Project Management

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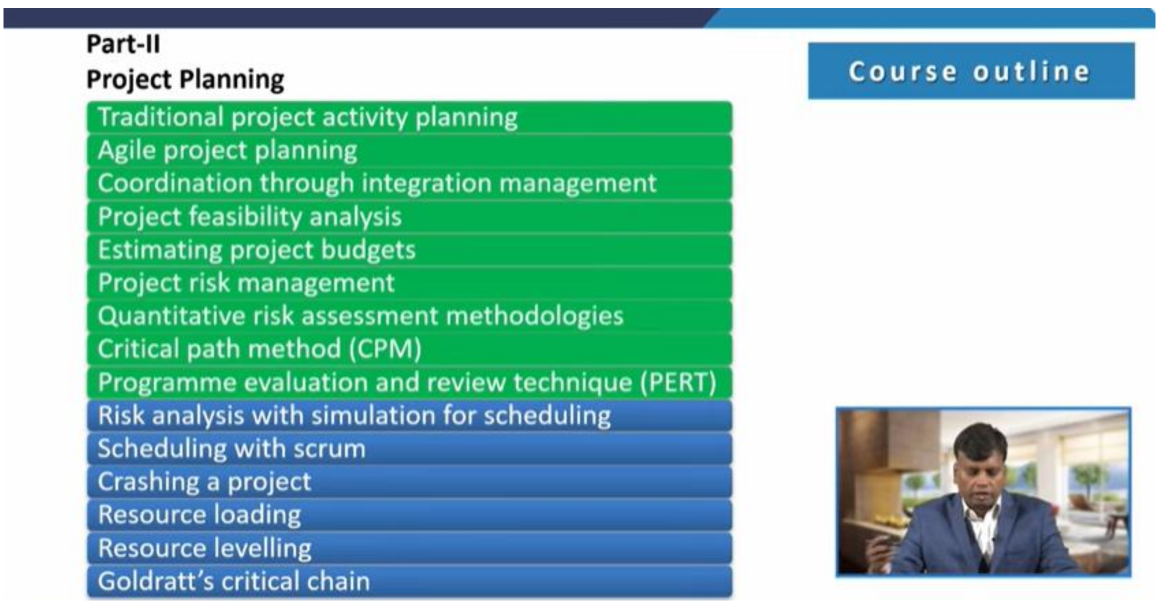
Week: 5

Lecture 21- Programme Evaluation And Review Technique PERT

Dear student, in the previous lecture, I have discussed about scheduling by using technique called critical path method. There, the activity times are deterministic. If the activity times is probabilistic, that means not deterministic, we should go for some other technique that techniques called program evaluation and review technique that we are going to discuss today. So, this slide shows as usual in the previous lecture was critical path method. Now, we are discussing about PERT, program evaluation and review technique. So, the agenda for this lecture is I will explain project scheduling for uncertain activities.



Agenda

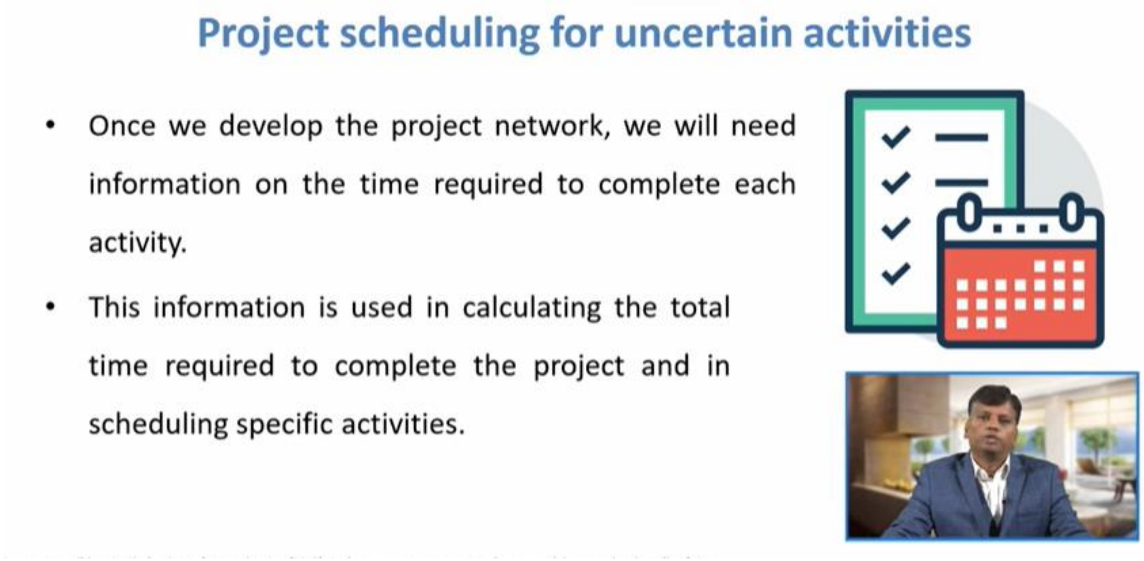
- Project scheduling for uncertain activities
- Expected time for an activity
- Variance of an activity
- Variability in Project Completion Time
- Probability of the path meeting the deadline

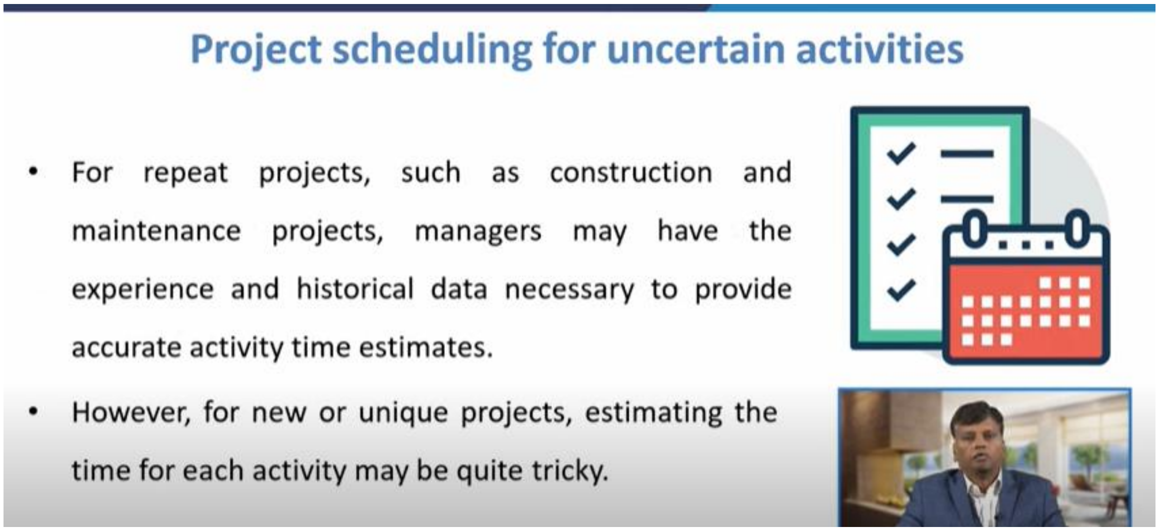
Here uncertain means the activity time is in probabilistic in nature, activity time is not fixed, then how to find out the expected time of an activity, then how to find the variance of the activity, then how to find out the variability in project completion time. Finally, we will discuss about probability of path meeting the deadline. PERT was developed in the late 1950s by the navy specifically for the Polaris missile project to handle uncertain activity times. Once we develop the project network, we will need information on the time required to complete each activity.

Introduction

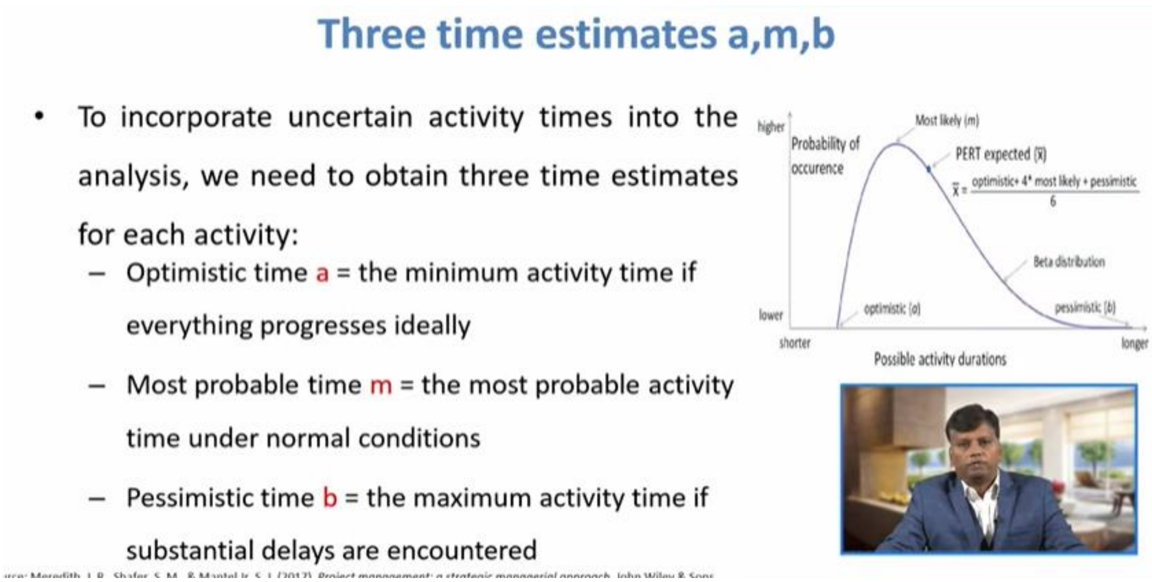
- PERT was developed in the late 1950s by the Navy specifically for the Polaris missile project to handle uncertain activity times.

The information is used in calculating the total time required to complete the project and in scheduling specific activities. For repeat projects such as construction and maintenance projects, managers may have the experience and historical data necessary to provide accurate activity time estimates. If it is very repeated very repeated projects, the estimation of activity time is very easy by seeing historical data we can use as it is. However, the challenge comes for new or unique project, the estimating time for each activity may be quite tricky. If the time is not deterministic, if the activity time is probabilistic in nature, then instead of giving only one time estimate, there are three time estimate will be given.



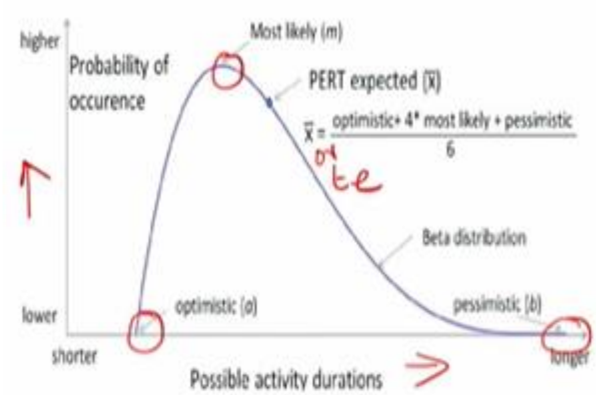


We call it as A optimistic time, M most likely time, B is a pessimistic time. So to incorporate uncertain activity times into the analysis, we need to obtain three time estimate for each activity. For example, optimistic time, what is the meaning of this optimistic time called A? The minimum activity time if everything progresses ideally, that is the optimistic time. The next time is most probable time, we call it as M. What is the meaning of this M? The most probable activity time under normal conditions, then pessimistic time, we call it as B, that is the maximum time if substantial delays are encountered.



So these three activity time follow beta distribution. I have brought the beta distribution for your understanding. You see this is the right skewed distribution. So this A represents the optimistic time. In the extreme right, B represents the pessimistic time.

You know as usual in any distribution in x axis is the possible activity duration, in y axis it is a probability. So this M is the most likely time. So here what we have to do, we can call it as x bar or t, t e expected time. We know that for any distribution, there are two things are important for us. One is we have to know what is the mean of the distribution and what is the variance of the distribution.



So for our further calculations, even though there are three time estimate is given, we are going to use only the mean of that three time estimate. That I will explain in the next slide. So this is the given problem. For example, activity A, so optimistic time is 4 units, maybe assume that it is a week or days, 4 units. You see the pessimistic time is 12 units and most likely time is 5 units.

Three time estimates a,m,b

Activity	Optimistic (a)	Most Probable (m)	Pessimistic (b)
A	4	5	12
B	1	1.5	5
C	2	3	4
D	3	4	11
E	2	3	4
F	1.5	2	2.5
G	1.5	3	4.5
H	2.5	3.5	7.5
I	1.5	2	2.5
J	1	2	3

Activity	Description	Predecessor (s)
A	Product design	–
B	Market research	–
C	Product analysis	A
D	Product model	A
E	Sales brochure	A
F	Cost accounting	C
G	Product testing	D
H	Sales survey	B, E
I	Price and demand report	H
J	Project report	F, G, I

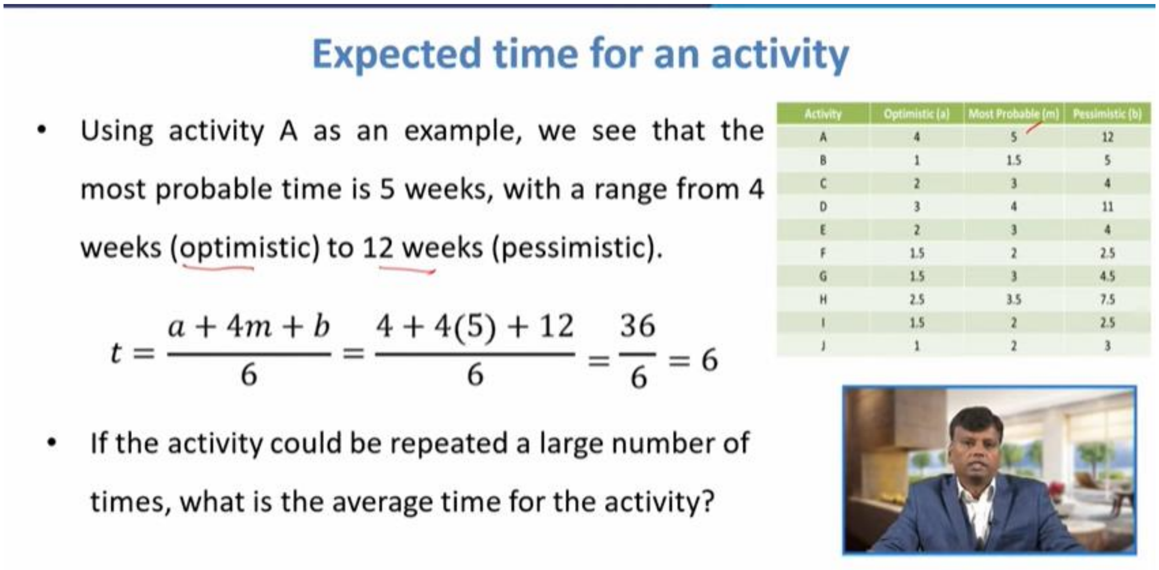
So for each activity A, B, C, D, E, F, G, H, I, J, there are three time estimate is given. So here you know what is the difference between critical path method and PERT method. In the critical path method, there was only one time estimate. But for the PERT, there are three time estimate for the given problem, what we discussed in the previous class. The first task is converting that three time estimate into a single time estimate that is called expected time for an activity.

Expected time for an activity

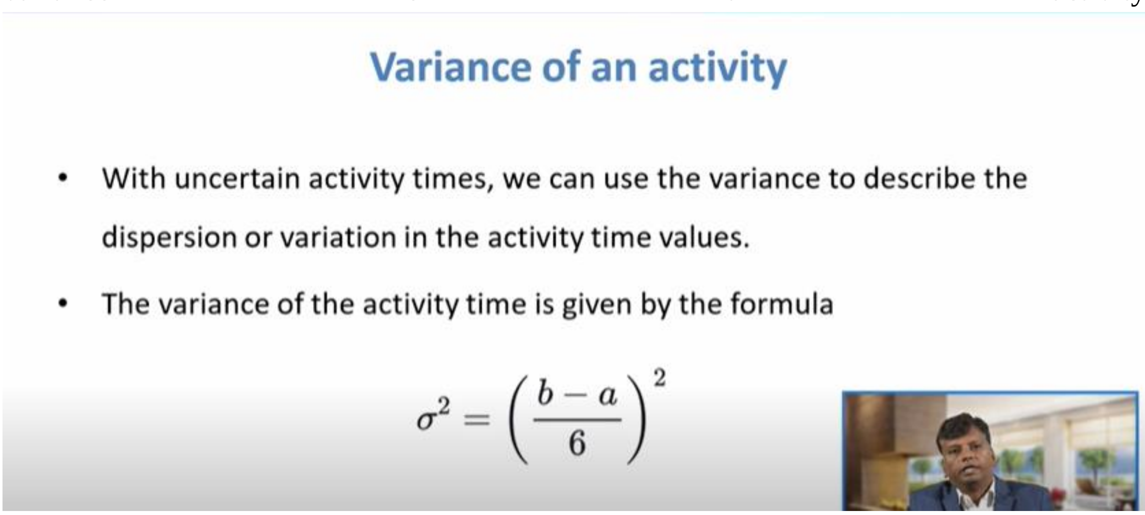
- To illustrate the PERT/CPM procedure with uncertain activity times, let us consider the optimistic, most probable, and pessimistic time estimates for the project activities

$$t = \frac{a + 4m + b}{6}$$

To illustrate the PERT procedure with uncertain activity times, let us consider optimistic most probable and pessimistic time estimate for the project activity. So what we are doing, we are estimating t that is t_e expected time for each activity is called A plus 4M plus B upon 6. This is nothing but the mean formula of a beta distribution. Now I will tell you how to find out mean time estimate for each activity. For example, you say using activity A as an example, we see that most probable time is 5 weeks, this one M with a range of 4 weeks optimistic and 12 weeks is a pessimistic time.



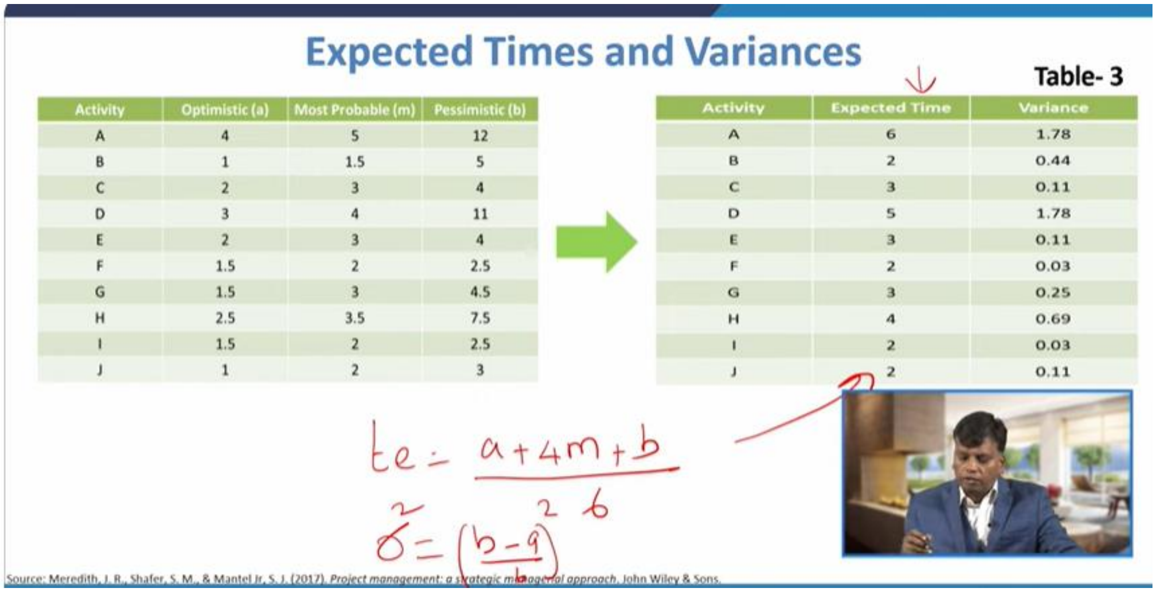
So we will use this formula that is (a+4m+b)/6. We are getting 36 upon 6, equal to 6. If the activity could be repeated a large number of times, what is the average time for that activity that is nothing but the tE expected time for example 6. The next aspect of the beta distribution is finding the variance of the distribution. So next we go for finding variance of an activity.



With uncertain activity times, we can use the variance to describe the dispersion or variation in the activity time values. So the variance of the activity time is given by this formula that is range B minus A that is pessimistic time minus optimistic time upon 6 to the power 2. So variance is range upon 6 whole square.

Variance = $\left(\frac{b-a}{6}\right)^2$

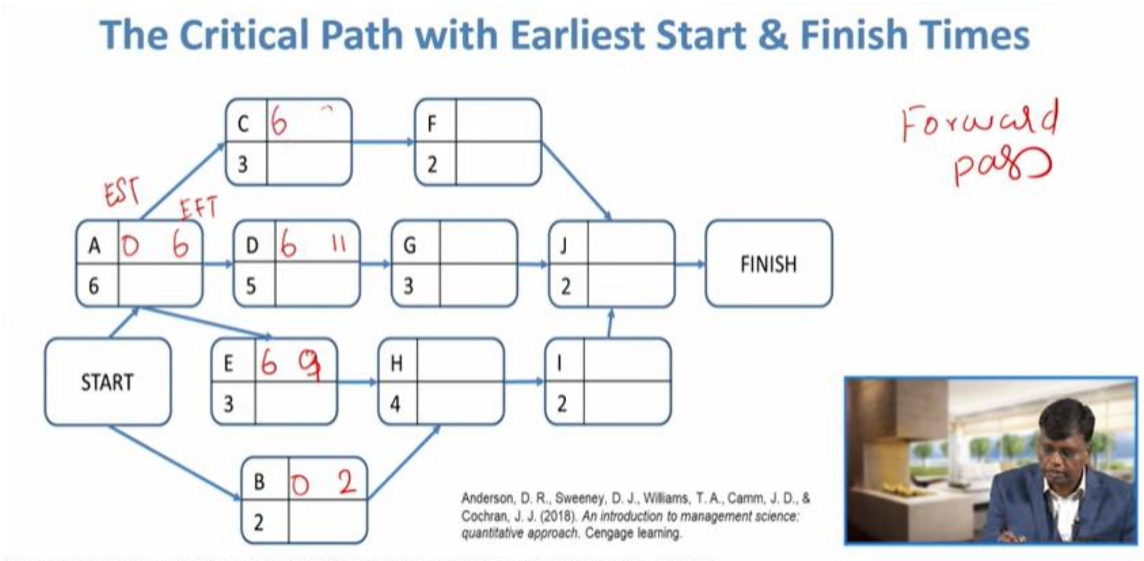
So what important task you need to do is, so every 3 time estimate has to be converted into 1 time estimate by using this formula optimistic time. Similarly, you have to find out the variance of each activity.



So this was by using this formula we got expected time. Similarly, we know the variance formula. The variance formula is for each activity. So what we have to do is whenever there are 3 time estimate, you have to convert that 3 time estimate into a 1 time estimate and you have to find out the variance of that each activity. So now here after we will be using only this table for our further calculation.

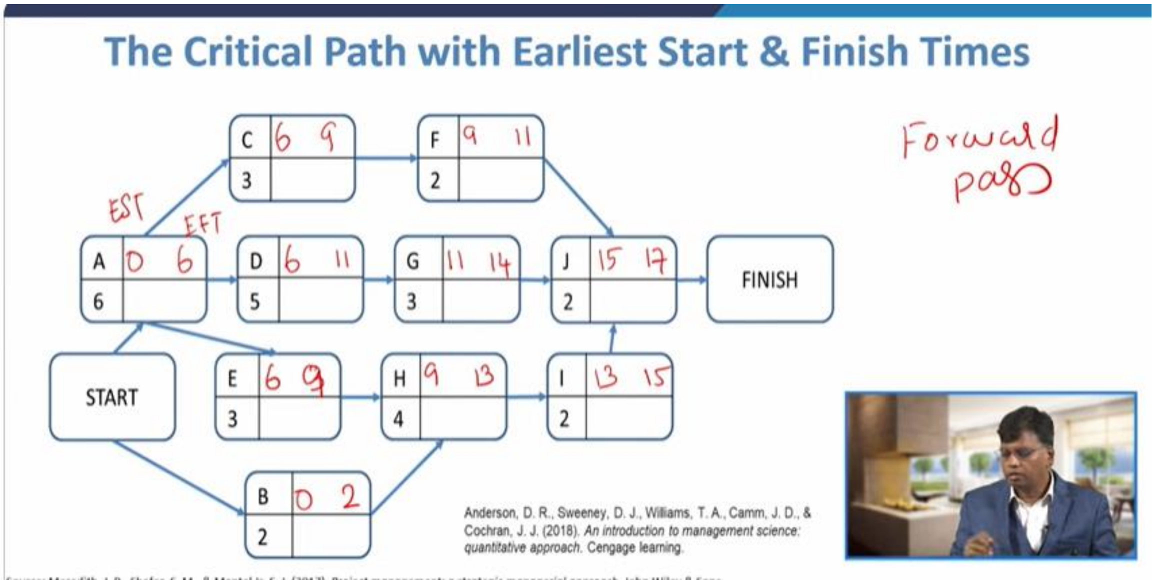
Now I have drawn the table into the network form by considering all the precedence activities. So there is activity A is there CDEFGHIJ. Now by using forward pass, I am going to find out earliest start time and earliest finish time for each activity. For example, activity A it is the first activity. So it is 0 plus 6, = 6.

So the 0 represents the earliest starting time, 6 represents the earliest finishing time. Similarly, for activity B there is no preceding activity, so it will start with 0, so it will be 2. For activity E, it will start with 6 because the earliest finishing time of activity A is 6. So 6 plus 3, 6 plus 3, 9. For activity D, it starts with 6, 6 plus 5, 11.

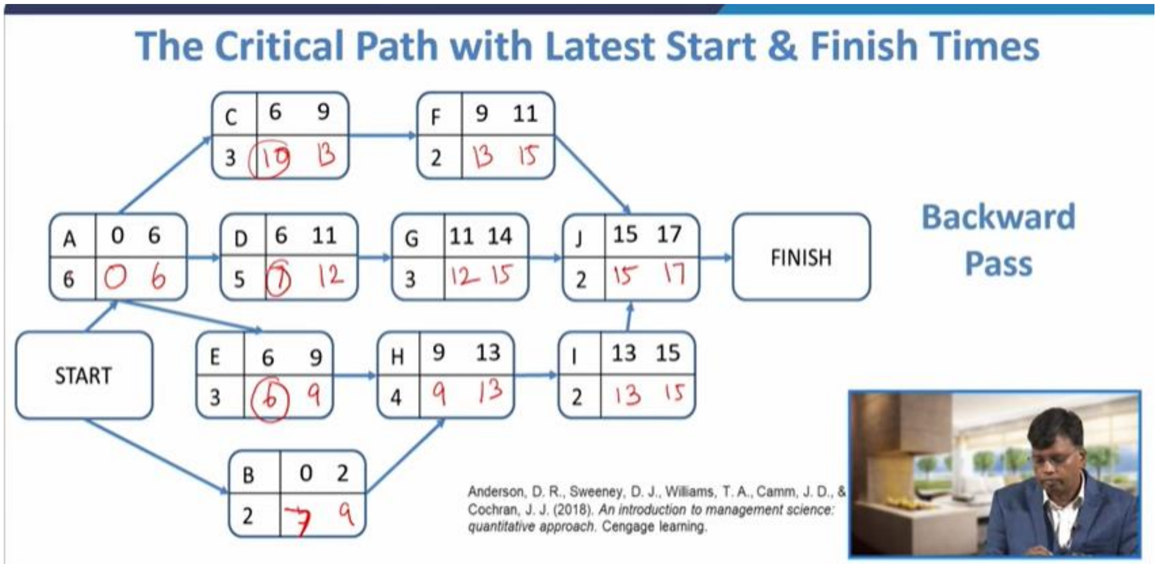


So for activity C, it will start on 6, 6 plus 3, 9. Then activity F starts with 9, 9 plus 2, 11. For activity G, earliest starting time 11, 11 plus 3, 14. For activity H, now you see there are 2 activities here, 9 and 2 is there. There is our earliest latest finish time for activity A, E is 9, activity B is 2, so we have to take the largest number 9, so 9 plus 4, 13.

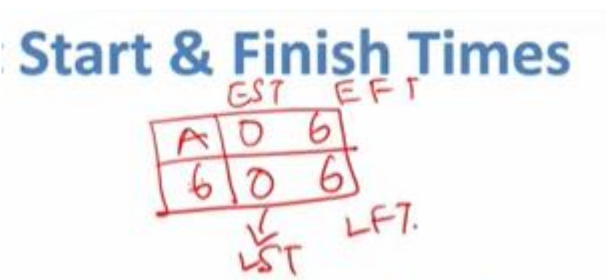
For activity I, it will start on 13 plus 2 is 15. For activity J, you see there are 3 possibilities, 11, 14, 15. So we have to take the maximum value, so 15. So 15 plus 2 is 17. So now I have got earliest start time and earliest finishing time for all the activities, so that is called forward pass.



Previously I have found earliest start time and earliest finish time by using forward pass algorithm. Now I am going to find out latest start time and latest finish time by using backward pass algorithm. So for the backward pass algorithm, we have to start from 17, 17 minus 2 is 15. So 15 minus 2, 13, then 15 minus 2, 13, then again 15 minus 3, 12, then 13 minus 4, it is 9. Now again 13 minus 3, 10, then 12 minus 5, 7, then it is 9 minus 3 is 6, then it is 9 minus 2 is 7.



Now for activity A, there are 3 possibilities here, 10 is there, 7 is there, 6 is there. So the backward pass algorithm says that we have to take the lowest value, lowest value is 6, so 6 minus 6, 0. So what it represents, for example when I say 15, the 15 is latest start time, 17 is latest finish time because the notation what I have used is like this. So this is activity name, whatever value which is written at the bottom, for example A it is 6 is the duration, the 0 represents earliest start time, 6 represents earliest finish time. How I got by adding the duration? So this 0 and 6, for this represents latest start time, this is latest finish time.



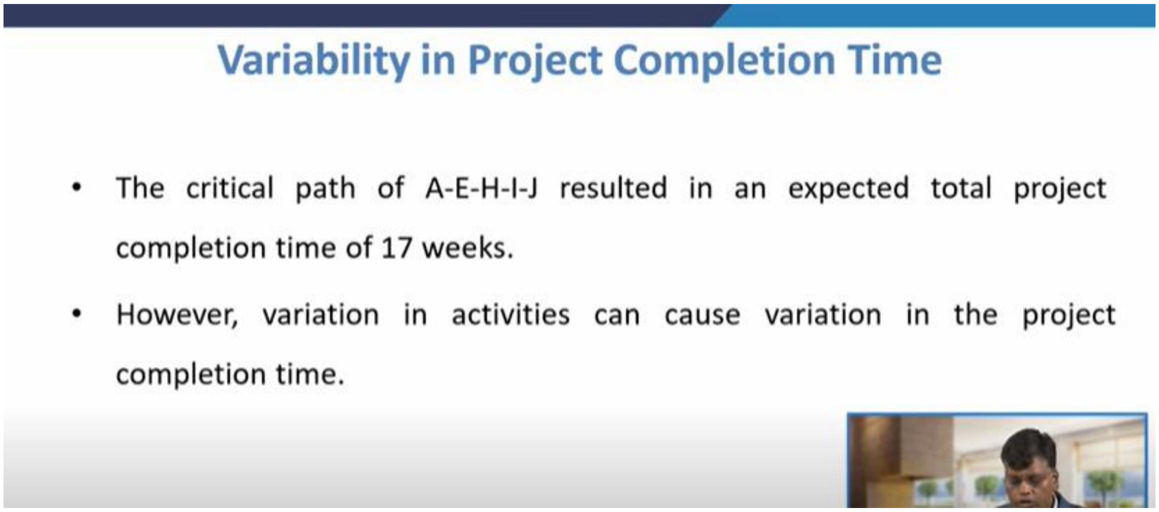
Now I have brought the complete picture. So for each activity I have found the earliest start time, earliest finish time and the latest start time and latest finish time. This is the table, I wrote earliest start time for each activity and earliest finish time, you see earliest start time, earliest finish time, then latest start time, latest finish time. Then I found the slack for each activity. Slack is latest start time minus earliest start time, otherwise latest finish time minus latest start time.

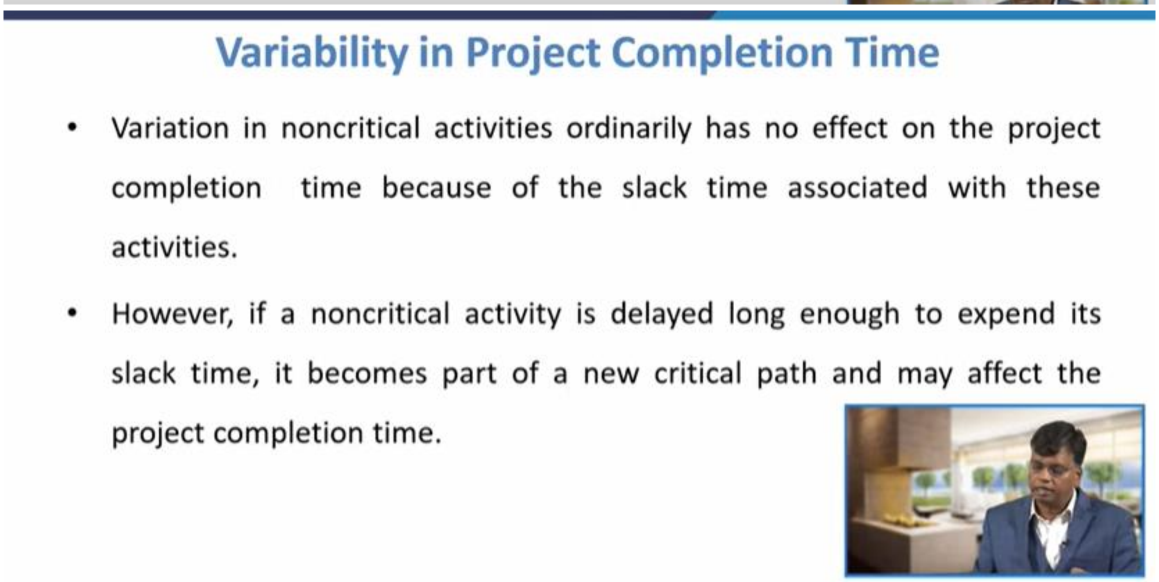
Activity	ES	LS	EF	LF	Slack (LS-ES)	Critical Path?
A	0	0	6	6	0	YES
B	0	7	2	9	7	
C	6	10	9	13	4	
D	6	7	11	12	1	
E	6	6	9	9	0	YES
F	9	13	11	15	4	
G	11	12	14	15	1	
H	9	9	13	13	0	YES
I	13	13	15	15	0	YES
J	15	15	17	17	0	YES

Table- 4

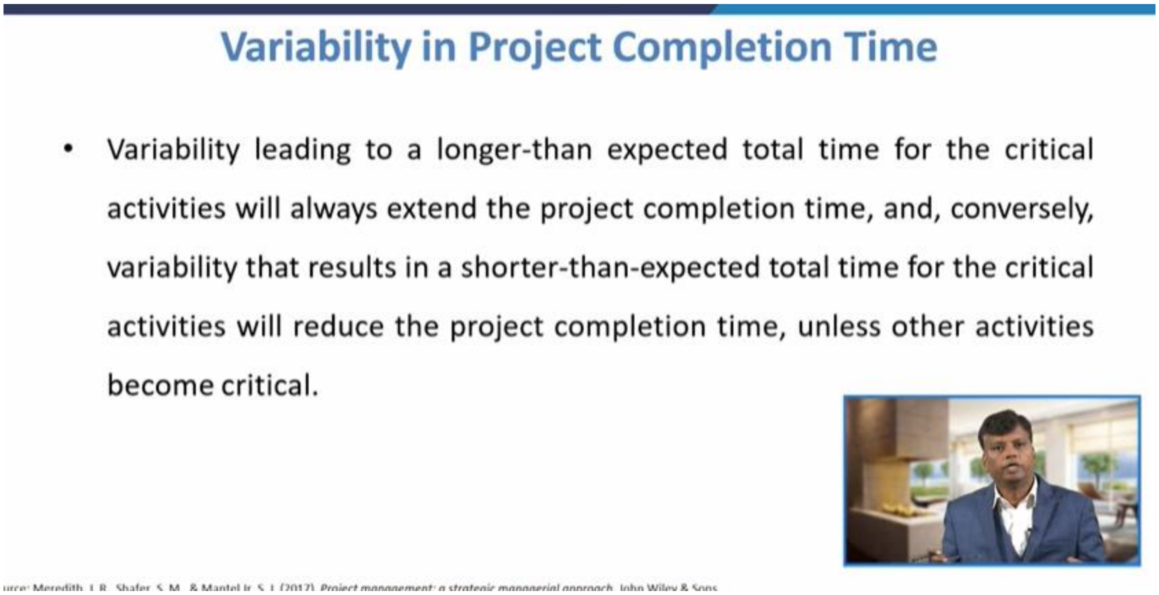
urce: Meredith, J. R., Shafer, S. M., & Mantel Jr, S. J. (2017). *Project management: a strategic managerial approach*. John Wiley & Sons

Here what I have written is latest start time minus earliest start time. So wherever the slack is 0, so we say that is the critical path. Now you see the critical path is A E H I J. I go back to the table, so A E H IJ, this is the critical path. So the duration is, you see it is take 17 weeks.

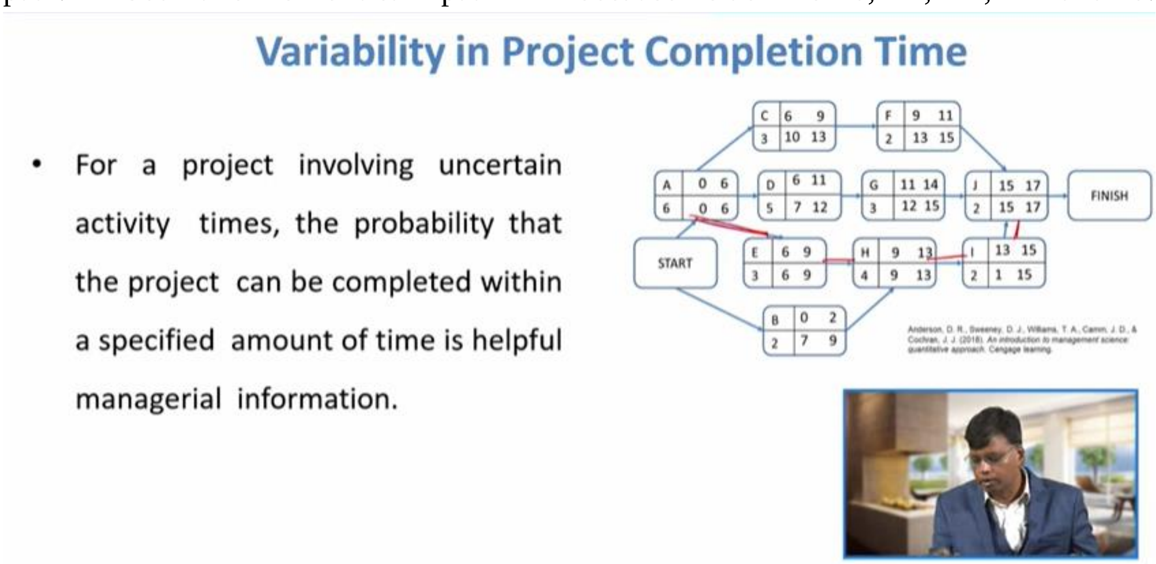




So however the variation in activities can cause variation in the project completion time. So because this is the expected total project completion time, but there is a variability, so we have to consider the variance of all critical activities to consider whether it can be finished within 17 weeks or not. So variation in non-critical activity ordinarily has no effect on the project completion time because of the slack time associated with these activities. So we need not bother about the variance for the non-critical activities. However, if a non-critical activity is delayed long enough to expand its slack time, it becomes part of a new critical path and that may affect the project completion time.



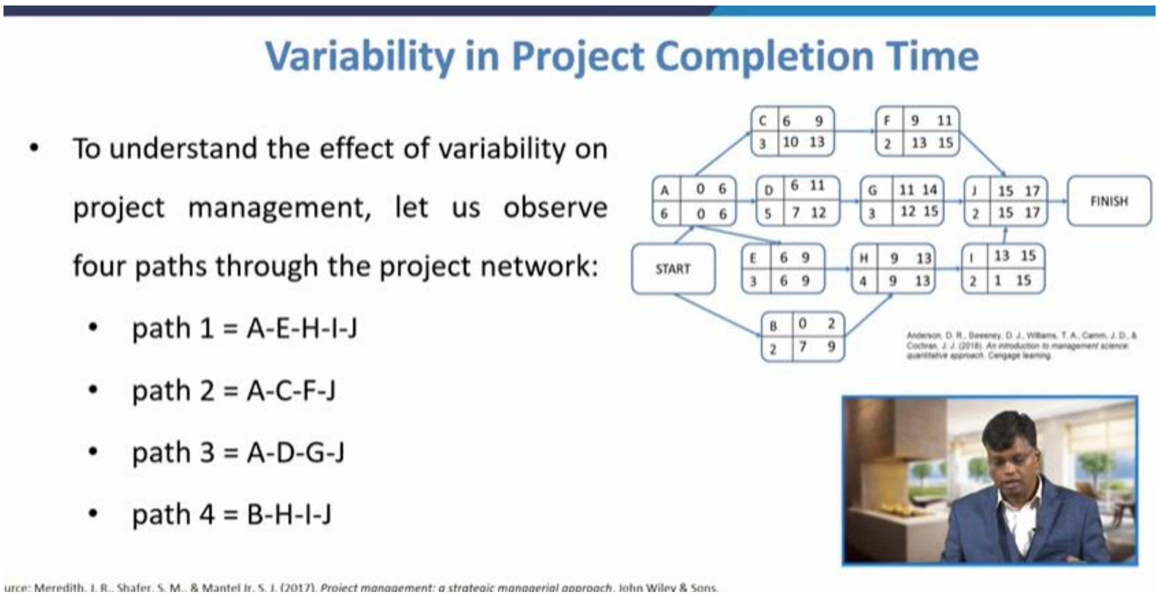
The variability leading to a longer than expected total time for the critical activities will always extend to the project completion time and conversely variability that result in a shorter than the expected total time for a critical activities will reduce the project completion time unless other activities become critical. For a project involving uncertain activities, the probability that the project can be completed within a specified amount of time is helpful for managerial information. For example, we know this is the critical path. See this is critical path A because slack is 0, E, H, I and J.



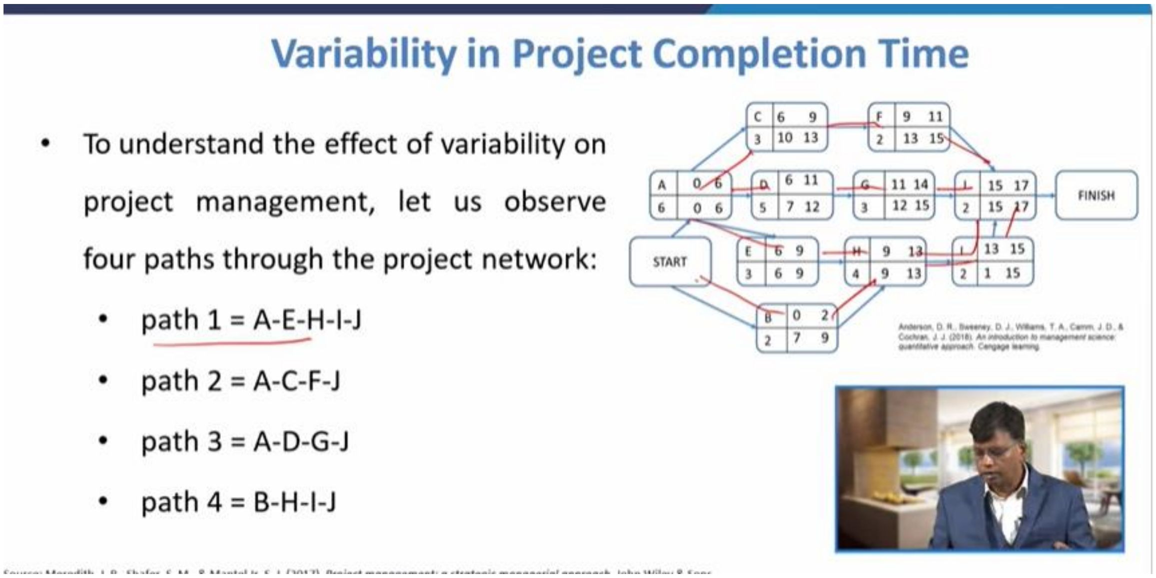
So this path is critical path. So how much days? It will take 15 unit of time but the probability of this, not 15, it is a 17. So the probability of completion of this project in 17 weeks is only 50 percentage because we used the concept of mean. So the project completion time follow normal distribution. So it is 17. Dear students, you should remember whenever you are studying part, there are two type of distribution will come into play here.

One is beta distribution. So the activity time follow beta distribution but the project completion time will follow normal distribution. So I say it is 17 weeks or 17 unit of time. So this 17, the probability of completion of this project 17 is only 50 percentage, 0.5. There are other way to find out the critical path.

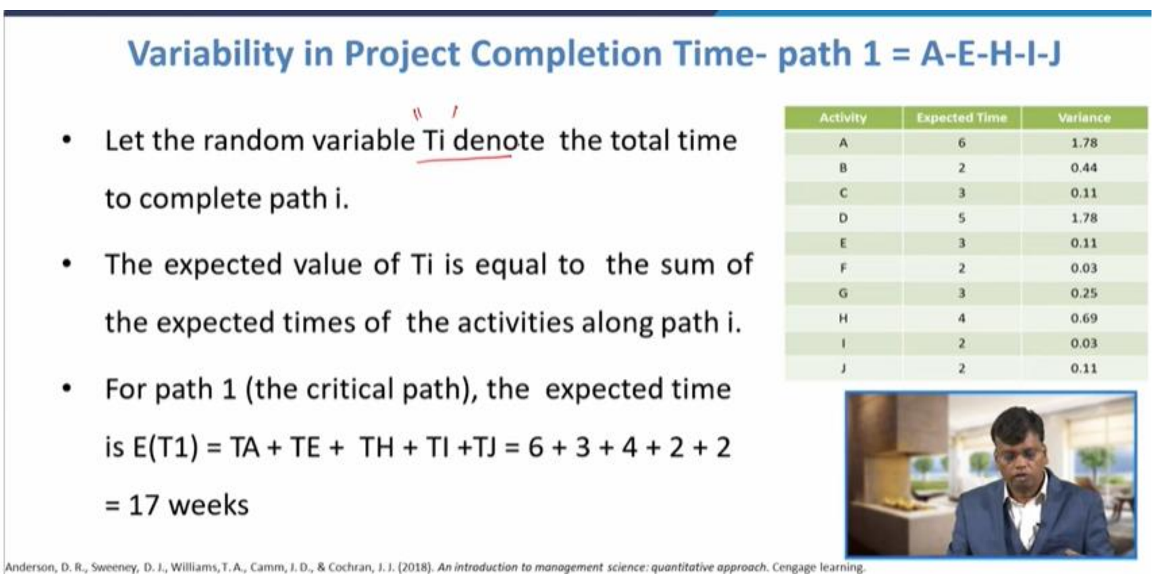
To understand the effect of variability on the project management, let us observe 4 path throughout the project. So I have used forward pass and backward pass. Instead of that you can see all possibility AE HIJ is one possibility path 1, this route. Then ACFJ, ACFJ another path.



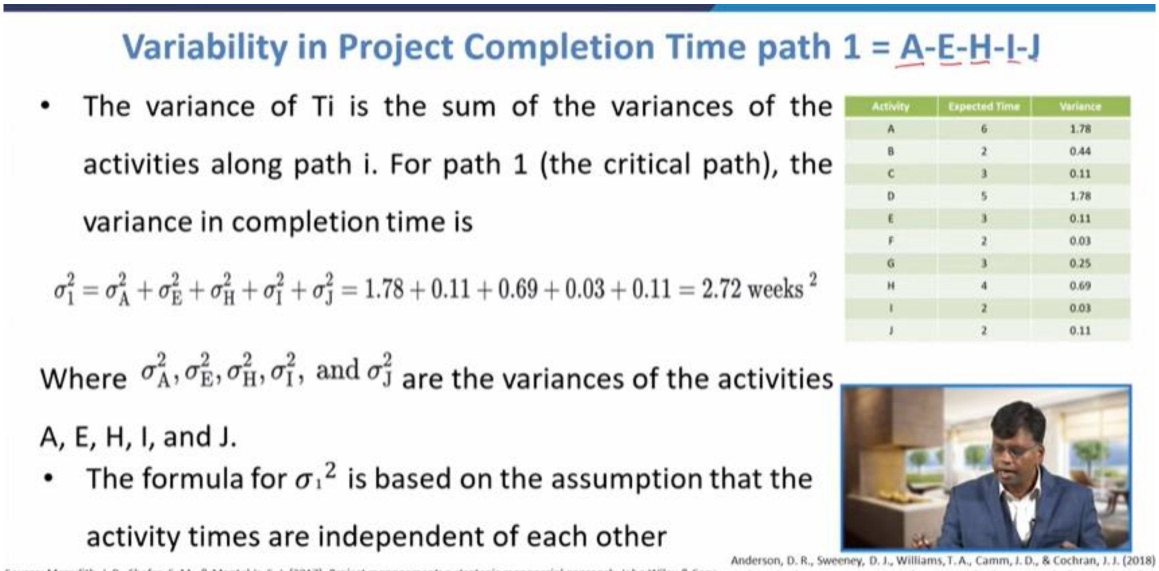
Then path 3, AD GJ, AD GJ. Path 4, BH IJ, here BH IJ. So we can find out which is the longest duration. So that path is critical path. But for a small network, it is very easy to manually to find out. But if there are so many activities, you have to follow this forward and backward pass to find out the critical path.



Now we will consider the variability in project completion time AE HIJ. So let the random variable Ti denote, here Ti denote the total time to complete the path I. The expected value of Ti is equal to the sum of expected time of activities along the path. That is what we got 17. For example, path 1 that is a critical path, the expected time is TA, time for activity AE HIJ.



So 6+3+4+2+2, we are getting 17 weeks. Now we will find out the variability in the project completion time along the path AE HIJ. To find out the variability, you need to add variance of activity AE HIJ. So that is variance of AE HIJ.



When you add it, we will get in 2.72 weeks square. The formula for variance is based on the assumption that activity times are independent of each other. If two or more activities are dependent, the formula provides only an approximation of the variance of path completion time. The closer the activities are to be being independent, the better the approximation. Knowing that the standard deviation is the square root of the variance, we compute the standard deviation sigma 1 for the path 1 completion time as square root of 2.72, we are getting 1.65. So 17 is the mean, the standard deviation of that normal distribution is 1.65. So as I told you, the project completion time follow normal distribution.

Variability in Project Completion Time

- Knowing that the standard deviation is the square root of the variance, we compute the standard deviation σ_1 for the path 1 completion time as

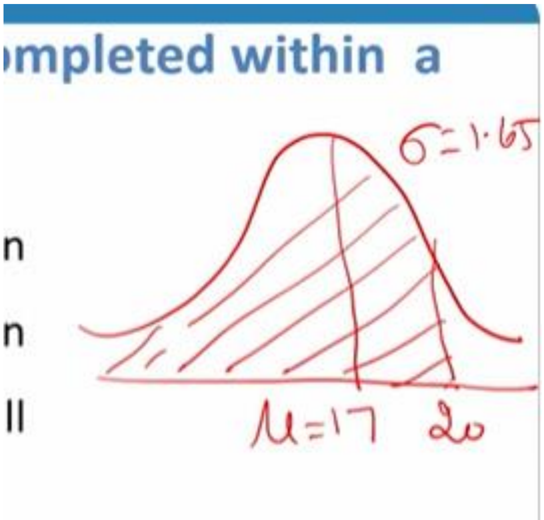
$$\sigma_1 = \sqrt{{\sigma_1}^2} = \sqrt{2.72} = 1.65$$

Probability that a path of activities will be completed within a specified time

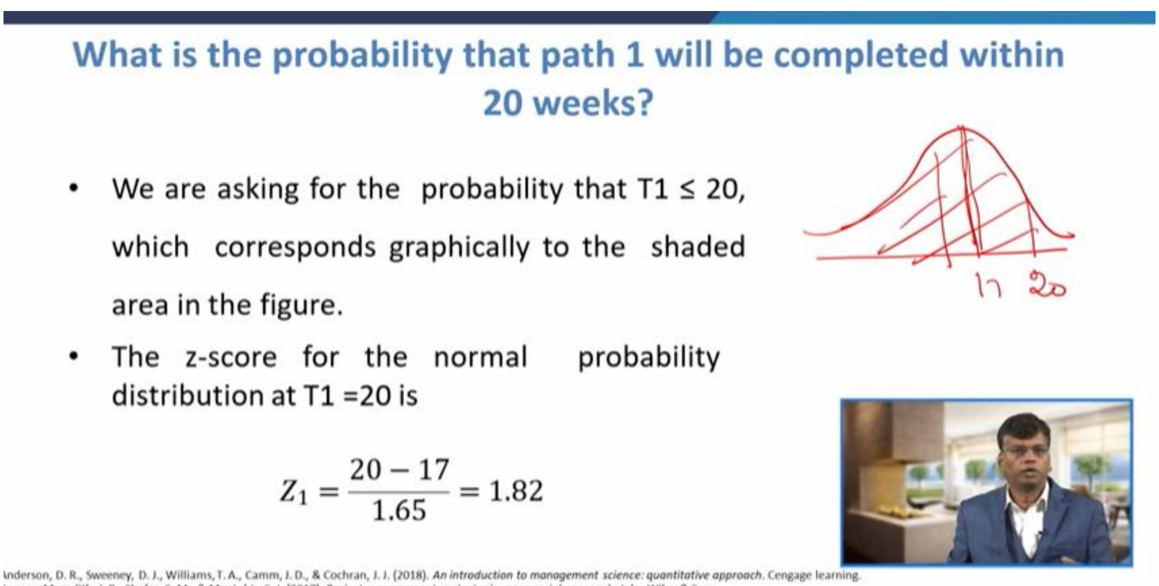
- Assuming that the distribution of the path completion time T1 follows a normal distribution, we can compute the probability that a path of activities will be completed within a specified time.
- For example, suppose that management allotted 20 weeks for the project.

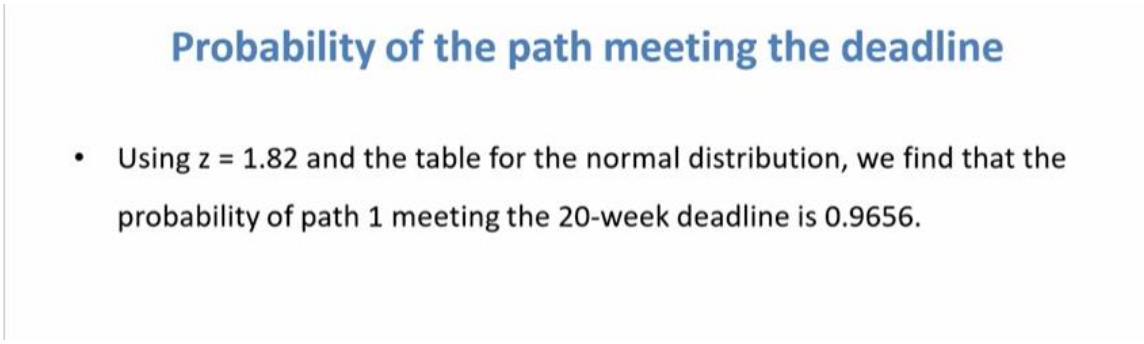
nderson. D. R.. Sweeney, D. J., Williams, T. A., Camm, J. D., & Cochran, J. J. (2018). *An introduction to management science: quantitative approach*. Cengage learning.

It is 17, it is 1.65. Assuming that the distribution of path completion time t1 follow normal distribution, we can compute the probability that path activities will be completed within the specified time. For example, suppose that the management allotted 20 weeks for the project, we know that this is project completion time 17 weeks and the standard deviation is 1.65, this is the mean. Suppose the project need to be completed within 20 weeks, so 20 will come here, so here 20. So we have to find out what is the probability of completing this project within 20 weeks.



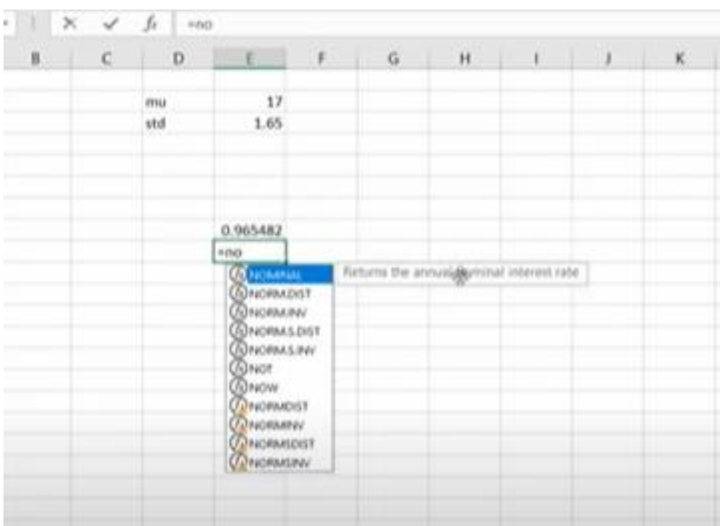
See without going any further calculation always it will be more than 50 percentage because 17 itself it will take 50 percentage chance to finish it, but there is additional time is there, so there is a more chance to complete the project within 20 days. So what we are going to do, we need to find out the area of this normal distribution. This is 17, this is 20, so we need to find out the area of the normal distribution. There are two way we can do it with the help of statistical table, you can find out the area of the normal distribution. If you are using statistical table, you have to find out the convert into the Z scale, we know that this is 20 minus 17 upon 1.65 equals to 1.82,

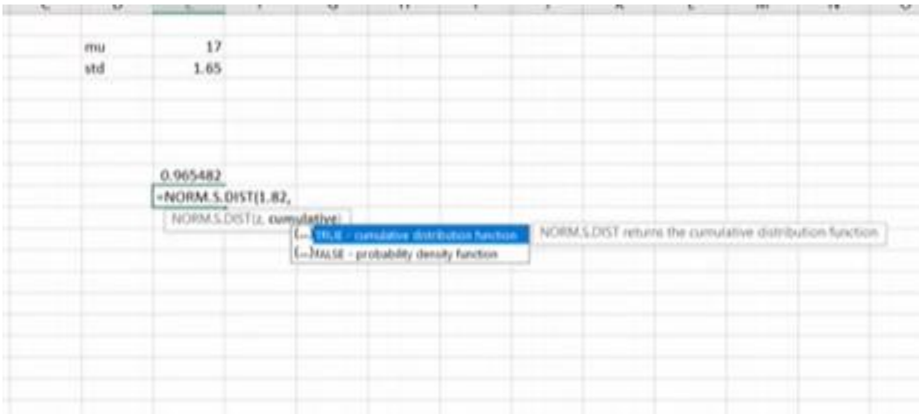




so you can refer the standard normal probability table, from there you can find out what is the probability. Otherwise you can use excel to find out the area of the normal distribution. Now I am going to use excel to find out the area of normal distribution. Now we are going to find out area of a normal distribution, we know the mean is 17 and standard deviation is 1.6 5, we can use excel function "NORM.DST", so x is 20 because we have to find out from what is the probability of completing the project in 20 days.

So mean is 17 and standard deviation is 1.65, then it is true, so the probability of completing the project within 20 days is 96.542 percentage. In excel there is another option, if you want to find out area of a standardised normal distribution, so what you have to do, there norm dot S dot DST, so here you have to type the Z value, so in our problem the Z value is 1.82, so 1.82, then you have to choose true, again we are getting the same answer.





So this is the way to use excel to find the value of normal distribution. So now using Z equal to 1.82, we can use a table for the normal distribution or we can use excel also like what we have discussed just now, we can find the probability of path meeting 20 week deadline is 96 percentage. In this lecture, I have discussed about project scheduling for uncertain activities, then I have explained what is the formula for finding expected time of an activity and variance of an activity, then by using forward pass and backward pass, again I have explained how to find out the critical path slack, then how to find out the variability in project completion time, then I have explained probability of path meeting the deadline that also I have explained. So one more point I want to emphasize here is that the project activity follow beta distribution and the project completion time follow normal distribution that you have to keep in mind. Thank you. Thank you.