Quantitative Investment Management
Professor J. P. Singh
Department of Management Studies
Indian Institute of Technology Roorkee
Lecture 57
Stock Index futures - 1

Welcome back. So, let us continue today we will start a new topic, we will be talking about two different types of futures, the index futures and the interest rate futures. These futures have widespread applications on the one hand on the other hand they have some nuances, which the learner should be acquainted with and that is why I have picked these two specific type of futures to be discussed in detail.

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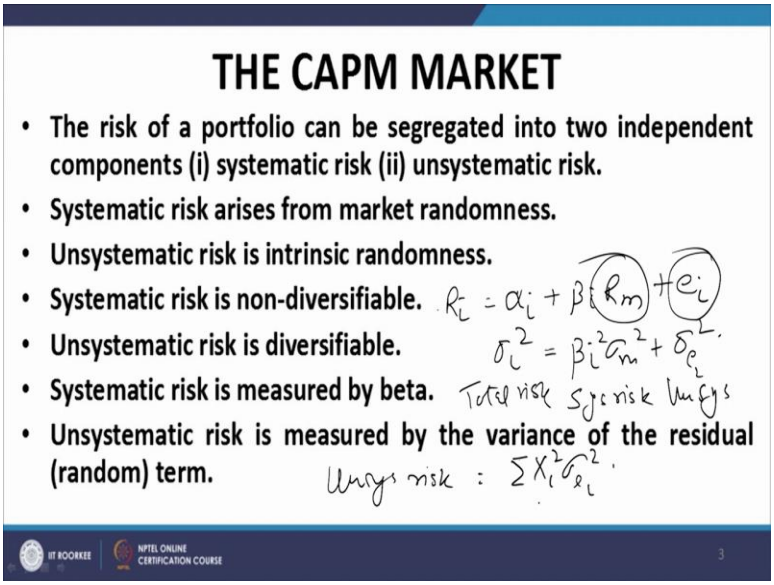

So, let us start with the index futures. But before I talk about index futures, I need to recap or revise certain aspects of the CAPM model of stock prices or stock returns. And therefore, we will have a quick review of CAPM and then proceed to the theory of index futures. The CAPM market, let us start with the CAPM market, the risk of a portfolio can be segregated into two independent components.

And these two components of risk are called systematic risk and unsystematic risk, systematic risk is the risk that percolates term to the individual security or the portfolio due to the randomness that prevails in market returns or market prices.

Unsystematic risk is intrinsic to the various securities that form the portfolio that is something which is specific to the securities which is which is independent of the flow of or the randomness of the market.

Systematic risk is non diversifiable because it relates to a common index, which is the market index, whereas, unsystematic risk can be diversified by having a collection of securities to form a portfolio. In fact, diversification is one of the very common approach approaches to risk mitigation in the context of portfolio management.

So, if you combine a number of securities, the larger the number of securities in fact, the greater would be the the reduction in unsystematic risk, because, you see, the point is unsystematic risk is basically random. And if you come may have a large enough combination of random variables, it is quite likely that you end up with lesser variance on the unsystematic risk thereby goes down.

Therefore, the systematic risk is an diversifiable the unsystematic risk diversifiable. The systematic was measured by the regression coefficient between the market returns and the and the individual security or portfolio returns. With the market return being obviously, the independent variable and the this security of the portfolio return being the dependent variable, it is called beta.

So, systematic risk is measured in terms of beta unsystematic risk, on the other hand, is the variance associated with the residual term, when we write the CAPM equation, let me just explain it in a little bit more detail in the context of the CAPM model, or for that model, the single index model, we can write the return on individual security Ri, R is the security as comprising of the alpha let us call it alpha i plus beta i Rm I am using the thing single index model here plus a random error term Ei.

Now, the variance of this Ei is supposed to generate the unsystematic risk that is intrinsic to the security please note that, whereas, the variance of Rm is deemed to generate the systematic risk

component of the risk, and please note, both these risk are orthogonal, they are independent of each other.

In fact, we have the relation sigma i squared, which is the total risk of the portfolio is equal to beta i squared sigma m squared plus sigma of e i square this so this is the unsystematic risk and this is this systematic risk, this is the total risk.

So, let us continue unsystematic risk is measured by the variance of the residual term. And in the context of a portfolio unsystematic risk of a portfolio is measured in terms of summation of xi squared sigma of e i square the summation over all the securities constituting the portfolio,

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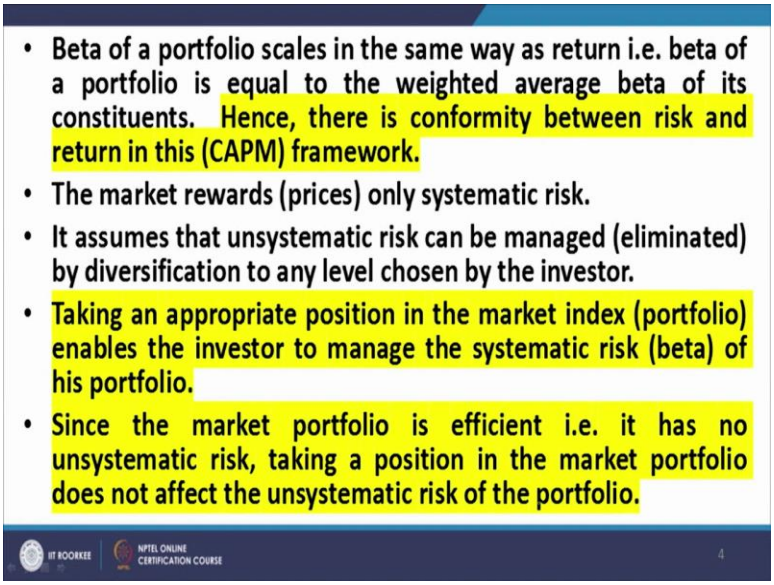

Beta of a portfolio scales in the same way as return that is the beta of a portfolio of securities is equal to the weighted average beta of the constituent securities, just like expected return of a portfolio is equal to the weighted average expected return of the constituent securities. Hence there is conformity between risk and return in the CAPM Framework.

This is one of the very important outcomes of the research of William Sharpe, which he built upon in the context of the mean variance portfolio optimization model propounded by propounded by Markowitz.

The market rewards or prices only systematic risk, this is the fundamental assumption of the CAPM model or the fundamental outcome rather of the CAPM model that market will give you expected returns provided you take systematic risk if your if your portfolio is devoid of systematic risk, and you have a huge amount of unsystematic risk, you will not get any market expected returns, or you will get any do not get any rewards from the market market rewards only systematic risk, the higher the systematic risk level of your portfolio, the higher your expected returns.

But please note the word expected returns is not actual returns, it is expected returns. So, higher systematic risk means what means you are likely to get higher returns. But there is also a higher likelihood because the systematic risk part is higher. So, there is also higher likelihood of the returns not materializing. So the expected returns are higher, the possibility of the returns not materializing is also higher if you are taking high systematic risk.

And conversely, if you are taking low systematic risk, then you will have low expected returns, but those expected returns would be very likely to mature to materialize, because the risk is less. So, that is the relationship between risk and return in fact.

But the important outcome of the CAPM model insofar as the CAPM is concerned is that market will only consider systematic risk when it is pricing a security, security that equilibrium will be priced in in context of or in conformity with their respective levels of systematic risk and so, also the investor is higher the systematic risk taken by an investor higher would be his expected returns.

It assumes so, there in continuation of the previous postulate, it assumes that unsystematic risks can be managed or eliminated for that matter by adequate diversification to any level chosen by the investor. So, it assumes that the unsystematic risk can be managed or by diversification to any level chosen by the investor and as a result of it the market does not give you any benefit by taking for taking any unsystematic risk you may take as much as unsystematic risk as you like but the market will not reward you for that.

Taking an appropriate position in the market index market portfolio enables the investor to manage the systematic risk of portfolio that is quite simple. You see, the basic thing is that the

beta which is a measure of systematic risk of the portfolio is equal to the weighted average beta of the constituents.

So, if one of the constituents has the market portfolio, then obviously, that (mark) the amount that you put in into the market portfolio enables you to manage the systematic risk of the overall portfolio. The added point is that because the market portfolio is efficient as we will come to in a minute, the unsystematic risk of the market portfolio is 0 and therefore, addition of the market portfolio to your portfolio will not change the level of unsystematic risk of your portfolio.

So, while on the one hand, the investment in the market portfolio will enable you to manage the systematic risk of your portfolio, it will let the unsystematic risk remain unchanged. So, taking an appropriate position in the market index or the market portfolio enables the investor to manage the systematic risk that is beta of the portfolio. Since the market portfolio is efficient that is it has low unsystematic risk taking a position in the market portfolio does not affect the unsystematic risk of the portfolio that is what I explained just now.

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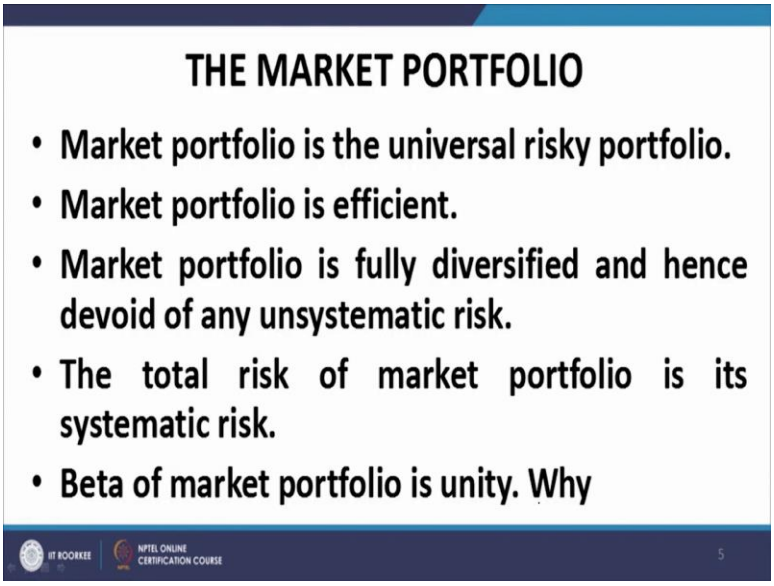

Now, talking about the market portfolio the market portfolio is the universal risky portfolio I do not have time to go into the entire theory of the CAPM model but basically the CAPM model envisages two investment or model or two asset model, where one of the asset is the risk free

asset and the other asset is the market portfolio and it says that any level of risk envisaged by the investor can be created using these two components.

In other words, every investor in the market would be investing some money or some component of his money some component of his investment in the market portfolio and some component of the investment in the risk free portfolio in order to generate a portfolio of his desired risk level. So, that that is basically what we also call the two point theorem.

So, market portfolio is the universal risky portfolio, market portfolios is efficient it is devoid of any unsystematic risk because it is fully diversified, you cannot diversify it further and as a result of which it has no unsystematic risk market (port) portfolio is fully diversified and hence devoid of any unsystematic risk that is what I mentioned just now.

The total risk of the (mark) for market portfolio is a systematic risk and beta of the market portfolios is unity because you are aggressing the market portfolio against itself and therefore, beta would be equal to 1.

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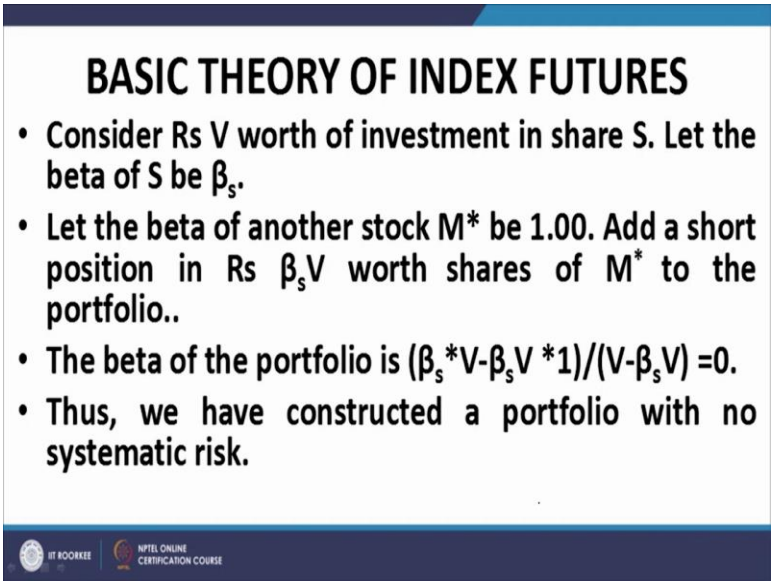

Now, let us talk about basic theory of index futures. Consider rupees V rupees V worth of investment in a particular share, let us call the share is let the beta of SB beta S let the beat of another stock M star stock M star so we have we have an investment of V in stock S the beta of

stock S as beta S and we add to that stock M star which has a beta of 1 and a short position in M star is taken worth rupees beta S into V.

So, now, we have a portfolio comprising of V value of share S with beta equal to beta S and beta SV value short of a stock M star and beta M star is equal to 1 if you work out the beta of this combined portfolio, you will find a very interesting result because the beta of the portfolio is equal to the weighted average beta of its constituents what we find is that the beta of the portfolio consisting of S and M star is equal to 0.

And therefore, by using this mechanism, this simple elementary mechanism, we have been able to construct a portfolio which is devoid of systematic risk a portfolio completely devoid of systematically. We have this portfolio we have this security S with an investment V in S with a beta of beta S, we short stock M star with a beta of M with a value equal to V S in V into beta S and what we find is that the beta of this combined portfolio consisting of a long position in S and a short position in M star is equal to 0.

And therefore, this combined portfolio as 0 systematic risk. Now, what happens if this this stock M star that I have been talking about is replaced by the market portfolio as we saw just now, the beta of the market portfolio is also equal to 1 and systematic risk is measured only in terms of beta and therefore, by replacing M star by M, we are not affecting the systematic risk of the portfolio and the systematic risk continues to be 0.

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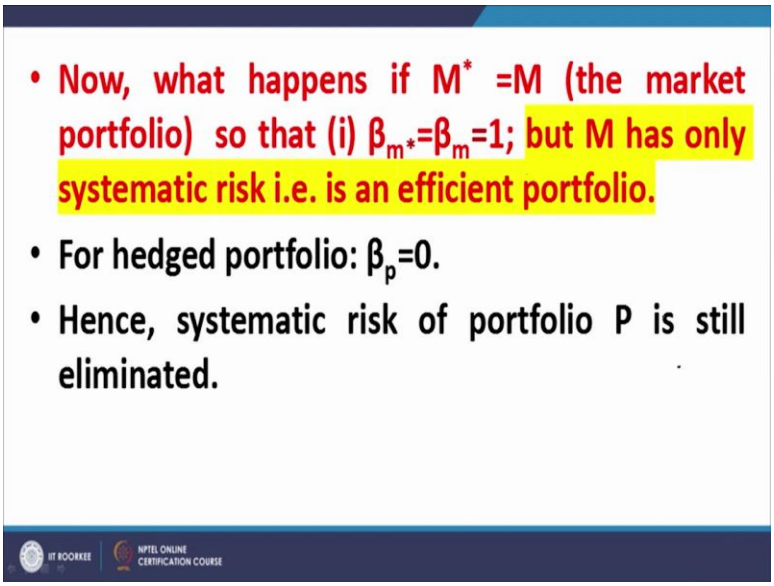

So, what happens if M star is equal to M the market portfolio so, that beta M star is equal to beta M is equal to 1, but M. Now, let us talk about the unsystematic risk, as far as the unsystematic risk is concerned, the unsystematic risk of M is equal to 0 why because it is the market portfolio it is fully diversified and therefore, there is no unsystematic risk insofar as M is concerned.

And because the portfolio M has no unsystematic risk, the addition of M to S will not change the unsystematic risk of S that is the interesting part. So, on the one hand by using this market combination of market portfolio and S which we started with, we have been able to construct a portfolio that has 0 systematic risk, but has unsystematic risk which is identical to the original portfolio of S.

So, let me read it out for you. Now, what happens if M star is equal to M the market portfolio so, that beta M star is equal to beta M is equal to 1, but beta M has but M has only systematic risk, because it is a efficient portfolio. As far as the hedge portfolio is concerned, beta p is equal to 0 systematic risk of the portfolio continues to be 0 here on the replacement of M star by M.

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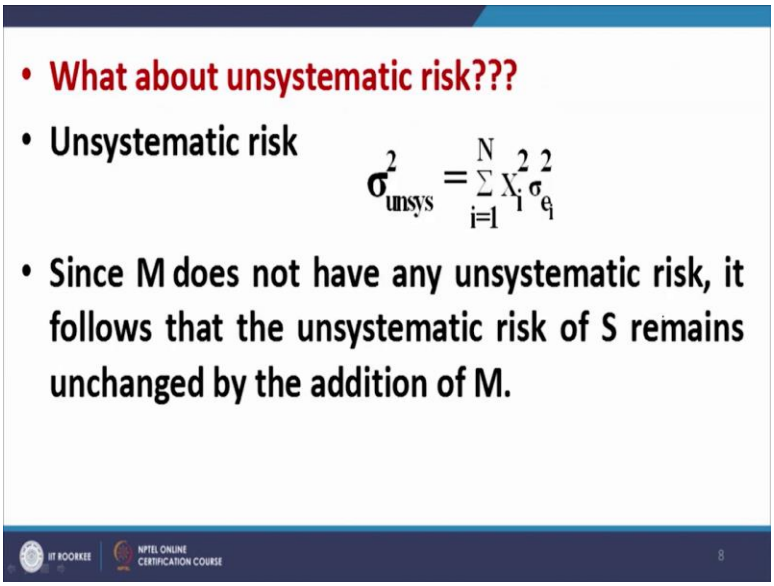

However, what about unsystematic risk? The unsystematic risk is unchanged because the addition of M to the portfolio will not change the unsystematic risk of S, because M has no unsystematic risk. Since M does not have any unsystematic risk, it follows that the unsystematic risk of S remains unchanged by the addition of M.

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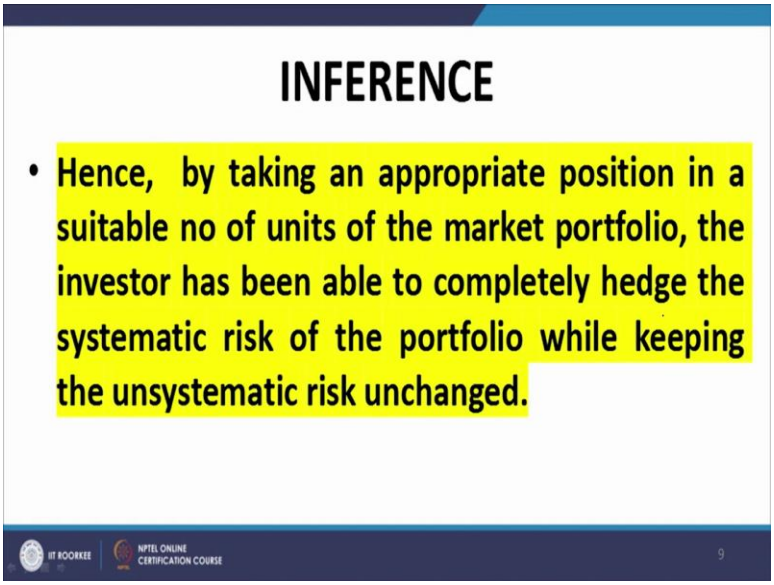

The inferences by taking an appropriate position in a suitable number of units of the market portfolio, the investor has been able to completely hedge the systematic risk of the portfolio,

while keeping the unsystematic risk unchanged. So, systematic risk has been eliminated unsystematic risk has been unaffected.

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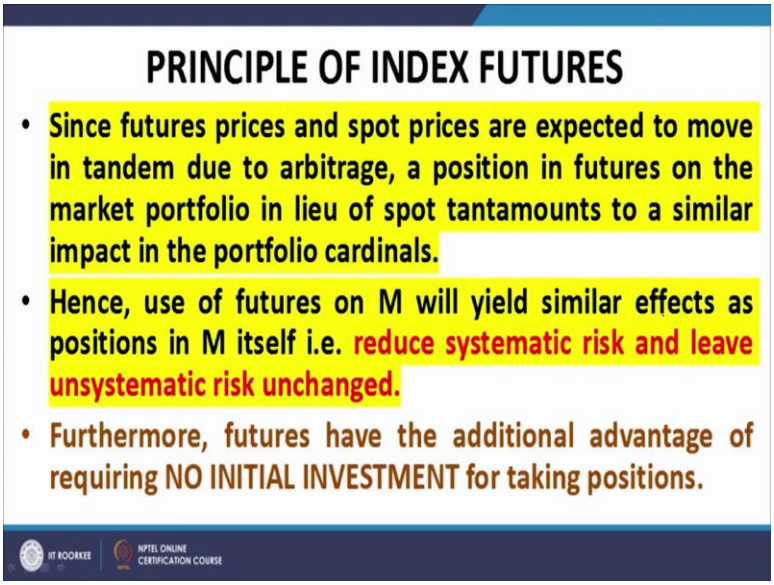

So, principle of index futures, this is this forms a backdrop on the basis of it the index futures have been created in the market. Since futures prices and spot prices are expected to move in tandem due to arbitrage considerations, a position in futures on the market portfolio you are now replacing the market portfolio by futures on the market portfolio you started with M star.

And arbitrary portfolio with a beta of 1 it may or may not have unsystematic risk, we replaced M star by M the market portfolio which has only systematic risk and no (un) which has only a systematic risk and no unsystematic risk, yes, and now, what we are doing is we are replacing M that is the market portfolio by futures on the market portfolio.

So, since futures prices and spot prices are expected to move in tandem due to arbitrage position in futures on the market portfolio in new spot amounts to a similar impact on the market (card) on the portfolio cardinals. So, if you replace the investment in the market portfolio by the appropriate position in the futures on that market portfolio, you are likely to get a similar impact on your original portfolio comprising of stockers.

So, that is what is the importance or that is what is the philosophy rather of index futures. Hence, use of futures on M will yield similar effects as positions in M itself instead of using the spot investment and the market index we are now investing in the futures on the market index that is the it reduces systematic risk and leaves the unsystematic risk unchanged.

So, if you take futures position in the market portfolio as part of your portfolio, then what happens that is the systematic risk is reduced or systematic risk is managed rather whether it is reduced or increased would depend on the nature of your position nature of your position in the futures on the market portfolio and the unsystematic risk remains unchanged. Furthermore, futures have the additional advantage of requiring no initial investment other than margins of course, for taking positions.

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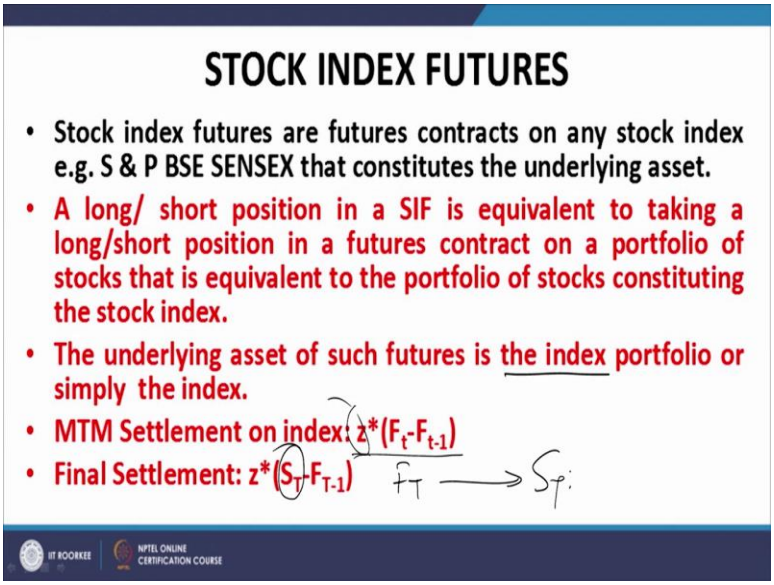

So, stock index futures, what are stock index, stock index futures or futures contracts or any stock index like the S & P BSE index, Sensex that constitute the underlying asset along oblique short position in a stock index futures is equivalent to taking a long oblique short position in a futures contract on a portfolio of stocks that is equivalent to the portfolio of stocks constituting the stock index.

Now, what we are basically doing is we are replacing you see let us store talk about a stock future a stock future is a futures contract written on an underlying stock let us say stock on our

RIL. So, we have a future written on RIL now what we are doing is we are replacing RIL by a portfolio of stocks which is equivalent to the portfolio of the stock index for example, the BSE, sensex.

So, a stock index future is a futures which is written on a portfolio of stocks, which is equivalent to the stock index. So, what does it mean? It means that on the date of maturity of the futures contract, you should be delivering this portfolio of stocks to the party who is long in this futures contract so the short party should be delivering this to the long party. However, actually, of course, the delivery of the index is impractical so, we talk about cash settlement, we will come back to it.

A long oblique short position in a stock index futures equivalent to take a long oblique short position in a futures contract or a portfolio or stock that is equivalent to the portfolio stocks constituting the stock index. The underlying asset of such futures is the index. So, instead of a single stock, we are now replacing that single stock by a portfolio of stocks, which portfolio of stocks the portfolio stocks which is representative of the or which is equivalent to the index that is supposed to be the underlying index of the futures contracts for example, the BSE, Sensex.

Now, the MTM settlement is given by z star into ft minus ft minus 1 this is as usual the differentiate between the two settlement prices, t minus 1 that is yesterday's yesterday's settlement price and t that is today's settlement price and you multiply it by z what is z, z is the lot size or the contract multiple.

And the final settlement is done by what the final settlement is done by setting the value of the futures price to equivalent to the spot value of the index, which is the underlying index of the futures contract, so that is what is given in this final settlement instead of f t, we replace f t by ST this is justified because of the principle of convergence, which I have alluded to a number of times.

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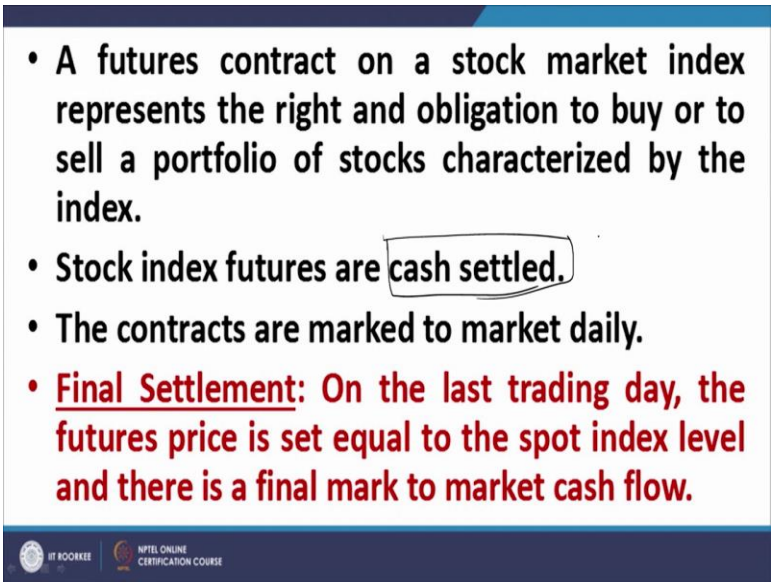

A futures contract on a stock market index represents the right and obligation to buy or to sell a portfolio of stocks characterized by the index, stock index futures are cash settled. So, that is important that is what I briefly mentioned earlier, the it is obviously impractical to make make a physical settlement of stock index futures the delivery of the index is impractical.

In fact, it is absurd and therefore, the settlement of a futures contract as always cash settled. In other words, instead of delivering the index the, the futures price as on the date of maturity of the future is set equal to the spot value of the index on that date at that point of settlement, and the differential therefore, is is transferred to the respective margin accounts as the final M to M transfer market marking to market transfer.

The contracts are marked to market daily as is the usual case for most of the futures contracts. Final settlement this is what I mentioned just now, on the last trading day, the futures price is set equal to the spot index level on the last trading day, the futures price is set equal to the spot index level and there is a final mark to market settlement.

This is interesting, this is important that is the premise of cash settlement. On the last trading day, the futures price is set equal to the spot index level and there is a final mark to market cash flow.

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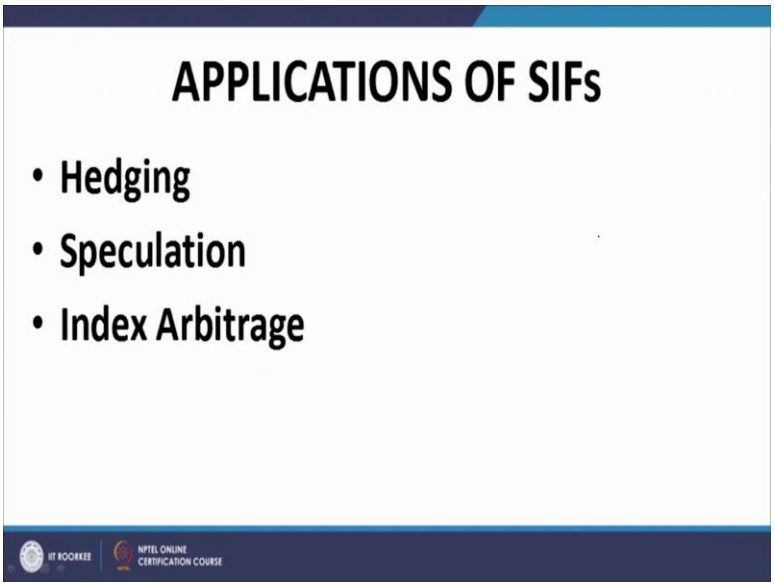

Now, we talk about applications of stock index futures and there are three fundamental applications of stock index futures as in fact of most other derivative contracts, we have hedging the most important application the most fundamental application, we have speculation where you take a position based on your perspective of the markets evolution and if the market evolution is as per your perception, then you tend to make a profit.

On the other hand, if the market evolution is not as per your perception, you tend to make a loss on your open position and that is why in fact it is called an open position. Then we have index arbitrage where if there is a differential between the arbitrage free price of the futures contract and the actual traded price, you can extract profit out of the system.

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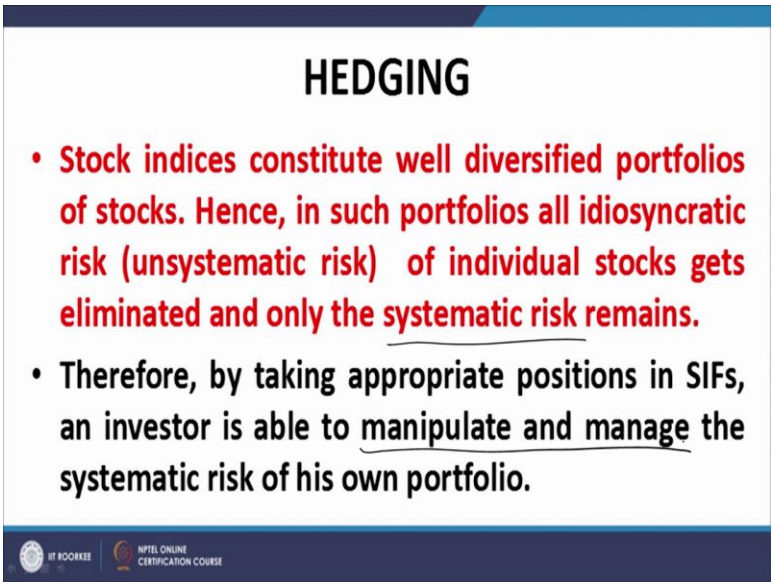

Now, let us talk about hedging. Stock indices constitute well diversified portfolios of stocks, hence, in such portfolios, all idiosyncratic risk that is the unsystematic risk of individual stocks gets eliminated and only the systematic risk remains.

So, when we talk about as the market index, we are talking about well diversified portfolios, and because the portfolios are well diversified, we assume or we understand that the unsystematic risk component is adequately diversified away and the risk that remains with such indices is systematic risk component. Therefore, by taking appropriate positions in stock index futures and investor is able to manipulate and manage the systematic risk while leaving the unsystematic risk of the portfolio unchanged.

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Typically, when we talk about hedging, we are talking about beta management because, as I mentioned, the application the application of stock index futures is basically that of the is basically that of managing the systematic risk, and systematic risk is measured by beta. So, it is basically the management of beta which is achieved by the taking of various positions in stock index futures.

The beta management could well manifest itself in various modes, various forms beta management could manifest itself as market risk reduction, volatility management or investment management, then you also have the opportunity of locking in the benefit of stock picking if you have some special specific some insider information about a particular stock, you can derive advantage from that information by insulating your portfolio from the market insofar as the systematic risk is concerned, but taking an appropriate position in futures contracts or index futures, we shall be looking at an example on this.

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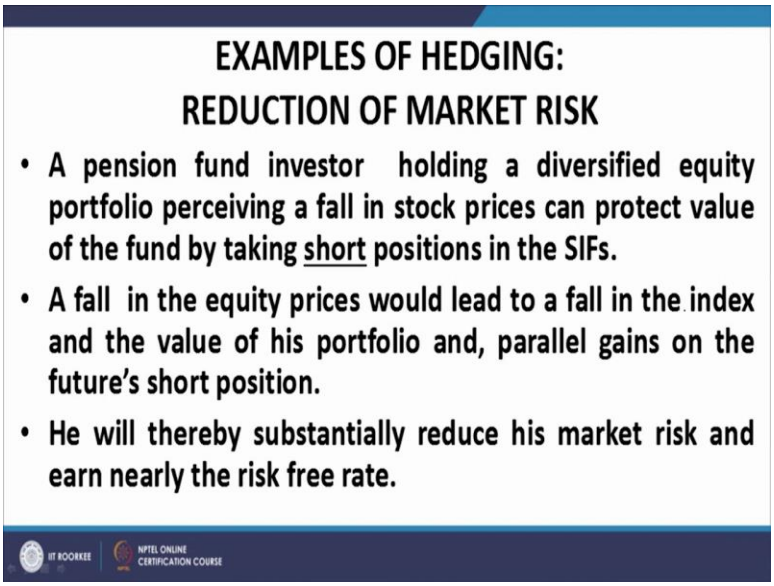

So, examples of hedging reduction of market risk, a pension fund investor holding a diversified equity portfolio, perceiving a fall in stock prices can protect the value of the fund by taking the short position in stock index futures, because if there is going to be a fall in stock prices as long portfolios are going to incur a loss.

But by taking a short position in stock index futures, if there is a fall in stock in stock prices, there is likely to be a fall in the indices also and if there is a fall in the indices, the short position on stock index futures would generate profits for him, which could compensate him for the loss on his investment portfolio.

Let me repeat that this is important. If you have an investment portfolio consisting of long position in stocks, and you perceive that the market is likely to go down, you could take a short position in index futures to mitigate the damage or mitigate the fall in value due to the possibility of the market going down.

Now, how this would operate. If the market falls, as per as per your perception, the futures the futures prices would also fall and if there is a fall in the futures prices, your short position in index futures would gain in value and this gain in value could compensate you substantially for the loss that you incurred on your investment position.

So, a fall in the equity prices would lead to a fall in the index in the value of the portfolio parallel gains on the future short position. So, fall in the index would result in a fall in the futures on the index and because they have a short position, you will make a profit on the short position that will compensate you for the loss in value on your primary investment. You will thereby substantially reduce his market risk and nearly the risk free rate.

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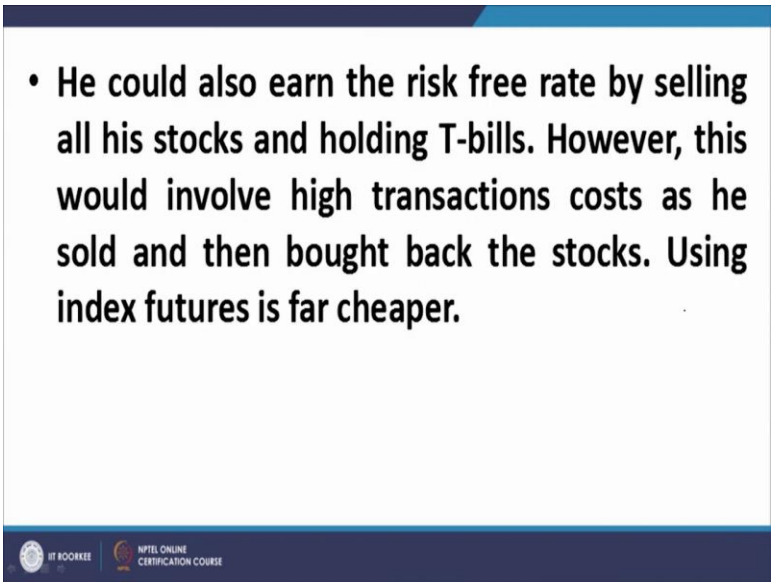

He could also earn the risk free rate by selling all the stocks and holding T bills however, this would involve high transaction costs as he sold and then bought back the stocks using index futures is far cheaper. So, this is another advantage of using index futures, the frictional costs would be mitigated.

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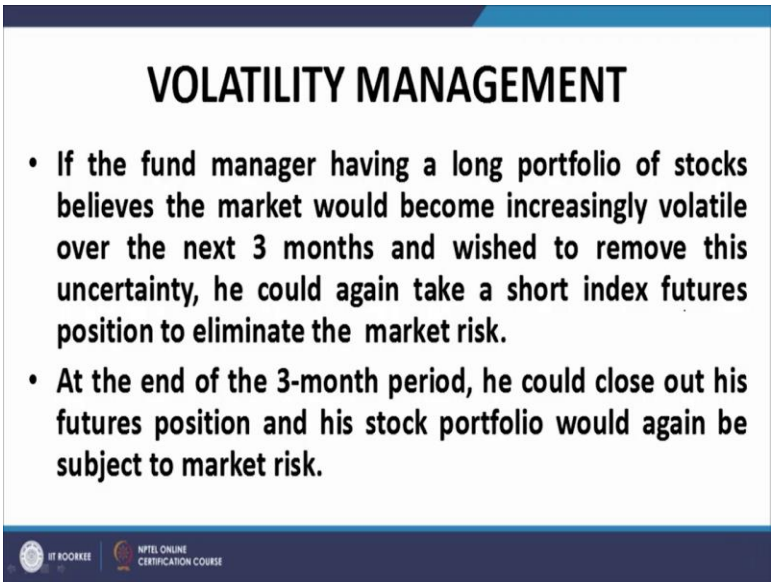

Now, we talk about a situation where volatility is managed by index futures. If the fund manager having a long portfolio of stocks believes that the market would become increasingly volatile over the next 3 months, and wished to remove the uncertainty, he could again take a short index futures position to eliminate the market risk.

Now, because his long in the portfolio long in the portfolio of stocks, if he takes a short position in index futures, he could mitigate the volatility that the volatility in his primary position or his investments will be counteracted or will be annulled by the volatility at the short position in index futures. At the end of 3 months period, it will close out the futures position and his stock portfolio would again be subject to the market volatility.

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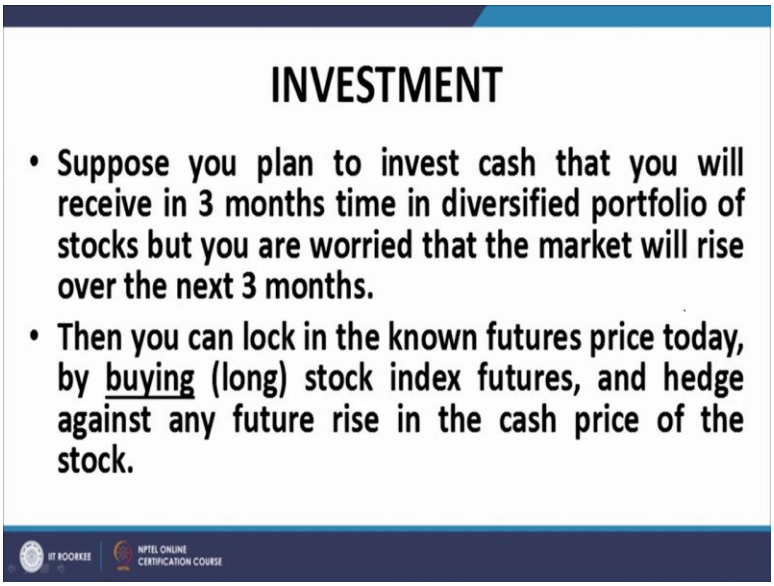

Then, talking about investment, suppose you plan to invest cash that you will receive in 3 months time in a diversified portfolio stock but you are worried that the market will rise over the next 3 months. Now, if the market rises over the next 3 months, your investment would be over a lesser face value of stocks or bonds, whatever the case may be, and as a result of which your returns would be less over the remaining period of your or investment horizon.

Now, to compensate that what you do you take a long position in index futures, so that if the stock prices rise, and your investments worth declines, that could be compensated by the increase in value of your long position as well and as a result of which your loss is mitigated to some extent. Then you can lock in the known futures price today by buying stock index futures and hedge against any future rise in the cash price of the stock.

So, that is how you can manage your investment position in the future also, if you are going to make an investment after 3 months of (base) on the basis of received in cash at that point in time, and you believe that the stocks could rise in price their indices could rise in value in the end of remaining period of 3 months, you could reduce the detriment to you on account of the rise in value of the stocks at the point at which you are going to make the investment by taking a long position in futures.

Your futures will also gain due to the rise in value of the stock and as a result of which your loss on your at the point of investment 3 months hence, would be compensated.

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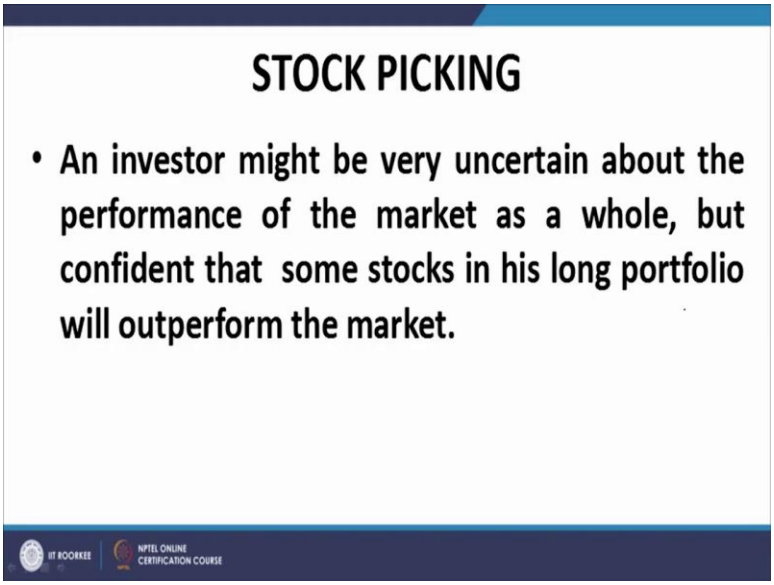

Stock picking, we will continue in the next lecturer. Thank you.