Quantitative Investment Management
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Lecture 06
Arbitrage Free Pricing of Bonds

Let us move on to the very fundamental topic in the context of investment management and that is the concept of intrinsic value.

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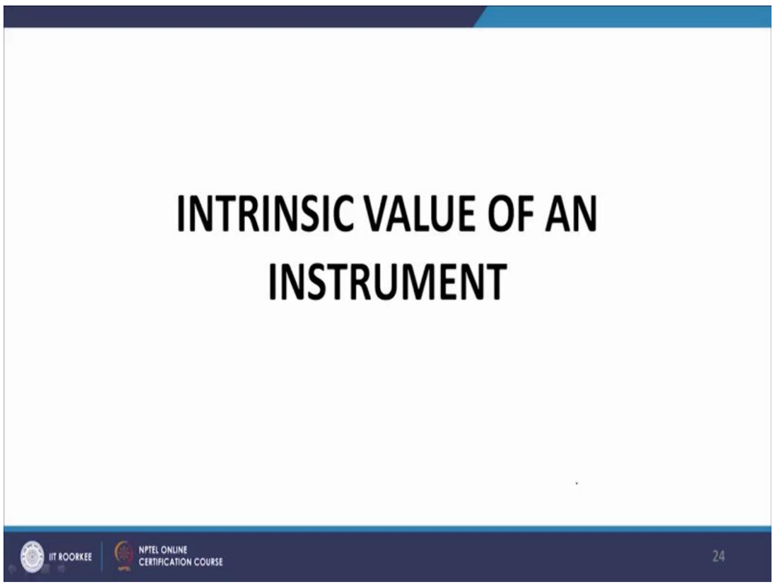

What we mean by the intrinsic value of a financial instrument?

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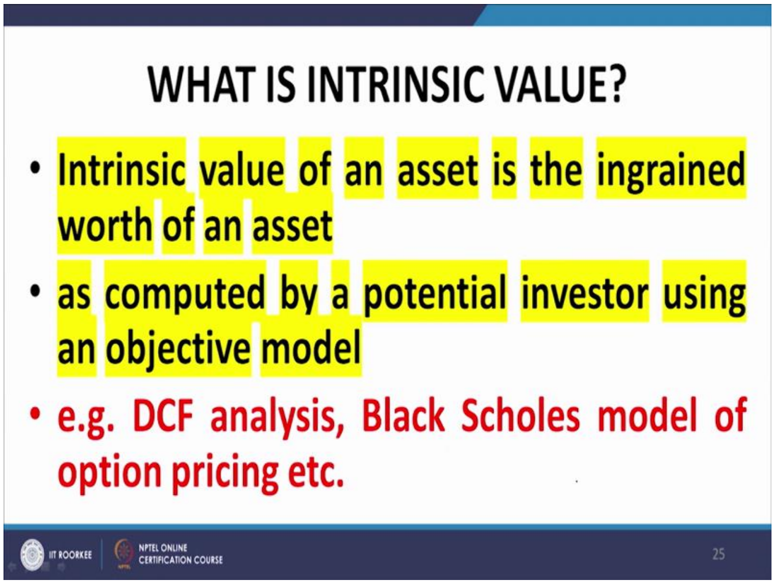

An intrinsic value of an asset, the intrinsic value of an asset is the ingrained worth of an asset as computed by a potential investor, this is important, using an objective model. So, there are two fundamental characteristics of intrinsic value, number 1, it is computed by a potential investor that is, it is investor specific. I will come back to this point in a few minutes. And the second is that it is computed with reference to an objective model like the DCF model or the Black Scholes model of option pricing. So, let me repeat, intrinsic value of an asset is the ingrained worth of the asset is computed by a potential investor using an objective model.

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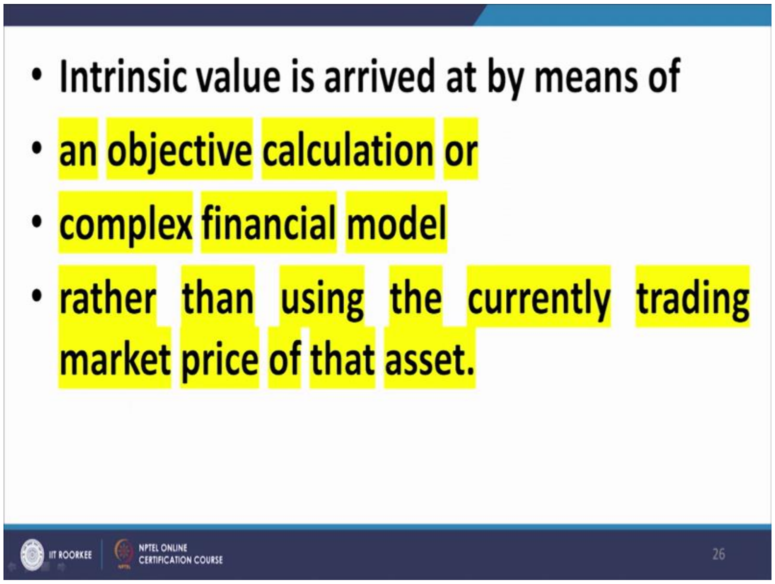

The intrinsic value is arrived at by virtue of an objective calculation or a complex financial model rather than using the currently traded market price of that asset. So, it is in some sense it is insulated from the market and the price does not determine the intrinsic value. In fact, it is the value that is determined by the investor using a model and it comprises or it is a value which is more basic than the market price.

Basic in the sense that it takes into account the fundamental inputs as determined by the investors as ascertained by the investor and on that basis using an appropriate model again that is determined by the investor to arrive at a certain estimate of the worth of the asset and that estimate is called the intrinsic value.

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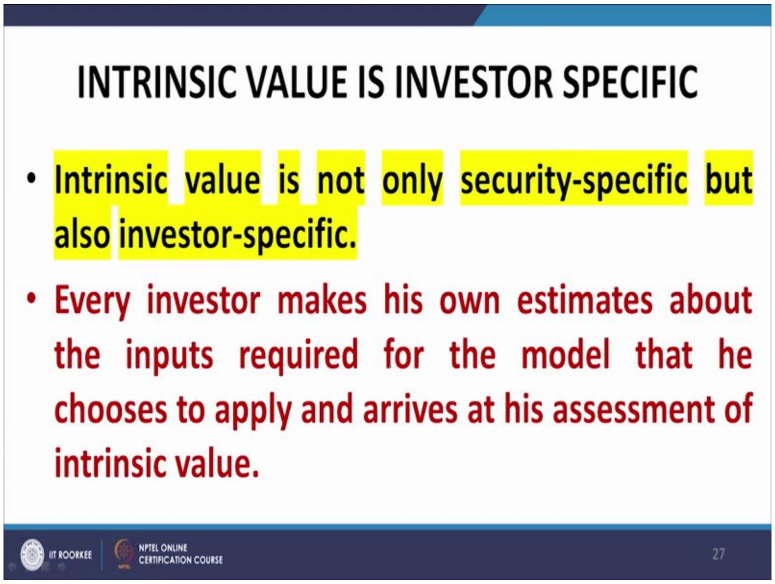

So, intrinsic value is not only security specific, it is naturally security specific, but it is also investor specific, since, intrinsic value if you any if a person x computes the intrinsic value or another person y computes the intrinsic value they may not necessarily be the same, although market prices as identified by x and as identified by y need to be identical will be identical in fact.

And, but I repeat intrinsic value ascertained by two different investors need not necessarily be the same, because I repeat intrinsic value is determined on the basis of inputs as ascertained by the investor as appropriately estimated assessed by the investor and using a model which the investor deems to be appropriate for valuating that particular instrument.

So, both the things not only the inputs, but also the objective model that is using are investor specific are as per the discretion as per the choice of the investor as to their appropriateness. Every investor makes his own estimates about the inputs required for the model that he chooses to apply and arrives at his assessment of the intrinsic model.

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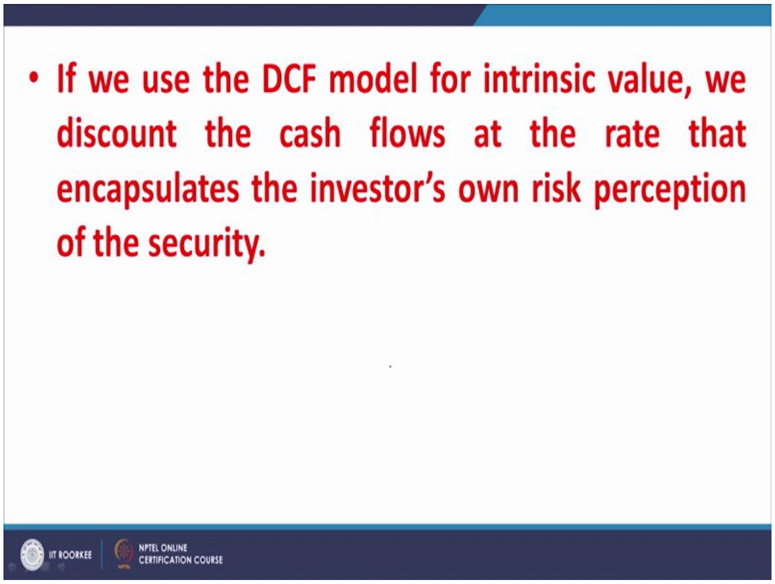

If we use the DCF model for intrinsic valuation, we discount all future cash flows attributable to that particular security at a rate which encapsulates the investor's risk perception of the realizability of those cash flows. Let me repeat, if we use the DCF model for intrinsic valuation, in other words, in the DCF model, the discounted cash flow model for intrinsic valuation what we do is, we work out the present value of a discount all future cash flows attributable to that particular security to that instrument at the appropriate risk adjusted rate as deemed justifiable by the investors.

The estimation of the cash flows I repeat, as well as the estimation of the discount rate are both done by the investor, and therefore, are investors specific as I have strongly empathized.

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Intrinsic value and market trades, now what happens is you see I identify a particular investable asset and I determine its intrinsic value, If I calculate the cash flows relating to that particular asset, particular let us say equity. And I also work out or estimate the riskiness in the realizability of those cash flows and attribute or ascribe discount rate for the to justify or to epitomise the particular risk profile of the reliability of those cash flows.

And then at discount all those cash flows to arrive at a certain value. That value to me is the worth of the particular investment that is how I have assess the worth of the investment, so that is my assessment of the worth of that investment or if the market assessment is different, if the market is pricing the asset more or less than my assessment, it gives rise to a feeling of mispricing a perceived mispricing as far as I am concerned, which consequently may induce me to enter into a sale transaction or by transaction in relation to that security.

If the market prices are higher than my assessment of the worth of that asset, I may sell that asset if I own it, and conversely, or even short sell that asset. And conversely, if my perception is that the market price is lower than my worth, I would rather buy the treasure from the market. So, intrinsic value and market trades are very much interrelated in the sense that intrinsic value is investor’s worth market value is the collective wisdom.

And if the investor's worth, or investor perceived worth is different from the market perception, the investor may undertake transaction in that particular asset, that asset may constitute investment opportunities from the perspective of that investor.

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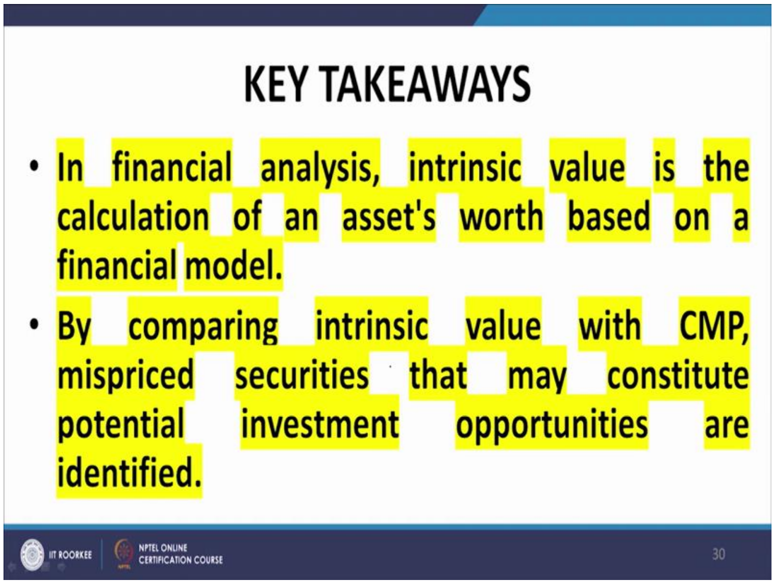

So, key takeaways, in financial analysis intrinsic value is the calculation of an asset's worth based on a financial model. By comparing the intrinsic value with the current market price, mispriced securities is perceived by that investor that may constitute potential investment opportunities are identified, but I repeat intrinsic value is investor's specific.

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Approaches to bond valuation arbitrate free pricing.

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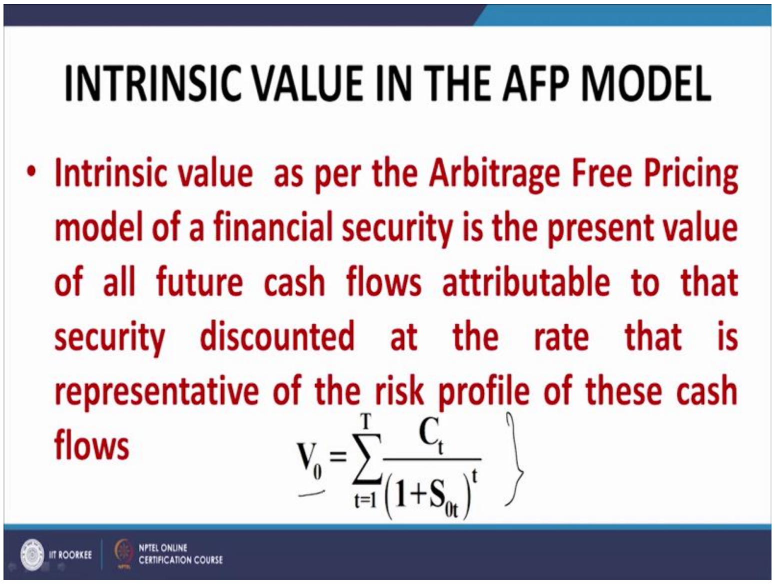

Now, in this particular section, what I will bring to you is that how the origin or the rationale behind the discounted cash flow based valuation or pricing or the estimation of intrinsic value of bond securities. So, intrinsic value as per the arbitrage free pricing model of a financial security is the present value, I have talked about it just now, is the present value of all future cash flows attributable to that security discounted at the rate that is representative of the risk profile of these cash flows.

So, that is given by this equation, capital C t is the cash flow occurring at the point t equal to t which is arbitrary and S0t is the interest rate is the appropriate risk adjuster discounted in tandem with the riskiness of the realizability of the cash flows, and we discount each of the cash flows that is going to be realized by the security to arrive at its today's value.

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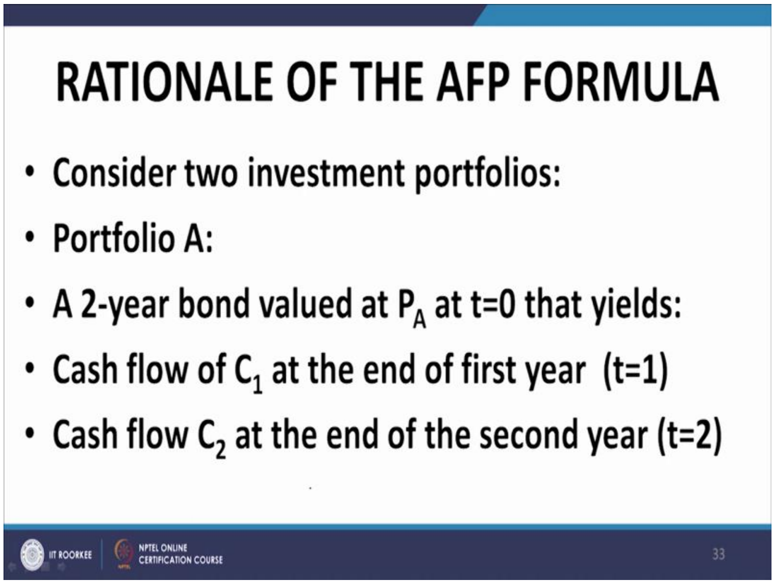

What is the rationale behind this formula? Let us see, this is interesting. Consider two investment portfolios, portfolio A, a 2 year bond valued at PA, PA is the current price of that bond at t equal to 0. And what does it give you? It gives you a cash flow of C1, capital C1 at the end of the first year that is t equal to 1 year and a cash flow C2 at the end of the second year t equal to 2. So, we have a bond, it is a 2 year bond.

So, it will give cash flow at the end of the first year which is C1 and it will give a cash flow at the end of the second year which is C2, t equal to 1 cash flow C1, t equal to 2 cash flow C2 and the current price of the bond or the current value of the bond is PA. What is the current point in time? It is t equal to 0.

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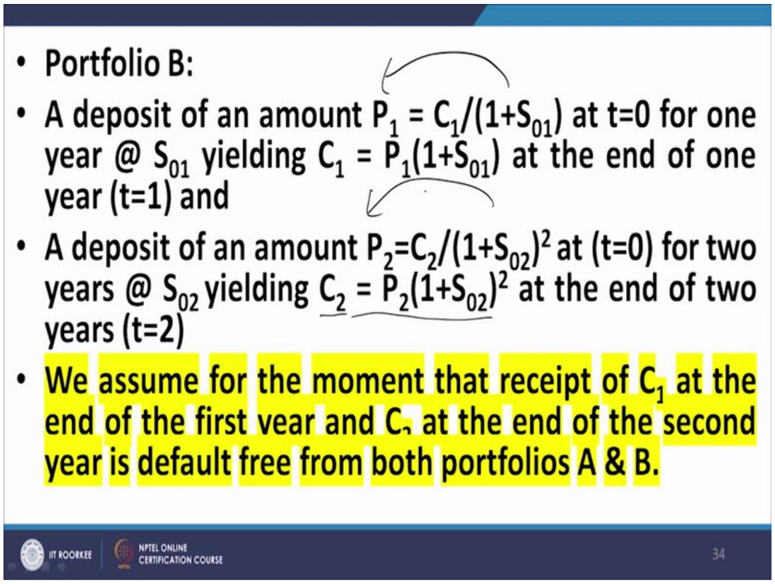

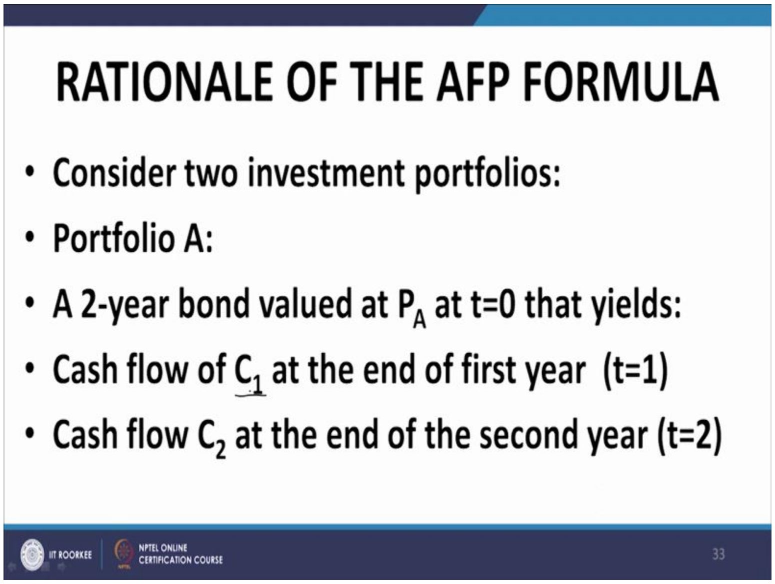

We construct another Portfolio B, Portfolio B comprises of two deposits, what are those deposits, a deposit of an amount P1 equal to C1. What is C1? C1 is defined here, this is C1. So, what is P1? P1 is equal to C1 divided by 1 plus S01, S01 is the relevant interest rate and P1 is the amount that is deposited at t equal to 0, and it is deposited for 1 year at the relevant rate S01. Therefore, the cash flow on account of this would be P1 into 1 plus S01, which will be equal to C1 by virtue of this formula, simply transposing this quantity to the left hand side.

Similarly, in addition to this deposit, there is a second deposit in this portfolio, this Portfolio B, there is a second deposit that deposit is of an amount P2, which is equal to C2, C2 is the second year cash flow from the bond in Portfolio A divided by 1 plus S02, S02 is the annual rate for a deposit of 2 years. So, 1 plus S02 whole squared, why squared because it is an annual rate and the period of deposit is 2 years. So, at t equal to 0, you make a deposit of P2, P2 is given by C2 divided by 1 plus S02 whole square and the deposit is for 2 years.

Therefore, at the end of 2 years the cash flow that we are going to get a C2 that is given by this expression, again simply transposing this to the left hand side. For the moment, let us assume that the receipt of C1 at the end of first year and the receipt to C2 at the end of the second year is default free in both the cases in the case of Portfolio A as well as in the case of Portfolio B. So, S01 and S02 are both risk free rates, but with different terms S01 represents to a deposit for 1 year, S02 represent a deposit for 2 years, we will come back to this issue again.

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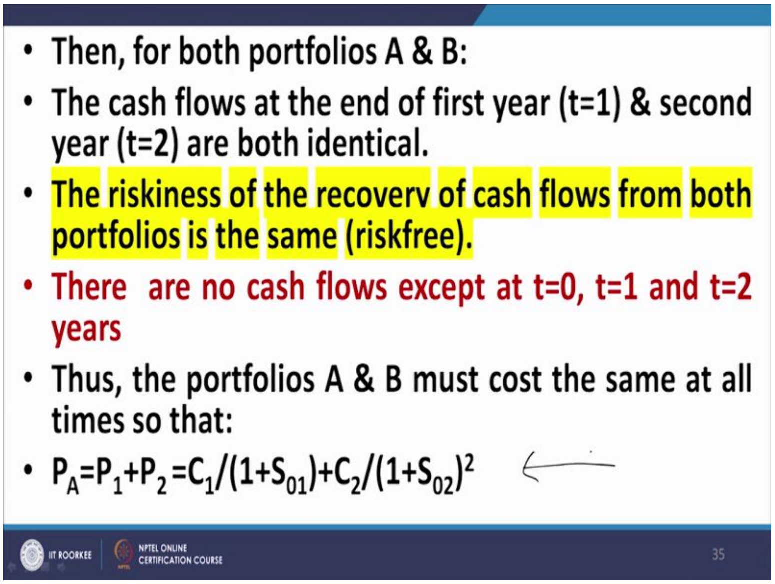

Then for both the portfolio's A and B the cash flows at the end of the first year t equal to 1 and at the end of the second year t equal to 2 are both identical, both Portfolio A is giving you a cash flow, it is a bond 2 year bond, it gives you a cash flow of C1 at the end of year 1 and it gives you a cash flow C2 at the end of year 2.

Portfolio B consists of two deposits, the first deposit is a 1 years deposit, it gives you a cash flow of C1 at the end of year 1 and nothing at the end of year 2, portfolio 2 is a 2 year deposit, it gives you no cash flow at t equal to 1 at the end of the first year that is, but it gives you a cash flow C2 at the end of the second year.

So, the aggregate cash flow of Portfolio B is identical to the aggregate cash flow of Portfolio A. And furthermore, we also note, what do we note, we note that all these cash flows are certain there is no possibility of default. So, there are no cash flows. And furthermore, another point there are no cash flows except to t equal to 0, t equal to 1 and t equal to 2.

Therefore, the portfolio's A and B must cost the same at all types, because their maturity cash flows are identical, their maturity cash flows are all certain there is no risk embedded in either Portfolio A or in Portfolio B, therefore, they are identical in terms of the risk profile, they are identical in terms of the magnitude of the cash flows, and therefore, the cost of portfolio A must be equal to the cost of Portfolio B and that gives us this fundamental equation for the intrinsic value of a bond, PA is equal to P1 plus P2.

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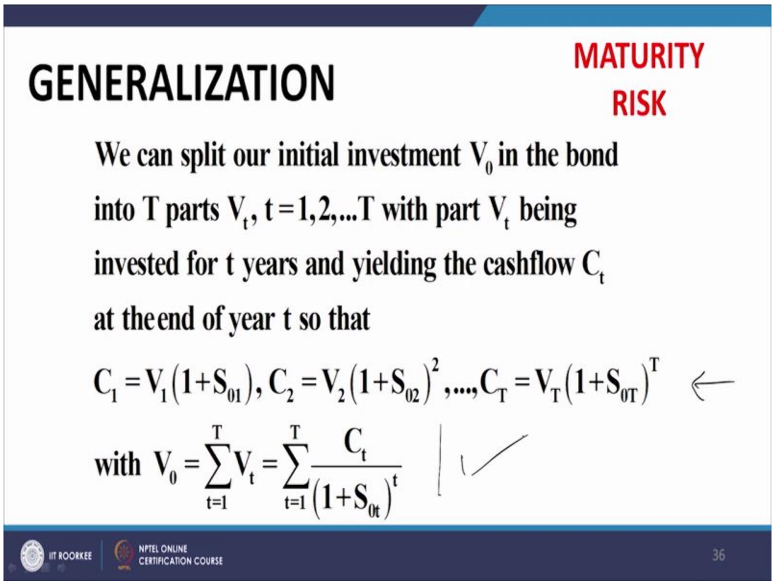

Now, we can generalize this, we can generalize this to in two falls, number 1, the maturity issue instead of a 2 year bond, if we have a t year bond, then naturally we can construct a portfolio a absolutely similarly, to what we had done in the earlier case, instead of using two deposits, we have deposits or we make a sequence of deposits in consonance with the cash flows that are generated from the bond as shown in this equation.

And the second generalization that we need to make is the incorporation of risk, the risk can be incorporated by adjusting the discount rate as we have discussed in an earlier lecture also, the discount rate usually consists of three components, the risk free rate, which represents the postponement of satiation or postponement of satisfaction, deferment of consumption, the inflation premium and the risk premium.

So, in consonance with the level of risk that is attached to these cash flows that we are having in the numerator that this security or the bond is to provide over its life we can adjust the discount rate, we can increase the discount rate appropriately to reflect the riskiness of the realizability of the cash flows from the bond. So, we finally end up with this formula. This is a very fundamental formula that we are going to use again and again, for the valuation of financial instruments.

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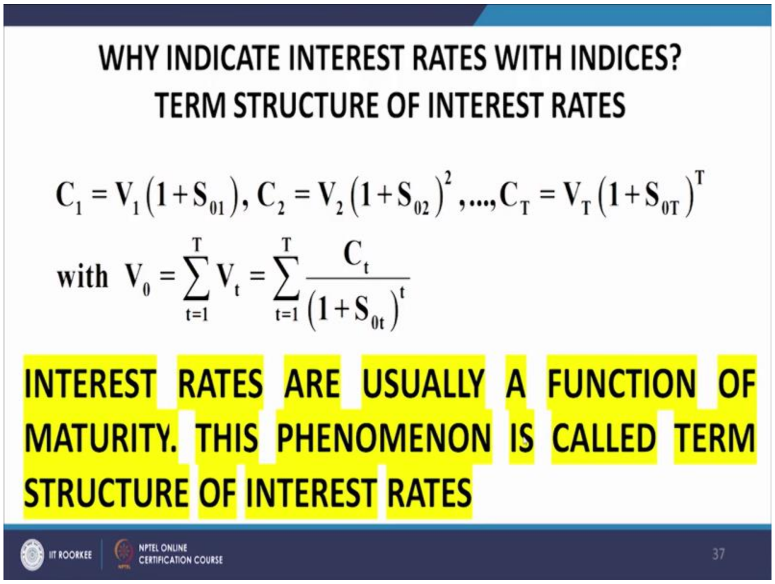

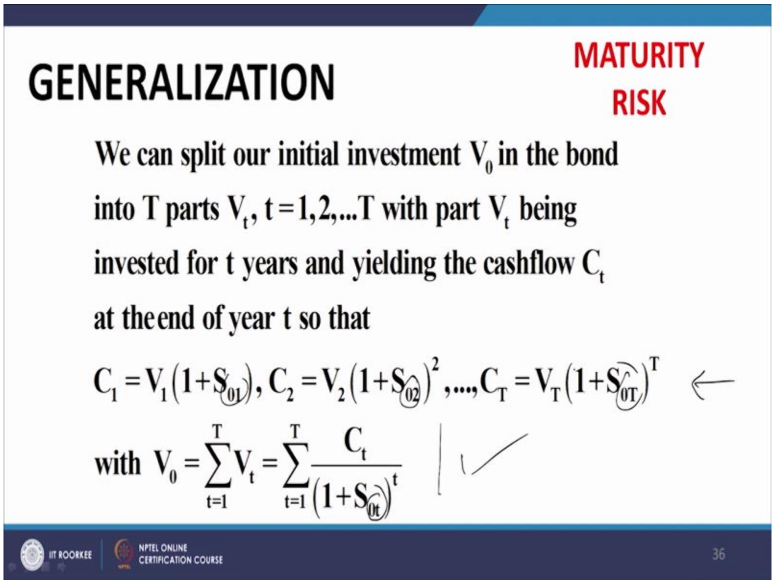

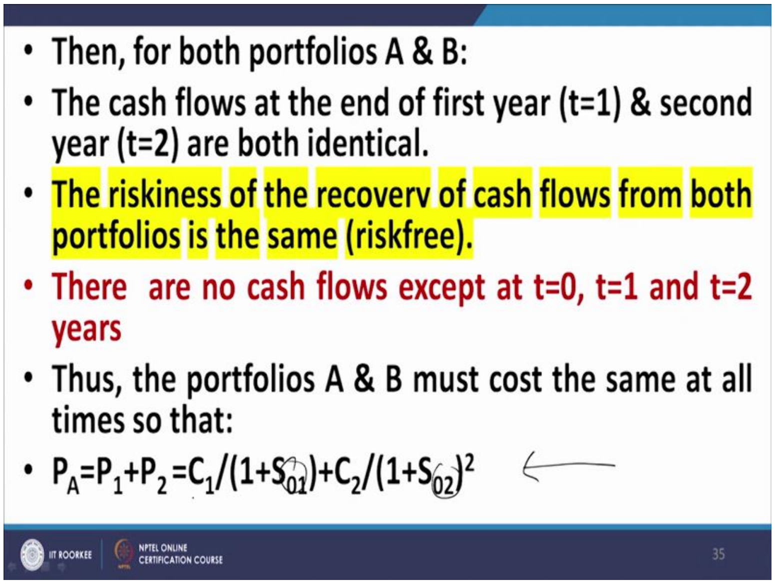

Now, we come to an interesting point, which I had mentioned just a few minutes back, that we are using two indices here, if you look at this formula, if you look, we are using two indices here, we are using two indices here, two indices here, similarly we are using two indices here. And even in the earlier calculation that we did, we had two indices. Why are we using two indices? That is a good question.

The answer to this is that the interest rates that we get on deposits, or borrowings for that matter, the interest that we get on deposits or borrowings are a function of the maturity of the deposit. For example, if I go to the bank today, and make a deposit of 6 months, I will get a different rate from a deposit if I make for 5 years, if I make a deposit for 5 years, the interest rate is usually not necessarily but usually likely to be slightly higher than the interest that that I would get for a deposit of 6 months.

So, the interest rates as well as deposit rate or borrowing rates are all functions of maturities, and as a result of this as a result of this, because S01 that is that spot rate, or the interest rate for a spot deposit of 1 years would be different from an interest rate for a spot deposit of 2 years and 3 years and so on, we need to identify this interest rate corresponding to the point at which the cash flow is occurring from the instrument. If the cash flow is occurring at the end of the first year from the bond, then we need to discount it at the 1 year deposit rate, 1 year spot rate, 1 year interest rate.

If the cash flow is occurring at the end of the second year, then we need to discount it at the second year, or the 2 year interest rate, and so on, as a result of which we need to identify the interest rate by two numbers. That is what we have done here S01, S02, S0t and so on. Now, there is another point why use 0, because we are talking about spot rates, you shall soon be acquainted with something called the forward rates.

And as a result of which, because we are identifying these spot rates, spot rates means you are going to make a deposit t at t equal to 0, that deposit that you are looking at the interest rate that you are looking at is irrelevant to our deposit that we are making at t equal to 0 and the second index represents the length of the deposit, first index when that deposit is made, the second index is when the deposit or the maturity of the deposit.

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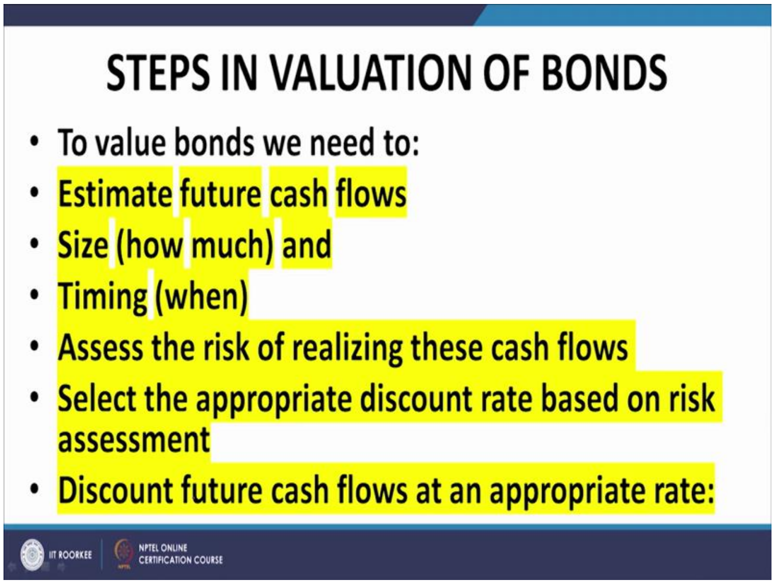

So, what are the steps in the valuation of bonds, let us quickly recap. To value bonds we need to estimate future cash flows, that is calculate the size of the future cash flows in fixed rates, this may be trivial, but in floating rate bonds, this may not be so trivial. In fixed rate bonds of course the cash flows are specified or the cash flows can be calculated directly from the issue document, which usually specifies a percentage and when by applying that percentage on the face value of the bond.

And taking note of the frequency of coupon payments, you can arrive at cash flows, the magnitude of cash flow at various instants of time. Then, of course, we need the timing of the cash flows, whether it is half yearly, quarterly, annual or whatever the case may be, then we need to assess the risk and realizability of these cash flows. Select an appropriate discount rate based on the assessment of riskiness of the realizability of those cash flows, and then finally, discount future cash flows at the appropriate rate.

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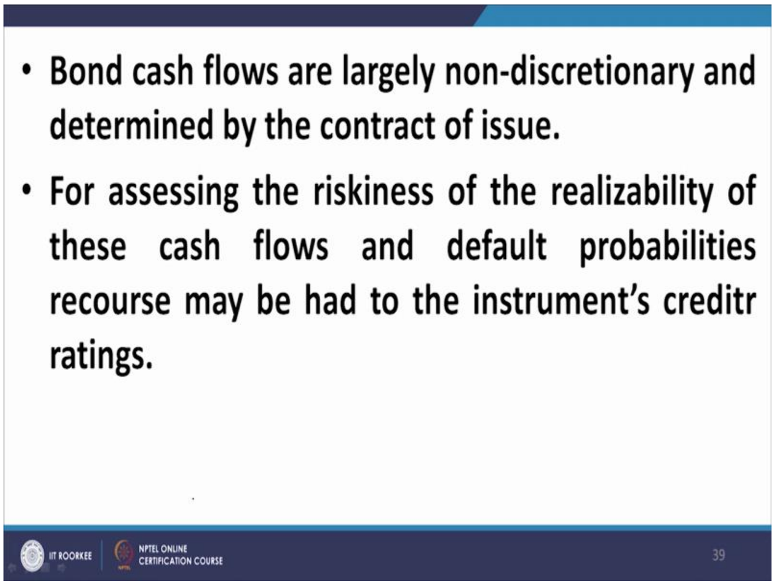

Well, there are two issues that I want to highlight before we move further. Bond cash flows are largely non discretionary. We know that interest payments are non discretionary, they are mandatory, there are charge against the profits of the company. And secondly, the second issue is that bond cash flows. As I mentioned just a couple of minutes back bond cash flows can be calculated directly from the issue document, the issue document usually specifies the coupon rate, the frequency of coupons as well as the face value.

And these are the three ingredients that are required for calculating the cash flows on the bond. So, as far as the cash flows are concerned, they are pretty much contractual, they have non-discretionary and they can be obtained from the contract official the issue document. So, there is a little bit of triviality as far as the calculation of the cash flows from a bond instrument is concerned, it is not a very cumbersome exercise.

As far as assessing the riskiness of the realizability of these cash flows are concerned that has the possibility of default, and the probabilities of default in the realizability of these cash flows. Some inference can be drawn from the credit ratings enjoyed by the instrument, higher the credit rating, obviously lesser would be the chance of default in these instruments. As I mentioned, in most cases, we have semiannual coupons. Although what we will be discussing, would be based on annual coupons to keep the exposition as simple as possible.

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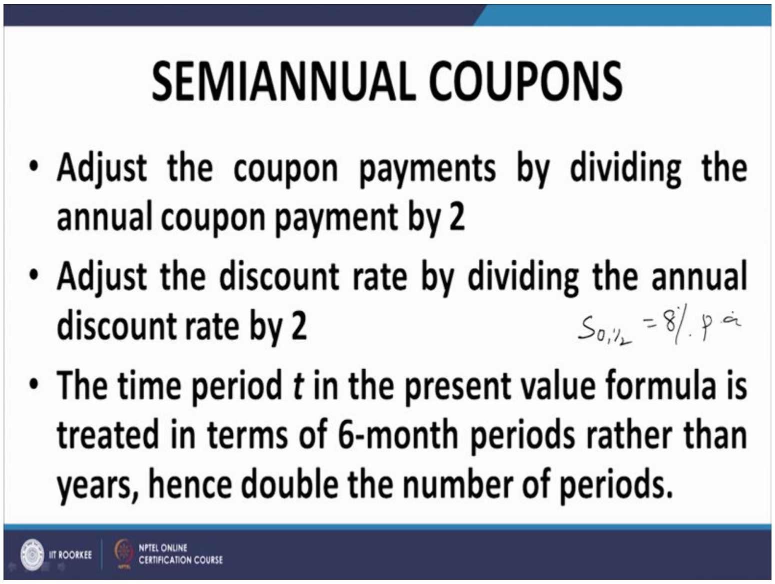

But the annual coupon based exposition can easily be translated to the slightly more cumbersome regime of semiannual coupons. What are the steps that we need to do when valuing a bond that gives us semiannual coupons? We need to adjust the coupon payments by dividing the annual coupon payment by 2 that is quite simple, because if the coupon rate is 10 percent per annum, the semiannual coupon rate will be 5 percent.

And if the face value of the bond is 1000, that means a 50 unit payment at the end of 6 months and a 50 unit payment at the end of 1 years. Adjust the discount rate by dividing the annual discount rate by 2, now please note, it is the convention in the markets that interest rates are always quoted on a per annum basis, because interest rates are always quoted on per annum basis and you are having 6, the discounting period that you are having a 6 monthly you need to half this discount rates when you do that discounting.

For example, if S01 is equal to if, let us say you want to work out the discount rate for the first 6 months, if there is a cash payment of 50 at the end of 6 months, you will use S0 comma 1 by 2, but suppose S0 comma 1 by 2 is 8 percent that means this is 8 percent per annum and since you have to use the half year discount rate, you will use 4 percent, you will half that particular rate.

Although please note this important thing, let us take this example of S01 by 2, let us say this is 8 percent, it is understood that this S01 by 2 is the half yearly discount rate, but it is expressed as an annual rate, please note, that means what, that means if you are going to make a deposit in a bank for 6 months, you will get 8 percent per annum on that rate that means the rate that is quoted is per annum basis, but it would be applied for half year that means if the deposit amount is 100, you will get 4 percent for at the end of 6 months that is you will get 100 and 4.

So, adjust the discount rate by dividing the annual discount rate by 2. The time period t in the present value formula is treated in terms of 6 month periods rather than in years. And as you have to double it, you have to double the number of periods because cash flows are occurring at the end of every 6 months for a semiannual payment.

So, because they are occurring at the end of every 6 months, the number of discounting periods or compounding periods you may like whatever you call it will be 6 months and as a result of it, if it is a 5 year bond, that number of discounting periods will be 10. Discounting periods will be doubled the coupon rate will be halved and then applied on the face value to get the interest payment at the end of every 6 months. And it is the most complex thing is the discount rate. Although the discount rate pertains to 6 monthly intervals, it is expressed on an annual basis and as a result of which it needs to be halved as well.

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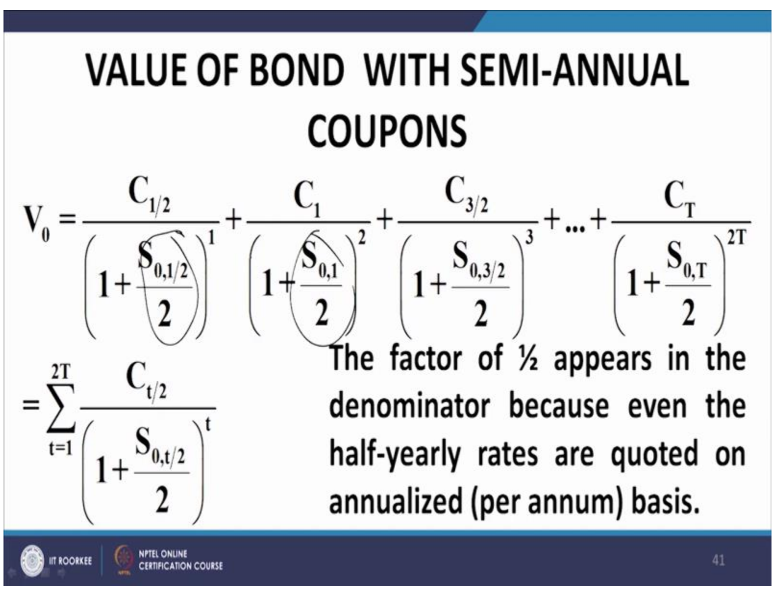

This is the formal expression, the factor of 1 by 2 that is here, this factor 1 by 2 here occurs because even the half yearly rates, the rate that will on a half yearly deposit is expressed as a per annum basis by doubling that rate. So, when you do the discounting you have to undo this and that means you have to half the annual rate. Although that rate pertains to a half year deposit.

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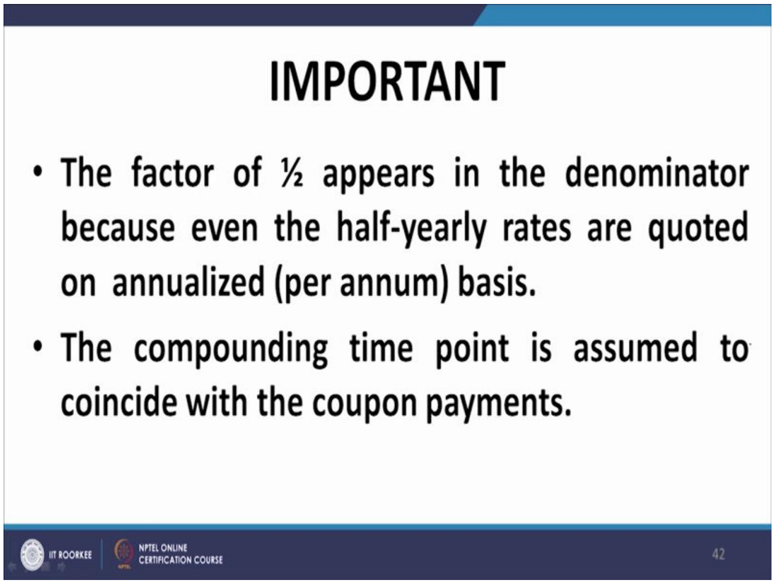

And the second thing is you have to keep track of the compounding time point if the compounding time point coincide with the point at which the cash flow takes place. This is very important, this part.

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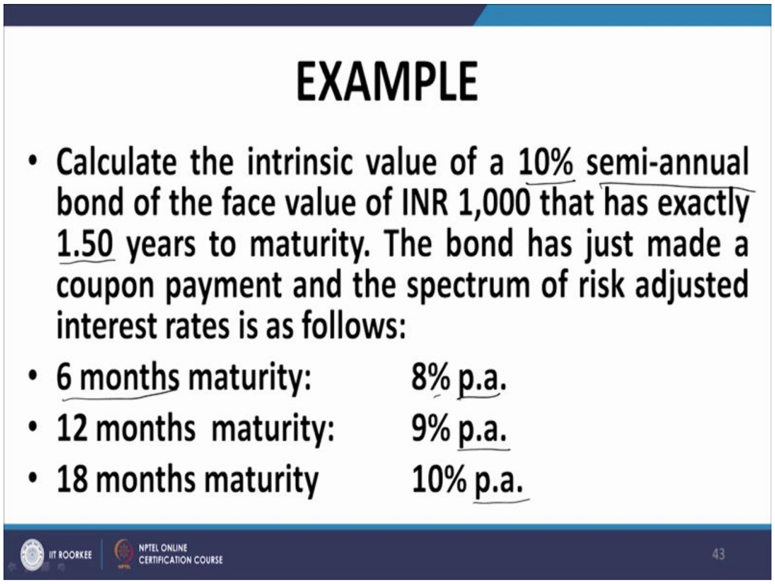

Let us do an example. Calculate the intrinsic value for 10 percent semiannual bond of the face value of 1000 which is exactly 1.50 years to go to maturity. There are two important things here, 10 percent, this is the coupon rate per annum basis, please note this, semiannual cash flow and the frequency is half yearly and the life of the bond is 1.50 years. The bond has just made a coupon payment and the spectrum of appropriate risk adjusted rates is as follows.

Please note this as an 8 percent per annum, 9 percent per annum, 10 percent per annum. Now, here it is 6 months maturity, but the rate is expressed as 8 percent per annum. So, when you will discount it, you will discount this cash flow at 6 months, you will use 4 percent not 8 percent, please note this.

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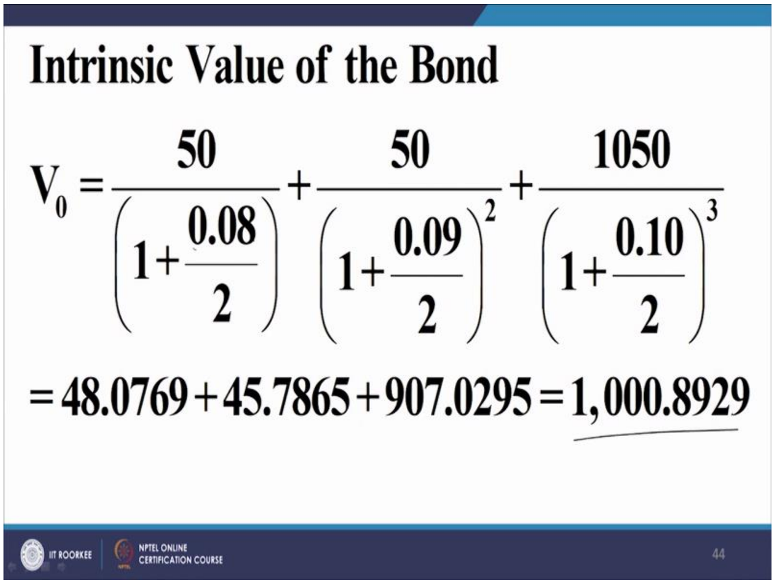

So, this is the solution to the problem. As you can see here, I have divided this by 2, divided this by 2 because this is the annual rate and divide this by 2 as well because again, a 6 month rates are doubled and then expressed as annual rates. This is the net present value and this is also called intrinsic value of the instrument.

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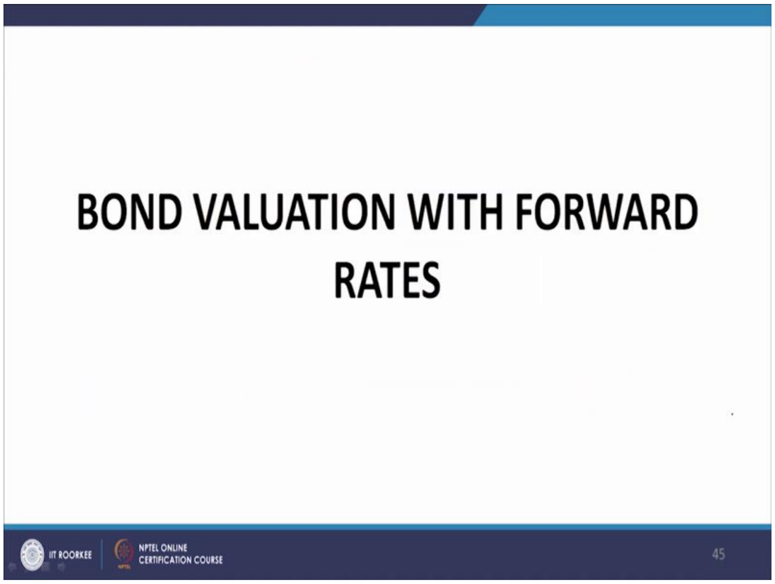

Bond valuation with forward rates:

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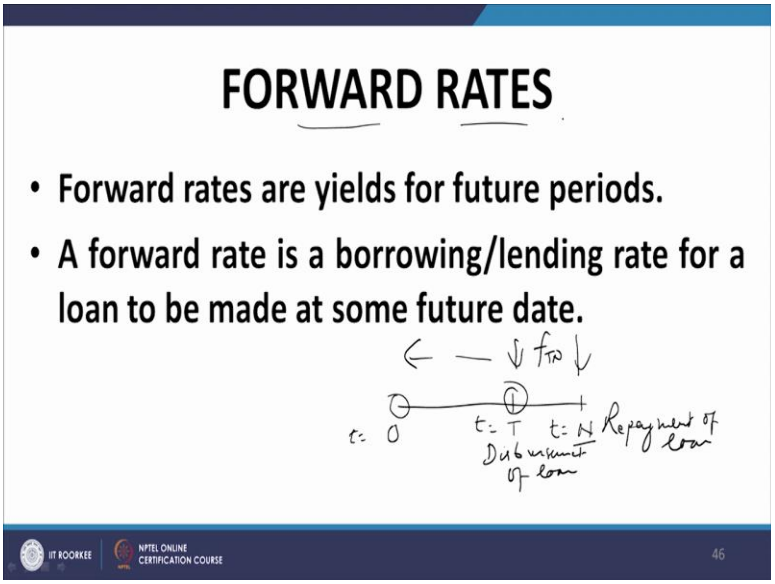

Now, before I talk about bond valuation with forward rates let me explain what is the forward rate? We all know what is the forward contract. A forward contract is a contract, which is negotiated at t equal to 0 and which is settled at t equal to capital T which is the maturity of the contract. All the terms are agreed at t equal to 0, the actual cash flows occur at t equal to capital T, this is a forward contract.

A forward loan and the related forward rate is a extension of the concept of a forward contract. What is the forward rate? We have three time points here, we have t equal to 0 that is today, then we have another time point in the future let us say this is capital T, and then we have a third time point let us call it capital N, t equal to T, t equal to N.

Now, what happens in the case of a forward loan is that all the terms in relation to a loan are agreed upon a t equal to 0, the rate of interest repayment schedule the collaterals and whatever it is, whatever is relevant to the loan transaction is agreed upon a t equal to 0. However, the actual loan is not dispersed, it does not take place at t equal to 0 it does not take place at t equal to 0, it takes place at a future date t equal to capital T.

Please note, this future date t equal to capital T when is that loan going to be initiated when it is going to be dispersed is also agreed upon at t equal to 0. For example, 1 year later, I need 50,000 rupees, I go to my bank and I tell them that 1 years later, I need 50,000 rupees. And please let me know what interests you are going to charge on that 50,000 rupees.

I do not want to take any interest rate risk and as a result of which I want that this transaction should be settled at t equal to 0 as far as the interest rate is considered as far as the repayment is concerned, I need that loan for 1 years that is from t equal to 1 years to t equal to 2 years, what is the rate that you are going to charge me.

That rate which is crystallized at t equal to 0 for a loan that is initiated at t equal to capital T and repaid at t equal to N, this is repayment of loan, this is dispersement. So, that loan is dispersed at t equal to capital T, it is repaid at t equal to N both these dates I repeat, I reiterate this point and this point are agreed at t equal to 0.

And not only these two agreed a t equal to 0 the interest rate fTN and is also agreed at t equal to 0. This transaction of a loan at a future date for a predetermined period at a predetermined rate is called a forward loan. And that related interest rate that is also agreed at t equal to 0, this is fundamental. The rate interest rate is also agreed at t equal to 0, that rate also is agreed upon a t equal to 0 and that is called the forward rate.

This transaction is called the forward loan. And the rate that operates over this forward loan, the rate of interest that is agreed upon at t equal to 0 to operate on this forward loan is called a forward rate. So, forward rates are yield for future periods or forward rate is a borrowing lending rate for a loan to be made at some future date. It is an extension of the basic concept of a forward contract.

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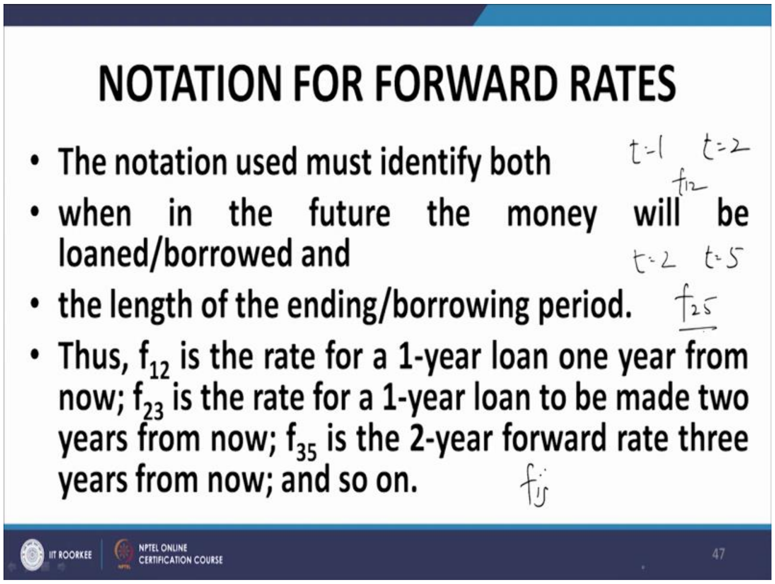

Now, as far as the notation for forward rates are concerned, I just made an intro to that. If we have a loan, which starts at t equal to 1 and ends at t equal to 2, the relevant rate of interest that is going to apply is usually represented by f12. And suppose we have a rate that operates for a loan that takes place that is initiated at t equal to 2 and it is a 3 year loan. So, it matures at t equal to 5 and that is written as t25.

So, t25 is a 3 year loan that starts at t equal to 2 years and that matures at t equal to 5 with the interest rate that would operate on that loan is represented by f25. And similarly f, let us make it more general fij is the interest rate that is going to operate on a forward loan, forward loan means that terms are agreed at t equal to 0, but the loan is going to be dispersed at ti and the loan is going to operate, or loan is going to be repaid at tj. So, I shall continue from here on in the next lecture in describing to you the arbitrage free relationship between spot and forward rates.

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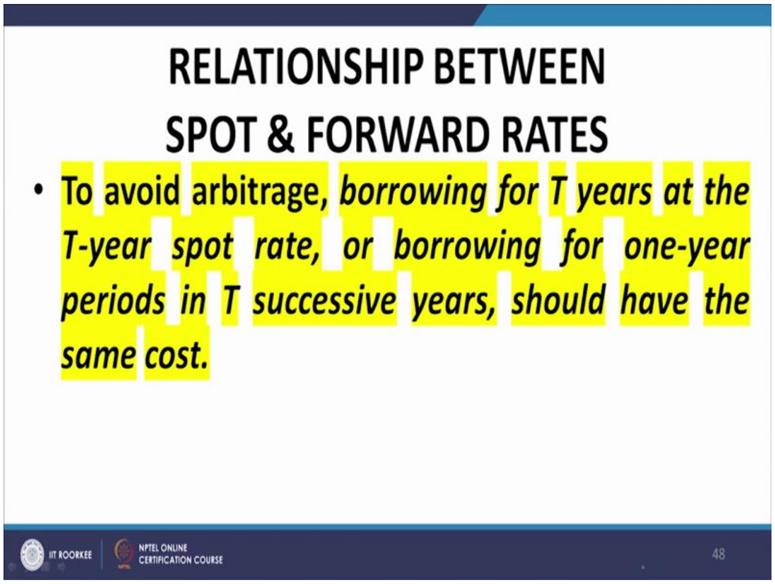

And then we will use that relationship to do valuation of a bond using forward rates instead of spot rates as we have done just know. In today's lecture, I have done the valuation of a bond using spot rates S01, S02, S03, S0t all these spot rates please note, then, we shall now do the valuation of bonds using forward rates. Thank you.