Quantitative Investment Management
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Lecture: 55
Issues in Futures Hedging

Welcome back. So, let us continue where we left off, towards the end of the last lecture I was talking about the sources or the causes that cause imperfections in the hedging process using futures contracts.

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As I mentioned, hedging by a forward contract is a perfect hedge. Why it is a perfect hedge? Because all cash flows at maturity of the forward contracts are predetermined and there is in the absence of default when we ignore the aspect of default in forward contracts, all the cash flows are there with certainty.

And as a result of which when we hedge using a forward contract that turns out to be perfect hedge. What is the perfect hedge? A perfect hedge is a hedge where the cash flows are predetermined and are devoid of any possibility of uncertainty. Now, however, the same situation does not prevail in the context of futures contracts.

Futures contracts have what is known as the basis risk, which I have alluded to earlier. In addition to that, there is the issue of marking to market and if the interest rates change, during the life of the futures contracts during the term to maturity of the futures contract, then that would also influence the hedging process, or that would also cause imperfections in the Hedge.

Then basis risk could be due to difference in the underlying asset. That is the asset on which the futures contracts are written is different from the asset on which the exposure is created. Non-identical maturity, this aspect I have already dealt with, if the maturity of the hedge and the maturity of the futures contract do not coincide.

Then the basis as on the date of maturity of the hedge is a random variable and could take different values, or there is always the possibility of it taking different values. Now, the important thing here is that if the important thing that I want to emphasize at this point is that and emphasize once again, is that the maturity of the hedge need not necessarily coincide with the maturity of the futures contract, the maturity of the hedge is at the choice of the investor choice of the hedger, whereas the maturity of the futures contract is determined by the specifications of the contract, as released by the exchange at which they are listed for trading. So, that is a very important point that we need to keep in mind throughout.

Then there is the issue of lot size. Futures contracts are traded in lots of units of the underlying, we do not write a futures contract or we do not have a futures contract. Usually, on one unit of the underlying, it is usually the case that one futures contract is written on a number of units of the underlying. So, these are possible sources of imperfections and futures hedges, we shall take them up one by one in detail.

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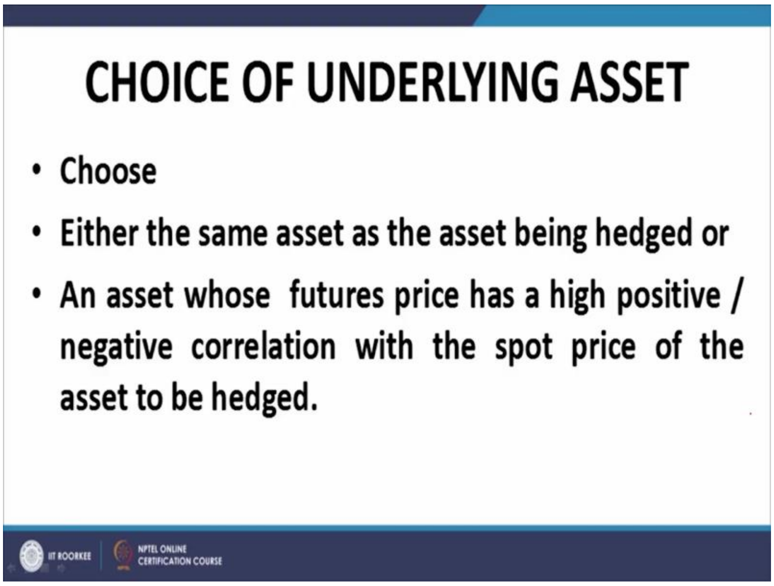

Choice of underlying assets. Now, obviously, the best choice and the immediate choice would be to use the same asset as the underlying asset as the asset on which we have the exposure. However, in the event, that futures on the exposed assets are not available, then we need to take exposure or we need to take a futures contract on a different asset. And that could cause of course, imperfections in the hedge, but we should choose the, choose the asset as the underlying asset we of which the futures prices bear a strong correlation, either positive or negative with the spot prices of the exposed assets.

Let me repeat, we should choose the underlying asset or the futures contract as an asset that whose futures prices be at a strong correlation with the spot price. Because at the end of the day, it is the futures prices which are going to annul the changes in the spot prices.

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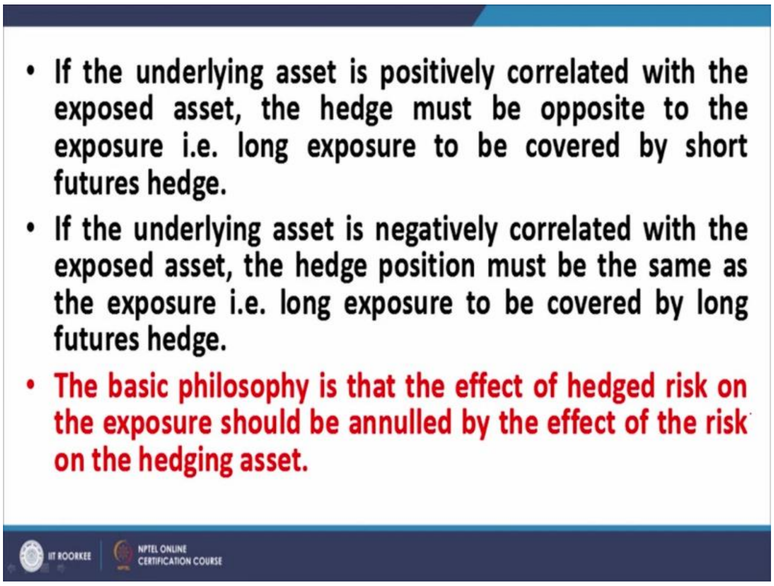

If the underlying asset is positively correlated with the exposed asset, the hedge must be opposite to the exposure that is if the exposure is long, the hedge should be short and vice versa, if the underlying asset is negatively correlated with the exposed assets, which is not usually the case but could happen. And in that event, the hedge position must be the same as the exposure that is, if the exposure is the long exposure if the portfolio is long then the hedge should also belong. The basic philosophy is that the effect of the hedged risk on the exposure should be annulled by the effect of the risk on the hedging asset.

Let me repeat this is the fundamental of futures hedging, and in fact, any type of hedging for that matter, the basic philosophy is that the effect of the hedged risk, this is the stimulus, which is causing changes in prices of the exposure as well as the hedge. And the point is, that the stimulus when it acts on the exposure results in a certain variation in price of the exposure, so variation in the value of the exposure.

And the same stimulus when it acts on the hedge acts in the opposite creates a similar change in price or value, but the change is in the opposite direction to the change in price or value of the exposed assets. So, that the two changes tend to annul each other rather than reinforce each other.

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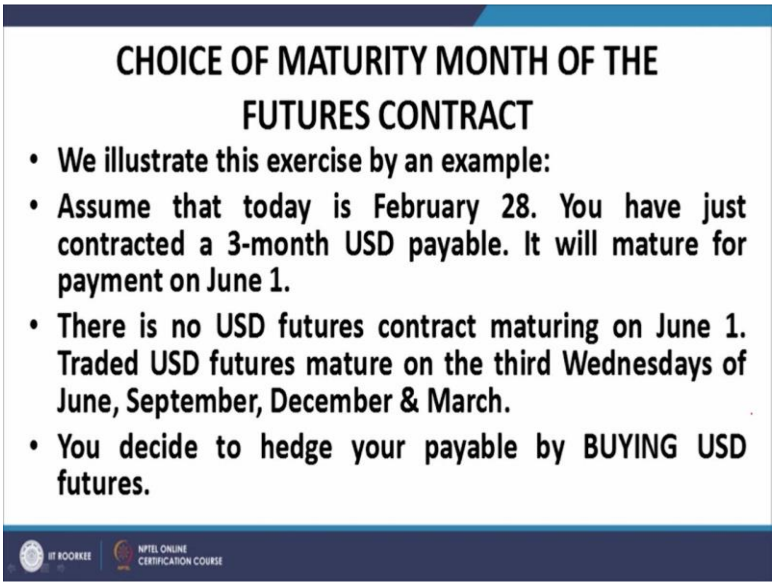

Choice of maturity month of the futures contract. Now, the immediate reaction here would be of the layman and that we should choose the futures contract whose maturity is closest to the maturity of the exposure or the term of exposure or the hedge period that is. However, that is not necessarily the case, as is shown exemplified by the example that I am going to use now.

We illustrate this exercise by an example, assume that today is February 28, we have just contracted a three-month US dollar payable, it will mature for payment on June 1. There is no US dollar future contracts maturing on June 1. Traded US dollars futures mature on the third Wednesdays of June, September, December and March, this is given data we assume this data.

You decide to hedge your payable by buying the futures. Why you will buy the futures? Because your exposure is two- or three-month US dollar payable. You need to, you need US dollars you need to buy US dollars from the market in order to meet your obligation created under the contract. Therefore, you will take a long position in US dollars futures because you want to buy US Dollars.

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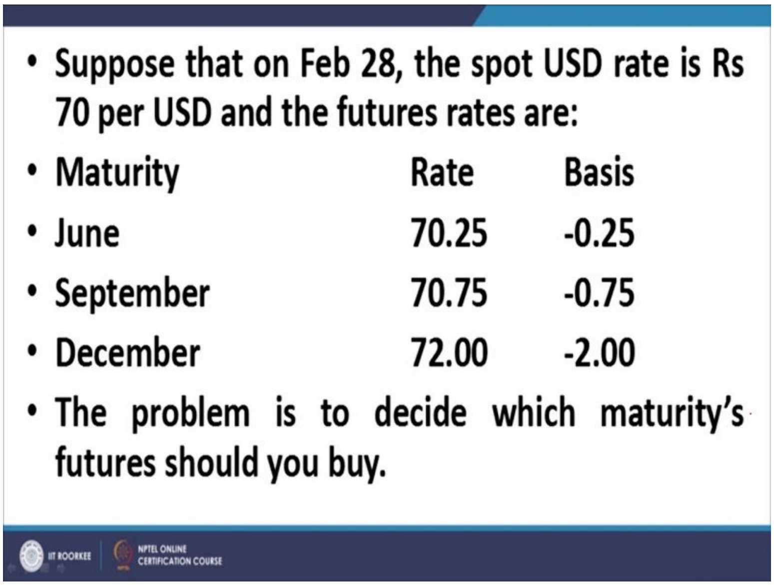

Suppose that on February 28, the spot US dollar rate is rupees 70 per US dollar and the futures rates are June 70.25, the basis is minus 0.25, 70 minus 70.25. That is equal to minus 0.25. The September futures price is 70.75 the basis is minus 0.75. And the December futures price is 72. And the basis is minus 2.00. Our problem is to decide which futures contract we should use for implementing or long hedge.

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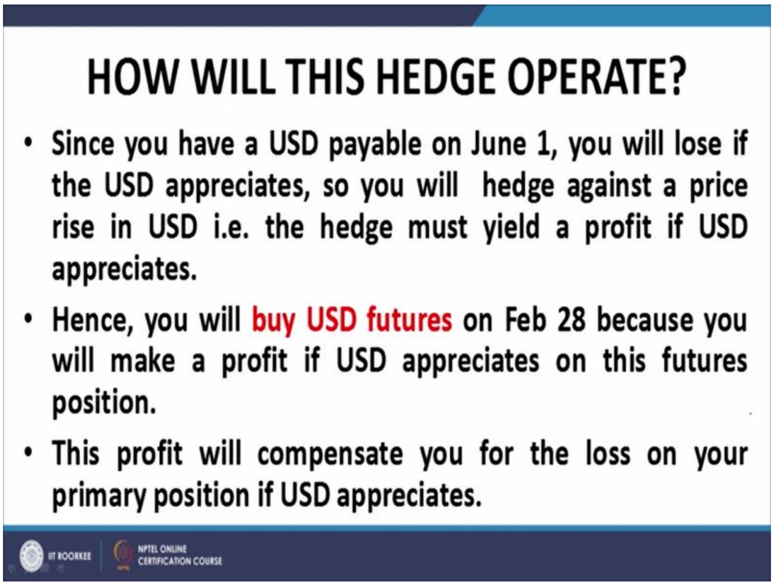

How will the hedge operate? Since you have a US dollar payable on June 1, you will lose if the US dollar appreciates because your cash outflow on account of INR will increase higher the rate at which you are going to purchase US dollars in the market on June 1 higher is your cash outflow on account of rupees and therefore, higher is the expenditure that you will incur in terms of rupees.

So, hence, therefore, the hedge should be such that if the US dollar price appreciates, it brings you a profit and that profit would compensate you for the excess payment or excess cash flow that would occur on account of appreciation of US dollar prices in the spot market when you buy US dollars on June 1.

So, you will hedge against a price rise in US dollars. That is the hedge must yield a profit if US dollar appreciate because you are incurring a loss on your primary position on your cash position if the US dollar appreciates, for that simple reason that you have to buy US dollars in the market for riveting them to the supplier.

Hence, you will buy US Dollars futures on February 28. Because you will make a profit if US dollar appreciates in the futures markets. The profit will compensate you for the loss on your primary position on your cash position or your position when you buy US dollars in the market in the spot market that is for remitting to your US supplier.

So, let me repeat your cash position is such, you cash exposure is such that you incur a loss if the US dollar appreciates, you have to buy US dollar in the spot market for remitting to your US supplier. And therefore, you take a long futures hedge the long futures hedge would generate profits if the futures prices of US dollar appreciates as that profit on the futures will compensate you for the loss if any on your cash position or spot position.

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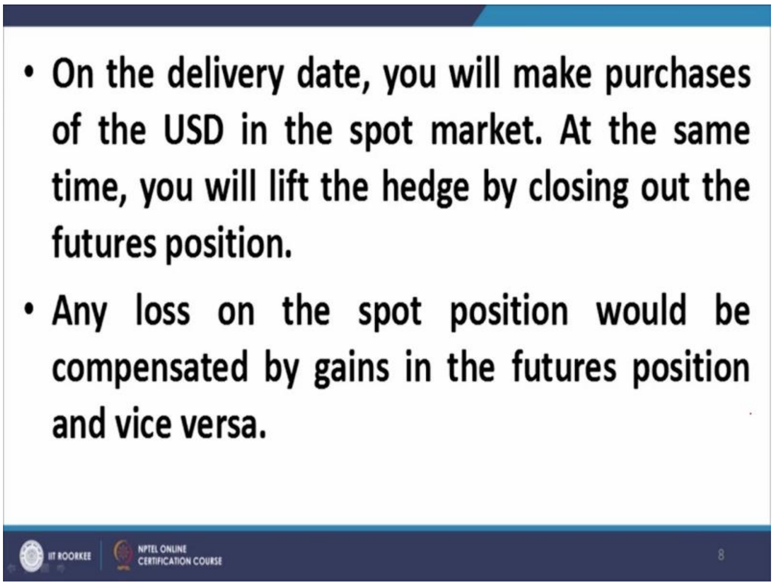

On the delivery date you will make purchases of US dollars in the spot market. At the same time, you will lift the hedge by closing out the futures position. Any loss in this spot position as I mentioned, just know. Due to dollar appreciation would be compensated by gains in the futures position due to appreciation of dollars in the futures market, which is quite likely, because of the correlation that prevails between spot and futures market of the same asset.

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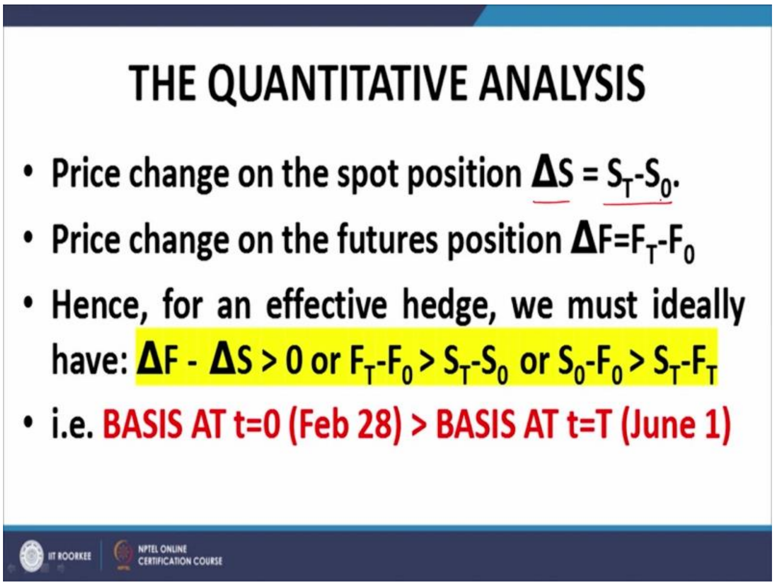

Now, let us look at it in a quantitative manner. Price change in the spot position delta S is equal to is S T minus S 0, what is S T? S T is the spot price on the date of maturity of the hedge and S 0 is the today's spot price. Price changes the futures position delta F is equal to F T minus F 0 please note this is a long position. So, the profit on the futures will be the ending price minus the beginning price that is equal to F T minus F 0. Therefore, for an effective hedge we want that the price escalation in the futures market which is the profit relevant should be higher than the price escalation in the spot market which is the loss element.

So, we must have delta F minus delta S greater than 0, so that we do not lose out on account of the transaction or the set of hedge transactions that gives us F T minus F 0 greater than S T minus S 0 or S 0 minus F 0 greater then S T minus F T, that is equal, that is equivalent to saying that the basis at t equal to 0 that is February 28 should be greater than the basis of at t equal to capital T. That is the basis on June 1.

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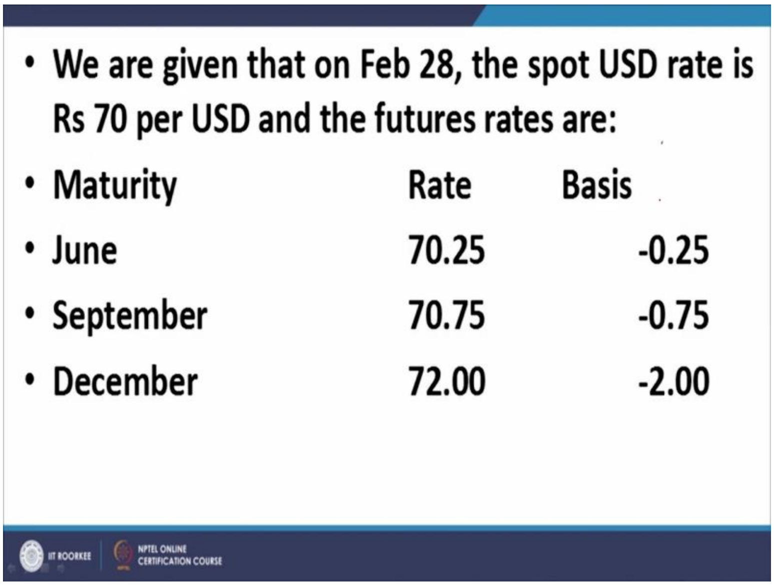

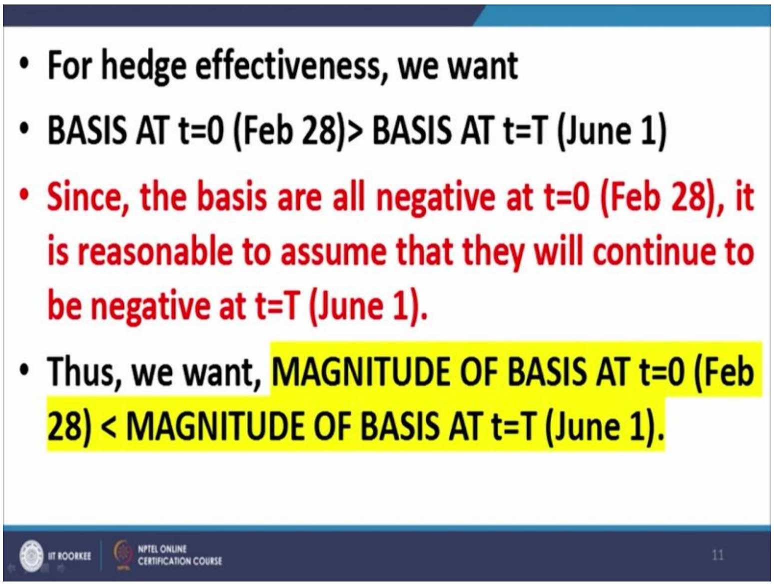

We are given that on February 28, the US dollar rate is just 70. And the futures rates are as follows that was given that I have already alluded to. The important thing is that for hedge effectiveness recall that the basis at t equal to 0 that is February 28 should be greater than the basis at t equal to capital T that is on June 1. Now, the important thing to notice that basis is negative throughout and as on February 28 as on February 28. If you look at the basis of all the three contracts, the June, the September and the December contracts, all the three contracts carry a negative basis.

So, it is logical to presume that the basis as of June 1 when the hedge is going to be lifted is also likely to be negative. Please note we do not know the basis on June 1 the we are sitting at

February 28 that is t equal to 0 when we are going to determine the hedge and the implement the hedge process.

We do not know the basis as of June 1 which is a random variable, but we can make an assumption a logical assumption that says the basis as on February 28 for all the futures contract is negative. Therefore, it is likely that it is likely but please note it is not certain, it is likely but it is not certain that the basis on June 1 will also be negative.

On the presumption that the basis of June 1 turns out to be negative or we on the presumption that we make that the basis of June 1 is negative the hedge effectiveness criterion modifies to magnitude of basis at t equal to 0 should be less than the magnitude of basis at t equal to capital T because both the basis are the negative.

So, if the basis at t equal to 0 has to be greater than basis at t equal to capital T, then the magnitude of basis at t equal to 0 must be less than the magnitude of basis at t equal to capital T. Because why? Because the basis are negative or assumed negative, at least for the case of June basis, June 1 basis.

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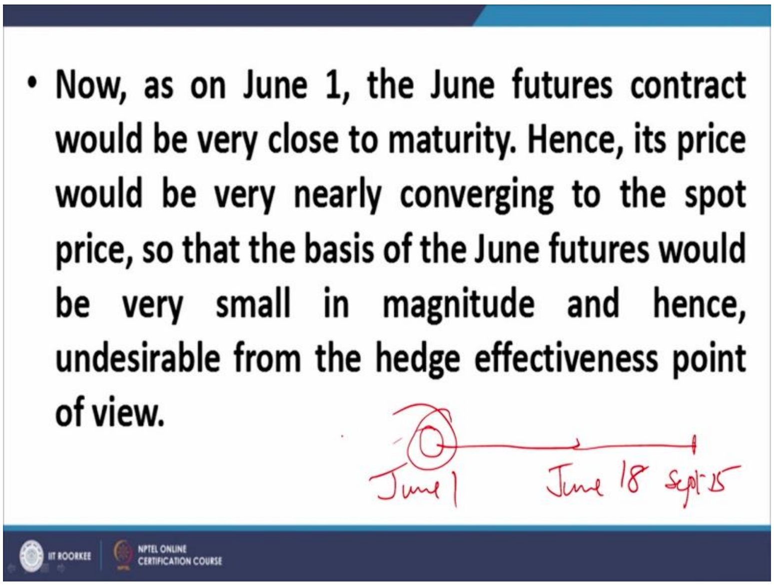

However, now is the catch to the problem. Let us look at it very carefully. Now as on June 1, the June futures contract will be very close to maturity, the June futures contract matures on the third Wednesday of June and we are at June 1. So, we are pretty much close to the maturity of the futures contract and therefore, its price would be very close to the spot price and as a result of which the basis of the June futures contract is going to be very small in magnitude and hence undesirable for the from the hedge effectiveness perspective. Let me explain it a bit.

See, sitting at June 1 when you are sitting at June 1 and the maturity of this is let us say this is June 1 and this is the third Wednesday of June, let us say it is June 18. So, there are only 18 days between June 1 and June 18. And on June 18 we know because June 18 is the maturity of the futures contract. The futures price must converge to the spot price as on that date and the basis should converge to 0. That is that is important. So, therefore sitting here at July 1 is quite likely that the convergence process will also, would already have initiated. And as a result of which the basis as on June 1 would be very small in magnitude.

It would be pretty much converging to the spot, because the actual convergence is a very few days away. So, that being the case, but what is our hedge effectiveness criterion? We want that the basis as of June 1 should be as wide as possible. The basis as of June 1, the magnitude of the basis as of June 1 should be more than the magnitude of basis as of February 2018. In other words, the basis should widen. On the other hand, when we look at the June 1 contract, we find that the basis does not widen it narrows down because it is approaching convergence very soon.

So, that is the important reason that June contract may not be the optimal contract. Now, if we look at the September contract, let us say it is September 15. If we look at the September contract, then there is still sufficient time remaining, still about three and a half months remaining to for the basis to converge to 0 for the futures price to converge to the spot price. And as a result of which the randomness as at this point, is substantially uninhibited. And the basis could take a significant number of random values.

And that implies that there is a significant possibility that the magnitude of the basis as on June 1 of the September contract could satisfy the hedge effectiveness criterion with greater probability. In other words, we could say that the basis could possibly widen as on June 1 of the September contract, and then gradually narrow down later on when it approaches September 15. So, in with that perspective, we can say that June contract the near contract may not be the best option, when we look at the hedging of such situations using futures contracts.

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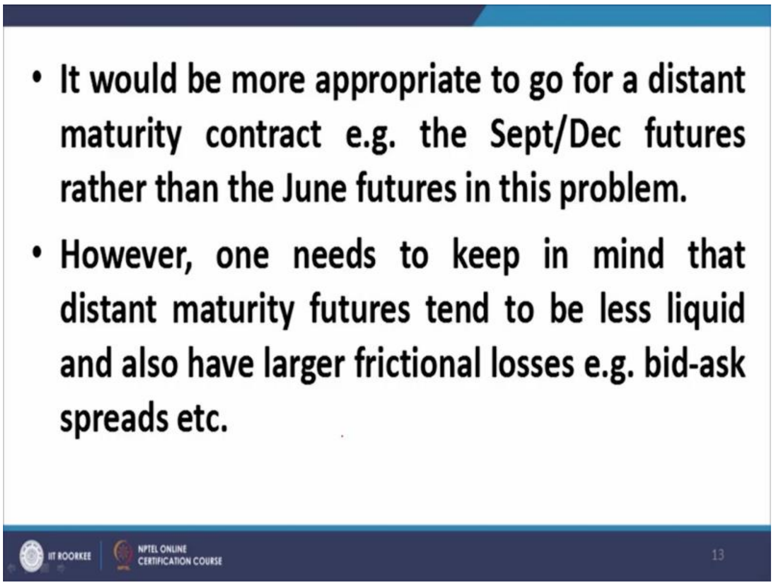

So, it would be more appropriate to go for a distance maturity contract that is the September oblique December futures, rather than the June futures in this particular problem. Please note that this is not a universal rule, it is not always true that the distant contracts are more viable or more optimal than the close contract.

In this particular problem. When we are having a long hedge, and we are having a negative basis, the combination of the long hedge and the negative basis mandates that we will look at a distant contract as the optimal contract. However, one needs to keep in mind other issues also. What are those other issues?

That distant maturity futures tend to be less liquid, and also have large frictional losses, that is bid ask spreads. So, this is an aspect which also needs to be considered. When we decide upon the optimality of the contract, we cannot immediately react and say that I will go for a far or distant contract, you also have to look into liquidity issues and the cost effectiveness of such distant contracts.

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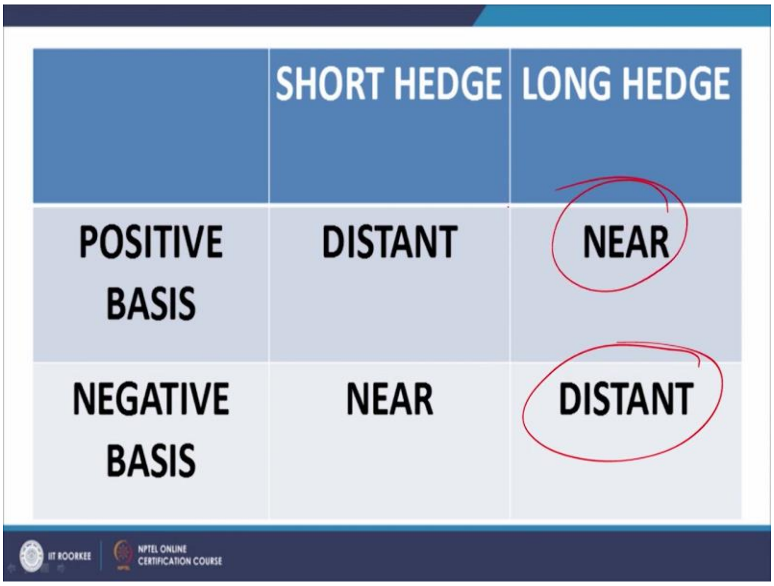

This is a matrix of what I have discussed just know. If you have a negative basis, and you are implementing a long hedge, then a distant contract turns out to be more viable. However, if the if you look at the same problem, if you do the same problem, but with a positive basis, you will find that the near contract turns out to be viable.

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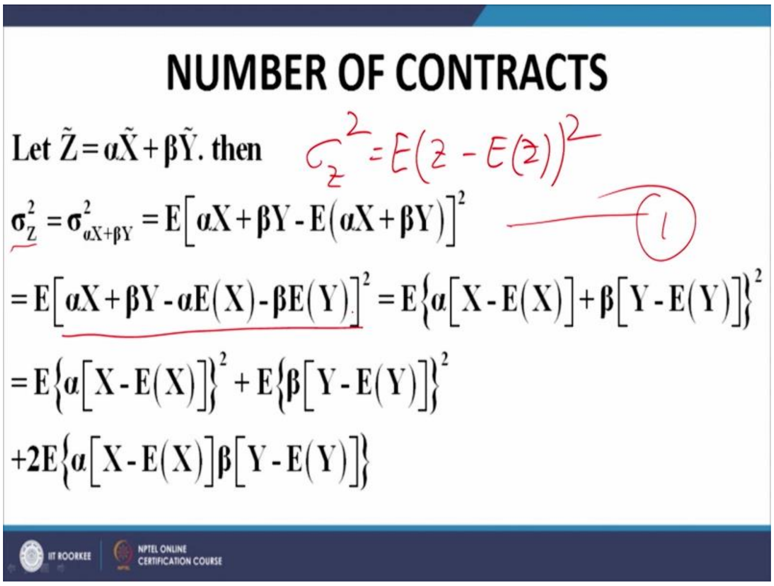

So, that is as far as the maturity of the contracts is concerned. It is a very interesting problem. And it is a problem where the intuition and the acumen of the analyst comes into the fore. Now, let us look at the number of contracts. But before I venture into the issue of number of contracts, I need to establish a simple statistical proposition X and Y are two random variables. And we define a variable Z as the linear combination of X and Y with coefficients alpha and beta which are real numbers. So, we define Z, Z is a random variable, which is a linear combination of two random variables X and Y with coefficients alpha and beta which are real numbers. Then the variance of Z, the variance of Z is given by this expression here which I write as equation number 1.

A variance of Z is equal to E of Z square minus E Z whole square. And when we substitute the value of Z and E of Z, what we end up is the expression that we have on this slide. So, this is equal to this can be written as like we put it here, sigma of Z square is equal to E of Z minus E Z whole square.

So, that is precisely what is written in equation number 1, we have substituted the expressions for Z and E of Z, please not E of Z is equal to alpha E of X plus beta E of Y. So, we can simplify this expression, as you have in the equation that I have underlined. And on simplifying all this is simply algebra and nothing more and using the property of the expectation operator. Throughout we have been using the property of the expectation operator.

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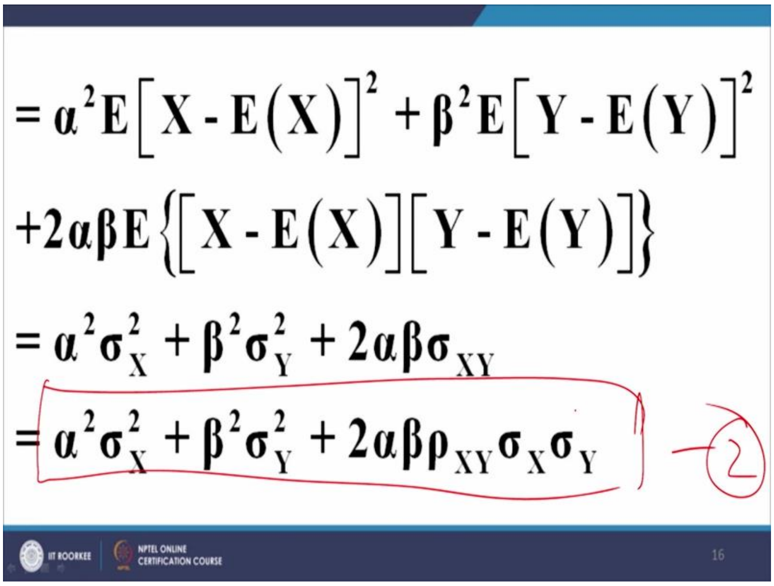

The result that we get here is final result that we get here is, which I have included in a box. And what is it? The variance of Z square is equal to alpha square variance of X square beta square variance of Y square plus 2 alpha beta, the correlation coefficient between X and Y into sigma X, that is standard deviation of X and standard deviation of Y.

This is a very important result. And this is the result that I will carry forward when I tackle the issue of determining the optimal number of contracts. So, let us now move to the optimal number of contracts. This result that we have here, let us call it equation number 2 will enable us to work through it.

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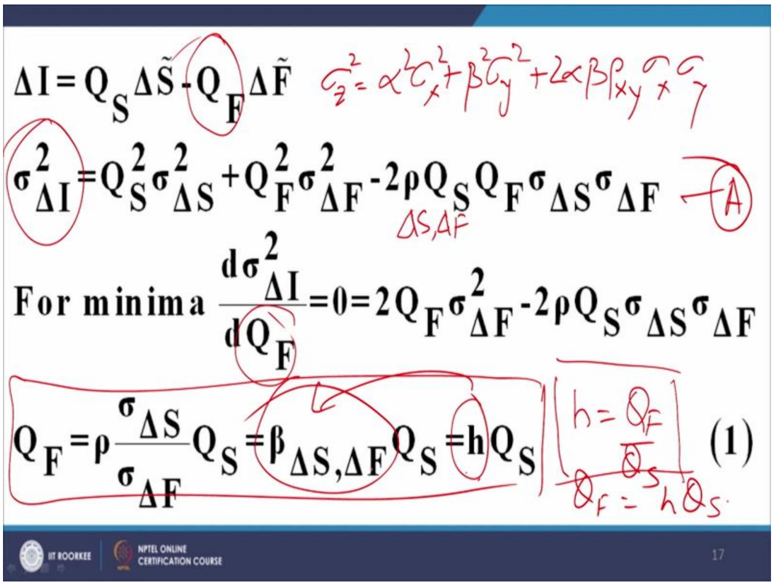

Now, the change in the value of an investment between t equal to 0 and at t equal to whatever is the hedge period, let us call it t equal to capital H is given by delta I, delta is the value, change in value of the invest hedged assets. Please note, it is the combination of the exposure and the hedge the change in value of the combination of the exposure and the hedge.

So, that is equal to Q S, Q S is the spot quantity the cash quantity of the asset into delta S where delta S is the change in the spot price between t equal to 0 and t equal to capital H plus Q F, Q F is the quantity that is used for hedging in the futures market into delta F, where delta F is the change in price of the futures between t equal to 0 and t equal to h.

So, let me repeat delta I is the value, change in value rather of the hedged investment and that is equal to Q S that is the quantity of the exposure into the change in spot price of the exposure plus Q F that is the quantity that we have taken up in the futures market for implementing the hedge into change in the futures price. So, using that formula that we use arrived at just know what was that formula? Let me write it down sigma Z square is equal to alpha square sigma X square plus beta square sigma Y square plus 2 alpha beta row X Y sigma X sigma Y.

So, using this formula, we write the variance of delta I as equation number, let us call it equation number A. Sigma delta I square is equal to Q S square sigma delta S square plus Q F square sigma delta F square plus 2 rho Q S Q F sigma delta S sigma delta F. What is rho?

Rho is the coefficient of correlation between what? Between delta S and delta F. Now, for minima we can differentiate this expression with respect to what? We have only one degree of freedom here Q S is given 2S, sigma S is the spot price we have no control over that dissimilar delta F is the change in the futures prices markets.

Again, we have no control over that, the only factors that we have control over in this out of these four quantities is Q F, that is our exposure in the futures market, that is the exposure that we are going to use for the purpose of hedging of our principal position or the cash position.

So, we differentiate with respect to Q F, let me repeat this is the only free variable out of these four variables Q S, Q F, delta S and delta F. Out of these four variables Q S is given its value or the quantity of assets that constitutes our investment, delta S and delta F are market prices, we have no control over them.

And therefore, the only free variable, then the variable which will determine the hedge effectiveness from our perspective is delta is the Q F that is the quantity that we will take up in the futures market for implementing the hedge. So, for minima, we differentiate sigma delta I square with respect to Q F and equate it to 0.

So, that we can minimize the variance sigma delta I square, that is the total variance of the combined portfolio comprising of the cash position and the hedge and what we end up on simplifying all equating it to 0 and simplifying is given here in this box Q F is equal to rho into sigma delta S into Q S divided by Q F. Now, rho sigma delta S upon sigma delta F is nothing but the regression coefficient beta delta S comma delta F.

So, we can write Q F as beta delta S delta F into Q S, but using the definition of the hedge ratio. What is the hedge ratio? We use hedge ratio the general definition of the hedge ratio, the standard definition of the hedge ratio is h is equal to Q F upon Q S or Q F is equal to h into Q is.

Therefore, if we use the definition Q F is equal to h into Q S, what we find is? That h is equal to, h is equal to what? h is equal to the regression coefficient beta delta S delta F. So, let me explain this important result this is a very, very important result in the context of futures hedging. The hedge ratio the standard definition of the hedge ratio which operates for all hedges is given by this expression h is equal to Q F upon Q S.

The exposure that we take in the futures market divided by the exposure that we have in the cash market or the spot market, that is the hedge ratio, that is the standard definition the hedge ratio. In this special case, when we are talking about the minimum variance hedge ratio, when we were talking about the hedge ratio, which will minimize the variance of the hedge position that hedge ratio turns out to be equal to beta of delta S comma delta F. So, this hedge ratio this hedge h is equal to beta delta S delta F is a special situation when our measure of risk is the variance and therefore, we decide or we want to minimize the variance of our hedge position.

And when we do that, what the optimality condition for that is that the hedge ratio should be equal to the regression coefficient between spot and futures prices. The normal definition of hedge ratio which operates universally irrespective of the measure of risk and which is not specific to this case is that hedge ratio is the ratio of the futures position divided by the quantity in the futures market divided by the quantity in the spot market.

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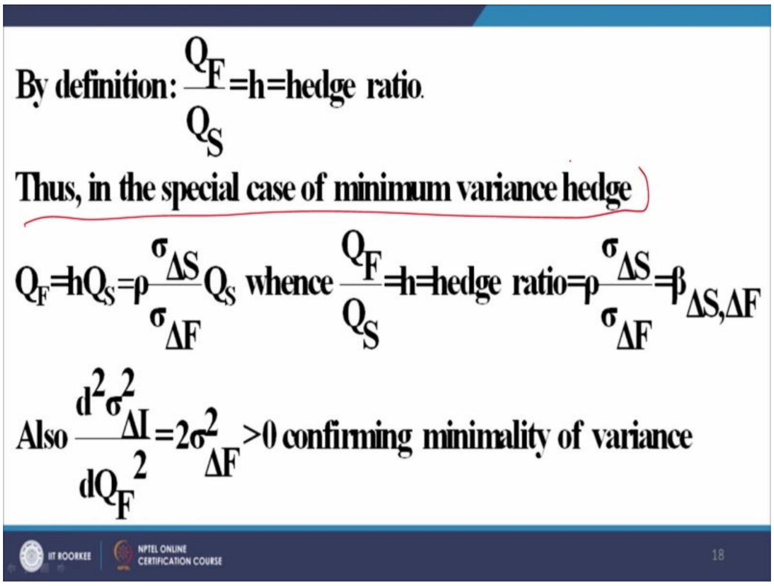

So, by definition quantity in the futures market upon quantity in the spot market is equal to the hedge ratio that is in the special case of minimum variance. This is important, I emphasize this again and again what we are saying, what we are going to say now, that is the hedge ratio is equal to beta delta S delta is operate operable only in the context of when the minimum variance hedge ratio when we are going to minimize the variances. And the second derivative turns out to be positive showing the minimality of variance.

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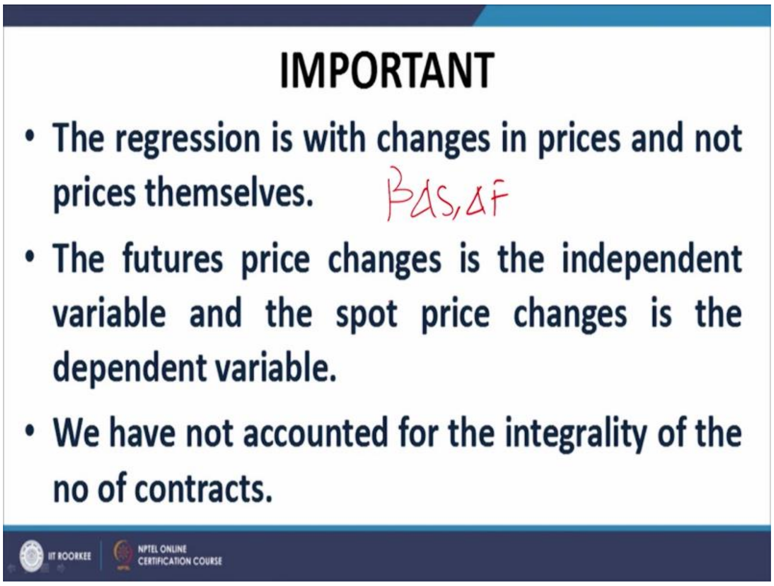

So, now, we talk about some important observations on this. The regression is with respect to changes in prices and not prices themselves. If you look at this, we have talked about beta delta S comma delta F. So, we are talking about regression involving changes in prices, not prices themselves, it is not beta S comma F, it is beta delta S comma delta F. So, where the regression equation has to be arrived at for whatever the hedge ratio the regression equation has to be arrived at using changes in prices.

Number two, the futures price change that is delta F is the independent variable along the X axis and the spot price changes are along the Y axis that represent the dependent variable and then when you work out the regression line, that regression line, the slope of that regression line will give you the, give you the optimal hedge ratio if you are planning to minimize the variance of the hedge position. We are not accounted for the integrality of the number of contracts.

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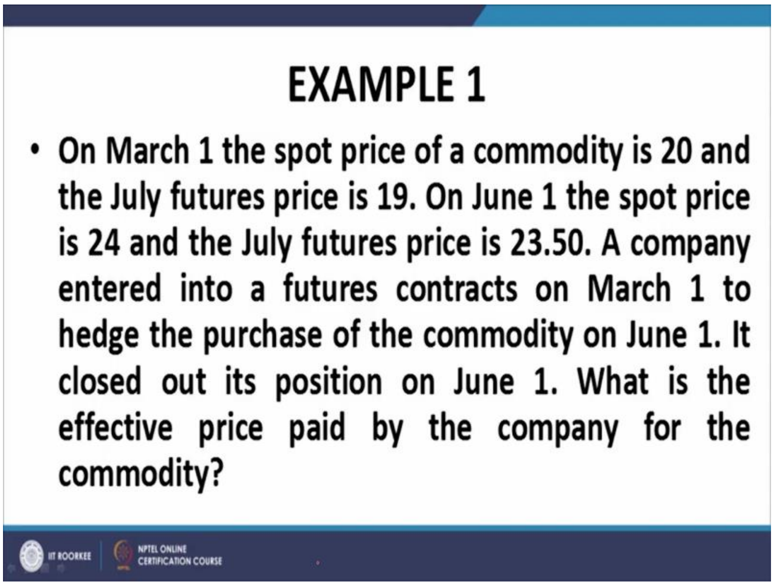

One or two examples, which I will take up in the next lecturer. Thank you.