Quantitative Investment Management
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Lecture 56
Perfect Futures Hedge, Cross Hedge, Tailing the Hedge

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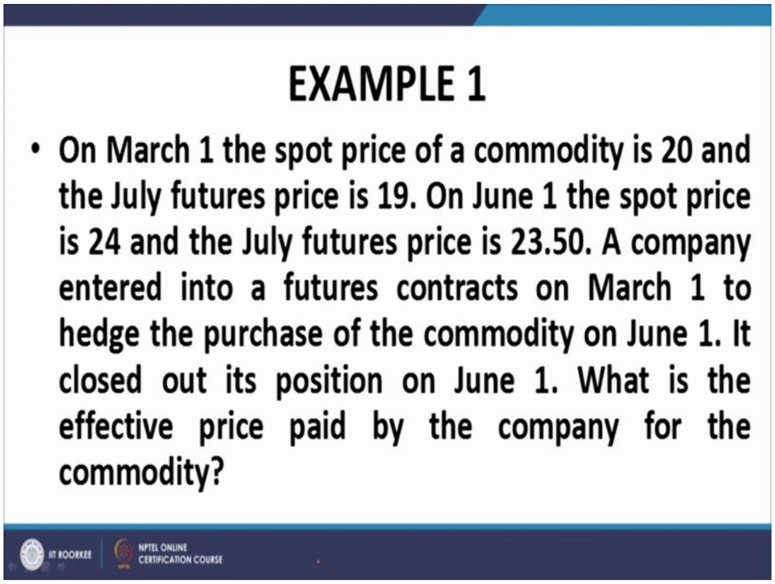

Welcome back. So, let us continue with the example on March 1 the spot price of a commodity is 20 and the July futures price is 19. On June 1, the spot price is 24 and the July futures price is 23 point 50. A company entered into a futures contract on March 1 to hedge the purchase of a commodity on June 1, it closed out its position on June 1, what is the effective price paid by the company for the commodity.

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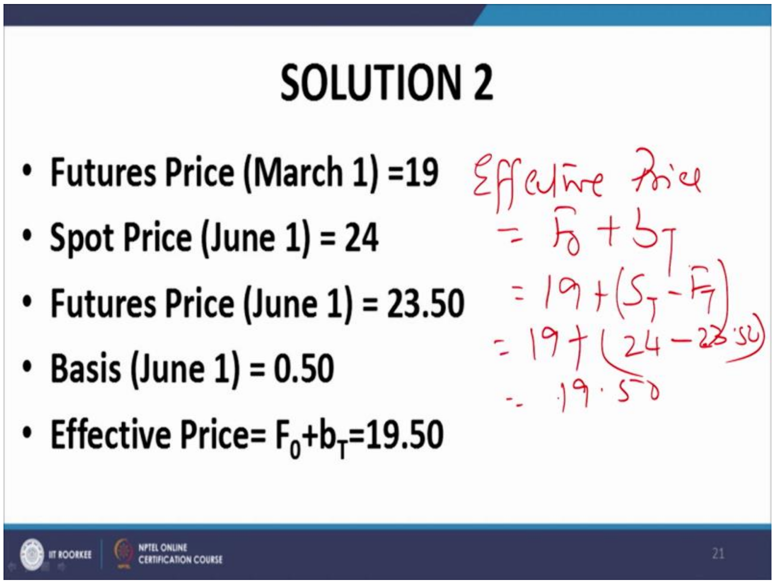

It is quite simple, the effective price is paid equal to the spot price in the futures price rather as on the date of inception of the hedge plus the basis as on date of the maturity of the hedge the effective price is equal to F0 plus bT what is F0, F0 is the future price at t equal to 0 that is March 1 that is equal to 19 plus bT what is bT, bT is equal to ST minus FT what is ST, ST is equal to ST is equal to spot price as on June 1 is 24 this is 24 minus FT, what is the futures price 23 point 50. So, that is equal to 19 point 50, simple problem to start with.

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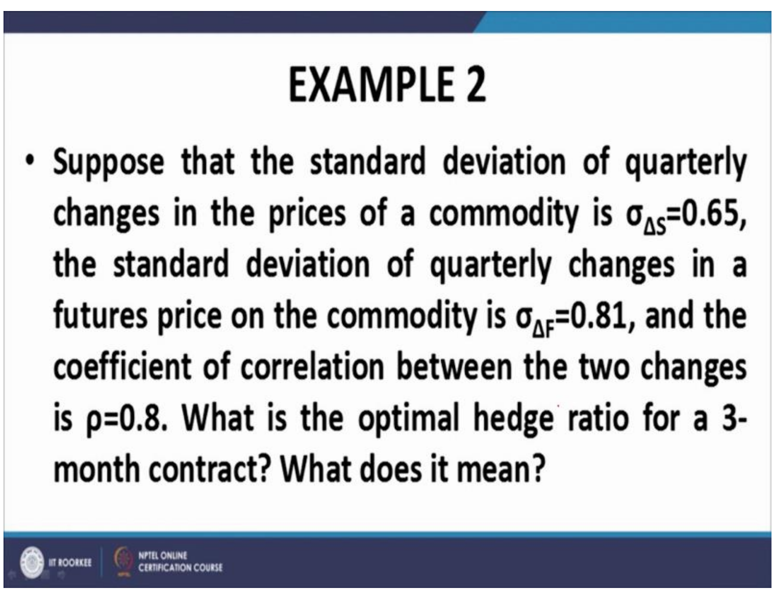

Let us do another simple problem. Suppose that the standard deviation of quarterly changes in prices of a commodity sigma delta S is equal to 0 point 65 the standard deviation of quarterly changes in a futures price on the commodity sigma delta F is equal to 0 point 81 and the coefficient of correlation between the two changes row is equal to the o point 8 what is the optimal hedge ratio?

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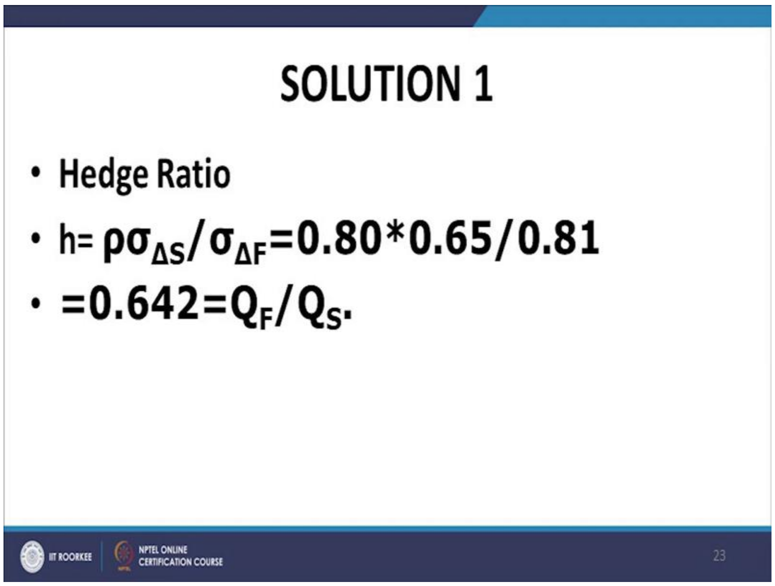

It is s quite simple optimal edge ratio because we are talking about minimum variances when you substitute the values is equal to 0 point 80 into 0 point 65 divided by 0 point 81 that is equal to 0 point 642 that is equal to QF upon QS that is for hedging 1 unit in quantity of the hedged asset we need 0 point 642 units of the hedging asset in the futures market.

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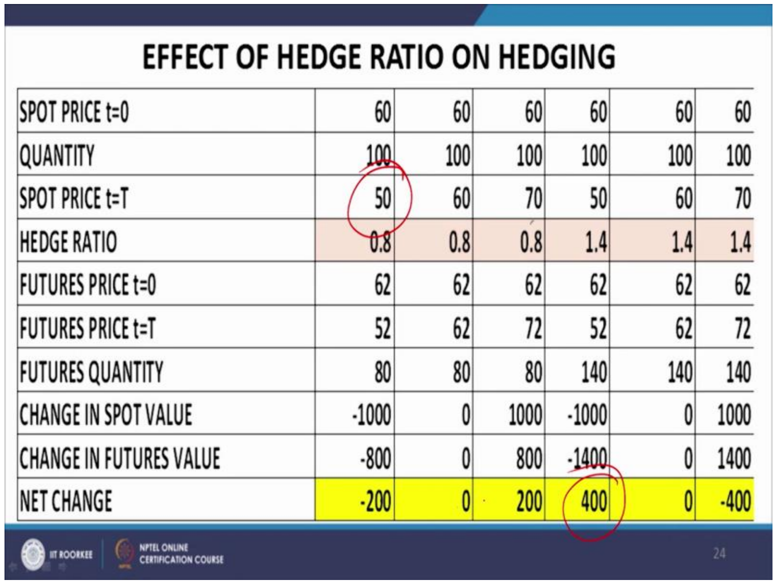

EFFECT OF HEDGE RATIO ON HEDGING

SPOT PRICE t=0	60	60	60	60	60	60
QUANTITY	100	100	100	100	100	100
SPOT PRICE t=T	50	60	70	50	60	70
HEDGE RATIO	0.8	0.8	0.8	1.4	1.4	1.4
FUTURES PRICE t=0	62	62	62	62	62	62
FUTURES PRICE t=T	52	62	72	52	62	72
FUTURES QUANTITY	80	80	80	140	140	140
CHANGE IN SPOT VALUE	-1000	0	1000	-1000	0	1000
CHANGE IN FUTURES VALUE	-800	0	800	-1400	0	1400
NET CHANGE	-200	0	200	400	0	-400

So, the hedge ratio turns out to be very seminal, very important when we talk about the hedging process and the effectiveness of the hedge. If there is any error in the estimation of the hedge ratio that will manifest itself in a drastic consequences as insofar as the hedging is concerned. That is evident from what we have on this particular slide, I have considered two hedge ratios hedge ratio of 0 point 8 and additional 1 point 4.

Let me quickly run through the through the data. The spot price is 60 the hedged quantity is 100 and the spot price at maturity of the hedge can take 3 values or any any of 3 values rather, it is a random variable, it can take any of 3 values 50, 60 or 70. The hedge ratio I have taken two sets, one set with the hedge ratio of 0 point 80 the second set with the hedge ratio 1 point 40. The futures price at t equal to 0 is 62, which is known which is not random.

The futures price at t equal to capital T, again can take 2, 3 values 52, 62 and 72 then the futures quantity based on the hedge ratio turns out to be 80 in the first case and 140 in the second case, when the hedge ratio is 1 point 40. If you look at the results, the net change of the hedged position that is the spot and the futures, if the hedge ratio is 0 point 80 under the 3 scenarios is minus 200, 0 and 200 corresponding to the spot prices of 50, 60 and 70.

Now, if you look at this same (scena) situation, with a hedge ratio of 1 point 40 the profit figures are completely reversed when the cost realized at the end of the hedge period was 50 the profit or the net change was minus 200 however, when you use the hedge ratio 1 point 40, the change turns out to be plus 400.

So, that is the importance of the hedge ratio even the direction has changed from minus 200 with the hedge ratio of 0 point 80 we are now having plus 400 with hedge ratio 1 point 40, that is the importance that is the ramifications of the hedge ratio that we are going to use in the hedging process.

The objective of this illustration is (pure) fairly and squarely or purely and squarely to bring to the learners the importance of the hedge ratio and care that needs to be exercised when selecting a hedge ratio.

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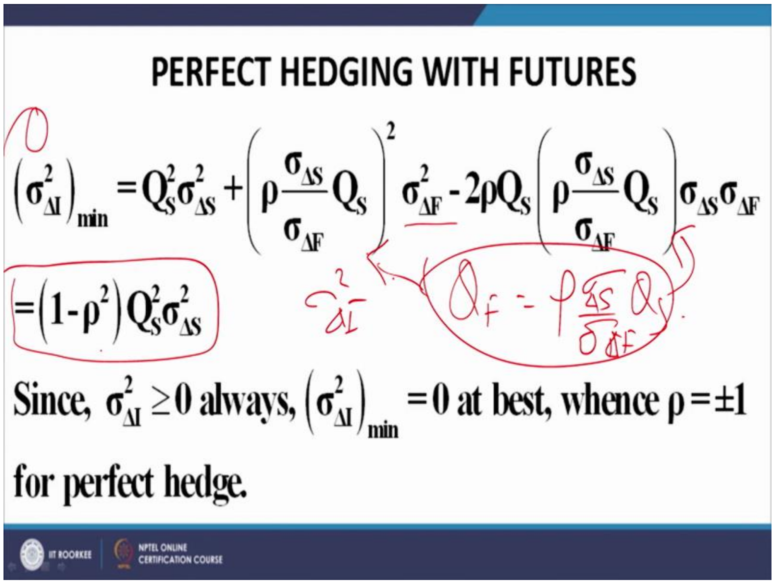

Perfect hedging with futures, now, we know that in the context of the minimum variance hedge is given by if you recall, QF is equal to rho sigma S upon sigma F sigma delta S I am sorry sigma delta S upon sigma delta F into QS.

So, I substitute this value of QF into the expression for the variance of the hedge position here as well as here then I simplify, and when I simplify, you get a very interesting result, the result that I get is the expression that I enclose within the box 1 minus rho square sigma sigma delta S square into Q squared.

Now, the variance is obviously bounded from below by 0. So, we when we talk about minimum variance, we would we cannot be more happy than the variance being 0. In other words, the best situation best case scenario that the variance can give us is the variance mean equal to 0. And if the variance is equal to 0 of the hedged position, that is, the left hand side is equal to 0 for the right hand side, what we get is rho is equal to plus minus 1.

In other words, we can achieve a variance of 0 for the hedged position in two situations, when rho is equal to plus 1 or row is equal to minus 1. That is, when the changes in spot prices and changes in futures prices are either perfectly correlated or are perfectly anti-correlated. Let us analyze this further, for a perfect futures as we must have rho is equal to plus 1 or minus 1, that is the changes in spot prices and this would changes in spot futures prices must either be perfectly correlated, or perfectly anti-correlated.

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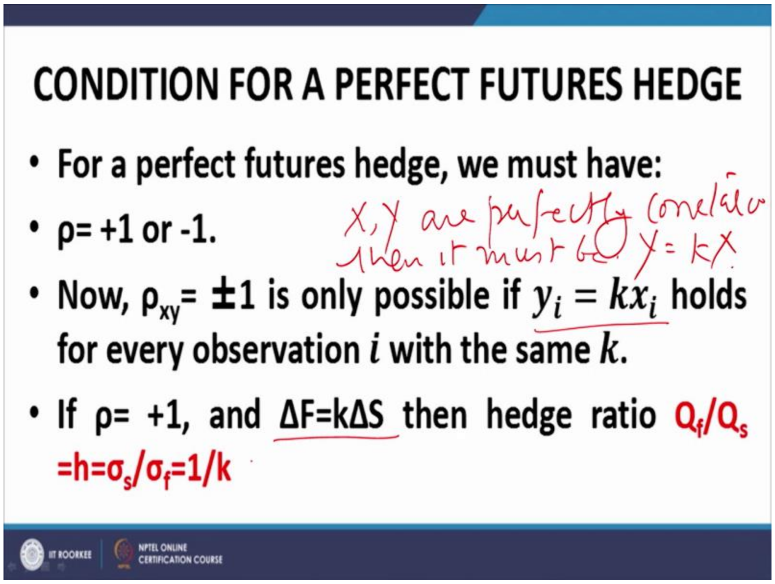

Now, if let us that implies what that implies that if the two variables, let us say x and y are perfectly correlated then it must be that y is equal to kx. In other words, yi is equal to k of xi, for every observation i, for every observation with the yi, yi must be equal to xi with the same k yi is equal to k xi with the same k, that means, the same constant should operate over all values, all observations between yi pairs of yi and xi.

If, for example, if x is equal to 1, y is equal to 2, if x is equal to 2, y is equal to 4, and so on, that would be a perfectly correlated series. So, in order that a (perfe) does a series being, be perfectly correlated a series between x and y be perfectly correlated, the all the observations must be related as yi is equal to xi, or the random variable, x and y must be related as y is equal to kx.

And if rho is equal to plus 1, then therefore we can write delta F is equal to k into delta S in view of what I mentioned just now, and then the hedge ratio turns out to be 1 upon k as you can see, so the net result of what I have been talking about is that for a perfect hedge rho must be equal to plus 1 or minus 1. Now, if rho is equal to plus 1 or minus 1 for that matter, the two random variables x and y must be perfectly correlated that is y must be equal to kx for some constant k.

In other words, every observation of y and the corresponding observation of x must necessarily satisfy yi is equal to k xi with the same k and if that is the situation, the hedge ratio that that can be used for having an optimal hedge a perfect hedge in this case is equal to 1 upon K.

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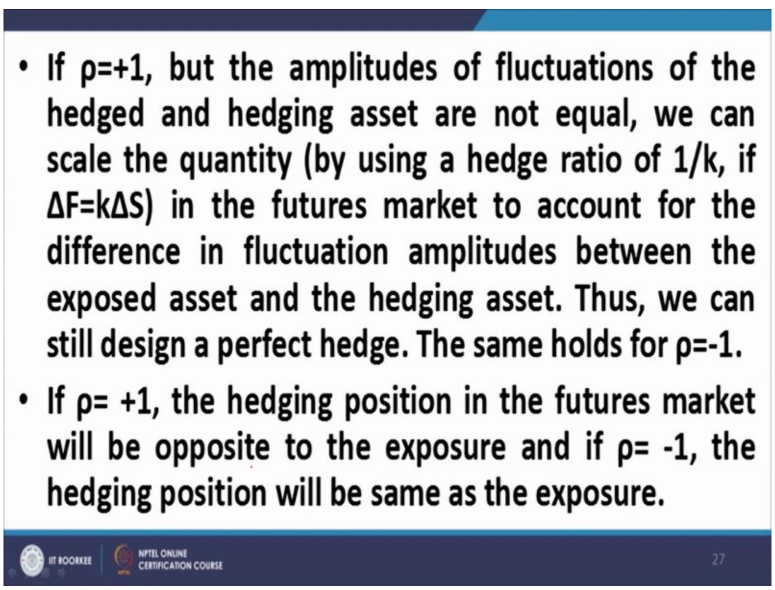

If rho is equal to plus 1, but the amplitudes of fluctuations of the hedge and hedging assets are not equal, we can scale the quantity by using hedge ratio of 1 upon k, if delta F is equal to k delta S, in the futures market to account for the difference in fluctuating amplitudes between the exposed assets and the hedging asset this is what I explained just now.

For example, if the changes in futures price is twice the changes in the spot price for every change, then the quantity that we use in the futures market for hedging will be half the hedge ratio will be half and if we use a hedge ratio half you will end up with a perfect hedge if the two assets are perfectly correlated, if the two price series are perfectly correlated.

If rho is equal to plus 1, the hedging position in the futures market will be opposite to the exposure and if rho is equal to minus 1, the hedging position will be same as exposure that is I have already mentioned.

Let me read out the previous paragraph once again, this is interesting if rho is equal to plus 1, but the amplitudes of the fluctuations of the hedged and the hedged assets are not equal, we can scale the quantity by using a hedge ratio of 1 upon k if delta F is equal to K delta S in the futures market to account for the difference in fluctuating amplitude between the exposed asset and the hedging asset.

Thus we can still design a perfect hedge notwithstanding the fact that amplitudes are not identical, but the amplitude of one series is multiple of the amplitude of the other for every amplitude. So, for example, as I mentioned, if the if the changes in futures prices are twice the changes in the spot price for every change, then you can have half as the hedge ratio, the quantity in the futures market will be half of the quantity in the spot market that you are planning to hedge.

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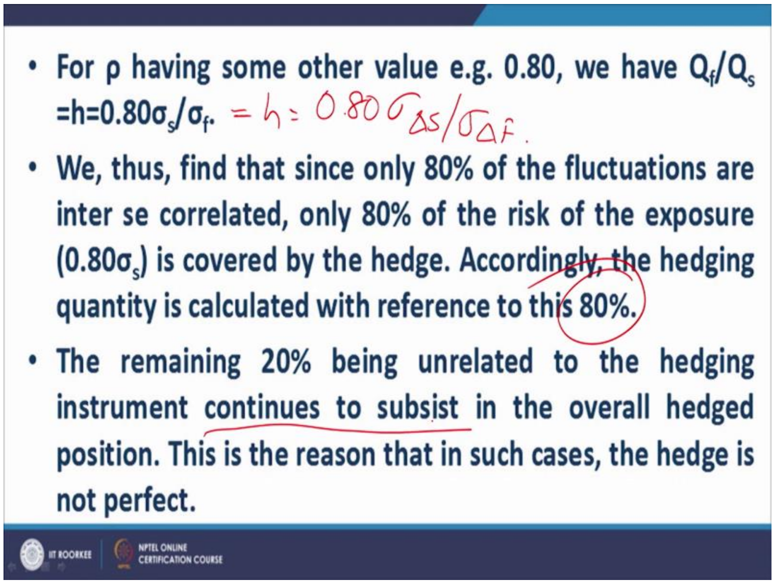

For rho having some other value now this is interesting, for rho having some other values for example 0 point 80 we have Qf upon Qs is equal to 0 point 80 sigma s upon sigma f actually it should be sigma delta s upon sigma delta f, let me write it down for you that is the correction error which is equal to 0 point 80 sigma delta s divided by sigma delta f.

We thus find that since only 80 percent of the fluctuations are inter se (relate) correlated only 80 percent of the risk of the exposure is covered by the hedge now, in this situation, when the correlation between the changes in spot prices and the changes in futures prices is 0 point 80.

What does it mean? It means that we can only hedge 80 percent of the changes in prices in the spot market by using the futures hedge that we are considering, the 20 percent that is not correlated between the two prices or that correlation is not there between the two prices will manifest itself as a random or unsystematic error, unsystematic fluctuations, which cannot be which are not being diversified away by this process of hedging.

We thus find that since only 80 percent of the fluctuations are inter se correlated only 80 percent of the risk of exposure is covered by the hedge see, because only 80 percent is the of the total variation in the spot position is being influenced by the hedge and therefore, only 80 percent can be eliminated by using the best possible hedge using that particular asset.

The remaining 20 percent which is totally uncorrelated with the with the hedging asset will manifest itself as random fluctuations that will manifest itself as inefficiencies in the hedge or imperfections in the hedge. Accordingly, the hedging quantity is calculated with respect to this 80 percent.

So, let me repeat we thus find that since only 80 percent of the fluctuations are inter se correlated, only 80 percent of the risk of the exposure is covered by the hedge accordingly, the hedged quantity is calculated as reference to this 80 percent.

Because this is this is what we are going to hedge 20 percent because it has no correlation with the hedging asset. The hedging asset will not manifest itself as a link the why the fluctuations of that 20 percent component, the remaining 20 percent been unrelated to the hedging instrument continues to subsist this is important in the overall hedged position this is the reason that in such cases, the hedge is not perfect.

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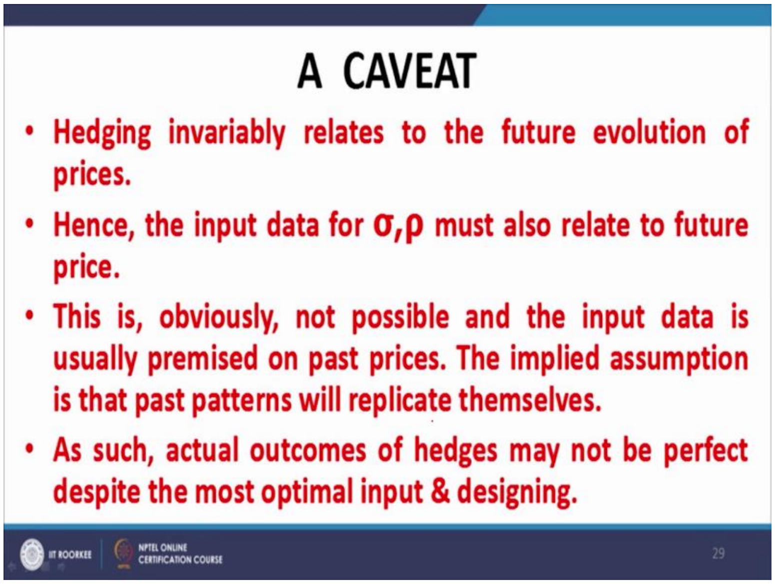

Caveat, hedging invariably relates to the future evolution of prices as the input data for sigma and rho must also relate to futures prices. This is obviously not possible and the input data is usually premised on past premises past prices, the implied assumption is that past patterns will replicate themselves as such actual outcomes of hedges may not, may not be perfect, despite the most optimal input hedge designing.

So, that is the important part you see what we are trying to say here is that, basically whatever optimality conditions are so on that we have derived, so, far, it was certain inputs, these inputs must relate to the future evolution of the prices of the hedged asset.

And of course, the correlation between the prices of the hedged asset and the hedging asset the future evolution is important, not what has happened in the past, but obviously, we cannot predict the future these are random variables.

Therefore, the best choice that we can make is to use past data insofar as it is possible the understanding is that the past patterns will replicate themselves in the future, but that may not necessarily happen, that may not happen always, at least and as a result of it, because your hedging process, hedging strategy is based purely an entirely on past data, the net result is that you may end up with inefficiencies in the hedge, if the past data is not replicated in the future.

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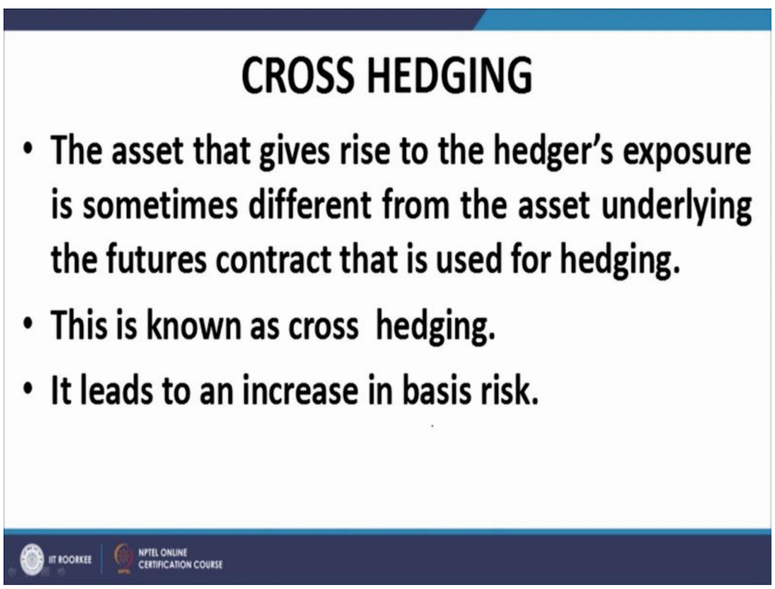

Now, we talk about cross hedging, what happens if your exposure is in one asset and the underlying asset of the futures contract that you are using for hedging your exposure is discussed here and the process is called cross hedging. Let me repeat your exposure is an asset A, the underlying asset of a (fu) of the futures contract that has been used for hedging is asset B, then the process is called cross hedging. Let us discuss it the asset that gives rise to the hedges exposure is sometimes different from the asset underlying the futures contract that is use for hedging this is known as cross hedging, it leads to increase in basic risk.

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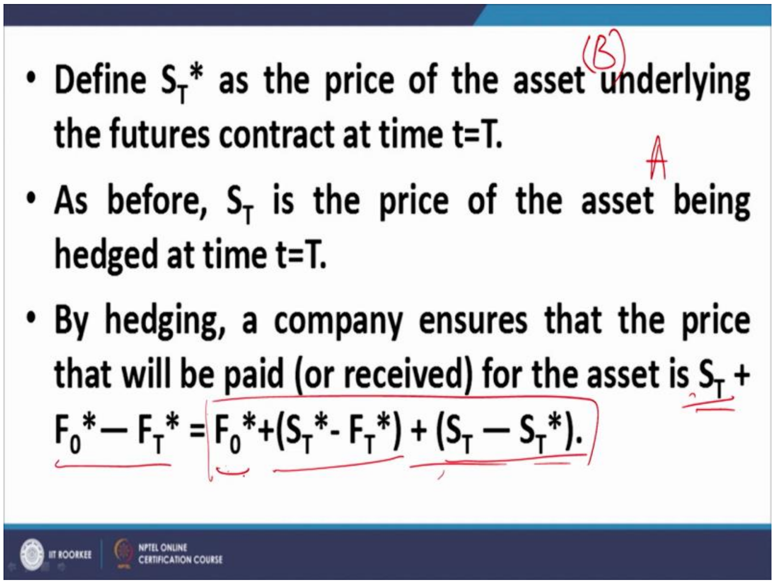

Let us defined ST star, ST star as the price of the asset underlying the futures contracts at time t equal to capital T, the asset B, let us call it asset B. ST star as the price of the asset underlying the futures contract at time t equal to capital as before ST is the price of the asset A that is the asset that has been hedged being hedged at t equal to capital T.

By hedging a company ensures that the price that that will be paid or received for the asset ST that is the spot price of asset A in the spot market at t equal to capital T plus the benefit of futures and the benefit of futures is F0 star minus FT star, because it is a short hedge, you are selling the asset so, you are receiving this price ST and this is the profit from the futures position.

I can write this in the form F0 star plus ST star minus FT star plus ST minus ST star simply by adding or rearranging, adding, subtracting and rearranging the terms, I can write this expression that is the price that I will receive as this expression.

Now, this has 3 components, F0 star, which is the futures price of the asset B, that is the asset, which is the underlying asset of the futures contract contract that is known that ST is equal to 0 price ST star minus FT star, this is the basis that is related to asset B and ST minus ST star is the new term is the additional term which arises due to the difference between the the spot asset and the spot asset that is asset A and the asset B.

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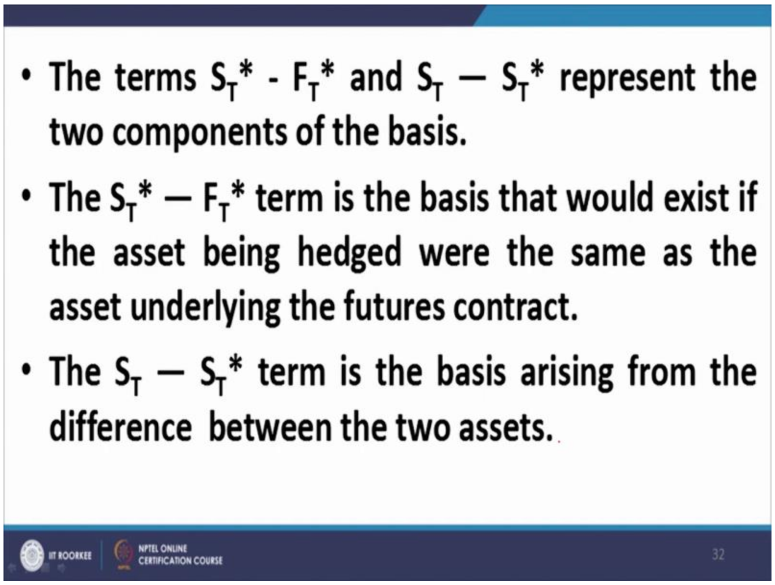

So, the terms is T star minus FT star and ST minus ST star represent the two components of the basis. ST star minus FT start is the term in the is the base term is the basis that would exist if the asset being hedged were the same as the asset underlying the futures contract that is it this relates to the asset that is asset B that is the underlying asset of the futures contract. And ST minus ST star term is the basis arising from the difference between the two assets A and B.

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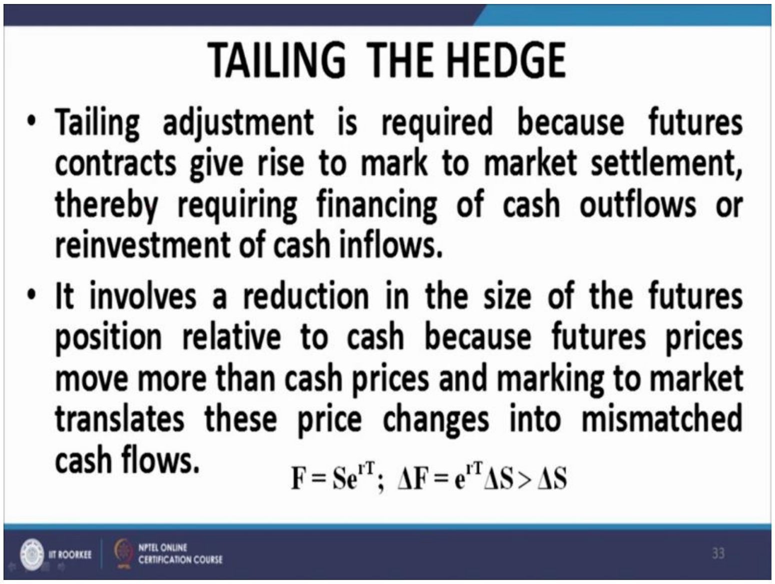

Now, we will talk about a very interesting strategy hedging strategy, which involves using a dynamic hedging ratio. And we can we will show that by using a dynamic hedge ratio assuming that the interest rates are non stochastic on the premise that interest rates are non stochastic, if we use a dynamic hedge ratio, we can achieve risk free rate of return on the hedged position.

So, tailing adjustment is required, because futures contracts give rise to marking to market adjustments marking to market settlements, thereby requiring financing of cash outflows with or reinvestment of cash inflows.

It requires reduction in the size of the futures position relative to cash, because the futures prices move more than cash prices and marking to market translates these price changes into mismatched cash flows, you can see here f is equal to Se to the power rt, the futures price is the future value of the spot price.

And therefore, delta f is equal to e to the power rt into delta S e to the power rt is the quantity greater than equal to 1 and therefore, delta f is greater than equal to delta S that is the change in the futures price markets should under the conditions of free arbitrage no free arbitration rather under the conditions of no free arbitrage would ensure that the futures price changes are magnified relative to the corresponding spot price changes and that that results in the differentials in a margin (dro) deposits and withdrawals and consequential mismatching of cash flows.

So, to manage these, these, these scenarios, we use a dynamic hedge ratio and we show that by using a dynamic hedge ratio, we can still arrive attain a risk free rate of return.

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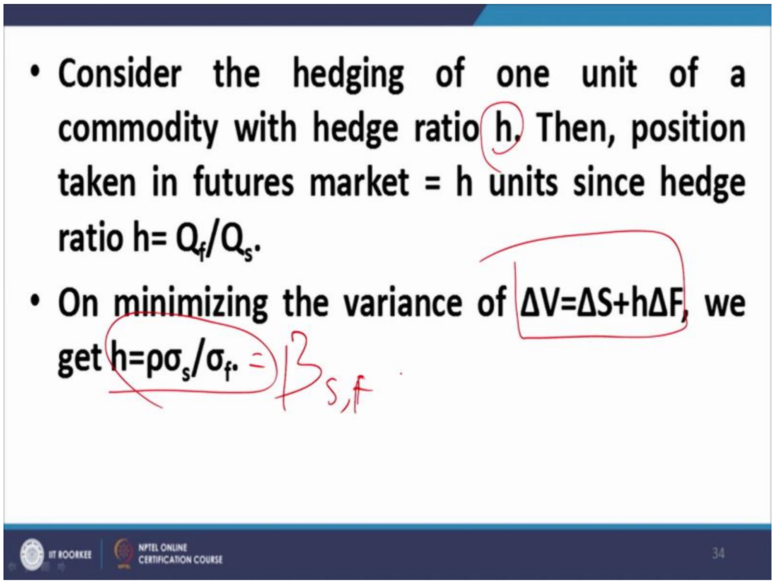

Consider the hedging of one unit of a commodity with hedge ratio h, then the position taken in the futures market is equal to h units and the corresponding to a spot market position of 1 units, the position taken in the futures market is h units or minimizing the variance of this expression we have already discussed that in an earlier section in today's lecture.

What we find is that h is equal to beta and beta is given by rho sigma s upon sigma h I shall be using sigma s for sigma delta s sigma f for sigma delta f for the purpose of brevity for the purpose to reduce the proliferation of symbols, we shall be using simply h is equal to beta that is equal to beta that is equal to rho sigma delta sigma s upon sigma f instead of sigma delta s upon sigma delta f.

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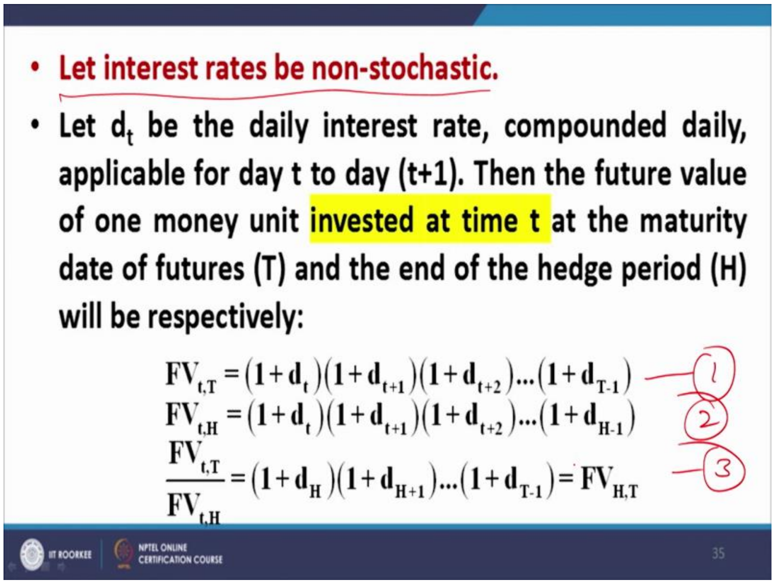

Now, let us say assume that the interest rates are non stochastic I mentioned it at the beginning that we are assuming that interest rates are not random that evolution is not random the evolution is deterministic. So, we know the interest rates that is going to (())(23:27) for between today and tomorrow let dt be the daily interest rate compounded daily applicable for dt to dt plus 1.

Then the future value of 1 money unit invested at time t at the maturity date of the futures capital T and at the end of the hedge period capital H will be respectively future value of 1 unit of money when the interest rate for day t to t plus 1 is dt t plus 1 to t plus 2 is dt plus 1 and so on.

The future value of 1 unit of money invested at t equal to the small t and maturing at t equal to capital T is given by equation number 1 here and similarly, an investment of 1 unit of money at t equal to small t small t is any arbitrary point in time please note this an investment of 1 unit of money at t equal to small t.

And maturing at t equal to capital h where h is the hedge period is given by equation number 2, dividing 1 by 2 we get equation number 3, which basically represents the future value of 1 unit of money which is invested at t equal to capital H and maturity at t equal to capital T.

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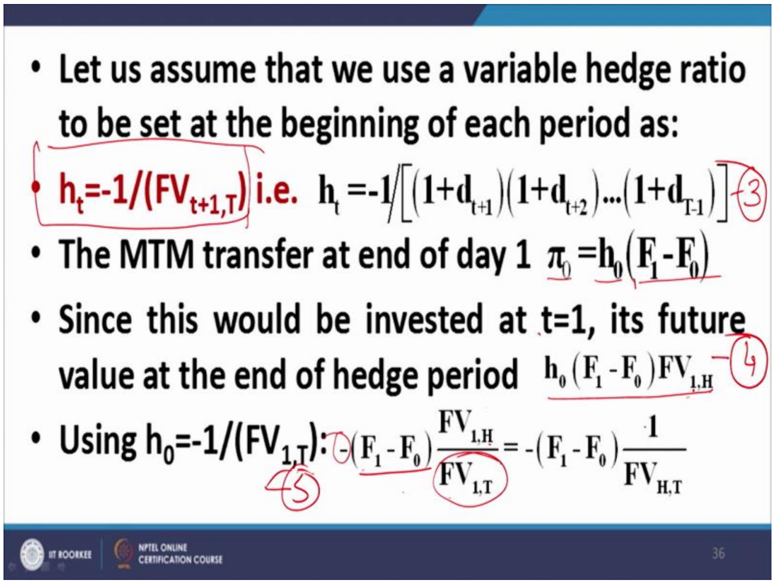

Let us assume that we use a variable hedge ratio to be set at the beginning of each period as ht is equal to minus 1 upon future value of F T plus 1 capital T, please note the future value of FT plus 1 and capital T will be known, because we are assuming interest rates to be non stochastic. So, interest rates are known up in advance. And this minus sign represents that the position that it takes in the futures market will be inverse to the position that I have insofar as my exposure is concerned.

Now, this can be written this expression, when it is substitute the value of future value of T plus 1 comma capital T can be written in the form of equation number 3, the mark to market transfer as at the end of day 1 is equal to what pi 0 is equal to 0 into F1 minus F0, because, F0 is my exposure in the futures market, remember, we are talking about hedging of 1 unit of their commodity and the hedge ratio is ho.

So, the exposure in the futures market is h0 and F1 minus F0 is the change in price since this would be invested at t equal to 1, its future value at the end of the hedge period will be equal to h0 f1 minus F0 into future value 1 comma h, let us call it equation number 4 ho, this is the amount h0 into f1 minus F0 is the amount and FV1 comma h is the future value factor.

So, future value factor 1 comma h using h0 is equal to minus 1 future value 1 comma t. And for the first day, when we substitute this let me call this equation number 5 when we substitute from equation number 5 in equation number 4, what I get is minus F1 minus F0 future value 1 comma h divided by 1 comma t, this 1 comma t is coming from the hedge ratio minus 1 future value 1 comma t this minus sign here also and FV 1 comma t, both this terms are coming from the hedge ratio h0. So, simplifying FV 1 comma h divided by FV 1 comma t, this is equal to 1 upon FV h comma t, the rest is as it is, let us call it equation number 5.

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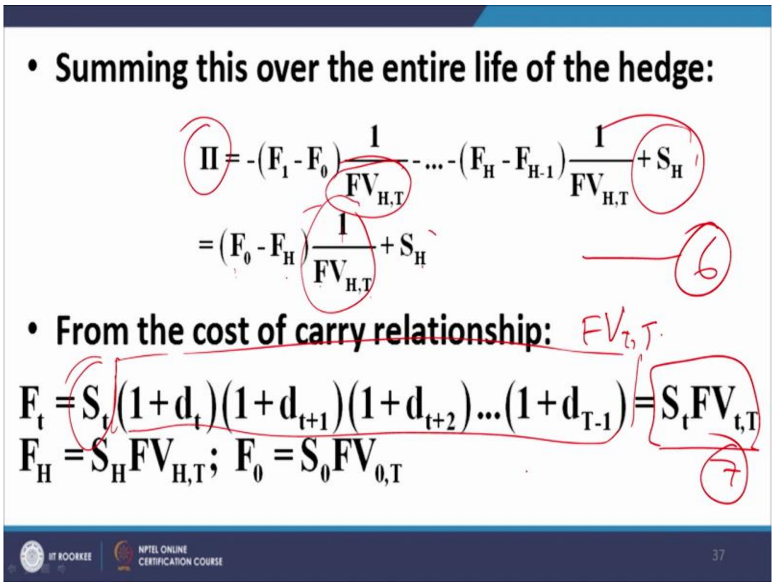

Now, summing this over the entire life of the hedge what will I get the total value pi capital pi is equal to minus f 1 minus F0 into 1 upon a FV ht, and then the second term would be minus f2 minus f1 bracket into 1 upon FV h comma t, you see please note this term this factor 1 upon FV h comma t is independent of small t.

So, it will remain the same for all the terms and when you do the addition the and this SH is when you at the end of the hedge period to dispose of the asset, you get or you receive a cash inflow of S comma SH.

So, the total value is equal to F0 minus FH, all the intermediate terms will cancel out into 1 upon FV h comma t plus SH, all the intermediate terms f2 minus f1, f3 minus f2, all of them will cancel out, you will be left with the last term and the first term that is F0 minus Fh and this 1 upon FV HT will remain with every term it is a common term is independent of t and this is as such as the maturity cash flow, cash flow at the termination of the hedge.

Now, from the cost of carry relationship what we get, we get the future price or the future value or the forward price for that matter is equal to the future value of this spot price. Therefore, f small t is equal to s small t into the expression this expression and what is this expression? If you recall, this is nothing but the future value factor that is FV t comma capital T.

So, the futures price at t equal to small t, now t equal to small t is any arbitrary time is equal to the spot price as on that date that is ST into the future value factor that is FV small t comma capital T that is what is here in this particular slide. Let us called this equation number 6 and let us call this equation number 7. Now, using the equation we can write F capital H is equal to SH FV h comma t and F0 is equal to F0, FV 0 comma t.

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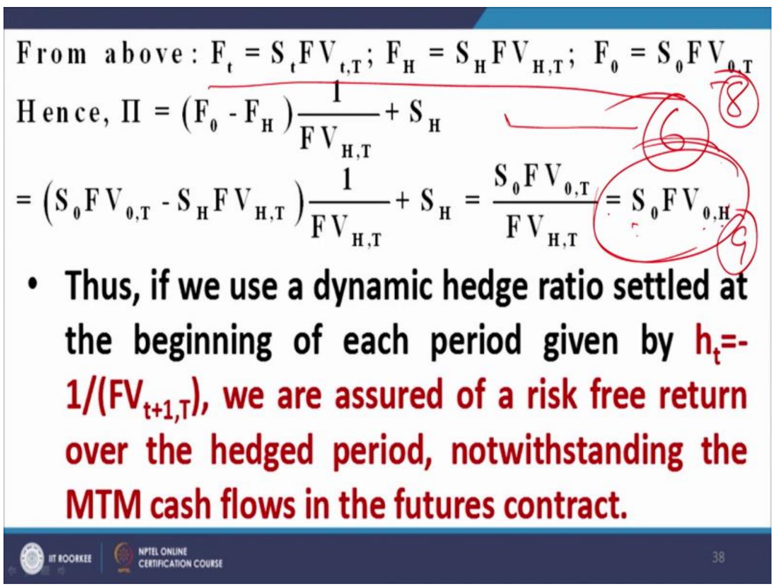

Now, it is only substitution and nothing else, we have these expressions from the previous slide, let us call them equation number 8. And we have this expression also from the previous slide, this is equation number 6, this is equation number 6.

And using equation number 8 and equation number 6, then we simplify this we get this expression, which you have here at the extreme right of equation number 9. And this shows what this shows S0 is today's price t equal to 0 price, which is in the deterministic which is known into the future value between 0 and H, which is also known, because interest rates are non stochastic.

So, we know the evolution of the interest rates and we know the future value factor for an investment at t equal to 0 and maturity at t equal to h with certainty. So, the entire expression here is known at t equal to 0 and that means we have risk plus hedge.

Thus is if you use a dynamic hedge ratio settled at the beginning of each period, given by HT equal to minus 1 upon FV t plus 1 comma t were assured of risk free return over the hedge period notwithstanding, the MTM, MTM cash flows in the futures contract.

This is the outcome that instead of a static hedge ratio, if we use a dynamic hedge ratio, we can achieve risk free rate of return, we can achieve perfect hedge in that sense, but with the caveat that the interest rates are non stochastic interest rates are deterministic they are known in advance over the entire life of the futures contracts. The next topic is index futures, which I will take up in the next lecture. Thank you.