashing cost is 80, so again I am adding 80. So, overall crashing cost is 440, overhead cost is 16 days, per day 160, 2, 5, 6, 0, so the total cost is total direct cost plus total indirect cost I am getting 9500, you see that now the cost has increased, so that implies that the optimal duration for this project is 17. But we are going to give you a whole picture of the cost time relationship up to 13 days for your managers. Even though we got the result but still we go for one more crashing for 15 days.



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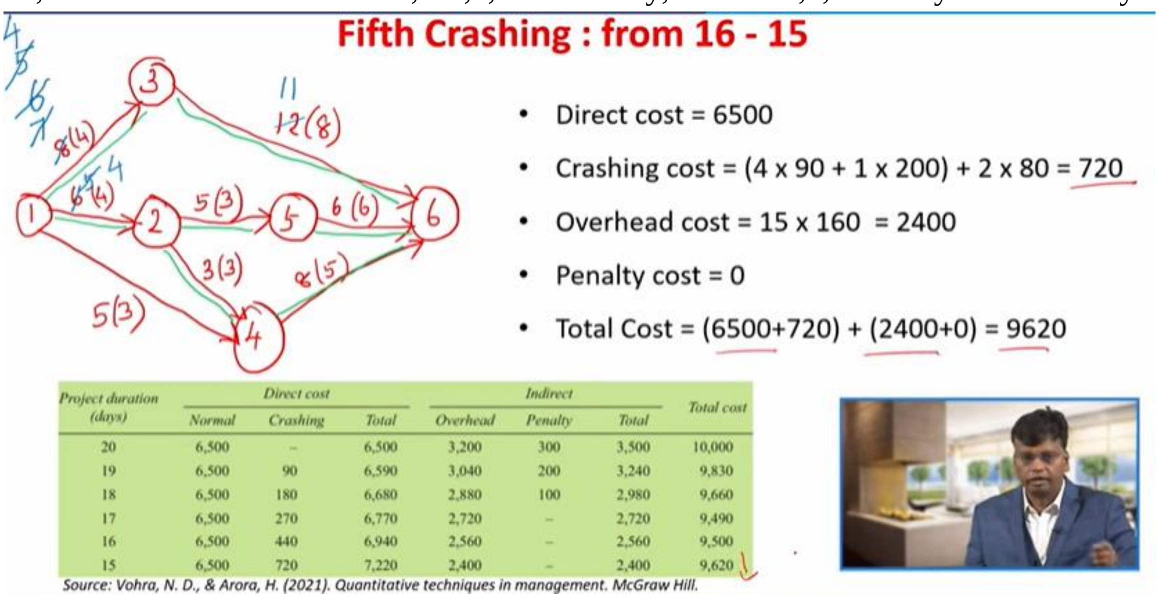
1–4	5	3	30
2–4	3	3	–
2–5	5	3	40
3–6	12	8	200
4–6	8	5	50
5–6	6	6	–

Alternative	Activities	Total crashing cost
1	3–6, 1–2	200 + 80 = 280
2	3–6, 4–6, 2–5	200 + 50 + 40 = 290

Now what will happen, if you want to do reducing the length of the project time to 15 days, you notice that activity 13 cannot be further reduced because we have consumed all 4. Now what are the other possibilities, 3,6,1,2 we can reduce, then 3,6,4,6 we can reduce, 2,5 you can reduce, because 5,6 we cannot reduce. So, the first alternative 3,6 per day 200, 12 per day 800, so 280, the second alternative 3,6 200, 4,6 500, this I got from this table, then 2,5 is 40, so total 290, so which is the lowest one, the first alternative. So, what I am going to do, I am going to use the first alternative 36 one day, 12, so what I have done, 3,2,6 one day, then 1,2, 5 days to 4 days.

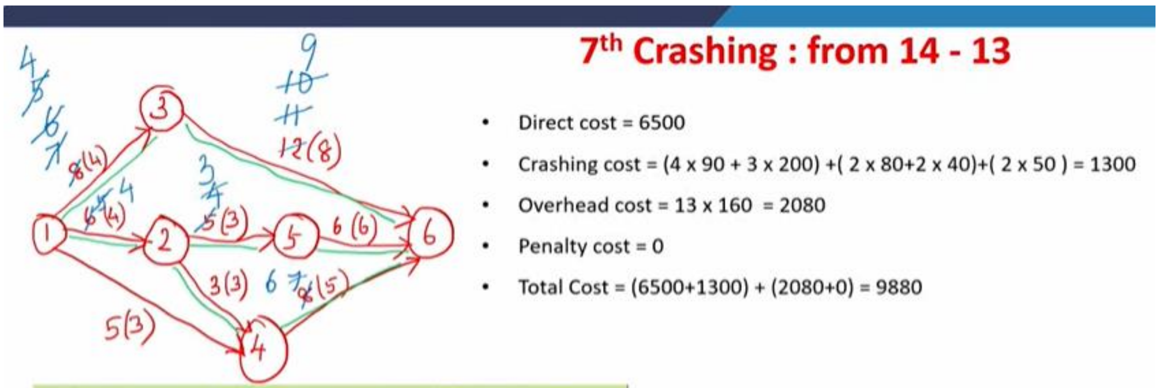


Project duration (days)	Direct cost			Indirect			Total cost
	Normal	Crashing	Total	Overhead	Penalty	Total	
20	6,500	–	6,500	3,200	300	3,500	10,000
19	6,500	90	6,590	3,040	200	3,240	9,830
18	6,500	180	6,680	2,880	100	2,980	9,660
17	6,500	270	6,770	2,720	–	2,720	9,490
16	6,500	440	6,940	2,560	–	2,560	9,500
15	6,500	720	7,220	2,400	–	2,400	9,620

Source: Vohra, N. D., & Arora, H. (2021). Quantitative techniques in management. McGraw Hill.

You see further 12 days we cannot reduce. Now I am again going to find the total cost. So, what I have done, the 1,3 already 4 multiplied by 90, then one day I have crashed 12 to 11, so 1 into 200 and 1 to 2 I reduced by 2 days, 6,4, 280, so crashing cost is 720, so overhead cost is 15160, the total direct cost and total indirect cost is 9620, you see that again the cost is increasing. Now we will go to 14 days. Here also if you want to do 14 days, so 1,3 is not possible, 1,2 also not possible, so what we can see, one day we can reduce on 3 to 6, so it will become 10, 14 and 2 to 5 we can reduce one day, so it will become 4+4=8, plus 6, 14. Another option in 4 to 6 we can reduce by 1 day, so 7 plus 3, 10 plus 4, 14.

So, now we will find the direct cost as it is, 1,3 I reduced by 4 days, 3,6 by 2 days, so what happened this much amount. The next 1,2 I reduced by 2 days, then 2,5 reduced by 1 day and 4 to 6 here bracket is required by 1 day, so the total cost is 1010. Now it is a 14 days, we will get the corresponding overhead cost. So, when you add it we are getting 9550. So, again we can go up to 13, now we will go for reducing to 13 days.



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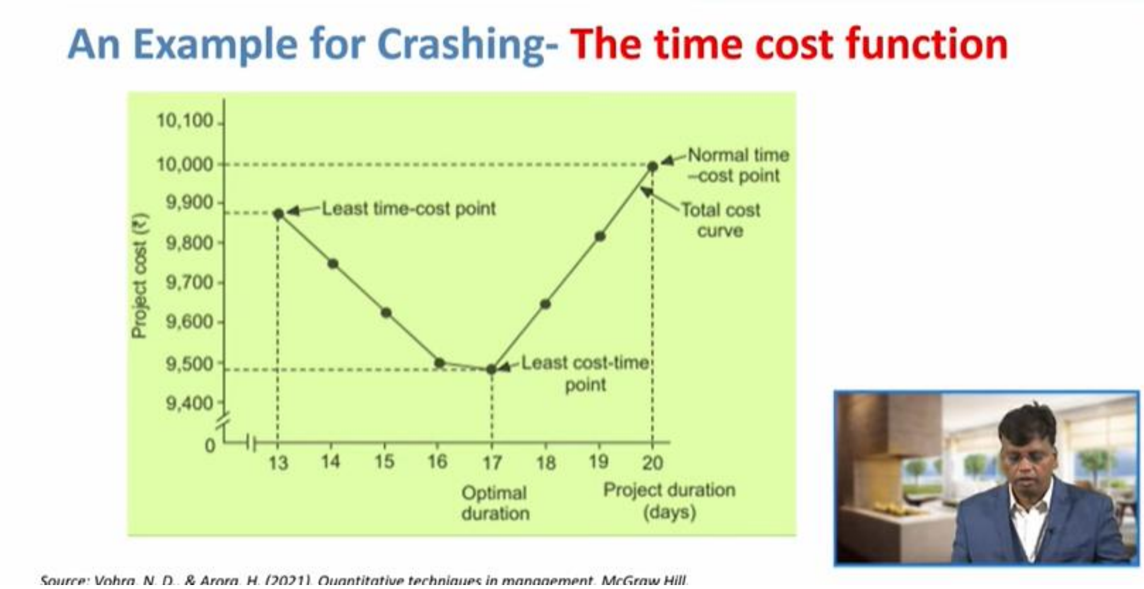
16	6,500	440	6,940	2,560	-	2,560	9,500
15	6,500	720	7,220	2,400	-	2,400	9,620
14	6,500	1,010	7,510	2,240	-	2,240	9,750
13	6,500	1,300	7,800	2,080	-	2,080	9,880

Source: Vohra, N. D., & Arora, H. (2021). Quantitative techniques in management. McGraw Hill

Now here for each alternatives, for each path we can reduce by 1 day. So, what we can do, 3 to 6 we can reduce by 1 day, 9 it will be 13, then 2 to 5 from 4 we can reduce to 3, so 7 plus 6, 13, then 4 to 6 from 7 we can reduce to 6, 6 plus 3, 9 plus 4, 13. So, how we got this cost, 1,3 cost and 3,6 cost. Similarly, this 1,2 cost, then 2 to 5 cost, then 4 to 6 cost, so we got 1300.



So, overhead cost is 13, so 2080, the overall cost is 9880. So, from 20 days we have reduced to 13 days, for each day reduction we found all possible total cost. So, when you plot it, this is very useful graph for our project managers. So, what is happening, start from here, when it is a 20 days, we got the cost is 10000. When you keep on reduce 19, 18, on 17 days our cost is below 9500.

So, that is the least cost time point. And if we go beyond below 17, when go 16, 17, the cost is started to increase. So, on 13 days the cost is around below 9900. So, this graph is used for managers to promise the client, if they ask you for reduction in time, we can say that when you reduce beyond 17, the cost of project will increase. So, this is very useful information for the project manager for bargaining with time cost trade off with the clients. In this lecture, I have explained what is the time cost trade off and concept of crashing.

Similarly, I have explained what is the crash time, crash cost and normal time and normal cost. I have taken a problem with the help of problem, I have explained how to do the crashing. From 20 days to 13 days, I have reduced from 20 days to 13 days and I have calculated corresponding cost. So, that I plotted in the form of time cost relationship figure. So, this figure is very useful information for a project manager to take the decision.

That is to find out what is the optimal time for completion of this project. Thank you. Thank you.