

Quantitative Investment Management
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Lecture: 54
Futures Hedging

So, let us continue from where we left off. Just before the break I explained that due to arbitrage considerations, the prices in the forward market and the futures market should align themselves in a tandem manner should be in phase that is. Now, that gives us the opportunity to introduce or implement the process of what is called hedging. What is Hedging? Let me give you an example to start with, suppose, today's T equal to zero and at this point in time you have received an admission offer from a US university let us call it Harvard, for that matter.

So, you have received an offer letter from for admission from Harvard University, which requires you to deposit your fee by at T equal to three months, that is three months from today. Let me repeat a T equal to zero, you have received a letter offer of a admission from Harvard University, and you need to make a payment of say 25,000 dollars at T equal to three months for the confirmation of admission. Now, you have got two choices really. Either you buy the US dollars in the spot market and keep them with you for three months or maybe invest them in dollar deposit and keep them for, there for three months.

And at the end of three months, you can make the payment of the fee as per the requirements of the letter of offer. The other thing is that you can keep your position uncovered. And at the end of three months, when the necessity arises. To make the payment you did you borrow the you buy the money or buy the dollars from the spot market and make the remittance to the university. Now, if you adopt the second course of action, you are obviously subject to the possibility or the risk that arises from the fact that the US dollar exchange rate may escalate. And if the US dollar exchange rate does escalate, you may have to shell out a higher amount in terms of Indian rupees.

There is a mechanism in place due to the behavior of futures prices, which we call future hedging, I think which enables you to protect yourself by taking positions in the futures market against the loss or the potential loss or the possible loss that can arise because of the escalation in price of the US dollars. What you do is instead of buying the US dollars in the spot market at T equal to zero and storing them with you. Decide to buy the US dollars at T

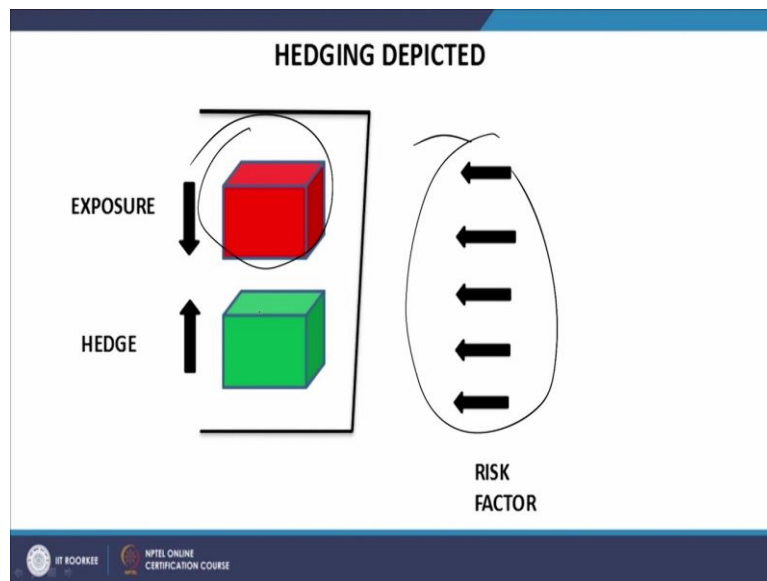
equal to three months at then prevailing market price. But at the same time to protect yourself against the possibility of a price escalation, you take a long position in US dollars futures at T equal to zero.

The advantage of this strategy is obvious that when you take a long position at in the few US Dollars futures at T equal to zero, you will not have to shell out the entire amount that is required for purchasing of the 25,000 US dollars in the spot market, you will have to shell out only the margin amounts that may be required by the relevant exchange for taking your position in the futures market. Now what will happen is at T equal to three months when your maturity date or the date for the payment of the fees arises you buy the US dollars in the spot market and make the remittance to the US university.

And if there is an escalation in the price of the US dollars, because of the positive relationship positive correlation between the spot prices and the futures prices, it is quite likely that the futures prices of US dollars would also have gone up. In other words, because you are long in the futures contract and the futures prices have also gone up in tandem with the spot prices. At least that is the positive probability, not certainty though, it would mean that you would make a profit on your futures contract and therefore the loss that you would encounter on due to an escalation in the US dollar exchange rate.

When you buy the US dollars in the spot market at T equal to three months would be compensated to some extent when you liquidate your future position, a long futures position, because it is quite likely that the futures price would also have gone up. And therefore, the price at which you would liquidate your future position would be higher than the price at which you entered the futures position. This is in very simple language; this is an illustration of what we mean by futures hedging.

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So, let us now look at it diagrammatically we have a risk factor here, this risk factor is acting on our exposure, let us take the case of an investment portfolio that you have you have a risk factor here, let us say it is an economic change, it is change due to the new budget introduced by the government. Or maybe it is some other economic factor, as a result of the value of my investment, which is represented by the red box goes down.

Now, we want to protect or I want to protect against this possibility, I believe that the there is going to be a change in the government policies, as a result of which the value of my investment is likely to go down. I want to protect myself against this possibility of a decline in the value of my investment.

What do I do? I hedge my portfolio against this risk factor that I have here. Whatever this risk factor is, I hedge my investment, which is the red box, how will I hedge this particular exposure? I will take a position in the futures market that would be such that if this risk factor actually materializes, that is, if the government change actually materializes, then the price of my portfolio will go down I have no control over that. But at the same time, I have taken a position or I take a position in the futures market such that the value of the futures position due to this change that has been introduced by the government goes up.

So, the combination of my exposure and the hedge they are on is to a large extent insulated isolated from the risk factor, if the risk factor results in a decline in the value of my exposure. At the same time, the same risk factor will result in an escalation or enhancement in the value



of the hedge and as a result of which the hedge portfolio, the hedged portfolio remains reasonably stable in value, this is the principle of the hedge.

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HEDGING

- A hedge is an investment position intended to offset potential losses or gains (that is changes in value whether negative or positive) that may be incurred by a companion investment.
- Hedging is the practice of taking a position in one market to offset and balance against the risk adopted by assuming a position in a contrary or opposing market or investment.

futures market
Spot?



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So, let us read out what we mean by a hedge? A hedge is an investment position intended to offset potential losses or gains, that is changes in value, whether negative or positive, that may be incurred by a companion investment, that is precisely what I have explained just now. Hedging is the practice of taking a position in one market. This is the important part, you are taking a position in one market, this is in our case, it is the futures market, futures market to offset and balance against the risk adopted by assuming a position in a contrary or opposing market as the spot market.

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BASIS & BASIS RISK

- Basis at any instant of time t is given by $b_t = S_t - F_t$.
- Since both S_t and F_t are stochastic processes and evolve in time with a random element, the basis is also a stochastic process.
- The basis is, therefore not precisely predictable at a future instant of time or that it will remain constant.
- This unpredictability of the basis causes imperfection in futures hedging termed as basis risk.
- Nevertheless, no arbitrage dictates that basis $b_T \rightarrow 0$ at maturity of the futures contract.

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Now, we talk about the technicalities, the nuances of hedging and the first thing that comes into play is a basis. A basis is a very fundamental feature of all hedging processes. And therefore, we need to understand what we mean by the basis? Basis at any instant of time t is given by the spot price operating at that time t minus the futures price of the same asset at that point t .

So, basis is the difference between the spot price of a given asset at a given point in time and the futures price of the same asset at the same point in time. Now, since both S_t and F_t are stochastic processes, they are random variables, they are sequences of random variables and evolve in time with a random element, the basis is also a stochastic process.

The basis is therefore not precisely predictable. Why? Because neither S_t nor F_t are precisely predictable. Both of them operate with an element of randomness. In other words, the future evolution of both the spot price as well as the futures price contains is embedded with a certain amount of randomness.

And as a result of which a difference, which is the basis will also contain an element of randomness. So, the basis is, therefore not precisely predictable at a future instant of time, or that it will remain constant. So, we do not, we cannot say that the basis will remain constant. We cannot even predict its value precisely at a future point in time.

This unpredictability of the basis causes imperfections in futures hedging, termed as basis risk. But what is the catch? Let me come to that. Nevertheless, no arbitrage dictates that the basis b capital T tends to zero on the maturity of the futures contract. I have already mentioned this in an earlier lecture that, there operates the phenomenon of convergence in futures market, what is the phenomenon of convergence?

The phenomenon of convergence is that the futures price on the date of maturity of the futures price must be in close vicinity of the spot price. The two prices, that is the futures price on its date of maturity of the futures contract and the spot price as on that date must converge because of arbitrage consideration.

And therefore, what would be the basis? The basis as on the date of maturity of the futures contract be close to zero, this is a catch other than that, it is difficult to predict precisely the basis at any point in time. And this unpredictability of the basis gives rise to a risk element, which is not completely eliminated, when we do futures hedging.

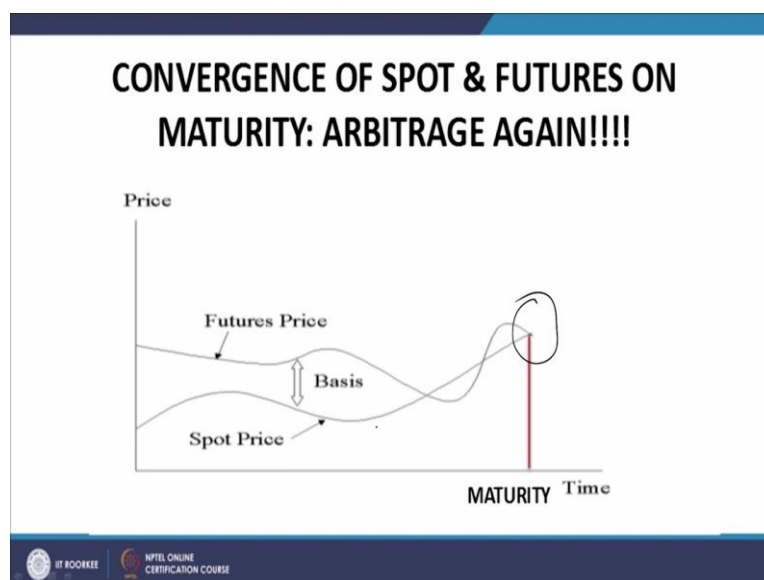
And therefore, futures hedging is imperfect. And the risk arising from the unpredictability of basis is called the basis risk. So, let me quickly read it out again, it is a very important slide, so let us read it out again. Basis at any instant of time is given by the difference between the spot price and the futures price of a given commodity at a given point in time.

But, since both S_t and F_t are stochastic processes and evolve in time with a random element, the basis is also a stochastic process. The basis is therefore, not precisely predictable at a future instant of time or that it will remain constant you cannot say, you cannot predict its value, you cannot even say that it is constant.

This unpredictability of the basis causes imperfections in futures hedging, termed as basis risk, because the basis is unpredictable. And therefore, we do not know exactly how the hedge is going to operate. We do not know exactly, we do not know precisely. Although we are qualitatively assured that the hedge would operate in to our benefit otherwise, we will not do the hedging.

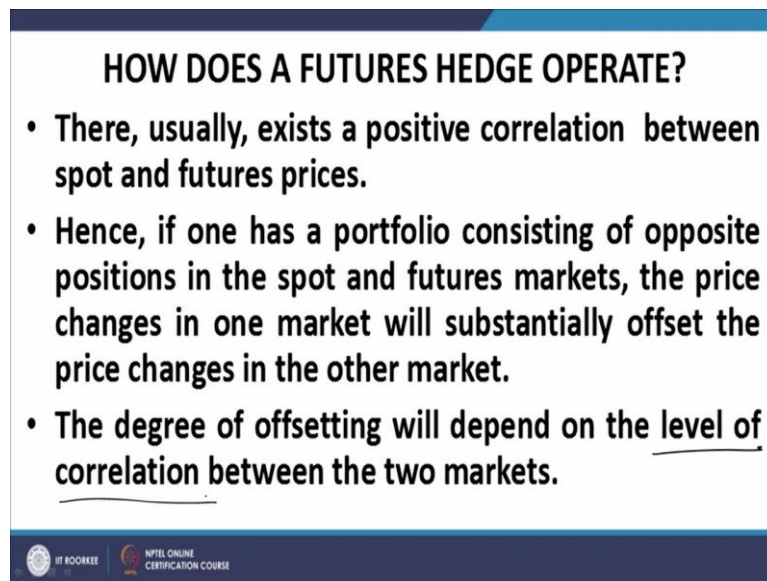
But the point is the precise operation of the hedge how much net result of the hedge position is going to materialize that is not precisely predictable. Nevertheless, no arbitrage dictates that the basis b capital T , that is, where capital T is what? It is the date of maturity of the futures contract tends to zero at the maturity of the futures contract.

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This is what I was trying to say, as on the date of maturity of the futures contract, the spot prices and the futures prices must converge and therefore, the basis must approach zero.

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HOW DOES A FUTURES HEDGE OPERATE?

- There, usually, exists a positive correlation between spot and futures prices.
- Hence, if one has a portfolio consisting of opposite positions in the spot and futures markets, the price changes in one market will substantially offset the price changes in the other market.
- The degree of offsetting will depend on the level of correlation between the two markets.

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How does the future hedge operate? I have given you an idea already. Let us read through this slide quickly. There usually exists a positive correlation between spot and futures markets we have established in fact, in the earlier lecture in the previous lecture of today hence, if one has a portfolio consisting of opposite positions in the spot and futures market, let us say you have a portfolio that consists of long positions in a given asset and a short position in futures on that asset.

So, what will happen if there is a risk factor or if there is an external stimulus, as a result of which the price of your asset declines your spot asset declines, then that stimulus will also cause the future, the price, or the futures price rather, of the hedge to increase and as a result of which some sort of stability will be obtained some the variation in the prices will be minimalized due to the hedging process.

So, hence, if one has a portfolio consisting of opposite position in the spot and futures markets, the price changes in one market will substantially offset the price changes in the other market, the degree of offsetting will depend on the level of correlation. This is very important, the degree of offsetting will depend on the level of correlation, the more strongly they are correlated, the stronger is going to be the hedge effect, the stronger or the greater is the offsetting of the price between the exposure and the hedge. The lesser is the correlation lesser would be the impact of this common risk factor on the two positions. The degree of offsetting will depend on the level of correlation between the two markets.

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HEDGING: BETTER VS CERTAIN OUTCOME

Hedging of a position leads to a more certain outcome.

It may or may not ultimately result to a better outcome.

HEDGING: BETTER VS CERTAIN OUTCOME

SPOT PRICE $t=0$	60	60	60
QUANTITY	100	100	100
SPOT PRICE $t=T$	50 ✓	60 ✓	70 ✓
FUTURES PRICE $t=0$	62	62	62
FUTURES PRICE $t=T$	50 ✓	60 ✓	70 ✓
FUTURES QUANTITY	100	100	100
CHANGE IN SPOT VALUE <i>Unhedged</i>	-1000	0	1000
CHANGE IN FUTURES VALUE	-1200	-200	800
NET CHANGE	200	200	200

This is what I have been emphasizing see the point let me illustrate this with an example. Let us look at this example. The spot price of an asset is 60 at t equal to zero, the quantity is 100 units whatever the units may be, the spot price at maturity of the hedge and maturity of the investment horizon, let us say is, we have taken three possibilities 50, 60 and 70. And the futures price at t equal to zero, we have taken 62. That is obviously known at because it is the price at t equal to zero, the futures price at t equal to capital T this is a random variable. So, we take three values 50, 60 and 70. Let us say the hedge ratio is one.

So, this what quantity is under this limit by the futures quantity of 100. In other words, you take a position of 100 units in the futures market, you have a exposure of 100 units, which you have already acquired in this spot market and again to protect the impact of price

fluctuation on that spot position, you take up position of 100 units in the futures market, opposite position. Now, let us say let us see the dynamics. Suppose we had the open position unhedged position, this is the unhedged position then what could have happened, if the spot price had gone down to 50, the value of portfolio would have gone down by 1000.

If this spot price had gone, had remained unchanged, the value of my portfolio would have remained unchanged. And if the spot price had gone up to 70, the value of my portfolio would have gone up by 1000. So, if I keep my portfolio unhedged, there are possibilities that I will make a loss of 1000 but there is also a possibility, there is also a possibility that I will make a profit of 1000.

Now, let us look at what happens due to the hedging a post editing, what is the situation of the hedge portfolio, that is the exposure plus the total position. In that case, what we find is that the net change in price is 200 irrespective of whatever is the change in the spot price, that is the important difference. That is the difference what is the difference?

In the case of the unhedged situation, I had the opportunity to make a profit of 1000. In the case of the hedge portfolio, I lost that opportunity of making 1000. What is the compensation for that? The compensation for that is I am having a certain profit of 200. So, that is the important part.

Hedging of a portfolio does not necessarily give you a better outcome, it only gives you a more certain outcome, it has given you a certain outcome, the profit has crystallized to stabilize to 200 irrespective of market position. In other words, the profit is now insulated from the market. However, you have lost that opportunity that had the market moved in the direction of favorable to you then you would have made a profit of 1000. But what was the counter to that, if the market had moved in an unfavorable direction, you would have incurred a loss of 1000.

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PERFECT VS IMPERFECT HEDGE

- A hedge is deemed perfect when the cash flows at maturity are known with certainty.
- Forward is a perfect hedge.
- Futures is imperfect hedge.



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Perfect versus Imperfect hedge. Now, a hedge is deemed to be perfect, when the cash flows at maturity are known with certainty. A forward is a perfect hedge, because you know the forward price that it is insulated from the market at a price that you have agreed at t equal to zero will be paid and received at t equal to capital T and the underlying asset which will form part of the substratum of the forward contract and which will be delivered at maturity, it all is also agreed upon at t equal to zero. So, the hedge is absolutely certain, the cash flows are certain and therefore, it is a perfect hedge. Futures is not a perfect hedge. As we saw just now, we will be seeing more about it as the lecture progresses.

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LONG HEDGE

- Hedges that involve taking a long position in a futures contract are known as long hedges.
- A long hedge is appropriate when a company knows it will have to purchase a certain asset in the future and wants to lock in a price now.

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A long hedge, hedges that involve taking a long position in a futures contract are known as long hedge. Hedges that involve taking a long position in the futures contract known as long hedges. When will you take a long hedge? You will take a long hedge when you want to purchase a certain asset like the example that I gave you, when I motivated the discussion on the hedging process using the US dollar for Harvard University example. You wanted to buy US dollars, you wanted to purchase the asset which was US Dollars 25,000 of them and so when you are trying to purchase an asset. And you want to protect yourself against price escalation of that asset, what we will do is you will take a long hedge on that.

So, that if there is a price escalation, to your detriment and so far as your purchase process is concerned, the long position in the futures will also increase because of the positive correlation between spot and futures market and as a result of which you will make some kind of a profit in the futures market, which will compensate you largely for the loss that you make in the spot market due to price escalation of your exposure.

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SHORT HEDGE

- A short hedge is a hedge that involves a short position in futures contracts.
- A short hedge is appropriate when the hedger already owns an asset and expects to sell it at some time in the future. *farmer growing corn*
- A short hedge can also be used when an asset is not owned right now but will be owned at some time in the future.

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A short hedge is a hedge that involves a short position in futures contract. A short hedge is appropriate, when the hedger already owns the asset and expects to sell it at some time in the future. For example, the farmer growing corn or maize, he knows that he is going to get his product at the end of the harvest season and he wants to sell it, he wants to crystallize the price at which he can sell it, then he can take a short hedge under which the as a result of which as a result of hedging the price that he will receive of the produced duly hedged price would be reasonably stable.

So, a short hedge is appropriate when the hedger already owns the asset and expected to sell it sometime in the future. A short hedge can also be used when an asset is not owned right now, but will be owned at some time in the future, like I gave you the example of the farmer.

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SHORT HEDGING & BASIS

- Price at which underlying is sold in the spot market at maturity of hedge period = S_T
- Profit from the short futures position, that was created at F_0 at $t=0$ and closed out at F_T at $t=T$ is $F_0 - F_T$
 $F_0 - F_T$ Total cash flow at $t=T$.
 $S_T + F_0 - F_T = F_0 + b_T$
- Net proceeds from the hedged transaction = $S_T + F_0 - F_T = F_0 + (S_T - F_T) = F_0 + b_T$

Short hedging and basis. So, let us try to understand what is the relation, what is the role of bases in the context of hedging, this is where the actual game starts. Price at which underlying is sold in the spot market at the maturity of the hedge period is S_T , please note it is positive. Because you are selling the asset, the hedger is selling the asset, therefore, he is receiving the price. What is the price that is receiving? Because it is selling that asset on the date of maturity that is t equal to capital T , the price that he is receiving is the spot price as on that date, which is equal to S capital T .

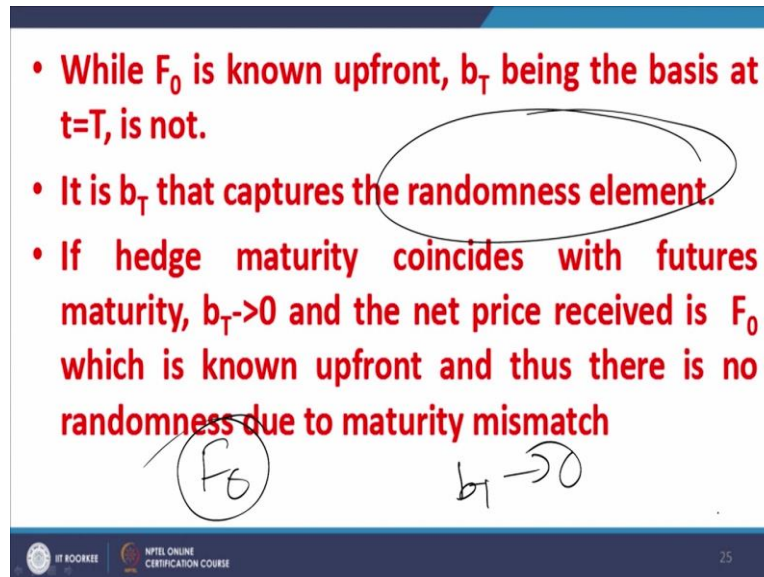
Now, to protect or to hedge this position of having to sell the asset at t equal to capital T , what had he done? He had taken a short position in futures in the same underlying asset. Let us assume that he had taken the short position at F_0 and let us assume that the futures price as on the date of maturity of the hedge is F capital T .

And let us further assume that the maturity of the hedge coincides with the horizon at which the farmer is planning to sell the crop. So, the profit on the futures position please note because it is a short position is given by F_0 minus F_T please note it is not F_T minus F_0 . F_T minus F_0 is the profit on a long position please note this.

And on a short position it is multiplied by minus 1 and therefore, the profit on a short position is equal to F_0 minus F_T . So, the total cash flow, total cash flow at t equal to capital T is

equal to S_T plus F_0 minus F_T that is equal to F_0 plus b_T . This is the role of basis short hedging.

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The slide contains three bullet points in red text. The second bullet point, 'It is b_T that captures the randomness element.', has 'randomness element' circled in black. The third bullet point, 'If hedge maturity coincides with futures maturity, $b_T \rightarrow 0$ and the net price received is F_0 which is known upfront and thus there is no randomness due to maturity mismatch', has ' F_0 ' circled in black. Below the text, there are handwritten annotations: ' F_0 ' circled in black and ' $b_T \rightarrow 0$ ' written in black. The slide footer includes the IIT Kharagpur logo, 'NPTEL ONLINE CERTIFICATION COURSE', and the number '25'.

- While F_0 is known upfront, b_T being the basis at $t=T$, is not.
- It is b_T that captures the randomness element.
- If hedge maturity coincides with futures maturity, $b_T \rightarrow 0$ and the net price received is F_0 which is known upfront and thus there is no randomness due to maturity mismatch

Similarly, now, F_0 is obviously known upfront, F_0 is known upfront. But b_T being the basis at t equal to capital T is not, now please note I will I need to emphasize one very important point at this juncture. The maturity of the futures need not necessarily coincide at all with the maturity of the hedge. The investment horizon and the hedge period usually coincide, usually coincide.

Again, that is also not absolutely necessary, but that is usually the practice that the investment horizon and the hedge period usually coincide. But the maturity of the future need not necessarily coincide with the hedge period. It may coincide, it may not coincide, in the event that it does not coincide. You will liquidate your hedge by selling of the futures in the futures market by taking a converse position in the futures market by liquidating the futures in the futures market.

So, it is absolutely not necessary that the futures maturity must be coincide with the hedge period it has not necessary. So, let us continue while F_0 is not upfront b capital T being the basis is not, it is b capital T which captures the randomness element which makes the futures an imperfect hedge. Because of the randomness of b capital T the futures may not be a perfect hedge, there is a catch to it, which is given in the next paragraph. So, let me read out the paragraph.




If hedged maturity coincides with the futures maturity, in the rare instance, that your hedge maturity, your investment horizon hedge maturity, and futures maturity all coincide. Then b_T will approach 0. Because of no arbitrage consideration. We have already discussed that the principle of convergence because of the principle of convergence b_T will tend to 0 only in the event, only the exception not the rule that the maturity of the futures is equal to the maturity of the hedge.

If that is so, then b_T coincides with it is close to 0, and as a result of which the price received by the short hedger is equal to F_0 , which is known upfront, and therefore there is no randomness. So, this is a rare instance where randomness may not be present in the event that the futures maturity coincides with the hedge maturity.

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LONG HEDGE & BASIS

- Similarly, the net price paid by a long hedger = $S_T + F_T - F_0 = -F_0 - (S_T - F_T) = -(F_0 + b_T)$. — $-(b_T + F_0)$
- Thus, if the basis weakens unexpectedly,
- Long hedger gains & short hedger loses & vice versa.
 if basis weakens

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In the case of a long hedge, the converse is the situation and the price that you pay, the price that the long hedger pays for acquiring the asset at t equal to capital T is given by this expression minus of b_T plus F_0 . What is this minus sign? This minus sign represents cash outflow, because you are buying the asset, this minus sign is a cash outflow.

Therefore, if the basis weakens, what happens? If this b_T becomes smaller, then you have to pay a lesser price. Therefore, long hedger gains, long hedger gains if basis weakens and short hedger obviously loses, because at the end of the day, we are talking about zero sum games. So, because if b_T becomes smaller, if b_T becomes weaker, then the price that you have to pay becomes less and as a result of which the long hedger gains.

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WHY FUTURES HEDGE IS NOT A PERFECT HEDGE?

- Marking to market.
- **Basis risk due to:**
- Different underlying asset.
- Non-identical maturity.
- Lot size issue.

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Why futures hedge is not a perfect hedge? First of all, the first reason is marking to market at the end of every day, the futures position will be marked to market and particularly if there are interest changes during the life of the hedge, then what happens? Then, the marking to market effect takes place.

Basis risk also arises due to different underlying assets, if the underlying asset that constitutes your exposure and the asset that is the underlying asset of the futures position is not the same. Let me repeat, if the asset that is your exposure and the underlying asset, which constitute of the futures contract that you have used for hedging is not the same.

Please note, your choice of the asset and so far as your exposure is concerned is yours is your will, but it is not necessary that corresponding to every asset. There would be futures contracts written on that asset and freely tradable in appropriate exchanges, there may be assets, where you do not have access to futures contracts on those. And those asset in that case, because of the difference between the asset in which you have the exposure, and the asset, which was the underlying asset of the future, there could be imperfections introduced into the hedge, because obviously, the price behavior would not be absolutely parallel.

Then non-identical majority I have discussed this point in some detail. And finally, there is a lot size issue. Futures one futures contract does not cover one unit of the underlying asset. In most cases, in most cases, there is a lump sum number of units of the underlying asset that are covered by one futures contract.

And this also results in imperfection, because trading has to be done in terms of whole numbers of the futures contract and not in terms of the number of units of the underlying asset. Trading is done in terms of the number of contracts, in terms of integrality of the contracts and not integrality of the units of the underlying asset. So, that is another issue that can cause basis risk.

So, let us let me quickly repeat difference between the asset in which you have the exposure and the asset which forms the underlying asset of the futures. Number two, difference between maturities of your investment horizon, hedge period and the maturity of the futures contract. This results in randomness and finally, the lot size issue. So, we shall take it up from here in the next lecture. Thank you.