Quantitative Investment Management
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Lecture 59
Interest Rate Futures – 1

Welcome back. Today what I will do is I will talk about interest rate futures. But before I start discussing this particular topic, a brief review a very brief reviews skeletal review of certain issues in relation to money markets, because they are relevant to the topic of interest rate futures. In the money market, we traditionally encountered two types of instruments one is the traditional instruments like certificates of deposits, where the issue of the instrument is at face value and the redemption is at face value plus the amount of interest for the tenure of for which the instrument is issued.

However, there is a different type of set of instruments as well like treasury bills for that matter, these bills are issued at a discount, they are sold at a discount to face value and they are redeemed at face value.

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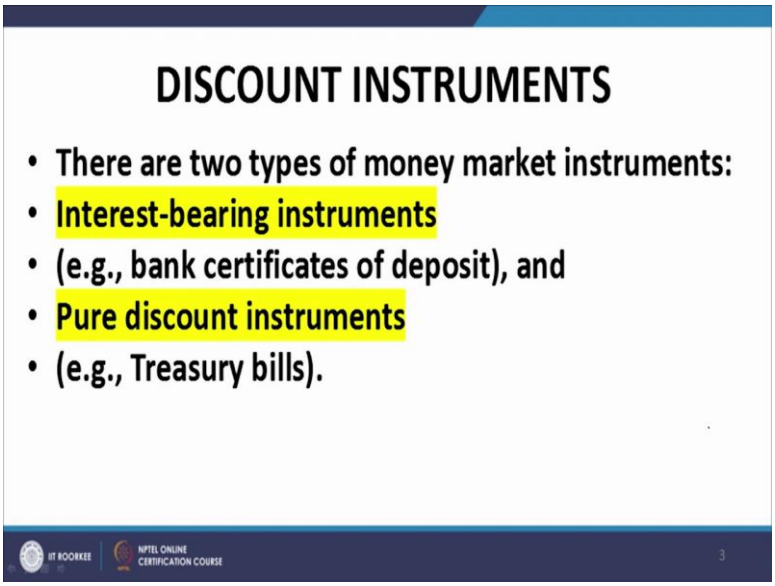

And these types of instruments are called discount instruments. So, let me repeat. In the money market scenario that is in the markets, where short term money is traded or short term instruments are traded tenure up to 1 year traded there are two types of instruments one are the conventional instruments which are issued at face value.

And redeemed at face value plus the interest plus the return over the period of the instrument over the tenure of the instrument over the term to maturity of the instrument. And the other is the discount instruments, which are issued at a discount of face value and are redeemed at face value law.

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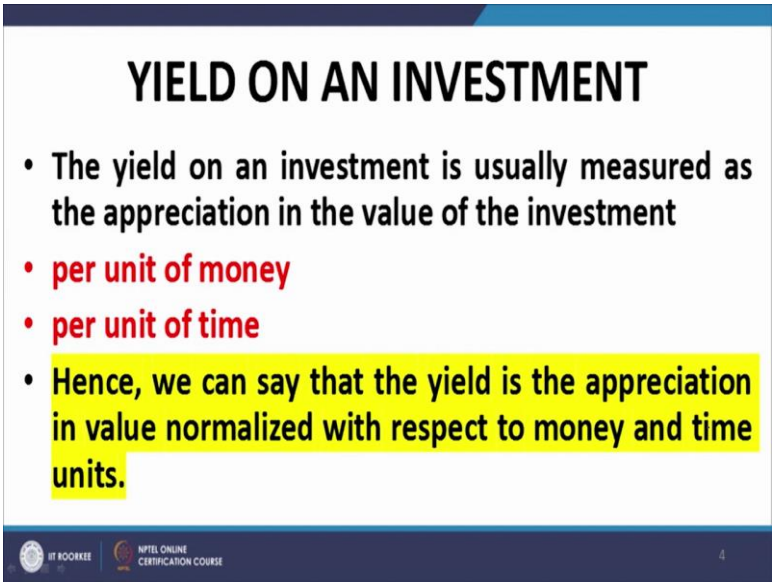

Traditionally, we define the yield on an investment or investment in a financial instrument for that matter, as the appreciation in the value of the instrument per unit of money and per unit of time. Let me repeat. We define the yield on an instrument as the appreciation in the value of that instrument per unit of money per unit of time.

Thus, in a sense, we normalize the appreciation and value of the instrument with respect to money or the amount of money amount of the investment and also with respect to the holding period that tenure or the term of the investment. Hence, we can say that the yield is the appreciation in value normalized with respect to money and time units.

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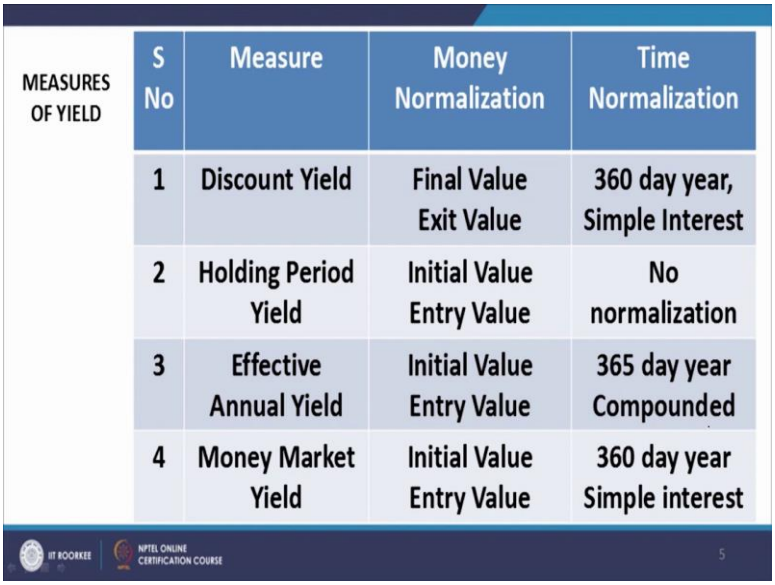

Now, here are the common methods or common approaches to the measurement of return or the measure of field in money markets. I will quickly refer to the in the first one is the discount yield, which will be very relevant for the topics that we are going to discuss today. Discount yield is the appreciation in the value of the investment per unit of face value, the normalization with respect to money is with respect to face value, that is the final value or the exit value of the investment and the normalization with respect to time is on the basis of a 360-day year and not a 365-day year.

And the method of computing this discount yield on a simple interest basis. Then we have the holding period yield which is actually the return earned over the holding period, not normalized with respect to time, there is no normalization with respect to time, it is the actual return that is earned by the investor over the holding period of its investment measured by the normalization in terms of money is in terms of the initial value, as indeed it should be.

We have the effective annual yield which is the traditional measure of return, the common measure of return which is used across various segments of the financial markets, including money markets, capital markets, derivative markets and so on. The normalization with respect to money as with respect to the initial value and that is the entry value of the investment as indeed it should be.

And the normalization with respect to time is with respect to a 365-day year. And the process of computing of this effective annual yield is through a compounding process and not a simple interest basis. So, in this case, interest is also considered. Then we have the money

market yield which is similar to the discount yield, except for the fact that it is normalized with respect to entry value. In other words, it is the appreciation in value of the investment per unit of the investment. The time normalization is with respect to a 360-day year, as in the case of the discontinued.

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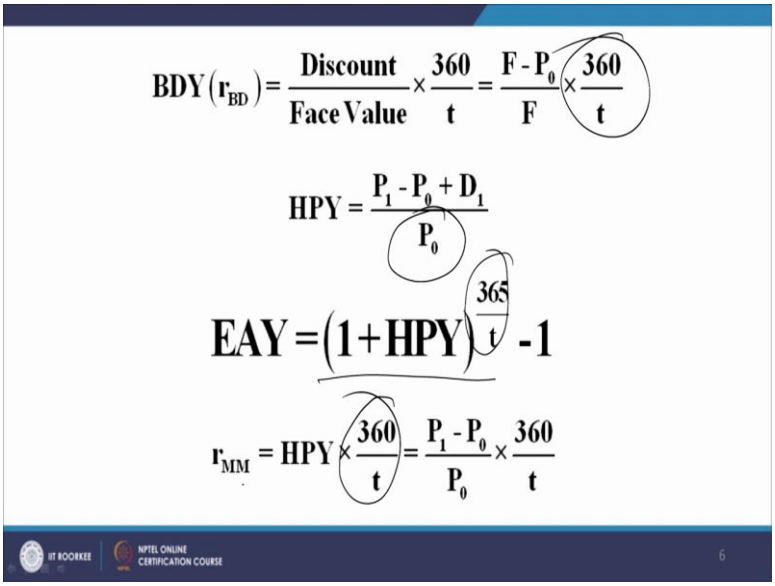

In this slide, we have the formula for the various types of yield, the discount yield is given by the appreciation in value that is F minus P0, where F is the maturity value of the investment, P0 is the entry value of the investment expressed as a fraction of F that is the terminal value and normalization with respect to time is 360 upon t as you can see here. The holding period yield is simply the appreciation in value per unit of money, the unit of money calculated with respect to the initial investment.

The effective annual yield is equal to 1 plus holding period yield compounded for the whole year by this factor 365 upon t using a 365-day year. Because we are talking about the yield, we subtract 1 from the figure that we arrived at in the first part. Then the money market return is obtained as the holding period yield annualized holding period yield, but annualized with respect to a 360-day year, and therefore, the factor use for analyzation is 360 upon t. So, the money market yield is equal to the holding period yield into 360 upon t.

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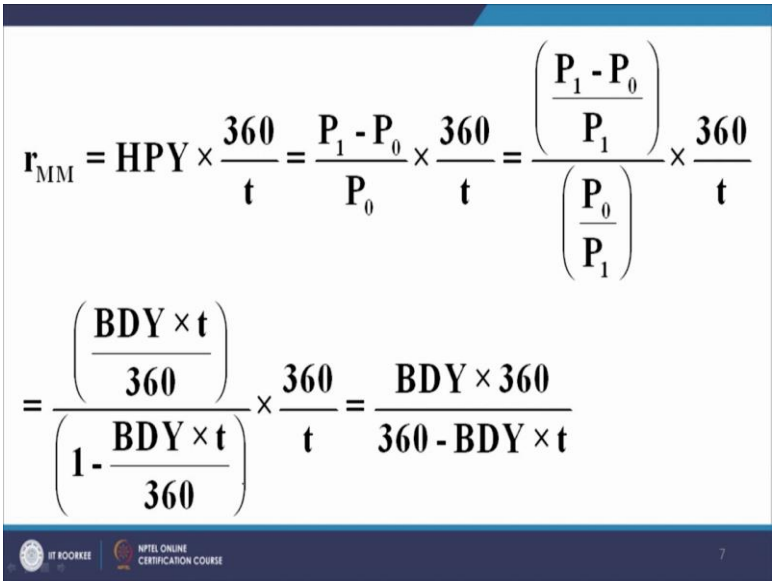

This is a relationship between the money market yield and the bankers discount yield, it is simply algebra so I will not devote time to this.

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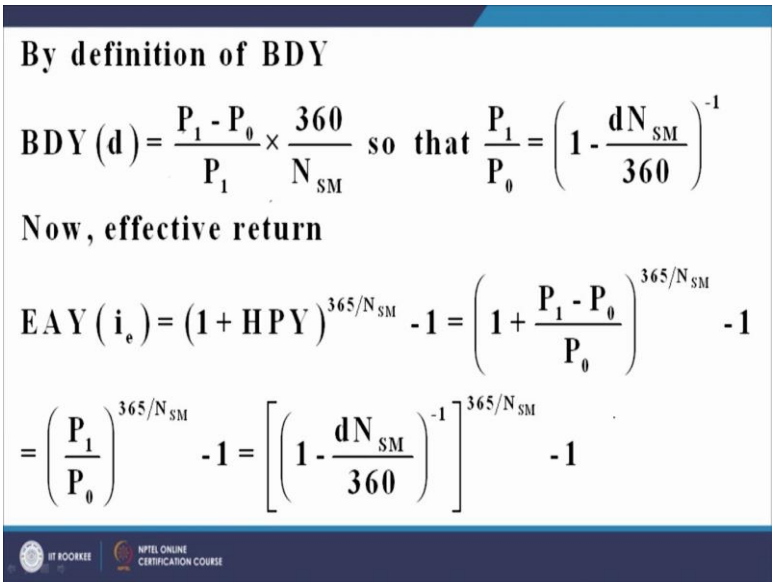

And here we arrive at a relationship between the banker's discount yield that is the discount yield and the effective annual yield. So, this is an important derivation.

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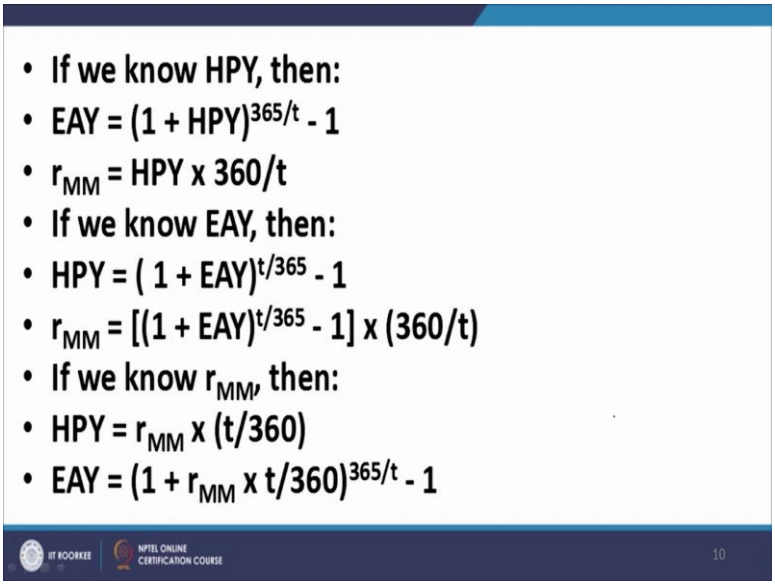

So, here we again have relationships between the effective annual yield and the holding period yield, money market yield and the holding period yield, holding period yield as a function of the effective annual yield, money market yield as a function of the effective annual yield, holding period yield as a function of the money market yield, effective annual yield as a function of the money market yield. So, these are important relationships, but there derivation is not very difficult, not very cumbersome, simple algebraic manipulations and you can arrive at these results.

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Now, we move on to interest rate futures. Let us first try to understand the philosophy of interest rate futures before we get into the nitty gritty, the new answers of interest rate futures.

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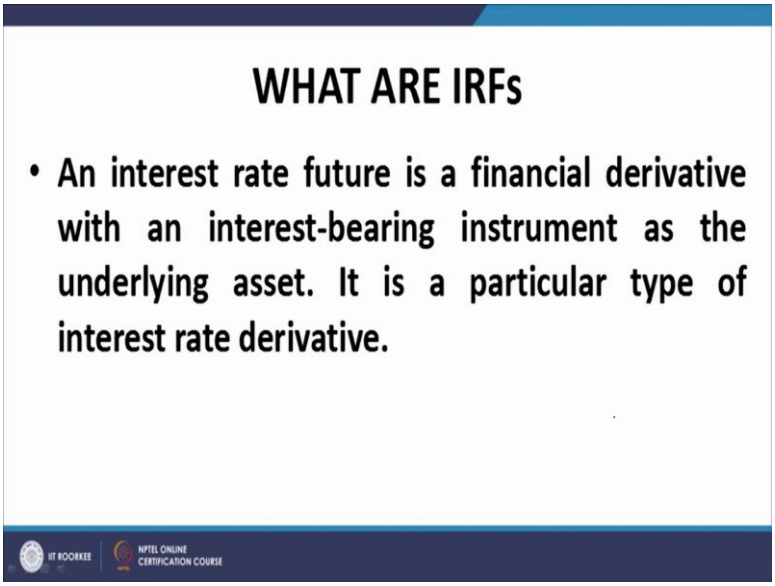

So, when I talk about interest rate futures, an interest rate future is the financial derivative with an interest bearing instrument as the underlying asset. You see, we are so far quite accustomed to futures on currencies, to futures on stocks in particular, to futures on commodities, where the underlying asset is the commodity or the stock or the currency as the case may be, the foreign currency as the case may be.

Here, we are talking about delivery of an asset, which is a bond or a treasury bill or a Eurodollar deposit. It is an interesting instrument which forms the underlying of the futures contract. So, this introduces certain nuances certain specialties insofar as interest rate futures are concerned. Let me try to explain this.

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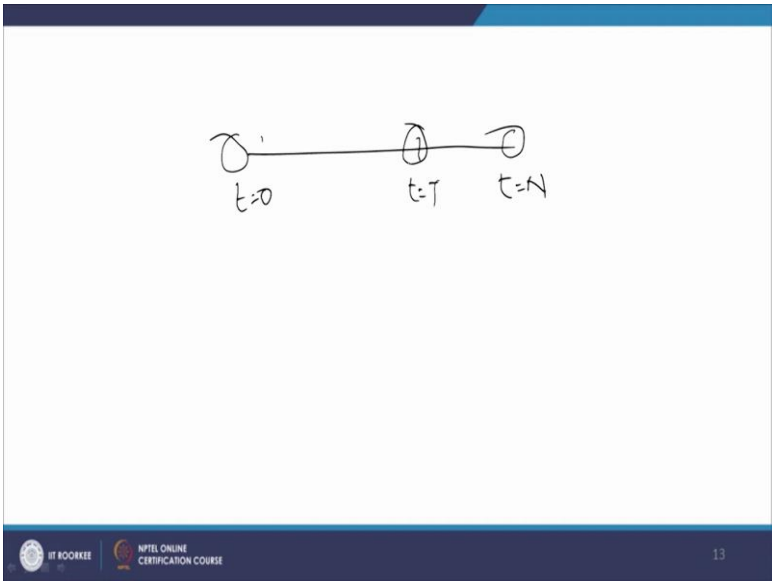

Let us say we are at t equal to 0. And we have contracted a treasury bill future. Now, what is the treasury bill future? It is a future wherein the underlying asset is a treasury bill. What does it mean? It means that at the maturity date of the future, let us call it t equal to capital T. What will happen? The party who is long in the future will receive a treasury bill from the party was short in the future.

The party who is short in the future will deliver a treasury bill to the party was long in the future. And because the underlying of that future is the treasury bill, so it is a treasury bill which will change ends. And of course, the payment would be made in the other direction. The party who is long in the future will make a payment to the party will short in the future.

So, the fact that the underlying of treasurable future is interest bearing instrument introduces a specialty a special feature in this future. Normally, when we talk about futures, there are two time points that are relevant, the time point of entry of the investment or for that matter, the time point at which the contract is initiated, and the point at which the delivery of the underlying asset is made.

So, the point at which you take the position in the futures and the maturity of the futures when the delivery of the underlying asset is made. These are the two important point time points insofar as taking positions and futures are concerned. However, in this case, there is a third time point, which let us call t equal to N. What is the time point?

The time point is the maturity of the underlying instrument. For example, if you are talking about a T-bill, then the T-bill that will be delivered a t equal to capital T, for the price which is agreed upon at t equal to 0, you see the price will be paid at t equal to capital T for the getting of the T-bill, the T-bill also change at t equal to capital T, but the T-bill when it matures with the party has taken possession of the T-bills.

Who was long in the future will now get the payment will now get the cash at the data of maturity of the T-bill, Let us call it capital N. So, there are three important time points when we talk about T-bill futures or for that matter any interested future. Number 1, the point of initialization of the contract or the point at which you enter the contract. Number 2, the point it with the contract matures the futures contract matures, that is when the T-bill will change ends, so the underlying interest-bearing instrument will change ends, and the payment will be made to the party was short by the party who is long.

And then finally, the maturity of the underlying instrument, but not the future, maturity of the underlying instrument, when the party who is long in the instrument, the party who has bought the underlying instrument at under the futures contract will now receive the payment from the issuer of the instrument.

So, in this case, there are 3 time points which become relevant t equal to 0, t equal to capital T that is the maturity of the futures, and t equal to capital N which is the maturity of the underlying instrument whether it be a T-bill or a Eurodollar deposit or a T-bond as well. So, this is special to the interest rate futures, which was not encountered when we talked about futures or other underlying assets.

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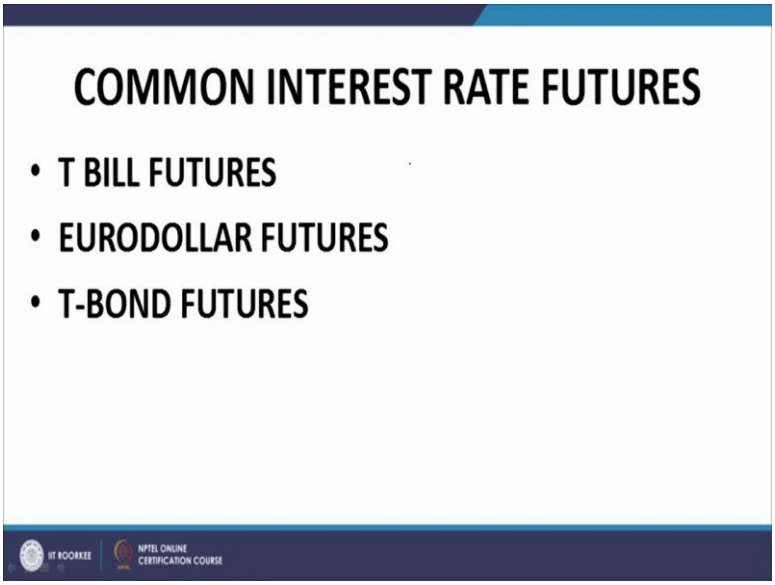

Common interest rate futures as I mentioned briefly, there are T-bill futures which are quite common, we have T-bill futures in India as well, on the 91 day T-bills, I will come back to that in a minute. Then, we have Eurodollar futures which are futures contracts written on Eurodollar deposits. And then we have T-bond futures which are futures contracts written on T-bonds, long term interest wherein government securities.

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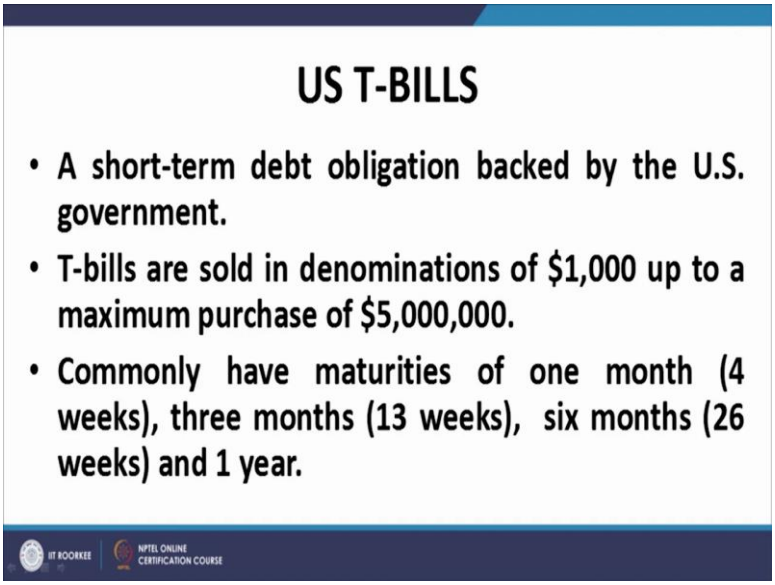

So, a quick recap or a quick review of the properties of US T-bills, features of US T-bills. A short-term debt obligation backed by the US government is similar to what we have the T-bills in India. I shall be covering primarily the provisions of US T-bill futures. So, in that context, a quick review about US T-bills. T-bills are sold in denominations of US dollars

1000 up to a maximum purchase of US dollars 5 million. Commonly have maturities of 1 month that is 4 weeks, 3 months that is 13 weeks, 6 months that is 26 weeks and 1-year.

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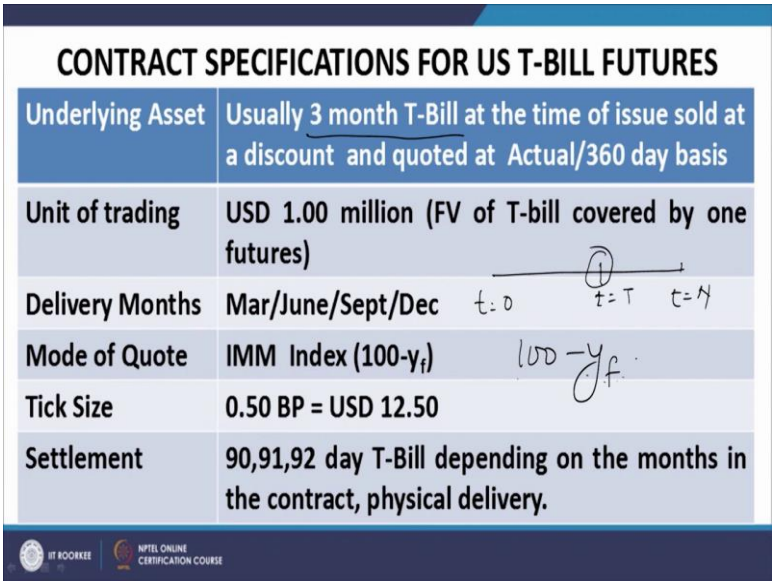

These are the contract specifications of T-bill futures. Let us quickly read through this. Underlying asset is usually a 3-month T-bill. Now, a 3-month T-bill can have 90 days, 91 days or even 92 days depending on the timing of its issue. So, usually a 3 month T-bill at the time of issue sold at a discount and quoted at actual upon 360-day basis. The unit of trading is US dollars 1 million.

Face value of the T-bills covered by one futures contract as US dollars 1 million. Delivery months are March, June, September and December. IMM index, now this is how the quotations, now please note this point I have a word quotations, this is how the quotations of the prices or the futures prices are made in the T-bill futures markets. How it is done? It is done by the IMM index. And what is the IMM index?

The IMM index is given by 100 minus yf. And what is yf? yf is the futures discount yield. How do we work out the futures discount yield? That is the yield worked out on the basis of the futures price. So, let us say this is t equal to 0, this is t equal to capital T, and let us say this is t equal to N. Now, this is at t equal to 0. Suppose you enter into a futures contract to buy a T-bill at t equal to capital T.

On the basis of the price that is embedded in that futures contract that is agreed upon in the futures contracts, the corresponding discount yield is called the futures discount yield and

expressed as a percentage and deducted from 100 gives you the IMM index of the futures contract. So, the IMM index is equal to 100 minus futures yield, futures yield is worked out on the basis of the price at which the futures contract is entered into at which the delivery will take place at t equal to capital T.

On that basis, we work out the discount yield and that gives us the futures discount yield. The tick size is equal to 0.50 basis points that corresponds to US dollars 12.50. When we note that the time of the futures is 3 months, and the credit amount is 1 million 0.50 change in basis points of the interest rate translates to an amount of 12.50 US dollars per T-bill futures contract. The settlement is achieved through a 90, 91, or 92-day T-bill depending on the months of the contract and physical deliveries envisaged.

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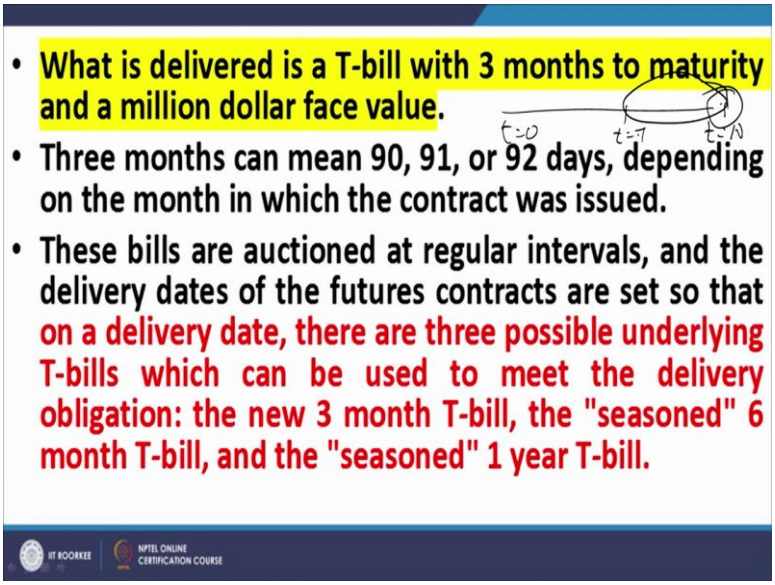

What is delivered is a T-bill with 3 months to maturity and a million-dollar face value. So, usually again coming back to this timeline t equal to 0, t equal to capital T, and T equal to N. Now, this particular period is pretty much fixed, it has this period as usually 3 months. The delivery that takes place at t equal to capital T is of a bill which matures in 3 months from that point in time that is the difference between t equal to capital T and t equal to capital N is equal to 3 months.

So, what is delivered is a T-bill delivered at what point, delivered a t equal to capital T. What is delivered is a T-bill with 3 months to maturity. Therefore, time difference between t equal to capital T and t equal to N is equal to 3 months and it has a million-dollar face value. So,

the party who received the T-bill was long in the future, will receive 1 million dollars at t equal to N.

The bill will be transferred in his name at t equal to capital T but the actual cash received the funds on account of the holding of the T-bill will be transferred to his account on at t equal to N of 1 million US dollars. Three months can mean 90, 91, or 92 days, depending on the month in which the contract was issued.

These bills are auctioned at regular intervals and the delivery dates of the futures contracts are set so that on a delivery date there are three possible underlying T-bills which can be used to meet the delivery obligation. The new 3-month T-bill, the seasoned 6-month T-bill and the seasoned 1-year T-bill.

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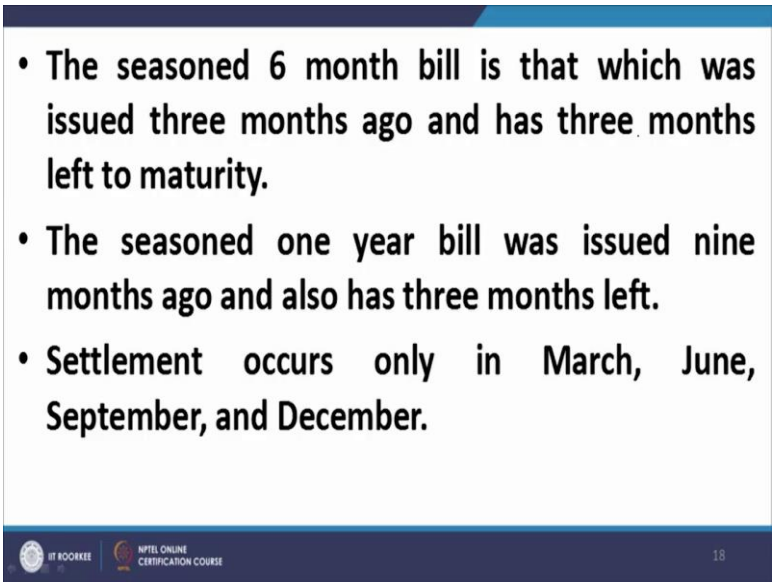

The season 6-month T-bill is that which was issued 3 months ago so that it has 3 months more to go when it is delivered under to meet the delivery obligation under the futures contract. It has 3 months to maturity. The seasoned 1-year T-bill has, again 9 months have passed since the issue of that season 1-year T-bill, 3 months are still to go. And this can be also used to meet a delivery obligation under a T-bill futures.

So, the seasoned 1-year T-bill was issued nine months ago and it has 3 months left. The basic point is that whatever bill is delivered, whether it is a new bill, a 6-month bill or a 1-year bill, the remaining tenure of the bill should be 3 months. That is the important part. The bill that is

delivered under the futures contract that forms a substratum, the underlying asset of the futures company is a 3-month bill and it has should have 3 months to maturity.

It may be a 1-year bill issued 9 months ago and 3 months left. It may be a 6-month bill issued 3 months ago and 3 months left. Or it may be a fresh bill. So, settlement occurs only in March, June, September and December.

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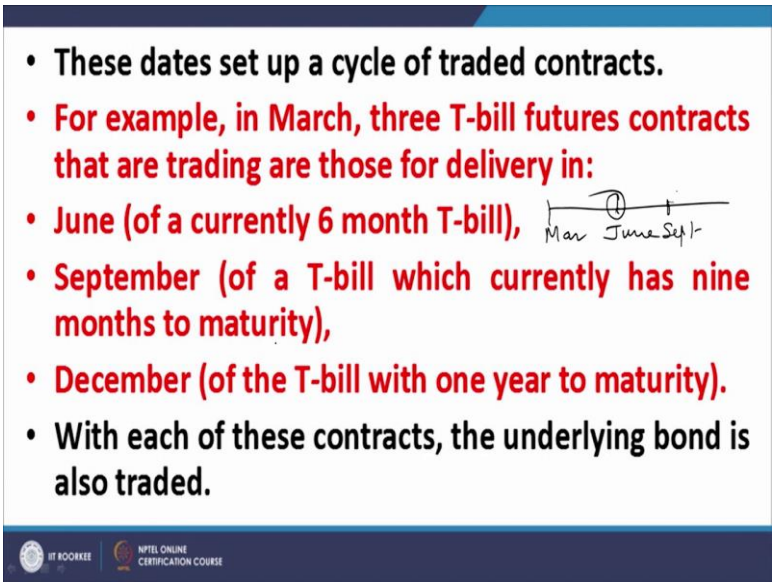

These dates set up a cycle of traded contracts. For example, in March three T-bill futures contracts that are trading are those for delivery in June of a currently 6-month T-bill because then when you have delivery in June, then you are in March, please note, you are in March, this is March, This is June. And at the point in June, when your futures contract is going to mature for delivery, that means, at June, you should be able to deliver a T-bill with a maturity of 3 months.

So, this period between June and September, that bill should be maturing in September, you are in March, that means it is a 6-month T-bill, it has 6 months to go and it will be delivered 3 months from now under the futures contract, at which point in time, it will be a 3-month bill. Similarly, you can have a bill in March, there are 3 T-bill futures contract that are trading those for delivery in June for a current 6 moths bill that means what I explained.

That is a bill which has 6 months to go, which will be delivered in June, when it has 3 months to go. Or you can have delivery in September of a bill which has 3 months to go when it is delivered in September. That means it maturity should be December and that means it has 9

months to go as of now. So, it has 9 months to maturity as of now. And then we have delivery in December of a bill which matures in March next year. So, that Bill has 1-year to go as of now. With each of these contracts, the underlying bond is also traded.

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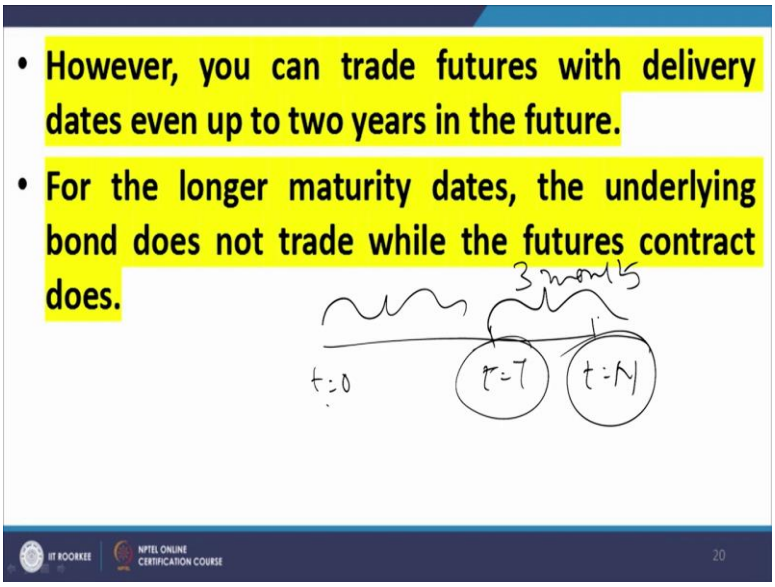

However, we can trade futures for delivery dates even up to two years in the future. For the longer delivery dates, the underlying bond does not trade while the futures contract does. So, please note this special feature t equal to 0, t equal to capital T, t equal to N. Now, this period is pretty much fixed this is equal to 3 months. But this need not be fixed, this need not be 3 months. Please note this point.

Between t equal to 0 and t equal to capital T need not be fixed. The term of the futures contract need not necessarily be 3 months, it can be more than 3 months obviously, it can be up to 2 years as you can see here on this slide. But what is to be delivered is to be a bill of 3 months maturity. So, between t equal to capital T, which is the maturity of the futures, and the maturity of the underlying asset, this period is pretty much fixed by the terms of the futures contract by the document of issue of the futures contract to be 3 months.

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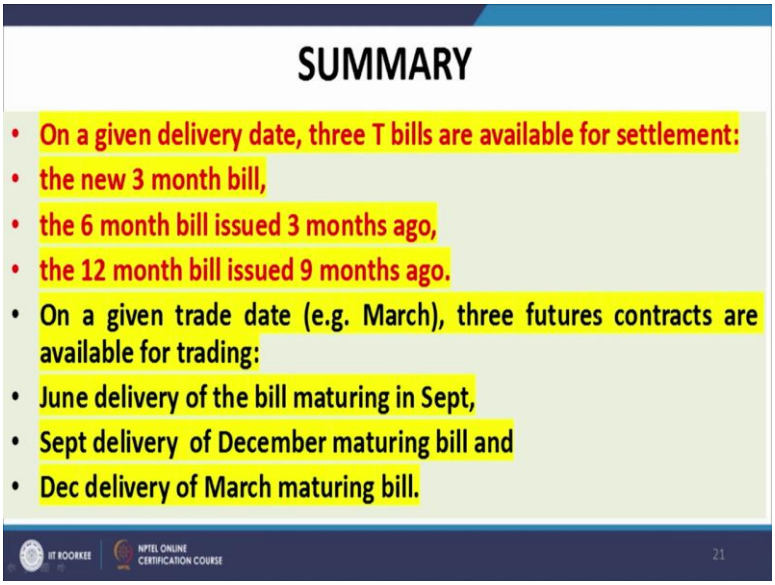

So, summary. On a given delivery date, three T-bills are available for settlement. The new 3 months bill, the 6-month bill issued 3 months ago, the 12-month bill issued 9 months ago. On a given trade date, for example March, three futures contracts are available for trading. The June delivery of a bill maturing in September, September delivery of a December maturing bill, and December delivery of a march maturing bill.

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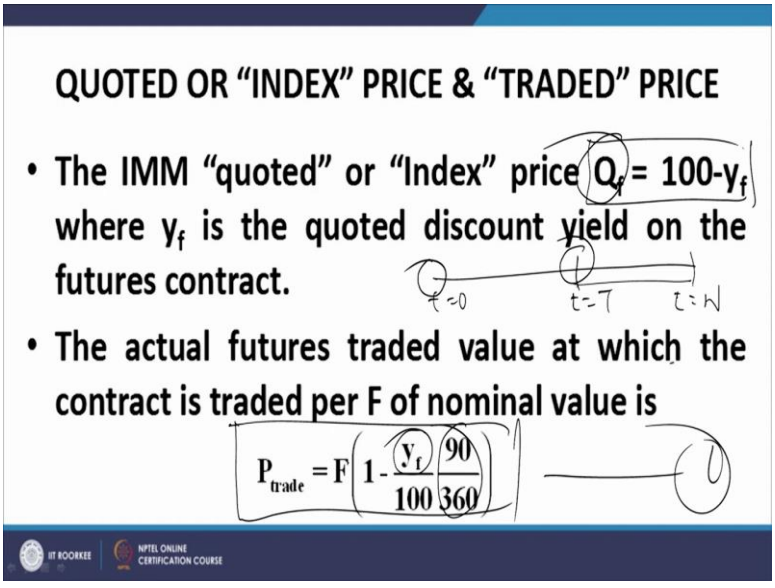

So, we talk about the IMM index, I briefly referred to the IMM indexes a few minutes ago. The IMM index is calculated as per this formula, what is rf I explained that, it is the futures discount yield, it is the discount yield worked out on the basis of the futures price. I repeat. Let us say this is t equal to 0, this is t equal to capital T, this is t equal to N.

Now, on the basis of the price that is agreed upon a t equal to 0, but which will be transacted which will be settled at t equal to capital T the yield that is calculated on the investment over this period that gives us the futures discount yield. So, now please note this is the method of quotation Qf is the quotation parameter, but it is not the basis of arriving at the mark to market settlement or the final settlement or the traded price, Qf is not the traded price.

How do we work out the traded price? We work out the traded price by the formula that I enclosed within the box, let us call this equation 1, P trade is equal to F, what is F, F is the face value of the treasury bills that form the substratum of the futures contract into 1 minus yf, yf is the futures discount yield I refer to it just now, upon 100. Please note, this yf that we are using here will be in the form of a percentage.

So, to convert it to a number we divide by 100 and we multiply it by 0.24 that is 90 upon 360. So, this is the formula that we use for working out the contract price that will form the basis of mark to market settlement, and that will also form the basis of the final settlement of the contract on the date of maturity of the futures contract.

Number 1, we have the IMM index, which is equal to 100 minus the futures discount rate expressed as a percentage. And then we have the traded price, which is obtained by equation number 1.

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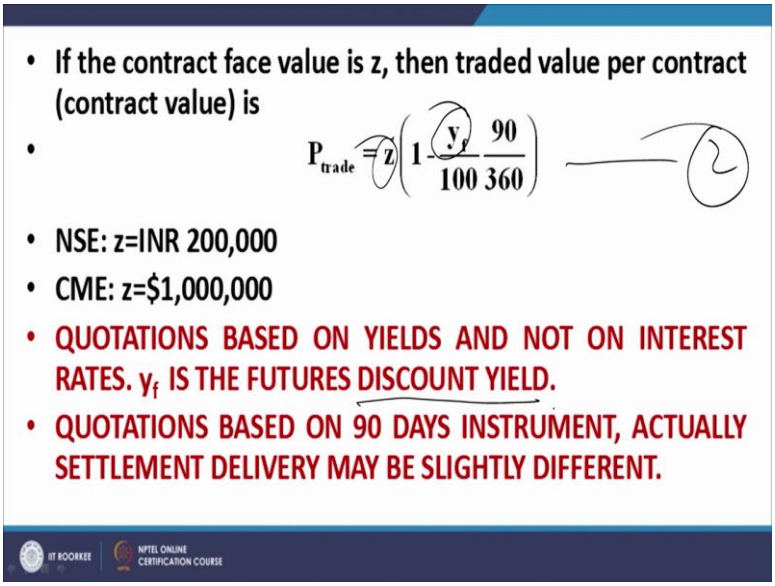

Now, depending on the contract size if the contract value is z then the traded price per contract will be given by equation number 2 where z is the face value of the treasury bills

which are covered by the futures contract. For the NSE 91-day T-bill futures contract. The z value is equal to two lakhs or 200,000. And in the case of the US T-bill futures contract, the z value is equal to 1 million dollars.

Quotations are based on yields and not on interest rates. Please note this, yf is the discount yield, it is not the futures interest rate it is a discount yield. Please note this fact very carefully, we are talking about discount yield. And that is the reason that before starting this topic, I talked about discount yields, which are prevalent which are rampant, in fact, in the T-bills markets throughout the world.

This is yf it is the discount yield calculated on the basis of the appreciation value per unit of the final value of the investment not the initial value and normalized with respect to a 360-day year. So, quotations are based on yields and not on interest rates, yf is the futures discount yield, this discount yield is important, not interested. Quotations are based on 90 days instruments, actual settlement delivery may be of 90, 91, or 92 days, as I mentioned earlier.

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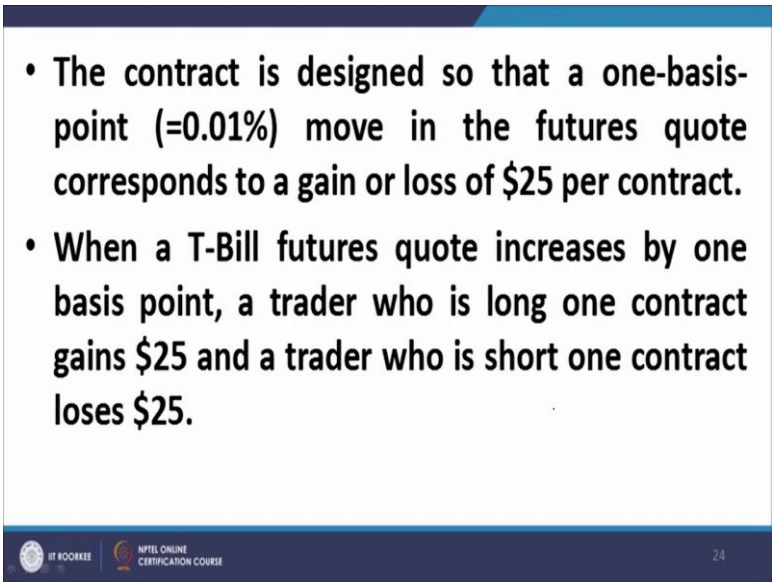

The contract is designed so that one basis point 0.01 percent move in the futures court corresponds to a gain or loss of US dollars 25 per contract, if there is a change of one basis point in the discount yield, then that corresponds to a change of 25 US dollars in the value of the contract. As I mentioned, the tick size is equal to half of a basis point.

And that corresponds to 12.5 US dollars in the contract value. When a T-bill futures quote increases by one basis point, that trader who is long in the contract gains dollars 25 and the trader who is short in the contract loses dollars 25. Let me repeat. When a T-bill futures quote increases by one basis point, a trader who is long one contract gains dollars 25 and a trader who is short one contract loses 25. It is quite simple.

When you know the two facts, first fact the value of the face value of the T-bills covered by one contract is equal to 1 million. And the second fact is that the prices or the indices are calculated with reference to a 90 upon 360 factor, day count factor. Using these two factors, we can easily show that one basis point corresponds to a 25 US dollar change in the price of the contract.

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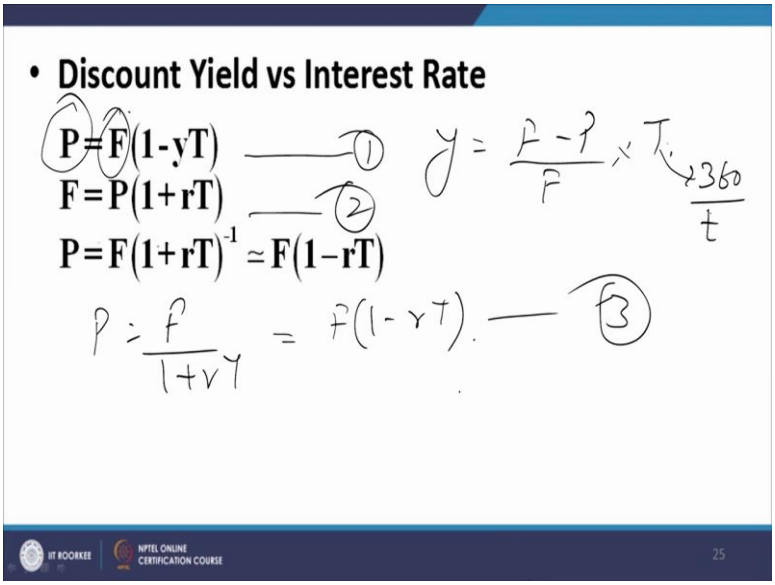

Now, this is the slide which is slightly a digression, but this brings to you the difference between the discount yield and the interest rate. Equation number 1 here is the formula for the discount yield. P is the value at which you enter the investment, F is the final value of the investment. So, when we talk about discount yield, we are basically trying to use the current price as the determinant of the discount yield.

But the appreciation and value of the of the investment is worked out on the basis of F you can see here, if I rearrange this, what they get is y is equal to F minus P divided by F into T. So, this F minus P is the appreciation and divided by F that is normalized with respect to the exit value the final value of the investment and multiplied by T to extrapolate it to 1-year.

However, you may do it the convention is to use T is equal to 360 upon small t, where small t is the holding period. This is the traditional formula, equation number 2 is the traditional formula for simple interest. Interestingly, if you use equation number 2 and invert equation number 2 you can write it as P is equal to F divided by 1 plus rT, and this to a first order approximation gives you F into 1 minus rT.

So, to a first order approximation if you compare equation number 1 and equation number 3, you find that they are the y and the r factors, that is the discount yield and the interest rates are pretty much close to each other. But the important thing is, this is a first order approximation, it breaks down when the values of I and r are large.

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Now, I shall discuss the short-term interest rate futures applications in the next lecture. Thank you.